

BAHIR DAR INSTITUTE OF TECHNOLOGY
FACULTY OF CIVIL AND WATER RESOURCE ENGINEERING

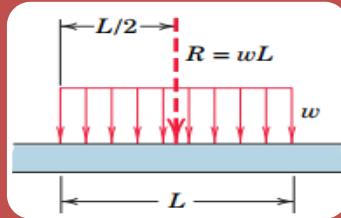
ENGINEERING MECHANICS I (STATICS)

CHAPTER 5

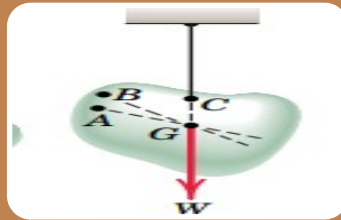
DISTRIBUTED FORCES

SEFINEW M. (MSC.)

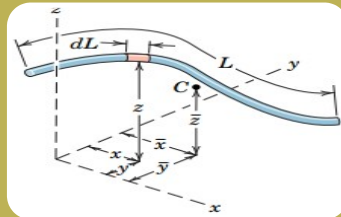
Contents



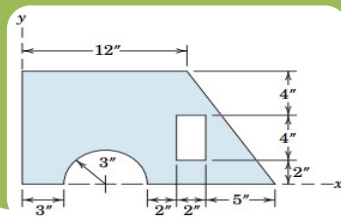
Introduction



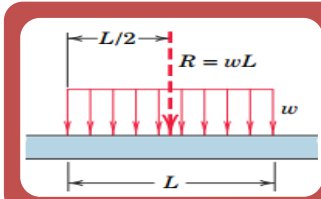
Centers of gravity, center of Mass and Centroids



Centroids of line, area and volume



Composite bodies

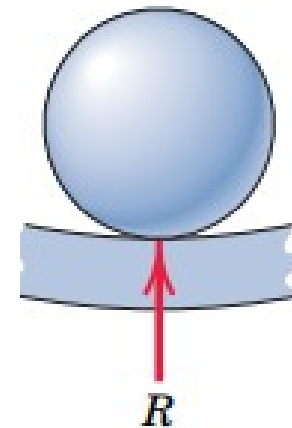
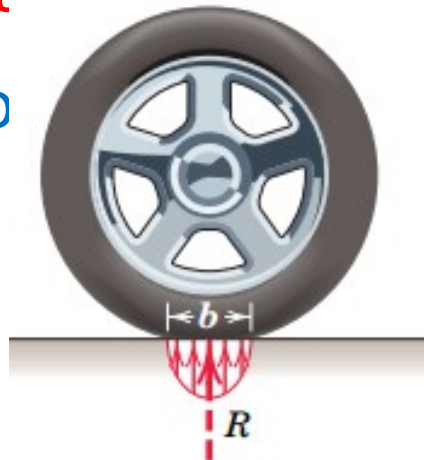


Introduction

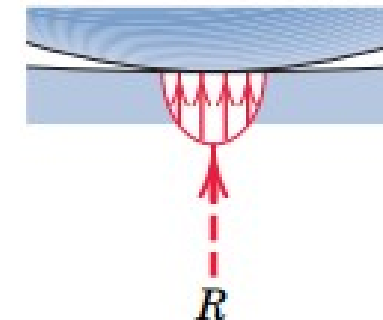
- Concentrated forces do not exist since every external force applied mechanically to a body is distributed over a finite contact area.
- Forces are distributed throughout volumes, over areas, and along lines.
- Resultant of the distributed forces and moments

has to

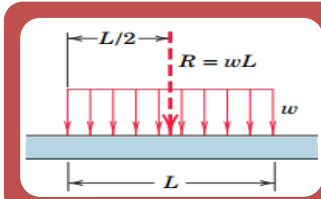
ed



Enlarged view
of contact



Resultant

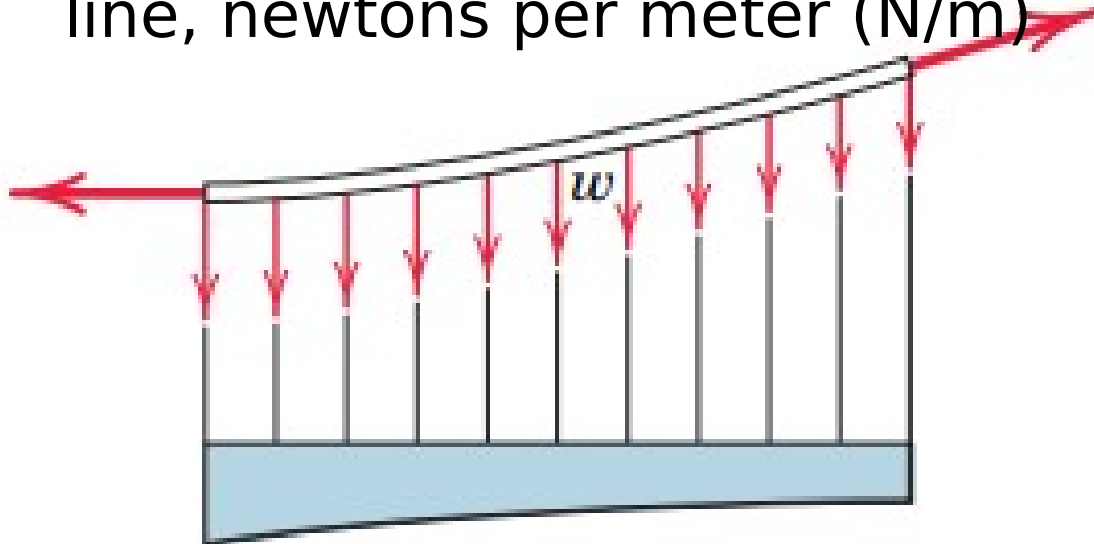


Introduction

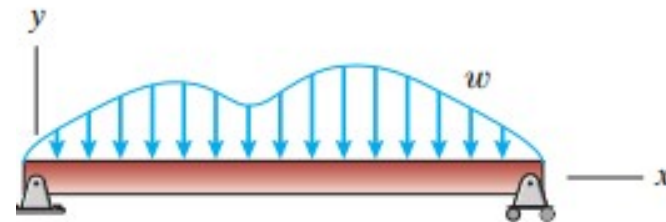
Force distribution

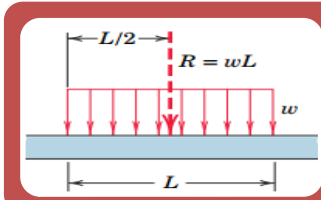
1. Line Distribution:

