

Addis Ababa Science & Technology University
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Probability and Random Process (EEEG-2114)

Chapter 1: Basic Concepts of Probability Theory

Basic Concepts of Probability Theory

Outline

- *Introduction*
- *Sample Space and Events*
- *Basic Set Operations*
- *Axioms and Properties of Probability*
- *Conditional Probability*
- *Independence of Events*

Introduction

- Events can in general be classified as:
 - ***Random***-the event may or may not occur
 - ***Certain***- the occurrence of the event is inevitable
 - ***Impossible***-the event will never occur
- Probability is the study of **randomness** and **uncertainty**.
- Probability theory provides powerful tools to explain, model and design real world physical systems with some degree of uncertainty.
- Some application areas of probability theory include:
 - Data communication systems
 - Wireless communication systems
 - Control systems, etc...

Sample Space and Events

i. Random Experiment

- A random experiment is an experiment in which the outcome varies in an unpredictable manner when the experiment is repeated under the same conditions.

Examples:

- Tossing a coin
- Rolling a die
- Picking a card from a deck

ii. Sample Space

- The sample space is the set of all possible outcomes of a random experiment.

Sample Space and Events Cont'd....

- The sample space is denoted by Ω and the possible outcomes are represented by

$$\Omega = \{\omega_1, \omega_2, \dots, \omega_n\}$$

iii. Event

- An event is any subset of the sample of the sample space, Ω
- Events can be represented by $A, B, C, \dots$

Example-1:

Consider a random experiment of rolling a die once.

iv. Sample Space

$$\Omega = \{1, 2, 3, 4, 5, 6\}$$

Sample Space and Events Cont'd....

ii. Some possible events

- An event of obtaining even numbers

$$A = \{2, 4, 6\}$$

- An event of obtaining numbers less than 4

$$B = \{1, 2, 3\}$$

Example-2:

Consider a random experiment of flipping a fair coin twice.

i. Sample space

$$\Omega = \{HH, HT, TH, TT\}$$

Sample Space and Events Cont'd....

ii. Some possible events

- An event of getting exactly one head

$$A = \{ HT, TH \}$$

- An event of getting at least one tail

$$B = \{ HT, TH, TT \}$$

- An event of getting at most one tail

$$C = \{ HH, HT, TH \}$$

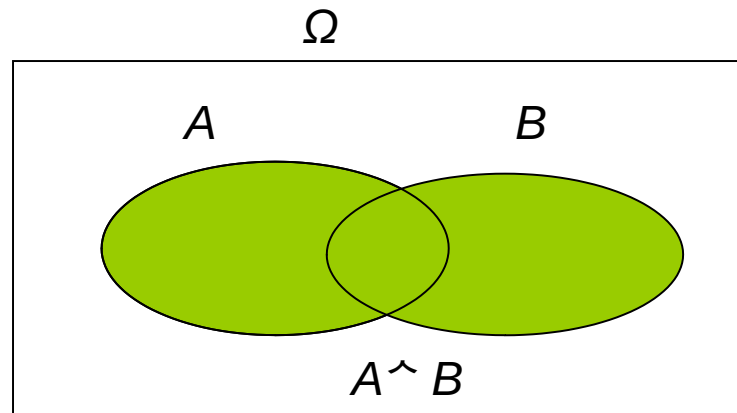
Basic Set Operations

- We can combine events using set operations to obtain other events.

1. Union

The union of two events A and B is defined as the set of outcomes that are either in A or B or both and is denoted by $A \cup B$.

$$A \cup B = \{\omega : \omega \in A \text{ or } \omega \in B\} \Leftrightarrow A \cup B = \{\omega : \omega \in A \vee \omega \in B\}$$

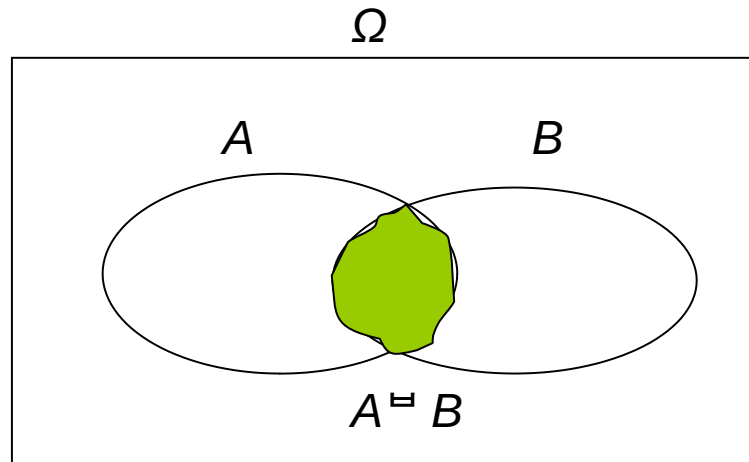


Basic Set Operations Cont'd.....

