Chapter 5 effective stress principle

Introduction

The effective stress principle enunciated by Karl Terzaghi in 1936 forms an extremely useful basis of the most important theories in soil engineering. The effective stress principle consists of two parts:

1. Definition of the effective stress
2. Importance of the effective stress in engineering behavior of soil

The methods for determination of effective stress in soils for hydrostatic conditions and for steady seepage conditions are discussed separately.

5.1 EFFECTIVE STRESS PRINCIPLE

(1) Definition of effective Stress

Figure 5.1 shows a soil mass which is fully saturated. Let us consider a prism of soil with a cross-sectional area A. the weight P of the soil in the prism is given by

$P = \gamma sat\, hA$

Where γsat the saturated weight of the soil, and h is is the height of the prism

Total stress (σ) on the base of the prism is equal to the force per unit area. Thus

$$\sigma = \frac{P}{A} = \gamma sat\, h$$

While dealing with stresses, it is more convenient to work in terms of unit weights rather than density.

$\gamma = \rho \,.\, g$

Where γ is in N/m^3 and ρ is in kg/m^3, g = $9.81 m/sec^2$

Thus, $\gamma sat = \rho sat * g = 9.81 \rho sat$

Generally, the unit weights are expressed in kN/m^3. In this case,

$\gamma sat = \rho sat * g/1000 = 9.81 * 10^{-3} \rho sat$

For example, if $\rho sat = 2000 kg/m^3$

$\gamma sat = 9.81 * 10^{-3} * 2000 = 19.62 kN/m^3$

Pore water pressure (u) is the pressure due to pore water filling in the voids of the soil. Thus

$u = \gamma w h$

Pore water pressure is also known as neutral pressure or neutral stress, because it cannot resist shear stresses. Pore water pressure is taken as zero when it is equal to atmospheric pressure, because in soil engineering the pressures used are generally gauge pressure and not absolute pressure.

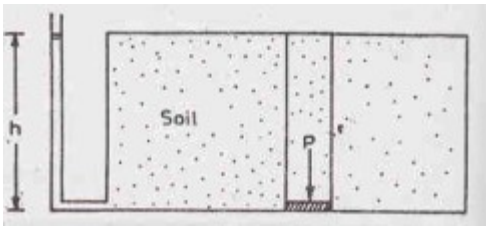


Figure 5.1 saturated soil mass

The effective stress (σ') at a point in the soil mass is equal to the total stress minus the pore water pressure. Thus,

$\sigma' = \sigma - u$

For saturated soils, it is obtained as

$\sigma' = \gamma sat\, h - \gamma w h$

$\sigma' = (\gamma sat - \gamma w ¿ h$

$\sigma' = \gamma' h$

Where, γ' is the submerged unit weight

(2) Importance of effective stress

The effective stress controls the engineering properties of soils. Compression and shear strength of a soil are dependent on the effective stress. Thus,

Compression = $f(\sigma')$

And shear strength = $\varphi ¿ \sigma')$

Where f and φrepresent some functions

As the effective stress in a soil increases, the compression of the soil occurs. This causes settlement of structures built on soils.

The shear strength of a soil depends on its effective stress. As the effective stress is changed, the shear strength changes. The stability of slopes, the earth pressures against retaining structure and the bearing capacity of soils depend up on the shear strength of the soil and hence, the effective stress. The importance of shear strength in soil engineering problem cannot be over-emphasized. It is one of the most important properties of soils.

5.1.2 NATURE OF EFFECTIVE STRESS

Let us consider a physical model of a soil mass, fully saturated, as shown in figure 5.2 (a). Let us take a wavy plane x-x passing through the point of contact of solid particles. On the microscopic scale, the wavy plane cannot be distinguished from a true horizontal plane as the individual particles are of relatively small size. Therefore, for all practical purposes, the plane x-x can be assumed as horizontal.

The total normal force P acting on the soil model is resisted partly by the interparticle forces at the points of contact (Pm) and partly by the pore water pressure force (Pw) (figure 5.2 (b)).

