

Addis Ababa Science and Technology University



Chapter 7

MOMENT OF INERTIA

Engineering Mechanics

Content

- Introduction
- Moment of inertia of area
- Radius of gyration
- Parallel axis theorem or transfer of axes
- Moment of area of composite bodies
- Product of inertia
- Principal Axes and Principal Moments of Inertia
- Mohr's Circle

Introduction

- Quantities called moments of **inertia** **arise** repeatedly in analyses of engineering problems.
- Moments of inertia of areas are used in the study of **distributed forces** and in calculating **deflections** of **beams**.
- The moment exerted by the pressure on a submerged flat plate can be expressed in terms of the moment of inertia of the plate's area.

Moment of Inertia of an Area

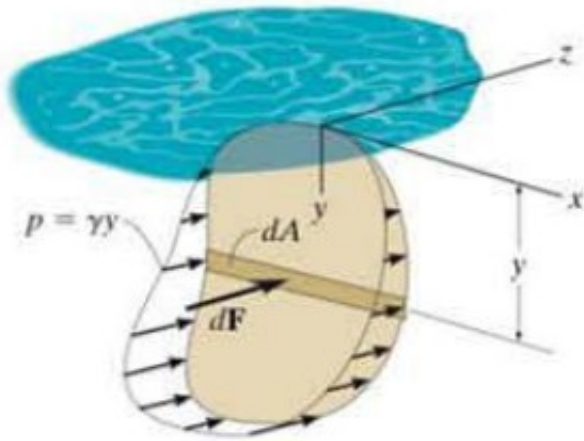


Fig. 10-1

- Whenever a distributed loading acts perpendicular to an area and its intensity varies linearly, the computation of the moment of the loading distribution about an axis will involve a quantity called the moment of inertia of the area.
- For example, consider the plate fig 10.1 which is subjected to a fluid pressure p .
- This pressure varies linearly with depth, such that $p = \gamma y$
- Thus , the force acting on the differential area dA of the plate is $dF = p dA = (\gamma y) dA$

Moment of Inertia of an Area

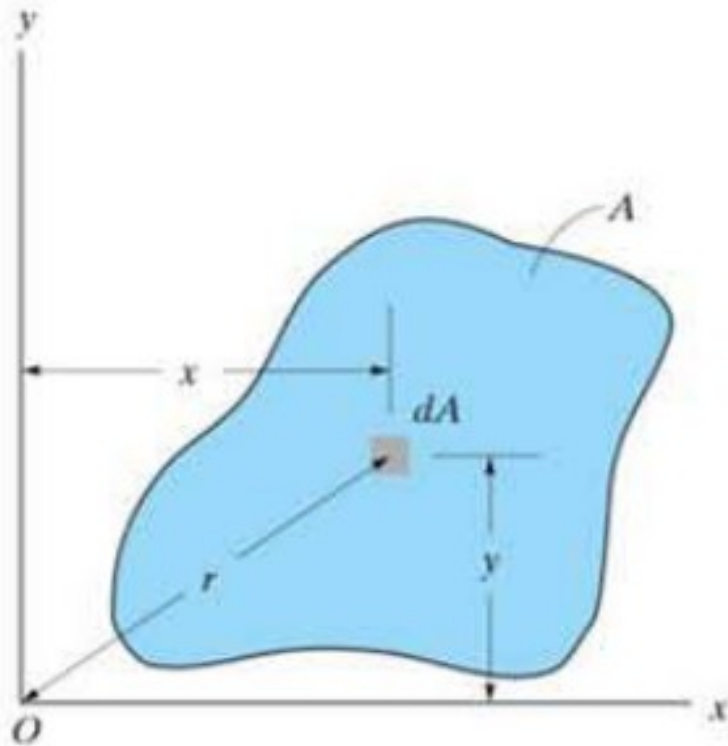


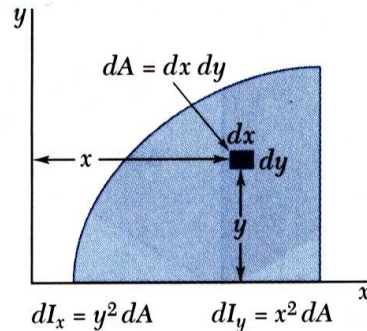
Fig. 10-2

$$I_x = \int_A y^2 dA$$

$$I_y = \int_A x^2 dA$$

$$J_O = \int_A r^2 dA = I_x + I_y$$

Moment of Inertia of an Area by Integration



- *Second moments or moments of inertia* of an area with respect to the x and y axes,

$$I_x = \int y^2 dA \quad I_y = \int x^2 dA$$

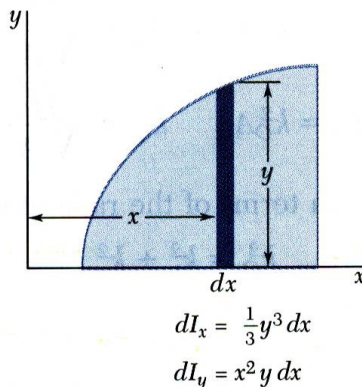
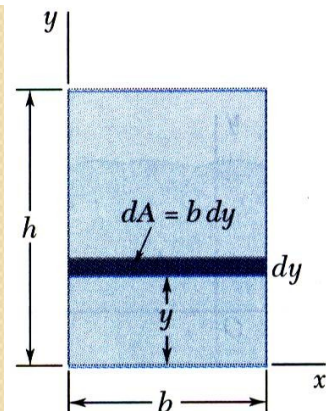
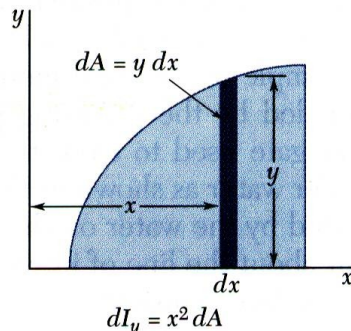
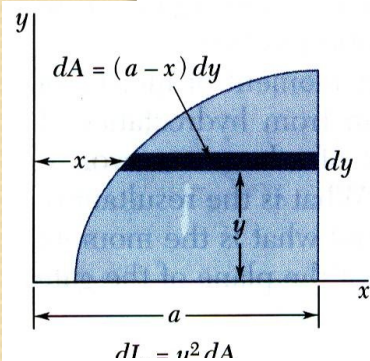
- Evaluation of the integrals is simplified by choosing dA to be a thin strip parallel to one of the coordinate axes.

- For a rectangular area,

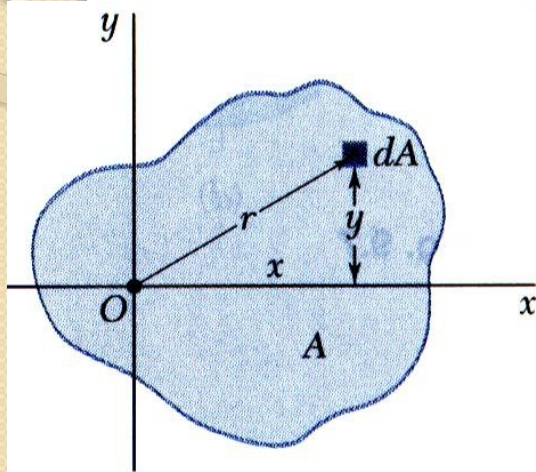
$$I_x = \int y^2 dA = \int_0^h y^2 b dy = \frac{1}{3} b h^3$$

- The formula for rectangular areas may also be applied to strips parallel to the axes,

$$dI_x = \frac{1}{3} y^3 dx \quad dI_y = x^2 dA = x^2 y dx$$



Polar Moment of Inertia



- The *polar moment of inertia* is an important parameter in problems involving torsion of cylindrical shafts and rotations of slabs.