- force is distributed along a **line**.
- intensity w of the loading is expressed as force per unit length of line, newtons per meter (N/m)



(a)



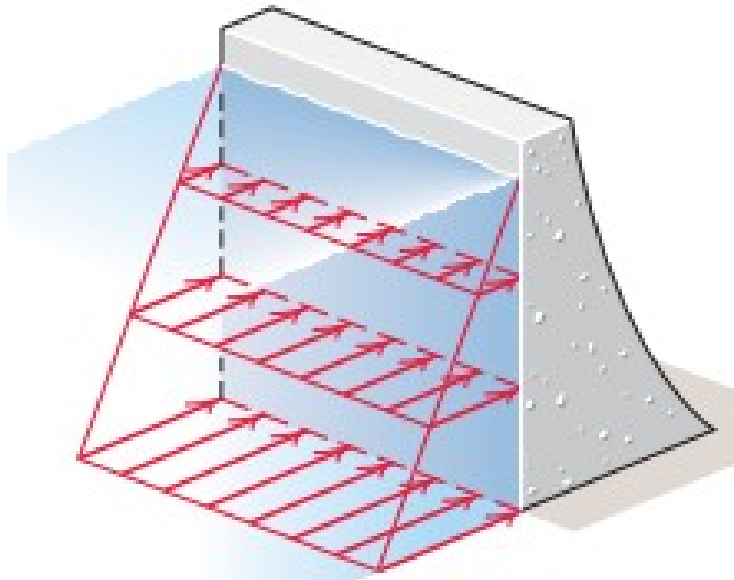


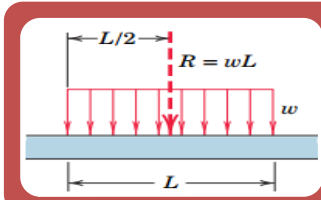
Introduction

Force distribution

2. Area Distribution:

- force is distributed over an area
- intensity w of the loading is expressed as force per unit area, newton per square





Introduction

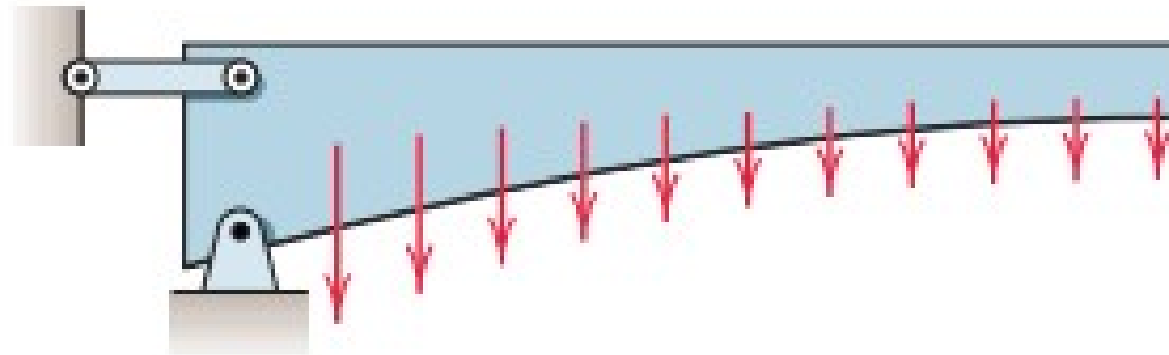
Force distribution

3. Volume Distribution:

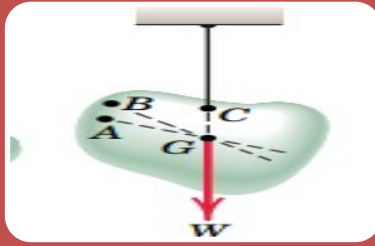
- A force distributed over the volume of a body is called a body force.

e.g. gravitational attraction

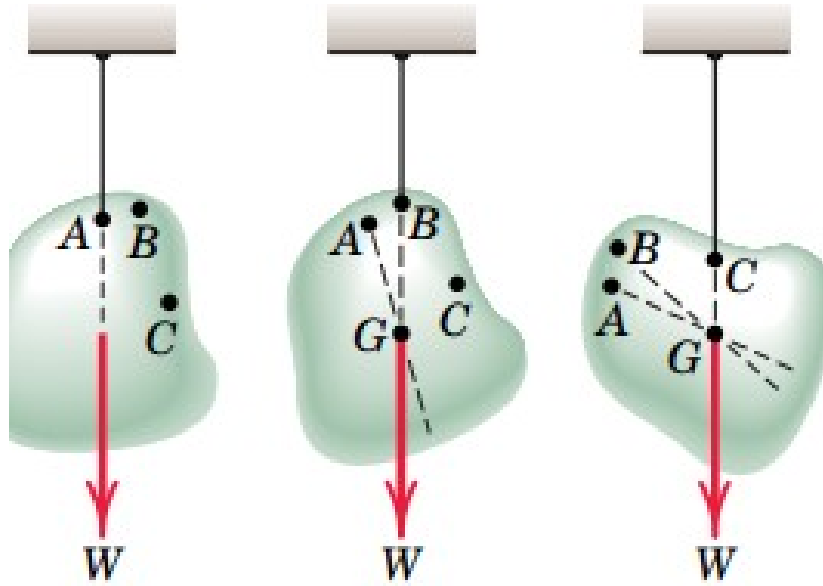
- intensity w of the force per unit volume, newton per cubic



volume,

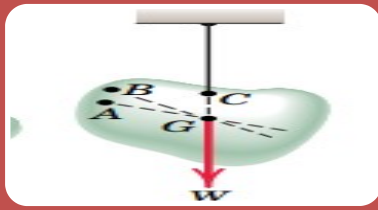


Centers of gravity, center of Mass and Centroids



If we suspend the body from any point A, B and C:

- Point **G**, a point where lines of action of all the three cases will be concurrent at is the **Center of gravity of the body**