Thus, $P = P\text{m} + P\text{w}$

At every point of contact, the interparticle force F can be resolved into the normal component (N) and the tangential component (T) to the plane x-x (figure 5.2c). The interparticle forces are random in both magnitude and direction throughout the soil mass. The tangential components, however, neutralize one another and the resultant of all the normal component is downward.

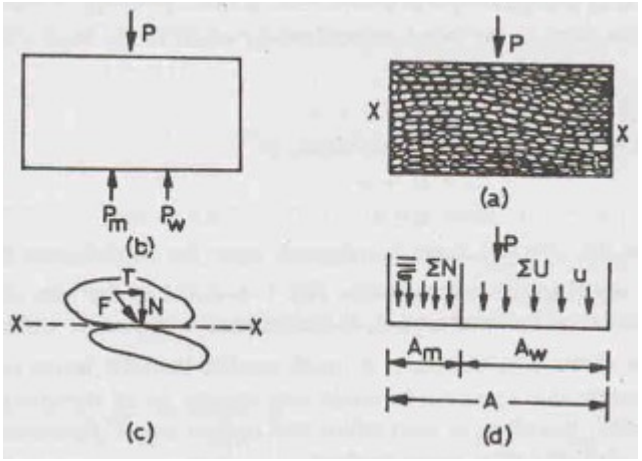


Figure 5.2 physical model of a soil mass.

The effective stress is the normal stress transmitted through the particles,

and is given by,

$$\sigma' = \frac{\sum of\ all\ N - components}{area\ of\ x-x\ plane}$$

Or $\sigma' = \frac{\sum N}{A}$

5.1.3 EFFECTIVE OF WATER TABLE FLUCTUATIONS ON EFFECTIVE STREE

Let us consider a soil mass in figure 5.3. The depth of the water table (W.T) is H1 below the ground surface. The soil above the water table is assumed to be wet, with a bulk unit weight of γ. The soil below the water table is saturated, with a saturated weight of γsat.

Total downward force (P) at section x-x is equal to the weight (W) of the soil. Thus,

$$P = W = \gamma H1 A + \gamma sat H2 A$$

Where, A is the area of cross-section of the soil mass.

Dividing by A throughout,

$$\frac{P}{A} = \gamma H1 + \gamma sat\, H2$$

The left hand side is equal to the total stress

Therefore, $\sigma = \gamma H1 + \gamma sat\, H2$

The pore water pressure (u) is given by,

$u = \gamma w\, H2$

Thus, $\sigma' = \sigma - u = (\gamma H1 + \gamma sat\, H2 ¿ - \gamma w\, H2$

Or $\sigma' = \gamma H1 + (\gamma sat - \gamma w) H2$

Or $\sigma' = \gamma H1 + \gamma' H2$

This equation gives the effective stress at section x-x and the following figure shows the directions of σ' and u at x-x.

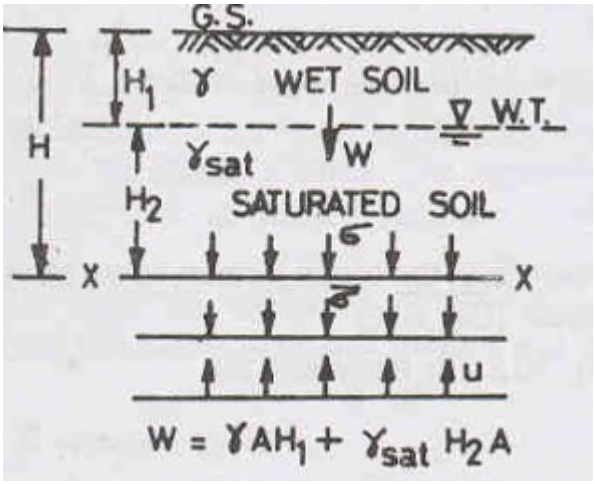


Fig 5.3

a) If the water table rises to the ground surface, the whole of the soil is saturated, and