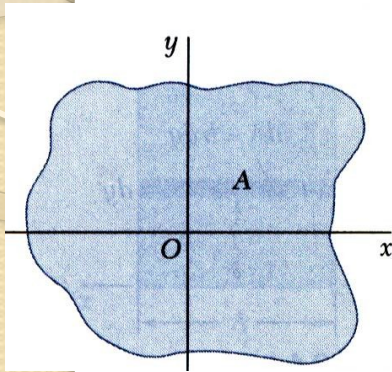
$$J_0 = \int r^2 dA$$

- The polar moment of inertia is related to the rectangular moments of inertia.

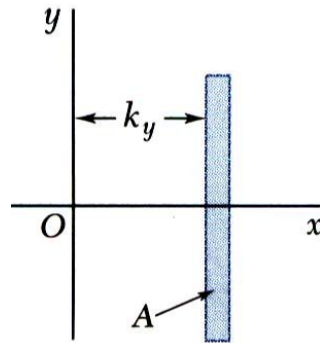
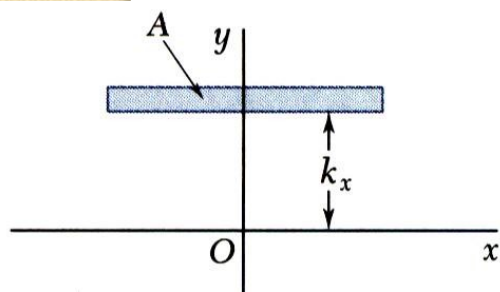
$$J_0 = \int r^2 dA = \int (x^2 + y^2) dA = \int x^2 dA + \int y^2 dA$$

$$= I_y + I_x$$

Radius of Gyration of an Area



- Consider area A with moment of inertia I_x .
Imagine that the area is concentrated in a thin strip parallel to the x -axis with [equivalent I_x]
 $k_x = \text{radius of gyration with respect to the } x \text{ axis}$

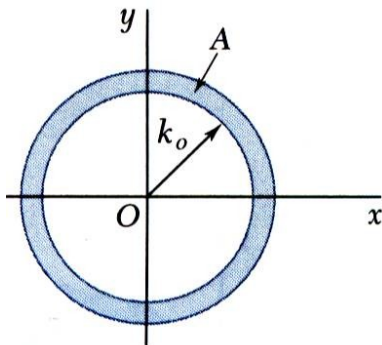


- Similarly,

$$I_y = k_y^2 A \quad k_y = \sqrt{\frac{I_y}{A}}$$

$$J_O = k_O^2 A \quad k_O = \sqrt{\frac{J_O}{A}}$$

$$k_O^2 = k_x^2 + k_y^2$$



Sample Problem 9.1

SOLUTION:

- A differential strip parallel to the x axis is chosen for dA

$$dI_x = y^2 dA \quad dA = l dy$$

- For similar triangles,

$$\frac{l}{b} = \frac{h-y}{h} \quad l = b \frac{h-y}{h} \quad dA = b \frac{h-y}{h} dy$$

- Integrating dI_x from $y = 0$ to

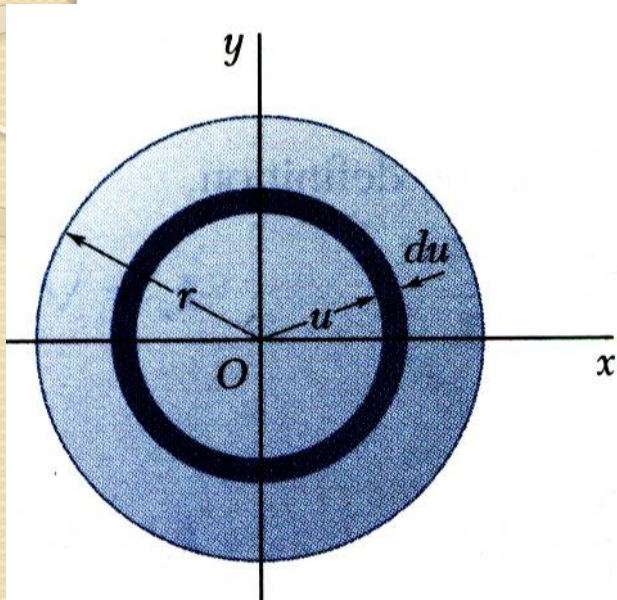
$$I_{I_x} = \int y^2 dA = \int_0^h y^2 b \frac{h-y}{h} dy = \frac{b}{h} \int_0^h (hy^2 - y^3) dy$$

$$= \frac{b}{h} \left[h \frac{y^3}{3} - \frac{y^4}{4} \right]_0^h$$

$$I_x = \frac{bh^3}{12}$$

Determine the moment of inertia of a triangle with respect to its base.

Sample Problem 9.2



SOLUTION:

- An annular differential area element is chosen,

$$dJ_O = u^2 dA \quad dA = 2\pi u du$$

$$J_O = \int dJ_O = \int_0^r u^2 (2\pi u du) = 2\pi \int_0^r u^3 du$$

$$J_O = \frac{\pi}{2} r^4$$

- a) Determine the centroidal polar moment of inertia of a circular area by direct integration.

- b) Using the result of part a, determine the moment of inertia of a

- From symmetry, I_x

$$\bar{J}_O = I_x + I_y = 2I_x \quad \frac{\pi}{2} r^4 = 2I_x$$

$$I_{diameter} = I_x = \frac{\pi}{4} r^4$$

Parallel Axis Theorem

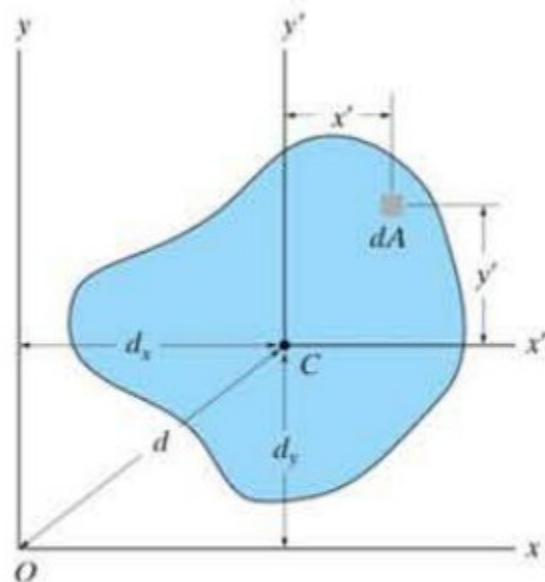


Fig. 10-3

$$\begin{aligned} I_x &= \int_A (y' + d_y)^2 dA \\ &= \int_A y'^2 dA + 2d_y \int_A y' dA + d_y^2 \int_A dA \end{aligned}$$

$$I_x = \bar{I}_x + Ad_y^2$$

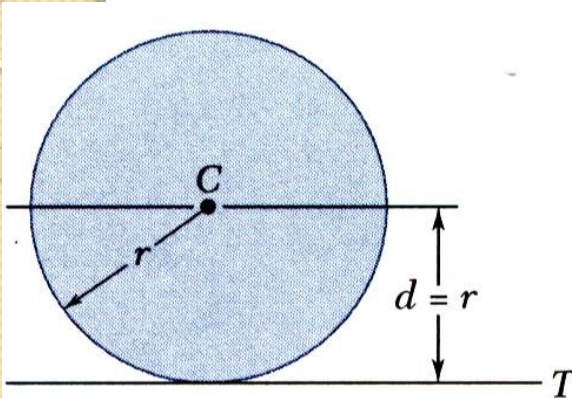
$$I_y = \bar{I}_y + Ad_x^2$$

And finally, for the polar moment of inertia, since $\bar{J}_C = \bar{I}_x + \bar{I}_y$ and $d^2 = d_x^2 + d_y^2$, we have

$$J_O = \bar{J}_C + Ad^2$$

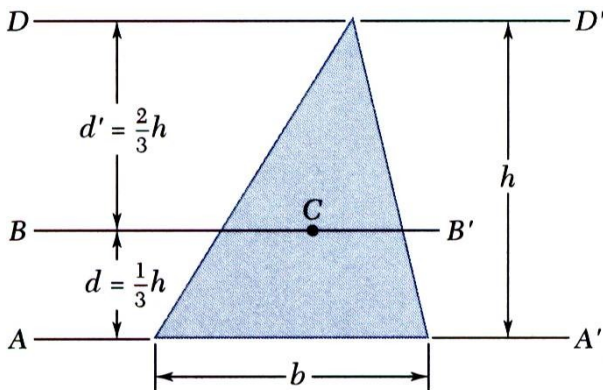
(10-5)

