

CHAPTER 6

CAMERA MODELING

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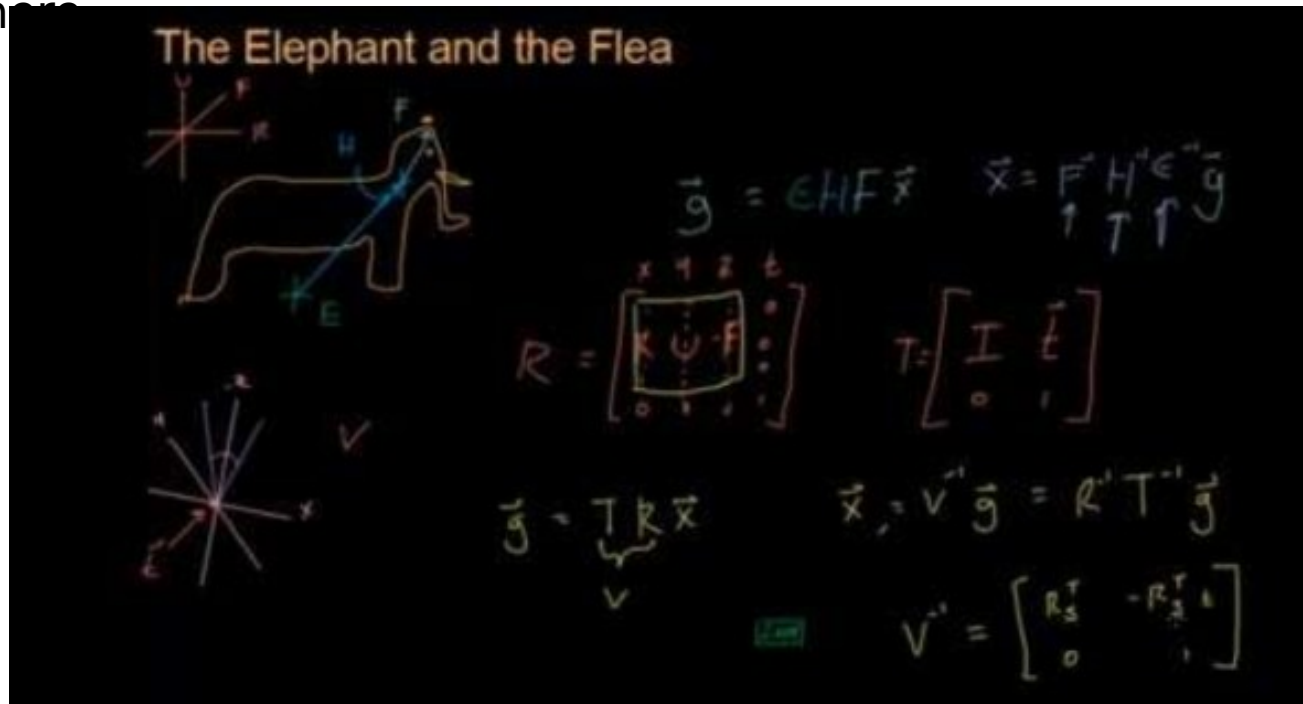
3D modeling

- Models may be created automatically or manually. The manual modeling process of preparing geometric data for 3D computer graphics is similar to plastic arts such as sculpting.
- In case of 3D camera modelling the camera view transform matrix is necessary for making games using automatic modelling process.



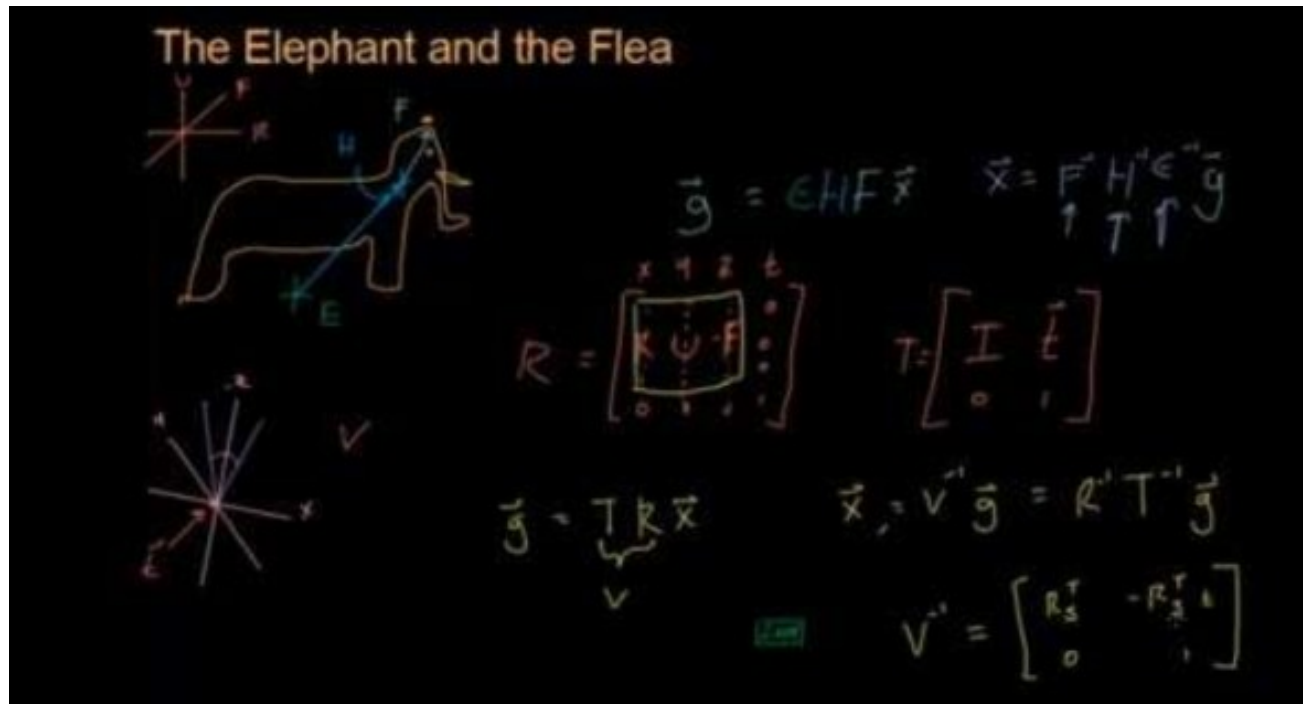
The camera view transform matrix

- When making a game, an object can be positioned anywhere in the world scene. Consider an example of an elephant and the flea shown in the diagram.
- Elephant can face any direction. A flea on head can be in any position.
- The position and the orientation of the head relative to the elephant body and position and the orientation the flea relative to the head is known here

