Chapter 2

Basic Laws

Basic Laws - Chapter 2

2.1 Ohm’s Law.
2.2 Nodes, Branches, and Loops.
2.3 Kirchhoff’s Laws.
2.4 Series Resistors and Voltage Division.
2.5 Parallel Resistors and Current Division.
2.6 Wye-Delta Transformations.

Chapter 2-Introduction

- basic concepts such as current, voltage, and power in an electric circuit.
- To actually determine the values of these variables in a given circuit requires that we understand some fundamental laws that govern electric circuits.
- These laws, known as Ohm’s law and Kirchhoff’s laws.

Introduction

- Two of the common Circuit Elements
- Resistor: is an element that resists the flow of electricity.
  - Resistors provides resistance
  - They oppose the flow of electricity
- Capacitor: is an element that stores charge for use later.
  - They store energy in an electricity field.
  - Provides capacitance

Resistance

- All materials resist the flow of current
- Resistance is usually represented by the variable R
- Depends on geometry and resistivity of the material
  - Copper 1.673e-8 ohm-meters
  - Lead 20.648e-8 ohm-meters
- Ohms per square for sheet.

A cylinder of length ℓ and cross-sectional area A has a resistance:

$$R = \rho \frac{\ell}{A}$$

where

$R =$ resistance of an element in ohms (Ω)
$\rho =$ resistivity of the material in ohm-meters
$\ell =$ length of cylindrical material in meters
$A =$ Cross sectional area of material in meters^2

Resistance: Basic Concepts and Assumptions

- We will always measure resistance in Ohms
- Ohms are denoted by the Greek letter Omega: Ω
- Examples: 50 Ω, 1 kΩ, 2.5 MΩ
- Conductors (e.g. wires) have very low resistance ($< 0.1\ \Omega$) that can usually be neglected (i.e. we will assume wires have zero resistance)
- Insulators (e.g. air) have very large resistance (> 50 MΩ) that can usually be ignored (omitted from circuit for analysis)
- Resistors have a medium range of resistance and must be accounted for in the circuit analysis
- Conceptually, a light bulb is similar to a resistor
- Properties of the bulb control how much current flows and how much power is dissipated (absorbed & emitted as light and heat)

Resistance

- If we increase the voltage, we increase the current.
- V is promotional to the I.
- The constant of proportionality we call the resistance, R. V= I * R---------- Ohm's law
- Units R= V/I, ohms=Volts/Amp and I= V/R
- The amount of resistance does not depends on the voltage or current. V=IR relates voltage to current.
- If you double the voltage you will double the current, not change the resistance.
- The amount of resistance depends on the material and shape of the wires.

Ohms Law

- Materials in general have a characteristic behavior of resisting the flow of electric charge.
- This physical property, or ability to resist current, is known as *resistance* and is represented by the symbol *R*.
- The resistance of any material with a uniform cross-sectional area *A* depends on *A* and its length *l*.

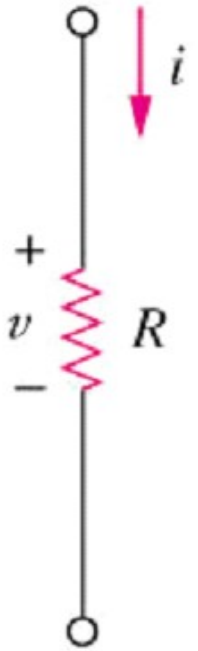
2.1 Ohms Law (1)

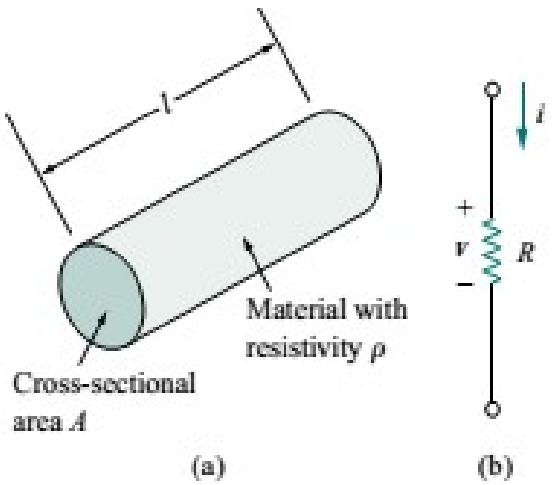
- Ohm's law states that the voltage across a resistor is directly proportional to the current I flowing through the resistor.

- Mathematical expression for Ohm's Law is as follows:

$$v = iR$$

- Two extreme possible values of R:
  0 (zero) and ∞ (infinite) are related with two basic circuit concepts: short circuit and open circuit.

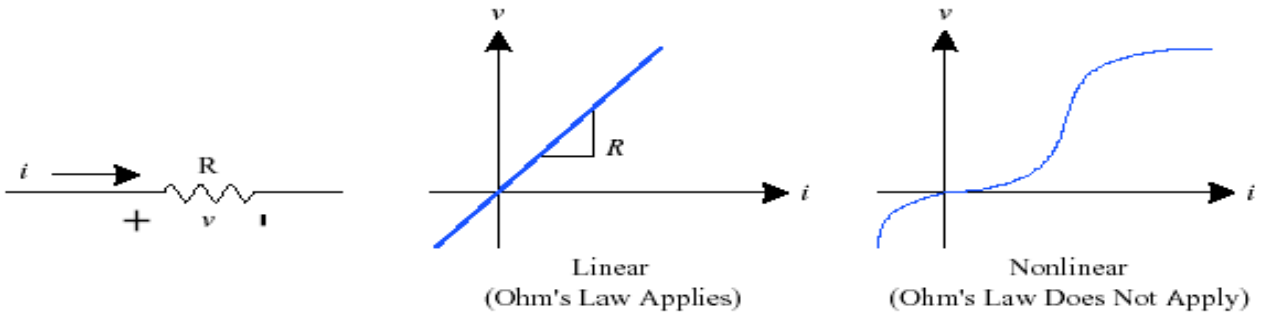




$$R = \rho \frac{\ell}{A}$$

- where ρ is known as the *resistivity* of the material in ohm-meters.
- Good conductors, such as copper and aluminum, have low resistivity's While insulators, such as mica and paper, have high resistivity's.

Ohm's Law



- As with all circuit elements, we need to know how the current through and voltage across the device are related
- Many materials have a complicated nonlinear relationship (including light bulbs): $v = \pm f(i)$
- Materials with a linear relationship satisfy Ohm's law: $v = \pm mi$
- The slope, m, is equal to the resistance of the element
- Ohm's Law: $v = \pm iR$
- Sign, $\pm$, is determined by the passive sign convention (PSC)

Ohms Law

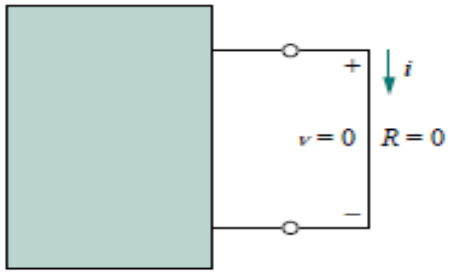
- Ohm's law states that the voltage *v* across a resistor is directly proportional to the current *i* flowing through the resistor. $v \propto I$

- Ohm defined the constant of proportionality for a resistor to be the resistance, *R*. (The resistance is a material property which can change if the internal or external conditions of the element are altered, e.g., if there are changes in the temperature.)

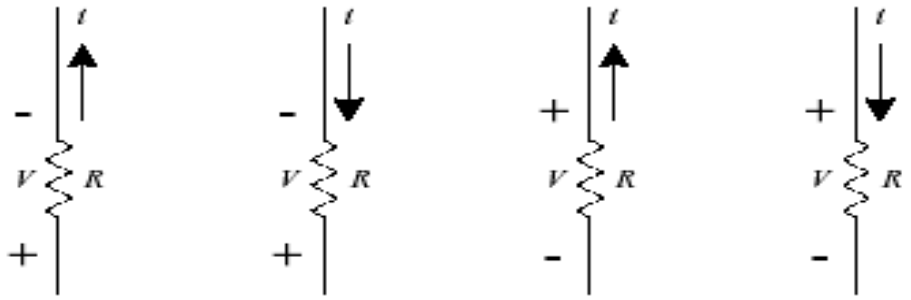
- The resistance *R* of an element denotes its ability to resist the flow of electric current; it is measured in ohms.

$$R = \frac{v}{i}$$

$$1\ \Omega = 1\ \text{V/A}$$

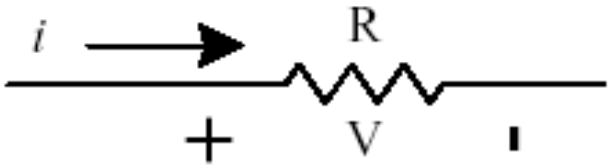


Resistors & Passive Sign Convention



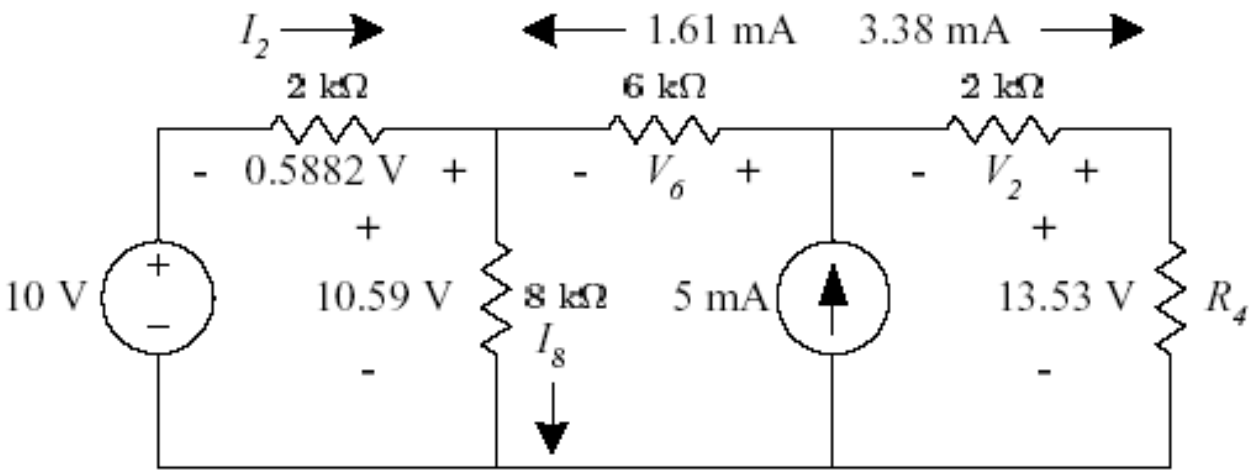
- Recall that relationships between current and voltage are sign sensitive
- Passive Sign Convention: Current enters the positive terminal of an element
  - If **PSC** satisfied: $v = iR$
  - If **PSC** not satisfied: $v = -iR$

Other Eq. derived from Ohm's Law



- Ohm's law implies: $i = \frac{v}{R}$
- Recall $p = \pm vi$. Therefore
  - $p = v\frac{v}{R} = \frac{v^2}{R}$
  - $p = (iR)i = i^2R$
- Resistors cannot produce power so the power absorbed by a resistor will always be positive
- $1\ \Omega = 1\ \mathrm{V/A}$