Centers of gravity, center of Mass and Centroids

Location of the center of gravity

Moment due to dw

moment due

$$W = \int dW$$

to w_x dW

$$\bar{x}W$$

My=

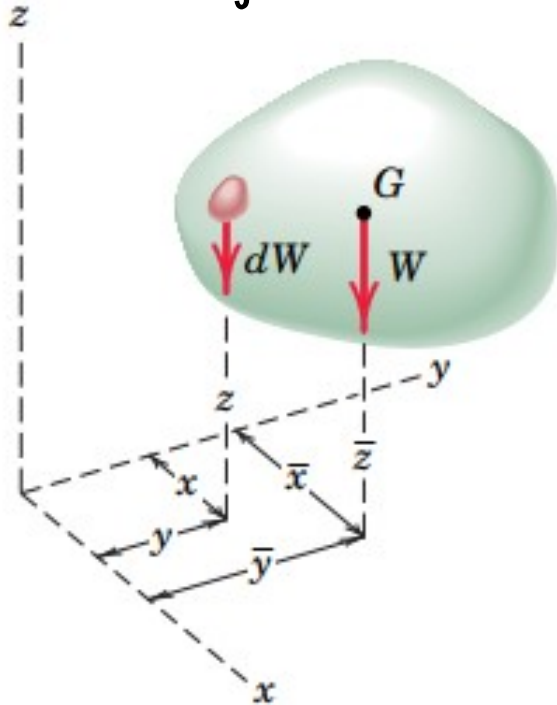
My=

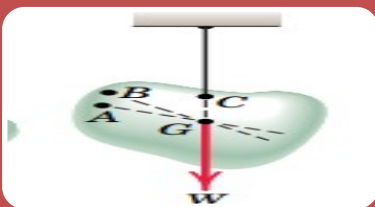
sum of my = $\int x dW$

- From the principle of moments: The moment of the resultant gravitational force W about any axis equals the

$$\bar{x}W = \int x dW$$

$$\bar{x} = \frac{\int x dW}{W} \quad \bar{y} = \frac{\int y dW}{W} \quad \bar{z} = \frac{\int z dW}{W}$$





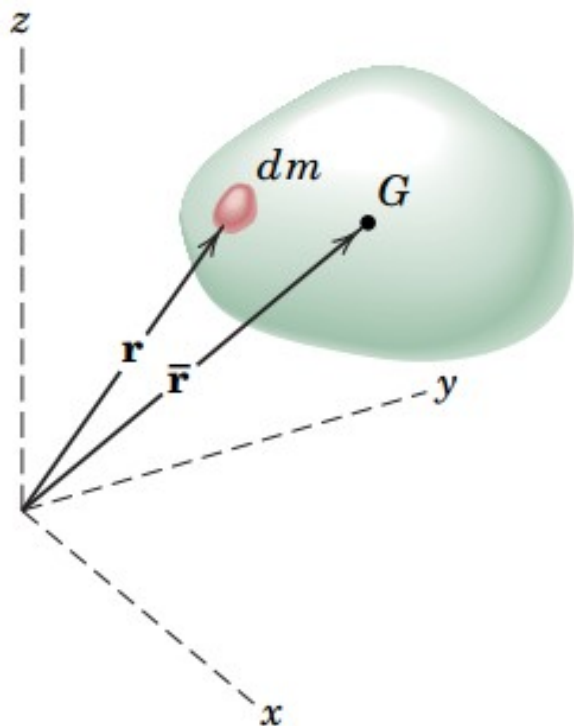
Centers of gravity, center of Mass and Centroids

$$\bar{x} = \frac{\int x dW}{W} \quad \bar{y} = \frac{\int y dW}{W} \quad \bar{z} = \frac{\int z dW}{W}$$

$$W = Mg$$

$$dW = dMg \rightarrow$$

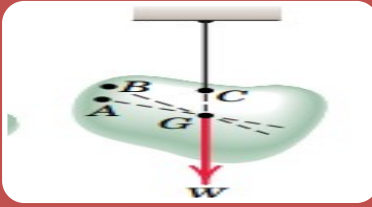
$$\bar{x} = \frac{\int x dm}{m} \quad \bar{y} = \frac{\int y dm}{m} \quad \bar{z} = \frac{\int z dm}{m}$$



$$\mathbf{r} = x\mathbf{i} + y\mathbf{j} + z\mathbf{k}$$

$$\bar{\mathbf{r}} = \bar{x}\mathbf{i} + \bar{y}\mathbf{j} + \bar{z}\mathbf{k}.$$

$$\bar{\mathbf{r}} = \frac{\int \mathbf{r} dm}{m}$$

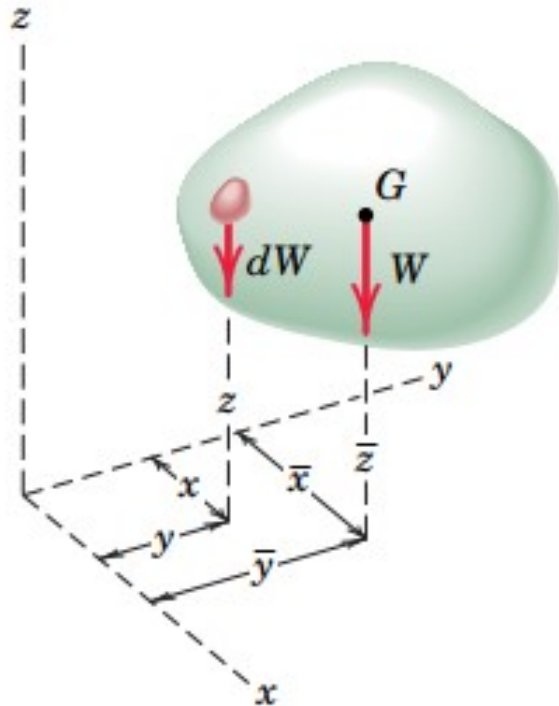


Centers of gravity, center of Mass and Centroids

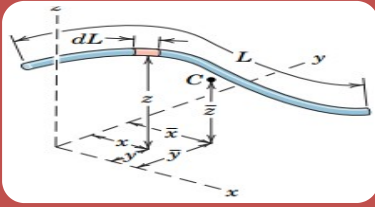
$$\bar{x} = \frac{\int x \, dm}{m} \quad \bar{y} = \frac{\int y \, dm}{m} \quad \bar{z} = \frac{\int z \, dm}{m}$$

density =
Mass/volume
 $dm = \rho \, dV$

$$\bar{x} = \frac{\int x \rho \, dV}{\int \rho \, dV} \quad \bar{y} = \frac{\int y \rho \, dV}{\int \rho \, dV} \quad \bar{z} = \frac{\int z \rho \, dV}{\int \rho \, dV}$$

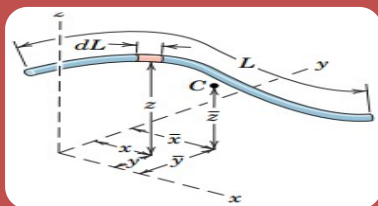


- Unique point in the body which is a function solely of the distribution of mass is called the center of mass.
- If gravity is treated as uniform and parallel then, the center of mass coincides with the