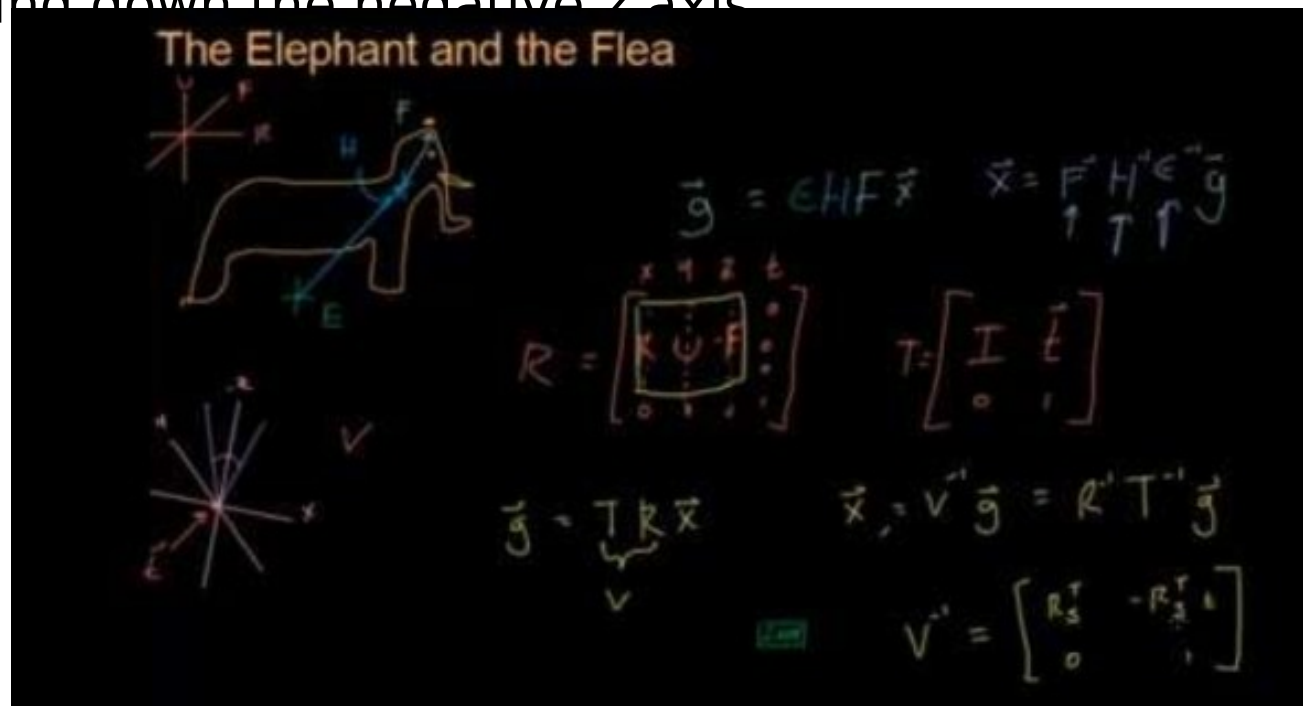


Camera view transform matrix Illustration

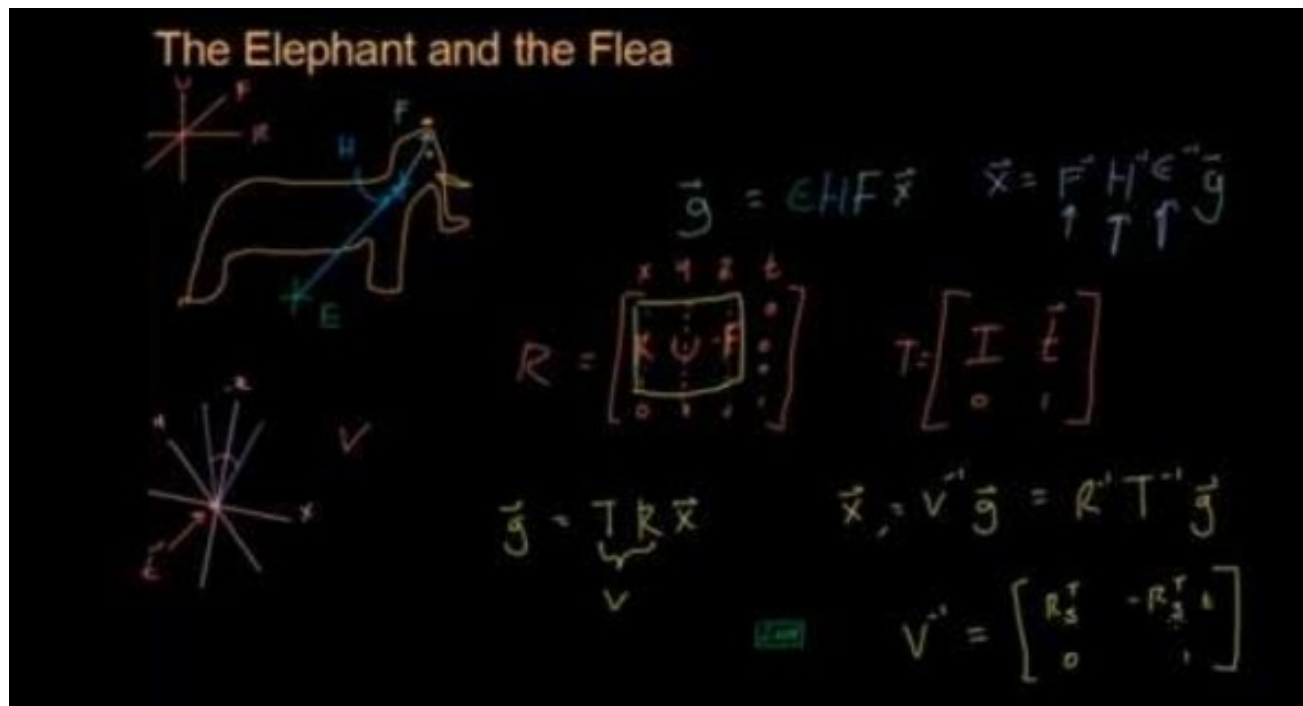
- The question is what is the position of the flea relative to the entire world?
- The local coordinates of the elephant are known. A matrix that transforms the elephant to its coordinates called E. Coordinates of the head in comparison to the elephant is known. So there is a vector call H from E to the head that gives the position of the head relative to the body. There is one last matrix F that is the position of the flea on the head.



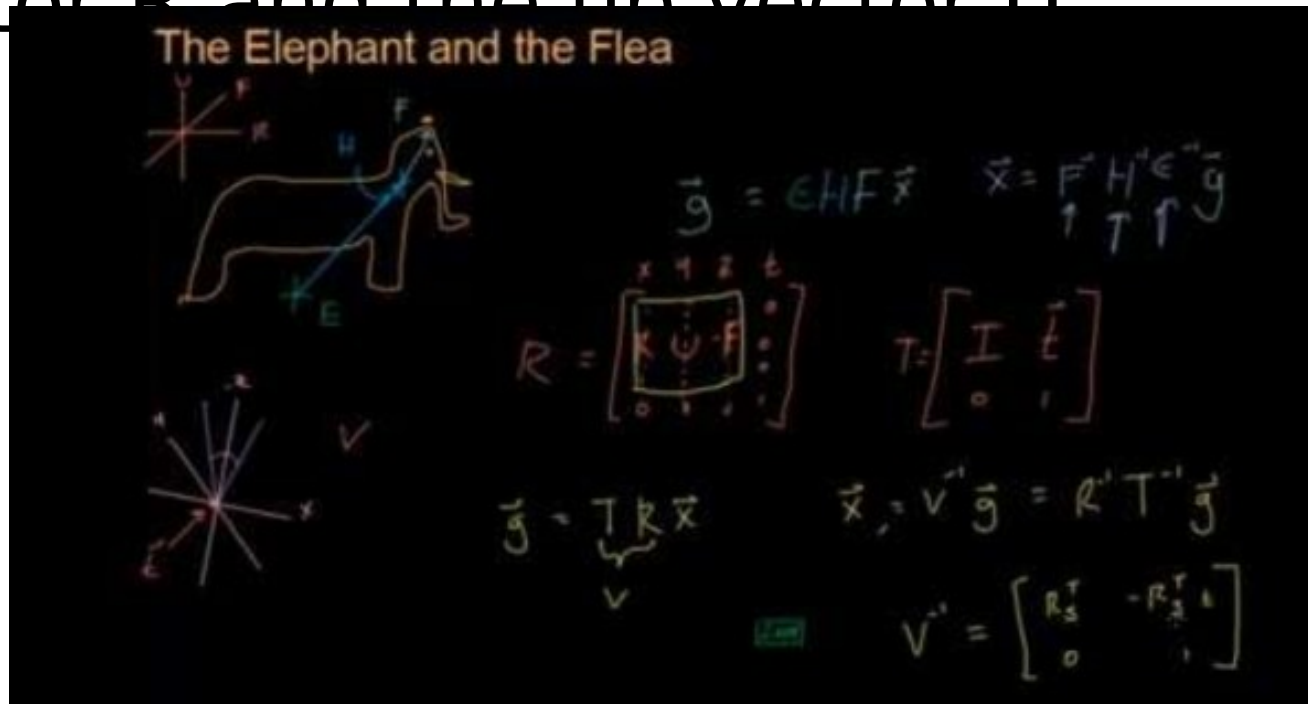
- The vector X is in the local space of the flea in order to obtain the global vector that corresponds to the local vector. When there is global vector other matrix F, H, E are inversed.
- In order to send the scene to the video card the entire scene has to be transformed so that the camera is at the origin. It is looking down the negative z axis.



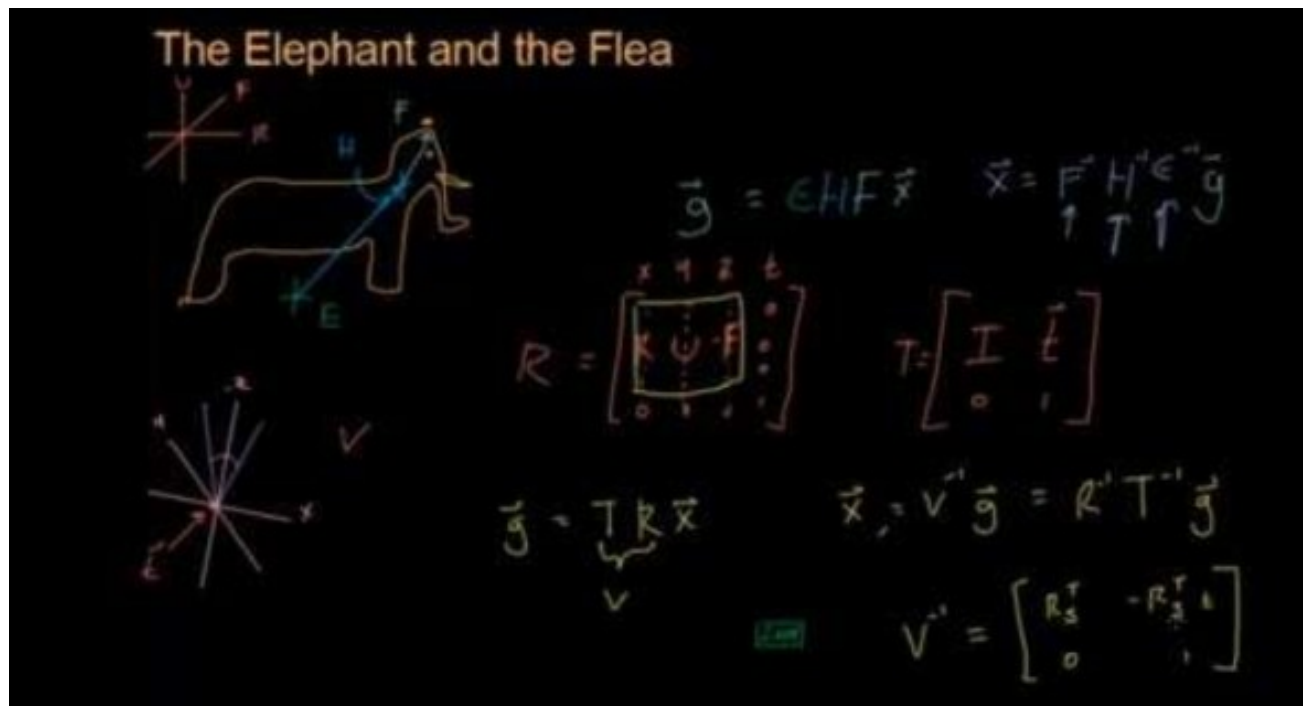
- In pink graph, the camera looking at the scene similar to eye. It wants to be the local coordinates.
- The global vectors are put to the local space of the camera such that looking at the negative z axis. Assemble a view matrix V that transform all of these vectors into the local space of the camera.