Example: Ohm's Law



I_2 =

V_6 =

R_4 =

V_2 =

I_8 =

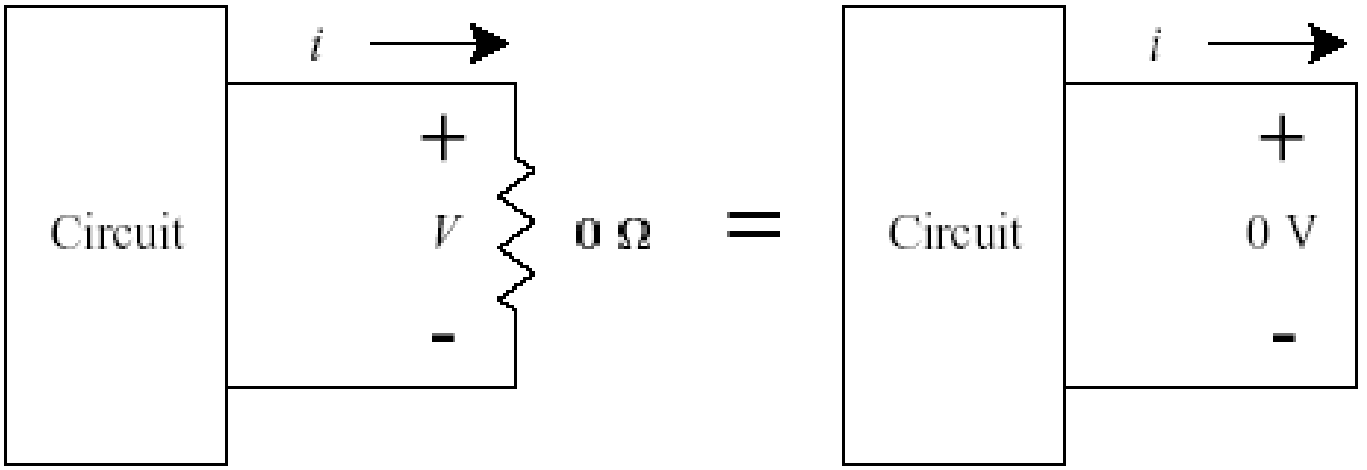
An electric iron draws 2 A at 120 V. Find its resistance.

Solution:

From Ohm's law,

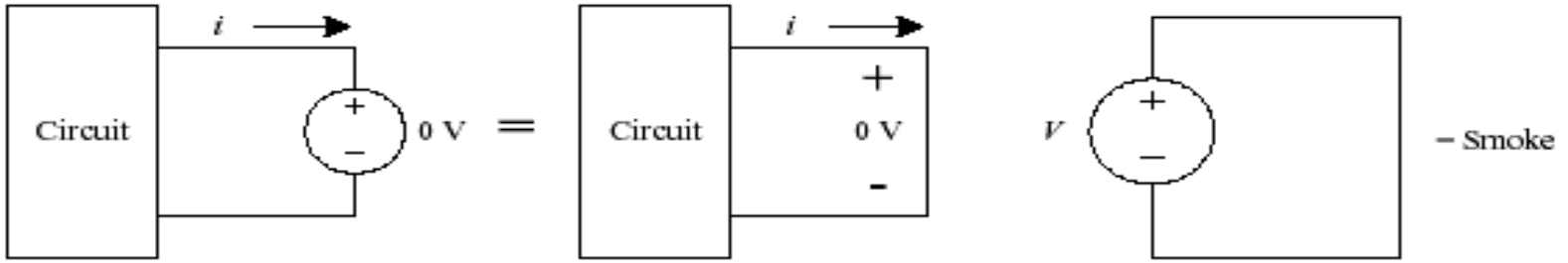
$$R = \frac{v}{i} = \frac{120}{2} = 60\ \Omega$$

Short Circuit as Zero Resistance



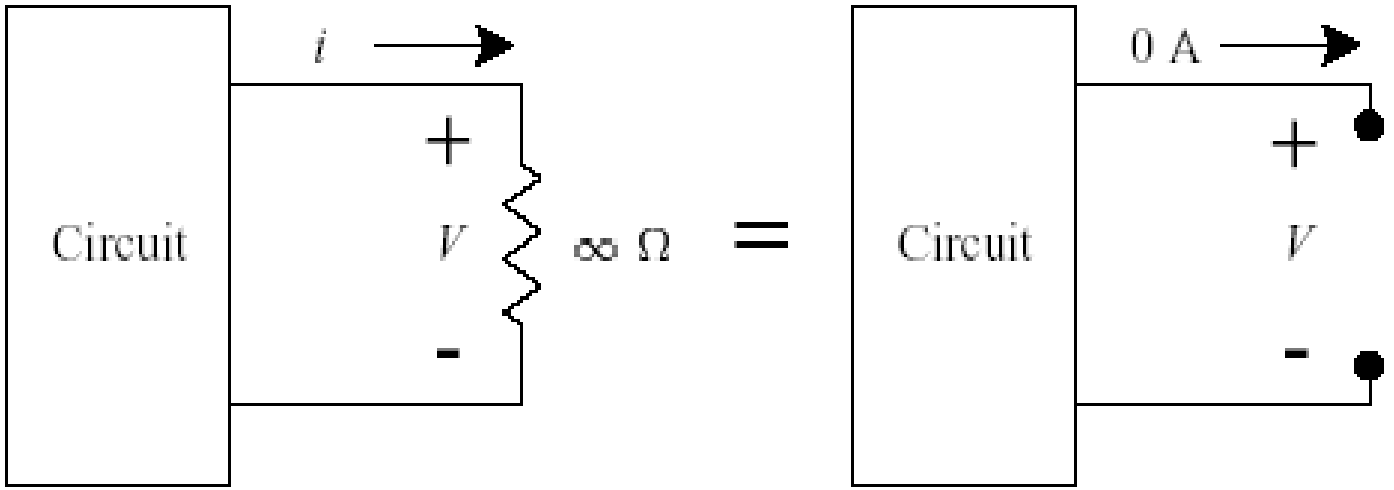
- An element (or wire) with $R = 0$ is called a short circuit
- Just drawn as a wire (line)

Short Circuit as Voltage Source (0V)



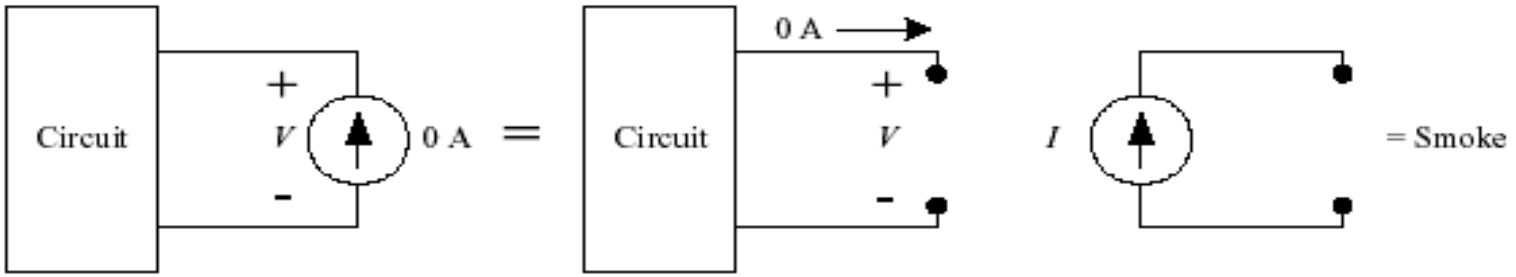
- An ideal voltage source $V_s = 0$ V is also equivalent to a short circuit
- Since $v = iR$ and $R = 0$, $v = 0$ regardless of i
- Could draw a source with $V_s = 0$ V, but is not done in practice
- Can not connect a voltage source to a short circuit
- Irresistible force meets immovable object
- In practice, the wire usually wins and the voltage source melts (if not protected)

Open Circuit



- An element (or wire) with $R = \infty$ is called a open circuit
- Just omitted

Open Circuit as Current Source (0 A)



- An ideal current source $I = 0$ A is also equivalent to an open circuit
- Could draw a source with $I = 0$ A, but is not done in practice
- Cannot connect a current source to an open circuit (spark coil)
- Irresistible force meets immovable object
- In practice, you blow the current source (if not protected)
- The insulator (air) usually wins. Else, sparks fly.

2.1 Ohms Law (2)

- *Conductance* is the ability of an element to conduct electric current; it is the reciprocal of resistance R and is measured in mhos or siemens.

$$G = \frac{1}{R} = \frac{i}{v}$$

- The power dissipated by a resistor:

$$p = vi = i^2 R = \frac{v^2}{R}$$

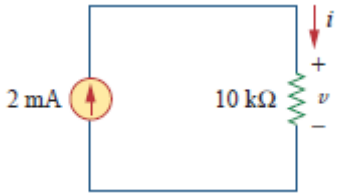
Conductance

- Sometimes conductance is specified instead of resistance
- Inverse of resistance
- $G = \frac{1}{R} = \frac{i}{v}$
- Units: siemens (S) or mhos ($\mho$)
- $1\ \mathrm{S} = 1\ \mho = 1\ \mathrm{A/V}$
- In words: the ability of an element to conduct electric current

$$v = Ri \qquad p = vi = i^2R = \frac{v^2}{R}$$

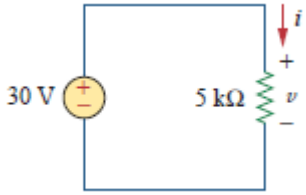
$$i = Gv \qquad p = vi = v^2G = \frac{i^2}{G}$$

For the circuit shown in Fig. 2.9, calculate the voltage v, the conductance G, and the power p.



Answer: 20 V, 100 μS, 40 mW.

In the circuit shown in Fig. 2.8, calculate the current i, G, and the power p.



Solution:
The voltage across the resistor is the same as the sourc
because the resistor and the voltage source are conne
pair of terminals. Hence, the current is

$$i = \frac{v}{R} = \frac{30}{5 \times 10^3} = 6 \text{ mA}$$

The conductance is

$$G = \frac{1}{R} = \frac{1}{5 \times 10^3} = 0.2 \text{ mS}$$

We can calculate the power in various ways using either Eqs. (1.7), (2.10), or (2.11).

$$p = vi = 30(6 \times 10^{-3}) = 180 \text{ mW}$$

or

$$p = i^2R = (6 \times 10^{-3})^2 5 \times 10^3 = 180 \text{ mW}$$

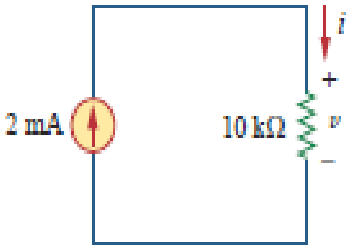
or

$$p = v^2G = (30)^2 0.2 \times 10^{-3} = 180 \text{ mW}$$

For the circuit shown in Fig. 2.9, calculate the voltage *v, the conductance G, and the power p.*

G= i/R , V= iR , p=vi

Answer: 20 V, 100 mS, 40 mW.



A voltage source of $20 \sin \pi t$ V is connected across a 5-kΩ resistor. Find the current through the resistor and the power dissipated.

Solution:

$$i = \frac{v}{R} = \frac{20 \sin \pi t}{5 \times 10^3} = 4 \sin \pi t \text{ mA}$$

Hence,

$$p = vi = 80 \sin^2 \pi t \text{ mW}$$

A resistor absorbs an instantaneous power of $20 \cos^2 t$ mW when connected to a voltage source $v = 10 \cos t$ V. Find i and R.

Answer: $2 \cos t$ mA, 5 kΩ.

