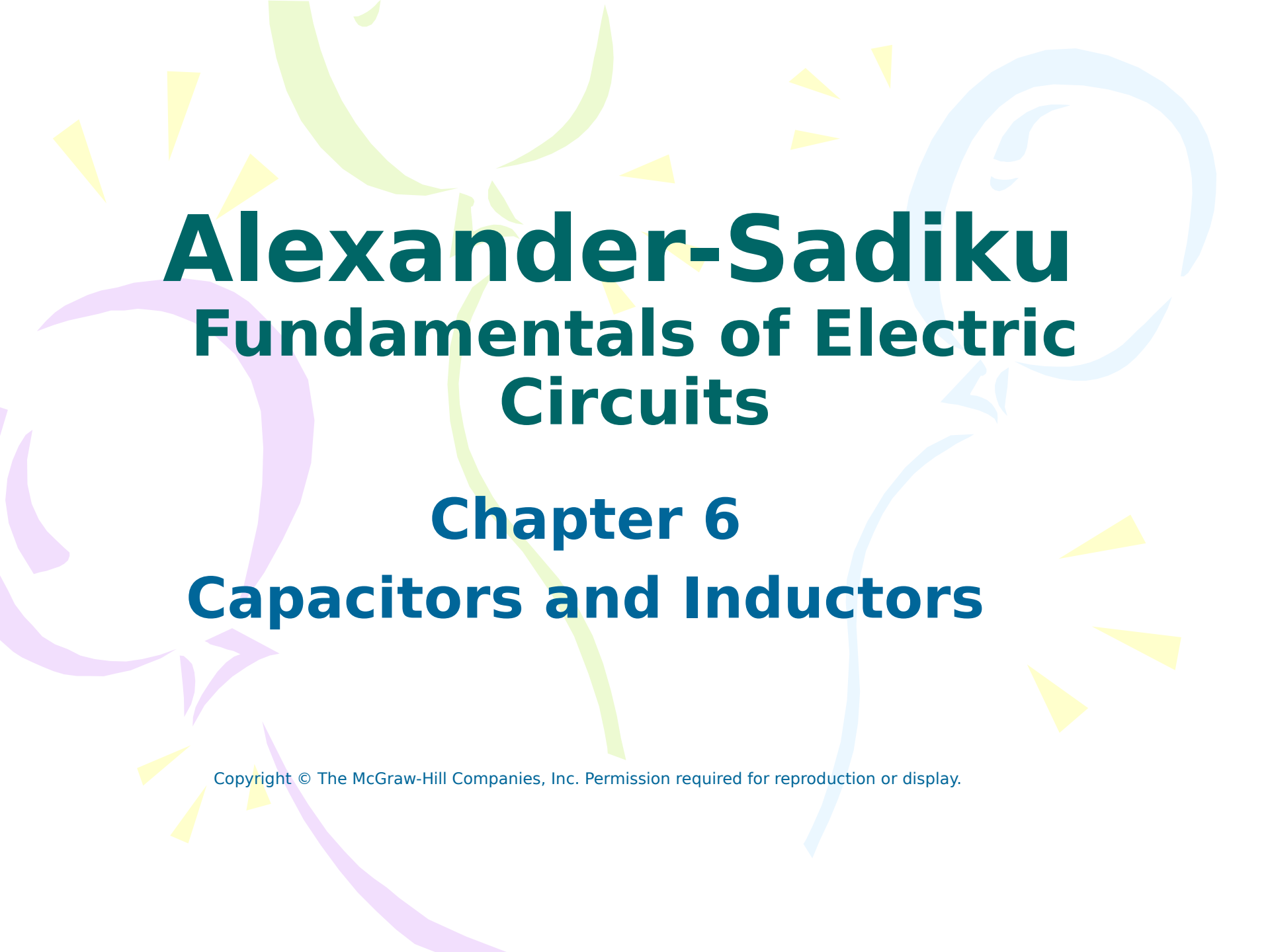


The background features several large, flowing, curved lines in shades of light green, light blue, and light purple. Interspersed among these curves are numerous small, yellow, triangular shapes, some pointing upwards and others downwards, creating a dynamic and celebratory feel.

