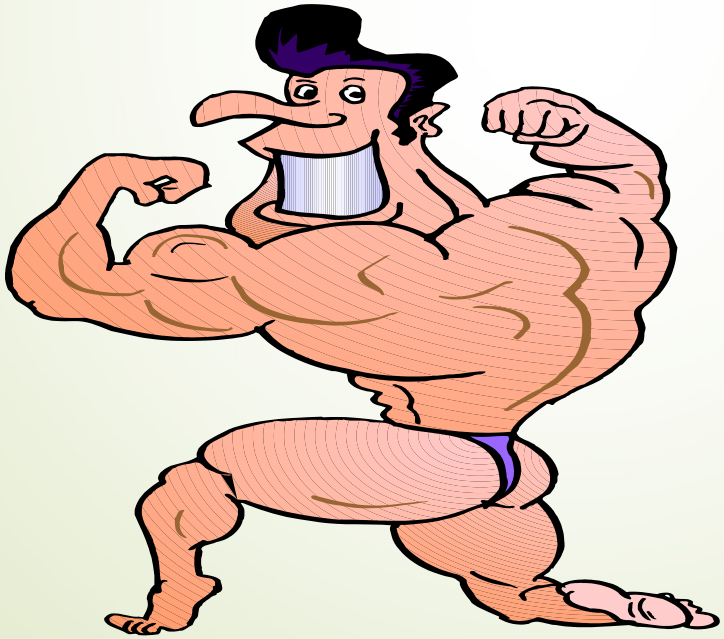
7. SHEARING RESISTANCE AND STRENGTH OF SOILS



Strength of different materials

Steel

Tensile strength

Concrete

Compressive strength

Soil

Shear strength

Presence of pore water

Complex behavior



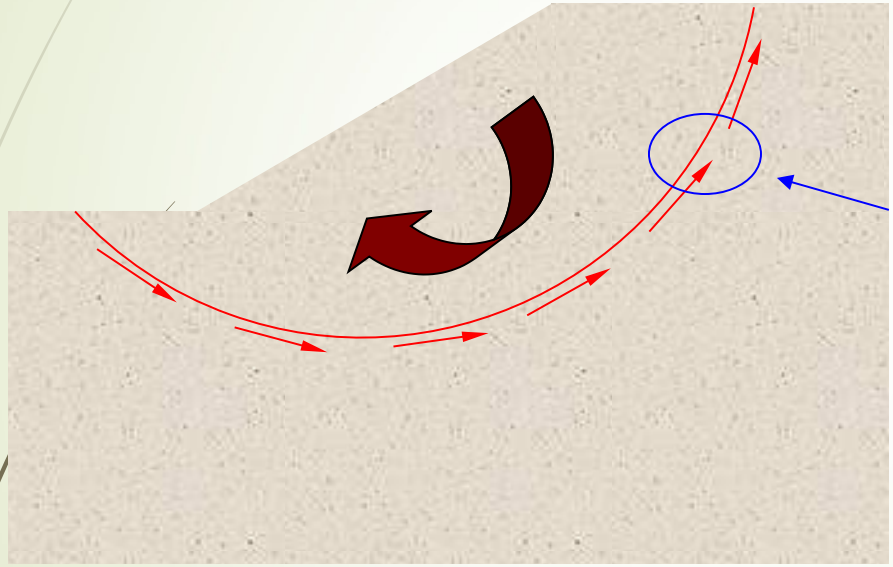
SHEARING RESISTANCE AND STRENGTH OF SOILS

- **The property that enables a material to remain in equilibrium when its surface is not level is called shearing strength. All solids have this property to some extent.**

- **Shearing strength is the major structural property of soils. It is this property, which provides supporting ability or bearing capacity, and permits slopes to be stable.**

- ***Generally, Shear strength is the magnitude of the shear stress that a soil can sustain.***

SHEAR FAILURE MECHANISM

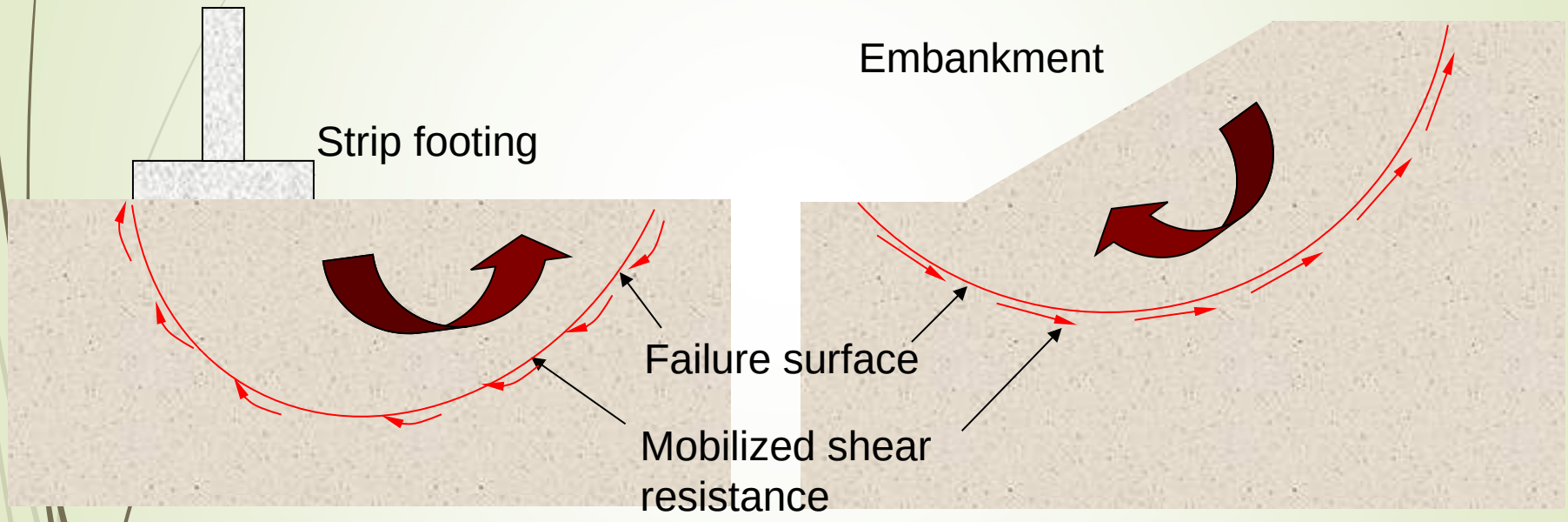


The soil grains slide over each other along the failure surface.

No crushing of individual grains.

Shear failure of soils

Soils generally fail in shear



At failure, shear stress along the failure surface (mobilized shear resistance) reaches the shear strength.

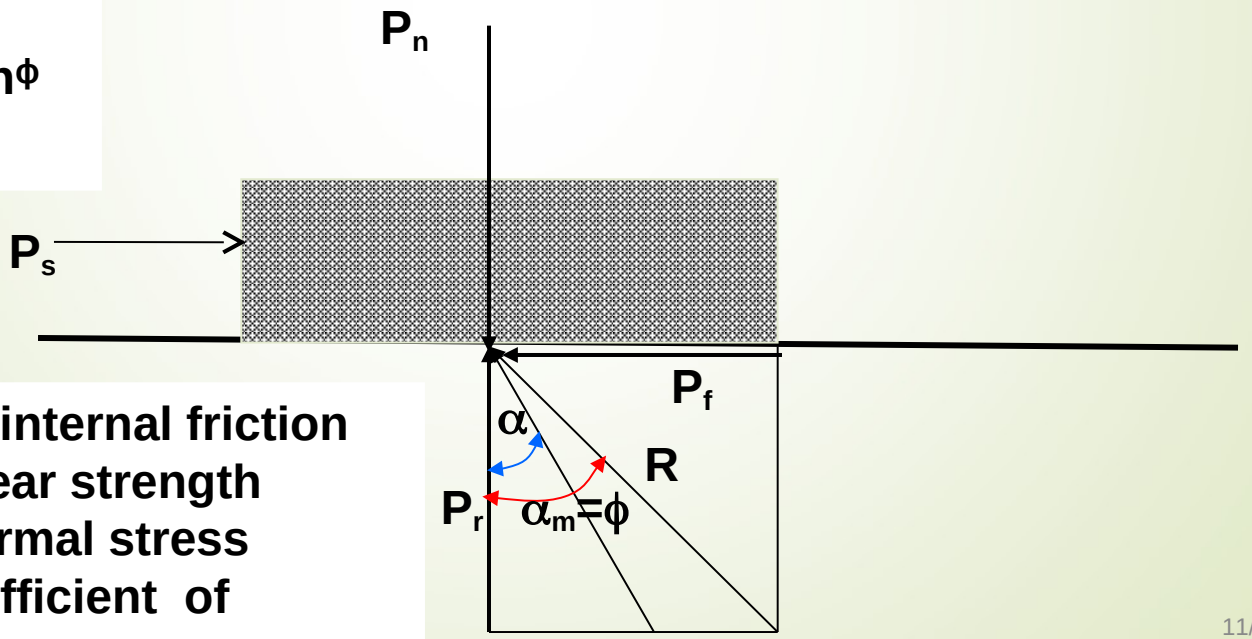
BASIC CONCEPT OF SHEARING RESISTANCE AND SHEARING STRENGTH

- **The basic concept of shearing resistance and shearing strength can be made clear by studying first the basic principles of friction between solid bodies.**

$P_s = P_n \tan\phi$

$(P_s / A) = (P_n / A) \tan\phi$

$S = \sigma \tan\phi$



Where: ϕ = angle of internal friction

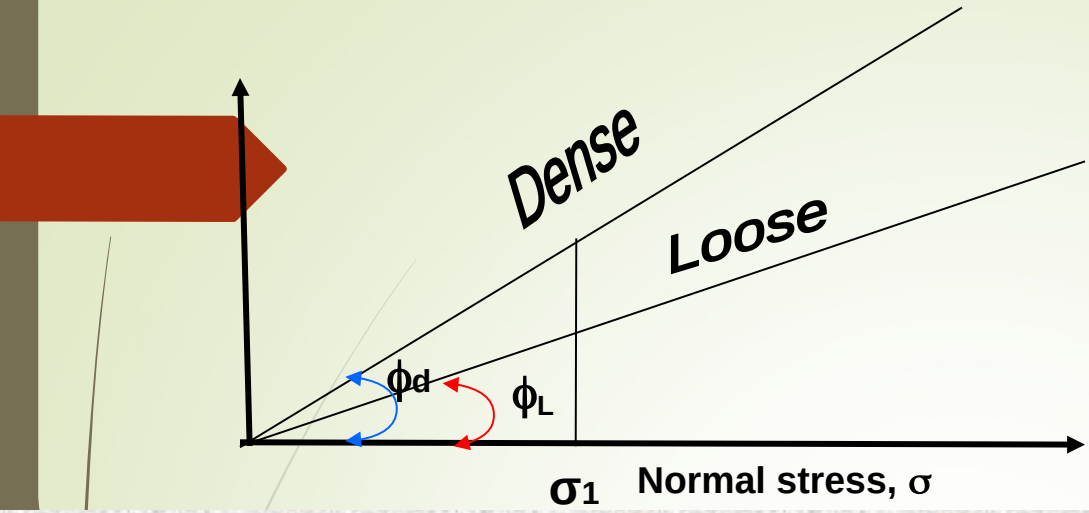
s= is the shear strength

σ= is the normal stress

tanϕ=is the coefficient of friction

SHEARING STRENGTH OF GRANULAR SOIL

- In sands and in other cohesionless granular materials the resistance to sliding on any plane through the material is made up of: -
- **a) Sliding friction :** is closely similar to frictional resistance of solids in contact
- **b) Rolling friction and**
- **c) Interlocking**

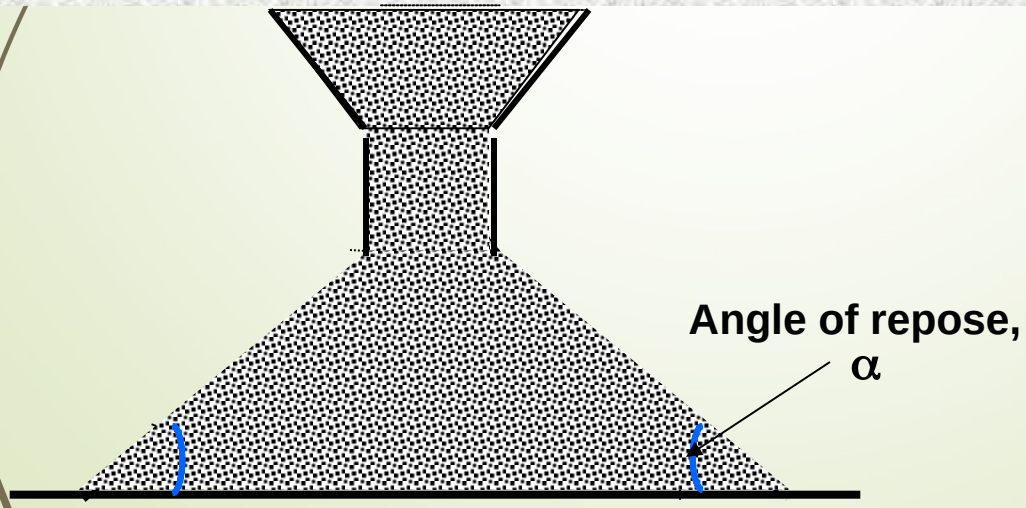


$S = \sigma_1 \tan\phi_d$

$S = \sigma_1 \tan\phi_L$

.**The angle of internal friction for dry granular soil is sometimes taken as identical with the angle of repose of soil.**

