ARITHMETIC AND GEOMETRIC EXPRESSIONS

Sequence

Sequence is a set of numbers listed in a well defined order with a specific rule that can be used to state the next numbers in that set.

1. Write the next three terms of each of the sequences below

(a) 1, 2,4,8,.....

Answers

16, 32,64

(b)-4,-1,2,5,8,11

Answers

14,17,20

Series

A series is the sum of all the terms of a sequence e.g

(i) 1+2+4+8+16+,.....

Answers

14,17,20

1. . For the AP, 2+5+8+..... find

(i) The 10^{th} term

(ii)

Answers

a=2, first term

d = 5 - 2

D = 3

T_{10} = a+ (n-1)d

= 2+ (10-1)3

= 29

(iii) The 51 st term

Answers

$T_{51} = 2 + (51 - 1)3$
- $= 2 + 50 \times 3$
$= 152$

(iv) The n^{th} term
(v)

Answers

$T_n = a + (n - 1)\, d$
$= 2 + (n - 1)3$
$= 2 + 3n - 3$
$= 3n - 1$

3. find the number of terms in the AP 3 + (-1) + (-5) +….+ (-53).

Answers

$a = 3, d = -4, \quad T_n = -53$
$T_n = a + (n - 1)d$
$-53 = 3 + (n - 1)\ -4$
$-53 = 3 - 4n + 4$
$4n = 53 + 7$
$\frac{1}{4} \times 4n = 60 \times \frac{1}{4}$

$n = 15$

4. The 10^{th} term of an AP is 37 and the 16^{th} term is 61, for this AP find:

(i) The common difference

Answers

$T_n = a + (n - 1)d$
$T_{10} = a + 9d$
$37 = a + 9d$…….eqn 1
and
$T_{16} = a + (16 - 1)d$
$16 = a + 15d$…….eqn 2 and solve the equations simultaneously.
$a + 9d = 37$
$-(a + 15d = 61)$

$-\frac{6}{4} = \frac{-24}{4}$
$d = 4$

(ii) The first term

Answers

First term

$a + 9d = 37$

$a + 9(4) = 37$

$a + 36 = 37$

$a = 37 - 36$

$a = 1$

(iii) the 30^{th} term

Answers

$Tn = a + (n - 1)$

$T_{30} = 1 + (30 - 1)4$

$T_{30} = \underline{117}$

5. The n^{th} term (Tn) of an AP is given by $Tn = 1/2(4n - 3)$.

(a) State (i) the 5^{th} term (ii) the 10^{th} term (iii) the 6^{th} term

Answers

The 5^{th} term

$T_5 = 1/2(4n - 3)$

$T_5 = 1/2(4 \times 5n - 3$

$T_5 = 1/2(17)$

$T_5 = 8.5$

Answers

$T_{10} = 1/2(4 \times 10 - 30)$

$T_{10} = 1/2(40 - 3)$

$T_{10} = 18.5$

Answers

$T_6 = 1/2(4 \times 6 - 3)$

$T_6 = 1/2(24 - 3)$

$T_6 = 1/2(21)$

$T_6 = 10.5$

(**b) the common difference**

Answers

$d = T_6 - T_5$

d = 10.5 - 8.5
d = 2

Therefore, the common difference is 2.

6. If x + 1, 2x - 1 and x + 5 are three consecutive terms, find the value of x.

Answers

X + 1 , 2x - 1 , x + 5
T_1 T_2 T_3

For an AP,

Common difference, d = $T_2 - T_1 = T_3 - T_1$
(2x - 1) - (x + 1) = (x + 5) - (2x - 1)
2x - 1 - x - 1 = x + 5 - 2x
2x - x - 1 - 1 = x - 2x + 5 + 1
X - 2 = -x + 6
X + x = 2 + 6
2x = 8
X=4

7. (i) if the numbers 3,m,n and 8 are three consecutive terms of an AP, find the values of m and n.

Answers

M - 3 = n - m and n - m = 18 - n
M + m = n + 3 n + n = 18 + m
2m = n + 3 2n = 18 + m
$\frac{n+3}{2}$ m= .....eq1 m = 2n -
18.....eq2

Equate m = m

n $\frac{3}{2}$ + = 2n - 18/1

n + 3 = 2(2n - 18)
n + 3 = 4n - 36
n-4n = -36 -3
-3n = -39

$\frac{-3n}{-3}$ $\frac{-39}{-3}$ =

n = 13

for m

m $\frac{n+3}{2}$ =

m = $\frac{13+3}{2}$

m $\frac{16}{2}$ =

m = 8

Therefore, m = 8 and n = 13

(ii) The numbers m - 1, 4m + 1 and 5m - 1 are three consecutive terms of an AP, find the numbers.