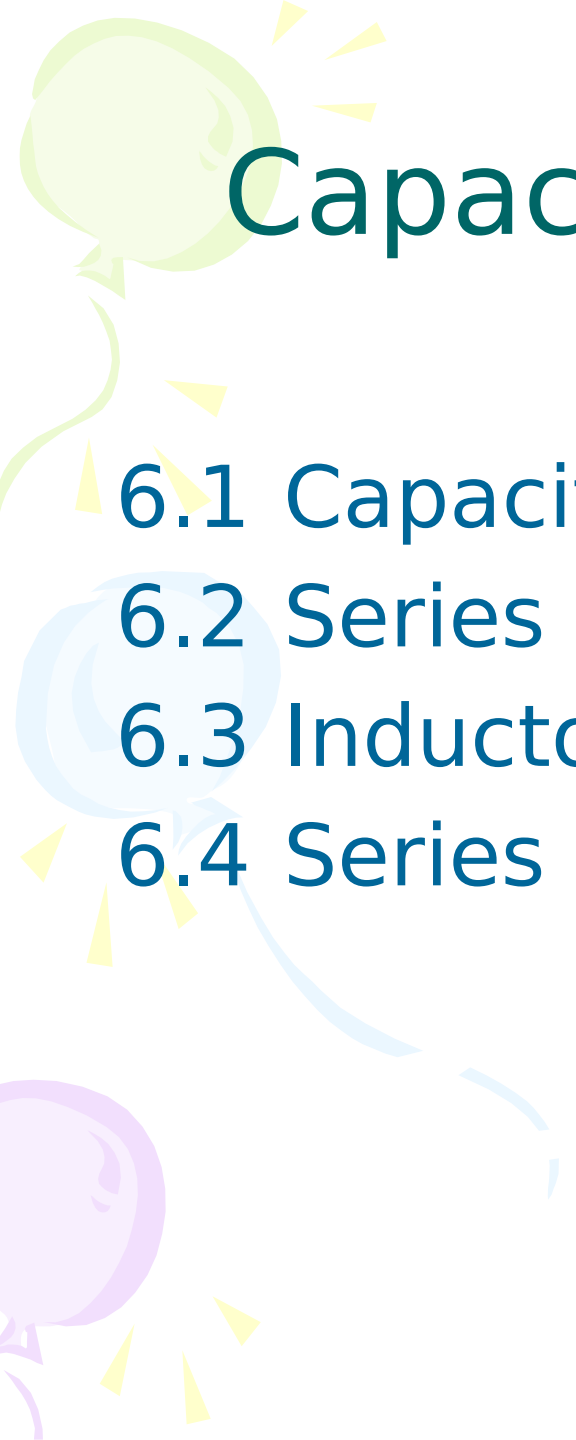
Alexander-Sadiku

Fundamentals of Electric Circuits

Chapter 6

Capacitors and Inductors

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Capacitors and Inductors

Chapter 6

6.1 Capacitors

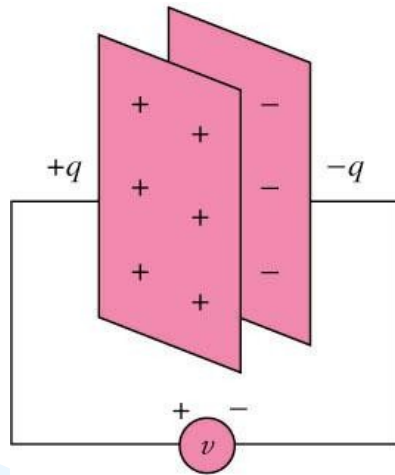
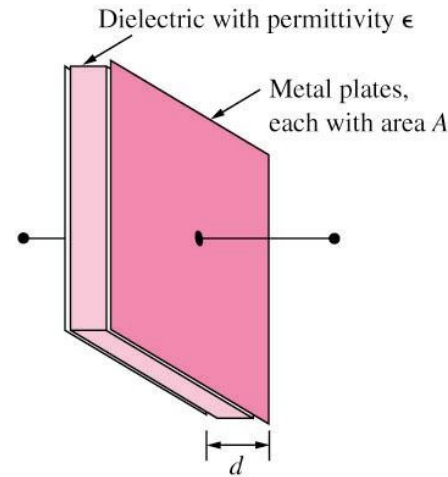
6.2 Series and Parallel Capacitors

6.3 Inductors

6.4 Series and Parallel Inductors

6.1 Capacitors (1)

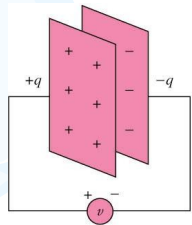
- A capacitor is a passive element designed to **store energy** in its **electric field**.



- A **capacitor** consists of two conducting plates separated by an insulator (or dielectric).

6.1 Capacitors (2)

- **Capacitance** C is the ratio of the charge q on one plate of a capacitor to the voltage difference v between the two plates, measured in farads (F).