Circuit Building Blocks

- Before we can begin analysis, we need a common language and framework for describing circuits
- For this course, networks and circuits are the same
- Networks are composed of nodes, branches, and loops

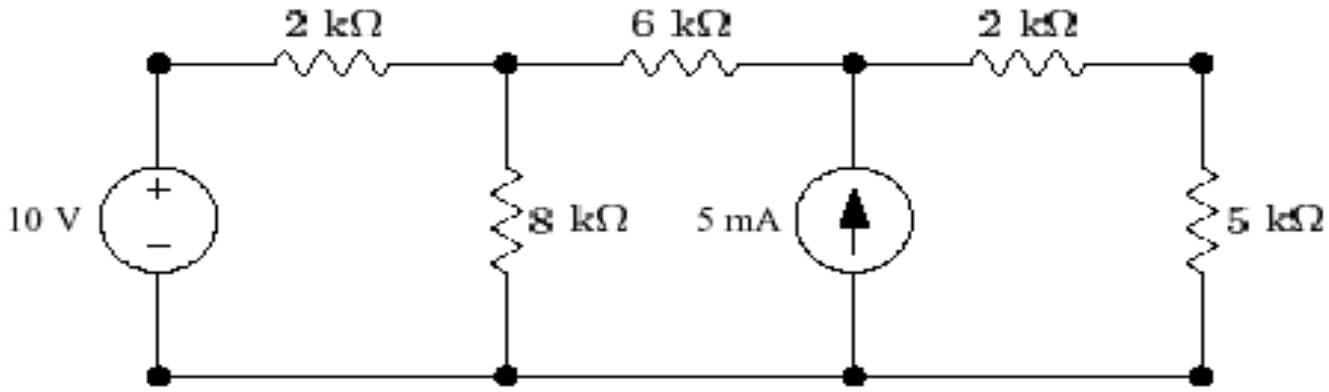
2.2 Nodes, Branches and Loops (1)

- A branch represents a single element such as a voltage source or a resistor.
- A node is the point of connection between two or more branches.
- A loop is any closed path in a circuit.

- A network with b branches, n nodes, and l independent loops will satisfy the fundamental theorem of network topology:

$$b = l + n - 1$$

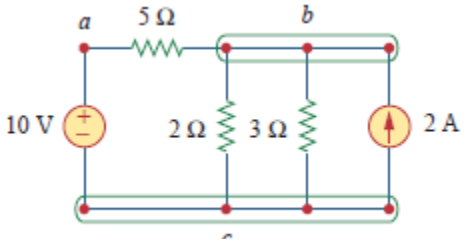
Branches



Example: How many branches?

- Branch: a single two-terminal element in a circuit
- Segments of wire are not counted as elements
- Examples: voltage source, resistor, current source

Nodes, branch & loop



In other words, a branch represents any two-terminal element. The circuit in Fig. 2.10 has five branches, namely, the 10-V voltage source, the 2-A current source, and the three resistors.

A node is the point of connection between two or more branches.

Type equation here.

A node is usually indicated by a dot in a circuit. If a short circuit (a connecting wire) connects two nodes, the two nodes constitute a single node. The circuit in Fig. 2.10 has three nodes *a*, *b*, and *c*. Notice that the three points that form node *b* are connected by perfectly conducting wires and therefore constitute a single point. The same is true of the four points forming node *c*. We demonstrate that the circuit in Fig. 2.10 has only three nodes by redrawing the circuit in Fig. 2.11. The two circuits in Figs. 2.10 and 2.11 are identical. However, for the sake of clarity, nodes *b* and *c* are spread out with perfect conductors as in Fig. 2.10.

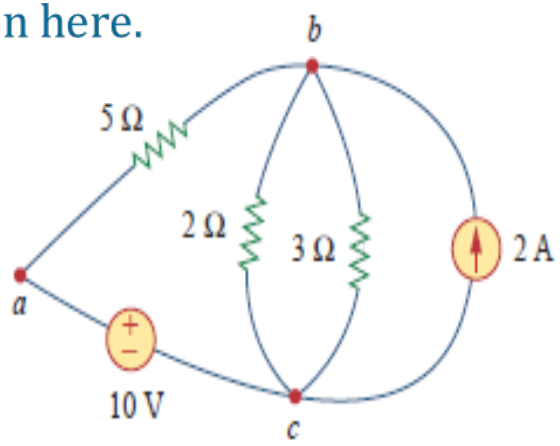


Figure 2.11
The three-node circuit of Fig. 2.10 is redrawn.

loops

A loop is any closed path in a circuit.

A loop is a closed path formed by starting at a node, passing through a set of nodes, and returning to the starting node without passing through any node more than once. A loop is said to be *independent* if it contains at least one branch which is not a part of any other independent loop. Independent loops or paths result in independent sets of equations.

It is possible to form an independent set of loops where one of the loops does not contain such a branch. In Fig. 2.11, *abca* with the 2Ω resistor is independent. A second loop with the 3Ω resistor and the current source is independent. The third loop could be the one with the 2Ω resistor in parallel with the 3Ω resistor. This does form an independent set of loops.

A network with b branches, n nodes, and l independent loops will satisfy the fundamental theorem of network topology:

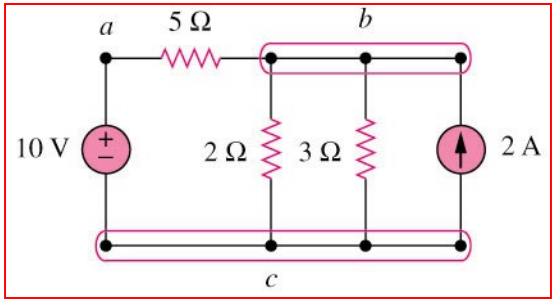
$$b = l + n - 1 \qquad (2.12)$$

As the next two definitions show, circuit topology is of great value to the study of voltages and currents in an electric circuit.

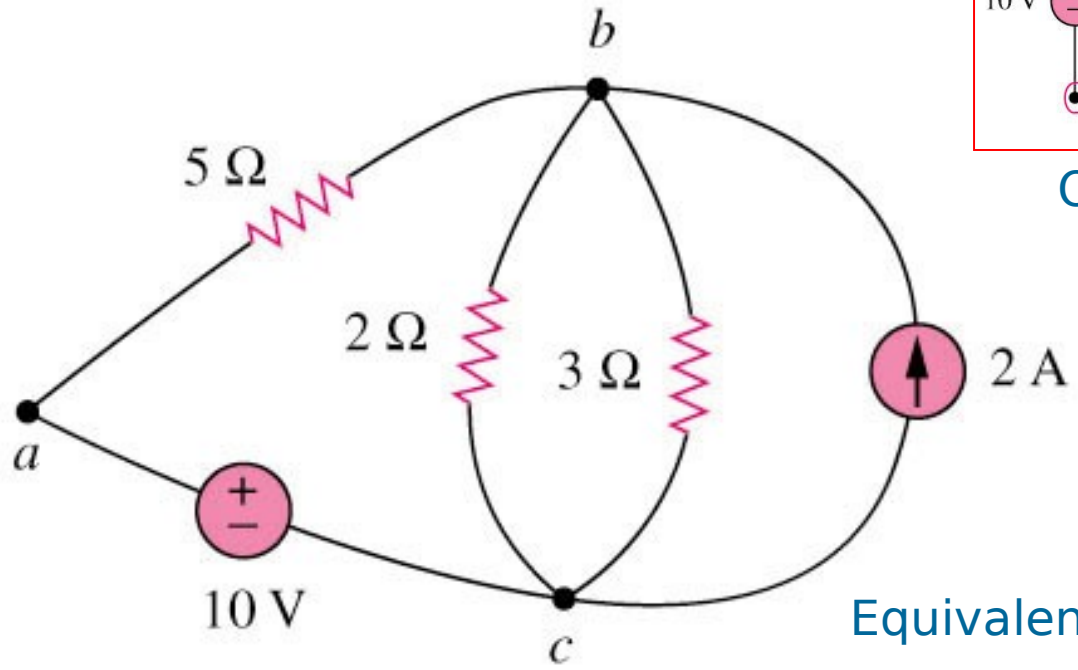
Two or more elements are in **series** if they exclusively share a single node and consequently carry the same current.
Two or more elements are in **parallel** if they are connected to the same two nodes and consequently have the same voltage across them.

2.2 Nodes, Branches and Loops (2)

Example 1



Original circuit



Equivalent circuit

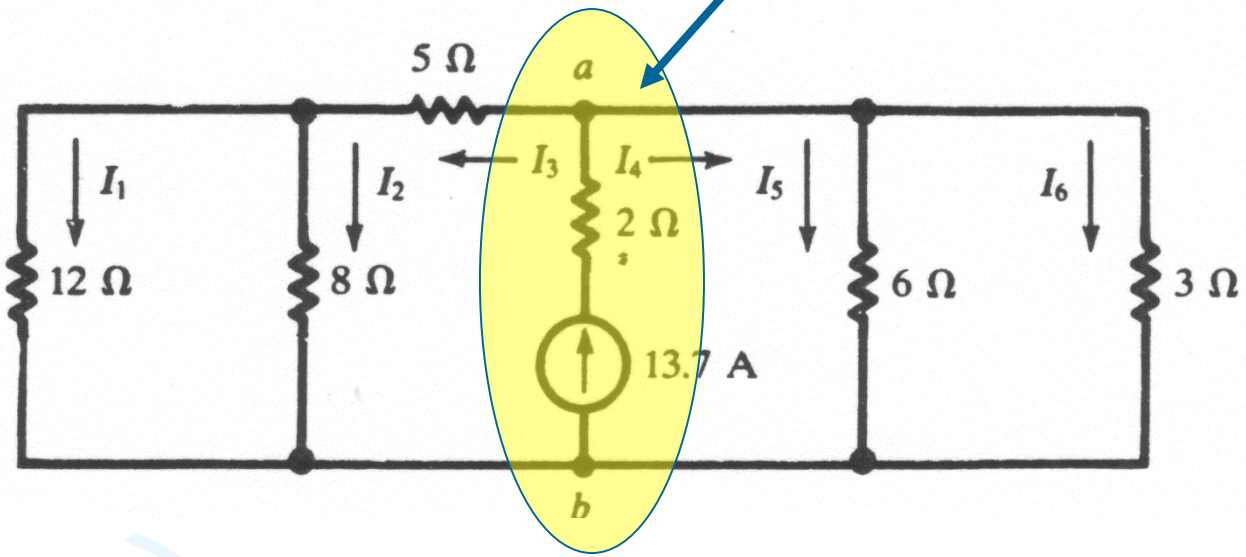
How many branches, nodes and loops are there?

- For example, the closed path abca containing the 2 resistor in Fig. 2.11 is a loop. Another loop is the closed path bcb containing the 3- resistor and the current source. Although one can identify six loops in Fig. 2.11, only three of them are independent.