2. Intersection

The intersection of two events A and B is defined as the set of outcomes that are common to both A and B and is denoted by $A \cap B$.

$$A \cap B = \{\omega : \omega \in A \text{ and } \omega \in B\} \Leftrightarrow A \cap B = \{\omega : \omega \in A \wedge \omega \in B\}$$

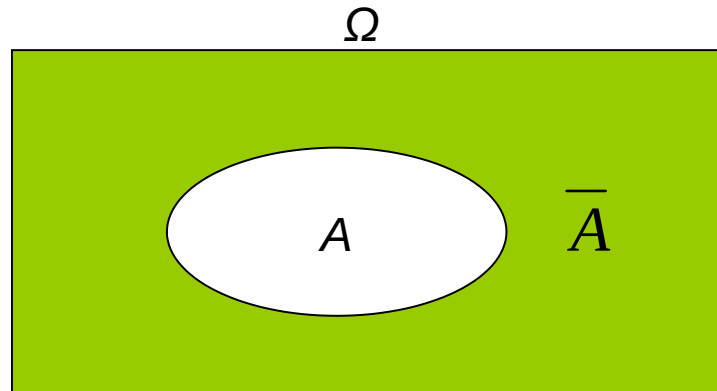


Basic Set Operations Cont'd.....

3. Complement

The complement of an event A is defined as the set of all outcomes that are not in A and is denoted by $\bar{A}$.

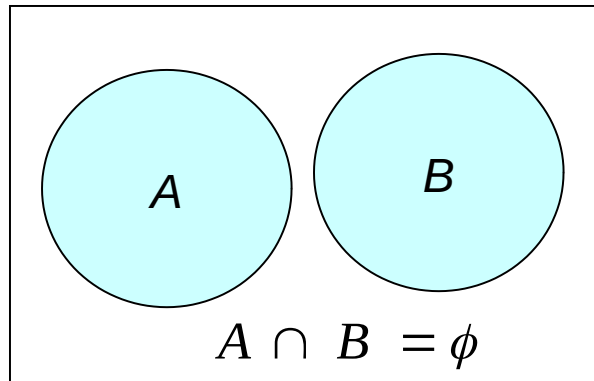
$$\bar{A} = \{\omega : \omega \in \Omega \text{ and } \omega \notin A\} \Leftrightarrow \bar{A} = \{\omega : \omega \in \Omega \wedge \omega \notin A\}$$



Basic Set Operations Cont'd.....

4. Mutually Exclusive (Disjoint) Events

Two events **A** and **B** are said to be mutually exclusive or disjoint if **A** and **B** have no elements in common, i.e., $A \cap B = \Phi$



5. Equal Events

Two events **A** and **B** are said to equal if they contain the same outcomes and is denoted by $A=B$.

Some Properties of Set Operations

1. Elementary Properties

- i. $\overline{\Omega} = \Phi$
- ii. $\overline{\Phi} = \Omega$
- iii. $\Omega \cup A = \Omega$
- iv. $\Omega \cap A = A$
- v. $A \cup \overline{A} = \Omega$
- vi. $A \cap \overline{A} = \Phi$
- vii. $\overline{\overline{A}} = A$

2. Commutative Properties

$$A \cup B = B \cup A$$

$$A \cap B = B \cap A$$

3. Associative Properties

$$A \cup (B \cup C) = (A \cup B) \cup C$$

$$A \cap (B \cap C) = (A \cap B) \cap C$$

Some Properties of Set Operations Cont'd.....