$\sigma' = \gamma'(H1+H2) = \gamma' H$

As $\gamma' < \gamma$, the effective stress is reduced due to rise of water table

b) If the water table is depressed below the section x-x

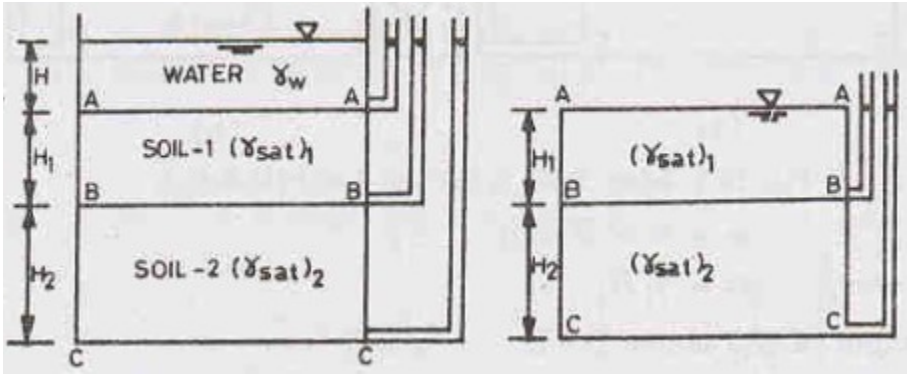
$\sigma' = \gamma H$

In this case, the effective stress is increased.

Thus, it is observed that the fluctuations in water table level cause changes in the pore water pressure and the corresponding changes in the effective stress.

5.1.4 EFFECTIVE STRESS IN A SOIL MASS UNDER HYDROSTATIC CONDITIONS

Figure 5.4 (a) shows a soil mass under hydrostatic conditions, wherein the water level remains constant. As the interstices in the soil mass are interconnected, water rises to the same elevation in different piezometers fixed to the soil mass.



a) b)

Figure 5.4 Water Table above soil

(1)**Water Table above the soil surface A-A**

Section A-A $\sigma = \gamma w H$, u = $\gamma w H$

Therefore, $\sigma' = \sigma - u = 0$

Section B-B $\sigma = \gamma w H + (\gamma sat)_1 H_1$, and u = γw¿H_1)

Therefore, σ'=¿$_1$ - γw] $H_1 = \gamma_1' H_1$

Where, γ' is the submerged unit of soil-1

Section C-C $\sigma = \gamma w H + (\gamma sat$¿$_1 H_1 + (\gamma sat$¿$_2 H_2$

u = γw¿$_1 + H_2$)

Therefore, $\sigma' = \gamma_1' H_1 + \gamma_2' H_2$

Where, γ_2' is the submerged unit weight of soil -2

(2)**Water Table at the soil surface A-A**

Figure 5.4b shows the condition when the depth H of water above the section A-A is reduced to zero. In this case, the effective stresses at various sections are determined as under:

Section A-A $\sigma = u = \sigma' = 0$

Section B-B $\sigma = (\gamma sat$¿$_1 H_1$, and u = $\gamma w H_1$

And $\sigma' = \gamma_1' H_1$

Section C-C $\sigma = ((\gamma sat$¿$_1 H_1 + ((\gamma sat$¿$_2 H_2$

u = γw¿$_1 + H_2$)

Therefore, $\sigma' = \gamma' H_1 + \gamma' H_2$

(3) Water table in soil -1

Figure 5.5a shows the case when the water table is at D-D in the soil-1 at depth H_1'. The effective stresses at various sections are determined as follows:

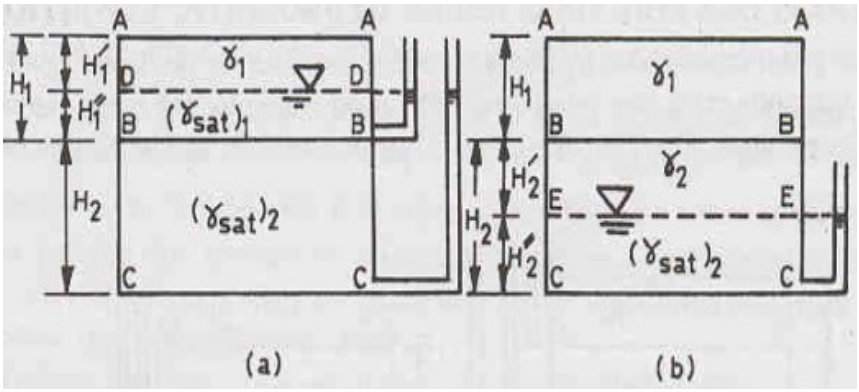


Figure 5.5 Water Table in (a) soil-1 and (b) soil-2

Section A-A $\sigma = u = \sigma' = 0$

Section D-D $\sigma = \gamma_1 H_1'$

Where γ_1 is the unit weight of soil above D-D u =0

Therefore, $\sigma' = \gamma_1 H_1'$

Section B-B $\sigma = \gamma_1 H_1' + (\gamma sat)\, H_1''$ (note. $H_1' + H_1'' = H_1$)

$u = \gamma_w H_1''$

therefore, $\sigma' = \gamma_1 H_1' + [(\gamma sat)_1 - \gamma w] H_1'' = \gamma_1 H_1' + \gamma_1 H_1''$

Section C-C $\sigma = \gamma_1 H_1' + (\gamma sat)_1 H_1'' + (\gamma sat)_2 H_2$

$u = \gamma_w (H_2 + H_1'')$

therefore, $\sigma' = \gamma_1 H_1' + \gamma_1 H_1'' + \gamma_2' H_2$

when, $H_1'' = 0$ $\sigma' = \gamma_1 H_1' + \gamma_2' H_2$

(4) Water table in soil-2

Figure 5.5b shows the condition when the water table is at EE in soil-2 at depth H_2'/ the effective stresses at various sections are as under:

Section A-A $\sigma = \mathbf{u} = \sigma' = \mathbf{0}$

Section B-B $\sigma = \gamma_1 H_1$, u =0, $\sigma' = \gamma_1 H_1$

Section E-E $\sigma = \gamma_1 H_1 + \gamma_2 H_2'$, u =0 (note. $H_2' + H_2'' = H_2$)

$\sigma' = \gamma_1 H_1 + \gamma_2 H_2'$

Section C-C $\sigma = \gamma_1 H_1 + \gamma_2 H_2' + (\gamma sat)_2 H_2''$

$u = \gamma w H_2''$

$\sigma' = \gamma_1 H_1 + \gamma_2 H_2' + \gamma_2' H_2''$, Where γ_2' is submerged unit weight of soil-2

(5) Water Table below C-C

Figure 5.6 shows the condition when the water table is below C-C. As the pore water pressure is zero everywhere, the effective stresses are also equal to the total stresses.

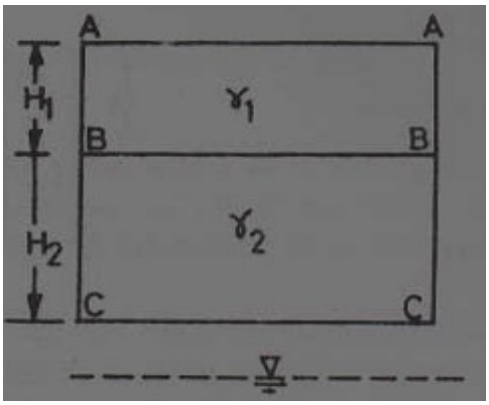


Figure 5.6 Water Table below soil

Section B-B $\sigma = \sigma' = \gamma_1 H_1$

Section C-C $\sigma = \sigma' = \gamma_1 H_1 + \gamma_1 H_2$

The following points are worth noting in the five cases studied above.

i. The effective stress at any section goes on increasing as the water table goes down
ii. The effective stress depends upon the bulk unit weight above the water table and the submerged unit weight below the water level
iii. The effective stresses in a soil mass can be determined from the basic definitions, without memorizing any formula.

