

# **3. Growth Models and Theories of Development**

- Every nation strives after development. Economic progress is an essential component, but it is not the only component. Development is not purely an economic phenomenon. In an ultimate sense, it must encompass more than the material and financial side of people's lives, to expand human freedoms. Development should therefore be perceived as a multidimensional process involving the reorganization and reorientation of entire economic and social systems.
- In addition to improvements in incomes and output, it typically involves changes in institutional, social, and administrative structures as well as in popular attitudes and even customs and beliefs. Finally, although development is usually defined in a national context, its more widespread realization may necessitate modification of the international economic and social system as well.

- **In this chapter, we explore the historical and intellectual evolution in scholarly thinking about how and why development does or does not take place. We do this by examining four major and often competing development theories. You will see that each offers valuable insights and a useful perspective on the nature of the development process**

# **Classic Theories of Economic Development: Four Approaches**

- The classic post-World War II literature on economic development has been dominated by four major and sometimes competing strands of thought:
  - (1) the linear-stages-of-growth model,
  - (2) theories and patterns of structural change,
  - (3) the international-dependence revolution, and
  - (4) the neoclassical, free-market counterrevolution.
- In recent years, an eclectic approach has emerged that draws on all of these classic theories.

# **1. Development as Growth and the Linear-Stages Theories**

## **(a) Rostow's Stages of Growth :**

**A theory of economic development, associated with the American economic historian Walt W. Rostow, according to which a country passes through sequential stages in achieving development.**

- The most influential and outspoken advocate of the stages-of-growth model of development was the American economic historian Walt W. Rostow. According to Rostow, the transition from underdevelopment to development can be described in terms of a series of steps or stages through which all countries must proceed.
- He distinguished five stages of economic growth:
  1. the traditional society
  2. the pre conditions for take-off
  3. the take-off
  4. the drive to maturity, and
  5. the age of high mass consumption

# 1. The Traditional society

- A traditional society has been defined as “one whose structure is developed with in limited production functions based on pre-Newtonian science and technology and as pre Newtonian attitudes towards the physical world”
- This does not mean that there was little economic change in such societies. It did not lack inventiveness and innovations, but lacked the tools and the outlook toward the physical world of the post-Newtonian era
- The social structure of such societies was hierarchical in which family and clan connections played a dominant role
- Political power was concentrated in the regions, in the hands of the landed aristocracy supported by a large retinue of soldiers and civil servants

- More than **75%** of the working population was engaged in agriculture
- Naturally agriculture happened to be the main source of income of the state and the nobles, which was dissipated on the construction of temples and other monuments, on the expensive funerals and weddings and on the prosecution of wars

## 2. The Pre conditions for take-off

- The introduction of the urge to development, which came from within the elite class and the elite classes of the foreign countries was very important. Hence, **an entrepreneurial class** emerged which mobilized savings for investment. Unification of the country and some **infrastructure facilities** like, transport, education and organized medical care were set up. **Agriculture started developing, death rates and birth rates** started to fall. The preconditions for take-off in the three non industrial sectors satisfied
- Change in the social overhead capital specially in construction of road
- Technical and technological revolution in agriculture
- Financing import by generating foreign exchange through export

### 3. The 'Take-off' stage

- Investment increased from around **5 percent to 10 percent** of the GNP. Quick emergence of political, social and institutional frame-work which exploited the high impulses to economic expansion in the modern sector to growth on an going character.
- Entrepreneurs, corporations and the government acquire the technique of mobilization of financial and real resources for further development. The take-off is defined as an industrial revolution tied **directly to radical changes in methods of production, having their decisive consequences over a relatively short period of time**. It is the interval during which, rate of investment, and real output per capita, the will and capacity to develop the entire institutional set up undergoes a change for further development and innovation

## 4. Stage of “Drive to Maturity”

- In this stage, the rate of investment increases from 10 percent of the GNP to higher limits up to 20 percent even. Important industries came up, including many imports replacement and exports replacement industries. Technical knowledge spreads to the rest of the economic sectors. Production of anything if not everything, becomes possible.

## 5. Age of high mass consumption

- In this stage, per capita real income becomes so high that the **consumption** grows beyond food, clothes and shelter to goods **of comforts and luxuries on a mass scale**. Urbanization and industrialization change the values of the society and development consciousness increases. Leading sectors change the basic structure of the society and new types of durable consumer goods industries become the leading sectors, very high levels of consumption and **physical quality of life are achieved**.

## **(b) Harod-Domar growth model:**

**A functional economic relationship in which the growth rate of gross domestic product ( $g$ ) depends directly on the national net savings rate ( $s$ ) and inversely on the national capital-output ratio ( $c$ ).**

- Every economy must save a certain proportion of its national income, if only to replace worn-out or impaired capital goods (buildings, equipment, and materials). However, in order to grow, new investments representing net additions to the capital stock are necessary.
- If we assume that there is some direct economic relationship between the size of the total capital stock,  $K$ , and total GDP,  $Y$ —for example, if \$3 of capital is always necessary to produce an annual \$1 stream of GDP—it follows that any net additions to the capital stock in the form of new investment will bring about corresponding increases in the flow of national output, GDP.

- Suppose that this relationship, known in economics as the capital-output ratio, is roughly 3 to 1. If we define the capital-output ratio as  $k$  and assume further that the national net savings ratio,  $s$ , is a fixed proportion of national output (e.g., 6%) and that total new investment is determined by the level of total savings, we can construct the following simple model of economic growth:

1. Net saving ( $S$ ) is some proportion,  $s$ , of national income ( $Y$ ) such that we have the simple equation

$$S = sY \quad 3.1$$

2. Net investment ( $I$ ) is defined as the change in the capital stock,  $K$ , and can be represented by  $\Delta K$  such that

$$I = \Delta K \quad 3.2$$

- But because the total capital stock,  $K$ , bears a direct relationship to total national income or output,  $Y$ , as expressed by the capital-output ratio,  $c$ , it follows that

$$\frac{K}{Y} = c$$

- Or  $\Delta K / \Delta Y = c$

- Or , finally,

$$\Delta K = c \Delta Y \quad 3.3$$

3. Finally, because net national savings,  $S$ , must equal net investment,  $I$ , we can write this equality as

$$S = I \quad 3.4$$

- But from Equation 3.1 we know that  $S = sY$ , and from Equations 3.2 and 3.3 we know that

$$I = \Delta K = c \Delta Y$$

- It therefore follows that we can write the “identity” of saving equaling investment shown by Equation 3.4 as

$$S = sY = c \Delta Y = \Delta K = I \quad 3.5$$

- or simply as

$$sY = c \Delta Y \quad 3.6$$

- Dividing both sides of Equation 3.6 first by  $Y$  and then by  $c$ , we obtain the following expression:

$$\frac{\Delta Y}{Y} = \frac{s}{c} \quad 3.7$$

- Note that the left-hand side of Equation 3.7,  $\Delta Y/Y$ , represents the rate of change or rate of growth of GDP.

- Equation 3.7, which is a simplified version of the famous equation in the Harrod-Domar theory of economic growth, states simply that the rate of growth of GDP ( $\Delta Y/Y$ ) is determined jointly by the net national savings ratio,  $s$ , and the national capital-output ratio,  $c$ .
- More specifically, it says that in the absence of government, the growth rate of national income will be directly or positively related to the savings ratio (i.e., the more an economy is able to save—and invest—out of a given GDP, the greater the growth of that GDP will be) and inversely or negatively related to the economy's capital-output ratio (i.e., the higher  $c$  is, the lower the rate of GDP growth will be).

- Equation 3.7 is also often expressed in terms of gross savings ,  $s^G$ , in which case the growth rate is given by

$$\frac{\Delta Y}{Y} = \frac{s^G}{c} - \delta \quad 3.7'$$

where  $\delta$  is the rate of capital depreciation

- The economic logic of Equations 3.7 and 3.7' is very simple. *To grow, economies must save and invest a certain proportion of their GDP.*
- *The more they can save and invest, the faster they can grow.* But the actual rate at which they can grow for any level of saving and investment—how much additional output can be had from an additional unit of investment—can be measured by the inverse of the capital-output ratio,  $c$ , because this inverse,  $1/c$ , is simply the output-capital or output-investment ratio.
- *It follows that multiplying the rate of new investment,  $s = I/Y$ , by its productivity,  $1/c$ , will give the rate by which national income or GDP will increase. This is the main point*

- In the context of the Harrod-Domar model, labor force growth is not described explicitly. This is because labor is assumed to be abundant in a developing-country context and can be hired as needed in a given proportion to capital investments **(this assumption is not always valid)**.
- In a general way, *technological progress can be expressed in the Harrod-Domar context as a decrease in the required capital-output ratio, giving more growth for a given level of investment*, as follows from Equation 3.7 or 3.7'.
- This is obvious when we realize that in the longer run this ratio is not fixed but can change over time in response to the functioning of financial markets and the policy environment. But again, the *focus was on the role of capital investment*.