4. Distributive Properties

$$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$$

$$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$$

5. DeMorgan's Rules

$$\overline{(A \cup B)} = \bar{A} \cap \bar{B}$$

$$\overline{(A \cap B)} = \bar{A} \cup \bar{B}$$

- The union and intersection operations can be repeated for an arbitrary number of events as follows.

$$\bigcup_{i=1}^n A_i = A_1 \cup A_2 \cup \dots \cup A_n$$

$$\bigcap_{i=1}^n A_i = A_1 \cap A_2 \cap \dots \cap A_n$$

Axioms and Properties of Probability

- Probability is a rule that assigns a number to each event **A** in the sample space, Ω .
- In short , the probability of any event **A** is given by

$$P(A) = \frac{n(A)}{n(\Omega)}$$

where

$n(A)$ - is the number of elements in the event A

$n(\Omega)$ - is the number of elements in the sample space Ω

Axioms and Properties of Probability Cont'd.....

- The probability of an event **A** is a real number which satisfies the following axioms.

1. Probability is a non-negative number, i.e.,

$$P(A) \geq 0$$

2. Probability of the whole set is unity, i.e.,

$$P(\Omega) = 1$$

From axioms (1) and (2), we obtain

$$0 \leq P(A) \leq 1$$

Axioms and Properties of Probability Cont'd.....

3. Probability of the union of two mutually exclusive (disjoint) events is the sum of the probability of the events, i.e.,

$$\text{If } A \cap B = \Phi, \text{ then } P(A \cup B) = P(A) + P(B)$$

- We can generalize axiom (3) for n pairwise mutually exclusive (disjoint) events.
- If $A_1, A_2, A_3, \dots, A_n$ is a sequence of n pairwise mutually exclusive (disjoint) events in the sample space Ω such that $A_i \cap A_j = \Phi$, for $i \neq j$, then

$$P\left(\bigcup_{i=1}^n A_i\right) = \sum_{i=1}^n P(A_i)$$

Axioms and Properties of Probability Cont'd.....

- By using the above probability axioms, other useful properties of probability can be obtained.

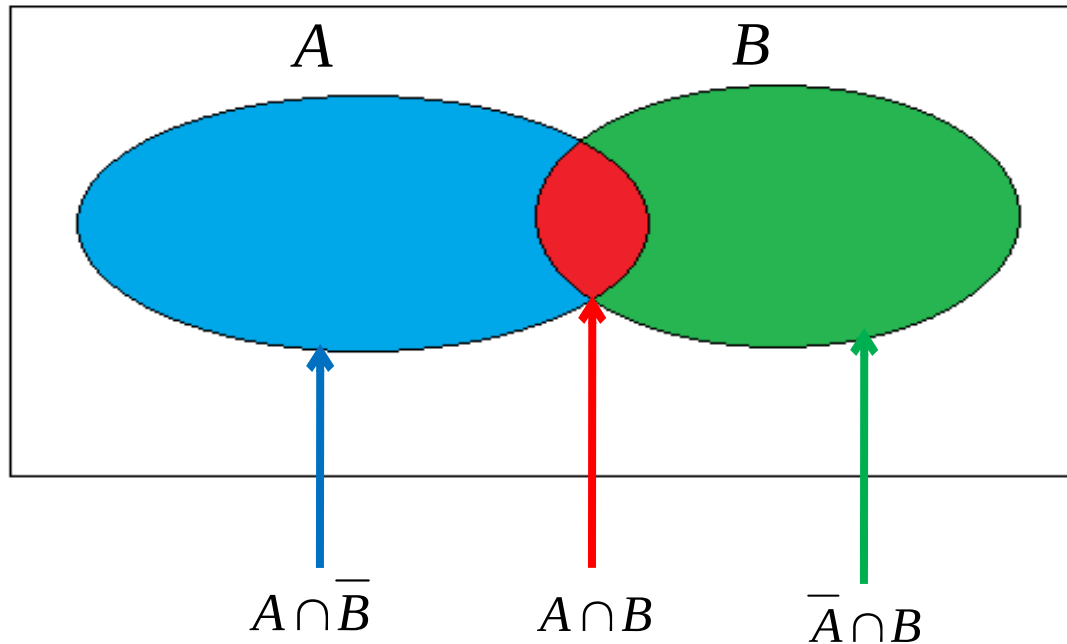
1. $P(\bar{A}) = 1 - P(A)$

Proof:

$$\begin{aligned} A \cap \bar{A} &= \Phi \Rightarrow P(A \cup \bar{A}) = P(A) + P(\bar{A}), \text{ but } A \cup \bar{A} = \Omega \\ \Rightarrow P(\Omega) &= P(A) + P(\bar{A}), P(\Omega) = P(A \cup \bar{A}) \\ \Rightarrow 1 &= P(A) + P(\bar{A}), P(\Omega) = 1 \\ \therefore P(\bar{A}) &= 1 - P(A) \end{aligned}$$

Axioms and Properties of Probability Cont'd.....