5.1.5 INCREASE IN EFFECTIVE STRESSES DUETO SURCHARGE

Let us consider the case when the soil surface is subjected to a surcharge

load of intensity q per unit area. Let us assume that the water table is at level B-B (figure 5.7). the stresses at various sections are determined as under:

Section A-A $\sigma = $ ¿**q,** **u =0,** σ' **=q**

i.e. all the points on the soil surface are subjected to an effective stress equal to q

Section B-B $\sigma = $ ¿**q** $+\gamma_1 H_1$, u =0

Therefore, $\sigma' = q + \gamma_1 H_1$

Section C-C $\sigma = $ ¿**q** $+ \gamma_1 H_1 + (\gamma sat)_2 H_2$, $u = \gamma w H_2$

Therefore, $\sigma' = $ ¿**q** $+\gamma_1 H_1 + \gamma_2' H_2$

From the above illustrations, it is clear that the effective stress throughout the depth is greater than the case with no surcharge discussed in the preceding section. The difference is equal to the intensity q. In other words, the effective stress is increased by q throughout.

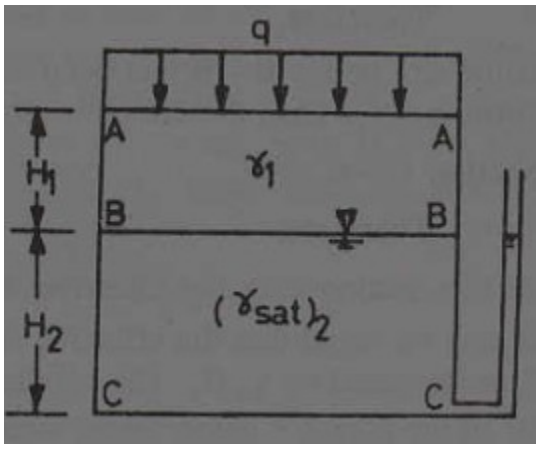


Figure 5.7 Effect of surcharge

5.2 SEEPAGE PRESSURE

As the water flows through a soil, it exerts a force on the soil. The force acts in the direction of flow in the case of isotropic soils. The force is known as the drag force or seepage force. The pressure induced in the soil is termed seepage pressure.

Let us consider the upward flow of water in a soil sample of length L and cross-sectional area A under a hydraulic head of h (figure 5.8a). the expression for seepage force and seepage pressure can be derived considering the boundary water pressures u_1 and u_2 acting on the top and bottom of the soil sample, as shown in figure 5.8(bi). The boundary water pressure can be resolved in to two components, namely, the hydrostatic pressure and the hydrodynamic pressure as shown in figure 5.8b(ii) and 5.8b(iii).

(1) The hydrostatic pressures u_1 (s) and u_2(s) are the components which would occur if there were no flow. If the samples were submerged under water to a depth of H_1, these pressures would have occurred.