Centroids of line, area and volume

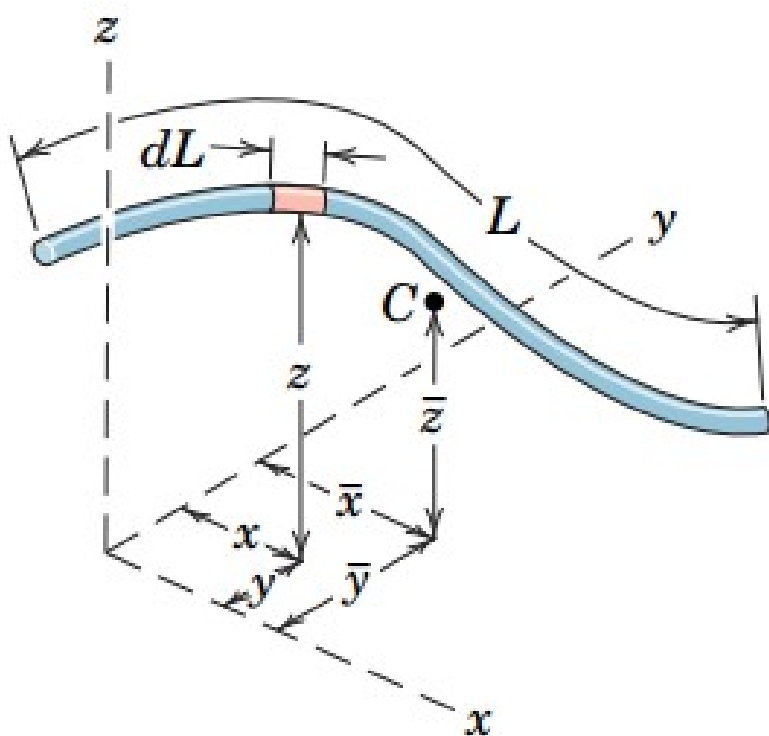
- **Center of mass** is used for an actual physical body.
- **Centroid** is used for the geometrical shape only
- If the **density is uniform** throughout the body, the positions of the **centroid** and **center of mass** are **identical**.
- If the **density varies**, centroid and center of mass will, in general, **not coincide**.
- The calculation of centroids falls within three distinct categories, depending on whether we can model the shape of the body involved as a **line**,



Centroids of lines

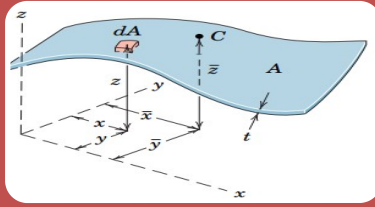
- For a slender rod or wire of length L , cross-sectional area A , and density ρ , the body approximates a line segment.

$$\text{Mass/volume} = \text{mass/area} \times \text{length} \quad dm = \rho A dL$$



- If density and Area are constant over the length of the rod, the coordinates of the center of mass also become the

$$\bar{x} = \frac{\int x dL}{L} \quad \bar{y} = \frac{\int y dL}{L} \quad \bar{z} = \frac{\int z dL}{L}$$

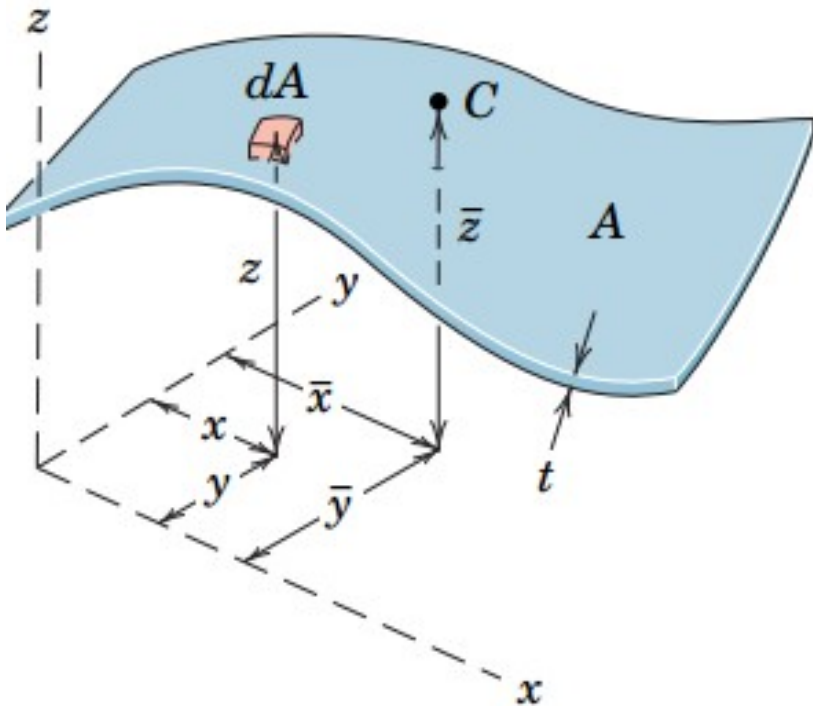


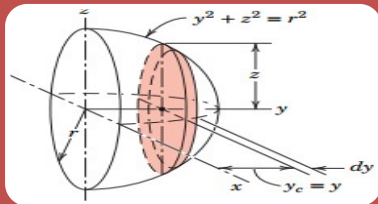
Centroids of areas

- When a body of density ρ has a small but constant thickness t , we can model it as a surface area A ,
 - density = ρ
 - Mass/volume = mass/(area * thickness) $dm = \rho t dA$

If density and thickness are constant, over the entire area, the coordinates of the center of mass of the body also become the coordinates of the centroid of the area.

$$\bar{x} = \frac{\int x dA}{A} \quad \bar{y} = \frac{\int y dA}{A} \quad \bar{z} = \frac{\int z dA}{A}$$



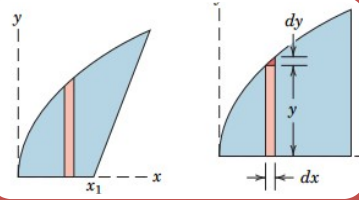


Centroids of volumes

- For a general body of volume V and density ρ , the element $dm = \rho dV$ has mass

The density cancels if it is constant over the entire volume, and the coordinates of the center of mass also become the coordinates of the centroid C of

$$\bar{x} = \frac{\int x dV}{V} \quad \bar{y} = \frac{\int y dV}{V} \quad \bar{z} = \frac{\int z dV}{V}$$