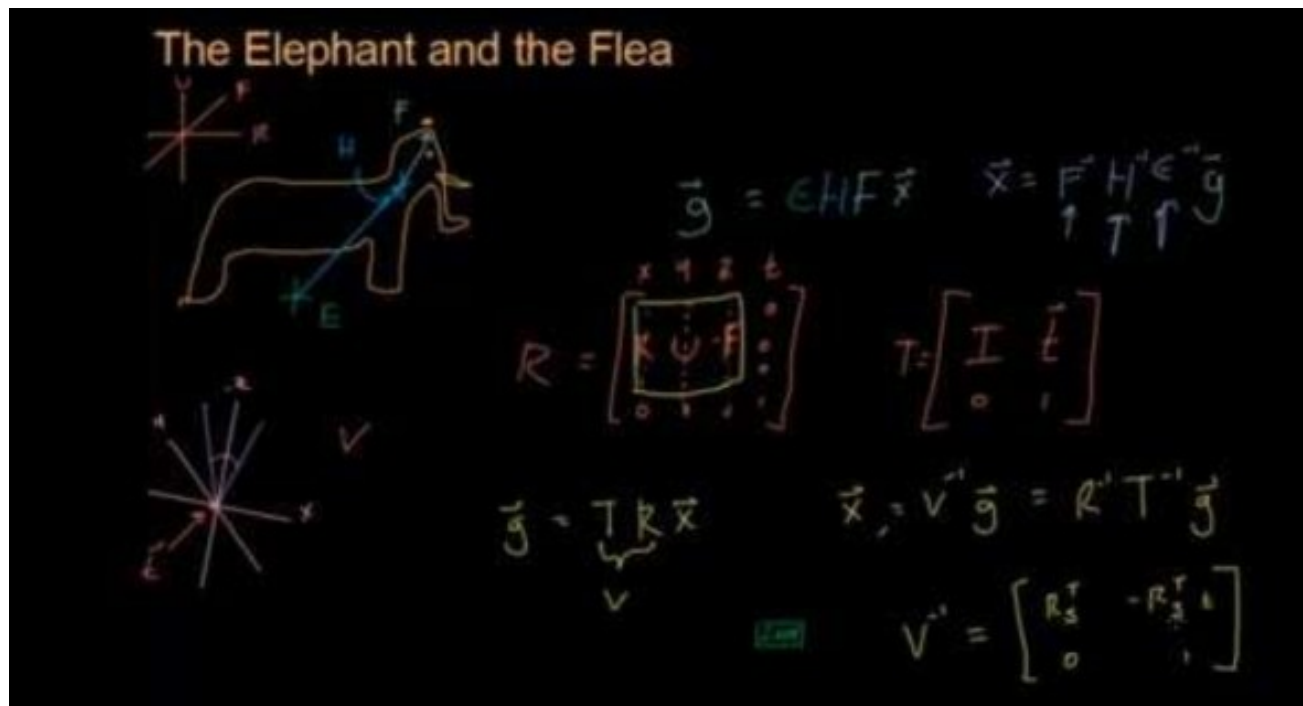
- In orange graph, the global space vectors can be drawn for a reference. The F is the forward vector, the right vector R and the up vector U



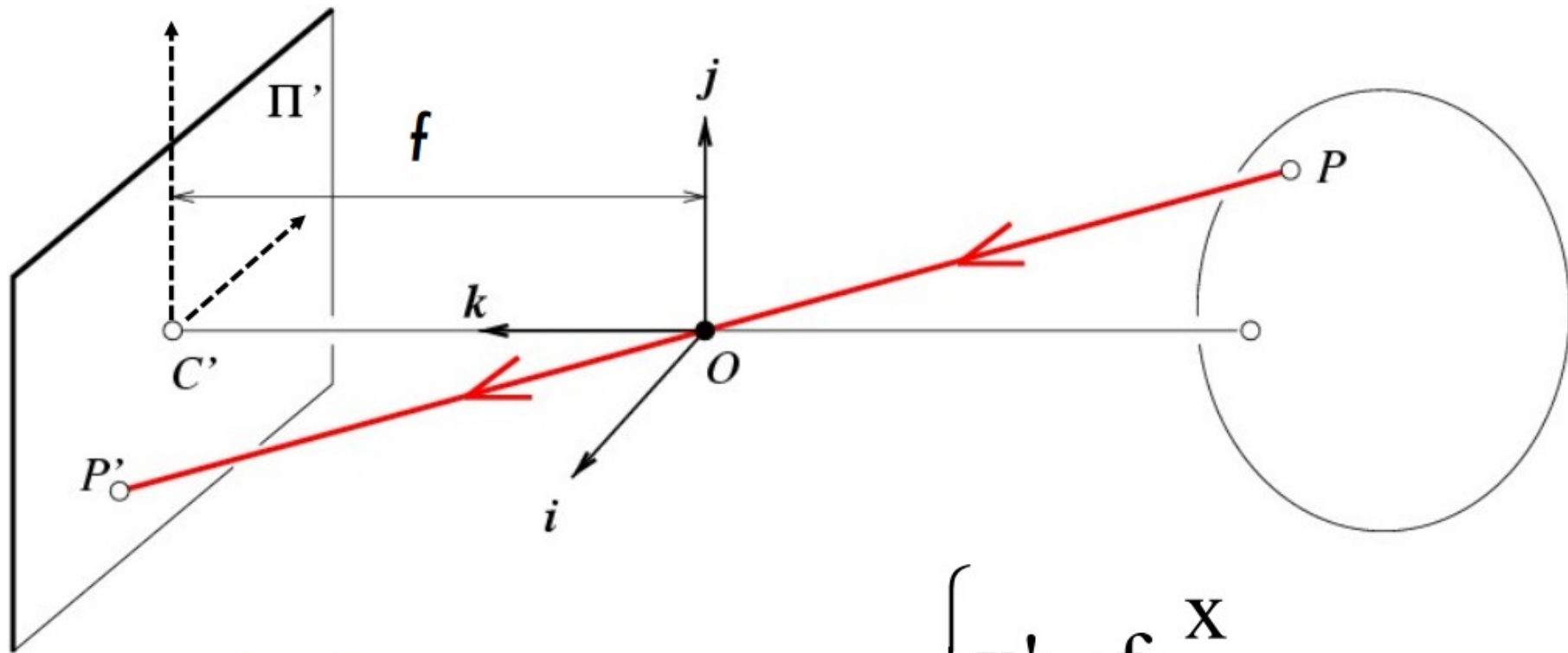
- When representing the camera rotation, rotation matrix F is negative z .
- In x, y, z matrix column. The negative f vector should go in the z direction. Up vector is going to be in the y direction so the y column gets the up vector. The right vector is in the x direction.



- Then the translation matrix is needed. The block matrix is used having identity and the origin of the camera T.
- Rotation and the translation of vector at the local camera space to find global space. T,R is v hence x vector and v inverse is determined.