2.2 Nodes, Branches and Loops (3)

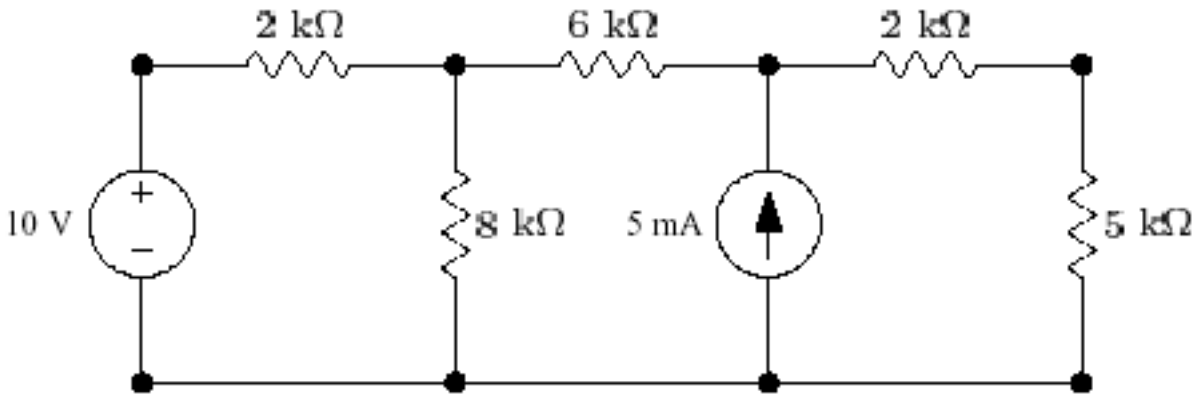
Example 2

Should we consider it as one branch or two branches?



How many branches, nodes and loops are there?

Nodes



Example: How many nodes? How many essential nodes?

- Node: the point of connection between two or more branches
- May include a portion of the circuit (more than a single point)
- Essential Node: the point of connection between three or more branches

Determine the number of branches and nodes in the circuit shown in Fig. 2.12. Identify which elements are in series and which are in parallel.

Solution:

Since there are four elements in the circuit, the circuit has four branches: 10 V, 5 Ω, 6 Ω, and 2 A. The circuit has three nodes as identified in Fig. 2.13. The 5-Ω resistor is in series with the 10-V voltage source because the same current would flow in both. The 6-Ω resistor is in parallel with the 2-A current source because both are connected to the same nodes 2 and 3.

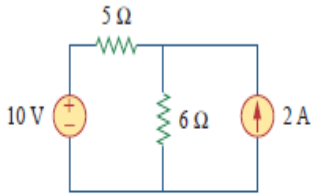


Figure 2.12
For Example 2.4.

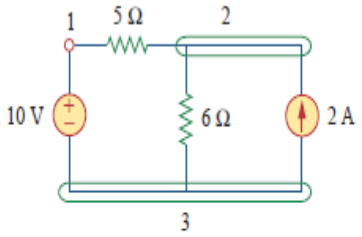
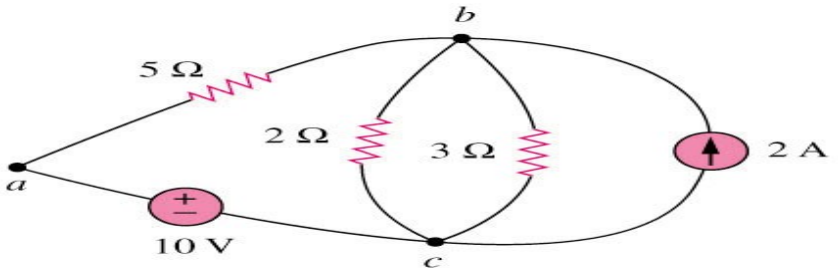


Figure 2.13
The three nodes in the circuit of Fig. 2.12.



It is possible to form an independent set of loops where one of the loops does not contain such a branch. In Fig. 2.11, *abca* with the 2Ω resistor is independent. A second loop with the 3Ω resistor and the current source is independent. The third loop could be the one with the 2Ω resistor in parallel with the 3Ω resistor. This does form an independent set of loops.

A network with b branches, n nodes, and l independent loops will satisfy the fundamental theorem of network topology:

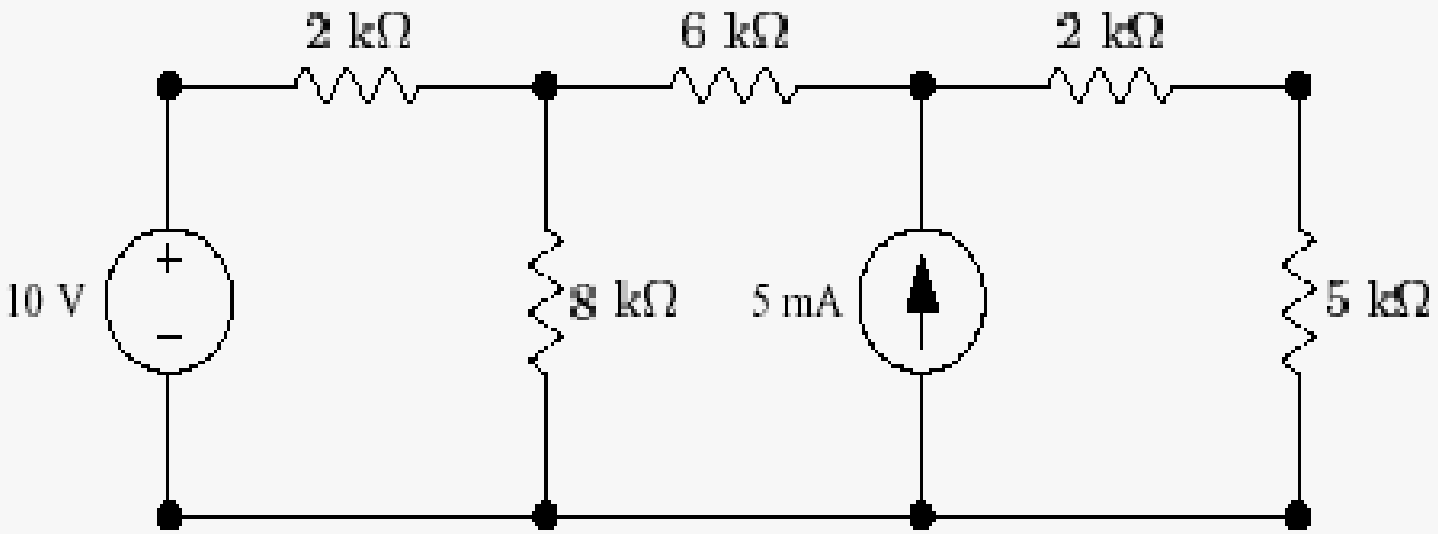
$$b = l + n - 1 \tag{2.12}$$

As the next two definitions show, circuit topology is of great value to the study of voltages and currents in an electric circuit.

Two or more elements are in series if they exclusively share a single node and consequently carry the same current.
Two or more elements are in parallel if they are connected to the same two nodes and consequently have the same voltage across them.

Elements are in series when they are chain-connected or connected sequentially, end to end. For example, two elements are in series if they share one common node and no other element is connected to that common node. Elements in parallel are connected to the same pair of terminals. Elements may be connected in a way that they are neither in series nor in parallel. In the circuit shown in Fig. 2.10, the voltage source and the 5-Ω resistor are in series because the same current will flow through them. The 2-Ω resistor, the 3-Ω resistor, and the current source are in parallel because they are connected to the same two nodes b and c and consequently have the same voltage across them. The 5-Ω and 2-Ω resistors are neither in series nor in parallel with each other.

Loops



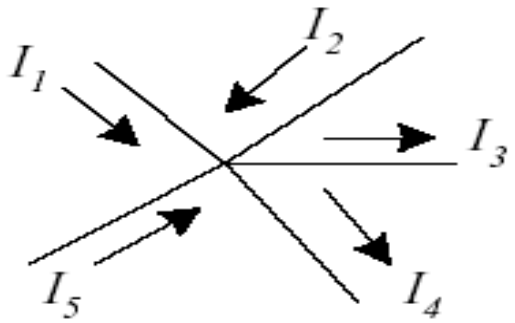
Example: How many loops?

- Loop: any closed path in a circuit

Overview of Kirchhoff's Laws

- The foundation of circuit analysis is
  - The defining equations for circuit elements (e.g. Ohm's law)
  - Kirchhoff's current law (KCL)
  - Kirchhoff's voltage law (KVL)
- The defining equations tell us how the voltage and current within a circuit element are related
- Kirchhoff's laws tell us how the voltages and currents in different branches are related

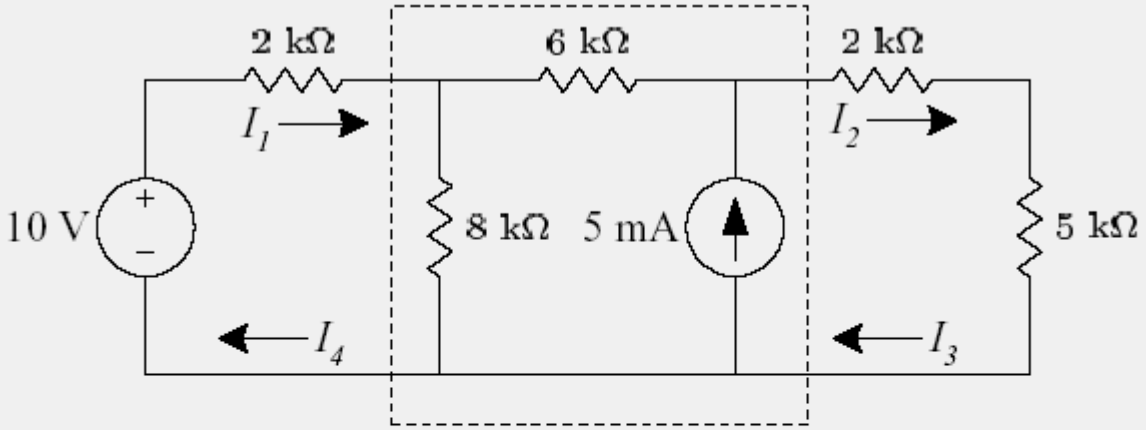
Kirchhoff's Current Law



$$I_1 + I_2 - I_3 - I_4 + I_5 = 0$$
$$I_1 + I_2 + I_5 = I_3 + I_4$$

- Kirchhoff's Current Law (KCL): the algebraic sum of currents entering a node (or a closed boundary) is zero
- The sum of currents entering a node is equal to the sum of the currents leaving a node
- Common sense:
  - All of the electrons have to go somewhere
  - The current that goes in, has to come out some place
- Based on law of conservation of charge and $\nabla j = \frac{\partial \rho}{\partial t} = 0$

Kirchhoff's Current Law for Boundaries



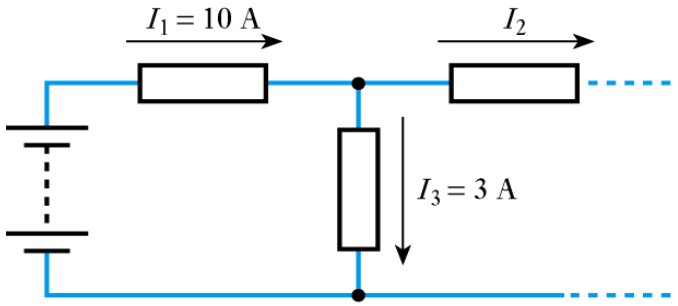
$$I_1 - I_2 + I_3 - I_4 = 0$$

$$I_1 + I_3 = I_2 + I_4$$

- KCL also applies to closed boundaries for *all* circuits

Cont…

- *At any instant the algebraic sum of the currents flowing into any junction in a circuit is zero*



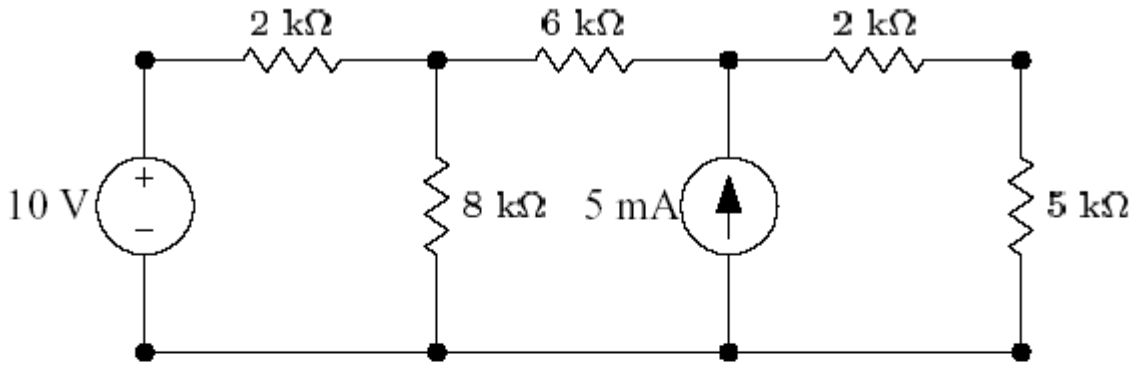
$$I_1 - I_2 - I_3 = 0$$

$$I_2 = I_1 - I_3$$

$$= 10 - 3$$

$$= 7 \text{ A}$$

KCL - Example



Apply KCL to each essential node in the circuit.