- We can decompose the events A , B and $A \cup B$ as unions of mutually exclusive (disjoint) events as follows.



We can see that $A \cap B$, $A \cap \bar{B}$ and $\bar{A} \cap B$ are disjoint events.

Axioms and Properties of Probability Cont'd.....

- From the above Venn diagram, we can write the following relations.

$$i. \quad A = (A \cap B) \cup (A \cap \bar{B})$$

$$\Rightarrow P(A) = P(A \cap B) + P(A \cap \bar{B})$$

$$\Rightarrow P(A \cap \bar{B}) = P(A) - P(A \cap B)$$

$$ii. \quad B = (A \cap B) \cup (\bar{A} \cap B)$$

$$\Rightarrow P(B) = P(A \cap B) + P(\bar{A} \cap B)$$

$$\Rightarrow P(\bar{A} \cap B) = P(B) - P(A \cap B)$$

$$iii. \quad A \cup B = A \cup (\bar{A} \cap B)$$

$$\Rightarrow P(A \cup B) = P(A) + P(\bar{A} \cap B)$$

$$iv. \quad A \cup B = B \cup (A \cap \bar{B})$$

$$\Rightarrow P(A \cup B) = P(B) + P(A \cap \bar{B})$$

$$v. \quad A \cup B = (A \cap \bar{B}) \cup (A \cap B) \cup (\bar{A} \cap B)$$

$$\Rightarrow P(A \cup B) = P(A \cap \bar{B}) + P(A \cap B) + P(\bar{A} \cap B)$$

Axioms and Properties of Probability Cont'd.....

$$2. \quad P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

Proof:

$$P(A \cup B) = P(A) + P(\overline{A} \cap B)$$

$$\text{But, } P(\overline{A} \cap B) = P(B) - P(A \cap B)$$

$$\therefore P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

- We can generalize the above property for three events **A**, **B** and **C** as follows.

$$\begin{aligned} P(A \cup B \cup C) = & P(A) + P(B) + P(C) - P(A \cap B) - P(A \cap C) - P(B \cap C) \\ & + P(A \cap B \cap C) \end{aligned}$$

Axioms and Properties of Probability Cont'd.....

- For n events $A_1, A_2, A_3, \dots, A_n$ the above property can be generalized as:

$$P\left(\bigcup_{i=1}^n A_i\right) = \sum_{j=1}^n P(A_j) - \sum_{j < k} P(A_j \cap A_k) + \dots + (-1)^{n+1} P(A_1 \cap A_2 \cap \dots \cap A_n)$$

3. $P(A \cup B) \leq P(A) + P(B)$

Proof:

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$\text{But, } P(A \cap B) \geq 0$$

$$\therefore P(A \cup B) \leq P(A) + P(B)$$

Axioms and Properties of Probability Cont'd.....

Example-1:

A box contains 10 identical balls numbered 0, 1, 2,...,9. A single ball is selected from the box at random. Consider the following events.

A: number of ball selected is odd

B: number of ball selected is multiple of 3

C: number of ball selected is less than 5

Find the following probabilities.

- | | |
|-----------|-------------------------|
| a. $P(A)$ | d. $P(A \cap B)$ |
| b. $P(B)$ | e. $P(A \cup B \cup C)$ |
| c. $P(C)$ | |

Axioms and Properties of Probability Cont'd.....