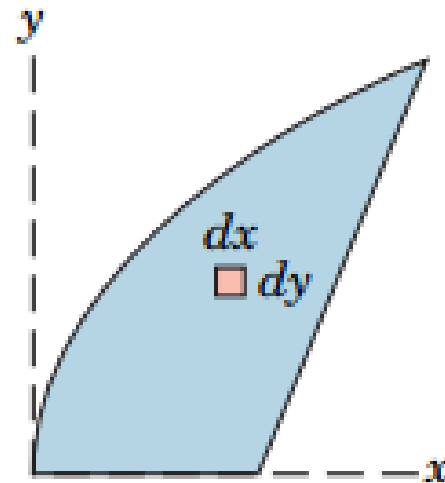
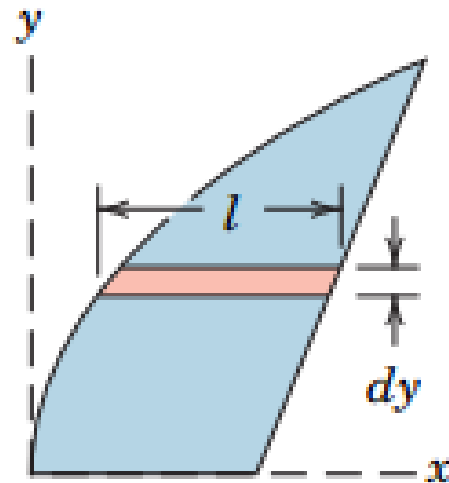


Choice of Element for Integration (guidelines)

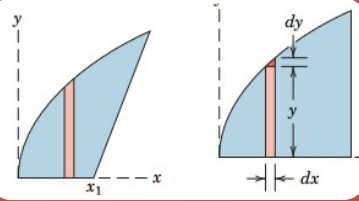
1. Order of Element

- Whenever possible, a **first order differential element** should be selected in preference to a **higher-order element** so that only one integration will be required to cover the entire figure.

$dA = l dy$ - first order



$dA = dx dy$ - second order



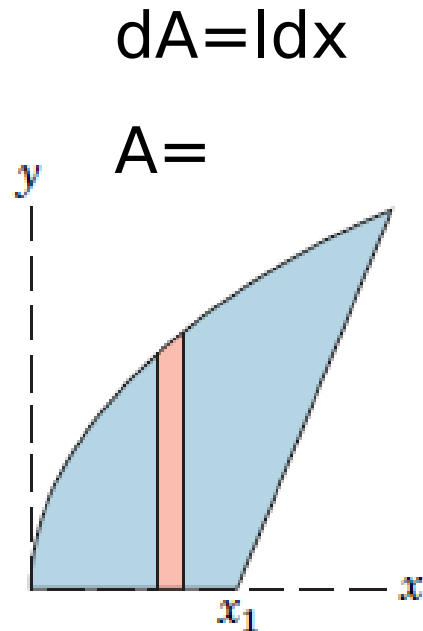
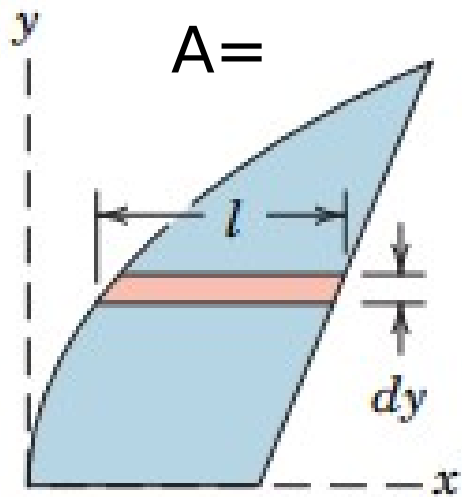
Choice of Element for Integration (guidelines)

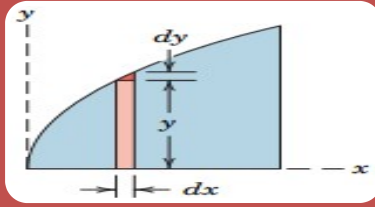
2. Continuity

- Choose an element which can be integrated **in one continuous** operation to cover the figure.

For the given shape:

- The **horizontal would be preferable** to the vertical strip in two separate integrals because of the discontinuity in the expression for the height of the strip at $x = x_1$.



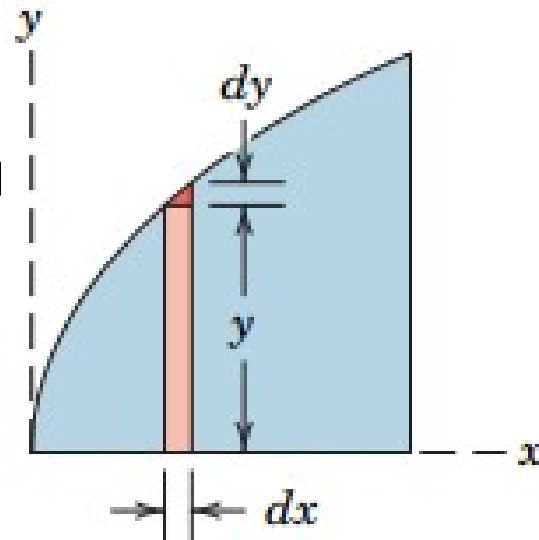


Choice of Element for Integration (guidelines)

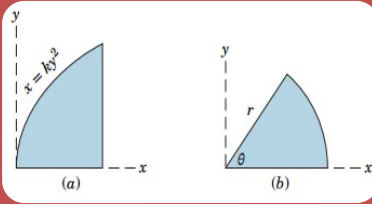
3. Discarding Higher-Order Terms.

- Higher order terms may always be dropped compared with lower-order terms.
- Thus, the vertical strip of area under the curve in is given by the first-order term

$dA = y \, dx$, and the
discarded.