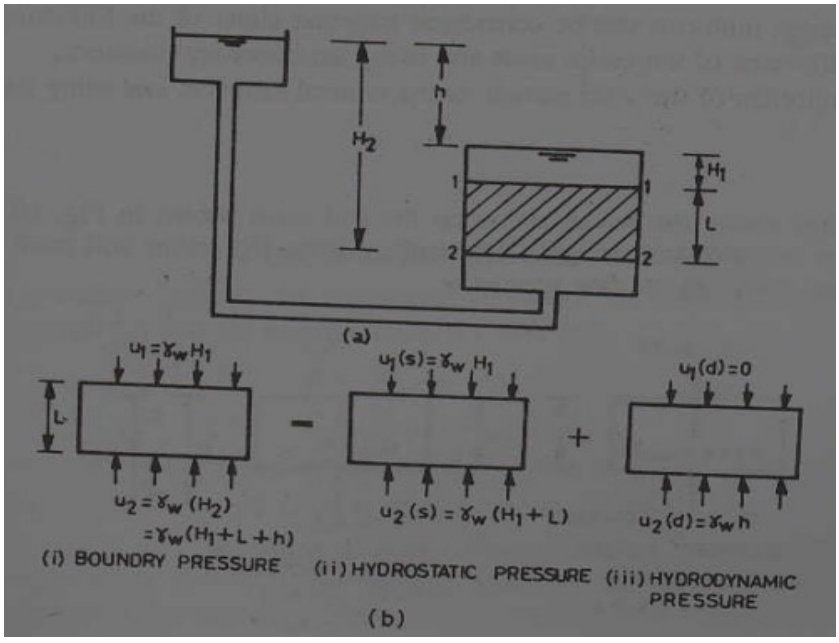


Figure 5.8 vertically upward flow

(2) The hydrodynamic pressures $u_1(d)$ and $u_2(d)$ are the components which are responsible for flow of water. This pressure is spent as the water flows through the soil. These components cause the seepage pressure.

At the top of the sample, $u_1 = u_1(s) + u_1(d)$

Or $$\gamma_1 H_1 = \gamma_1 H_1 + 0$$

At the bottom of the sample, $u_2 = u_2(s) + u_2(d)$

Or $$\gamma w(H_1 + L + h) = \gamma w(H_1 + L) + \gamma w h$$

The hydrodynamic pressure is due to hydraulic head h. the seepage force (J) acts on the soil skeleton due to flowing water through frictional drag. It is given by

$$J = \gamma w\ hA$$

The seepage pressure (Ps) is the seepage force per unit area.

$$Ps = J/A = \gamma w h$$

The seepage pressure (Ps) can be expressed in terms of the hydraulic gradient.

$$Ps = \gamma w h = \gamma w(h/L).L$$

$$Ps = i\gamma wL$$

The seepage force (J) can be expressed as the force per unit volume (j), as

$$j = \frac{J}{A*L} = \frac{\gamma w hA}{AL} = \gamma w \frac{h}{L} \quad \text{or } j = i\gamma w$$

Thus, the seepage force per unit volume is equal to the product of the hydraulic gradient (i) and the unit weight of water. As the hydraulic gradient is dimensionless, the seepage force per unit volume has the dimensions of the unit weight (i.e. F/L^3). It has the units of N/m^3. For isotropic soils, the seepage force acts in the direction of flow.

5.2.1 FAILURES OF HYDRAULIC STRUCTURES BY PIPING

Hydraulic structures, such as weirs and dams, built on pervious foundations sometimes fail by formation of a pipe-shaped channel in its foundation, known as piping failure. The failure occurs when water flowing through the foundation has a very high hydraulic gradient and it carries soil particles with it. There are two types of such failures:

(1) Backward-erosion piping failure
(2) Heave-piping failure

1. **Backward-erosion failure**: this type of piping begins with the removal of soil particles by flowing water near the exit points. A scour hole forms near the exit when the hydraulic gradient is high. The hole extends upstream along the foundation. The failure occurs as soon as the scour hole approaches the upstream reservoir. The failure mechanism of backward erosion piping is not amenable to theoretical solution. Backward-erosion of soil is caused by the percolating water, and the piping begins when the hydraulic gradient at exit, known as exit gradient, exceeds the critical gradient (i_c). The soil at the exit is removed by the percolating water. When the soil near the exit has been removed, the flow net gets modified. There is more concentration of the flow lines in the remaining soil mass, resulting in an increase of the exit gradient. This causes further removal of the soil. Backward erosion piping may also occur in the body of earth

structure, such as an earth dam.