## **(c) Solow growth model:**

- The Solow growth model shows how saving, population growth, and technological progress affect the level of an economy's output and its growth over time.

# The Accumulation of Capital

- Our first step is to examine how the supply and demand for goods determine the accumulation of capital. In this **first step**, we assume **that the labor force and technology are fixed**.
- We then relax these assumptions by introducing changes in the labor force later and then by introducing changes in technology next.

# The Supply and Demand for Goods

- By considering the supply and demand for goods, we can see what determines how much output is produced at any given time and how this output is allocated among alternative uses.
- **The Supply of Goods and the Production Function:**
- The supply of goods in the Solow model is based on the production function, which states that output depends on the capital stock and the labor force:

$$Y = F(K, L).$$

- The Solow growth model assumes that the production function has constant returns to scale. This assumption is often considered realistic, and, as we will see shortly, it helps simplify the analysis. Recall that a production function has constant returns to scale if

$$zY = F(zK, zL)$$

- for any positive number  $z$ . That is, if both capital and labor are multiplied by  $z$ , the amount of output is also multiplied by  $z$ .

- Production functions with constant returns to scale allow us to analyze all quantities in the economy relative to the size of the labor force. To see that this is true, set  $z = 1/L$  in the preceding equation to obtain

$$Y/L = F(K/L, 1).$$

- This equation shows that the amount of output per worker  $Y/L$  is a function of the amount of capital per worker  $K/L$ . (The number 1 is constant and thus can be ignored.) The assumption of constant returns to scale implies that the size of the economy—as measured by the number of workers—does not affect the relationship between output per worker and capital per worker.

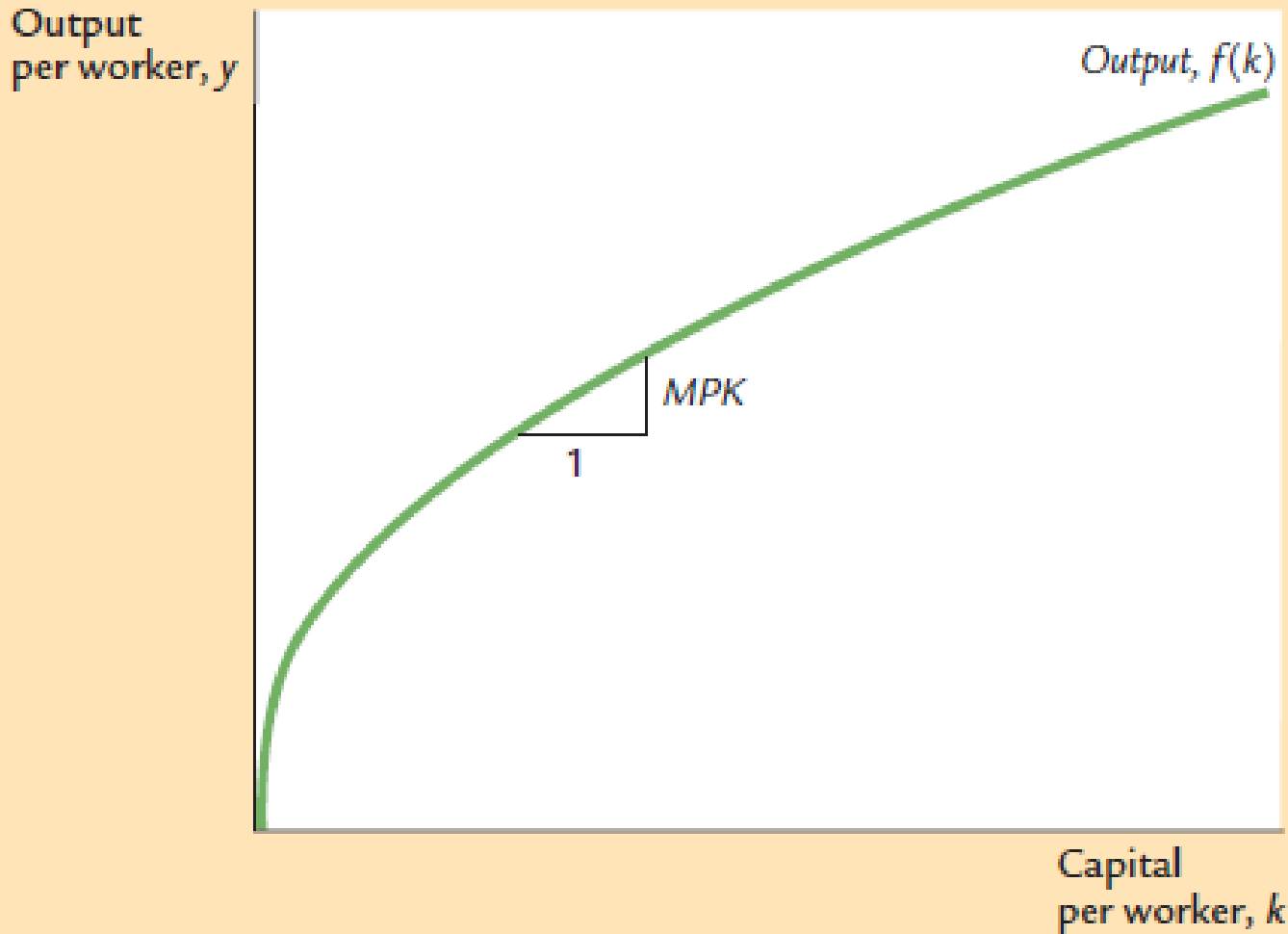
- Because the size of the economy does not matter, it will prove convenient to denote all quantities in per worker terms. We designate quantities per worker with lowercase letters, so  $y = Y/L$  is output per worker, and  $k = K/L$  is capital per worker. We can then write the production function as

$$y = f(k),$$

- where we define  $f(k) = F(k, 1)$ . Figure 7-1 illustrates this production function. The slope of this production function shows how much extra output a worker produces when given an extra unit of capital. This amount is **the marginal product of capital MPK. Mathematically, we write**

$$MPK = f(k + 1) - f(k).$$

**FIGURE 7-1**



**The Production Function** The production function shows how the amount of capital per worker  $k$  determines the amount of output per worker  $y = f(k)$ . The slope of the production function is the marginal product of capital: if  $k$  increases by 1 unit,  $y$  increases by  $MPK$  units. The production function becomes flatter as  $k$  increases, indicating diminishing marginal product of capital.

- **The Demand for Goods and the Consumption Function:**

- The demand for goods in the Solow model comes from consumption and investment. In other words, output per worker  $y$  is divided between consumption per worker  $c$  and investment per worker  $i$ :

$$y = c + i.$$

- This equation is the per-worker version of the national income accounts identity for an economy. Notice that it omits government purchases (**which for present purposes we can ignore**) and net exports (**because we are assuming a closed economy**).

- The Solow model assumes that each year people save a fraction  $s$  of their income and consume a fraction  $(1 - s)$ . We can express this idea with the following consumption function:

$$c = (1 - s)y,$$

- where  $s$ , the saving rate, is a number between zero and one. Keep in mind that various government policies can potentially influence a nation's saving rate, so one of our goals is to find what saving rate is desirable. For now, however, we just take the saving rate  $s$  as given.

- To see what this consumption function implies for investment, substitute  $(1 - s)y$  for  $c$  in the national income accounts identity:

$$y = (1 - s)y + i.$$

- Rearrange the terms to obtain

$$i = sy.$$

- This equation shows that investment equals saving. Thus, the rate of saving  $s$  is also the fraction of output devoted to investment.

- We have now introduced the two main ingredients of the Solow model— the production function and the consumption function— which describe the economy at any moment in time. For any given capital stock  $k$ , the production function  $y = f(k)$  determines how much output the economy produces, and the saving rate  $s$  determines the allocation of that output between consumption and investment.