- **The angle of repose is defined as the angle between the horizontal and the maximum slope at which a loose soil may remain stable.**



- **Shearing Strength of Saturated Granular Soils**

- **Since the normal stress is affected by pore water pressure, the shearing strength of sand in a saturated condition is always less than that of dry condition. WHY???**

  - $S = (\sigma - u) \tan \phi = \sigma' \tan \phi$

THE COULOMB EQUATION

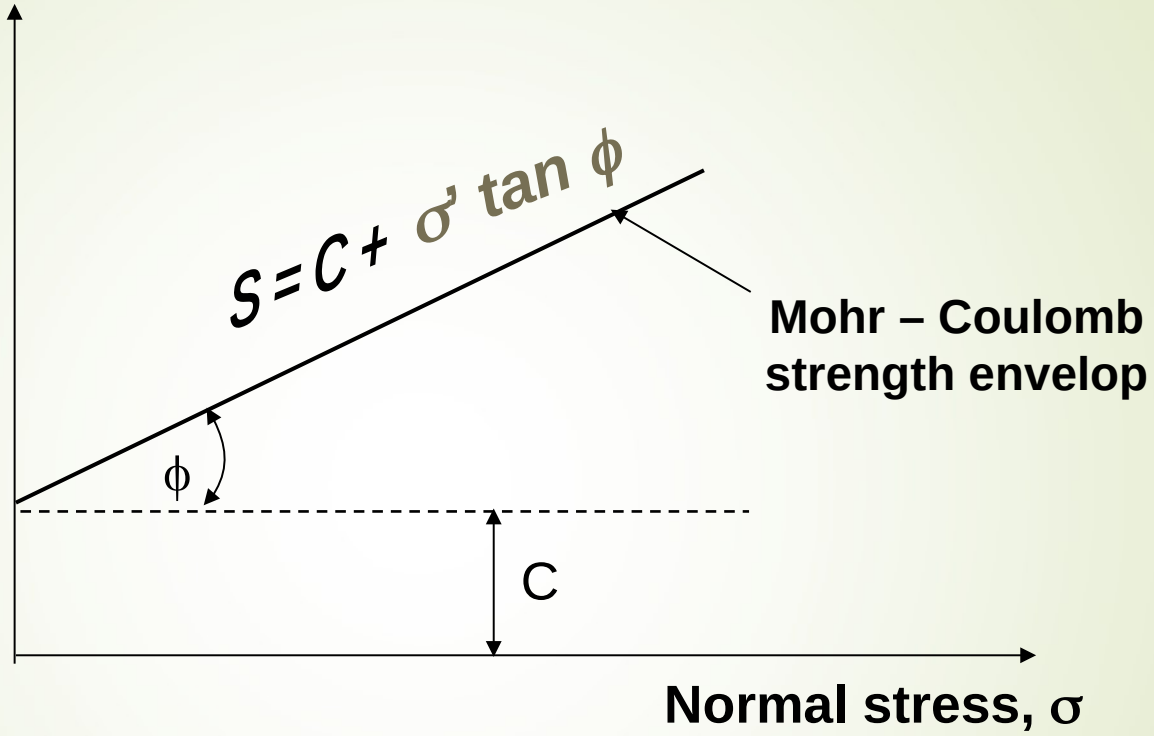
- The fundamental shear strength equation proposed by the French Engineer Coulomb is

$$S = C + \sigma \tan \phi$$

- This equation expresses the assumption that the **cohesion C is independent of the normal stress σ acting on the plane of failure**.
- At zero normal stress the shear strength of the soil is expressed as

$$S = C$$

- **C and ϕ are measures of shear strength**
- **Higher the values of C and ϕ, higher the shear strength.**



Shear strength consists of two components: **cohesive** and **frictional**.

MOHR STRESS CIRCLE

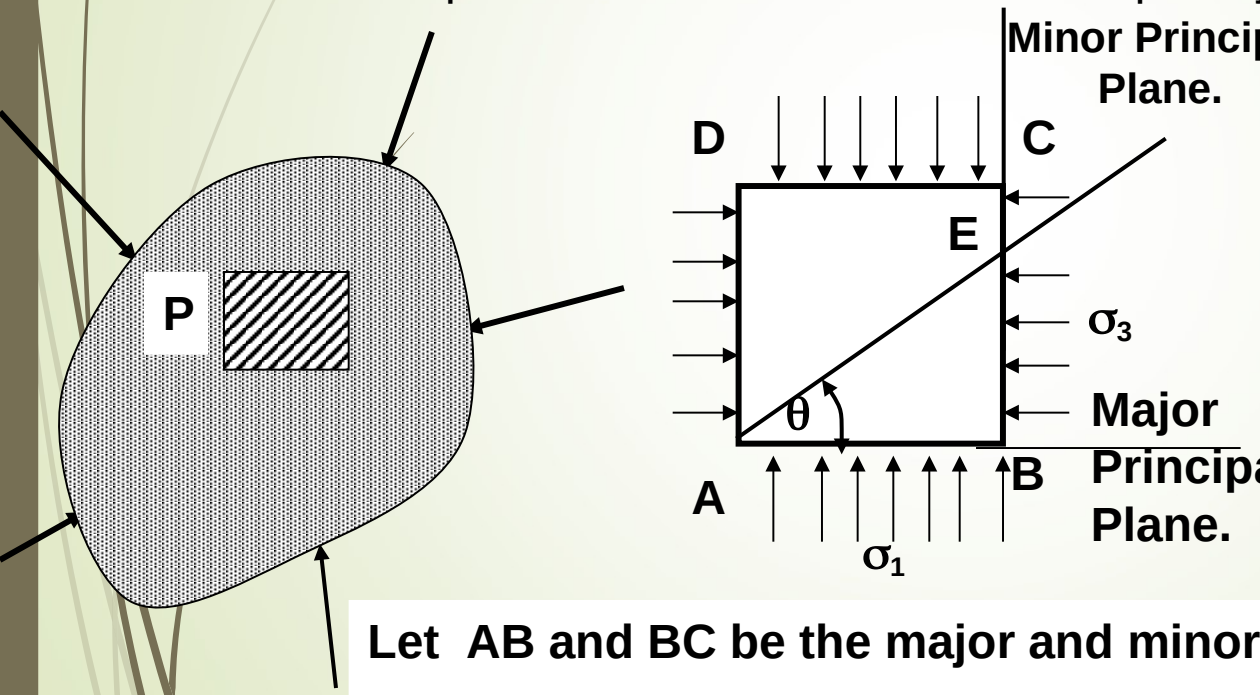
Principal Planes and Principal Stresses

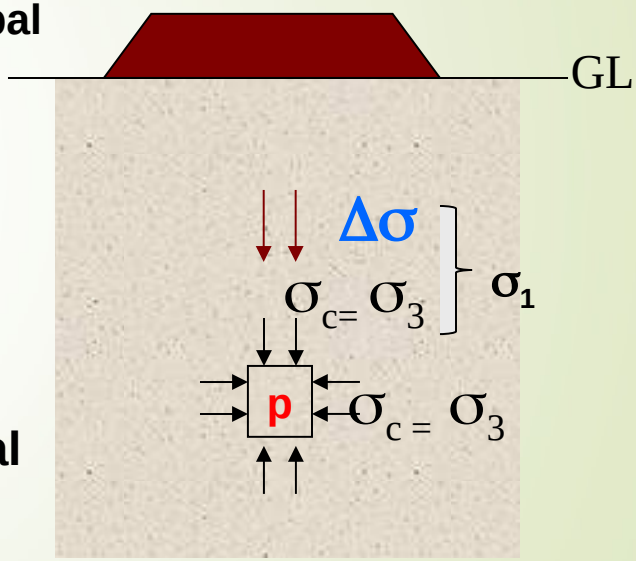
- **Through every point in a stressed body, there are three planes at right angles to each other, which are unique as compared to all other planes passing through the point, because they are subjected only by normal stresses with no accompanying shearing stresses.**
- **The three planes are**
  - **Major Plane**
  - **Intermediate Plane and**
  - **Minor principal planes**
- **And the corresponding stresses are;**
  - **Major principal stress (σ_1) ,**
  - **Intermediate principal stress (σ_2) ,**
  - **Minor principal stress (σ_3)**

- **Principal stresses at a point in a stressed body are important because, once they are evaluated the stresses on any other plane through the point can be determined.**

Stress at a Point

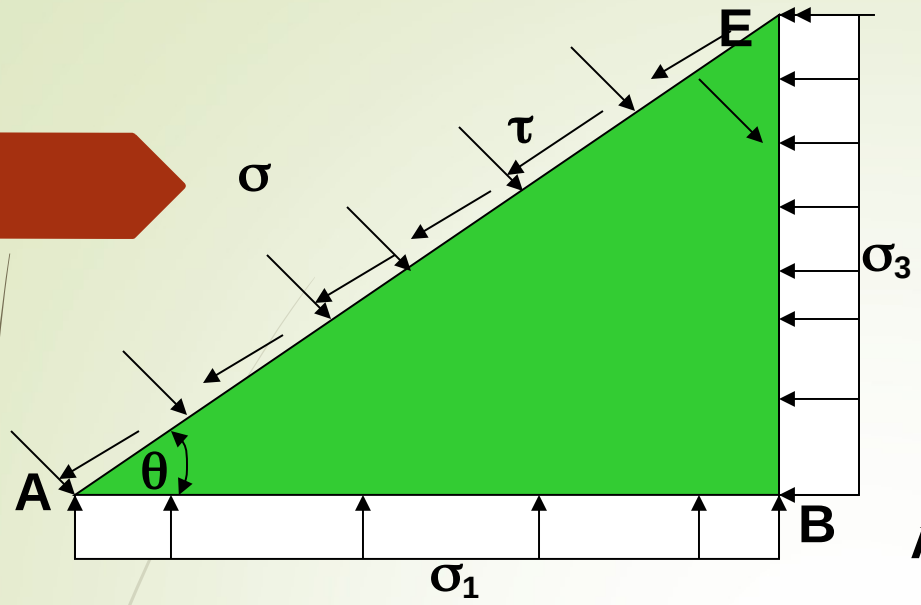
Consider a small prismatic soil element P acted upon by a system of forces

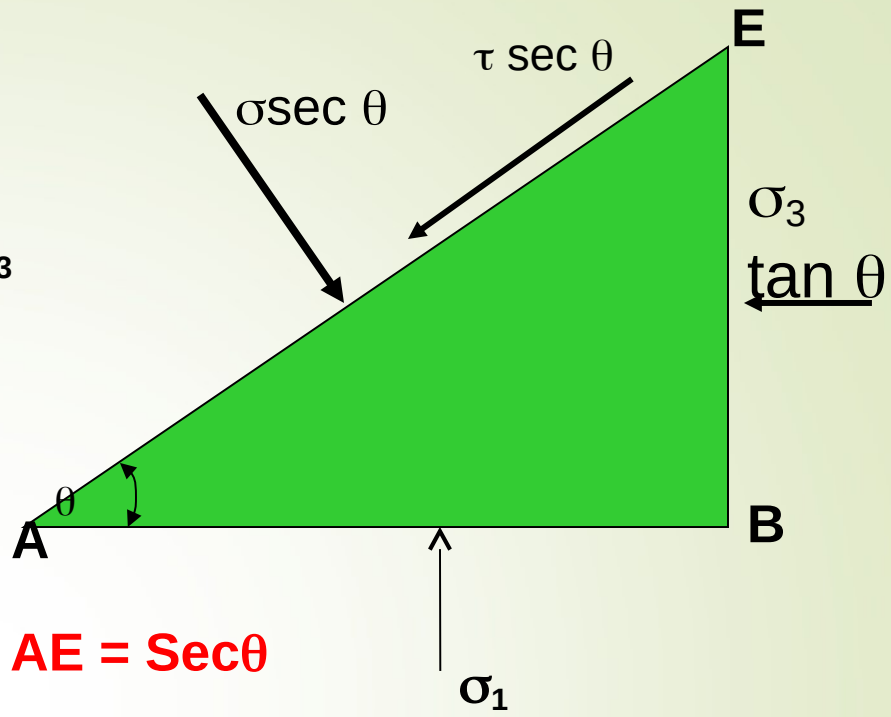




Let AB and BC be the major and minor principal planes

Consider a plane AE through the element making an angle θ With the major principal plane and let this be the plane on which the normal and shear stresses are needed.