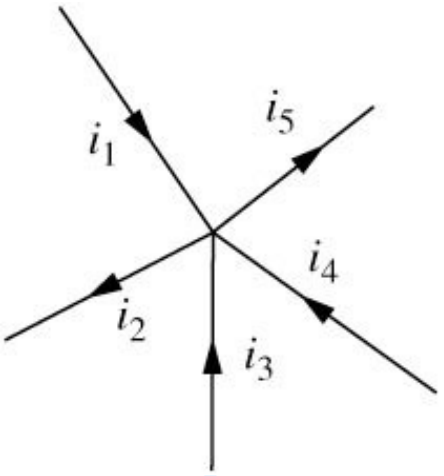
Essential Node 1:

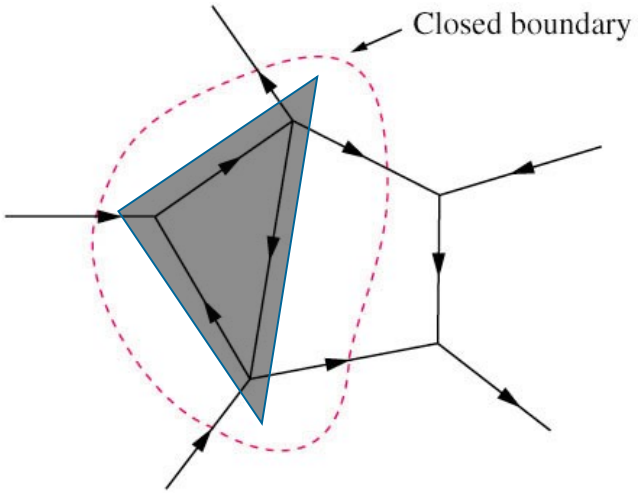
Essential Node 2:

Essential Node 3:

2.3 Kirchhoff's Laws (1)

- Kirchhoff's current law (KCL) states that the algebraic sum of currents entering a node (or a closed boundary) is zero.



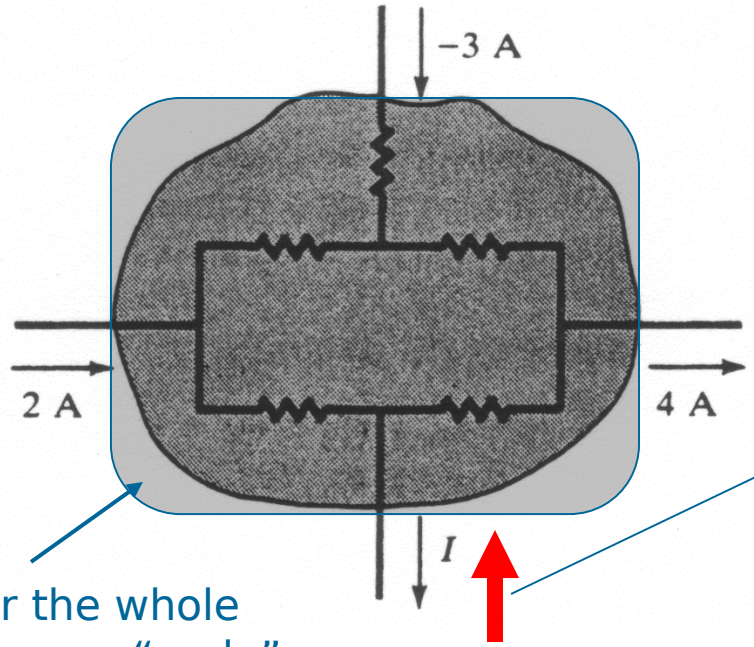


Mathematically, $$\sum_{n=1}^{N} i_n = 0$$

2.3 Kirchhoff's Laws (2)

Example 4

- Determine the current I for the circuit shown in the figure below.



We can consider the whole enclosed area as one "node".

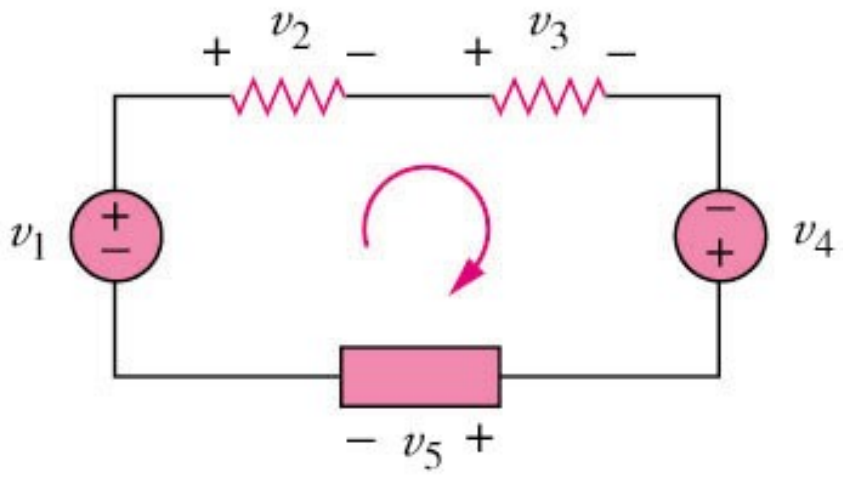
I + 4-(-3)-2 = 0

⇒I = -5A

This indicates that the actual current for I is flowing in the opposite direction.

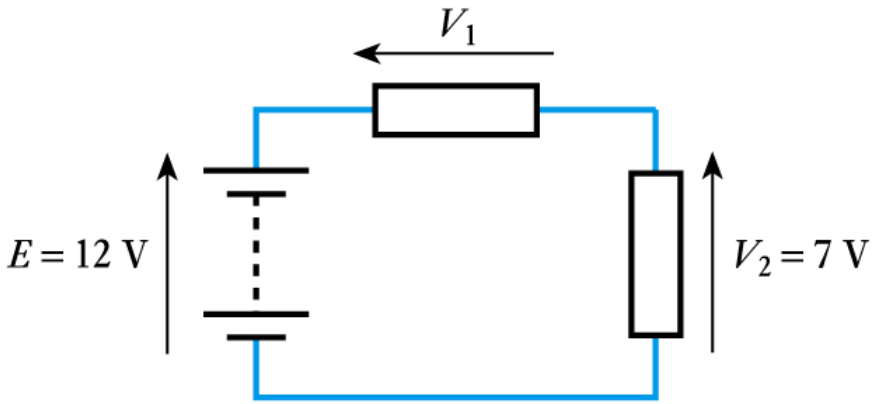
2.3 Kirchhoff's Laws (3)

- Kirchhoff's voltage law (KVL) states that the algebraic sum of all voltages around a closed path (or loop) is zero.



Mathematically, $$\sum_{m=1}^{M} v_n = 0$$

- *At any instant the algebraic sum of the voltages around any loop in a circuit is zero*
- For example



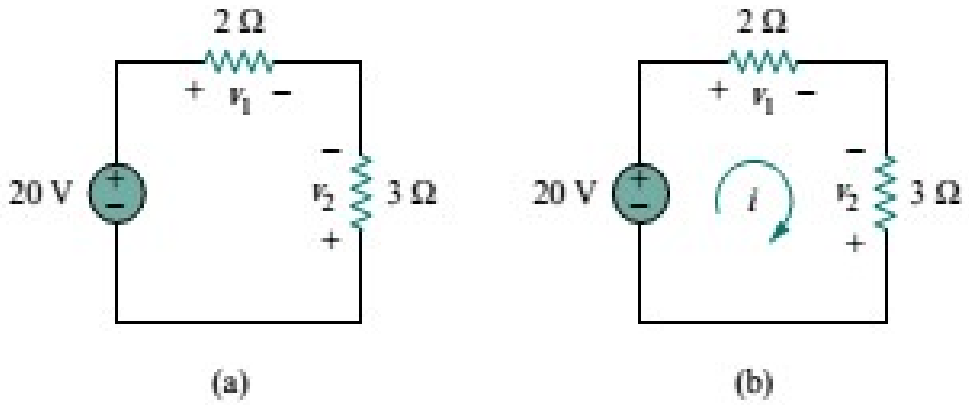
$$E - V_1 - V_2 = 0$$

$$V_1 = E - V_2$$

$$= 12 - 7$$

$$= 5 \text{ V}$$

For the circuit find voltage v1 and v2



Solution:

To find v_1 and v_2, we apply Ohm's law and Kirchhoff's voltage law Assume that current i flows through the loop

From Ohm's law,

$$v_1 = 2i, \qquad v_2 = -3i \tag{2.5.1}$$

Applying KVL around the loop gives

$$-20 + v_1 - v_2 = 0 \tag{2.5.2}$$

Substituting Eq. (2.5.1) into Eq. (2.5.2), we obtain

$$-20 + 2i + 3i = 0 \quad \text{or} \quad 5i = 20 \implies i = 4 \text{ A}$$

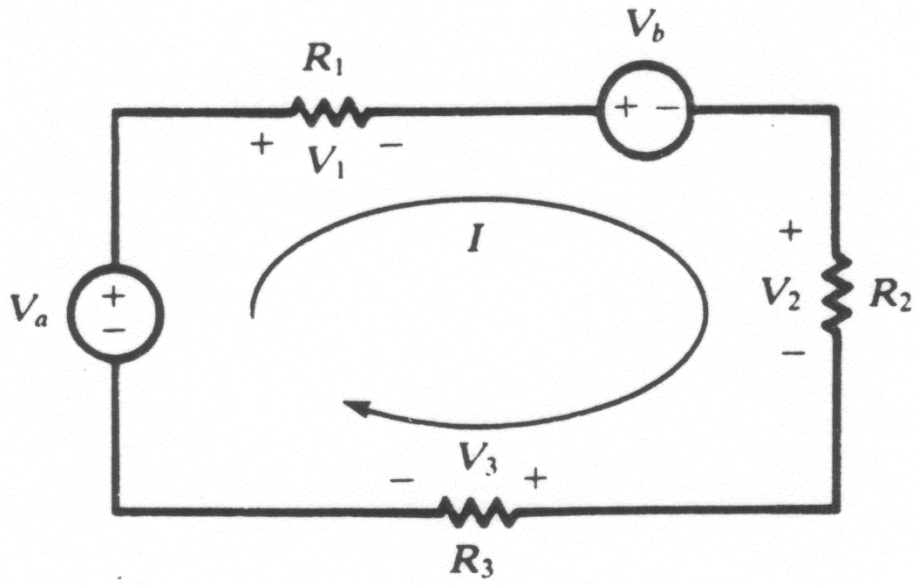
Substituting i in Eq. (2.5.1) finally gives

$$v_1 = 8 \text{ V}, \qquad v_2 = -12 \text{ V}$$

2.3 Kirchhoff's Laws (4)

Example 5

- Applying the KVL equation for the circuit of the figure below.