# Growth in the Capital Stock and the Steady State

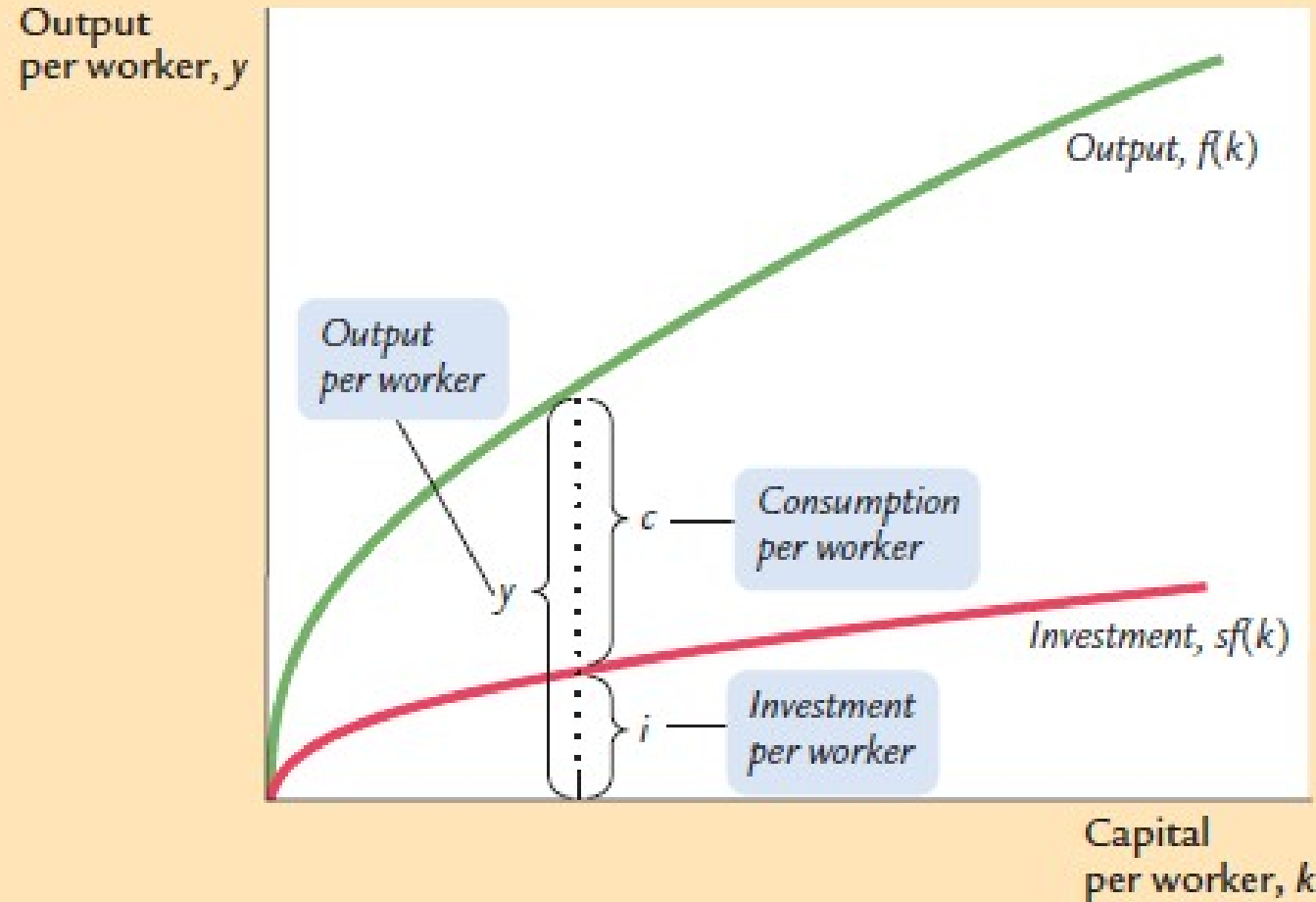
- At any moment, the capital stock is a key determinant of the economy's output, but the capital stock can change over time, and those changes can lead to economic growth. In particular, two forces influence the capital stock: investment and depreciation. *Investment* is expenditure on new plant and equipment, and it causes the capital stock to rise. *Depreciation* is the wearing out of old capital, and it causes the capital stock to fall. Let's consider each of these forces in turn.

- As we have already noted, investment per worker  $i$  equals  $sy$ . By substituting the production function for  $y$ , we can express investment per worker as a function of the capital stock per worker:

$$i = sf(k).$$

- This equation relates the existing stock of capital  $k$  to the accumulation of new capital  $i$ . Figure 7-2 shows this relationship. This figure illustrates how, for any value of  $k$ , the amount of output is determined by the production function  $f(k)$ , and the allocation of that output between consumption and saving is determined by the saving rate  $s$

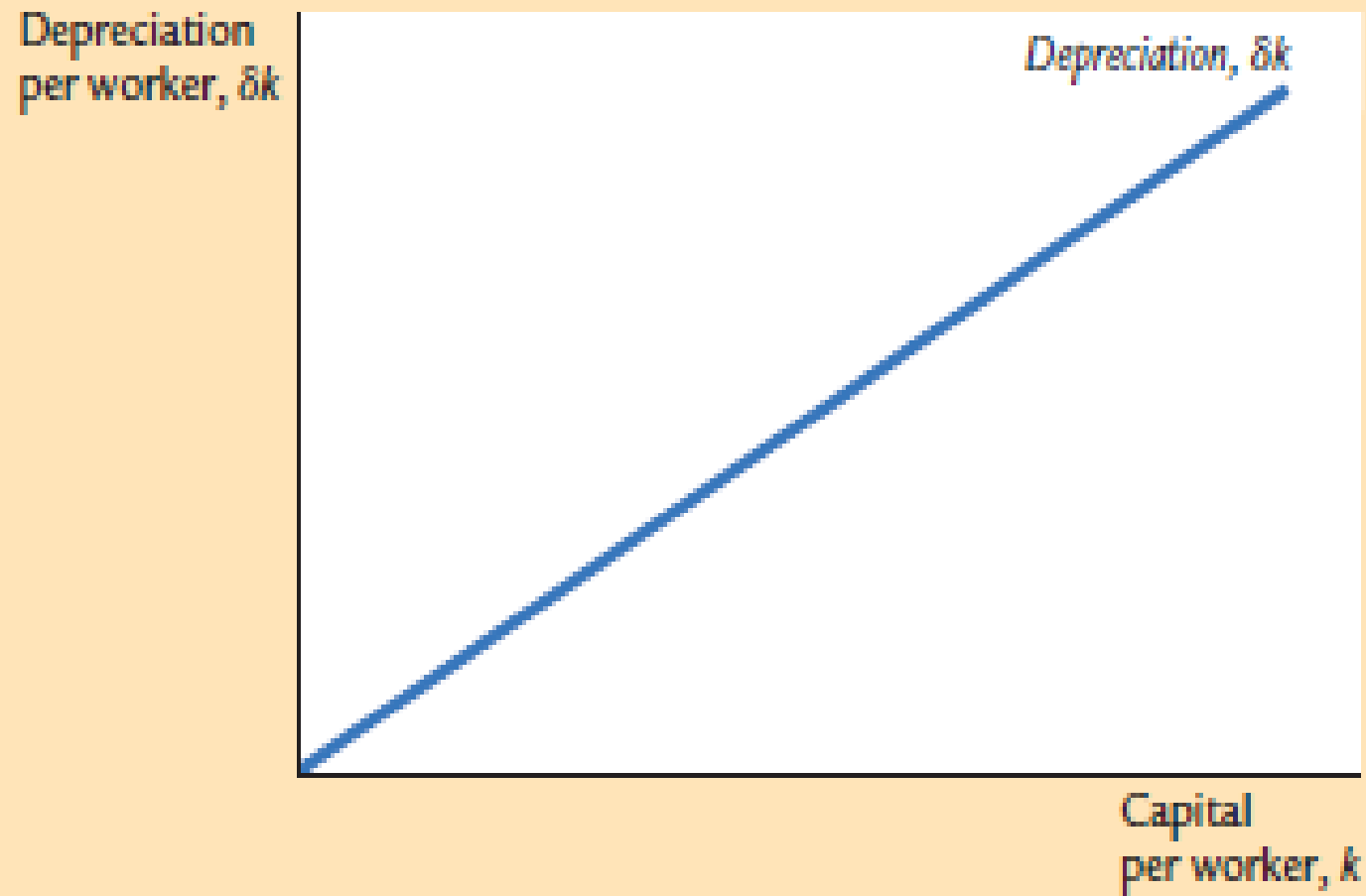
**FIGURE 7-2**



**Output, Consumption, and Investment** The saving rate  $s$  determines the allocation of output between consumption and investment. For any level of capital  $k$ , output is  $f(k)$ , investment is  $sf(k)$ , and consumption is  $f(k) - sf(k)$ .

- To incorporate depreciation into the model, we assume that a certain fraction  $d$  of the capital stock wears out each year. Here  $d$  (the lowercase Greek letter delta) is called the *depreciation rate*. For example, if capital lasts an average of 25 years, then the depreciation rate is 4 percent per year ( $d = 0.04$ ). The amount of capital that depreciates each year is  $dk$ . Figure 7-3 shows how the amount of depreciation depends on the capital stock.

**FIGURE 7-3**



**Depreciation** A constant fraction  $\delta$  of the capital stock wears out every year. Depreciation is therefore proportional to the capital stock.