Let AB= 1 unit Then BE= tan θ and AE = Secθ

Sum of all the forces along normal direction

$\sigma \sec\theta = \sigma_1 \cos\theta + \sigma_3 \tan\theta \cos(90-\theta)$

$= \sigma_1 \cos\theta + \sigma_3 \tan\theta \sin\theta$

$\sigma = \sigma_1 \cos^2\theta + \sigma_3 \sin^2\theta$

But $\cos^2\theta = \frac{1+\cos 2\theta}{2}$ and $\sin^2\theta = \frac{1-\cos 2\theta}{2}$

Thus, $\sigma = \sigma_1 \frac{(1+\cos 2\theta)}{2} + \sigma_3 \frac{(1-\cos 2\theta)}{2}$

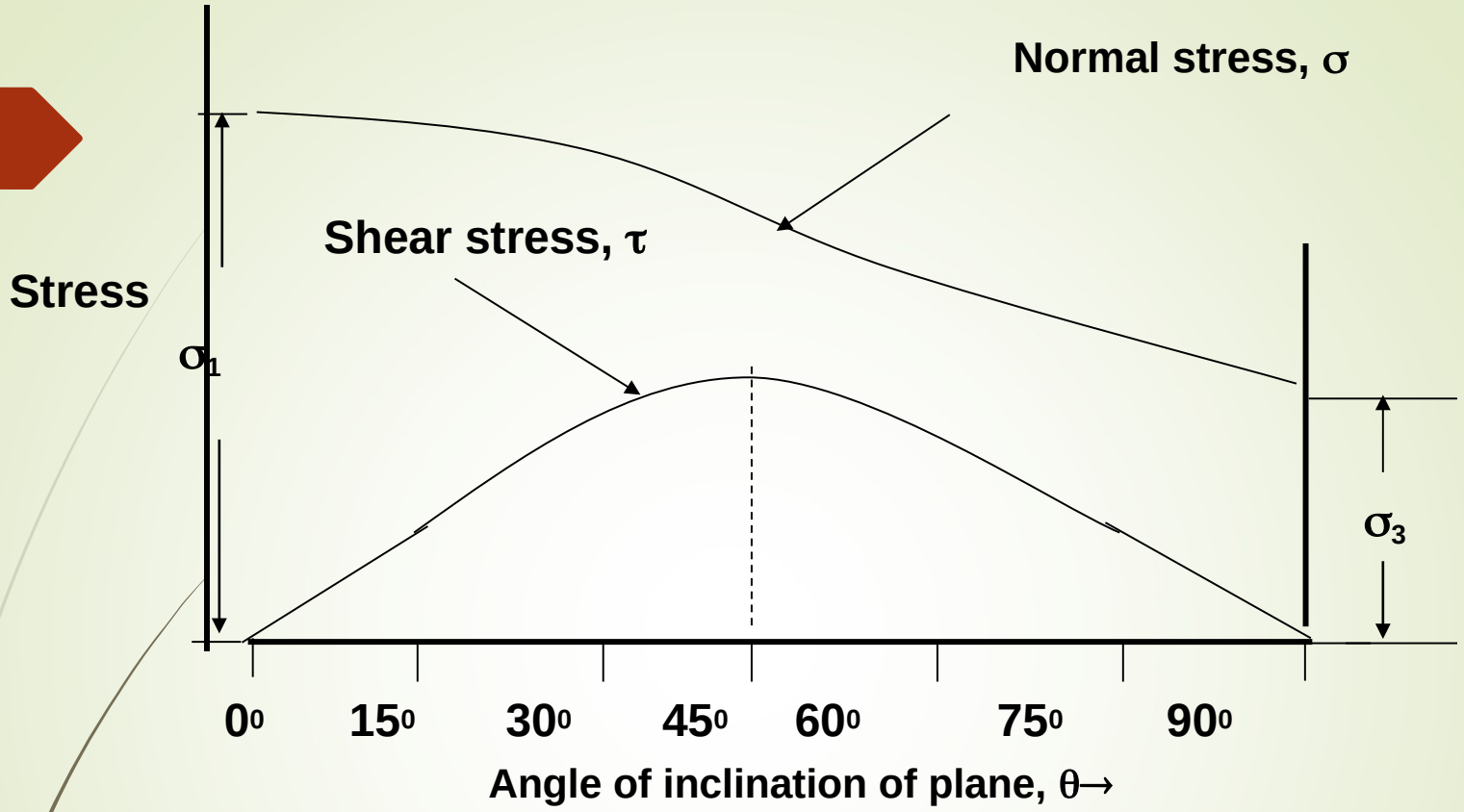
$$\sigma = \frac{\sigma_1 + \sigma_3}{2} + \left(\frac{\sigma_1 - \sigma_3}{2}\right)\cos 2\theta \quad \text{.....1}$$

- **Sum of all forces parallel to AE**

$\tau \sec\theta = \sigma_1 \sin\theta - \sigma_3 \tan\theta \cos\theta$

$\tau = \sigma_1 \sin\theta \cos\theta - \sigma_3 \sin\theta \cos\theta$

$$\tau = \left(\frac{\sigma_1 - \sigma_3}{2}\right)\sin 2\theta \quad \text{..... 2}$$



From the above curves it is clear that :

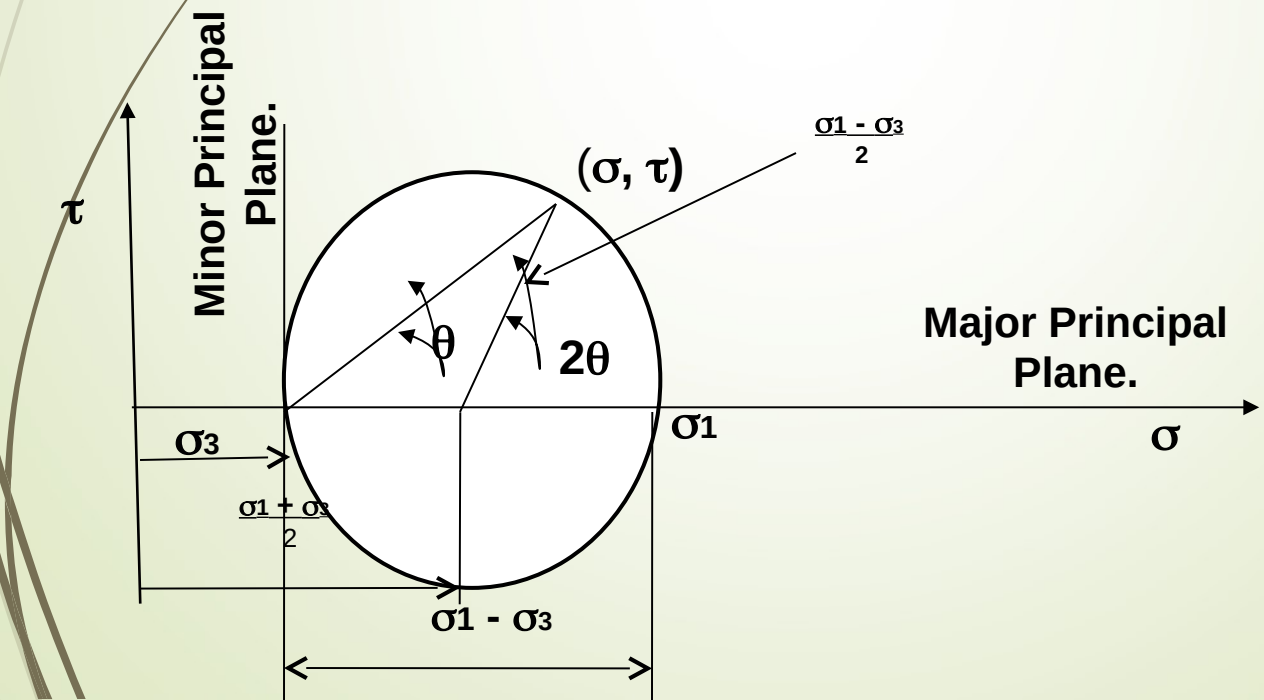
1. **The greatest and least principal stresses are respectively the max. and min. normal stresses on any plane through the point in question.**
2. **The max. shear stress occurs on plane 45^o to the principal planes**

Mohr Circle

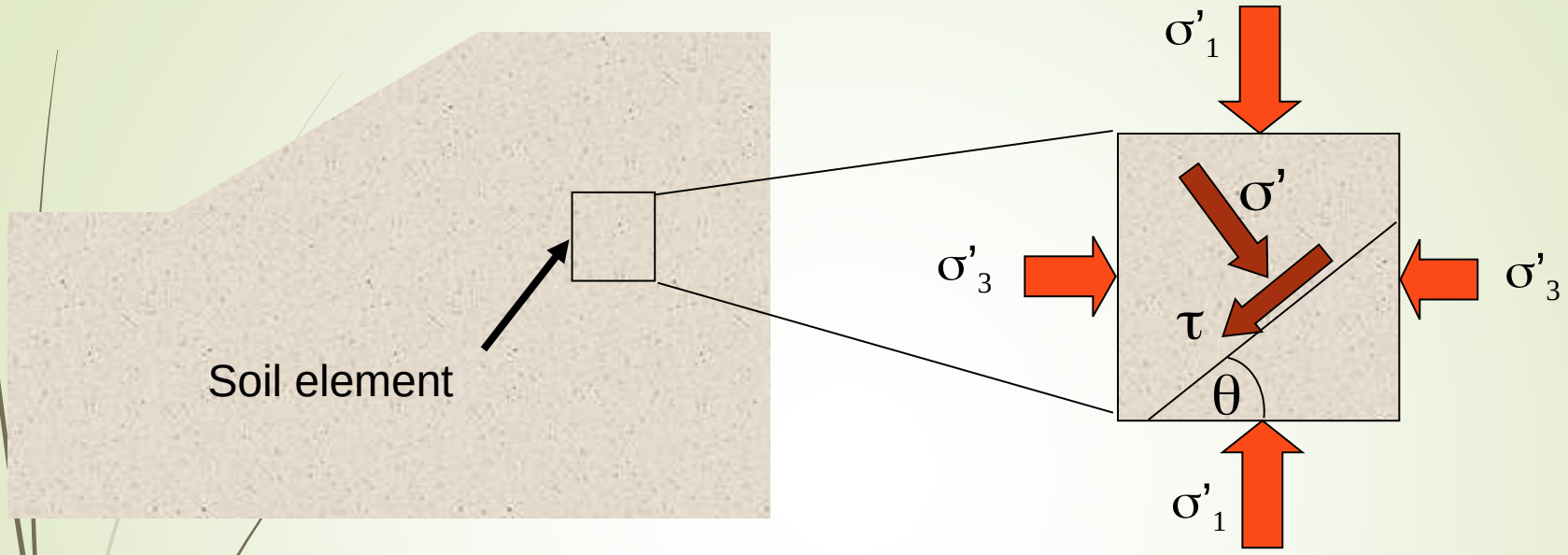
- **Squaring eqn 1 and 2 and adding the result gives**

$$\tau^2 + \left[\sigma - \frac{\sigma_1 + \sigma_3}{2}\right]^2 = \left(\frac{\sigma_1 - \sigma_3}{2}\right)^2$$

This is the equation of a circle whose center has the co-ordinates $\left[\frac{\sigma_1 + \sigma_3}{2}, 0\right]$ **and whose radius is** $\left(\frac{\sigma_1 - \sigma_3}{2}\right)$