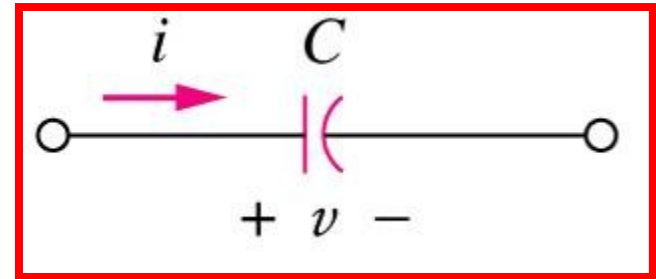
$$q = C v \quad \text{and}$$

$$C = \frac{\epsilon A}{d}$$

- Where ϵ is the permittivity of the dielectric material between the plates, A is the surface area of each plate, d is the distance between the plates.
- Unit: F, pF (10^{-12}), nF (10^{-9}), and **μF (10^{-6})**

6.1 Capacitors (3)

- If i is flowing into the +ve terminal of C
 - Charging $\Rightarrow i$ is +ve
 - Discharging $\Rightarrow i$ is -ve



- The current-voltage relationship of capacitor according to above convention is

$$i = C \frac{d v}{d t}$$

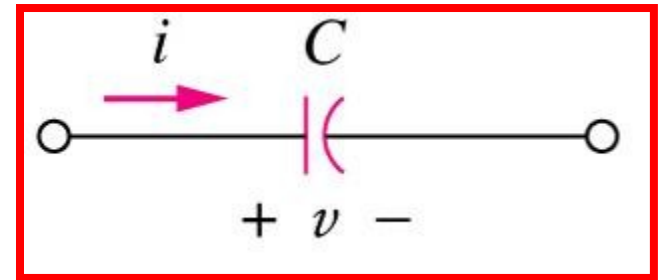
and

$$v = \frac{1}{C} \int_{t_0}^t i d t + v(t_0)$$

6.1 Capacitors (4)

- The energy, w , stored in the capacitor is

$$w = \frac{1}{2} C v^2$$



- A capacitor is
 - an **open circuit** to dc ($dv/dt = 0$).
 - its voltage **cannot change abruptly**.