Parallel Axis Theorem



- Moment of inertia I_T of a circular area with respect to a tangent to the circle

$$I_T = I_C + Ad^2 = \frac{1}{4}\pi r^4 + \pi r^2 r^2 = \frac{5}{4}\pi r^4$$



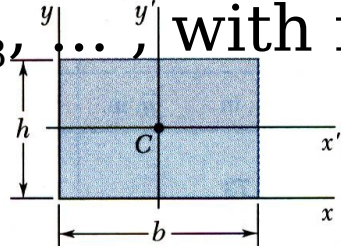
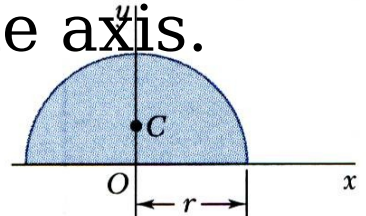
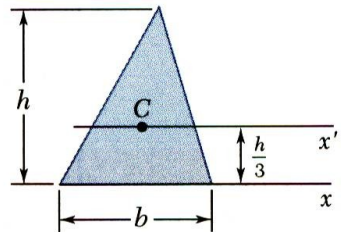
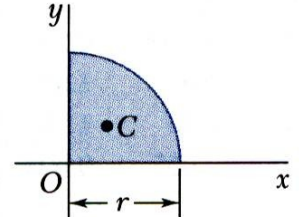
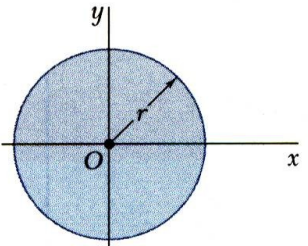
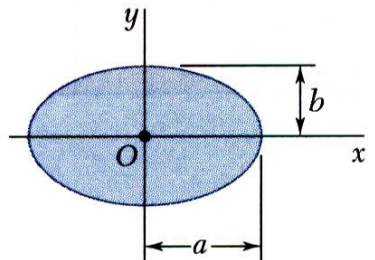
- Moment of inertia of a triangle with respect to a centroidal axis,

$$I_{AA'} = I_{BB'} + Ad^2$$

$$I_{BB'} = I_{AA'} - Ad^2 = \frac{1}{12}bh^3 - \frac{1}{2}bh\left(\frac{1}{3}h\right)^2 = \frac{1}{36}bh^3$$

Moments of Inertia of Composite Areas

- The moment of inertia of a composite area A about a given axis is obtained by adding the moments of inertia of the component areas $A_1, A_2, A_3, \dots$, with respect to the same axis.

Rectangle		$\bar{I}_{x'} = \frac{1}{12}bh^3$ $I_{x'} = \frac{1}{12}b^3h$ $I_y = \frac{1}{3}b^3h$ $J_C = \frac{1}{12}bh(b^2 + h^2)$	Semicircle		$I_x = I_y = \frac{1}{8}\pi r^4$ $J_O = \frac{1}{4}\pi r^4$
Triangle		$\bar{I}_{x'} = \frac{1}{36}bh^3$ $I_x = \frac{1}{12}bh^3$	Quarter circle		$I_x = I_y = \frac{1}{16}\pi r^4$ $J_O = \frac{1}{8}\pi r^4$
Circle		$\bar{I}_x = \bar{I}_y = \frac{1}{4}\pi r^4$ $J_O = \frac{1}{2}\pi r^4$	Ellipse		$\bar{I}_x = \frac{1}{4}\pi ab^3$ $\bar{I}_y = \frac{1}{4}\pi a^3b$ $J_O = \frac{1}{4}\pi ab(a^2 + b^2)$

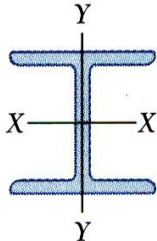
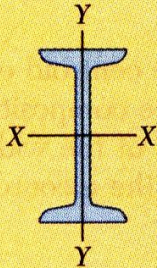
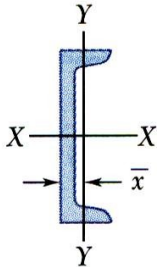
Moments of Inertia of Composite Areas

Part	<u>Area A</u>	dx	<u>dy</u>	$A dx^2$	$A dy^2$	$\bar{I}_x$	<u>$\bar{I}_y$</u>
1							
2							
3							
Sums				$\Sigma A dx^2$	$\Sigma A dy^2$	$\Sigma \bar{I}_x$	<u>$\Sigma \bar{I}_y$</u>

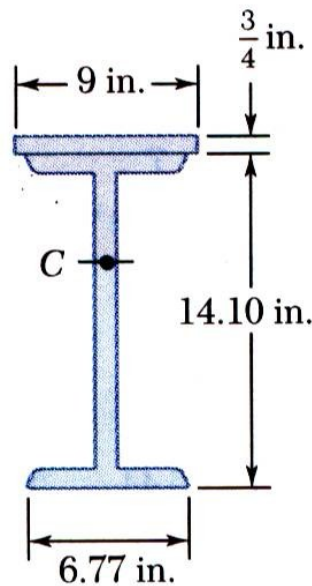
$$I_x = \sum \bar{I}_x + \sum A dx^2$$

$$I_y = \sum \bar{I}_y + \sum A dy^2$$

Moments of Inertia of Composite

		Designation	Area mm ²	Depth mm	Width mm	Axis X-X			Axis Y-Y		
						$\bar{I}_x$ 10 ⁶ mm ⁴	$\bar{k}_x$ mm	$\bar{y}$ mm	$\bar{I}_y$ 10 ⁶ mm ⁴	$\bar{k}_y$ mm	$\bar{x}$ mm
W Shapes (Wide-Flange Shapes)		W460 × 113†	14400	463	280	554	196.3		63.3	66.3	
		W410 × 85	10800	417	181	316	170.7		17.94	40.6	
		W360 × 57	7230	358	172	160.2	149.4		11.11	39.4	
		W200 × 46.1	5890	203	203	45.8	88.1		15.44	51.3	
S Shapes (American Standard Shapes)		S460 × 81.4†	10390	457	152	335	179.6		8.66	29.0	
		S310 × 47.3	6032	305	127	90.7	122.7		3.90	25.4	
		S250 × 37.8	4806	254	118	51.6	103.4		2.83	24.2	
		S150 × 18.6	2362	152	84	9.2	62.2		0.758	17.91	
C Shapes (American Standard Channels)		C310 × 30.8†	3929	305	74	53.7	117.1		1.615	20.29	17.73
		C250 × 22.8	2897	254	65	28.1	98.3		0.949	18.11	16.10
		C200 × 17.1	2181	203	57	13.57	79.0		0.549	15.88	14.50
		C150 × 12.2	1548	152	48	5.45	59.4		0.288	13.64	13.00

Sample Problem 9.4



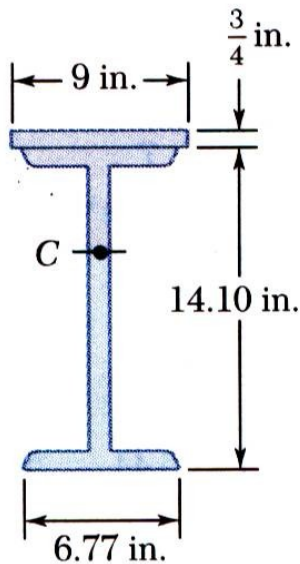
The strength of a W14x38 rolled steel beam is increased by attaching a plate to its upper flange.