Pinhole camera

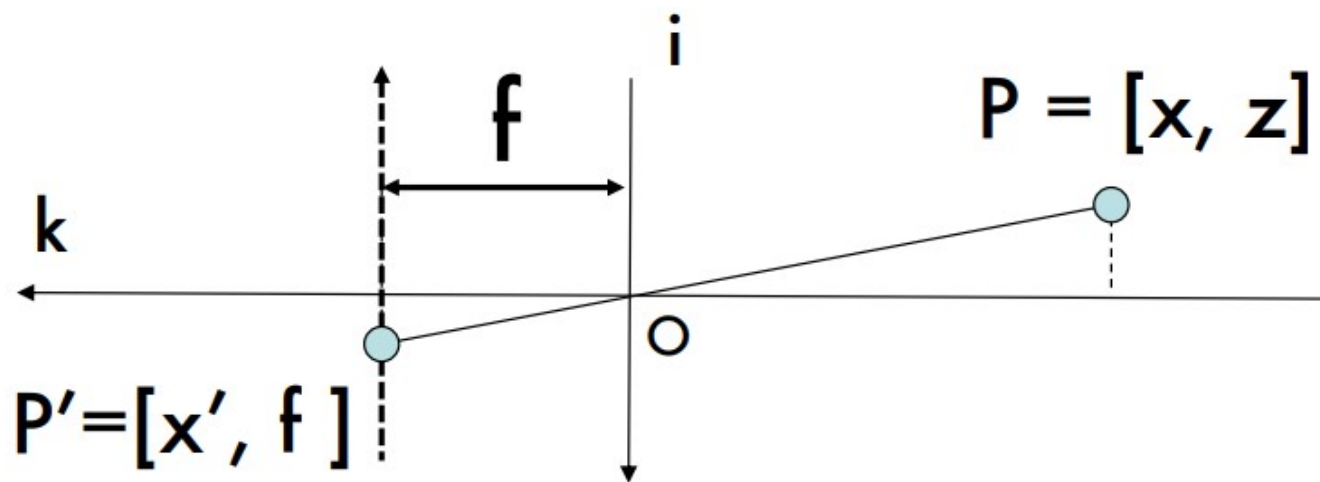
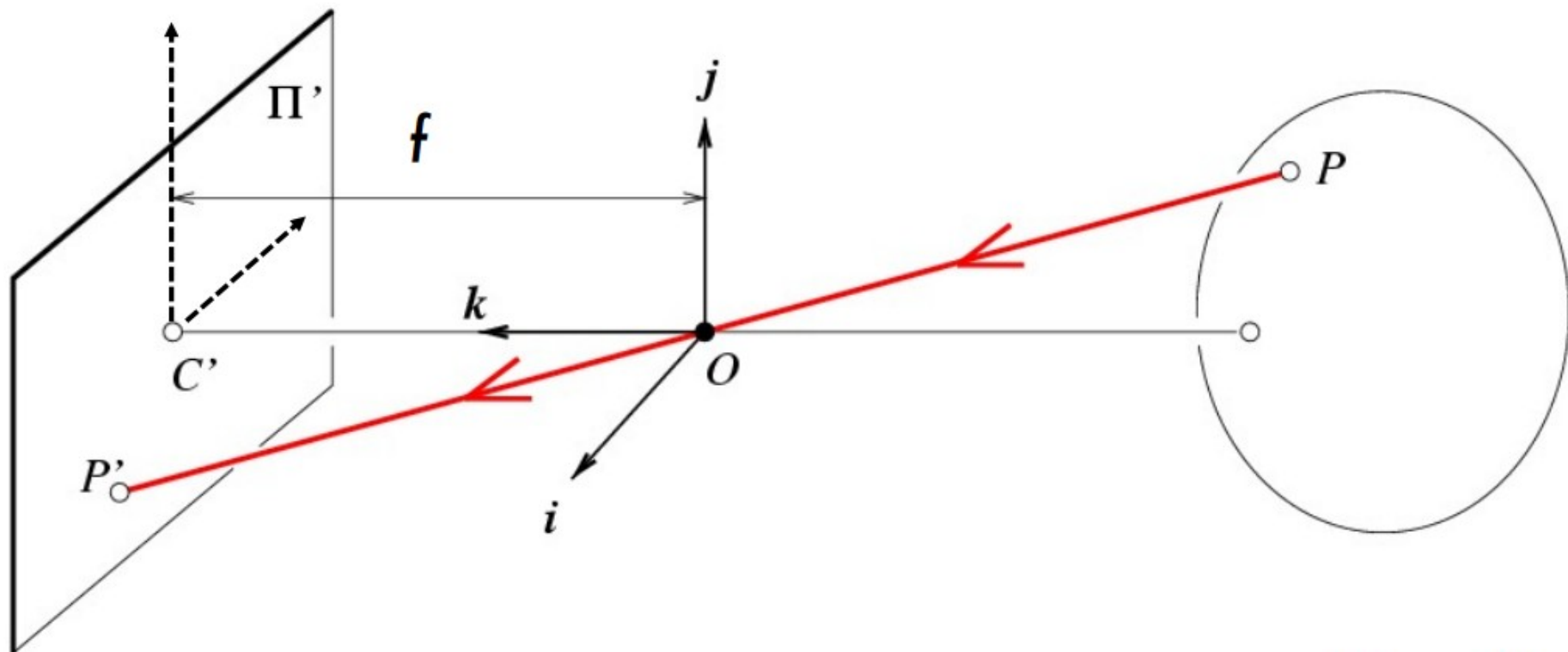


$$P = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \rightarrow P' = \begin{bmatrix} x' \\ y' \end{bmatrix}$$

$$\begin{cases} x' = f \frac{x}{z} \\ y' = f \frac{y}{z} \end{cases} \quad [\text{Eq. 1}]$$

Derived using similar triangles

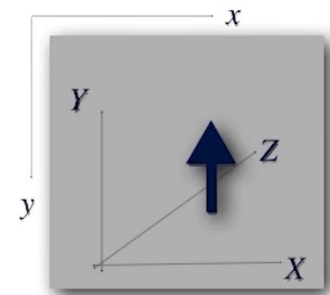
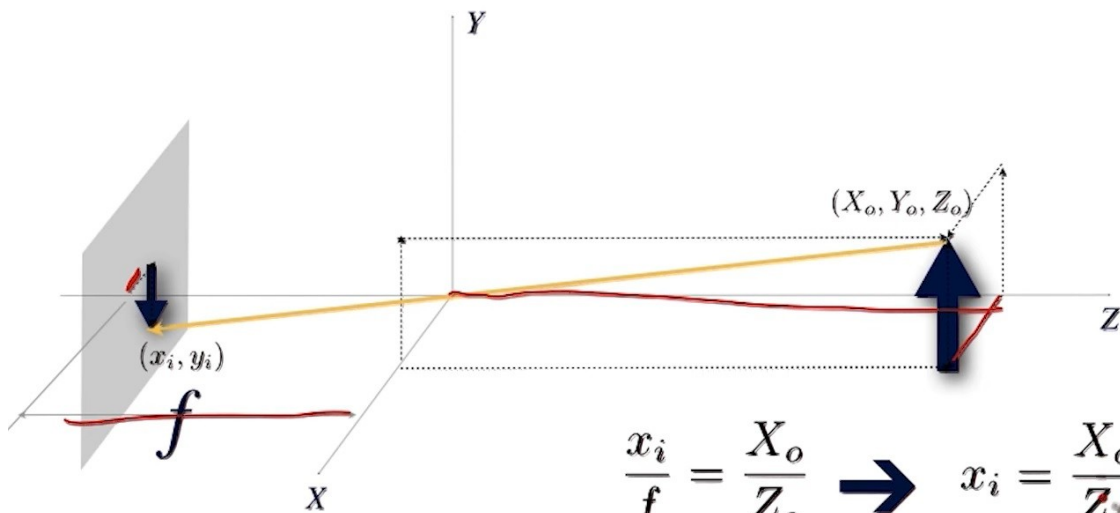
Pinhole camera



[Eq. 2]