6.1 Capacitors (5)

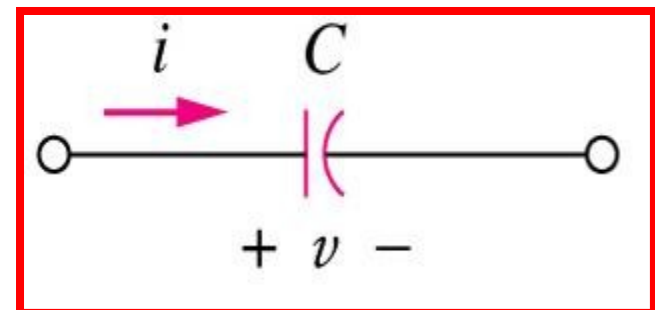
Example 1

The current through a 100- μF capacitor is

$$i(t) = 50 \sin(120 \pi t) \text{ mA.}$$

Calculate the voltage across it at $t = 1 \text{ ms}$ and $t = 5 \text{ ms}$.

Take $v(0) = 0$.



Answer:

$$v(1\text{ms}) = 93.14\text{mV}$$

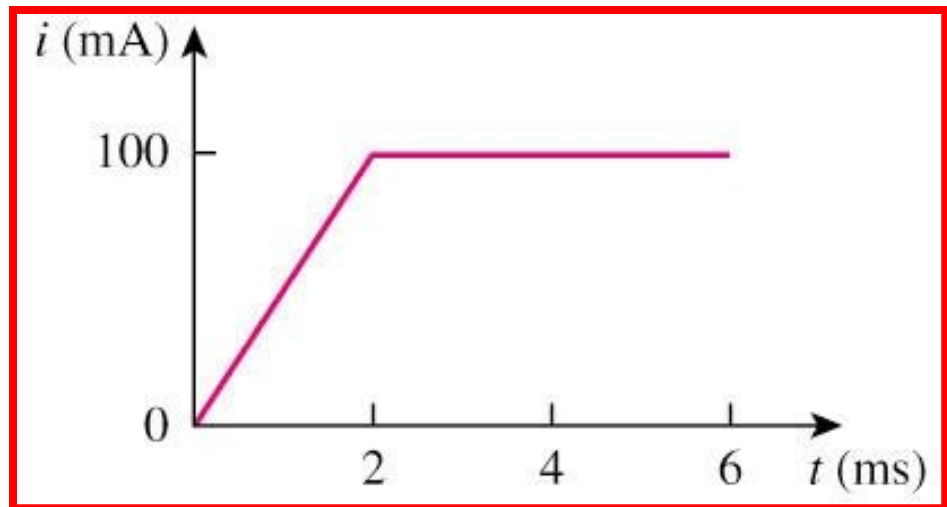
$$v(5\text{ms}) = 1.7361\text{V}$$

6.1 Capacitors (6)

Example 2

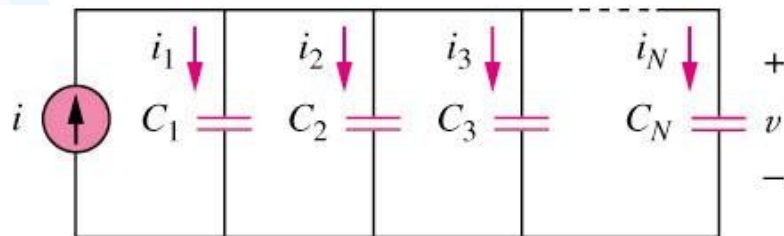
▶ An initially uncharged 1-mF capacitor has the current shown below across it.

Calculate the voltage across it at $t = 2$ ms and $t = 5$ ms.



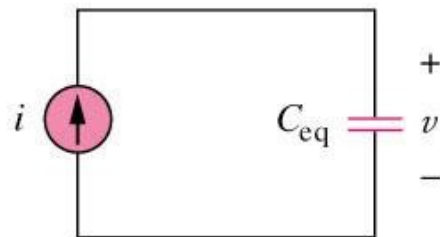
6.2 Series and Parallel Capacitors (1)

- The equivalent capacitance of N **parallel-connected** capacitors is the sum of the individual capacitances.