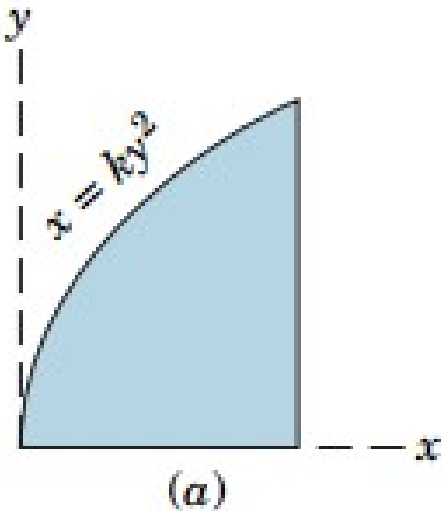
Angular area $\frac{1}{2} \, dx \, dy$ is
 $dA = y \, dx + \frac{1}{2} \, dx \, dy$



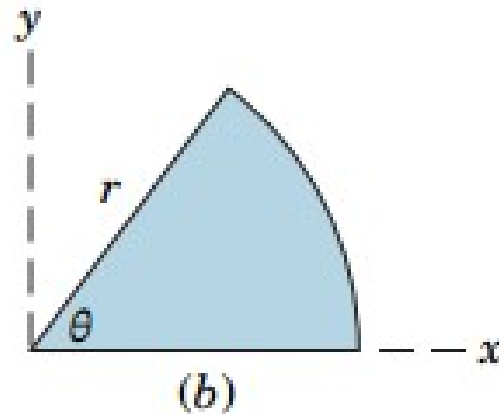
Choice of Element for Integration (guidelines)

4. Choice of Coordinates

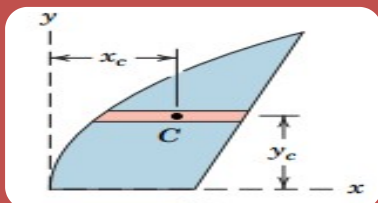
- choose the coordinate system which best matches the boundaries of the figure



Rectangular



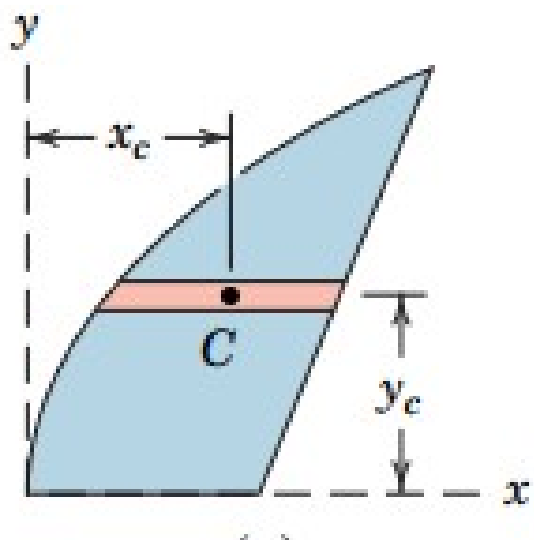
Polar



Choice of Element for Integration (guidelines)

4. Centroidal Coordinate of Element

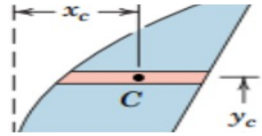
- Use the coordinate of the **centroid of the element** for the moment arm in setting up the moment of the differential element.



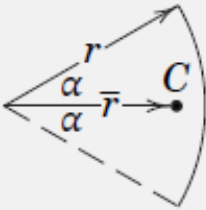
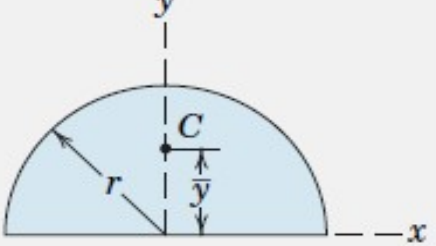
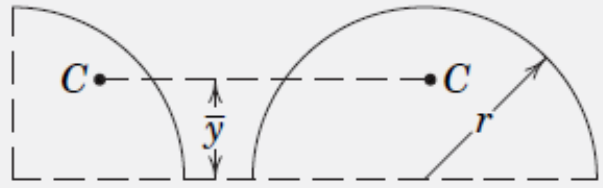
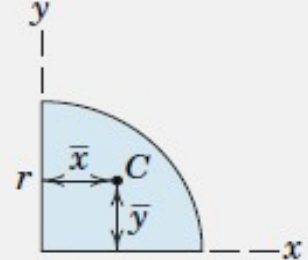
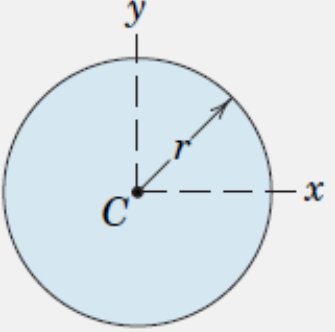
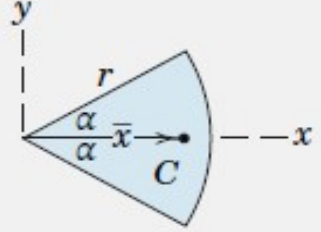
$$\bar{x} = \frac{\int x_c dA}{A} \quad \bar{y} = \frac{\int y_c dA}{A} \quad \bar{z} = \frac{\int z_c dA}{A}$$

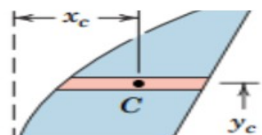
C - centroid

x_c - X coordinate of centroid of the differential element



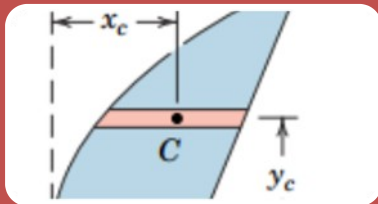
Common plane figures

Figure	Centroid	Figure	Centroid
Arc Segment 	$\bar{r} = \frac{r \sin \alpha}{\alpha}$	Semicircular Area 	$\bar{y} = \frac{4r}{3\pi}$
Quarter and Semicircular Arcs 	$\bar{y} = \frac{2r}{\pi}$	Quarter-Circular Area 	$\bar{x} = \bar{y} = \frac{4r}{3\pi}$
Circular Area 	—	Area of Circular Sector 	$\bar{x} = \frac{2}{3} \frac{r \sin \alpha}{\alpha}$