$$\frac{x'}{f} = \frac{x}{z}$$

A Camera Model



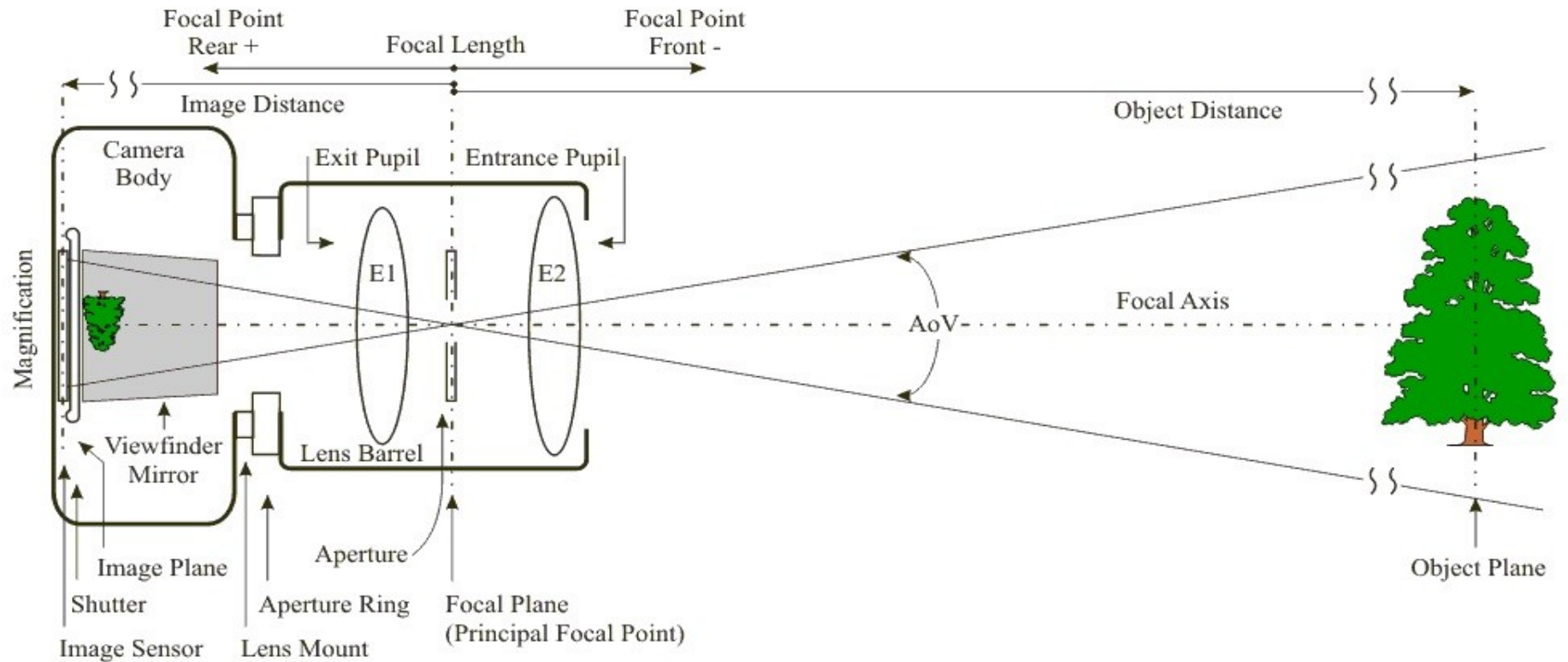
Using similar triangles

$$\frac{x_i}{f} = \frac{X_o}{Z_o} \rightarrow x_i = \frac{X_o}{Z_o} f$$

$$\frac{y_i}{f} = \frac{Y_o}{Z_o} \rightarrow y_i = \frac{Y_o}{Z_o} f$$

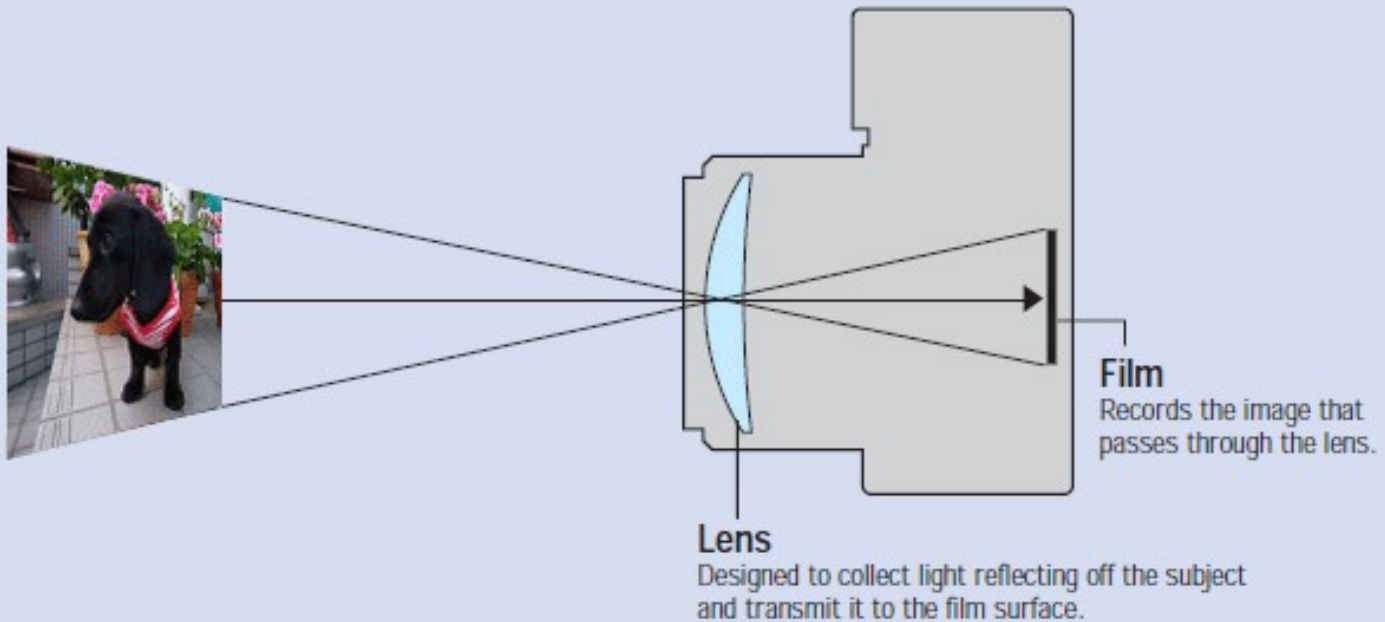


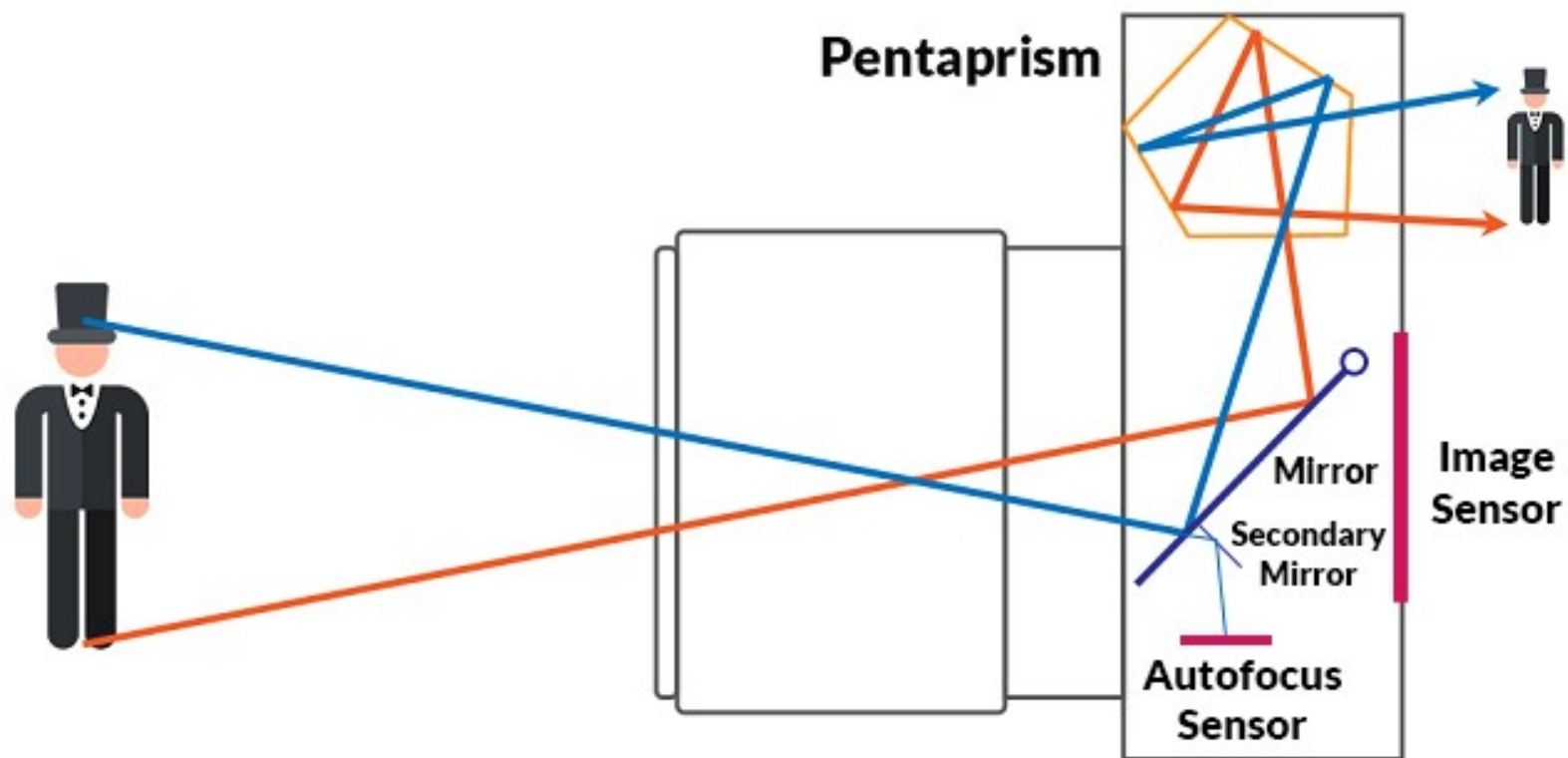
Digital Camera Model



Film Camera Model

- A camera that records images passing through the camera's lens on film is called a film camera.





Pinhole camera model

Discussion

- Imagine there is an infinite plane having a tiny pinhole.
- Pinhole is a point through which the object sends the rays from the top and bottom which is projected on the plane known as the image plane.
- The image in the image plane looks inverted and this is how the camera works as well.
- In real world these approximation of pinhole does not work because tiny aperture cameras with pinhole model there is deflection.
- Any light ray that enters the pinhole which is too small will hit the edge of the hole and through deflection image will be blur.

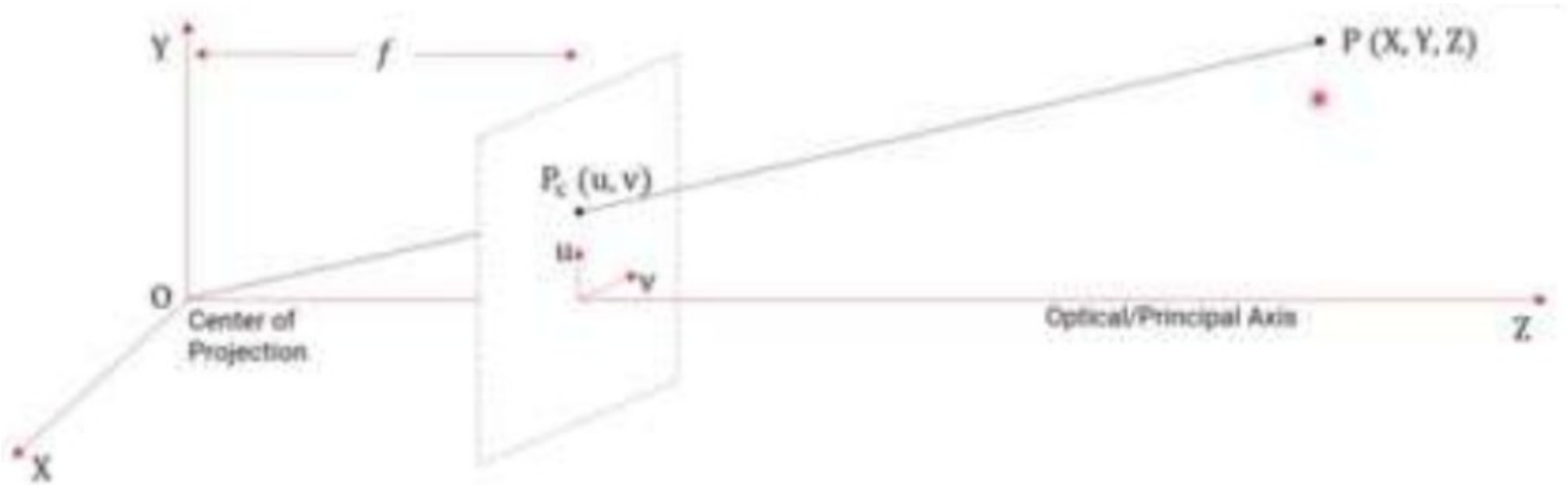
Pinhole diameter calculation

- The diameter is directly proportional to the square root focal length and the wavelength of light ($D=2$).
- Although smaller pinhole radius leads to diffraction, these formulation can be used. Pinhole model does not exist. In real world rather than having pinhole a convex lens can be used.
- Note f is the focal length for this camera.