(iii)

Answers

m - 1, 4m + 1, 5m - 1 and 4m + 1 is an arithmetic mean between m - 1 and 5m - 1

$\frac{a+c}{2}$ b=

4m + 1 $\frac{m-1+5m-1}{2}$ =

2(4m + 1)=m + 5m-1-1

8m+2=6m - 2

8m - 6m = -2 - 2

2m = -4

M = -2

Substitute for m = 1 in the series we get, -3, -7 and -11

8.(i) Find the arithmetic mean of the first 6 terms of 3 + 8 +.....

Answers

First term
a = 3
Common difference d = 8-3
d = 5

Therefore, the 6 terms are 3,8,13,18,23,28.

Arithmetic mean = $\frac{3 + 8 + 13 + 18 + 23 + 28}{6}$

Arithmetic mean = $\frac{93}{6}$

Arithmetic mean = $\underline{15.5}$

Or

Arithmetic mean = median

= $\frac{13+18}{2}$

= $\underline{15.5}$

(ii) Find the arithmetic mean and the geometric mean of 4 and 64.

Answers

Given 4 and 64

Arithmetic mean = $\frac{4+64}{2}$

$\frac{68}{2}$ =

= $\underline{34}$

Geometric mean = square root of 4 and 64
= 2 x 8

= 16

9. An arithmetic progression has a 1st term to be 2 and common difference of 2, show that the sum of the first nth terms of the AP is given by $S_n = n^2 + n$. hence find the sum of the 21st terms of an AP.

Answers

A = 2, d = 2

S_n $\frac{n}{2}(2a+(n-1)d)$ =

= $\frac{n}{2}(2x2+(n-1)2)$

$\frac{n}{2}(4+2n-2)$ =

$\frac{n}{2}(2+2n)$ =

$= n + n^2$

$S_n = n^2 + n$ is required

The sum of the first 21st terms

$S_{21} = 21^2 + 21$

$S_{21} = 441 + 21$

= 462

10. The sum S_n of the first n terms of an AP is given by $S_n = n^2 + n$, find (i) the first term (ii) common difference (iii) the formula for the sum of the first n - 1 terms

Answers

(i) $S_n = n^2 + 2n$

To find the first term we put n = 1 in the given sum

$S_1 = 1^2 + 2(1)$

$a = \underline{3}$

(ii) The common difference

$d = S_2 - 2S_1$

$= 8 - 6$

$= \underline{2}$

(iii) $S_n = n^2 + 2n$

$S_{n-1} = (n - 1)^2 + 2(n - 1)$

$= n^2 - n - n + 1 + 2n - 2$

$= n^2 - 2n + 2n + 1 - 2$

$= \underline{n^2 - 1}$

Geometric Progression (GP)

A geometric progression (GP) is a sequence in which each term is formed by multiplying the previous term by a constant amount.

n^{th} term of a Geometric Progression

The n^{th} term of a GP with first term a and common ratio r is: $T_n = ar^{n-1}$

1. For a GP, 2 + 6 + 18 + ..., find (i) the tenth term (ii) the 17^{th} term

Solution

First term (a) = 2

Common ratio $\frac{T_2}{T_1} = \frac{6}{2}$ r =

$r = 3$

n = 10

$T_n = ar^{n-1}$

$T_{10} = 2 \times 3^{10-1}$

$T_{10} = 39366$ So the 10^{th} term is 39 366

(ii) $T_{17} = 2 \text{ X } 3^{17-1}$

$= 2 \text{ x } 3^{16}$

$= 86\ 093\ 442$

2. The third term of a GP is 9 and the tenth term is 19 683, find;

(i) the common ratio
(ii) the 8^{th} term

Solutions

(i) $T_3 = ar^2$
$ar^2 = 9$.....eqn (i)

$T_{10} = ar^9$
$Ar^9 = 19\ 683$.....eqn (ii)

Dividing equation (i) by equation (ii)

$$\frac{ar^9}{ar^2}=\frac{19683}{9}$$
$$\sqrt[7]{r^7}=\sqrt[7]{2187}$$
$r=3$ Therefore the common ratio is 3

(ii) $ar^2 = 9$
A x $(3)^2 = 9$

$$\frac{9a}{9}=\frac{9}{9}$$ The first term is 1
$$a=1$$

(iii) $Tn = ar^{n-1}$
$T_8 = 1 \times 3^{8-1}$
$= 3^7$
$T_8 = 2187$ the 8^{th} term is 2 187

3. Given that x +2, x + 3 and x + 6 are the first three terms of a GP, find
(a) the value of x
(ii) the 5^{th} term of the GP.