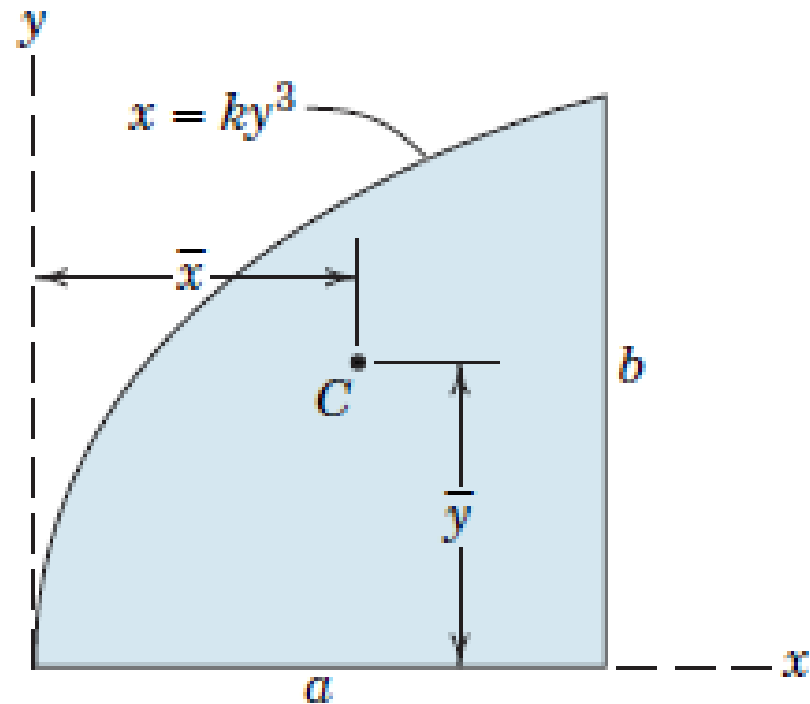
Common plane figures

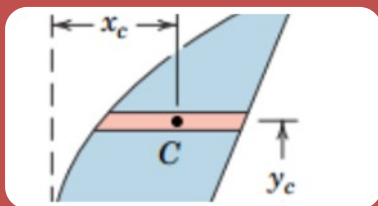
Figure	Centroid	Figure	Centroid
Rectangular Area 	—	Subparabolic Area 	$\bar{x} = \frac{3a}{4}$ $\bar{y} = \frac{3b}{10}$
Triangular Area 	$\bar{x} = \frac{a+b}{3}$ $\bar{y} = \frac{h}{3}$	Parabolic Area 	$\bar{x} = \frac{3a}{8}$ $\bar{y} = \frac{3b}{5}$



Example

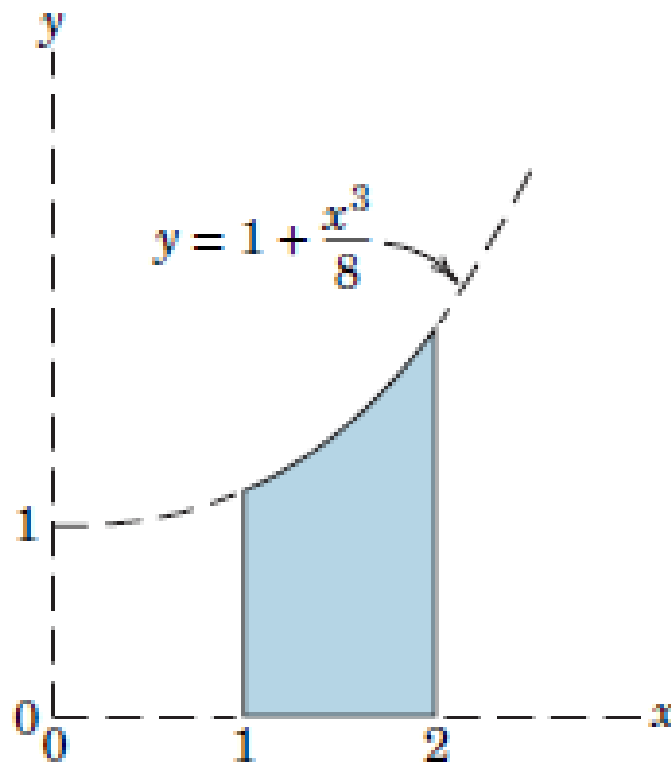
Example 1: Locate the centroid of the area under the curve $x = ky^3$ from $x = 0$ to $x = a$.

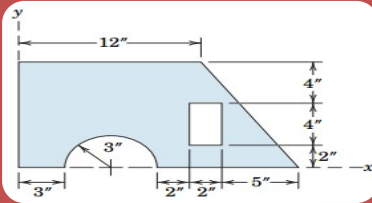




Example

Example 2: Determine the x- and y-coordinates of the centroid of the shaded area.





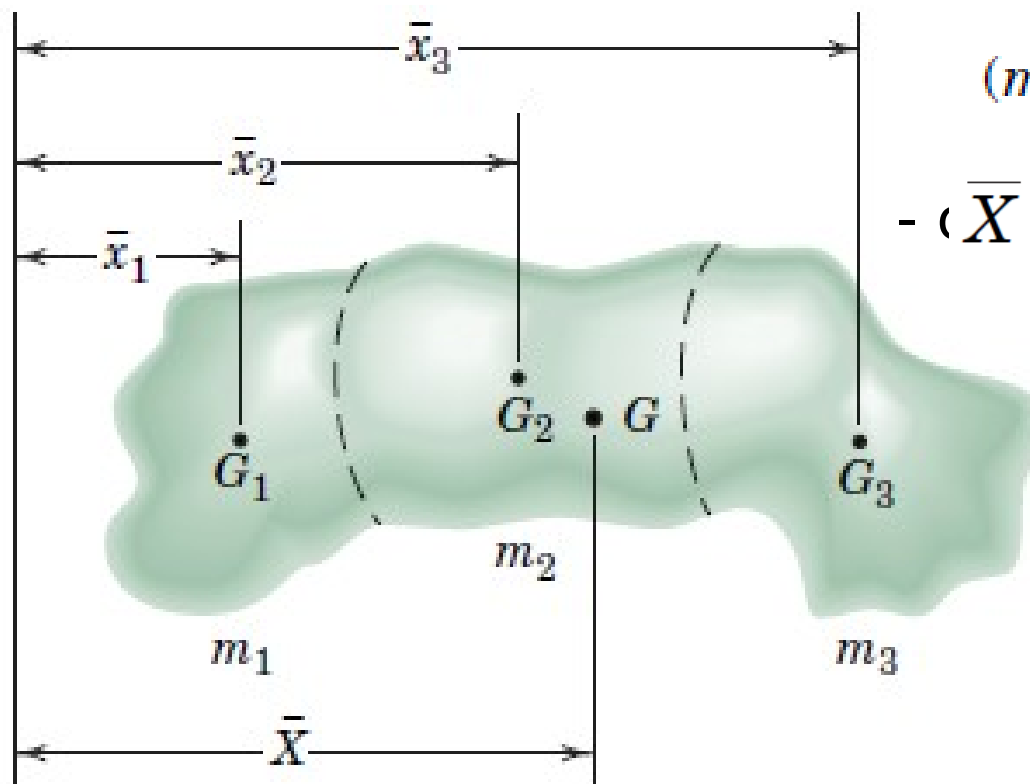
Composite bodies

Consider a body with masses M_1, M_2, M_3 with respective mass center coordinates $\bar{x}_1, \bar{x}_2, \bar{x}_3$