Solution:

- The sample space and the events are given by:

$$\Omega = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$$

$$C = \{0, 1, 2, 3, 4\}$$

$$A = \{1, 3, 5, 7, 9\}$$

$$A \cap B = \{3, 9\}$$

$$B = \{3, 6, 9\}$$

$$A \cup B \cup C = \{0, 1, 2, 3, 4, 5, 6, 7, 9\}$$

- The number of elements in the sample space and events are:

$$n(\Omega) = 10$$

$$n(C) = 5$$

$$n(A) = 5$$

$$n(A \cap B) = 2$$

$$n(B) = 3$$

$$n(A \cup B \cup C) = 9$$

Axioms and Properties of Probability Cont'd.....

➤ Thus, probabilities of the given events are given by:

$$a. P(A) = \frac{n(A)}{n(\Omega)} = \frac{5}{10} = \frac{1}{2}$$

$$d. P(A \cap B) = \frac{n(A \cap B)}{n(\Omega)} = \frac{2}{10} = \frac{1}{5}$$

$$b. P(B) = \frac{n(B)}{n(\Omega)} = \frac{3}{10}$$

$$e. P(A \cup B \cup C) = \frac{n(A \cup B \cup C)}{n(\Omega)} = \frac{9}{10}$$

$$c. P(C) = \frac{n(C)}{n(\Omega)} = \frac{5}{10} = \frac{1}{2}$$

Example-2:

Given $P(A) = 0.9$, $P(B) = 0.8$ and $P(A \cap B) = 0.75$, find :

$$a. P(A \cup B)$$

$$c. P(\bar{A} \cap \bar{B})$$

$$e. P(\bar{A} \cup B)$$

$$b. P(A \cap \bar{B})$$

$$d. P(\bar{A} \cup \bar{B})$$

$$f. P(\bar{B})$$

Axioms and Properties of Probability Cont'd.....

Solution:

$$a. P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$\Rightarrow P(A \cup B) = 0.9 + 0.8 - 0.75$$

$$\therefore P(A \cup B) = 0.95$$

$$b. P(A \cap \bar{B}) = P(A) - P(A \cap B)$$

$$\Rightarrow P(A \cap \bar{B}) = 0.9 - 0.75$$

$$\therefore P(A \cap \bar{B}) = 0.15$$

$$c. P(\bar{A} \cap \bar{B}) = P(\overline{A \cup B}) = 1 - P(A \cup B)$$

$$\Rightarrow P(\bar{A} \cap \bar{B}) = 1 - 0.95$$

$$\therefore P(\bar{A} \cap \bar{B}) = 0.05$$

$$d. P(\bar{A} \cup \bar{B}) = P(\overline{A \cap B}) = 1 - P(A \cap B)$$

$$\Rightarrow P(\bar{A} \cup \bar{B}) = 1 - 0.75$$

$$\therefore P(\bar{A} \cup \bar{B}) = 0.25$$

$$e. P(\bar{A} \cup B) = 1 - P(A) + P(A \cap B)$$

$$\Rightarrow P(\bar{A} \cup B) = 1 - 0.9 + 0.75$$

$$\therefore P(\bar{A} \cup B) = 0.85$$

$$f. P(\bar{B}) = 1 - P(B)$$

$$\Rightarrow P(\bar{B}) = 1 - 0.8$$

$$\therefore P(\bar{B}) = 0.2$$

Axioms and Properties of Probability Cont'd.....

Exercise:

1. If $A \cap B = \Phi$, then show that $P(A) \leq P(B)$.
2. If $P(A) = P(B) = P(A \cap B)$, then show that
$$P[(A \cap \bar{B}) \cup (\bar{A} \cap B)] = 0.$$
3. If $P(A) = P(B) = 1$, then show that $P(A \cap B) = 1$.
4. If $P(A) = 0.9$ and $P(B) = 0.8$, then show that $P(A \cap B) \geq 0.7$

Conditional Probability

- The conditional probability of an event **A** given **B**, denoted by **$P(A/B)$** , is defined as:

$$P(A/B) = \frac{P(A \cap B)}{P(B)}, \quad P(B) > 0 \quad (1)$$

- Similarly, the conditional probability of an event **B** given **A**, denoted by **$P(B/A)$** , is given by

$$P(B/A) = \frac{P(A \cap B)}{P(A)}, \quad P(A) > 0 \quad (2)$$

- From equations (1) and (2), we will get

$$P(A \cap B) = P(A/B)P(B) = P(B/A)P(A) \quad (3)$$

Conditional Probability Cont'd.....