Mohr Circle of stress



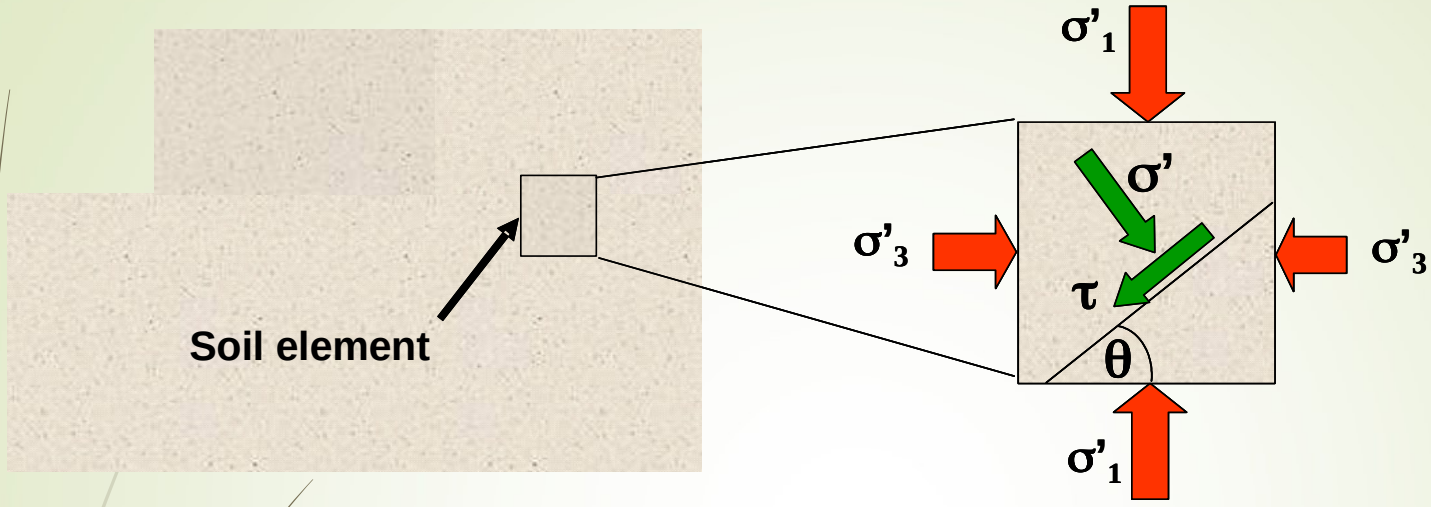
Resolving forces in σ and τ directions,

$$\tau = \frac{\sigma_1' - \sigma_3'}{2} Sin2\theta$$

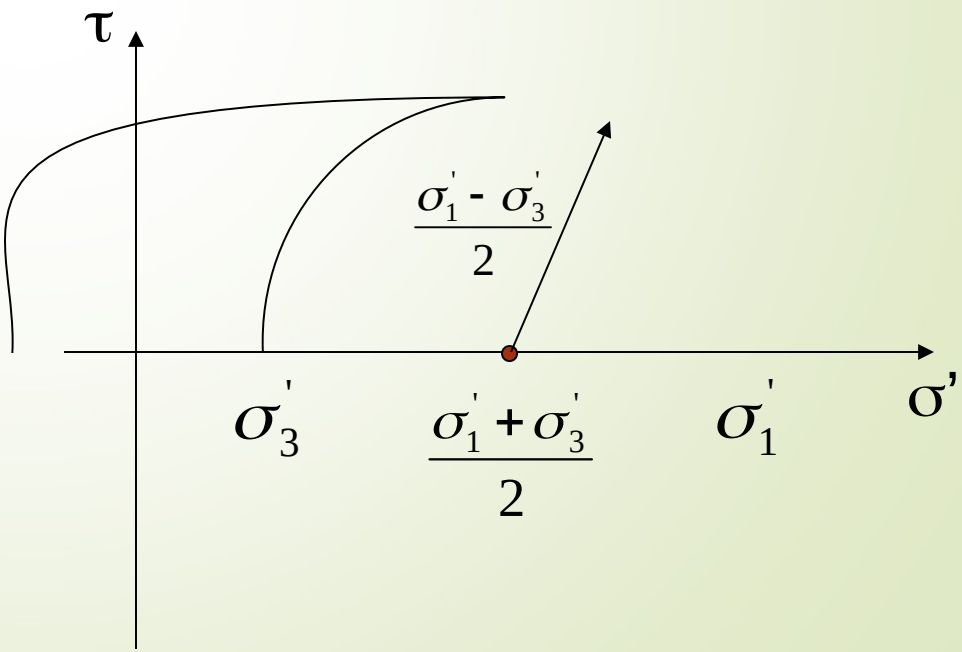
$$\sigma' = \frac{\sigma_1' + \sigma_3'}{2} + \frac{\sigma_1' - \sigma_3'}{2} Cos2\theta$$

$$\tau^2 + \left(\sigma' - \frac{\sigma_1' + \sigma_3'}{2}\right)^2 = \left(\frac{\sigma_1' - \sigma_3'}{2}\right)^2$$

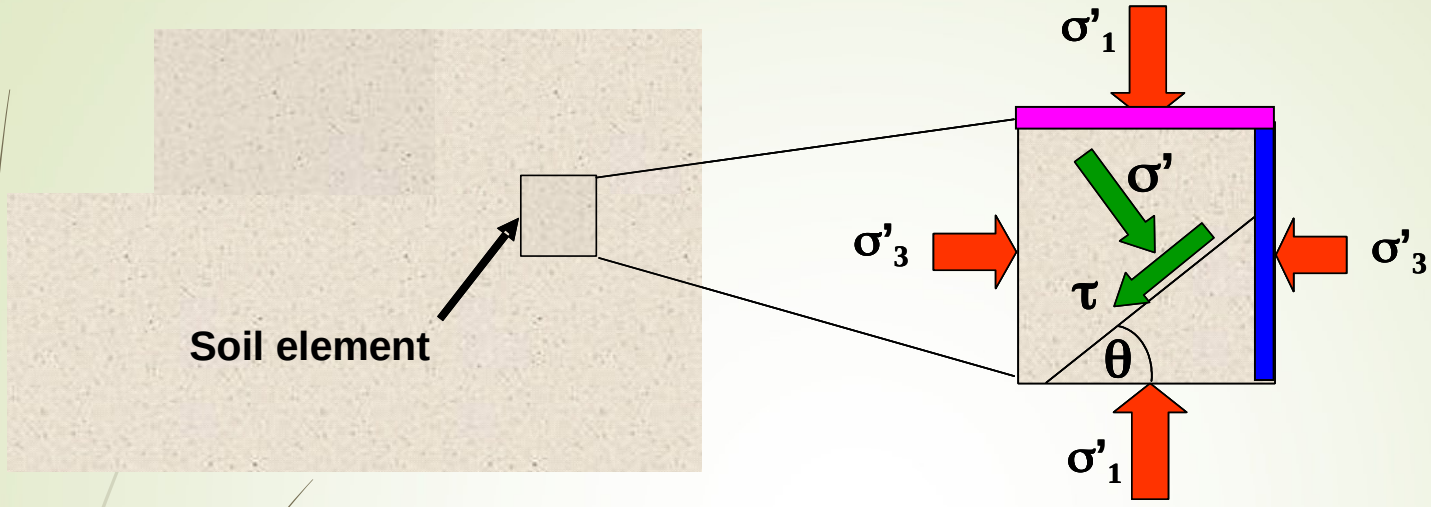
Mohr Circle of stress



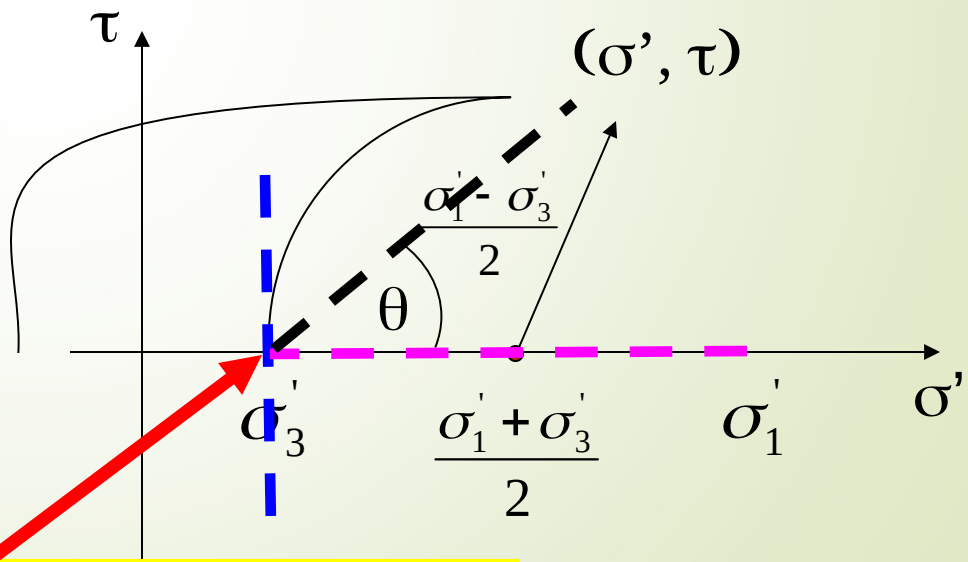
$$\tau^2 + \left(\sigma' - \frac{\sigma_1' + \sigma_3'}{2} \right)^2 = \left(\frac{\sigma_1' - \sigma_3'}{2} \right)^2$$



Mohr Circle of stress



$$\tau^2 + \left(\sigma' - \frac{\sigma_1' + \sigma_3'}{2}\right)^2 = \left(\frac{\sigma_1' - \sigma_3'}{2}\right)^2$$



P_D = Pole w.r.t. plane

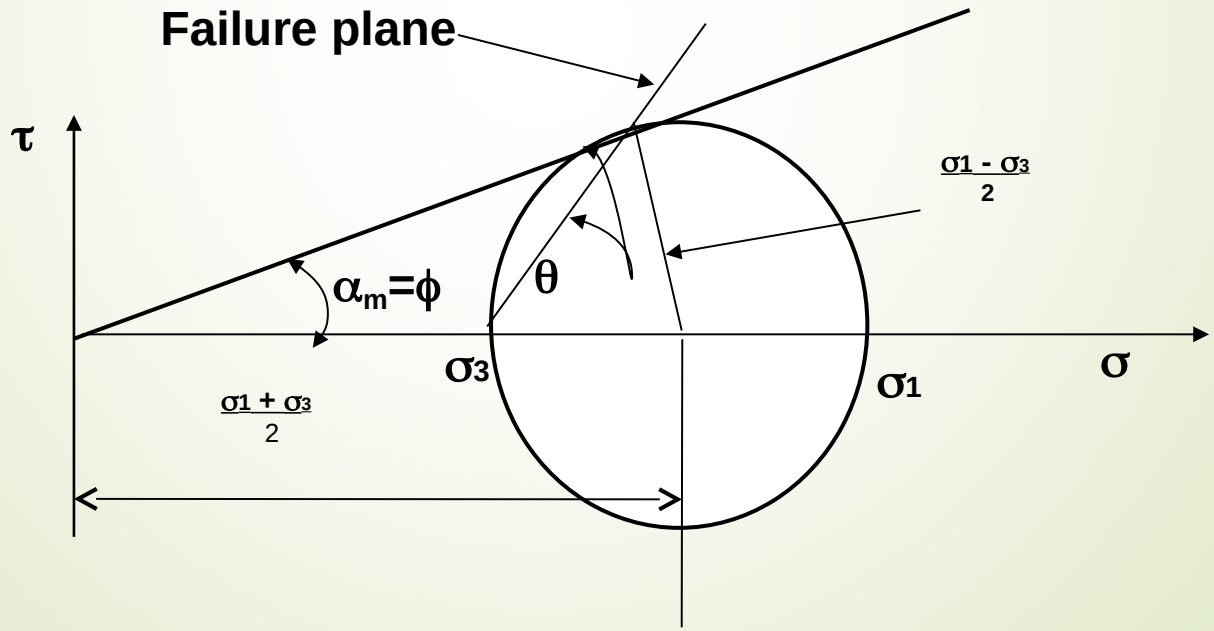
THE FOLLOWING FACTS ARE CLEAR FROM MOHR CIRCLE

1. **The maximum shearing stress, also called principal shearing stress is equal to ½ (σ_1-σ_3), which is also equal to the radius of the Mohr circle. The maximum shearing stress occurs on planes inclined at 45^0 either to the major or the minor principal plane.**

2. **Shearing stresses on planes inclined at 90^0 to each other are numerically equal but opposite in sign. These are also called conjugate shearing Stresses.**

3. **The magnitude of resultant stress on any plane is equal to $\sqrt{(\sigma^2 + \tau^2)}$ and its obliquity equal $\tan^{-1}(\tau/\sigma)$**

4. **The maximum value of obliquity angle is α_m. This can be determined by drawing a tangent to the circle through the origin ($\alpha_m=\phi$, if failure is developed).**



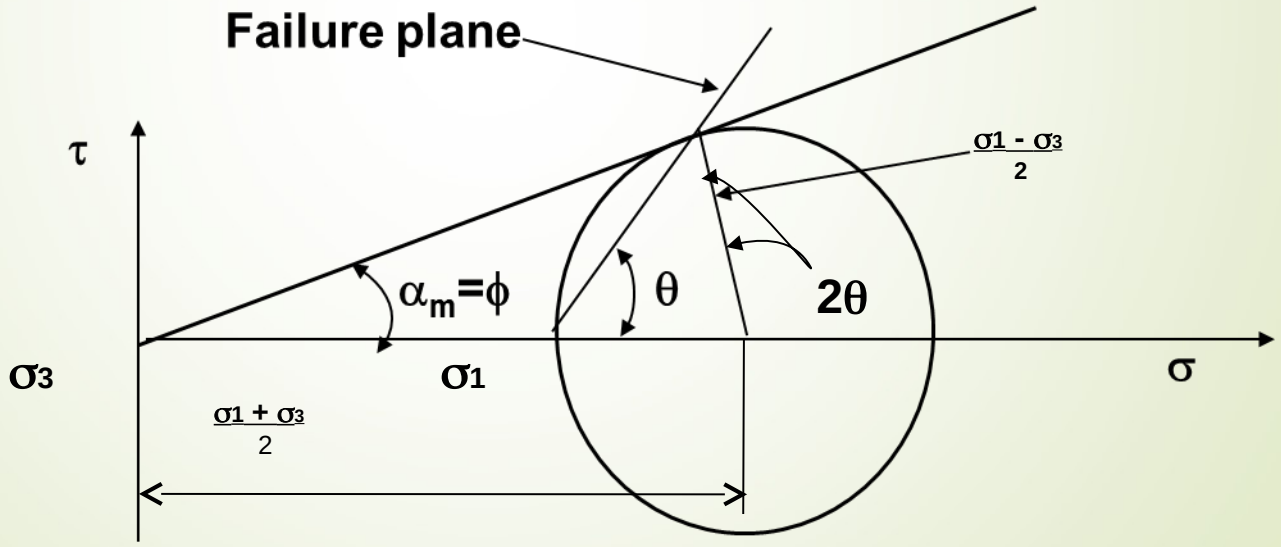
5. **A relationship between principal stresses on a plane of maximum obliquity can be derived as follows**

$$\sin\phi = \frac{\sigma_1 - \sigma_3}{\sigma_1 + \sigma_3} = \frac{\frac{\sigma_1}{\sigma_3} - 1}{\frac{\sigma_1}{\sigma_3} + 1}$$

$$\left(\frac{\sigma_1}{\sigma_3} + 1\right)\sin\phi = \frac{\sigma_1}{\sigma_3} - 1 \Rightarrow \frac{\sigma_1}{\sigma_3}(1 - \sin\phi) = 1 + \sin\phi$$

$$\Rightarrow \frac{\sigma_1}{\sigma_3} = \frac{1 + \sin\phi}{1 - \sin\phi}$$

6. **The angle at which the critical plane or the failure plane makes with the major principal plane (θ) can be expressed in terms of maximum obliquity (ϕ) as follows:**



$$\theta = 45 + \phi/2$$

- **The normal stress on a critical plan can be worked out in terms of principal stresses as follows:**
- **We know that**

$$\sigma = \frac{\sigma_1 + \sigma_3}{2} + \frac{\sigma_1 - \sigma_3}{2}\cos 2\theta$$