(a)

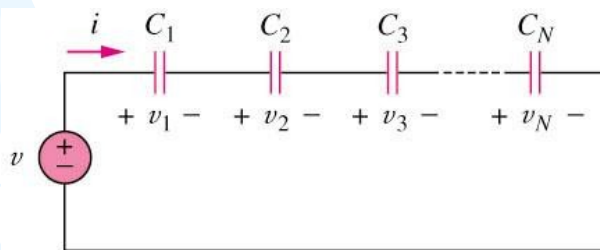
$$C_{eq} = C_1 + C_2 + \dots + C_N$$



(b)

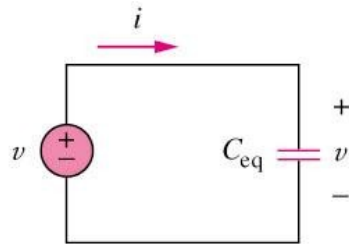
6.2 Series and Parallel Capacitors (2)

- The equivalent capacitance of N **series-connected** capacitors is the reciprocal of the sum of the reciprocals of the individual capacitances.



(a)

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \dots + \frac{1}{C_N}$$

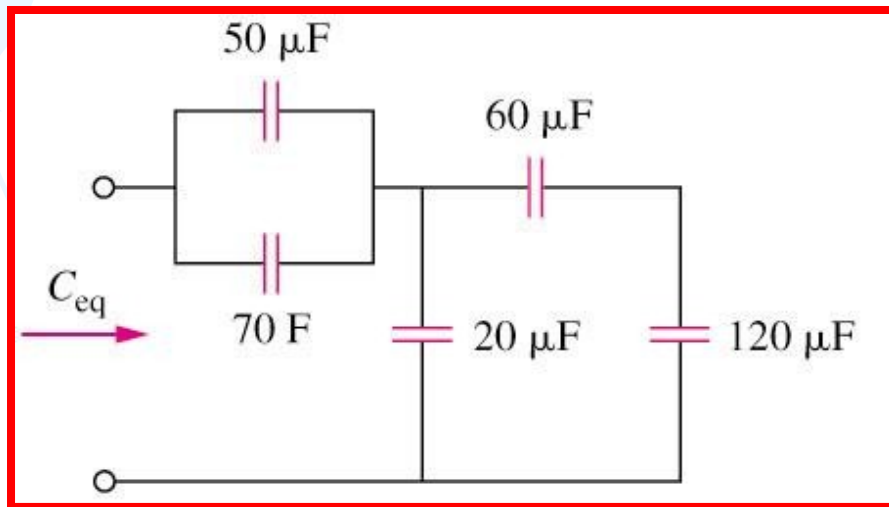


(b)

6.2 Series and Parallel Capacitors (3)

Example 3

Find the equivalent capacitance seen at the terminals of the circuit in the circuit shown below:



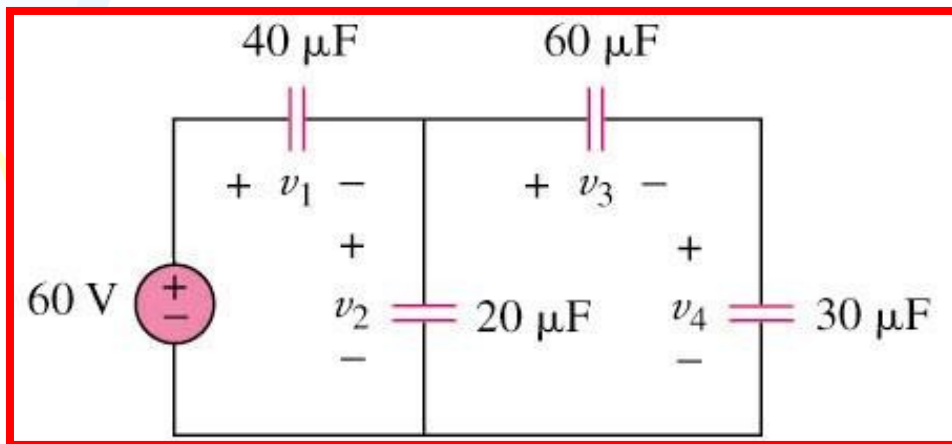
Answer:

$$C_{eq} = \underline{40\mu\text{F}}$$

6.2 Series and Parallel Capacitors (4)

Example 4

Find the voltage across each of the capacitors in the circuit shown below:



Answer:

$$v_1 = 30\text{V}$$

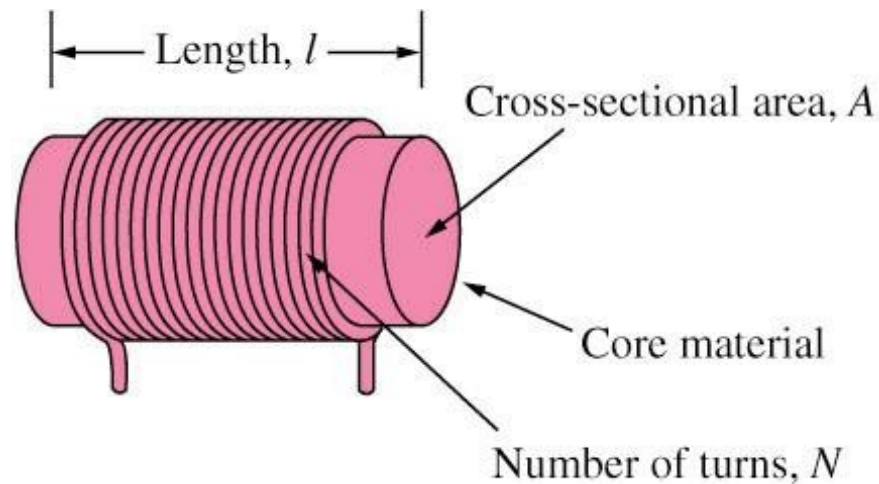
$$v_2 = 30\text{V}$$

$$v_3 = 10\text{V}$$

$$v_4 = 20\text{V}$$

6.3 Inductors (1)

- An inductor is a passive element designed to store energy in its magnetic field.



- An inductor consists of a coil of conducting wire.

6.3 Inductors (2)

- Inductance is the property whereby an inductor exhibits opposition to the change of current flowing through it, measured in henrys (H).

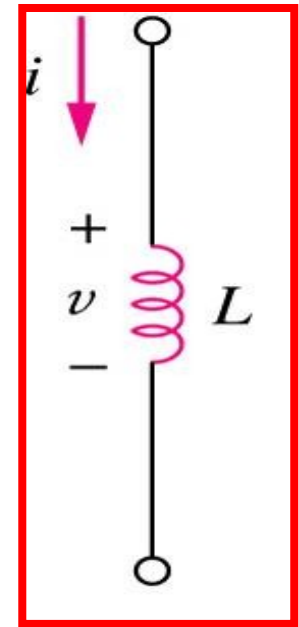
$$v = L \frac{d i}{d t} \quad \text{and} \quad L = \frac{N^2 \mu A}{l}$$

- The unit of inductors is Henry (H), mH (10^{-3}) and μH (10^{-6}).

6.3 Inductors (3)

- The current-voltage relationship of an inductor:

$$i = \frac{1}{L} \int_{t_0}^t v(t) dt + i(t_0)$$



- The power stored by an inductor:

$$w = \frac{1}{2} L i^2$$

- An inductor acts like a short circuit to dc ($di/dt = 0$) and its current cannot change abruptly.