Pinhole camera model working illustration

- Consider a point P in space $P(X,Y,Z)$. It is a 3D world point.
- X,Y,Z is a 3D world coordinate system and u,v is projected scheme on the image plane.
- $P(X,Y,Z)$ has the centre projection point O in the image plane. In previous image the virtual image and the image plane are identical and the only difference is the inversion.

Pinhole camera model working on virtual plane image



Using similar triangles,

$$\frac{f}{Z} = \frac{u}{X} = \frac{v}{Y} \quad u = \frac{fX}{Z} \quad v = \frac{fY}{Z} \quad w = 1$$

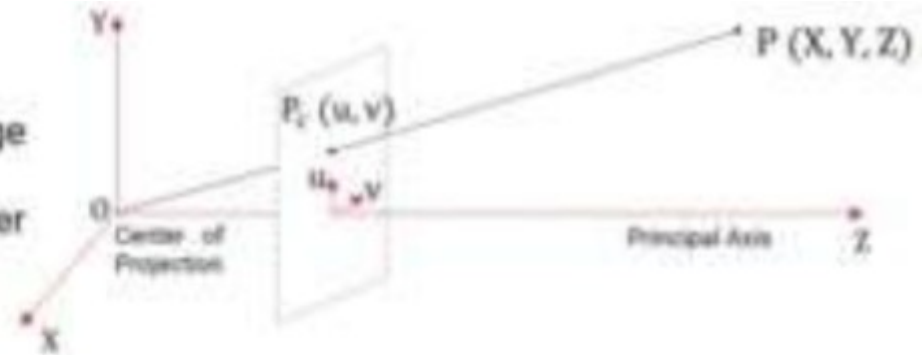
$$\Rightarrow \begin{pmatrix} u \\ v \\ w \end{pmatrix} = \begin{pmatrix} f & 0 & 0 \\ 0 & f & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} X \\ Y \\ Z \end{pmatrix}$$

Virtual plane image and its matrix

- In the virtual plane image, 3D point will be projected on the point P_c and position of the point is u, v . if the similar triangles are used, focal length upon the total depth X, Y, Z equals u/x in turn equals v/y . u and v can be discomputed as $u=fX/z$ and $v=fY/z$. u, v can be called as $w=1$ for converting into matrix form as shown in the figure.
- Hence equation on the left and right are same.

Coordinates offset from centre

$$\begin{pmatrix} u \\ v \\ 1 \end{pmatrix} = \begin{pmatrix} f & 0 & 0 \\ 0 & f & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} X \\ Y \\ Z \end{pmatrix}$$
 assumes that the 2D image coordinate system coincides with the center of projection O



Let offset from center be c_x and c_y

$$u = \frac{fX}{Z} + c_x \quad v = \frac{fY}{Z} + c_y$$

$$\begin{pmatrix} u \\ v \\ 1 \end{pmatrix} = \begin{pmatrix} f & 0 & c_x \\ 0 & f & c_y \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} X \\ Y \\ Z \end{pmatrix}$$

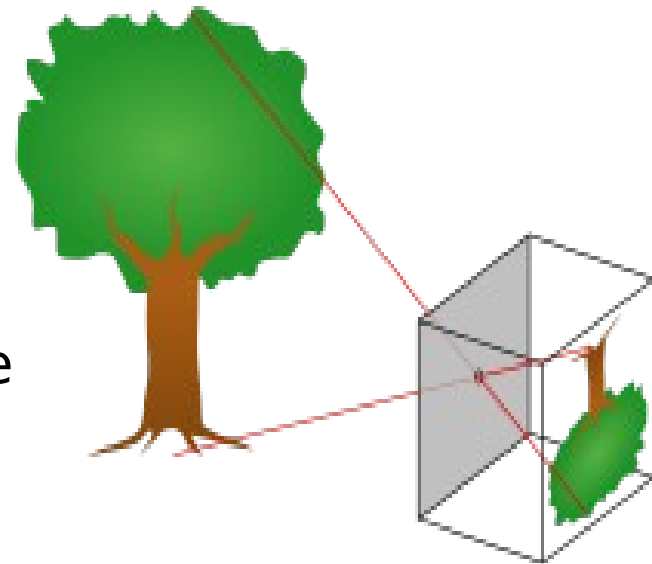
(c_x, c_y) is known as the principal point

Coordinates offset description

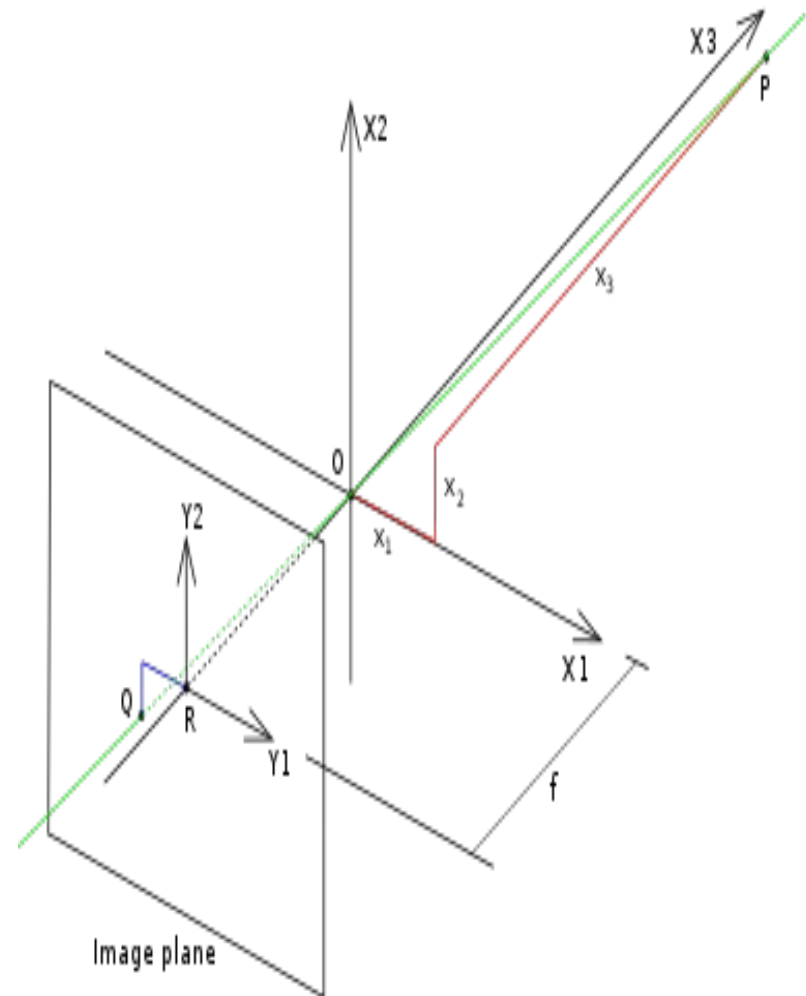
- If the centre of projection is away from the centre due to some manufacturing defects. To make sure that the O is the centre projection, axis starts join O to axis perpendicular to image plane is called principle axis.
- O need not be at the centre due to manufacturing defects. It can be shifted by C_x in the x direction and C_y in y direction.
- O is an offset from centre as given in the equation shown in figure. Hence the equation can be rewritten by replacing O 's with C_x and C_y . (C_x, C_y) are the principal points. This is the new equation where the centre of projection is not exactly at the centre.

Pinhole camera model

- The **pinhole camera model** describes the mathematical relationship between the [coordinates](#) of a point in three-dimensional space and its [projection](#) onto the image plane of an *ideal pinhole camera*, where the camera aperture is described as a point and no lenses are used to focus light.
- The model does not include, for example, [geometric distortions](#) or blurring of unfocused objects caused by lenses and finite sized apertures.
- A diagram of a [pinhole camera](#).



- The geometry and mathematics of the pinhole camera
- A 3D orthogonal coordinate system with its origin at **O**.
- This is also where the camera aperture is located.
- The three axes of the coordinate system are referred to as X_1 , X_2 , X_3 . Axis X_3 is pointing in the viewing direction of the camera and is referred to as the optical axis, *principal axis*, or *principal ray*.