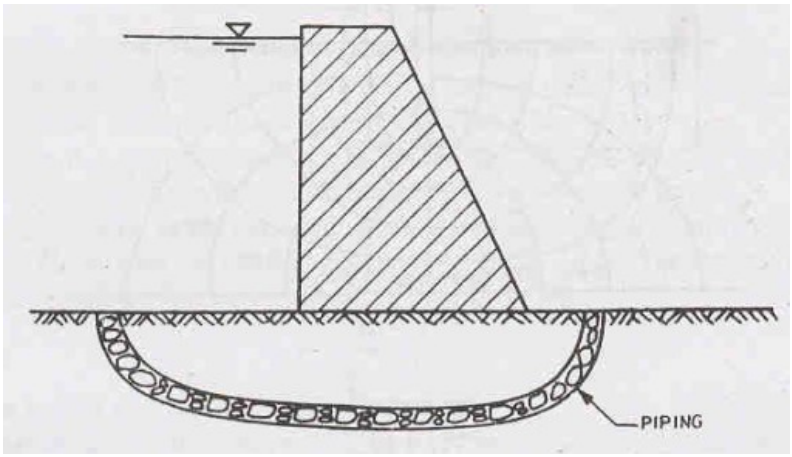


Figure 5.9 Backward erosion piping

2. **Heave piping:** failure by heave piping occurs in the form of a rise or a heave of a large mass of soil due to seepage pressure. When the seepage force due to upward flow of water at any level is greater than the submerged weight of the soil above that level, the entire soil mass in that zone heaves up and is blown out by the percolating water. Heave piping may occur on the downstream of sheet pile cutoff wall of hydraulic structure.

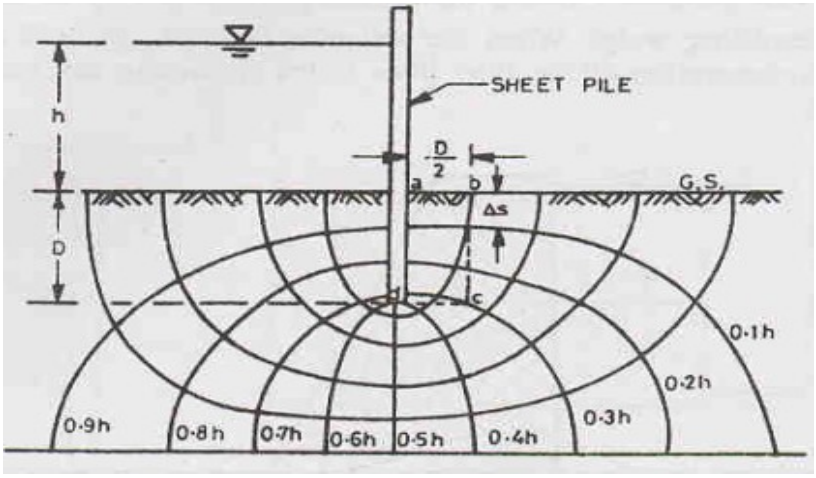


Figure 5.10 Heave piping

5.2.2 PREVENTION OF PIPING FAILURES

The occurrence of piping in and below a hydraulic structure such as an earth dam is disastrous. This may cause catastrophe. The following measures are generally adopted to prevent piping failures.

(1) **Increasing the path of percolation:** the hydraulic gradient (i) depends up on the path of percolation (L). If the length of the path is increased, the exit gradient will decrease to a safe value. The length of the path of percolation can be increased by adopting the following methods,

   (a) Increasing the base width of the hydraulic structure

   (b) Providing vertical cut off walls below the hydraulic structure

   (c) Providing an upstream impervious blanket

(2) **Reducing seepage**: with a reduction of seepage through the dam, the chances of piping failure through the body of the dam are considerably reduced. The quantity of seepage discharge is reduced by providing an impervious core, as shown in figure 5.11.

(3) **Providing drainage filter:** a drainage filter changes the direction of flow away from the downstream face. It prevents the movement of soil particles along with water. The drainage filter is properly graded. The drainage filter may be horizontal or in the form of a rock toe. It may also be in the form of a chimney drain, as shown in figure 5.11. a chimney drain is effective for stratified soil deposits in which the horizontal permeability is greater than the vertical permeability.

(4) **Loaded filter:** a loaded filter consists of graded sand and gravels. The function of the loaded filter is to increase the downward force without increasing the upward seepage force. The loaded filter is placed at the exit point where the water emerges from the foundation. For the sheet pile wall, the filter is placed over the affected zone.