Determine the moment of inertia and radius of gyration with respect to an axis which is parallel to the plate and passes through the centroid of the

SOLUTION:

- Determine location of the centroid of composite section with respect to a coordinate system with origin at the centroid of the beam section.
- Apply the parallel axis theorem to determine moments of inertia of beam section and plate with respect to composite section centroidal axis.
- Calculate the radius of gyration from the moment of inertia of the composite section.

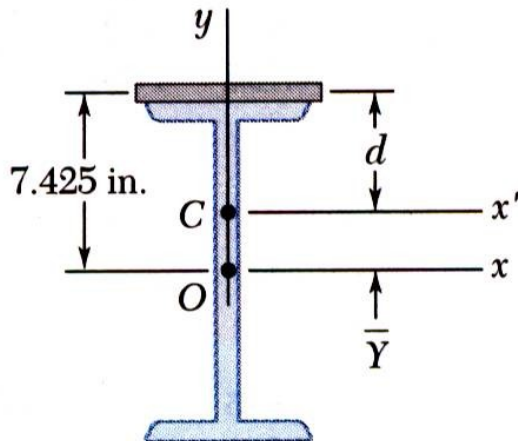
Sample Problem 9.4



SOLUTION:

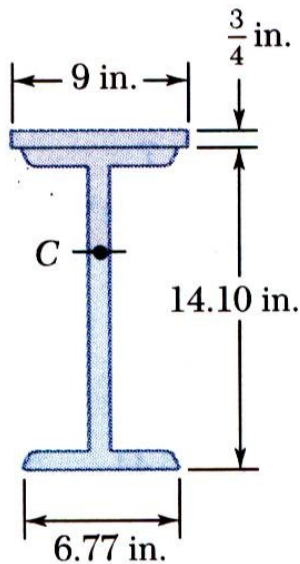
- Determine location of the centroid of composite section with respect to a coordinate system with origin at the centroid of the beam section.

Section	A, in^2	$\bar{y}, \text{in.}$	$\bar{y}A, \text{in}^3$
Plate	6.75	7.425	5012
Beam Section	11.20	0	0
	$\Sigma A = 17.95$		$\Sigma \bar{y}A = 5012$



$$\bar{Y} \Sigma A = \Sigma \bar{y}A \quad \bar{Y} = \frac{\Sigma \bar{y}A}{\Sigma A} = \frac{5012 \text{ in}^3}{17.95 \text{ in}^2} = 2.792 \text{ in.}$$

Sample Problem 9.4



- Apply the parallel axis theorem to determine moments of inertia of beam section and plate with respect to composite section centroidal axis.

$$I_{x', \text{beam section}} = \bar{I}_x + Ad^2 = 385 + (11.20)(2.792)^2$$

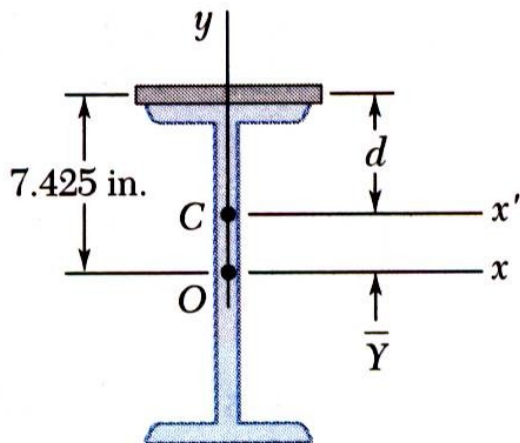
$$= 4723 \text{ in}^4$$

$$I_{x', \text{plate}} = \bar{I}_x + Ad^2 = \frac{1}{12}(9)\left(\frac{3}{4}\right)^3 + (6.75)(7.425 - 2.792)^2$$

$$= 1452 \text{ in}^4$$

$$I_{x'} = I_{x', \text{beam section}} + I_{x', \text{plate}} = 4723 + 1452$$

$$I_{x'} = 6175 \text{ in}^4$$



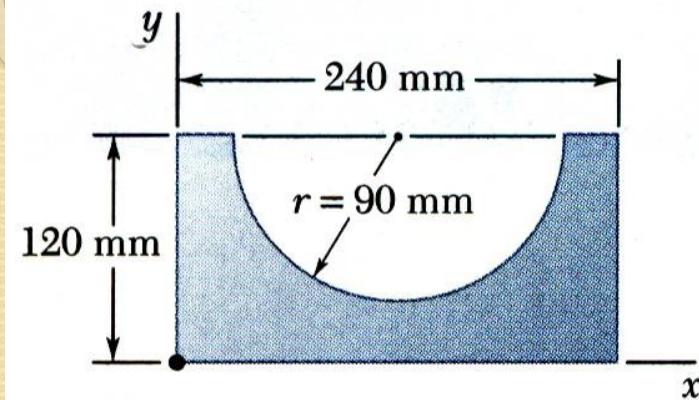
- Calculate the radius of gyration from the moment of inertia of the composite section.

$$k_{x'} = \sqrt{\frac{I_{x'}}{A}} = \frac{6175 \text{ in}^4}{17.95 \text{ in}^2}$$

$$k_{x'} = 5.87 \text{ in}$$

Sample Problem 9.5

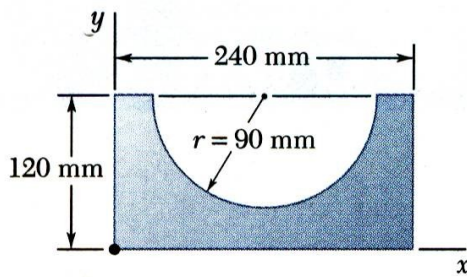
SOLUTION:



Determine the moment of inertia of the shaded area with respect to the x axis.

- Compute the moments of inertia of the bounding rectangle and half-circle with respect to the x axis.
- The moment of inertia of the shaded area is obtained by subtracting the moment of inertia of the half-circle from the moment of inertia of the rectangle.

Sample Problem 9.5



SOLUTION:

- Compute the moments of inertia of the bounding rectangle and half-circle with respect to the x

Rectangle axis $I_x = \frac{1}{3}bh^3 = \frac{1}{3}(240)(120)^3 = 1382 \times 10^6 \text{ mm}^4$

Half-circle:

moment of inertia with respect to

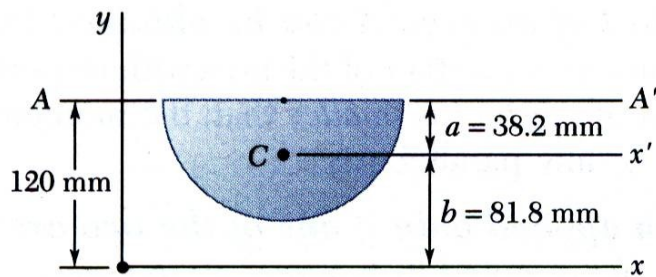
$$I_{AA'} = \frac{1}{8}\pi r^4 = \frac{1}{8}\pi (90)^4 = 2576 \times 10^6 \text{ mm}^4$$

moment of inertia with respect

$$\begin{aligned} I_{x'} &= I_{AA'} - Aa^2 = (2576 \times 10^6) - (1272 \times 10^3) \\ &= 7.20 \times 10^6 \text{ mm}^4 \end{aligned}$$

moment of inertia with

$$\begin{aligned} I_x &= I_x + Ab^2 = 7.20 \times 10^6 + (1272 \times 10^3)(81.8)^2 \\ &= 923 \times 10^6 \text{ mm}^4 \end{aligned}$$