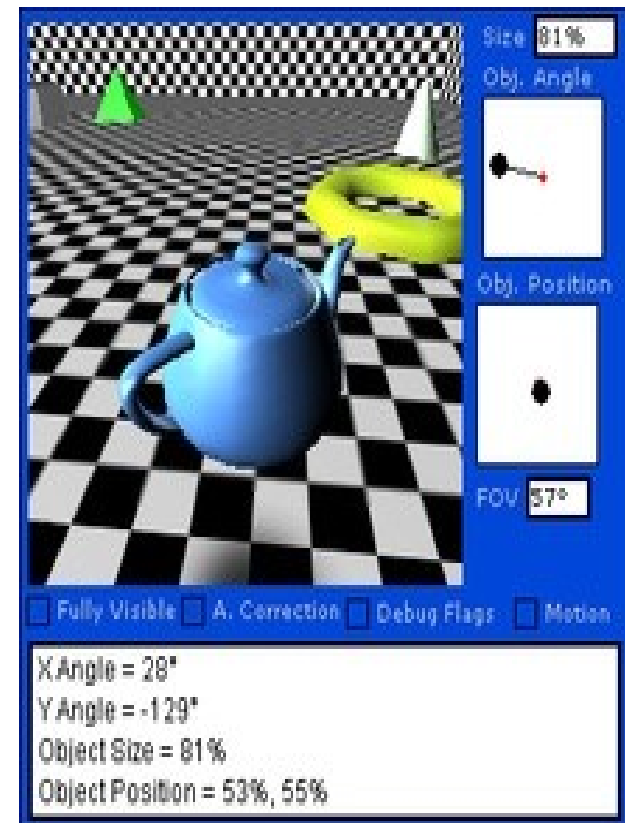
- The plane which is spanned by axes X_1 and X_2 is the front side of the camera, or *principal plane*
- An image plane, where the 3D world is projected through the aperture of the camera.
- The image plane is parallel to axes X_1 and X_2 and is located at distance f from the origin $\bullet$ in the negative direction of the X_3 axis, where f is the focal length of the pinhole camera. A practical implementation of a pinhole camera implies that the image plane is located such that it intersects the X_3 axis at coordinate $-f$ where $f > 0$.

- A point **R** at the intersection of the optical axis and the image plane. This point is referred to as the *principal point* or *image center*.
- A point **P** somewhere in the world at coordinate x_1, x_2, x_3 relative to the axes X_1, X_2, X_3 .
- The *projection line* of point **P** into the camera. This is the green line which passes through point **P** and the point **O**.

- The projection of point **P** onto the image plane, denoted **Q**. This point is given by the intersection of the projection line (green) and the image plane. In any practical situation we can assume that $x_3 > 0$ which means that the intersection point is well defined.
- There is also a 2D coordinate system in the image plane, with origin at **R** and with axes Y_1 and Y_2 which are parallel to X_1 and X_2 , respectively. The coordinates of point **Q** relative to this coordinate system is y_1, y_2 .

Virtual camera system

- In 3D video games, a **virtual camera system** aims at controlling a camera or a set of cameras to display a view of a 3D [virtual world](#).
- Camera systems are used in [video games](#) where their purpose is to show the action at the best possible angle; more generally, they are used in 3D virtual worlds when a third person view is required.
- Virtual camera system [demo](#) showing parameters of the camera that can be adjusted.



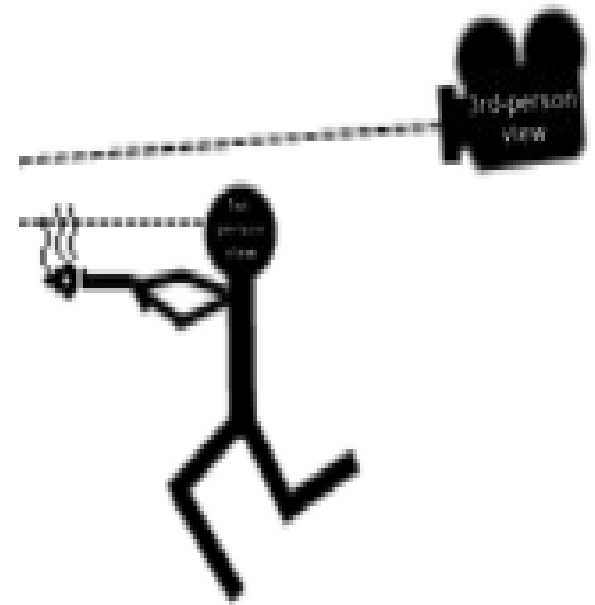
Fixed

- In this kind of system, the developers set the properties of the camera, such as its position, orientation or field of view, during the game creation.
- Selection of shots in Resident Evil 2 that aim at creating tension.



Tracking

- As the name says, a tracking camera follows the characters from behind. The player does not control the camera in any way - he/she cannot for example rotate it or move it to a different position. An illustration of a protagonist whom a player controls and a tracking camera just behind, slightly above, and slightly facing down towards that character.



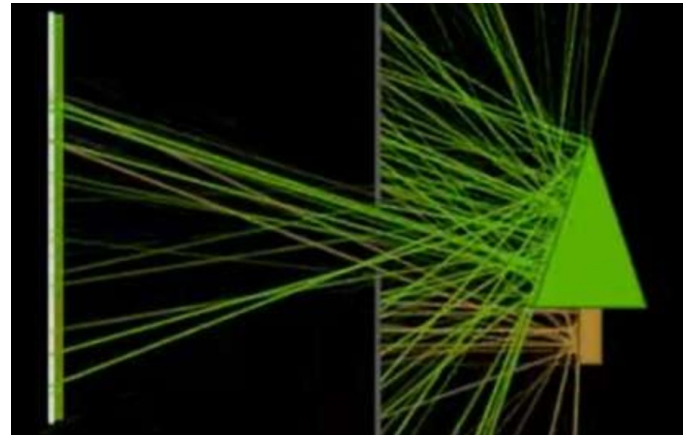
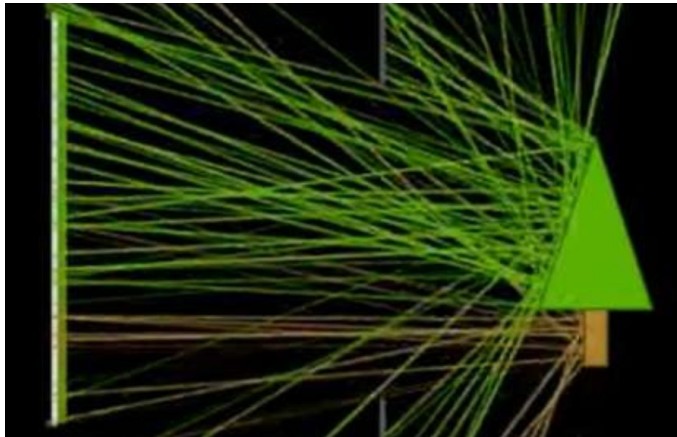
Interactive

- This type of camera system is an improvement over the tracking camera system.
- While the camera is still tracking the character, some of its parameters, such as its orientation or distance to the character, can be changed.
- Instead of staying behind Mario, the camera intelligently rotates to show the path (Super Mario 64).



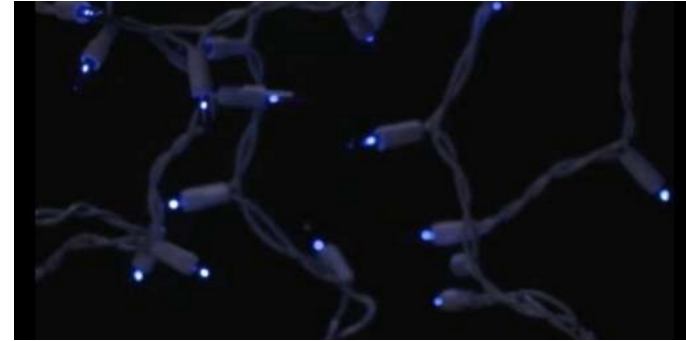
Virtual camera of a pinhole

- The geometric behaviour of cameras explore how light bounces off object and passes through small hole or aperture and hits image plane.