- We can express the impact of investment and depreciation on the capital stock with this equation:

Change in Capital Stock = Investment – Depreciation

$$\Delta k = i - dk,$$

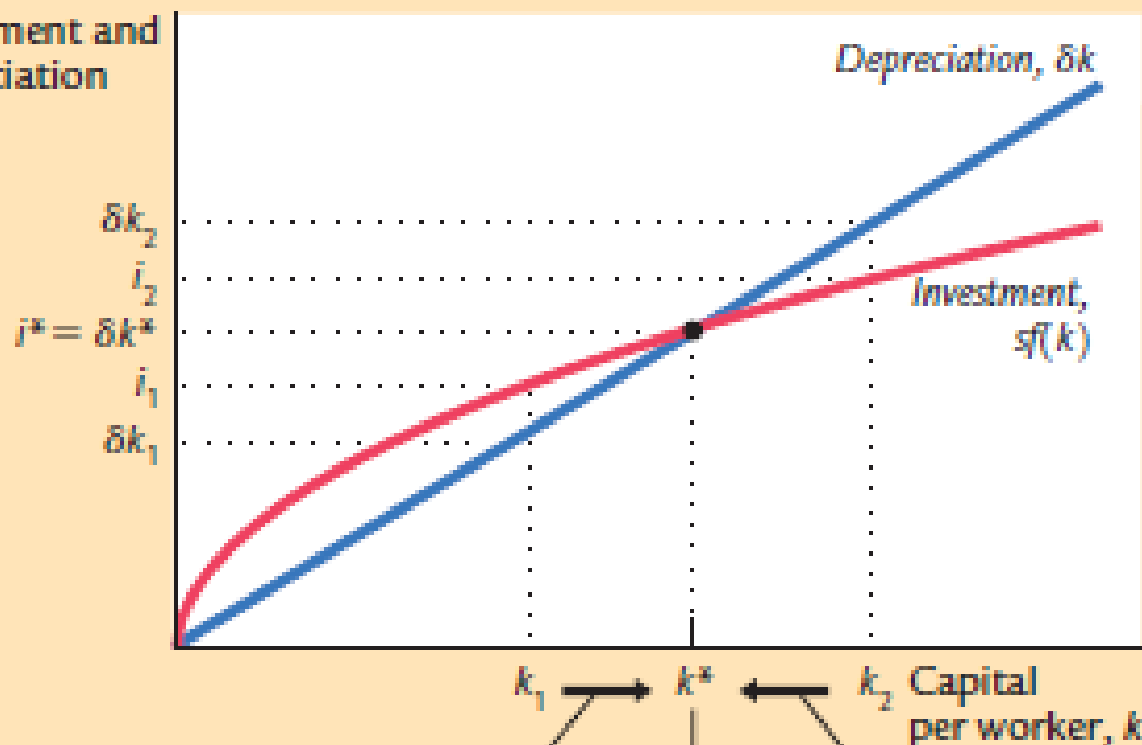
- where  $\Delta k$  is the change in the capital stock between one year and the next. Because investment  $i$  equals  $sf(k)$ , we can write this as

$$\Delta k = sf(k) - dk.$$

- Figure 7-4 graphs the terms of this equation—investment and depreciation—for different levels of the capital stock  $k$ . The higher the capital stock, the greater the amounts of output and investment. Yet the higher the capital stock, the greater also the amount of depreciation.

**FIGURE 7-4**

Investment and depreciation



*Capital stock increases because investment exceeds depreciation.*

*Steady-state level of capital per worker*

*Capital stock decreases because depreciation exceeds investment.*

**Investment, Depreciation, and the Steady State** The steady-state level of capital  $k^*$  is the level at which investment equals depreciation, indicating that the amount of capital will not change over time. Below  $k^*$  investment exceeds depreciation, so the capital stock grows. Above  $k^*$  investment is less than depreciation, so the capital stock shrinks.

- As Figure 7-4 shows, there is a single capital stock  $k^*$  at which the amount of investment equals the amount of depreciation. If the economy finds itself at this level of the capital stock, the capital stock will not change because the two forces acting on it—investment and depreciation—just balance. That is, at  $k^*$ ,  $\Delta k = 0$ , so the capital stock  $k$  and output  $f(k)$  are steady over time (rather than growing or shrinking). We therefore call  $k^*$  the steady-state level of capital.
- The steady state is significant for two reasons. As we have just seen, an economy at the steady state will stay there. In addition, and just as important, an economy *not* at the steady state will go there. That is, regardless of the level of capital with which the economy begins, it ends up with the steady-state level of capital. In this sense, *the steady state represents the long-run equilibrium of the economy.*

- Approaching the Steady State: A Numerical Example

- Let's use a numerical example to see how the Solow model works and how the economy approaches the steady state. For this example, we assume that the production function is

$$Y = K^{1/2}L^{1/2}.$$

- To derive the per-worker production function  $f(k)$ , divide both sides of the production function by the labor force  $L$ :

$$\frac{Y}{L} = \frac{K^{1/2}L^{1/2}}{L}.$$

- Rearrange to obtain  $\frac{Y}{L} = \left(\frac{K}{L}\right)^{1/2}.$

- Because  $y = Y/L$  and  $k = K/L$ , this equation becomes

$$y = k^{1/2},$$

- which can also be written as


$$y = \sqrt{k}$$

- This form of the production function states that output per worker equals the square root of the amount of capital per worker.
- To complete the example, let's assume that 30 percent of output is saved ( $s = 0.3$ ), that 10 percent of the capital stock depreciates every year ( $d = 0.1$ ), and that the economy starts off with 4 units of capital per worker ( $k = 4$ ). Given these numbers, we can now examine what happens to this economy over time

- We begin by looking at the production and allocation of output in the first year, when the economy has 4 units of capital per worker. Here are the steps we follow.
  - According to the production function  $y = k$ , the 4 units of capital per worker ( $k$ ) produce 2 units of output per worker ( $y$ ).
  - Because 30 percent of output is saved and invested and 70 percent is consumed,  $i = 0.6$  and  $c = 1.4$ .
  - Because 10 percent of the capital stock depreciates,  $dk = 0.4$ .
  - With investment of 0.6 and depreciation of 0.4, the change in the capital stock is  $dk = 0.2$ .
- Thus, the economy begins its second year with 4.2 units of capital per worker.

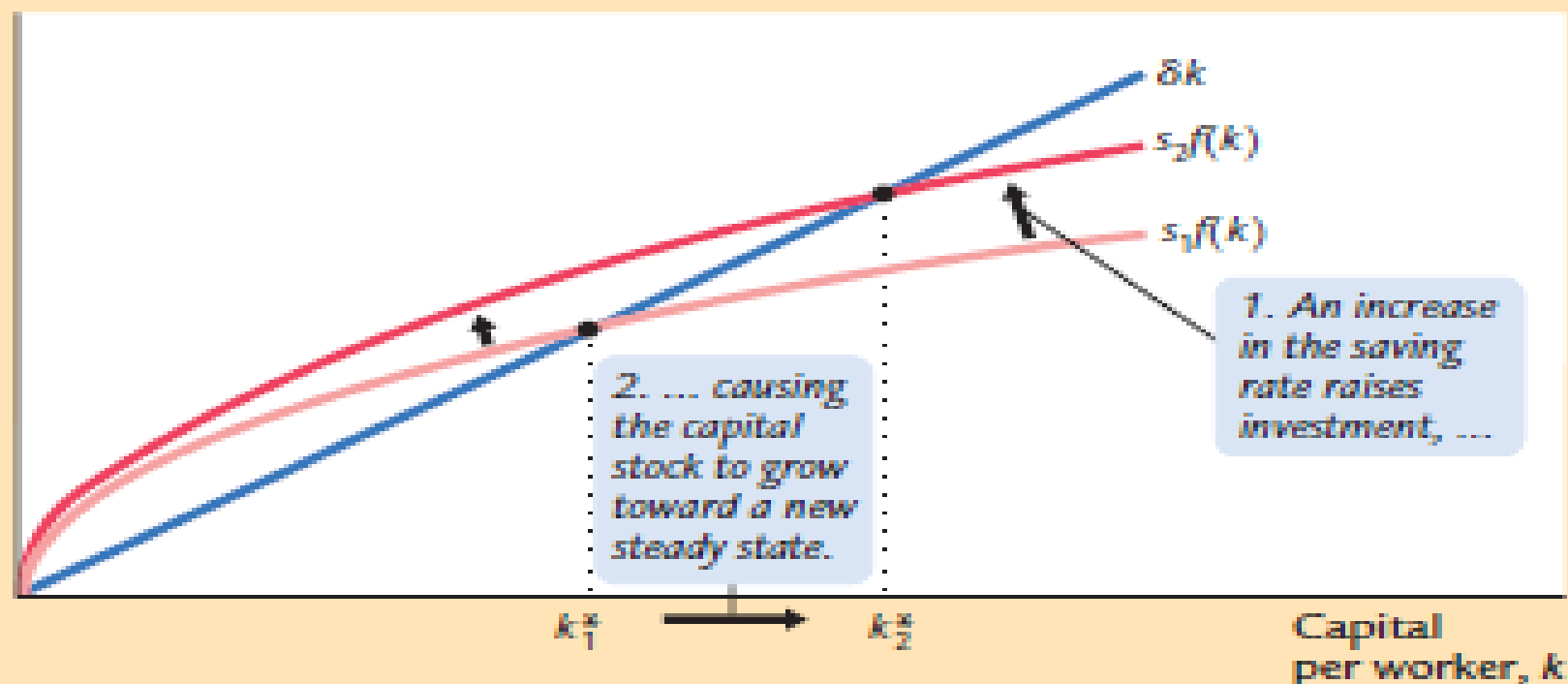
# • How Saving Affects Growth

- Consider what happens to an economy when its saving rate increases.
- Figure 7-5 shows such a change. The economy is assumed to begin in a steady state with saving rate  $s_1$  and capital stock  $k^*_1$ . When the saving rate increases from  $s_1$  to  $s_2$ , the  $sf(k)$  curve shifts upward. At the initial saving rate  $s_1$  and the initial capital stock  $k^*_1$ , the amount of investment just offsets the amount of depreciation. Immediately after the saving rate rises, investment is higher, but the capital stock and depreciation are unchanged. Therefore, investment exceeds depreciation. The capital stock will gradually rise until the economy reaches the new steady state  $k^*_2$ , which has a higher capital stock and a higher level of output than the old steady state.