- Then using equation (3), we will get

$$P(A/B) = \frac{P(B/A)P(A)}{P(B)} \quad \text{OR} \quad P(B/A) = \frac{P(A/B)P(B)}{P(A)} \quad (4)$$

- We know that

$$\begin{aligned} P(B) &= P(A \cap B) + P(\bar{A} \cap B) \\ \Rightarrow P(B) &= P(B/A)P(A) + P(B/\bar{A})P(\bar{A}) \end{aligned} \quad (5)$$

- Substituting equation (5) into equation (4), we will get

$$P(A/B) = \frac{P(B/A)P(A)}{P(B/A)P(A) + P(B/\bar{A})P(\bar{A})} \quad (6)$$

Conditional Probability Cont'd.....

- Similarly,

$$P(B / A) = \frac{P(A / B)P(B)}{P(A / B)P(B) + P(A / \overline{B})P(\overline{B})} \quad (7)$$

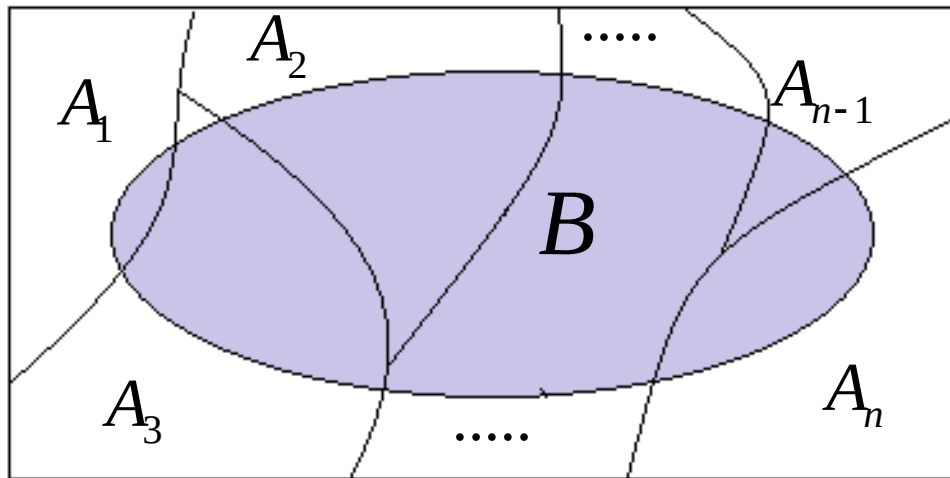
- Equations (6) and (7) are known as **Baye's Rule**.
- **Baye's Rule** can be extended for n events as follows.
- Let events $A_1, A_2, A_3, \dots, A_n$ be pairwise mutually exclusive (disjoint) events and their union be the sample space Ω , i.e.

$$A_i \cap A_j = \Phi \text{ and } \bigcup_{i=1}^n A_i = \Omega$$

$$\Rightarrow P\left(\bigcup_{i=1}^n A_i\right) = \sum_{i=1}^n P(A_i)$$

Conditional Probability Cont'd.....

- Let B be any event in Ω as shown below.



$$B = B \cap (A_1 \cup A_2 \cup \dots \cup A_n)$$

$$\Rightarrow B = (B \cap A_1) \cup (B \cap A_2) \cup \dots \cup (B \cap A_n)$$

$$\text{But, } A_i \cap A_j = \Phi \Rightarrow (B \cap A_i) \cap (B \cap A_j) = \Phi$$

Conditional Probability Cont'd.....

- The events $B \cap A_i$ and $B \cap A_j$ are mutually exclusive events.

$$\Rightarrow P(B) = P(B \cap A_1) + P(B \cap A_2) + \dots + P(B \cap A_n)$$

$$\Rightarrow P(B) = P(B / A_1)P(A_1) + P(B / A_2)P(A_2) + \dots + P(B / A_n)P(A_n) \quad (8)$$

- In short,

$$P(B) = \sum_{i=1}^n P(B \cap A_i) = \sum_{i=1}^n P(B / A_i)P(A_i) \quad (9)$$

- Then using equation (4), we will obtain

$$P(A_i / B) = \frac{P(B / A_i)P(A_i)}{\sum_{i=1}^n P(B / A_i)P(A_i)} \quad (10)$$

Conditional Probability Cont'd.....