Substitute $\theta = \left(45 + \frac{\phi}{2}\right)$

$$\sigma = \frac{\sigma_1 + \sigma_3}{2} + \frac{\sigma_1 - \sigma_3}{2}\cos 2(45 + \phi/2)$$

$$= \frac{\sigma_1 + \sigma_3}{2} + \frac{\sigma_1 - \sigma_3}{2}\cos(90 + \phi)$$

$$= \frac{\sigma_1 + \sigma_3}{2} - \left(\frac{\sigma_1 - \sigma_3}{2}\right)\sin\phi$$

$$\sigma = \frac{\sigma_1}{2}(1 - \sin\phi) + \frac{\sigma_3}{2}(1 + \sin\phi)$$

but $\sigma_1 = \sigma_3\left(\frac{1 + \sin\phi}{1 - \sin\phi}\right)$

$$\sigma = \frac{\sigma_3}{2}(1 + \sin\phi) + \frac{\sigma_3}{2}(1 + \sin\phi)$$

$= \sigma_3 (1 + \sin \phi)$

Similarly

$$\sigma_3 = \sigma_1 \left(\frac{1 - \sin\phi}{1 + \sin\phi} \right)$$

$$\sigma = \frac{\sigma_1}{2}(1 - \sin\phi) + \frac{\sigma_1}{2}(1 - \sin\phi)$$

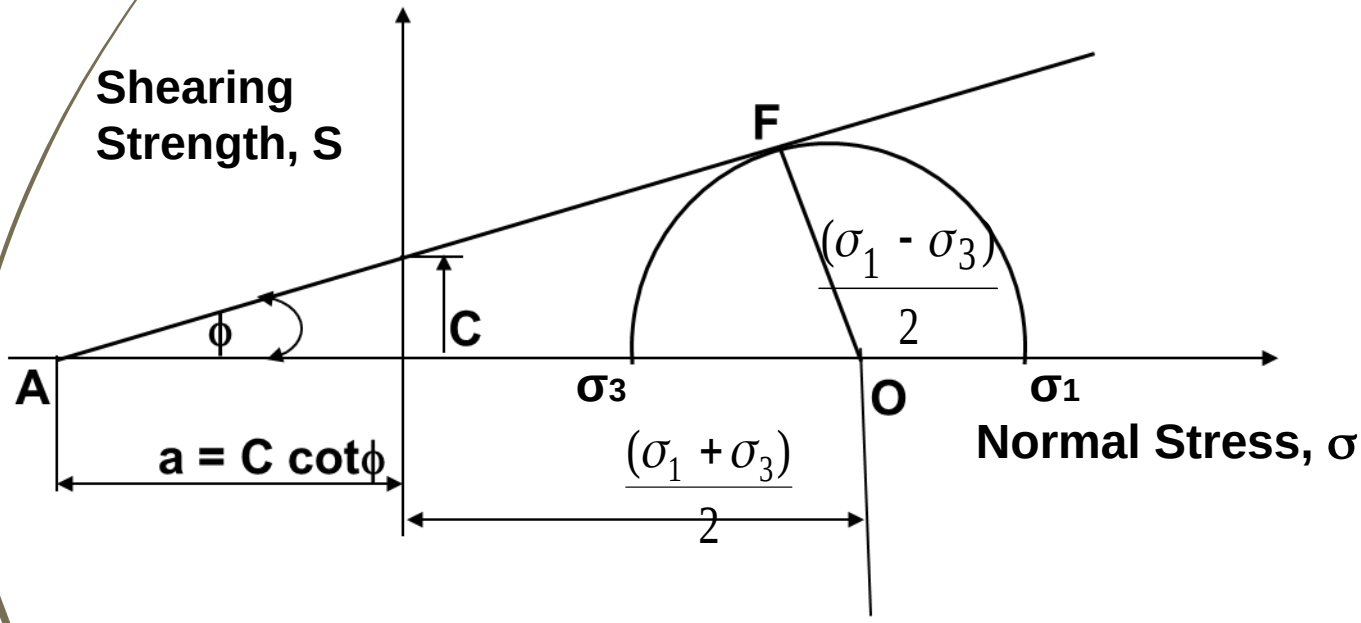
$$= \sigma_1 (1 - \sin \phi)$$

Therefore

$$\sigma = \sigma_1 (1-\sin\phi) = \sigma_3 (1+ \sin \phi)$$

Relationship Between the Principal Stresses and Cohesion (C)

- If the shearing resistance, S, of a soil depends on both friction and cohesion, sliding failure occurs in accordance with the coulomb equation, that is when $S = C + \sigma \tan \phi$



From ΔAOF

$$\sin\phi = \frac{\frac{\sigma_1 - \sigma_3}{2}}{\frac{\sigma_1 + \sigma_3}{2} + a} = \frac{\sigma_1 - \sigma_3}{\sigma_1 + \sigma_3 + 2a} = \frac{(\sigma_1 + a) - (\sigma_3 + a)}{(\sigma_1 + a) + (\sigma_3 + a)}$$

sinϕ

$$= \frac{\frac{\sigma_1 + a}{\sigma_3 + a} - 1}{\frac{\sigma_1 + a}{\sigma_3 + a} + 1} \Rightarrow \frac{\sigma_1 + a}{\sigma_3 + a} = \frac{1 + \sin\phi}{1 - \sin\phi}$$

$$\Rightarrow \sigma_1 + a = \frac{1 + \sin\phi}{1 - \sin\phi}(\sigma_3 + a) \Rightarrow \sigma_1 = \frac{1 + \sin\phi}{1 - \sin\phi}(\sigma_3 + a) - a$$

σ_1

$$= \sigma_3\left(\frac{1 + \sin\phi}{1 - \sin\phi}\right) + a\left[\frac{1 + \sin\phi}{1 - \sin\phi} - 1\right] \Rightarrow \sigma_1 = \sigma_3\left(\frac{1 + \sin\phi}{1 - \sin\phi}\right) + a\left(\frac{2\sin\phi}{1 - \sin\phi}\right)$$

but

$$a = C\cot\phi = C\frac{\cos\phi}{\sin\phi} = C\frac{\sqrt{1 - \sin^2\phi}}{\sin\phi}$$

then

$$\sigma_1 = \sigma_3\left(\frac{1+\sin\phi}{1-\sin\phi}\right) + C\frac{\sqrt{1-\sin^2\phi}}{\sin\phi}\left(\frac{2\sin\phi}{1-\sin\phi}\right) = \sigma_3\left(\frac{1+\sin\phi}{1-\sin\phi}\right) + 2C\sqrt{\frac{1+\sin\phi}{1-\sin\phi}}$$

$$\sigma_1 = \sigma_3 \tan^2\left(45+\frac{\phi}{2}\right) + 2C\tan\left(45+\frac{\phi}{2}\right)$$

Similarly

$$\sigma_3 = \sigma_1\left(\frac{1-\sin\phi}{1+\sin\phi}\right) - 2C\sqrt{\frac{1-\sin\phi}{1+\sin\phi}}$$

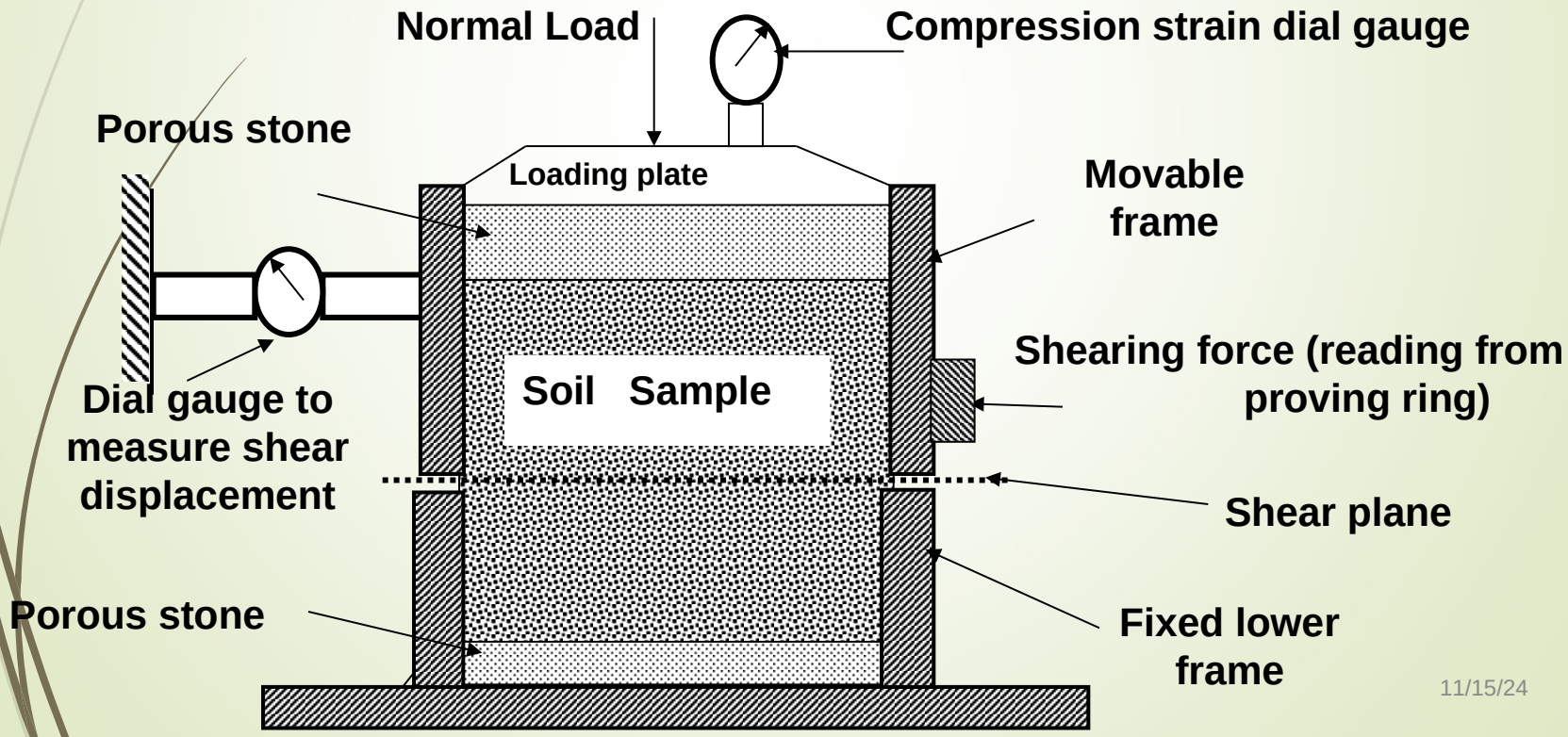
$$\sigma_3 = \sigma_1 \tan^2\left(45-\frac{\phi}{2}\right) - 2C\tan\left(45-\frac{\phi}{2}\right)$$

SHEAR TESTS

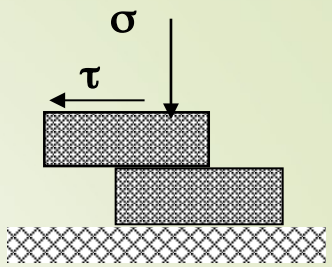
- The shear strength parameters C and ϕ of soils either in the undisturbed or remoulded states may be determined by any of the following methods
  - ***Direct shear test***
  - ***Triaxial shear test***
  - ***Unconfined compression test***