- The Solow model shows that the saving rate is a key determinant of the steady-state capital stock. *If the saving rate is high, the economy will have a large capital stock and a high level of output in the steady state. If the saving rate is low, the economy will have a small capital stock and a low level of output in the steady state.*
- Now we can see that the long-run consequences of a reduced saving rate are a lower capital stock and lower national income. This is why many economists are critical of persistent budget deficits.

**FIGURE 7-5**

Investment  
and depreciation



**An Increase in the Saving Rate** An increase in the saving rate  $s$  implies that the amount of investment for any given capital stock is higher. It therefore shifts the saving function upward. At the initial steady state  $k_1^*$ , investment now exceeds depreciation. The capital stock rises until the economy reaches a new steady state  $k_2^*$  with more capital and output.

- What does the Solow model say about the relationship between saving and economic growth? Higher saving leads to faster growth in the Solow model, but only temporarily. An increase in the rate of saving raises growth only until the economy reaches the new steady state. If the economy maintains a high saving rate, it will maintain a large capital stock and a high level of output, but it will not maintain a high rate of growth forever.

# The Golden Rule Level of Capital

- So far, we have used the Solow model to examine how an economy's rate of saving and investment determines its steady-state levels of capital and income. This analysis might lead you to think that higher saving is always a good thing because it always leads to greater income. Yet suppose a nation had a saving rate of 100 percent. That would lead to the largest possible capital stock and the largest possible income. But if all of this income is saved and none is ever consumed, what good is it?
- This section uses the Solow model to discuss the optimal amount of capital accumulation from the standpoint of economic well-being.

# • Comparing Steady States

- To keep our analysis simple, let's assume that a policymaker can set the economy's saving rate at any level. By setting the saving rate, the policymaker determines the economy's steady state. What steady state should the policymaker choose?
- The policymaker's goal is to maximize the well-being of the individuals who make up the society. Individuals themselves do not care about the amount of capital in the economy, or even the amount of output. They care about the amount of goods and services they can consume. Thus, a benevolent policymaker would want to choose the steady state with the highest level of consumption. **The steady-state value of  $k$  that maximizes consumption is called the Golden Rule level of capital and is denoted  $k^*$  gold**

- How can we tell whether an economy is at the Golden Rule level? To answer this question, we must first determine steady-state consumption per worker. Then we can see which steady state provides the most consumption. To find steady-state consumption per worker, we begin with the national income accounts identity

$$y = c + i$$

and rearrange it as

$$c = y - i.$$

- Consumption is output minus investment. Because we want to find steady-state consumption, we substitute steady-state values for output and investment. Steady-state output per worker is  $f(k^*)$ , where  $k^*$  is the steady-state capital stock per worker. Furthermore, because the capital stock is not changing in the steady state, investment equals depreciation  $dk^*$ .

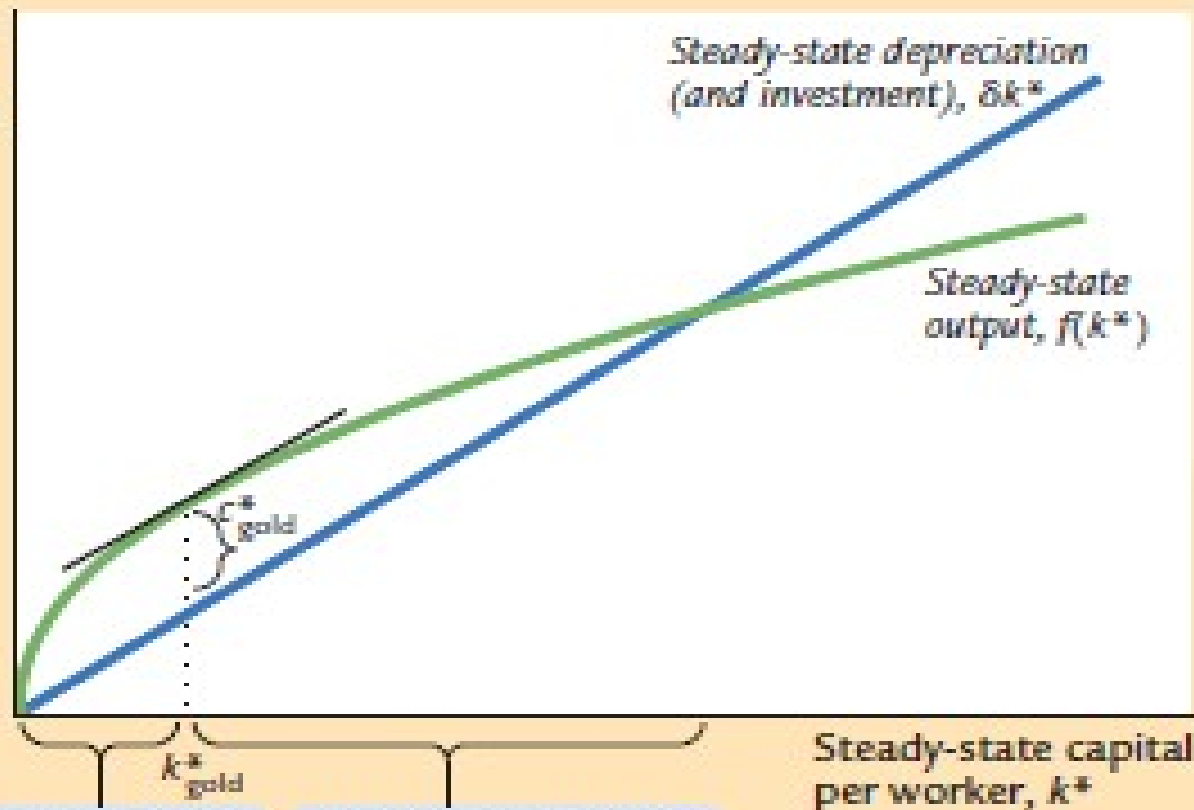
- Substituting  $f(k^*)$  for  $y$  and  $dk^*$  for  $i$ , we can write steady-state consumption per worker as

$$c^* = f(k^*) - dk^*.$$

- According to this equation, steady-state consumption is what's left of steady-state output after paying for steady-state depreciation. This equation shows that an increase in steady-state capital has two opposing effects on steady-state consumption. On the one hand, more capital means more output. On the other hand, more capital also means that more output must be used to replace capital that is wearing out.
- Figure 7-7 graphs steady-state output and steady-state depreciation as a function of the steady-state capital stock. Steady-state consumption is the gap between output and depreciation. This figure shows that there is one level of the capital stock—the Golden Rule level  $k^*$  gold—that maximizes consumption.

**FIGURE 7-7**

Steady-state  
output and  
depreciation



*Below the Golden Rule steady state, increases in steady-state capital raise steady-state consumption.*

*Above the Golden Rule steady state, increases in steady-state capital reduce steady-state consumption.*

Steady-state capital  
per worker,  $k^*$

### Steady-State Consumption

The economy's output is used for consumption or investment. In the steady state, investment equals depreciation. Therefore, steady-state consumption is the difference between output  $f(k^*)$  and depreciation  $\delta k^*$ . Steady-state consumption is maximized at the Golden Rule steady state. The Golden Rule capital stock is denoted  $k_{\text{gold}}^*$ , and the Golden Rule level of consumption is denoted  $c_{\text{gold}}^*$ .

- When comparing steady states, we must keep in mind that higher levels of capital affect both output and depreciation. If the capital stock is below the Golden Rule level, an increase in the capital stock raises output more than depreciation, so consumption rises. In this case, the production function is steeper than the  $dk^*$  line, so the gap between these two curves—which equals consumption—grows as  $k^*$  rises. By contrast, if the capital stock is above the Golden Rule level, an increase in the capital stock reduces consumption, because the increase in output is smaller than the increase in depreciation. In this case, the production function is flatter than the  $dk^*$  line, so the gap between the curves—consumption—shrinks as  $k^*$  rises. At the Golden Rule level of capital, the production function and the  $dk^*$  line have the same slope, and consumption is at its greatest level.