6.3 Inductors (4)

Example 5

The terminal voltage of a 2-H inductor is

$$v = 10(1-t) \text{ V}$$

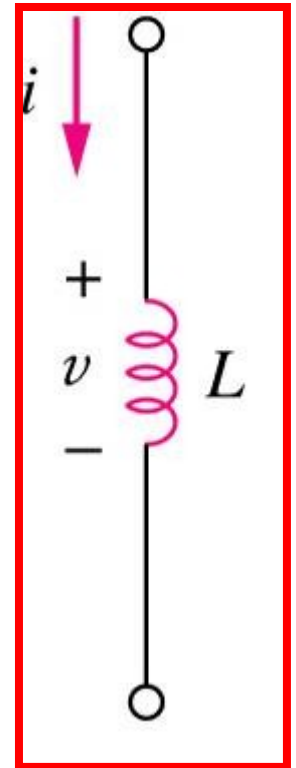
Find the current flowing through it at $t = 4 \text{ s}$ and the energy stored in it within $0 < t < 4 \text{ s}$.

Assume $i(0) = 2 \text{ A}$.

Answer:

$$i(4\text{s}) = -18\text{V}$$

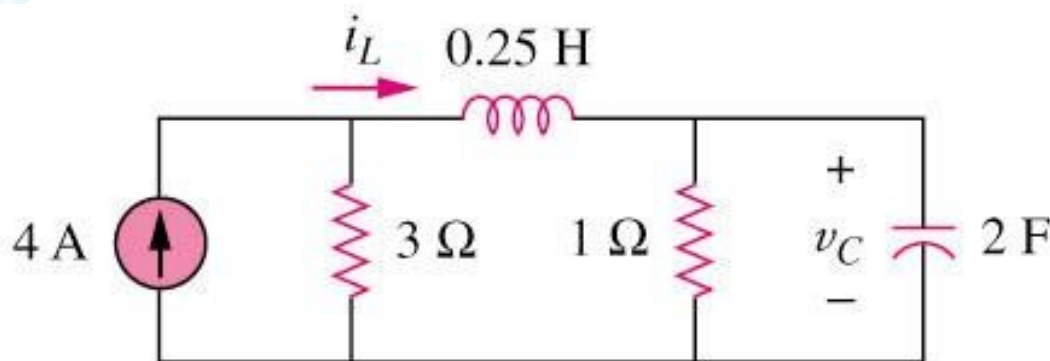
$$w(4\text{s}) = 320\text{J}$$



6.3 Inductors (5)

Example 6

Determine v_C , i_L , and the energy stored in the capacitor and inductor in the circuit of circuit shown below under dc conditions.



Answer:

$$i_L = 3A$$

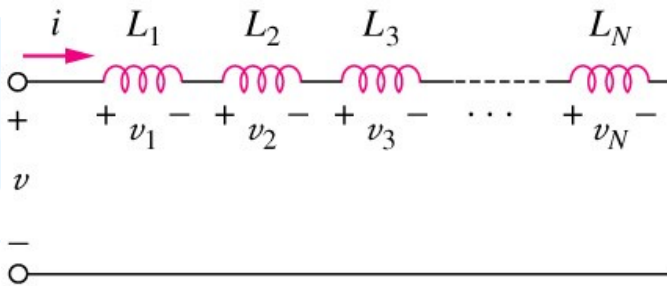
$$v_C = 3V$$

$$w_L = 1.125J$$

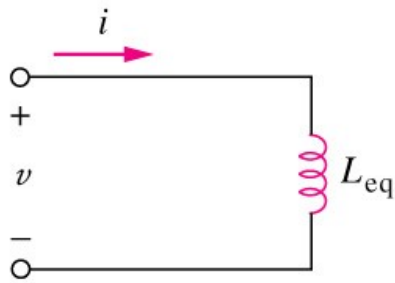
$$w_C = 9J$$

6.4 Series and Parallel Inductors (1)

- The equivalent inductance of **series-connected** inductors is the sum of the individual inductances.