Example-1:

Show that $P(\bar{A}/B) = 1 - P(A/B)$

Solution:

$$P(B) = P(A \cap B) + P(\bar{A} \cap B)$$

$$\Rightarrow P(B) = P(A/B)P(B) + P(\bar{A}/B)P(B)$$

Dividing both sides by $P(B)$, we obtain

$$1 = P(A/B) + P(\bar{A}/B)$$

$$\therefore P(\bar{A}/B) = 1 - P(A/B)$$

Conditional Probability Cont'd.....

Example-2:

Let A and B be two events such that $P(A)=x$, $P(B)=y$ and $P(B/A)=z$. Find the following probabilities in terms of x , y and z .

a. $P(A/B)$

b. $P(\bar{A} \cup \bar{B})$

c. $P(\bar{A}/B)$

Solution:

$$P(A \cap B) = P(B/A)P(A) = xz$$

a. $P(A/B) = \frac{P(A \cap B)}{P(B)} = \frac{xz}{y}$

b. $P(\bar{A} \cup \bar{B}) = P(\overline{A \cap B}) = 1 - P(A \cap B) = 1 - xz$

c. $P(\bar{A}/B) = \frac{P(\bar{A} \cap B)}{P(B)} = \frac{P(B) - P(A \cap B)}{P(B)} = 1 - \frac{xz}{y}$

Conditional Probability Cont'd.....

Example-3:

A box contains two black and three white balls. Two balls are selected at random from the box without replacement. Find the probability that

- a. both balls are black
- b. the second ball is white

Solution:

First let us define the events as follows:

B_1 : the outcome in the first selection is a black ball

Conditional Probability Cont'd.....

B_2 : the outcome in the second selection is a black ball

W_1 : the outcome in the first selection is a white ball

W_2 : the outcome in the second selection is a white ball

$$P(B_1) = 2/5 \quad P(B_2 / B_1) = 1/4 \quad P(W_2 / B_1) = 3/4$$

$$P(W_1) = 3/5 \quad P(B_2 / W_1) = 2/4 \quad P(W_2 / W_1) = 2/4$$

a. $P(B_1 \cap B_2) = P(B_2 / B_1)P(B_1) = (1/4)(2/5)$

$$\therefore P(B_1 \cap B_2) = 1/10$$

b.
$$\begin{aligned} P(W_2) &= P(W_2 \cap B_1) + P(W_2 \cap W_1) \\ &= P(W_2 / B_1)P(B_1) + P(W_2 / W_1)P(W_1) \\ &= (3/4)(2/5) + (2/4)(3/5) \end{aligned}$$

$$\therefore P(W_2) = 3/5$$

Conditional Probability Cont'd.....

Example-4:

Box **A** contains 100 bulbs of which 10% are defective. Box **B** contains 200 bulbs of which 5% are defective. A bulb is picked from a randomly selected box.

- a. Find the probability that the bulb is defective
- b. Assuming that the bulb is defective, find the probability that it came from box **A**.

Conditional Probability Cont'd.....

Solution:

First let us define the events as follows.

A : Box A is selected

B : Box B is selected

D : Bulb is defective



$$P(A) = P(B) = 1/2$$

$$P(D/A) = 1/10$$

$$P(D/B) = 1/20$$

$$\text{a. } P(D) = P(D/A)P(A) + P(D/B)P(B)$$

$$= (1/10)(1/2) + (1/20)(1/2)$$

$$\therefore P(D) = 3/40$$

$$\text{b. } P(A/D) = \frac{P(D/A)P(A)}{P(D)} = \frac{1/20}{3/40} = (1/20)(40/3)$$

$$\therefore P(A/D) = 2/3$$

Conditional Probability Cont'd.....

Example-5:

One bag contains 4 white and 3 black balls and a second bag contains 3 white and 5 black balls. One ball is drawn from the first bag and placed in the second bag unseen and then one ball is drawn from the second bag. What is the probability that it is a black ball?

Solution:

First let us define the events as follows.

B_1 : black ball is drawn from the first bag

W_1 : white ball is drawn from the first bag

Conditional Probability Cont'd.....