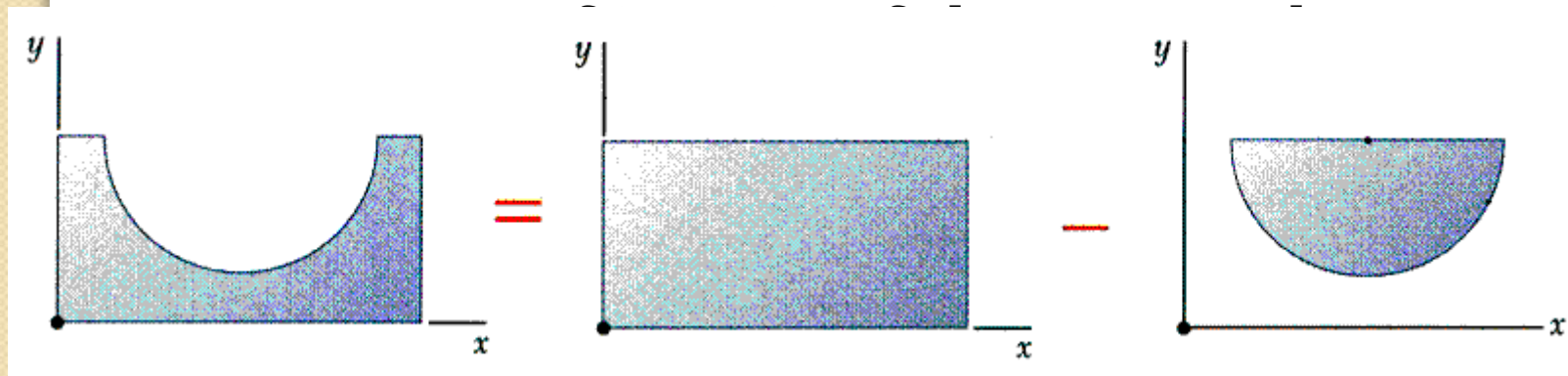
$$a = \frac{4r}{3\pi} = \frac{(4)(90)}{3\pi} = 38.2 \text{ mm}$$

$$b = 120 - a = 81.8 \text{ mm}$$

$$\begin{aligned} A &= \frac{1}{2}\pi r^2 = \frac{1}{2}\pi (90)^2 \\ &= 1272 \times 10^3 \text{ mm}^2 \end{aligned}$$

Sample Problem 9.5

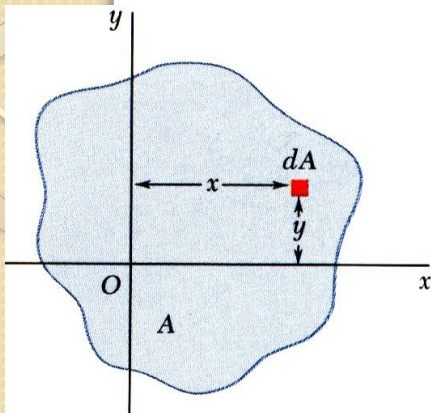
- The moment of inertia of the shaded area is obtained by subtracting the moment of inertia of the half-circle from the



$$I_x = 1382 \times 10^6 \text{ mm}^4 - 923 \times 10^6 \text{ mm}^4$$

$$I_x = 459 \times 10^6 \text{ mm}^4$$

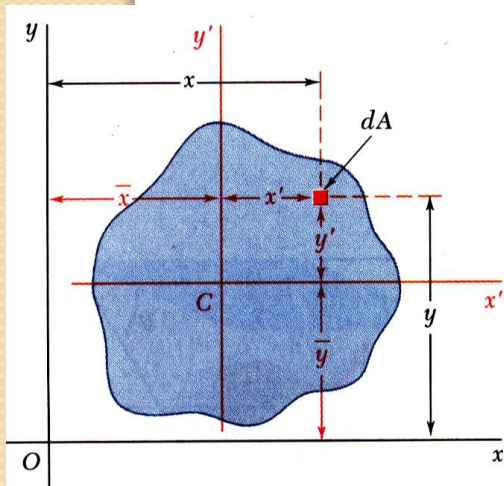
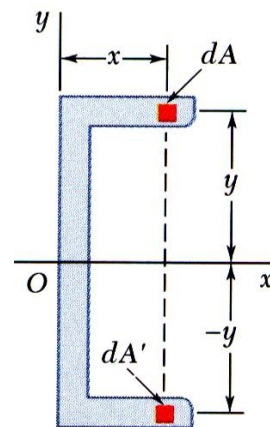
Product of Inertia



- *Product of Inertia:*

$$I_{xy} = \int xy dA$$

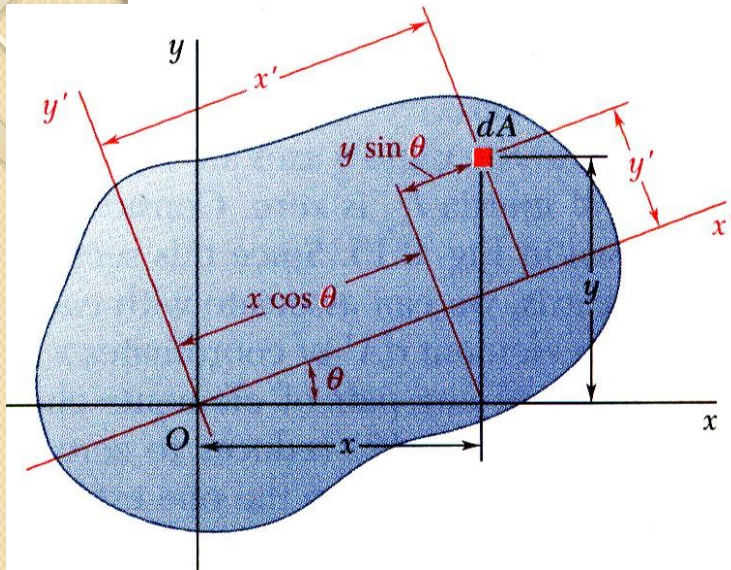
- When the x axis, the y axis, or both are an axis of symmetry, the product of inertia is zero.



- Parallel axis theorem for products of inertia:

$$I_{xy} = \bar{I}_{xy} + \bar{x}\bar{y}A$$

Principal Axes and Principal Moments of Inertia



Given

$$I_x = \int y^2 dA \quad I_y = \int x^2 dA$$

$$I_{xy} = \int xy dA$$

we wish to determine moments and product of inertia with respect to new axes x' and y'

Note:

$$x' = x \cos \theta + y \sin \theta$$

$$y' = y \cos \theta - x \sin \theta$$

The change of axes yields

$$I_{x'} = \frac{I_x + I_y}{2} + \frac{I_x - I_y}{2} \cos 2\theta - I_{xy} \sin 2\theta$$

$$I_{y'} = \frac{I_x + I_y}{2} - \frac{I_x - I_y}{2} \cos 2\theta + I_{xy} \sin 2\theta$$

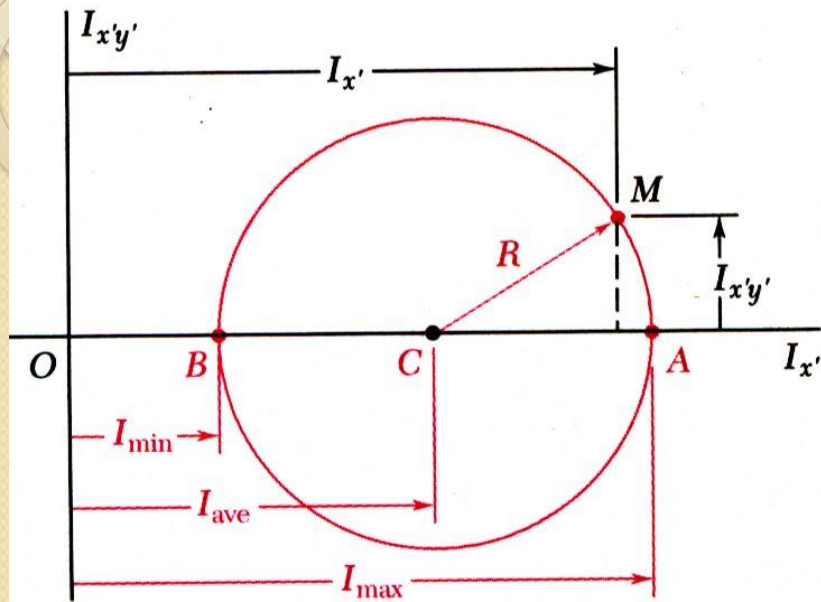
$$I_{x'y'} = \frac{I_x - I_y}{2} \sin 2\theta + I_{xy} \cos 2\theta$$

- The equations for $I_{x'}$ and $I_{y'}$ are the parametric equations for a circle,

$$I_{ave} = \frac{I_x + I_y}{2} \quad R = \sqrt{\left(\frac{I_x - I_y}{2} \right)^2 + I_{xy}^2}$$

- The equations for $I_{y'}$ and $I_{x'y'}$ lead to the same circle.

