LINEAR PROGRAMMING MODEL-A

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2.1. Introduction

- All organizations have at their disposal, men, machines, money, and materials, the supply of which may be limited

- The basic problem of any organization is how to **allocate scare resources** efficiently

- The management is, therefore, find the **best to allocation of resources** among the various activities so as to **optimize the measure of effectiveness**

- Therefore, linear programming model is the best method to use for such types of problems

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- **LP** is a mathematical method for determining a way to achieve the best outcome (such as maximum profit or lowest cost) in a given mathematical model for some list of requirements represented as linear relationships

- More formally, linear programming is a technique for the optimization of a linear objective function, subject to linear equality and linear inequality constraints

Components of LPM(LP MODEL)

1. Objective
2. Decision variables
3. Constraints
4. Parameters

Assumptions of LP

- **Linearity** – each decision variable has a linear impact on the objective function and in each constraint in which it appears. There is no squared, cubed, square root, products etc in the model

- **Divisibility/continuity** – non integer values are acceptable to the potential values of decision variables if they are continuous but not for discrete variables

- ***Certainty* :** Various parameters namely the objective function coefficients, the coefficients of inequality/equality constraints and the constraints (resource) values are known and constant. Hence, linear programming is deterministic in nature

Assumptions of LPP

- ***Proportionality:*** there has to be proportional contribution of the **decision variables** exists in the objective function and the constraint inequalities

- ***Finite choices –*** A linear programming model also assumes that a limited number of choices are available to the decision maker

- **Non-negativity** – all the decision variables will never assume negative values and hence we consider the first quadrant un the x-y coordinate

Structure of LPM

Optimize $Z = C_1x_1 + C_2x_2 + C_3x_3 + \ldots + C_nx_n$

Subject to

$$a_{11}x_1 + a_{12}x_2 + \ldots\ldots\ldots + a_{jj}x_j + \ldots\ldots a_{1n}x_n (\leq = \geq)\ b_1$$

$$a_{21}x_1 + a_{22}x_2 + \ldots\ldots\ldots + a_{2j}x_j + \ldots\ldots a_{2n}x_n (\leq = \geq)\ b_2$$

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$$a_{m1}x_1 + a_{m2}x_2 + \ldots\ldots\ldots + a_{mj}x_j + \ldots\ldots a_{mn}x_n (\leq = \geq)\ b_m$$

And non – negativity restrictions $x_j \geq 0$, $j = 1, 2, 3 \ldots\ldots n$

Where:

- ($\leq$ = $\geq$) means any of the three signs may be there
- Optimize means maximize or minimize objective function
- All constraints are equations except for non-negativity restrictions which are inequalities
- The right hand side (RHS) of each constraints are non-negative
- Inequality constraints can be changed to equality by adding or subtracting the LHS of each constraint by non-negative variable i.e., a slack variable S_1 is added to LHS of constraint with the sign $\leq$ and a surplus variable S_2 is subtracted on the LHS of constraint with the sign $\geq$) and also artificial variables in case of equal signs and $\geq$ signs
- a_{ij} and b_m are known coefficients and x_j is unknown variable

Formulation of LPM

- ***Step 1:*** Identify the unknown decision variables to be determined. These should be brought into algebraic relation form

- ***Step 2:*** Clearly define all the limitations for a given situation. These limitations or constraints also need to be expressed in algebraic form either as linear equations or inequalities in terms of the decision variables so identified in step 1

- ***Step 3:*** Identify the objective to be optimized and it also should be expressed in terms of linear function of decision variables

Graphical Solutions for LPP

To use the graphic method, the following steps are needed:

- **Step 1**: Formulate the LPM as explained before
- **Step 2: Draw a graph** the constraint sets on the x-y plane
  - Convert the inequalities in to equality
  - Identify the x and y intercepts and draw the line that joins them
  - shade the feasible solution region by considering the inequality sign
- **Step 3:** Identify the extreme/corner points
- **Step 4:** evaluate the objective function using the corner points

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- In case of maximizing objective function Z, the corner point of feasible region giving the maximum value of Z becomes the value of decision variables
- Similarly in minimizing case, the point of minimum value of Z gives the answer

Note: Graphical LPP is a two-dimensional model (it considers only two decision variables)

Example: Graphic Solution Approach