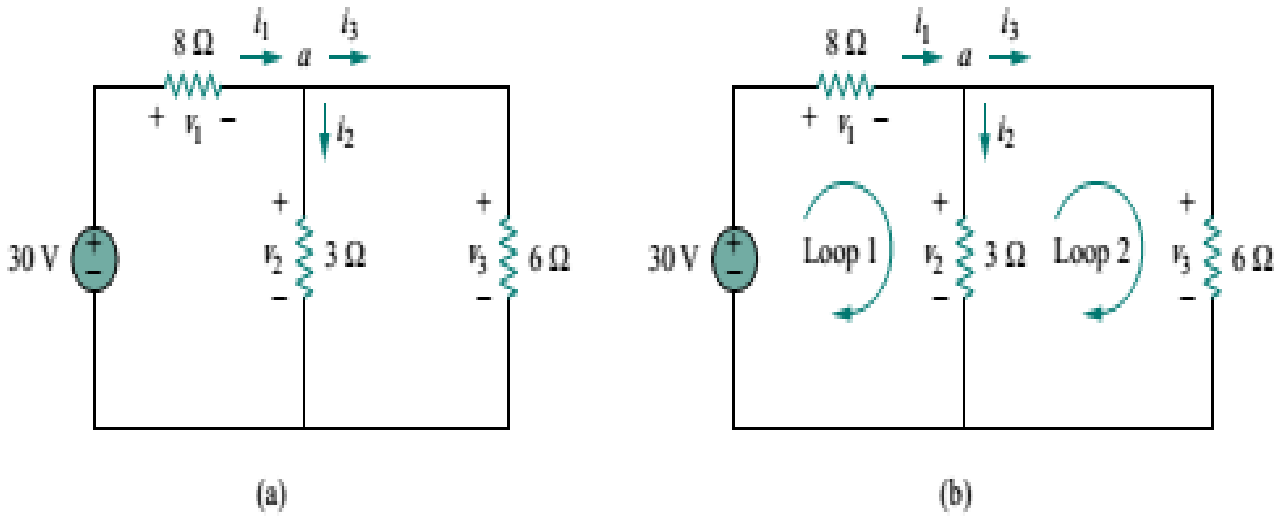
$v_a - v_1 - v_b - v_2 - v_3 = 0$

$V_1 = IR_1 \quad v_2 = IR_2 \quad v_3 = IR_3$

$\Rightarrow v_a - v_b = I(R_1 + R_2 + R_3)$

$$I = \frac{v_a - v_b}{R_1 + R_2 + R_3}$$

Find the currents and voltages in the circuit



Solution:

We apply Ohm's law and Kirchhoff's laws. By Ohm's law,

$v_1 = 8i_1$, $v_2 = 3i_2$, $v_3 = 6i_3$ (2.8.1)

Since the voltage and current of each resistor are related by Ohm's

law as shown, we are really looking for three things: (v_1 , v_2, v_3) or (i_1 , i_2, i_3). At node a, KCL gives

$i_1 - i_2 - i_3 = 0$ (2.8.2)

Applying KVL to loop 1 as in Fig. 2.27(b),

$-30 + v_1 + v_2 = 0$

We express this in terms of i_1 and i_2 as in Eq. (2.8.1) to obtain

$$-30 + 8i_1 + 3i_2 = 0$$

or

$$i_1 = \frac{(30 - 3i_2)}{8} \tag{2.8.3}$$

Applying KVL to loop 2,

$$-v_2 + v_3 = 0 \qquad \Longrightarrow \qquad v_3 = v_2 \tag{2.8.4}$$

as expected since the two resistors are in parallel. We express v_1 and v_2 in terms of i_1 and i_2 as in Eq. (2.8.1). Equation (2.8.4) becomes

$$6i_3 = 3i_2 \qquad \Longrightarrow \qquad i_3 = \frac{i_2}{2} \tag{2.8.5}$$

Substituting Eqs. (2.8.3) and (2.8.5) into (2.8.2) gives

$$\frac{30 - 3i_2}{8} - i_2 - \frac{i_2}{2} = 0$$

or $i_2 = 2$ A. From the value of i_2, we now use Eqs. (2.8.1) to (2.8.5) to obtain

$$i_1 = 3 \text{ A}, \qquad i_3 = 1 \text{ A}, \qquad v_1 = 24 \text{ V}, \qquad v_2 = 6 \text{ V}, \qquad v_3 = 6 \text{ V}$$

2.4 Series Resistors and Voltage Division (1)

- Series: Two or more elements are in series if they are cascaded or connected sequentially and consequently carry the same current.

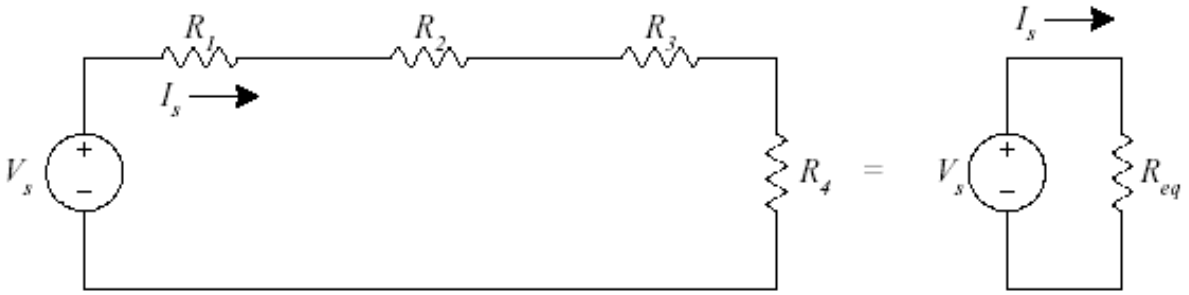
- The equivalent resistance of any number of resistors connected in a series is the sum of the individual resistances.

$$R_{eq} = R_1 + R_2 + \cdots + R_N = \sum_{n=1}^{N} R_n$$

- The voltage divider can be expressed as

$$v_n = \frac{R_n}{R_1 + R_2 + \cdots + R_N} v$$

Resistors in Series



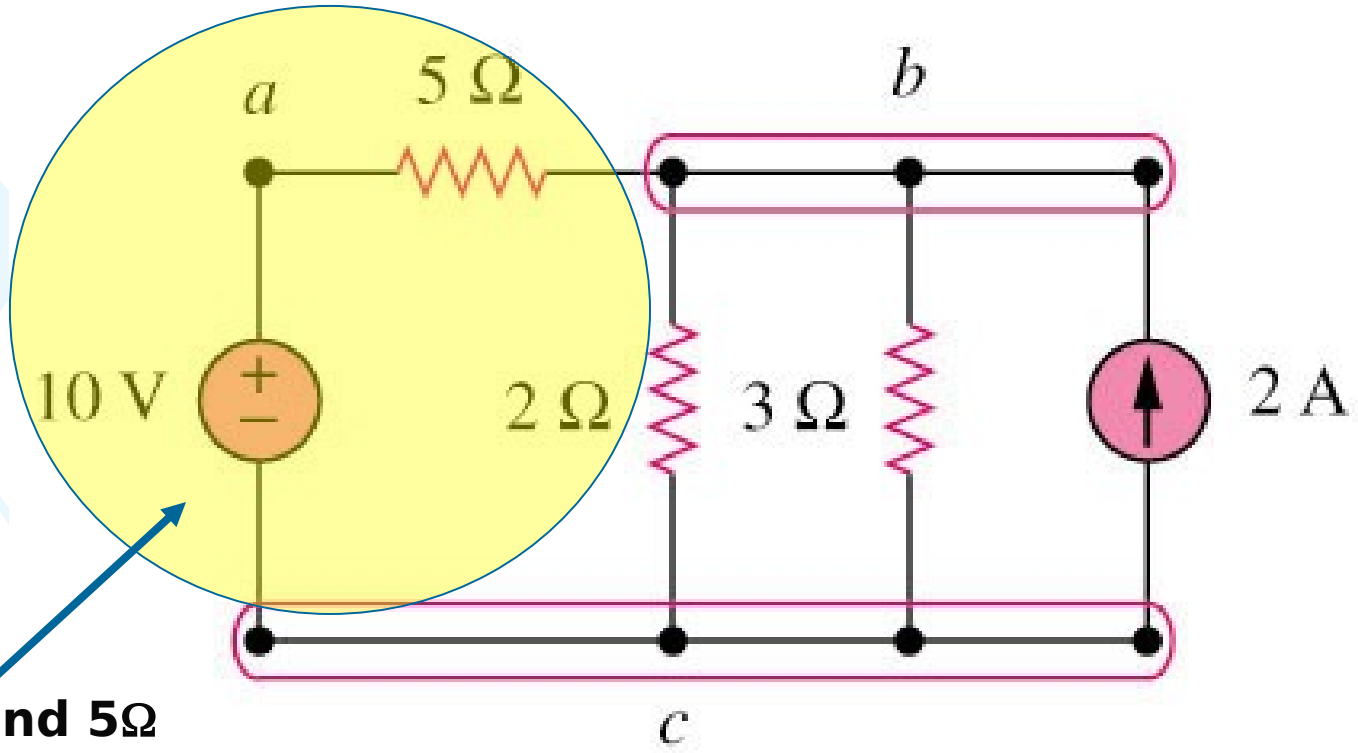
By KVL

$$\begin{aligned} V_s &= R_1 I_s + R_2 I_s + R_3 I_s + R_4 I_s \\ &= I_s (R_1 + R_2 + R_3 + R_4) \\ &= R_{eq} I_s \\ R_{eq} &= R_1 + R_2 + R_2 + R_4 \end{aligned}$$

- Resistors in series add

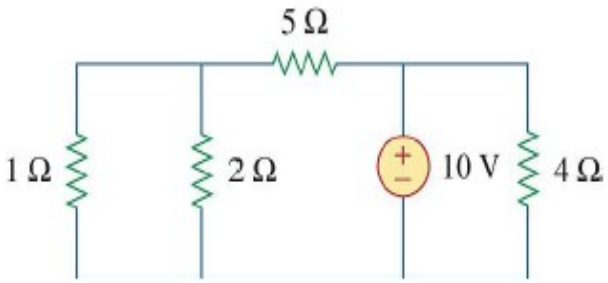
2.4 Series Resistors and Voltage Division (1)

Example 3



10V and 5Ω are in series

Definition . Series: Two or more elements are in series if they are cascaded or connected sequentially and consequently carry the same current.
Definition Parallel: Two or more elements are in parallel if they are connected to the same two nodes and consequently have the same voltage across them.
Elements may be connected in a way that they are neither in series nor in parallel.
Example How many branches and nodes does the circuit in the following gure have? Identify the elements that are in series and in parallel.



KIRCHHOFF'S LAWS

2.5 Parallel Resistors and Current Division (1)

- Parallel: Two or more elements are in parallel if they are connected to the same two nodes and consequently have the same voltage across them.