Principal Axes and Principal Moments of Inertia



$$(I_{x'} - I_{ave})^2 + I_{x'y'}^2 = R^2$$

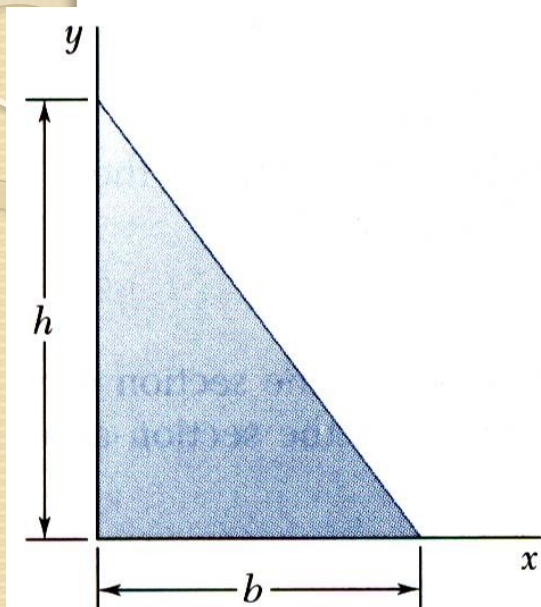
$$I_{ave} = \frac{I_x + I_y}{2} \quad R = \sqrt{\left(\frac{I_x - I_y}{2}\right)^2 + I_{xy}^2}$$

- At the points A and B, $I_{x'y'} = 0$ and $I_{x'}$ is a maximum and minimum, respectively. $I_{ave} \pm R$

$$\tan 2\theta_m = -\frac{2I_{xy}}{I_x - I_y}$$

- The equation for θ_m defines two angles, 90° apart which correspond to the *principal axes* of the area about O.
- I_{max} and I_{min} are the *principal moments of inertia* of the area about O.

Sample Problem 9.6



Determine the product of inertia of the right triangle
(a) with respect to the x and y axes and
(b) with respect to centroidal axes parallel to the x and y axes.

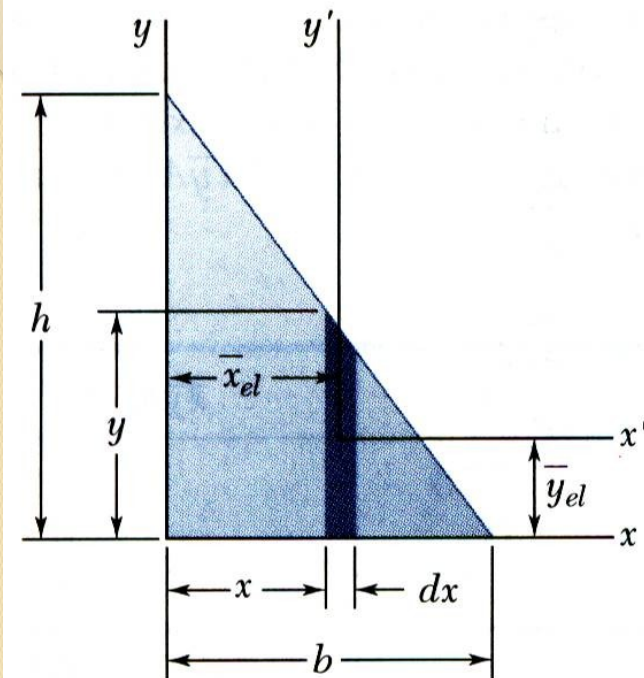
SOLUTION:

- Determine the product of inertia using direct integration with the parallel axis theorem on vertical differential area strips
- Apply the parallel axis theorem to evaluate the product of inertia with respect to the centroidal axes.

Sample Problem 9.6

SOLUTION:

- Determine the product of inertia using direct integration with the parallel axis theorem on vertical differential area strips



$$dA = y dx = h \left(1 - \frac{x}{b} \right) dx$$

$$\bar{x}_{el} = x \quad \bar{y}_{el} = \frac{1}{2} y = \frac{1}{2} h \left(1 - \frac{x}{b} \right)$$

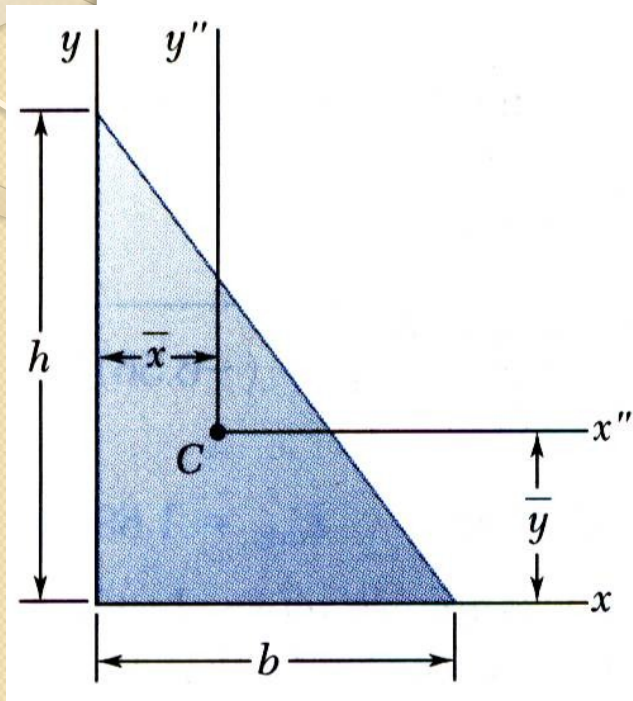
Integrating dI_x from $x = 0$ to $x = b$,

$$I_{xy} = \int dI_{xy} = \int \bar{x}_{el} \bar{y}_{el} dA = \int_0^b x \left(\frac{1}{2} h \right) \left(1 - \frac{x}{b} \right)^2 dx$$

$$= h^2 \int_0^b \left(\frac{x}{2} - \frac{x^2}{b} + \frac{x^3}{2b^2} \right) dx = h^2 \left[\frac{x^2}{4} - \frac{x^3}{3b} + \frac{x^4}{8b^2} \right]_0^b$$

$$I_{xy} = \frac{1}{24} b^2 h^2$$

Sample Problem 9.6



- Apply the parallel axis theorem to evaluate the product of inertia with respect to the centroidal axes.
- $$\bar{x} = \frac{1}{3}b \quad \bar{y} = \frac{1}{3}h$$

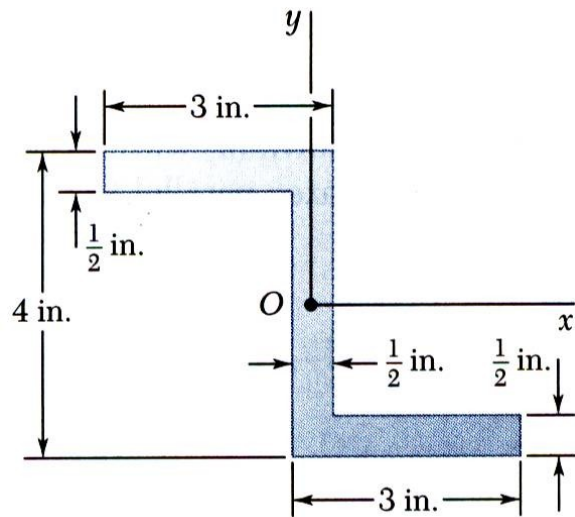
With the results from part a,

$$I_{xy} = \bar{I}_{x''y''} + \bar{x}\bar{y}A$$

$$\bar{I}_{x''y''} = \frac{1}{24}b^2h^2 - \left(\frac{1}{3}b\right)\left(\frac{1}{3}h\right)\left(\frac{1}{2}bh\right)$$

$$\bar{I}_{x''y''} = -\frac{1}{72}b^2h^2$$

Sample Problem 9.7



For the section shown, the moments of inertia with respect to the x and y axes are $I_x = 10.38 \text{ in}^4$ and $I_y = 6.97 \text{ in}^4$.