- We can now derive a simple condition that characterizes the Golden Rule level of capital. Recall that the slope of the production function is the marginal product of capital  $MPK$ . The slope of the  $dk^*$  line is  $d$ . Because these two slopes are equal at  $k^*$  gold, the Golden Rule is described by the equation

$$MPK = d.$$

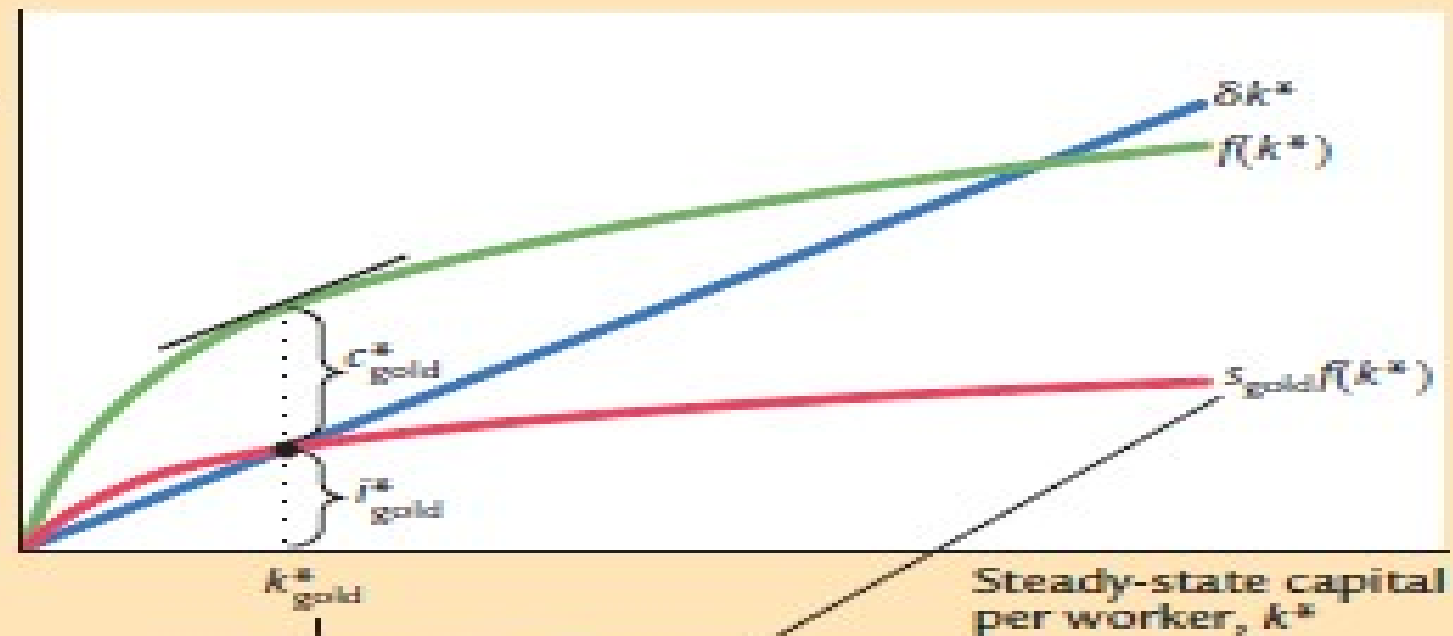
- At the Golden Rule level of capital, the marginal product of capital equals the depreciation rate.
- or

$$MPK - d = 0.$$

- At the Golden Rule level of capital, the marginal product of capital net of depreciation ( $MPK - d$ ) equals zero.

**FIGURE 7-8**

Steady-state output, depreciation, and investment per worker



1. To reach the Golden Rule steady state ...

2. ...the economy needs the right saving rate.

**The Saving Rate and the Golden Rule** There is only one saving rate that produces the Golden Rule level of capital  $k_{\text{gold}}^*$ . Any change in the saving rate would shift the  $sf(k)$  curve and would move the economy to a steady state with a lower level of consumption.

- Finding the Golden Rule Steady State:  
A Numerical Example

- **Population Growth**

- The basic Solow model shows that capital accumulation, by itself, cannot explain sustained economic growth: high rates of saving lead to high growth temporarily, but the economy eventually approaches a steady state in which capital and output are constant. To explain the sustained economic growth that we observe in most parts of the world, we must expand the Solow model to incorporate the other two sources of economic growth—population growth and technological progress. In this section we add population growth to the model.
- Instead of assuming that the population is fixed we now suppose that the population and the labor force grow at a constant rate  $n$ .

- # The Steady State With Population Growth
- How does population growth affect the steady state? To answer this question, we must discuss how population growth, along with investment and depreciation, influences the accumulation of capital per worker. As we noted before, investment raises the capital stock, and depreciation reduces it. But now there is a third force acting to change the amount of capital per worker: **the growth in the number of workers causes capital per worker to fall.**
- We continue to let lowercase letters stand for quantities per worker. Thus,  $k = K/L$  is capital per worker, and  $y = Y/L$  is output per worker. Keep in mind, however, that the number of workers is growing over time.
- **The change in the capital stock per worker is**

$$\Delta k = i - (d + n)k.$$

- This equation shows how investment, depreciation, and population growth influence the per-worker capital stock. Investment increases  $k$ , whereas depreciation and population growth decrease  $k$
- We can think of the term  $(d + n)k$  as defining *break-even investment*—the amount of investment necessary to keep the capital stock per worker constant. Break-even investment includes the depreciation of existing capital, which equals  $dk$ . It also includes the amount of investment necessary to provide new workers with capital. The amount of investment necessary for this purpose is  $nk$ , because there are  $n$  new workers for each existing worker and because  $k$  is the amount of capital for each worker. The equation shows that **population growth reduces the accumulation of capital per worker much the way depreciation does. Depreciation reduces  $k$  by wearing out the capital stock, whereas population growth reduces  $k$  by spreading the capital stock more thinly among a larger population of workers**

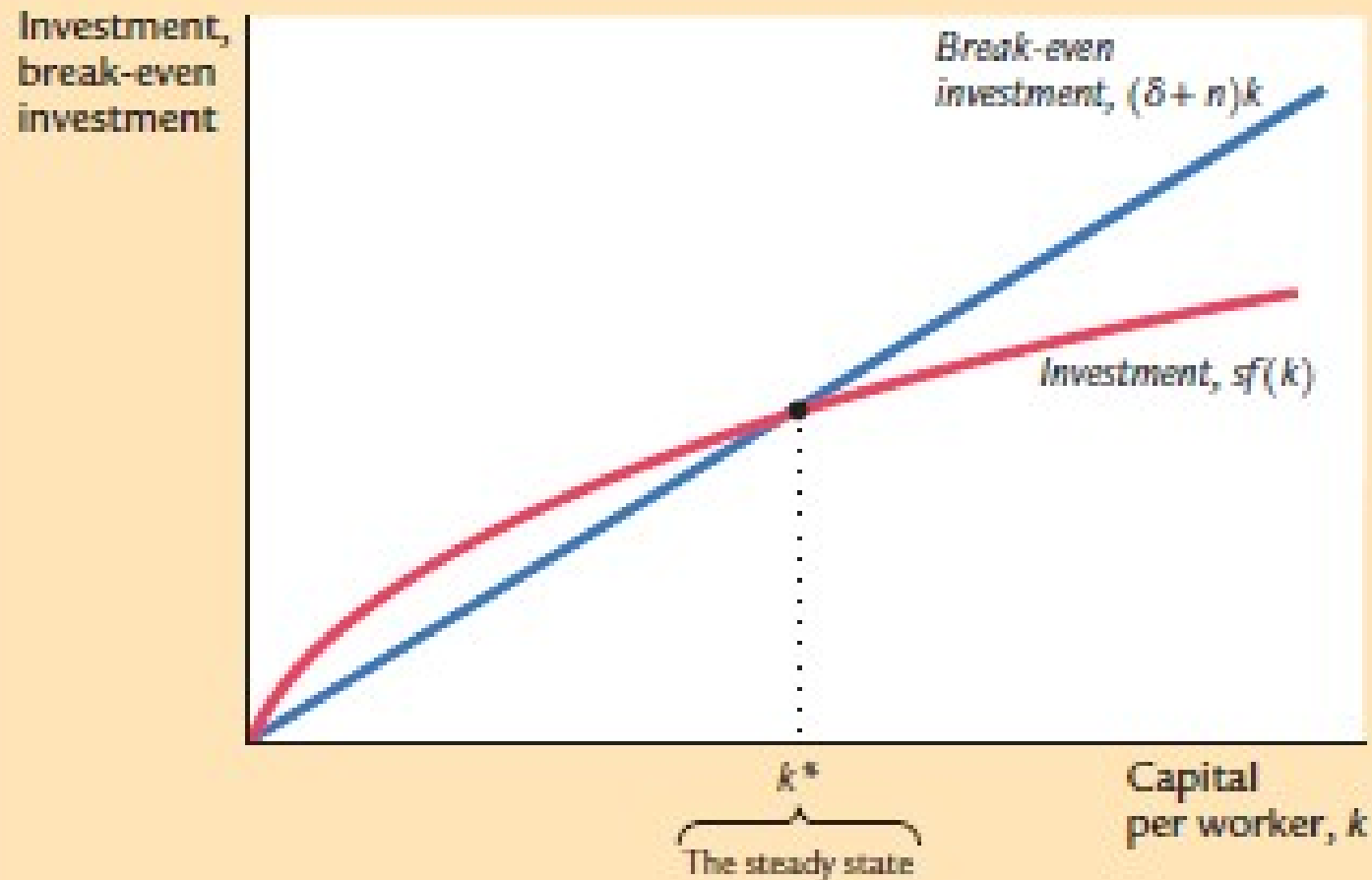
- Our analysis with population growth now proceeds much as it did previously. First, we substitute  $sf(k)$  for  $i$ . The equation can then be written as

$$\Delta k = sf(k) - (d + n)k.$$

- To see what determines the steady-state level of capital per worker, we use Figure 7-11, which extends the analysis of Figure 7-4 to include the effects of population growth. An economy is in a steady state if capital per worker  $k$  is unchanging. As before, we designate the steady-state value of  $k$  as  $k^*$ . If  $k$  is less than  $k^*$ , investment is greater than break-even investment, so  $k$  rises. If  $k$  is greater than  $k^*$ , investment is less than break-even investment, so  $k$  falls.