- The equivalent resistance of a circuit with N resistors in parallel is:

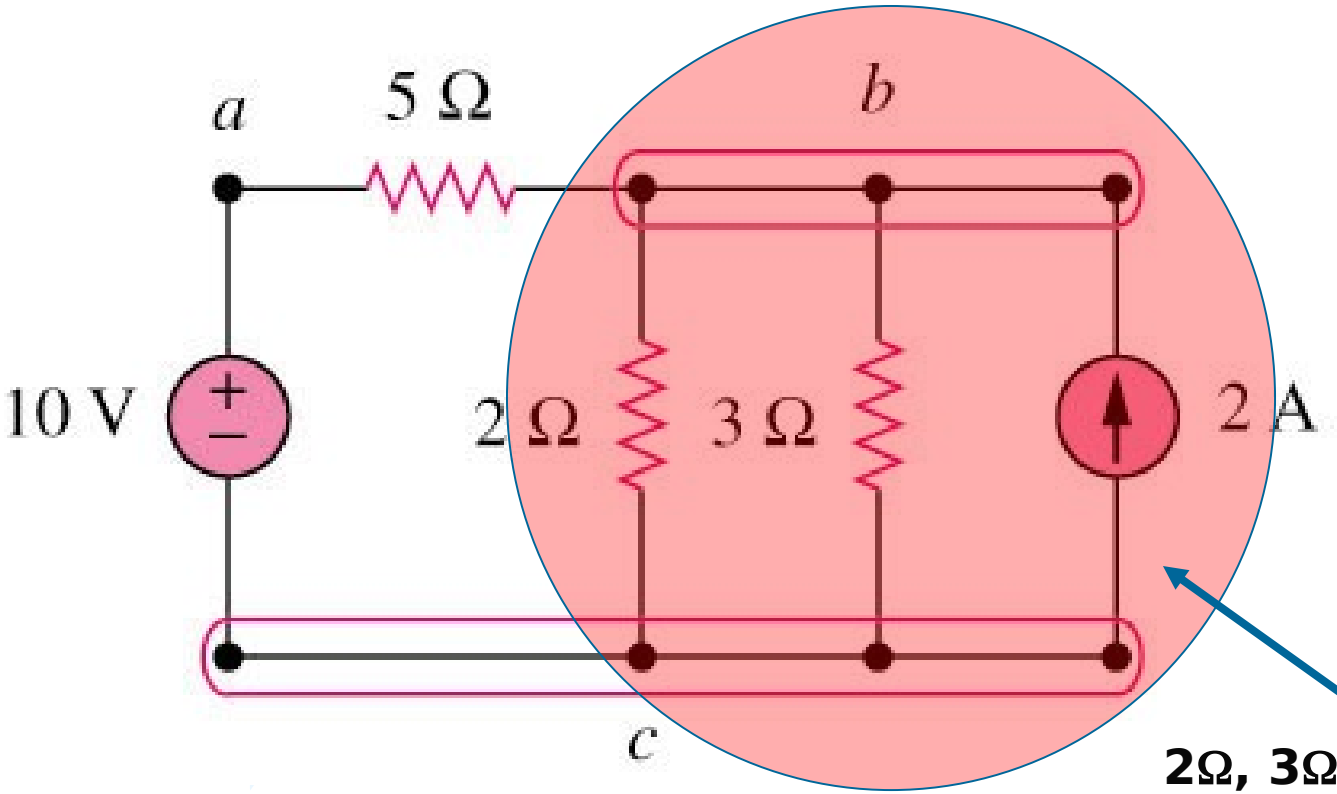
$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \cdots + \frac{1}{R_N}$$

- The total current i is shared by the resistors in inverse proportion to their resistances. The current divider can be expressed as:

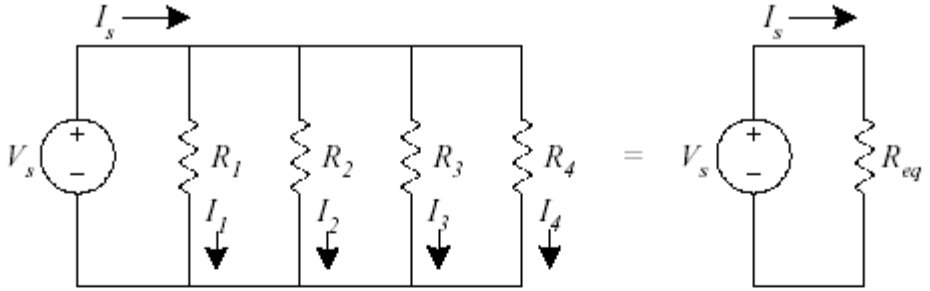
$$i_n = \frac{v}{R_n} = \frac{iR_{eq}}{R_n}$$

2.5 Parallel Resistors and Current Division (1)

Example 4

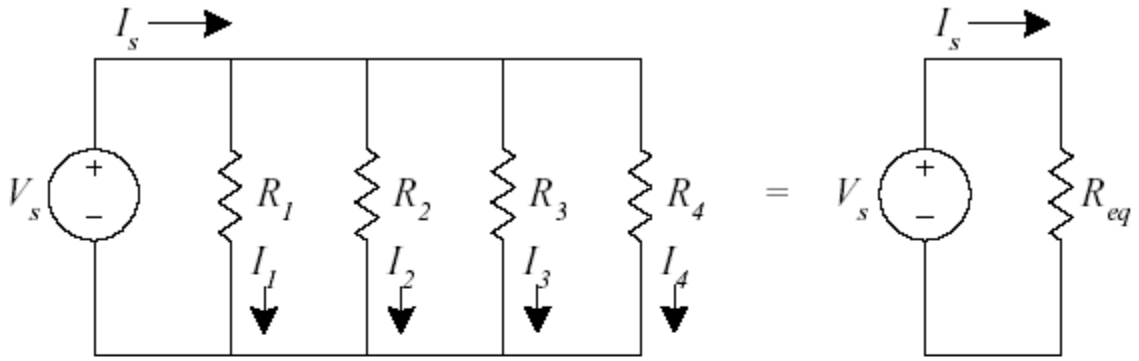


Resistors in Parallel



$$
\begin{aligned}
I_s &= I_1 + I_2 + I_3 + I_4 \\
&= V_s/R_1 + V_s/R_2 + V_s/R_3 + V_s/R_4 \\
&= V_s\left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \frac{1}{R_4}\right) \\
&= \frac{V_s}{R_{eq}} \\
\frac{1}{R_{eq}} &= \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \frac{1}{R_4} \\
R_{eq} &= \frac{1}{\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \frac{1}{R_4}}
\end{aligned}
$$

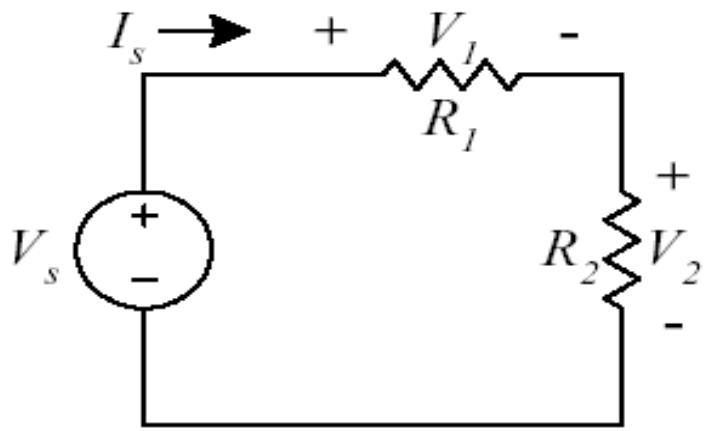
Resistors in Parallel



$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \frac{1}{R_4}$$
$$G_{eq} = G_1 + G_2 + G_3 + G_4$$

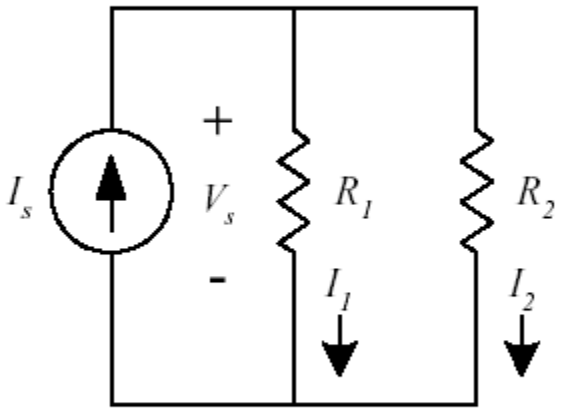
- Resistors in parallel have a more complicated relationship
- Easier to express in terms of conductance
- For two resistors: $R_{eq} = \frac{R_1 R_2}{R_1 + R_2}$

Voltage Divider



$$
\begin{aligned}
R_{eq} &= R_1 + R_2 \\
I_s &= \frac{V_s}{R_{eq}} \\
&= \frac{V_s}{R_1 + R_2} \\
V_2 &= I_s R_2 \\
&= V_s \frac{R_2}{R_1 + R_2} \\
V_1 &= I_s R_1 \\
&= V_s \frac{R_1}{R_1 + R_2}
\end{aligned}
$$

Current Divider



$$
\begin{aligned}
V_s &= I_s \frac{R_1 R_2}{R_1 + R_2} \\
I_2 &= \frac{V_s}{R_2} \\
&= I_s \frac{R_1}{R_1 + R_2} \\
I_1 &= \frac{V_s}{R_1} \\
&= I_s \frac{R_2}{R_1 + R_2}
\end{aligned}
$$

Find R_{eq} for the circuit shown in Fig. 2.34.

Solution:

To get R_{eq}, we combine resistors in series and in parallel. The 6-Ω and 3-Ω resistors are in parallel, so their equivalent resistance is

$$6\ \Omega \parallel 3\ \Omega = \frac{6 \times 3}{6+3} = 2\ \Omega$$

(The symbol ∥ is used to indicate a parallel combination.) Also, the 1-Ω and 5-Ω resistors are in series; hence their equivalent resistance is

$$1\ \Omega + 5\ \Omega = 6\ \Omega$$

Thus the circuit in Fig. 2.34 is reduced to that in Fig. 2.35(a). In Fig. 2.35(a), we notice that the two 2-Ω resistors are in series, so the equivalent resistance is

$$2\ \Omega + 2\ \Omega = 4\ \Omega$$

This 4-Ω resistor is now in parallel with the 6-Ω resistor in Fig. 2.35(a); their equivalent resistance is

$$4\ \Omega \parallel 6\ \Omega = \frac{4 \times 6}{4+6} = 2.4\ \Omega$$

The circuit in Fig. 2.35(a) is now replaced with that in Fig. 2.35(b). In Fig. 2.35(b), the three resistors are in series. Hence, the equivalent resistance for the circuit is

$$R_{eq} = 4\ \Omega + 2.4\ \Omega + 8\ \Omega = 14.4\ \Omega$$

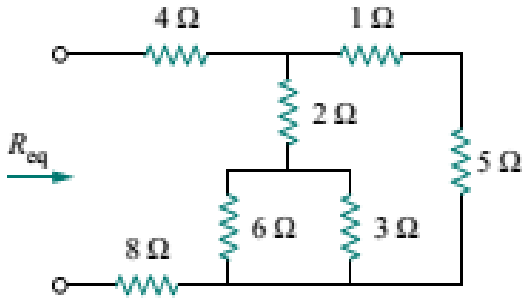
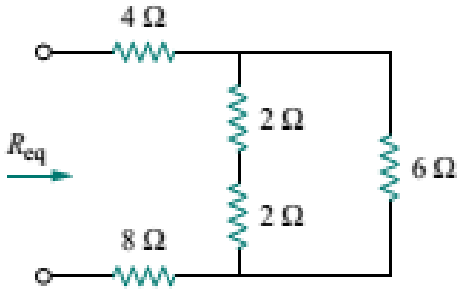
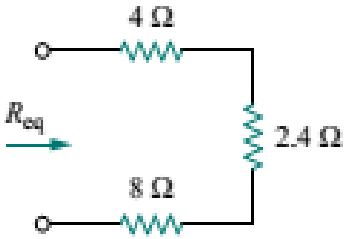


Figure 2.34 For Example 2.9.