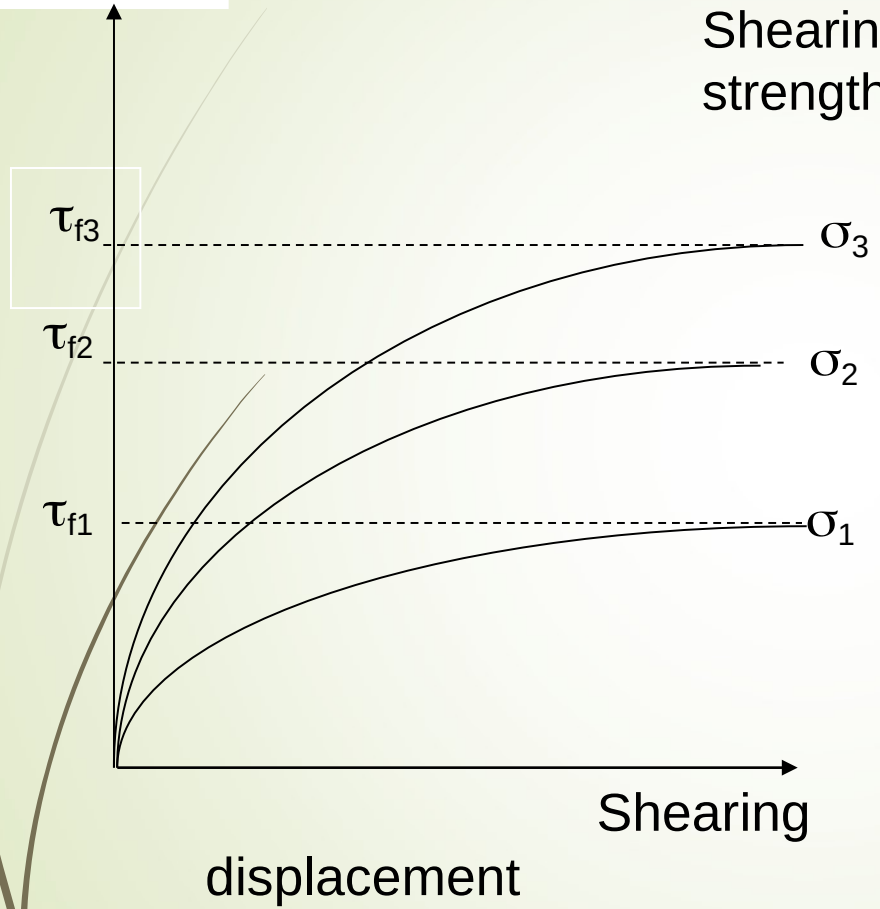
1. DIRECT SHEAR TEST

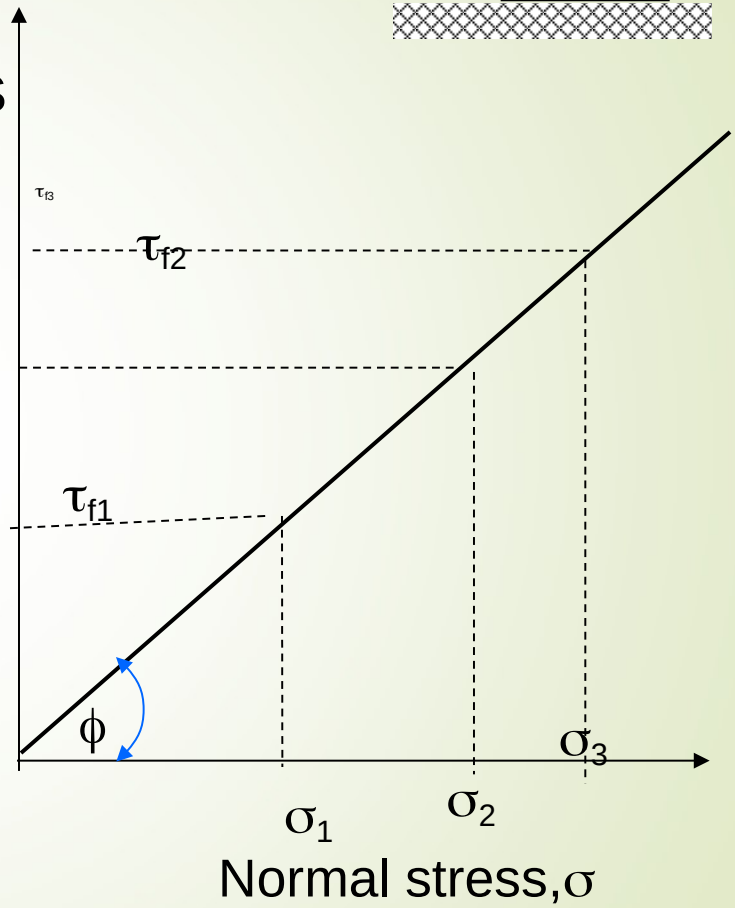
- The direct shear test is the **oldest and simplest shear test arrangement.**
- The test equipment consists of **a square brass box split horizontally** at the level of the center of the soil sample



Shear stress, τ

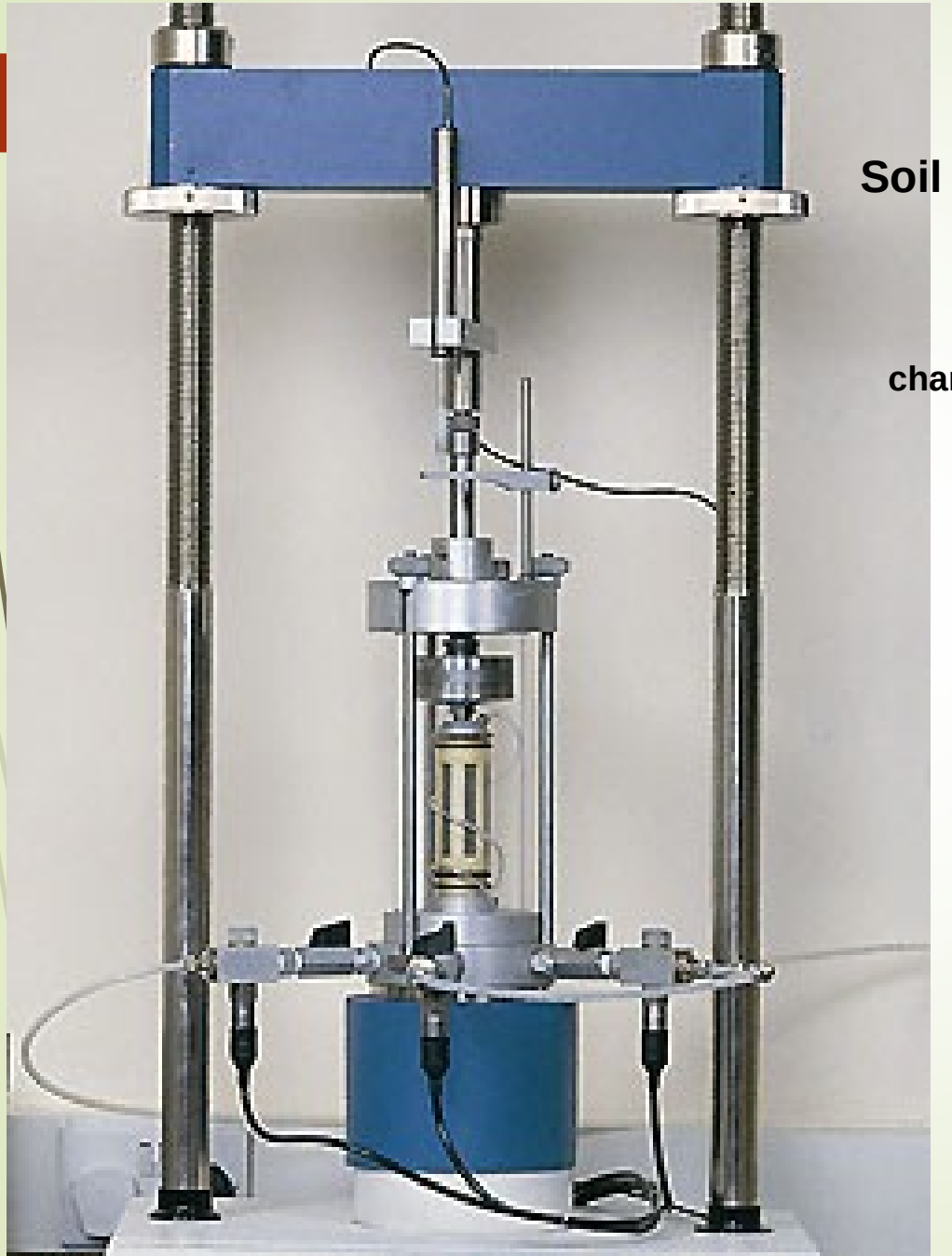


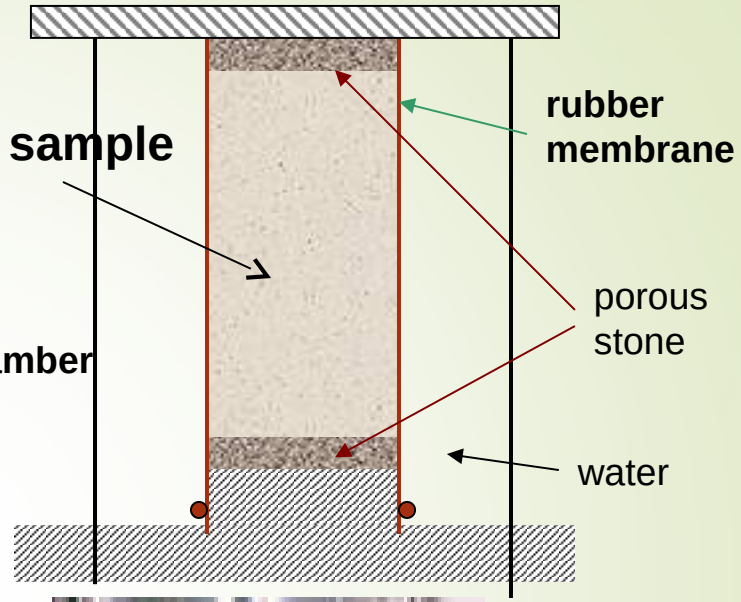




3. TRIAXIAL SHEAR TEST

- The triaxial shear test is the most reliable method now available for determination of shear strength parameters and is widely used for research and conventional testing.
- In this test, the sample to be tested is cylindrical in shape and is covered with rubber membrane.
- The sample is placed between two porous stones.
- Fluid pressure is applied within the chamber containing the sample gives uniform stress all over the surface of the sample.
- To cause shear failure in the sample, axial (or deviator) load is applied through a vertical loading ram.







soil sample at failure

- Deviator Stress = $\Delta\sigma_d$ =P/A

- In the triaxial shear test, **the applied confining pressure is equal to the minor principal stress (σ_3)**

- **The deviator stress plus the confining pressure is equal to the major principal stress (σ_1).**

i.e $\sigma_1 = \Delta\sigma + \sigma_3 = P/A + \sigma_3$

Corrected area

For drained test

$V_o = A_o L_o$,

$V = A L$

$\Delta V = V_o - V = A_o L_o - A L$

$$\frac{\Delta V}{V_o} = \frac{A_o L_o - A L}{A_o L_o}$$

$$\Rightarrow \quad \frac{\Delta V}{V_o} = 1 - \frac{A L}{A_o L_o}$$, but $L = L_o - \Delta L$

$$\Rightarrow$$

$$\Rightarrow \quad \frac{\Delta V}{V_o} = 1 - \frac{A (L_o - \Delta L)}{A_o L_o} = 1 - \frac{A}{A_o}\left(1 - \frac{\Delta L}{L_o}\right)$$

$$\Rightarrow$$

$$1 - \frac{\Delta V}{V_o} = \frac{A}{A_o}\left(1 - \frac{\Delta L}{L_o}\right)$$

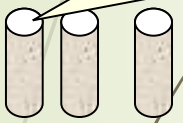
For Undrained test

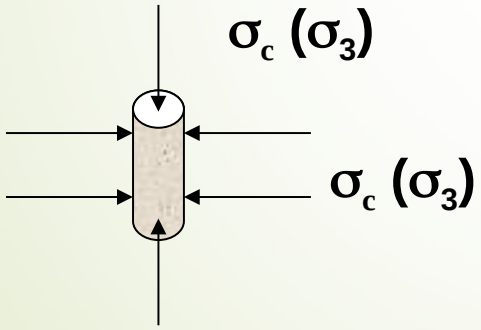
$$A = \frac{A_o}{1 - \frac{\Delta L}{L_o}}$$

$$A = A_o \left[\frac{1 - \frac{\Delta V}{V_o}}{1 - \frac{\Delta L}{L_o}}\right]$$

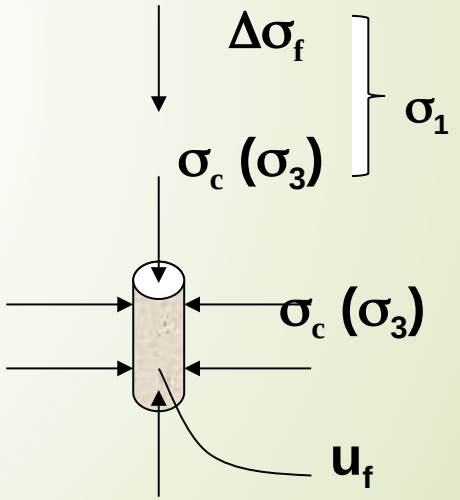
Triaxial Compression Test

Identical specimens initially subjected to different isotropic stresses (σ_3) and then loaded axially to failure