- In the steady state, the positive effect of investment on the capital stock per worker exactly balances the negative effects of depreciation and population growth. That is, at  $k^*$ ,  $\Delta k = 0$  and  $i^* = dk^* + nk^*$ . Once the economy is in the steady state, investment has two purposes. Some of it ( $dk^*$ ) replaces the depreciated capital, and the rest ( $nk^*$ ) provides the new workers with the steady-state amount of capital.

**FIGURE 7-11**



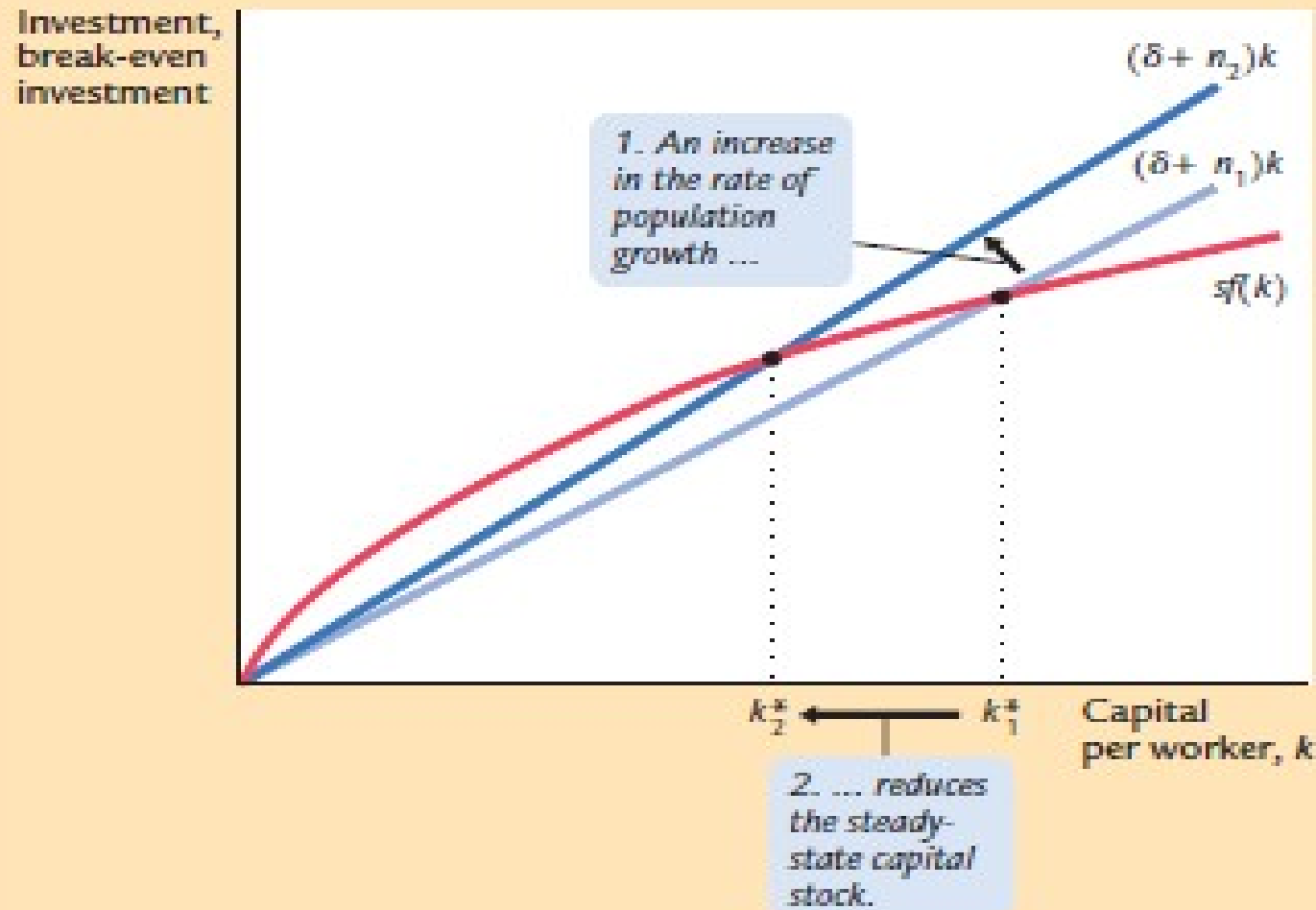
**Population Growth in the Solow Model** Depreciation and population growth are two reasons the capital stock per worker shrinks. If  $n$  is the rate of population growth and  $\delta$  is the rate of depreciation, then  $(\delta + n)k$  is *break-even investment*—the amount of investment necessary to keep constant the capital stock per worker  $k$ . For the economy to be in a steady state, investment  $sf(k)$  must offset the effects of depreciation and population growth  $(\delta + n)k$ . This is represented by the crossing of the two curves.

# • The Effects of Population Growth

- Population growth alters the basic Solow model in three ways.
- First, it brings us closer to explaining sustained economic growth. In the steady state with population growth, capital per worker and output per worker are constant. Because the number of workers is growing at rate  $n$ , however, *total* capital and *total* output must also be growing at rate  $n$ . Hence, although population growth cannot explain sustained growth in the standard of living (because output per worker is constant in the steady state), it can help explain sustained growth in total output.

- Second, population growth gives us another explanation for why some countries are rich and others are poor. Consider the effects of an increase in population growth. Figure 7-12 shows that an increase in the rate of population growth from  $n_1$  to  $n_2$  reduces the steady-state level of capital per worker from  $k^*_1$  to  $k^*_2$ . Because  $k^*$  is lower and because  $y^* = f(k^*)$ , the level of output per worker  $y^*$  is also lower. Thus, the Solow model predicts that countries with higher population growth will have lower levels of GDP per person. **Notice that a change in the population growth rate, like a change in the saving rate, has a level effect on income per person but does not affect the steady-state growth rate of income per person.**

**FIGURE 7-12**



### The Impact of Population Growth

An increase in the rate of population growth from  $n_1$  to  $n_2$  shifts the line representing population growth and depreciation upward. The new steady state  $k_2^*$  has a lower level of capital per worker than the initial steady state  $k_1^*$ . Thus, the Solow model predicts that economies with higher rates of population growth will have lower levels of capital per worker and therefore lower incomes.

- Finally, population growth affects our criterion for determining the Golden Rule (consumption-maximizing) level of capital. To see how this criterion changes, note that consumption per worker is

$$c = y - i.$$

- Because steady-state output is  $f(k^*)$  and steady-state investment is  $(d + n)k^*$ , we can express steady-state consumption as

$$c^* = f(k^*) - (d + n)k^*.$$

- Using an argument largely the same as before, we conclude that the level of  $k^*$  that maximizes consumption is the one at which

$$MPK = d + n,$$

- or equivalently,

$$MPK - d = n.$$

- In the Golden Rule steady state, the marginal product of capital net of depreciation equals the rate of population growth.

- **Technological Progress in the Solow Model**
- So far, our presentation of the Solow model has assumed an unchanging relationship between the inputs of capital and labor and the output of goods and services. Yet the model can be modified to include exogenous technological progress, which over time expands society's production capabilities.

- The Efficiency of Labor

- To incorporate technological progress, we must return to the production function that relates total capital  $K$  and total labor  $L$  to total output  $Y$ . Thus far, the production function has been

$$Y = F(K, L).$$

- We now write the production function as

$$Y = F(K, L \times E),$$

- where  $E$  is a new (and somewhat abstract) variable called the **efficiency of labor**. The efficiency of labor is meant to reflect society's knowledge about production methods: as the available technology improves, the efficiency of labor rises, and each hour of work contributes more to the production of goods and services

- The term  $L \times E$  can be interpreted as measuring the *effective number of workers*. It takes into account the number of actual workers  $L$  and the efficiency of each worker  $E$ . In other words,  $L$  measures the number of workers in the labor force, whereas  $L \times E$  measures both the workers and the technology with which the typical worker comes equipped. This new production function states that total output  $Y$  depends on the inputs of capital  $K$  and effective workers  $L \times E$ . The essence of this approach to modeling technological progress is that increases in the efficiency of labor  $E$  are analogous to increases in the labor force  $L$ .