Determine (a) the orientation of the principal axes of the section about O , and (b) the values of the principal moments of inertia about O .

SOLUTION:

- Compute the product of inertia with respect to the xy axes by dividing the section into three rectangles and applying the parallel axis theorem
- Determine the orientation of the principal axes (Eq. 9.25) and the principal moments of inertia (Eq. 9.27).

Sample Problem 9.7

SOLUTION:

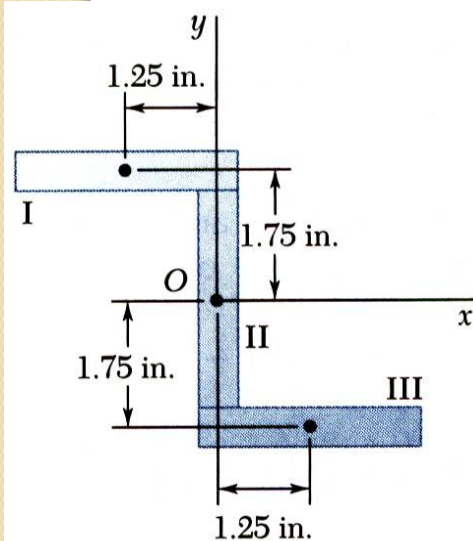
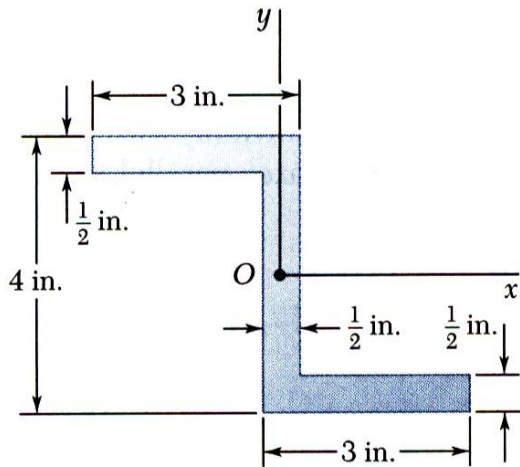
- Compute the product of inertia with respect to the xy axes by dividing the section into three rectangles.

Apply the parallel axis theorem to each rectangle $\sum (I_{x'y'} + \bar{x}\bar{y}A)$

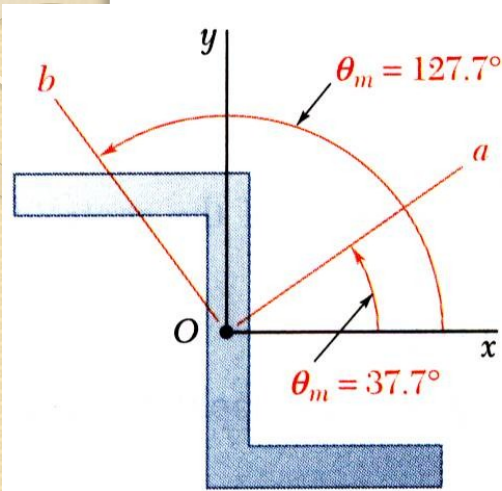
Note that the product of inertia with respect to centroidal axes parallel to the xy axes is zero for each rectangle.

Rectangle	Area, in ²	$\bar{x}$, in.	$\bar{y}$, in.	$\bar{x}\bar{y}A$, in ⁴
I	1.5	- 1.25	+ 1.75	- 3.28
II	1.5	0	0	0
III	1.5	+ 1.25	- 1.75	- 3.28
				$\sum \bar{x}\bar{y}A = - 6.56$

$$I_{xy} = \sum \bar{x}\bar{y}A = - 6.56 \text{ in}^4$$



Sample Problem 9.7



- Determine the orientation of the principal axes (Eq. 9.25) and the principal moments of inertia (Eq. 9.27).

$$\tan 2\theta_m = -\frac{2I_{xy}}{I_x - I_y} = -\frac{2(-6.56)}{1038 - 6.97} = +3.85$$

$$2\theta_m = 75.4^\circ \text{ and } 255.4^\circ$$

$$\theta_m = 37.7^\circ \text{ and } \theta_m = 127.7^\circ$$

$$I_x = 1038 \text{ in}^4$$

$$I_y = 6.97 \text{ in}^4$$

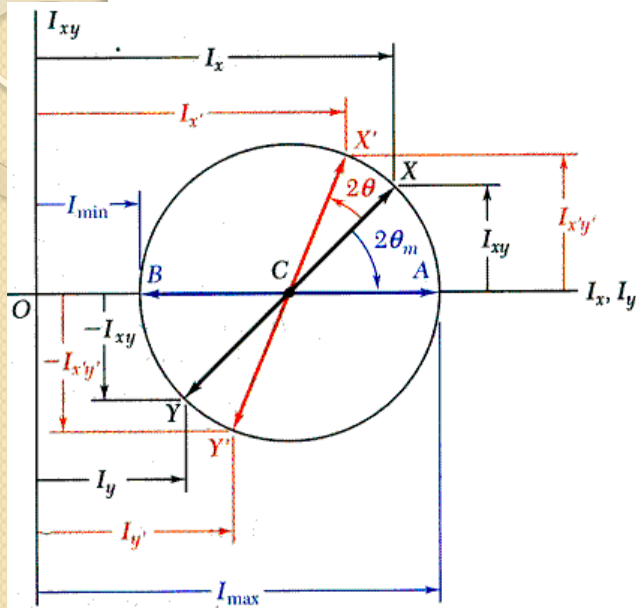
$$I_{xy} = -6.56 \text{ in}^4$$

$$\begin{aligned} I_{\max/\min} &= \frac{I_x + I_y}{2} \pm \sqrt{\left(\frac{I_x - I_y}{2}\right)^2 + I_{xy}^2} \\ &= \frac{1038 + 6.97}{2} \pm \sqrt{\left(\frac{1038 - 6.97}{2}\right)^2 + (-6.56)^2} \end{aligned}$$

$$I_a = I_{\max} = 1545 \text{ in}^4$$

$$I_b = I_{\min} = 1.897 \text{ in}^4$$

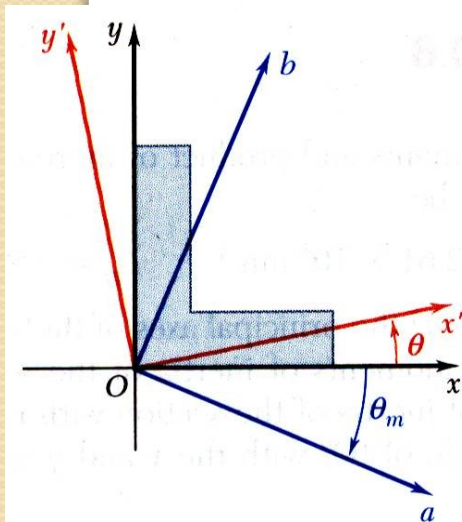
Mohr's Circle for Moments and Products of Inertia