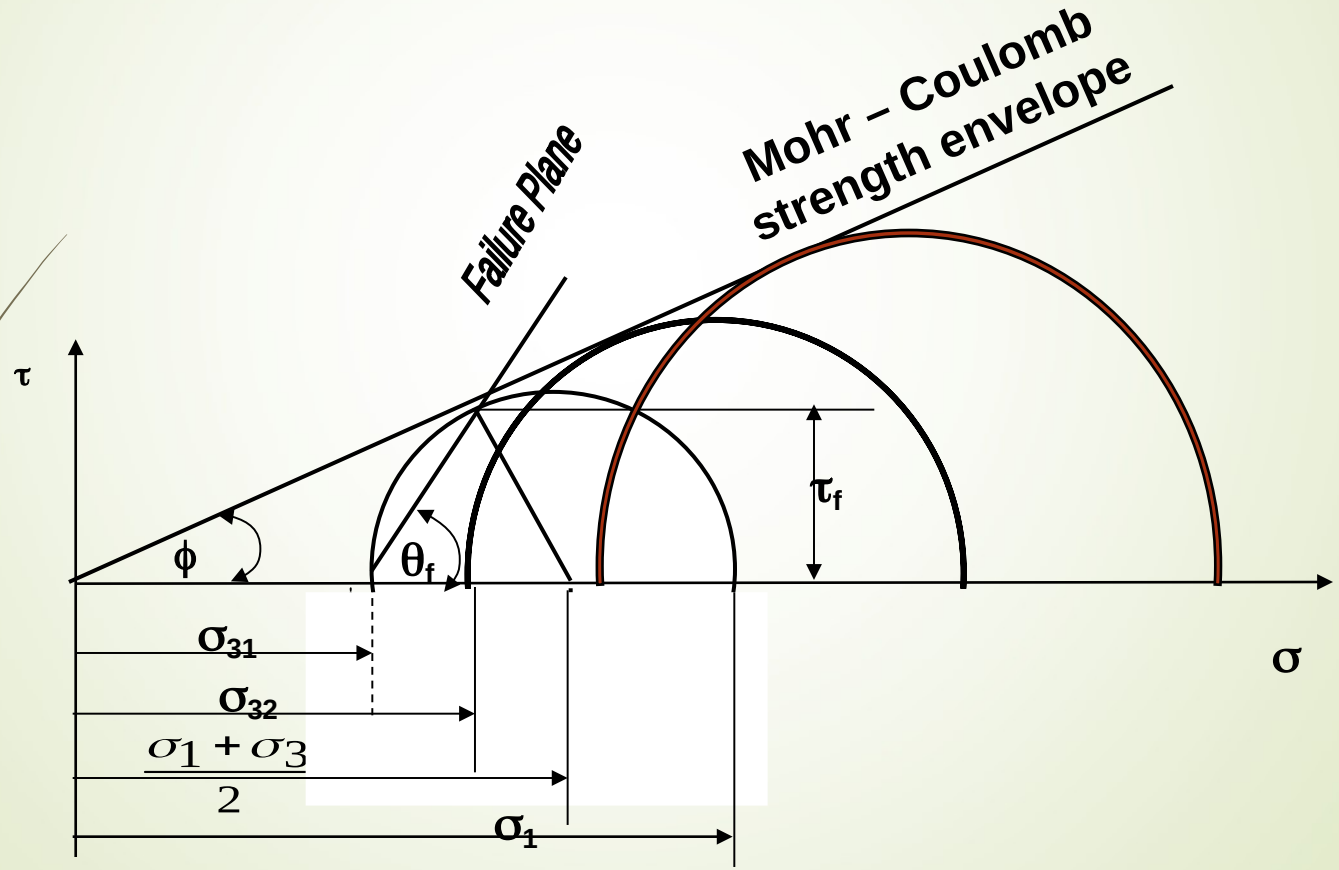


Initially…

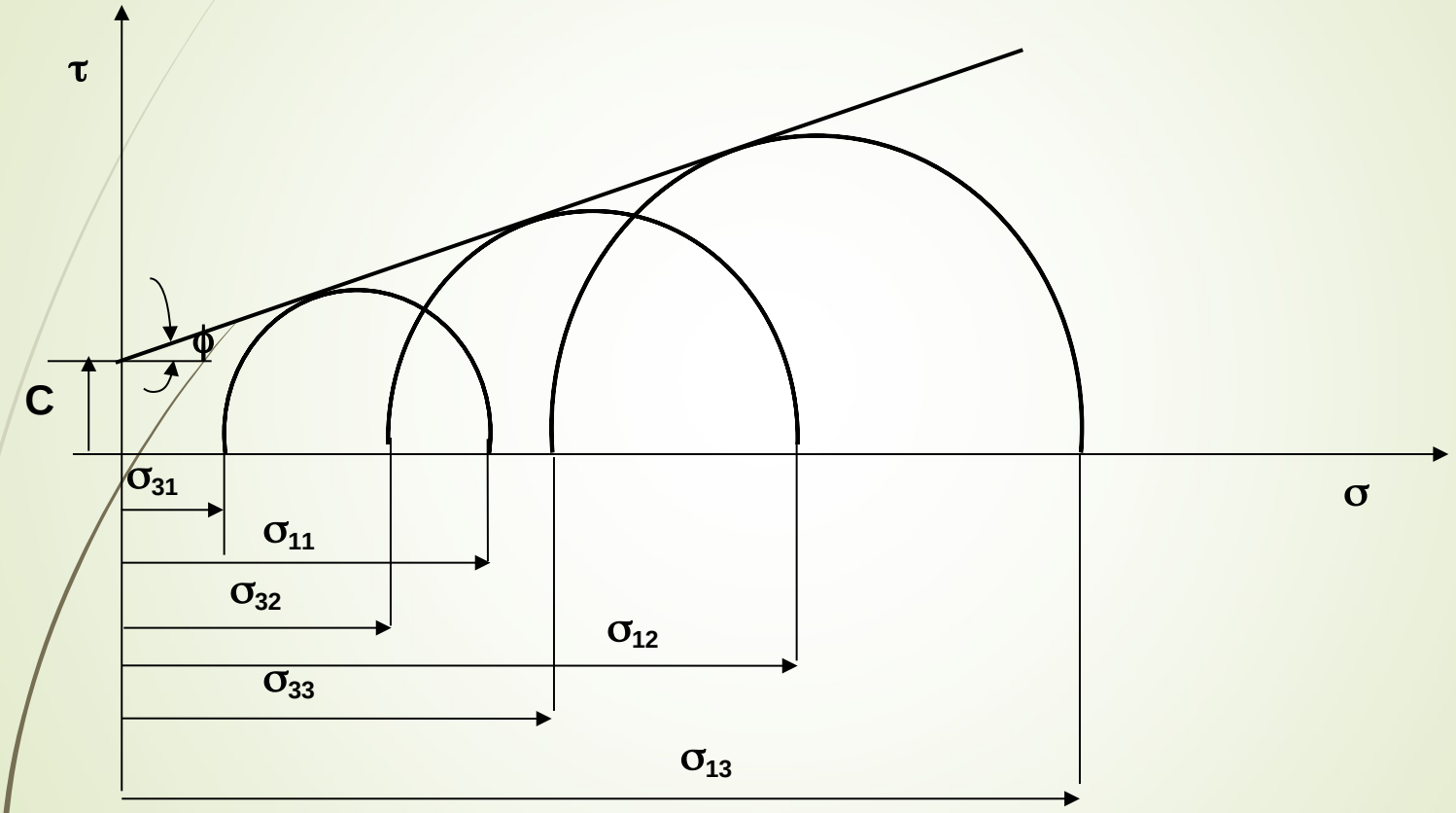


Failure

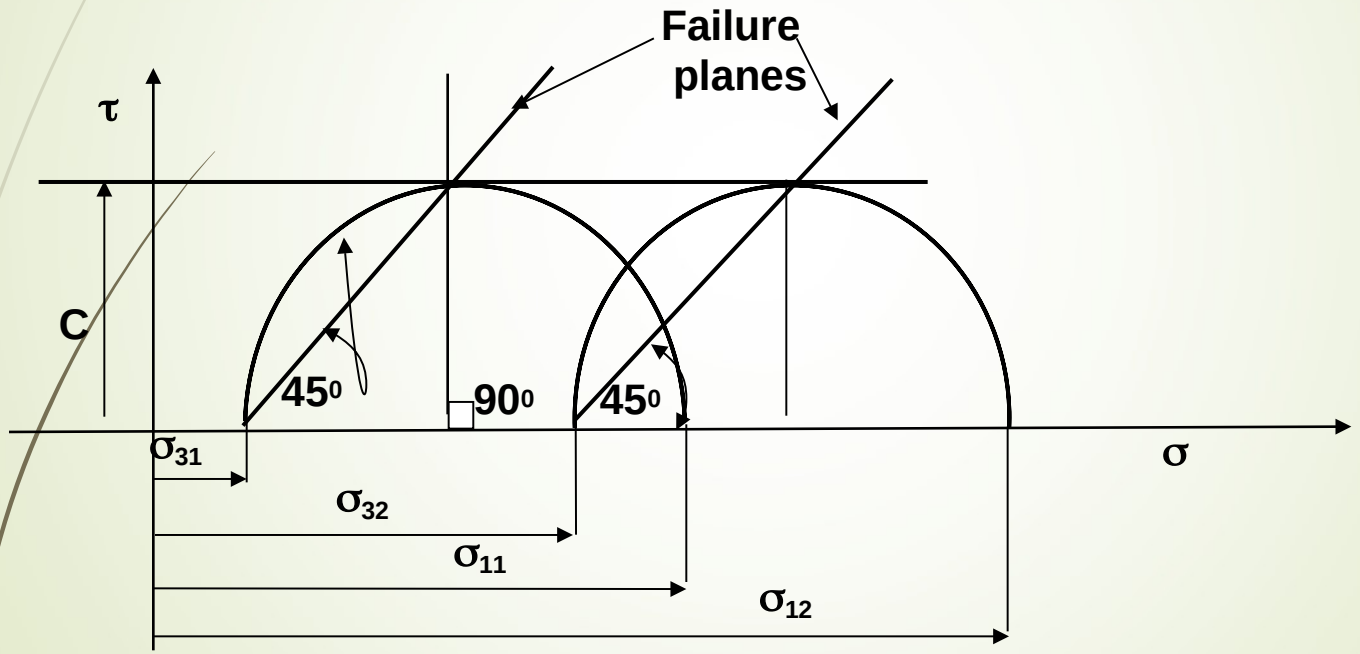
- **Mohr diagram for triaxial compression test for cohesionless soil.**



- **Mohr diagram for triaxial compression test for C-ϕ soil.**

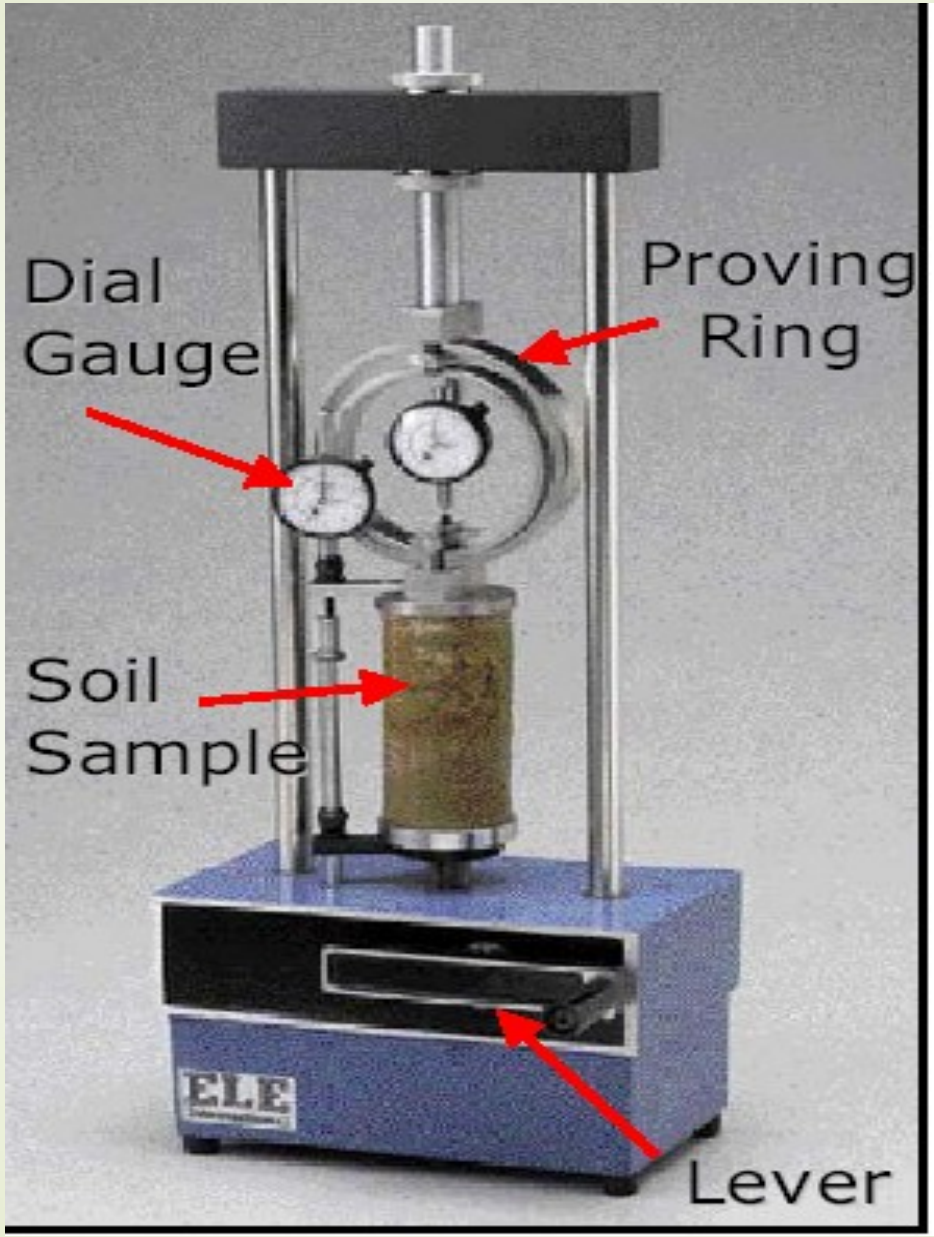


•**Mohr diagram for triaxial compression test for purely cohesive soil.**



3. UNCONFINED COMPRESSION

- **The unconfined compression test is a special case of a triaxial compression test in which the confining pressure $\sigma_3 = 0$.**
- **In this simple test, a cylindrical cohesive specimen without any lateral support is subjected to axial loading, till the sample fails either due to shear along a diagonal plane or by the lateral bulging.**