- The simplest assumption about technological progress is that it causes the efficiency of labor  $E$  to grow at some constant rate  $g$ . This form of technological progress is called *labor augmenting*, and  $g$  is called the rate of **labor-augmenting technological progress**. Because the labor force  $L$  is growing at rate  $n$ , and the efficiency of each unit of labor  $E$  is growing at rate  $g$ , the effective number of workers  $L \times E$  is growing at rate  $n + g$ .

# • The Steady State With Technological Progress

- Because technological progress is modeled here as labor augmenting, it fits into the model in much the same way as population growth. Technological progress **does not cause the actual number of workers to increase**, but because each worker in effect comes with more units of labor over time, technological progress causes the effective number of workers to increase.
- We begin by reconsidering our notation. Previously, when there was no technological progress, we analyzed the economy in terms of quantities per worker; now we can generalize that approach by analyzing the economy in terms of quantities per effective worker. We now let  $k = K/(L \times E)$  stand for capital per effective worker and  $y = Y/(L \times E)$  stand for output per effective worker. With these definitions, we can again write  $y = f(k)$ .

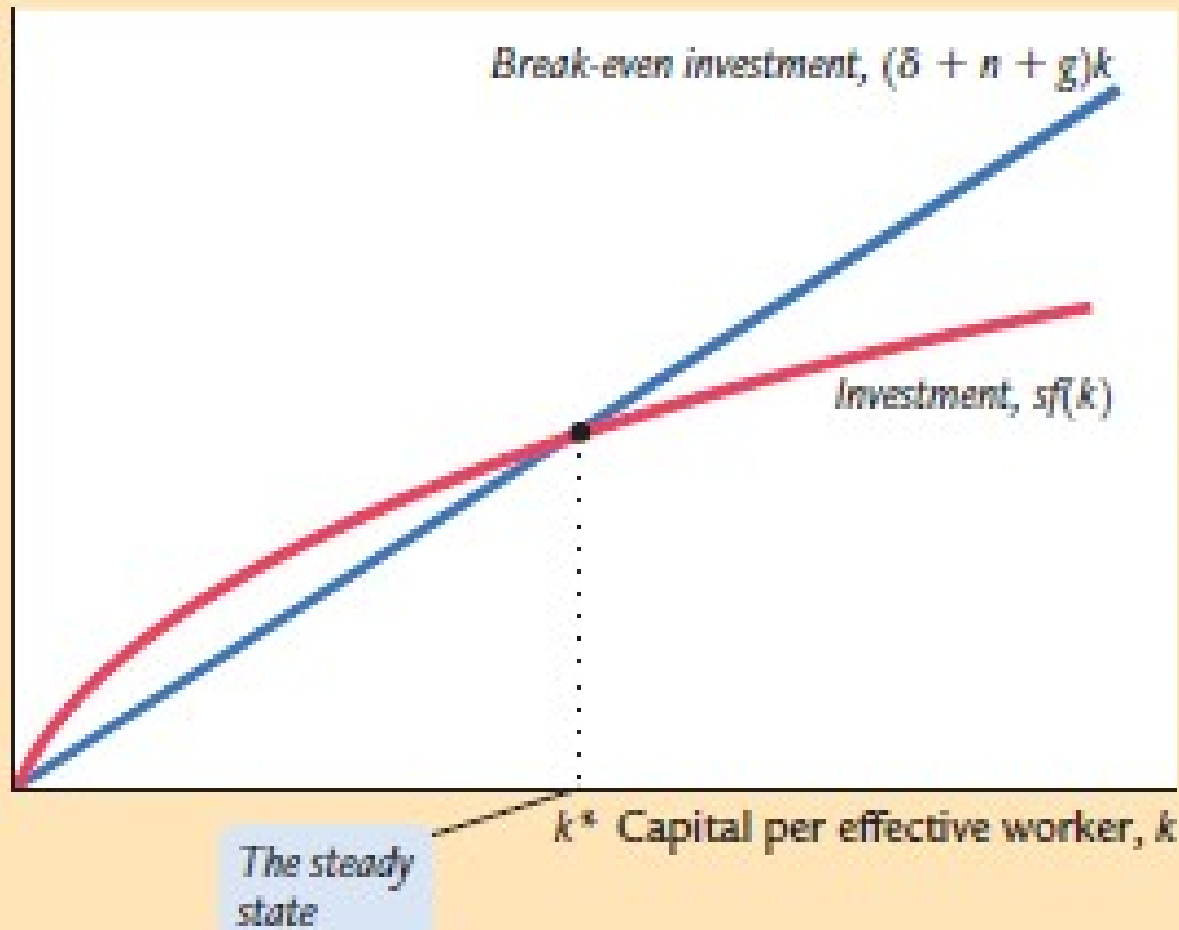
- Our analysis of the economy proceeds just as it did when we examined population growth. The equation showing the evolution of  $k$  over time becomes

$$\Delta k = sf(k) - (d + n + g)k.$$

- As before, the change in the capital stock  $\Delta k$  equals investment  $sf(k)$  minus breakeven investment  $(d + n + g)k$ . Now, however, because  $k = K/(L \times E)$ , break-even investment includes three terms: to keep  $k$  constant,  $dk$  is needed to replace depreciating capital,  $nk$  is needed to provide capital for new workers, and  $gk$  is needed to provide capital for the new “effective workers” created by technological progress.

**FIGURE 8-1**

Investment,  
break-even  
investment



**Technological Progress and the Solow Growth Model**  
Labor-augmenting technological progress at rate  $g$  enters our analysis of the Solow growth model in much the same way as did population growth at rate  $n$ . Now that  $k$  is defined as the amount of capital per effective worker, increases in the effective number of workers because of technological progress tend to decrease  $k$ . In the steady state, investment  $sf(k)$  exactly offsets the reductions in  $k$  attributable to depreciation, population growth, and technological progress.

- As shown in Figure 8-1, the inclusion of technological progress does not substantially alter our analysis of the steady state. There is one level of  $k$ , denoted  $k^*$ , at which capital per effective worker and output per effective worker are constant. As before, this steady state represents the long-run equilibrium of the economy.

# The Effects of Technological Progress

- Table 8-1 shows how four key variables behave in the steady state with technological progress. As we have just seen, capital per effective worker  $k$  is constant in the steady state. Because  $y = f(k)$ , output per effective worker is also constant. It is these quantities per effective worker that are steady in the steady state.
- From this information, we can also infer what is happening to variables that are not expressed in units per effective worker. For instance, consider output per actual worker  $Y/L = y \times E$ . Because  $y$  is constant in the steady state and  $E$  is growing at rate  $g$ , output per worker must also be growing at rate  $g$  in the steady state. Similarly, the economy's total output is  $Y = y \times (E \times L)$ . Because  $y$  is constant in the steady state,  $E$  is growing at rate  $g$ , and  $L$  is growing at rate  $n$ , total output grows at rate  $n + g$  in the steady state.

- With the addition of technological progress, our model can finally explain the sustained increases in standards of living that we observe. That is, we have shown that technological progress can lead to sustained growth in output per worker. By contrast, a high rate of saving leads to a high rate of growth only until the steady state is reached. Once the economy is in steady state, the rate of growth of output per worker depends only on the rate of technological progress. *According to the Solow model, only technological progress can explain sustained growth and persistently rising living standards.*

TABLE 8-1

Steady-State Growth Rates in the Solow Model  
With Technological Progress

| Variable                     | Symbol                      | Steady-State Growth Rate |
|------------------------------|-----------------------------|--------------------------|
| Capital per effective worker | $k = K/(E \times L)$        | 0                        |
| Output per effective worker  | $y = Y/(E \times L) = f(k)$ | 0                        |
| Output per worker            | $Y/L = y \times E$          | $g$                      |
| Total output                 | $Y = y \times (E \times L)$ | $n + g$                  |

- The introduction of technological progress also modifies the criterion for the Golden Rule. The Golden Rule level of capital is now defined as the steady state that maximizes consumption per effective worker. Following the same arguments that we have used before, we can show that steady-state consumption per effective worker is

$$c^* = f(k^*) - (d + n + g)k^*.$$

- Steady-state consumption is maximized if

$$MPK = d + n + g,$$

- or

$$MPK - d = n + g.$$

- That is, at the Golden Rule level of capital, the net marginal product of capital,  $MPK - d$ , equals the rate of growth of total output,  $n + g$ . Because actual economies experience both population growth and technological progress, we must use this criterion to evaluate whether they have more or less capital than they would at the Golden Rule steady state.