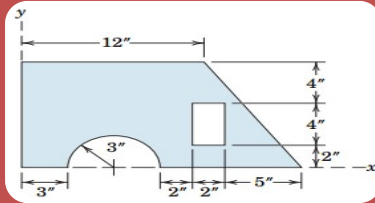
Moment principle

$$(m_1 + m_2 + m_3)\bar{X} = m_1\bar{x}_1 + m_2\bar{x}_2 + m_3\bar{x}_3$$

- ($\bar{X}$) ordinate of the center of mass of the whole body

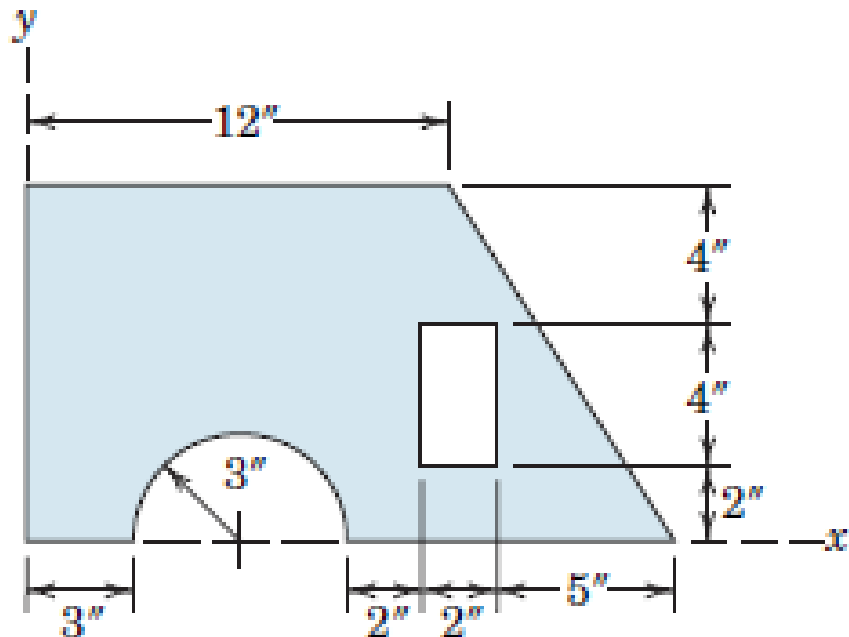


$$\bar{X} = \frac{\sum m\bar{x}}{\sum m} \quad \bar{Y} = \frac{\sum m\bar{y}}{\sum m} \quad \bar{Z} = \frac{\sum m\bar{z}}{\sum m}$$



Composite bodies

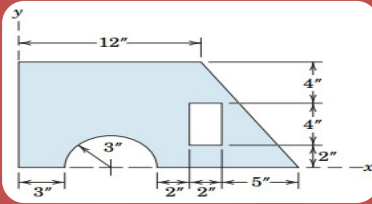
- If a hole is considered one of the component parts of a composite body, the corresponding mass represented by the hole is treated as



	A	$\bar{x}$	$\bar{y}$	$\bar{x}A$	$\bar{y}A$
PART					
1					
2					
3					
4					
TOTALS					

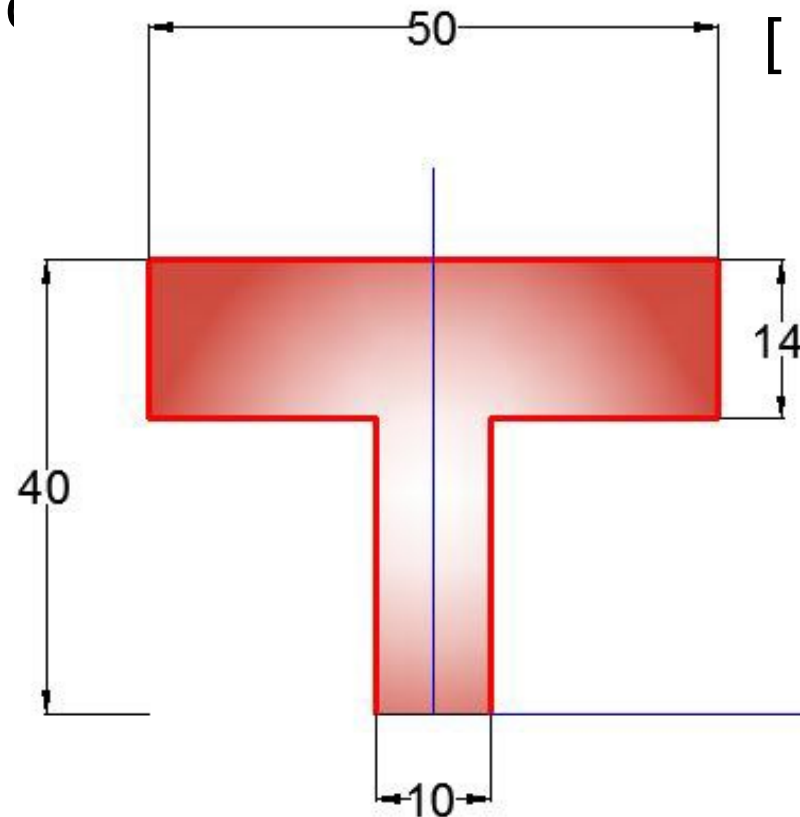
$$\left[\bar{X} = \frac{\Sigma A \bar{x}}{\Sigma A} \right]$$

$$\left[\bar{Y} = \frac{\Sigma A \bar{y}}{\Sigma A} \right]$$

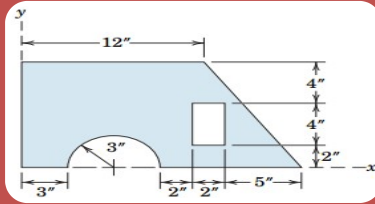


Composite bodies

Example: Determine the location of the centroid of the T-section beam. Dimensions



[Answer : $y = 27.58 \text{ cm}$, $x = 0 \text{ cm}$]



Composite bodies

Example: Compute the centroid of the plate as shown below with all the dimensions in m.

[Answer : $Y = 1.76$ m, $X = 1.32$ m]