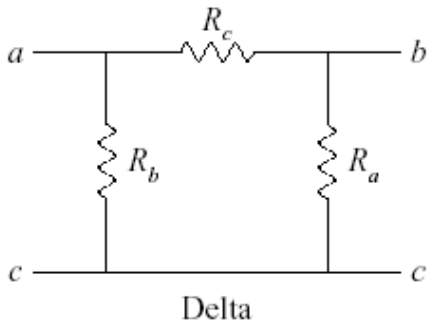


(a)

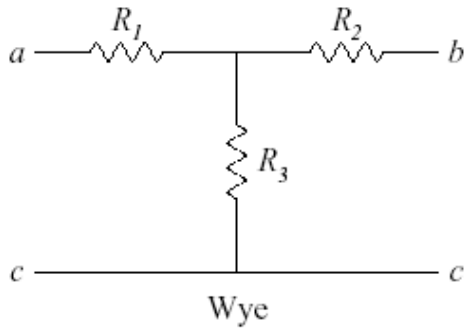


(b)

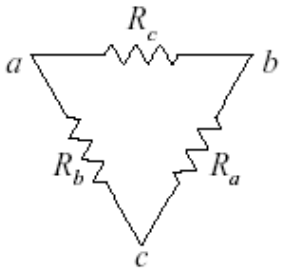
Delta ←→ Wye Transformations



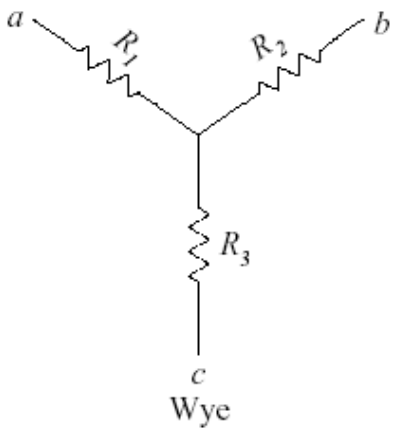
Delta



Wye



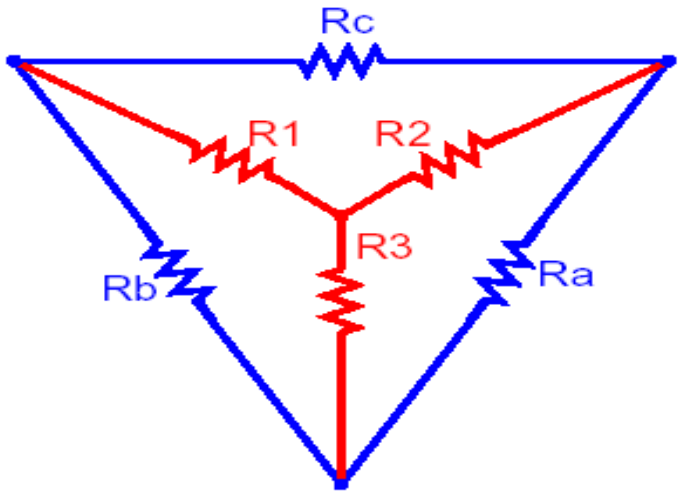
Delta



Wye

- Every Delta network is functionally equivalent to a Wye network (and vice versa).

Delta ←→ Wye Transformations



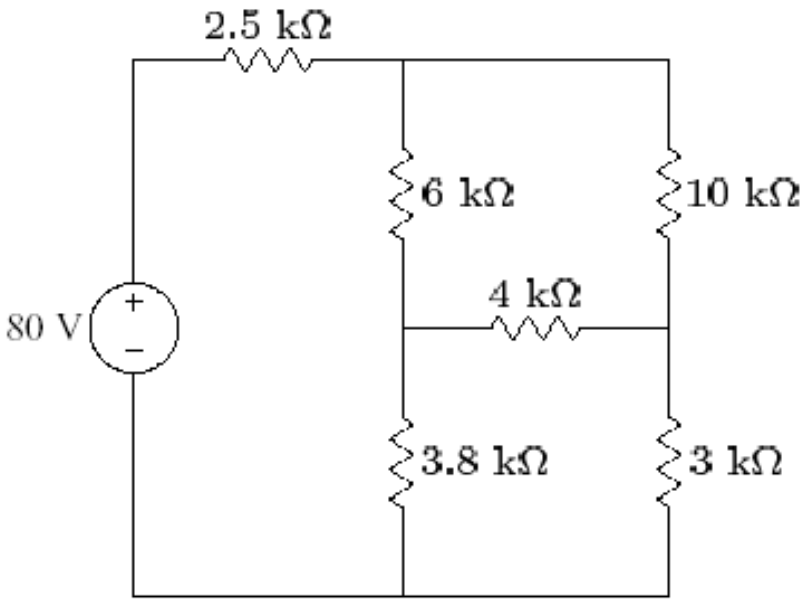
The following must be satisfied for equivalence

$$R_a = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_1} \qquad R_1 = \frac{R_bR_c}{R_a + R_b + R_c}$$

$$R_b = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_2} \qquad R_2 = \frac{R_cR_a}{R_a + R_b + R_c}$$

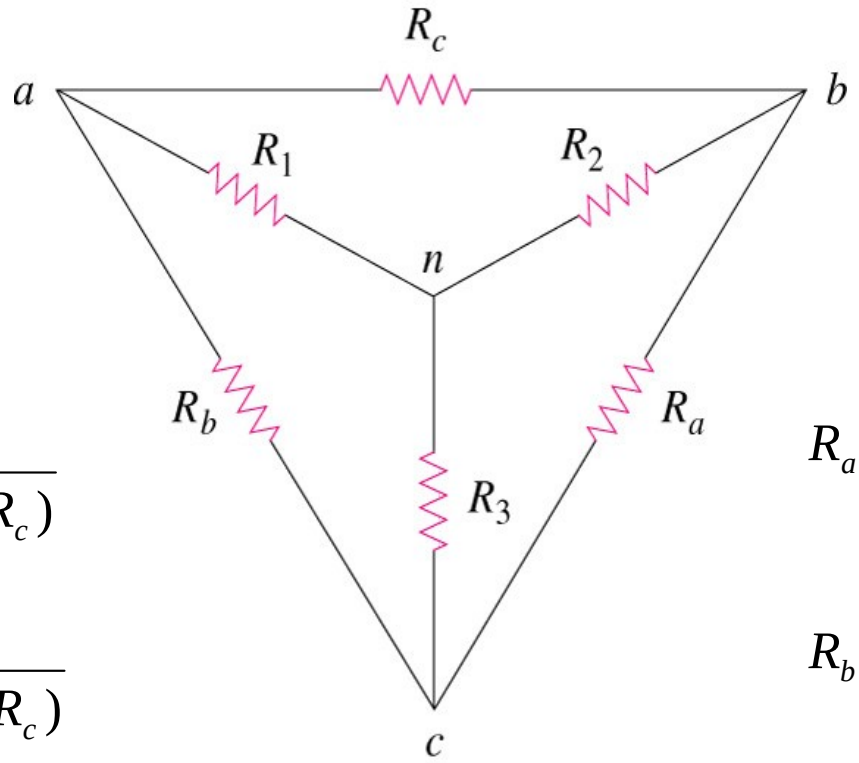
$$R_c = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_3} \qquad R_3 = \frac{R_aR_b}{R_a + R_b + R_c}$$

Example – Delta ←→ Wye Transformations



Find R_{eq} and Power delivered by source

2.6 Wye-Delta Transformations



Delta -> Star

$$R_1 = \frac{R_b R_c}{(R_a + R_b + R_c)}$$

$$R_2 = \frac{R_c R_a}{(R_a + R_b + R_c)}$$

$$R_3 = \frac{R_a R_b}{(R_a + R_b + R_c)}$$

Star -> Delta

$$R_a = \frac{R_1 R_2 + R_2 R_3 + R_3 R_1}{R_1}$$

$$R_b = \frac{R_1 R_2 + R_2 R_3 + R_3 R_1}{R_2}$$

$$R_c = \frac{R_1 R_2 + R_2 R_3 + R_3 R_1}{R_3}$$

Example 2.14

Convert the Δ network in Fig. 2.50(a) to an equivalent Y network.

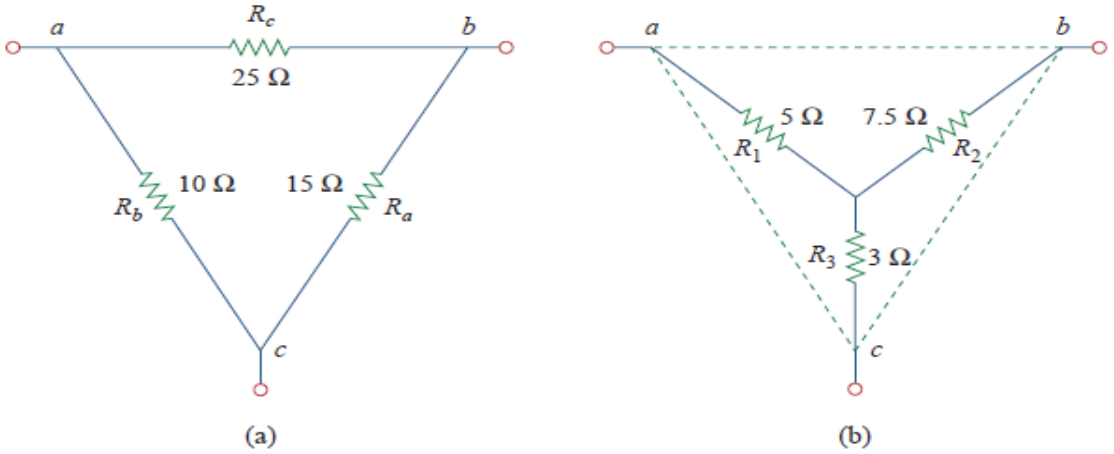


Figure 2.50
For Example 2.14: (a) original Δ network, (b) Y equivalent network.

Solution:
Using Eqs. (2.49) to (2.51), we obtain

$$R_1 = \frac{R_b R_c}{R_a + R_b + R_c} = \frac{10 \times 25}{15 + 10 + 25} = \frac{250}{50} = 5\ \Omega$$

$$R_2 = \frac{R_c R_a}{R_a + R_b + R_c} = \frac{25 \times 15}{50} = 7.5\ \Omega$$

$$R_3 = \frac{R_a R_b}{R_a + R_b + R_c} = \frac{15 \times 10}{50} = 3\ \Omega$$

The equivalent Y network is shown in Fig. 2.50(b).

Circuit Symbols

wire (conductor)

junctions



wires crossing
(no junction)



resistor



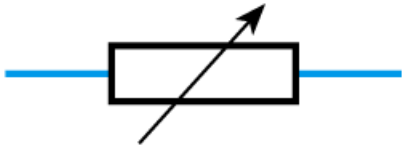
capacitor



inductor



variable resistor



switch



lamp