(a)

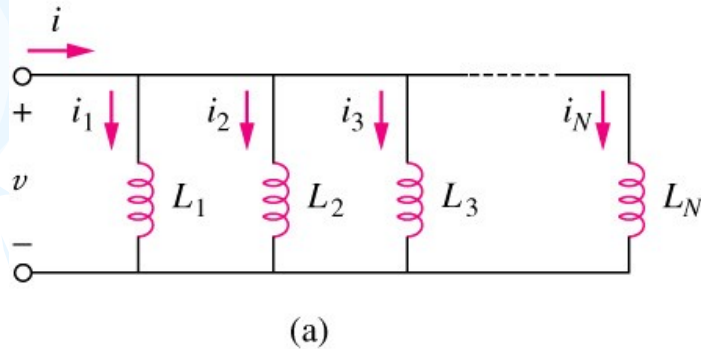


(b)

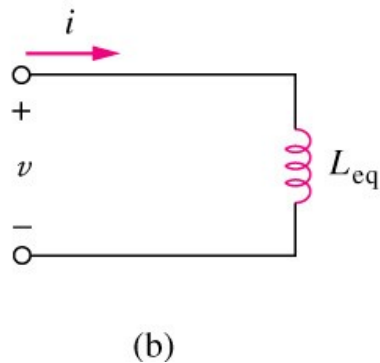
$$L_{eq} = L_1 + L_2 + \dots + L_N$$

6.4 Series and Parallel Inductors (2)

- The equivalent capacitance of **parallel** inductors is the reciprocal of the sum of the reciprocals of the individual inductances.



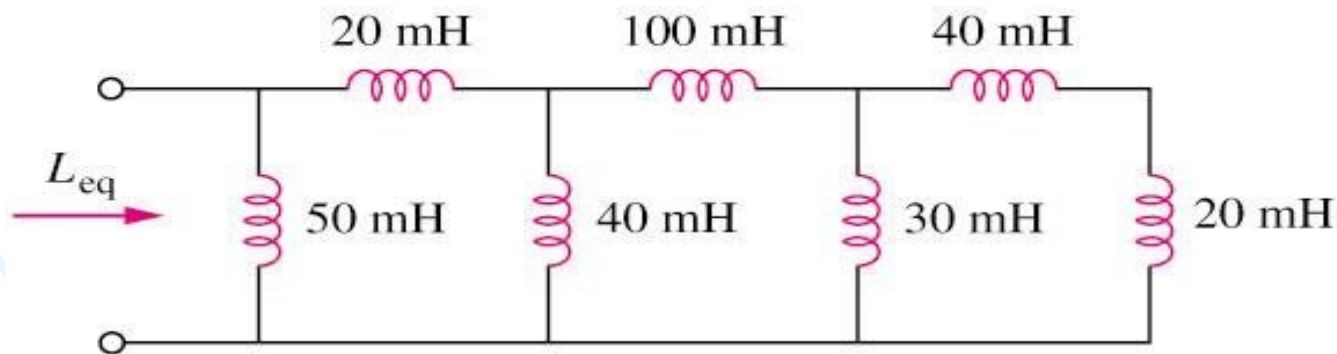
$$\frac{1}{L_{eq}} = \frac{1}{L_1} + \frac{1}{L_2} + \dots + \frac{1}{L_N}$$



6.4 Series and Parallel Capacitors (3)

Example 7

Calculate the equivalent inductance for the inductive ladder network in the circuit shown below:



Answer:

$$L_{eq} = \underline{25\text{mH}}$$

6.4 Series and Parallel Capacitors (4)

- Current and voltage relationship for R, L, C

Circuit element	Units	Voltage	Current	Power
 Resistance	ohms (Ω)	$v = Ri$ (Ohm's law)	$i = \frac{v}{R}$	$p = vi = i^2 R$
 Inductance	henries (H)	$v = L \frac{di}{dt}$	$i = \frac{1}{L} \int v dt + k_1$	$p = vi = Li \frac{di}{dt}$
 Capacitance	farads (F)	$v = \frac{1}{C} \int i dt + k_2$	$i = C \frac{dv}{dt}$	$p = vi = Cv \frac{dv}{dt}$