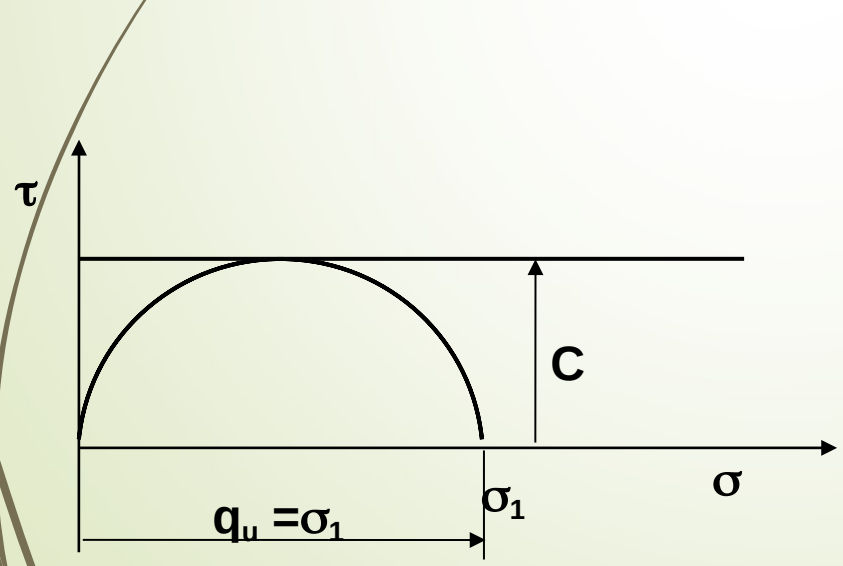

Dial
Gauge
Proving
Ring
Soil
Sample
ELE
Lever

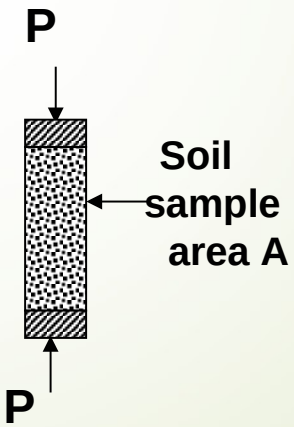
$$\sigma_1 = \frac{P_f}{A_f} = q_u = 2C \Rightarrow C = \frac{q_u}{2}$$

Where: P_f = axial load at failure

and

$$A_f = \frac{A_O}{1 - \frac{\Delta L}{L_O}}$$





DRAINAGE DURING SHEAR

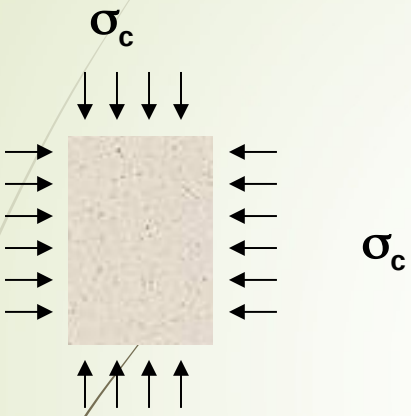
- **The shear strength of saturated soils is influenced by drainage conditions before and during shear.**
- **Hence, shear tests have been devised to measure shear strength of soils under three difference drainage conditions.**

A. **UU~Unconsolidated- Undrained Test or Quick Test**

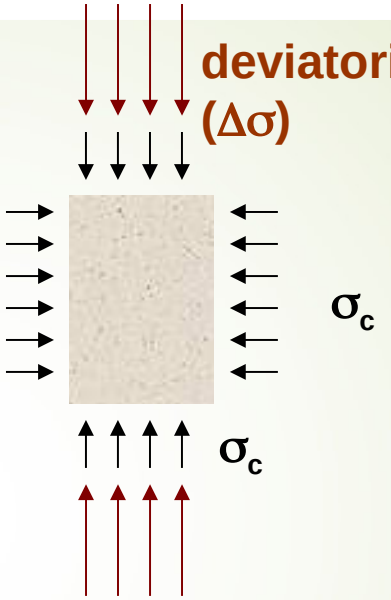
B. **CU~Consolidated – Consolidated-Undrained Test**

C. **CD~Consolidated-Drained (CD) or Slow Test**

TYPES OF SHEAR TESTS



Under all-around cell pressure σ_c



Shearing (loading)

Is the drainage valve open?

yes — no

Consolidated sample

Unconsolidated sample

Is the drainage valve open?

yes — no

Drained loading

Undrained loading

Unconsolidated Undrained (UU) Test

- pore pressure develops during shear

Not measured
$\therefore \sigma'$ unknown

= **0**; i.e., failure envelope is horizontal

- analyse in terms of σ ➔ gives c_u and ϕ_u
- very quick test

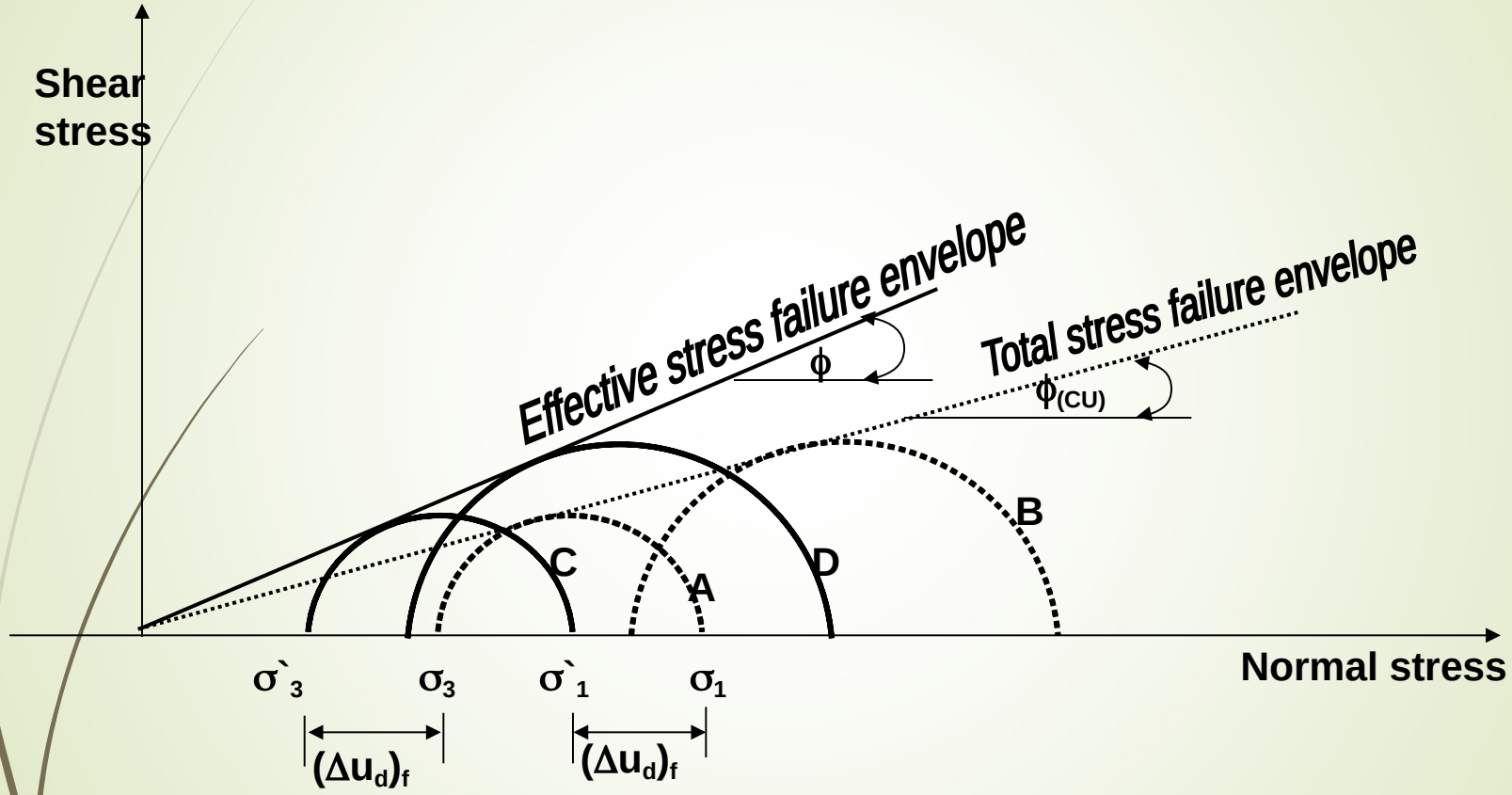
Use c_u and ϕ_u for analysing undrained situations (e.g., short term stability, quick loading)

Consolidated Undrained (CU) Test

- pore pressure develops during shear

Measure → σ'

- gives c' and ϕ'
- faster than CD (∴preferred way to find c' and ϕ')


Shear stress
Effective stress failure envelope
Total stress failure envelope
ϕ
ϕ(CU)
B
C
A
D
σ`3
σ3
σ`1
σ1
(Δud)f
(Δud)f
Normal stress

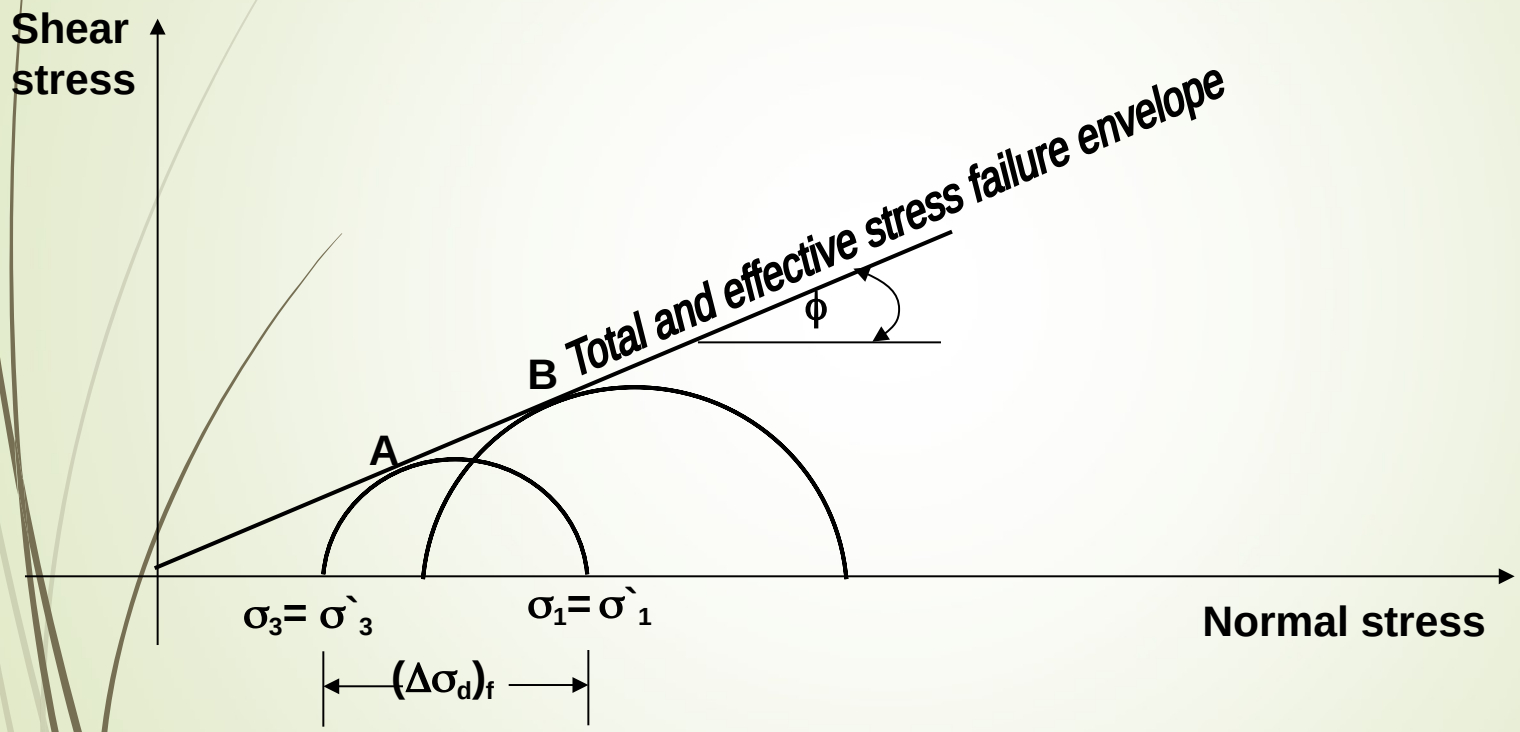
Consolidated Drained (CD) Test

- no excess pore pressure throughout the test
- very slow shearing to avoid build-up of pore pressure

Can be days!
∴ not desirable

- gives c' and ϕ'

Use c' and ϕ' for analysing fully drained situations (e.g., long term stability, very slow loading)


Shear stress
B Total and effective stress failure envelope
ϕ
A
σ3= σ`3
σ1= σ`1
Normal stress
(Δσd)f