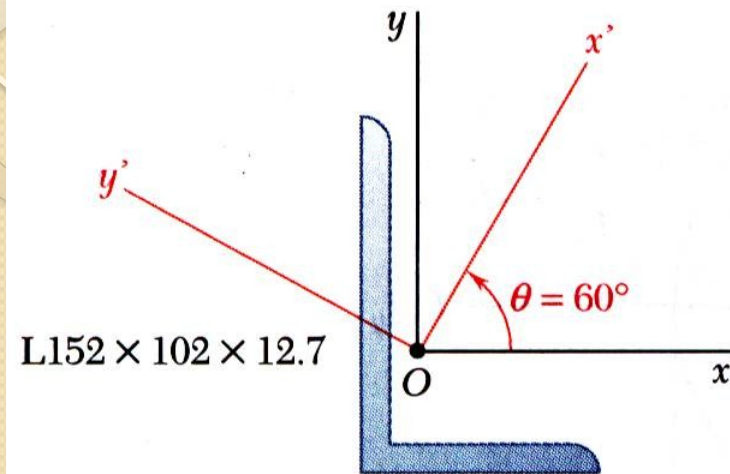
- The moments and product of inertia for an area are plotted as shown and used to construct *Mohr's circle*,

$$I_{ave} = \frac{I_x + I_y}{2} \quad R = \sqrt{\left(\frac{I_x - I_y}{2}\right)^2 + I_{xy}^2}$$

- Mohr's circle may be used to graphically or analytically determine the moments and product of inertia for any other rectangular axes including the principal axes and principal moments and products of inertia.



Sample Problem 9.8



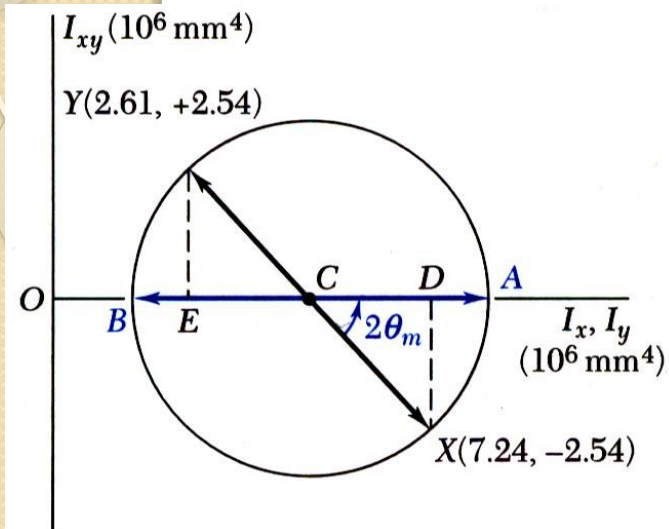
The moments and product of inertia with respect to the x and y axes are $I_x = 7.24 \times 10^6 \text{ mm}^4$, $I_y = 2.61 \times 10^6 \text{ mm}^4$, and $I_{xy} = -2.54 \times 10^6 \text{ mm}^4$.

Using Mohr's circle, determine (a) the principal axes about O , (b) the values of the principal moments about O , and (c) the values of the moments and

SOLUTION:

- Plot the points (I_x, I_{xy}) and $(I_y, -I_{xy})$. Construct Mohr's circle based on the circle diameter between the points.
- Based on the circle, determine the orientation of the principal axes and the principal moments of inertia.
- Based on the circle, evaluate the moments and product of inertia with respect to the $x'y'$ axes.

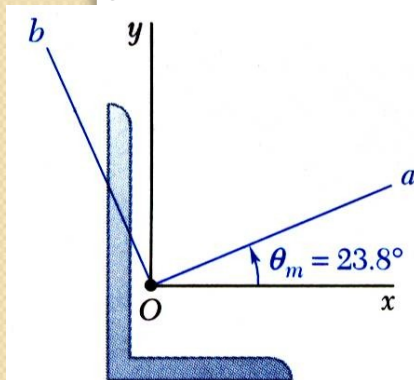
Sample Problem 9.8



$$I_x = 7.24 \times 10^6 \text{ mm}^4$$

$$I_y = 2.61 \times 10^6 \text{ mm}^4$$

$$I_{xy} = -2.54 \times 10^6 \text{ mm}^4$$



SOLUTION:

- Plot the points (I_x, I_{xy}) and $(I_y, -I_{xy})$. Construct Mohr's circle based on the circle diameter between the points.

$$OC = I_{ave} = \frac{1}{2}(I_x + I_y) = 4.925 \times 10^6 \text{ mm}^4$$

$$CD = \frac{1}{2}(I_x - I_y) = 2.315 \times 10^6 \text{ mm}^4$$

$$R = \sqrt{(CD)^2 + (DX)^2} = 3.437 \times 10^6 \text{ mm}^4$$

- Based on the circle, determine the orientation of the principal axes and the principal moments of inertia.

$$\tan 2\theta_m = \frac{DX}{CD} = 1.097 \quad 2\theta_m = 47.6^\circ$$

$$\theta_m = 23.8^\circ$$

$$I_{\max} = OA = I_{ave} + R$$

$$I_{\max} = 8.36 \times 10^6 \text{ mm}^4$$

$$I_{\min} = OB = I_{ave} - R$$

$$I_{\min} = 1.49 \times 10^6 \text{ mm}^4$$

Sample Problem 9.8

- Based on the circle, evaluate the moments and product of inertia with respect to the $x'y'$ axes.

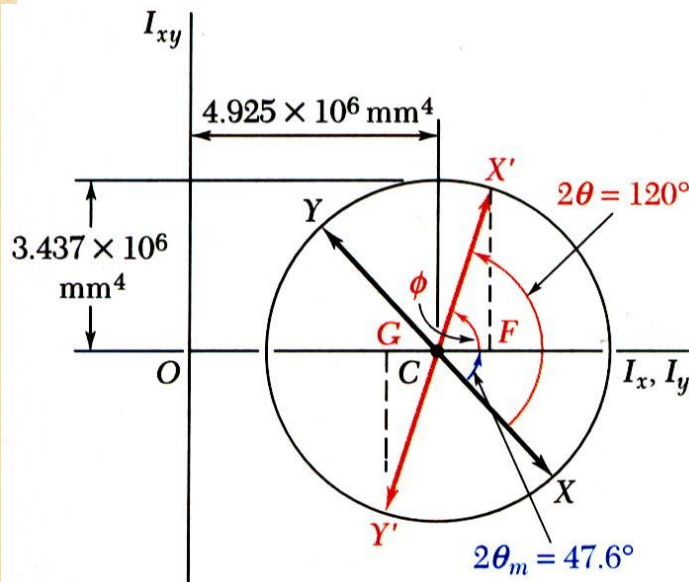
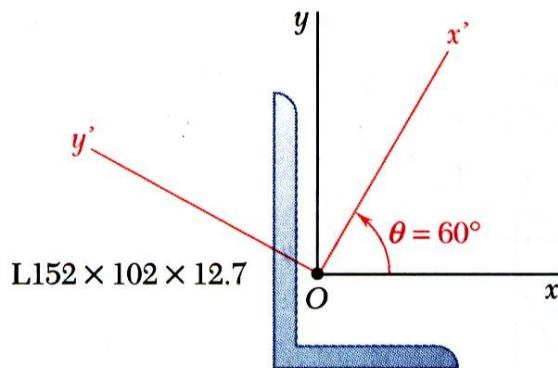
The points X' and Y' corresponding to the x' and y' axes are obtained by rotating CX and CY counterclockwise through an angle $\Theta = 2(60^\circ) = 120^\circ$. The angle that CX' forms with the x' axis is $\phi = 120^\circ - 47.6^\circ = 72.4^\circ$.

$$I_{x'} = OG = OC - CY' \cos \phi = I_{ave} - R \cos 72.4^\circ$$

$$I_{x'} = 3.89 \times 10^6 \text{ mm}^4$$

$$I_{x'y'} = FX' = CY' \sin \phi = R \sin 72.4^\circ$$

$$I_{x'y'} = 3.28 \times 10^6 \text{ mm}^4$$



$$OC = I_{ave} = 4.925 \times 10^6 \text{ mm}^4$$

$$R = 3.437 \times 10^6 \text{ mm}^4$$