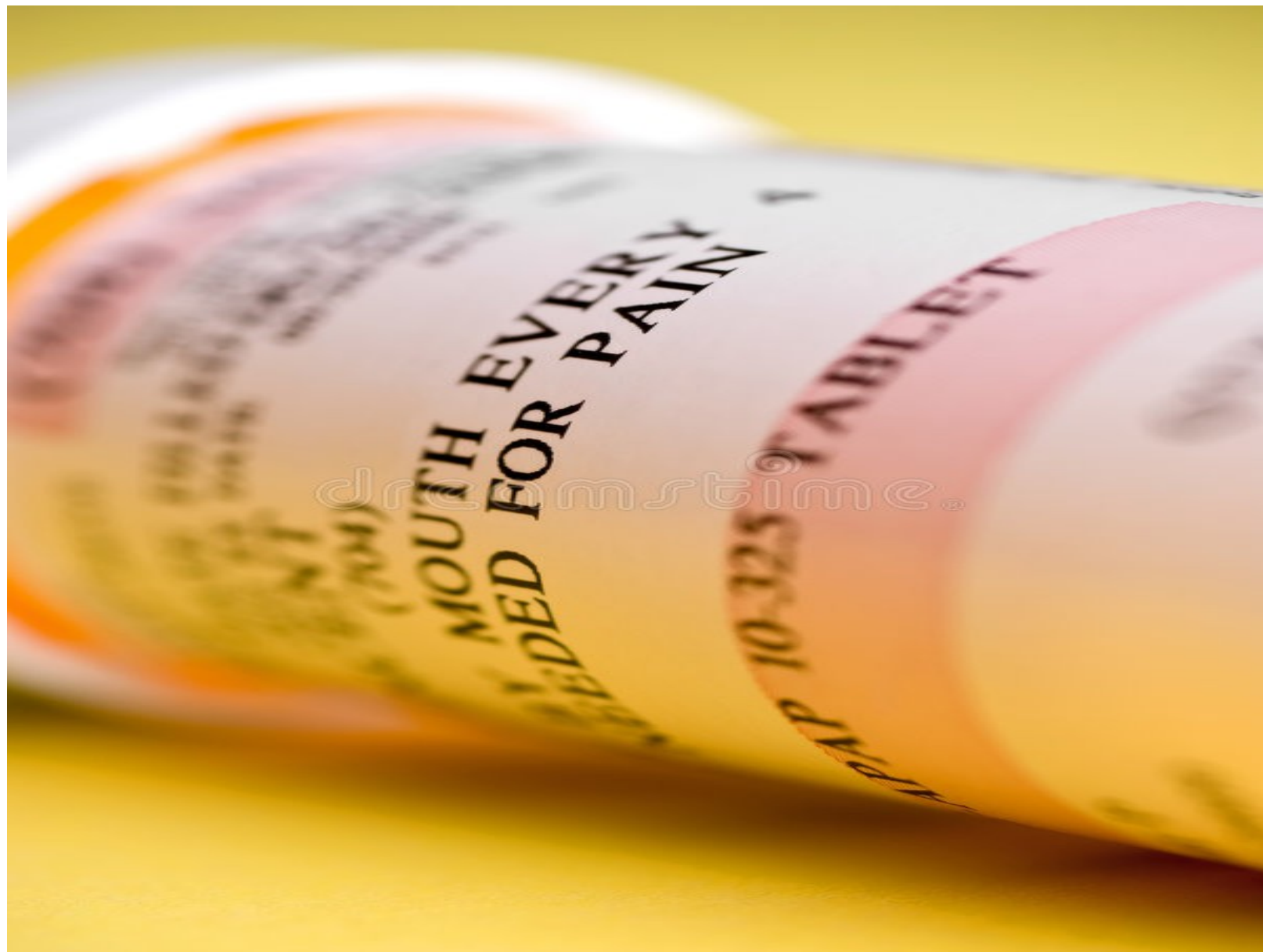


Region of objects

- One of the important concepts is the **depth field** is where region of objects appear in focus.
- Outside the region **objects appear blur**.
- When the image **out of focus** it goes into blurry circle called **circle of confusion**.

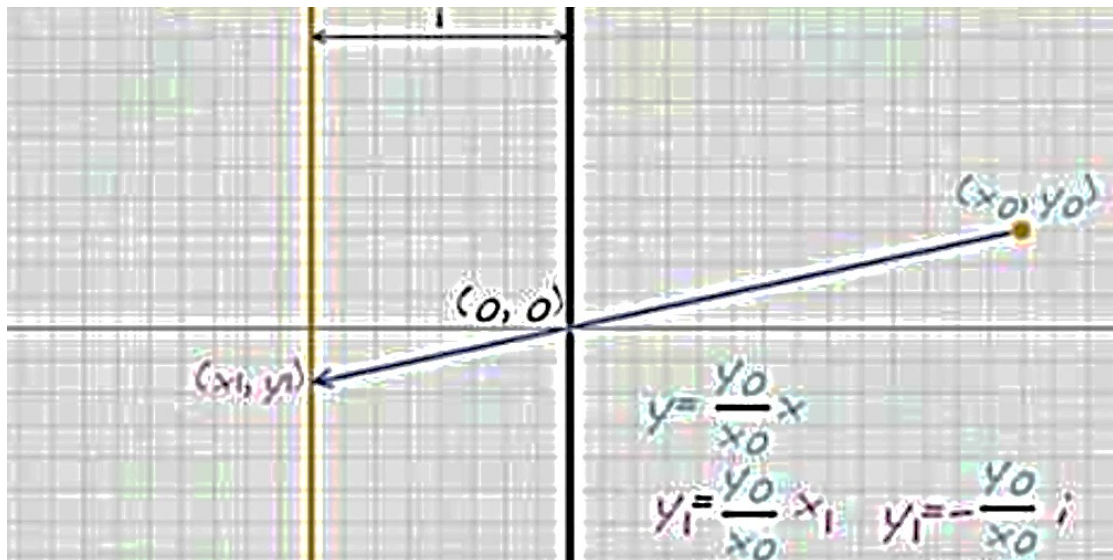






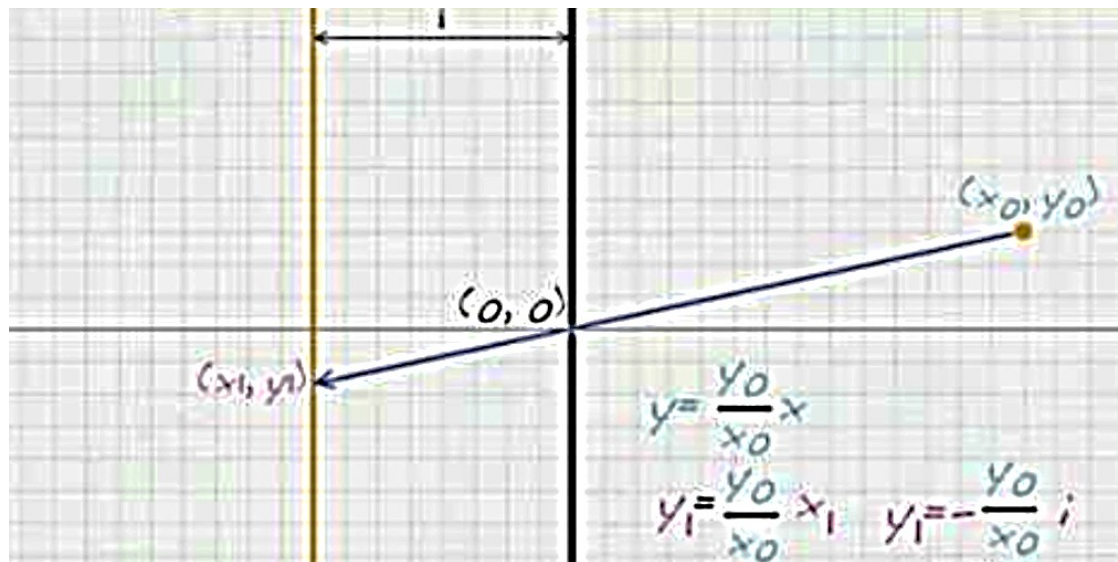
Coordinate system and image plane

- A coordinate system has to be introduced. Its convenient to choose the coordinate system so that the pinhole is the origin.
- Lets imagine the scene is off to the right. Suppose the image plane inside camera is at some distance to the left of the pinhole. Lets call this distance i .



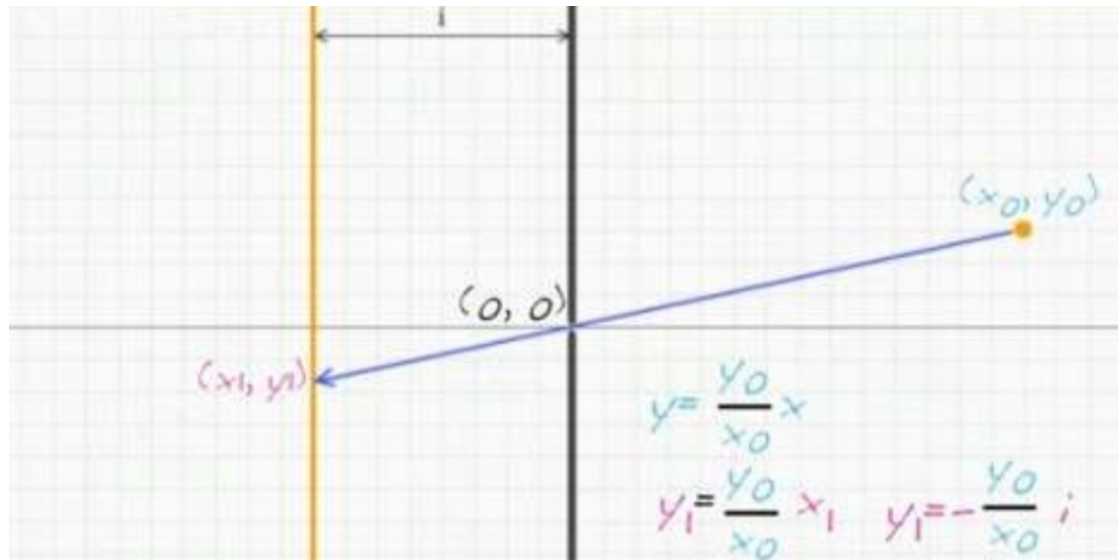
Light hit and bounce

- Now consider a point on some object scene. Call this point (X_0, Y_0) .
- When light in the environment hits this object point (X_0, Y_0) , some of them bounce towards the camera through the pinhole and hit the image plane. Let's call the place it hits the image plane (X_1, Y_1) .



Slope intercept form of a line satisfying ray

- First question is what is (X_1, Y_1) ? One way to solve this is to use slope intercept form of a line.
- The slope of the ray is Y_0/X_0 . Y intercept is 0 because the ray passes through the origin. That is the equation of the ray y as given in the diagram.



Resulting image flow

- The point (X_1, Y_1) is also on the ray which must satisfy the line equation by determining Y_1 .
- Notice the diagram where X_1 is negative and distance away from the origin. ie. $x_1 = -i$. Substitute the same in Y_1 equation.
- Notice that the Y_0 was additionally positive. But the corresponding point Y_1 in the plane is negative. That is the image flow.