Solutions

(i) Common ratio $\frac{T_2}{T_1}=\frac{T_3}{T_2}$ (r) =

$$\frac{x+3}{x+2}=\frac{x+6}{x+3}$$
$(x + 3)(x + 3) = (x + 2)(x + 6)$
$X^2 + 3x + 3x + 9 = X^2 + 6x + 2x + 12$
$6x + 9 = 8x + 12$

8x - 6x = 9 - 12

$$\frac{2x}{2}=\frac{-3}{2}$$

$$x=-1\tfrac{1}{3}$$

(ii) First term (a) = x + 2

$$=\frac{-3}{2}+2$$

$$=\frac{-3+4}{2}$$

$$\underline{\underline{a=\frac{1}{2}}}$$

Common ratio (r)=



$$T_5=ar^{n-1}$$

$$¿\frac{1}{2}\times 3^4$$

$$¿\frac{81}{2} or\ 40.5$$

The n^{th} term (T_n) of a GP is given by $T_n = 2^{9-n}$. Find

(i) the first term

(ii) the common ratio

(iii) the sum of the first 9 terms.

Solutions

(i) $T_n = 2^{9-n}$

$T_1 = 2^{9-1}$

$T_1 = 2^8$

$T_1 = 256$

$a = 256$

(ii) To find the common ratio, first calculate the second term (T_2)

$T_2 = 2^{9-2}$

$= 2^7$

$= 128$

Common ratio $= \frac{T_2}{T_1}$ (r)

$¿ \frac{128}{256}$

$¿ \frac{1}{2}$

(iii) Sum $= \frac{a(1-r^n)}{1-r}$

$¿ \frac{256[1-(\frac{1}{2})^9]}{1-\frac{1}{2}}$

$¿ \frac{256(0.998046875)}{\frac{1}{2}}$

$¿ \frac{255.5}{0.5}$

$Sum = 511$

Sum of a GP

6. Calculate, correct to three significant figures, the sum of the first 8 terms of the GP 12, 8, $5\frac{1}{3}$.....

Solutions

$$r=\frac{8}{12}$$

$$r=\frac{3}{4} or 0.75$$

First term a = 12

$$S_8=\frac{a(1-r^n)}{1-r}$$

$$¿\frac{12\left[(1-(\frac{3}{4})^8\right]}{1-\frac{3}{4}}$$

$$¿\frac{12(0.899837085)}{1-\frac{3}{4}}$$

$$¿\frac{10.79864502}{0.25}$$

$$¿43.19458008$$

= 43.2 correct to 3 significant figures

$\frac{1}{8}, \frac{1}{4}, \frac{1}{2}, \ldots\ldots\ldots$

7. Work out the sum of the first 10 terms of

Solution

Common ratio (r) $\frac{1}{4} \div \frac{1}{8}$ =

$$r=\frac{1}{4}\times 8$$

$$r=2 and a=\frac{1}{8}$$

$$S_{10}=\frac{a(r^n-1)}{r-1}$$

$$¿\frac{\frac{1}{8}(2^{10}-1)}{2-1}$$

$$¿\frac{\frac{1}{8}(1024-1)}{1}$$

$$¿\frac{1}{8}(1023)$$

$$S_{10}=127.875$$

Geometric Mean

8. Find the geometric Mean of 4 and 64.

Solution

$$
\begin{aligned}
&=\sqrt{4\times 64}\\
&=\sqrt{4}\times\sqrt{64}\\
&=2\times 8\\
&=16
\end{aligned}
$$

9. The sum of infinity of a certain GP is 28. if the first term is 37, find r

Sum to infinity

$$
\begin{aligned}
&S\infty=\frac{a}{1-r}=28\\
&a=37\\
&\frac{a}{1-r}=28\\
&\frac{37}{1-r}=28\\
&28(1-r)=37\\
&28-28r=37\\
&-28r=37-28\\
&\frac{28r}{28}=\frac{9}{-28}or-032
\end{aligned}
$$

10. Write down the number of terms in the following GP$_s$

(i) 2 + 4 + 8 + ……+512

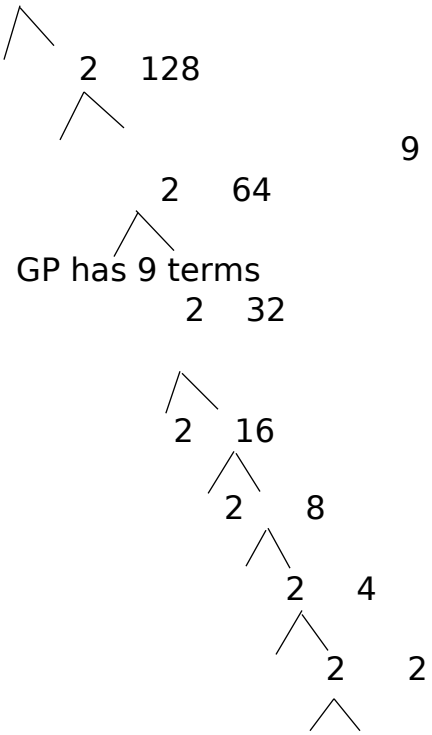
(ii) 81 + 27 + 9 + $\frac{1}{27}$ …… +

Solution

First term a = 2, r =2
Last term = 512

$$L=ar^{n-1}$$
$$\frac{512}{2}=\frac{2\times 2^{n-1}}{2}$$
$$256=2^{n-1}$$

Factorising 256



GP has 9 terms

$2^8 = 2^{n-1}$

8 = n - 1

n = 8 + 1

n = 9

$a=81, r=\frac{1}{3}, last=\frac{1}{27}$

The

$$n=1+\frac{\log\frac{L}{a}}{\log r}$$
$$¿\ 1+\frac{\log\frac{\frac{1}{27}}{81}}{\log\frac{1}{3}}$$
$$¿\ 1+\frac{\log(\frac{1}{27}\times\frac{1}{81})}{\log\frac{1}{3}}$$
$$n=1+\frac{\log(\frac{1}{3})^3\times(\frac{1}{3})^4}{\log(\frac{1}{3})}$$
$$n=1+\frac{\log(\frac{1}{3})^7}{\log(\frac{1}{3})}$$
$$n=1+\frac{7\log(\frac{1}{3})}{\log(\frac{1}{3})}$$
$$n=1+7$$
$$n=8$$

2 1

(ii) 1 + 27 + 9 + $\frac{1}{27}$ ..... +

The GP has 8 terms

ARITHMETIC PROGRESSION

PRACTICE QUESTIONS

1. The fourteenth term of an AP is 5.2 and the twenty fifth term of the AP is 105.find the sum to the first 79 term of AP?

T_{14}=a+(14-1)d=58.2 therefore the value of a+24d=105

T_{25}=a+(25-1)d=105

a+24(4.25)=105

a +13d=58.2 a=105-102

a+24d=105 a=**3**

subtract= -11d=-46.8 therefore the sum is S_{79}=79/2[2(3)+(79-1)4.25]

-11 -11 S_{79}=**13,331.25**

D=**4.25**

2. The twenty first of an AP is 184.9 and the twelfth is 104.8 calculate the sum of 81.

T_{21}=a+(21-1)d=184.9 therefore the value of a is a+11d=104.8

T_{12}=a +(12-1)d=104.8

a+11(8.9)=104.8

=a+20d =184.9 a=**6.9**

Subtract a+11d=104 therefore the sum is S_{81}=81/2[2(6.9)+(89-1)8.9]

=9d/9=80.1/9 =**35,466.5**

D=**8.9**

3.the nineth term of AP is 70.7 and the sixteenth term of the AP is 59.3 .find the T_{39}

T_{19}=a+18d=70.7 therefore a+18d=70.7 T_{39}=2.3+(39-1)3.8

T_{16}=a+15d=59.3 a+18(3.8)=70.7 T_{39}=2.3+144.4

3d/3=11.4/3 a=70.7-68.4 T_{39}=**146.7**

D=**3.8** **a=2.3**

4. the twenty third term of an AP is 159.5 and the eighteenth term of the AP is 123.5.find the sum of 18.

T_{23}= a +(23-1)d=159.5 a+22d=159.5 S_{18}=18/2[2(1.1)+(18-1)7.2]

T_{18}=a +(18-1)d=123.5 a+22(7.2)=159.5 **=1,121.4**

a +22d=159.5 a=159-158.4

a+17d=123.5 a=**1.1**

5d/5=36/5

D=**7.2**

5.the twenty first term of an AP is 42.6 and the nineteeth term of the AP is 39.find T_{46}

T_{21}=a+(21-1)d=42.6 a+20d=42.6 T_{46}=a+(n-1)

T_{19}= a+(19-1)=39 a+20(1.8)=42.6 = 6.6+(46.1)1.8

2d/2=3.6/2 a=42.6-81 **=87.6**

D=**1.8** **a=6.6**

GEOMETRICAL PROGRESSION

1. The third term of a G.P is 4096 and the fifth term is 1024. Calculate

   $T_3=ar^{3-1}$ $T_5= ar^{5-1}$ $S_{12}=$ 16384×4095/4096/1/2

   $T_3=ar^2= 4096$ $T_5= ar^4= 1024$ $S_{12}=$ **32760**

   $Tr/T_3= 1024/4096$ $ar^3= 4096$

   $Ar^2= ¼$ $a(1/2)^2= 4096$

   $\sqrt{r^2}=\sqrt{1/4}$ $a= 4096/(1/2)^2$

   r= ½ a=**16384**

2. The seventh term of a G.P is 729 and the fourth term is 19683. Find the fifth term.

   $T_7= ar^{7-1}$ $T_4=ar^{4-1}$

   $ar^6= 729$ $=ar^3= 19683$

   $T_7/T_4=r^6/r^3$ r= =**1/27**

   $r^3=729/19683$