B_2 : black ball is drawn from the second bag

W_2 : white ball is drawn from the second bag

Then, we will have:

$$P(B_1) = 3/7 \quad P(B_2 / B_1) = 6/9 \quad P(B_2 / W_1) = 5/9$$

$$P(W_1) = 4/7 \quad P(W_2 / B_1) = 3/9 \quad P(W_2 / W_1) = 4/9$$

$$P(B_2) = P(B_2 \cap B_1) + P(B_2 \cap W_1)$$

$$\Rightarrow P(B_2) = P(B_2 / B_1)P(B_1) + P(B_2 / W_1)P(W_1)$$

$$\Rightarrow P(B_2) = (6/9)(3/7) + (5/9)(4/7)$$

$$\therefore P(B_2) = 28/63$$

Conditional Probability Cont'd.....

Exercise:

1. For three events **A**, **B** and **C**, show that:
 - a. $P[(A \cap B) / C] = P[A / (B \cap C)]P(B / C)$
 - b. $P(A \cap B \cap C) = P[A / (B \cap C)]P(B / C)P(C)$
2. Box **A** contains 3 white and 2 red balls while another box **B** contains 2 red and 5 white balls. A ball drawn at random from one of the boxes turns out to be red. What is the probability that it came from box **A**?

Conditional Probability Cont'd.....

3. In a certain assembly plant, three machine A, B and C make 30%, 45% and 25% of the products respectively. It is known that 2%, 3% and 5% of the products made by each machine, respectively, are defective. Suppose that a finished product is randomly selected.
- a. What is the probability that it is defective?
 - b. If the product is known to be defective, what is the probability that it is made by machine A?

Independence of Events

- Two events **A** and **B** are said to be statistically independent if and only if

$$P(A \cap B) = P(A)P(B)$$

- Similarly, three events **A**, **B** and **C** are said to be statistically independent if and only if

$$P(A \cap B \cap C) = P(A)P(B)P(C)$$

- Generally, if $A_1, A_2, \dots, A_n$ are a sequence of independent events, then

$$P\left(\bigcap_{i=1}^n A_i\right) = \prod_{i=1}^n P(A_i)$$

Independence of Events Cont'd.....

- If **A** and **B** are independent, then we have

$$\text{i. } P(A/B) = \frac{P(A \cap B)}{P(B)} = \frac{P(A)P(B)}{P(B)} = P(A)$$

$$\Rightarrow P(A/B) = P(A)$$

$$\text{ii. } P(B/A) = \frac{P(A \cap B)}{P(A)} = \frac{P(A)P(B)}{P(A)} = P(B)$$

$$\Rightarrow P(B/A) = P(B)$$

Example-1:

If A and B are independent, then show that A and $\overline{B}$ are also independent.

Independence of Events Cont'd.....

Solution:

$$P(A) = P(A \cap B) + P(A \cap \bar{B})$$

$$\Rightarrow P(A \cap \bar{B}) = P(A) - P(A \cap B) = P(A) - P(A)P(B)$$

$$\Rightarrow P(A \cap \bar{B}) = P(A)[1 - P(B)] = P(A)P(\bar{B})$$

$\therefore$ By the definition of independent events, A and $\bar{B}$ are independent.

Example-2:

The probability that a husband and a wife will be alive 20 years from now are given by 0.8 and 0.9 respectively. Find the probability that in 20 years

a. both will be alive

c. at least one will be alive

b. neither will be alive

Independence of Events Cont'd.....

Solution:

- First let us define the events as follows.

H : Husband will be alive

W : Wife will be alive

- Then we will have

$$P(H) = 0.8 \Rightarrow P(\overline{H}) = 1 - P(H) = 1 - 0.8 = 0.2$$

$$P(W) = 0.9 \Rightarrow P(\overline{W}) = 1 - P(W) = 1 - 0.9 = 0.1$$

- The two events can be considered as independent.

Independence of Events Cont'd.....

Solution:

$$a. P(\text{both}) = P(H \cap W) = P(H)P(W)$$

$$\Rightarrow P(\text{both}) = P(H \cap W) = (0.8)(0.9)$$

$$\therefore P(\text{both}) = P(H \cap W) = 0.72$$

$$b. P(\text{neither}) = P(\overline{H} \cap \overline{W}) = P(\overline{H})P(\overline{W})$$

$$\Rightarrow P(\text{neither}) = P(\overline{H} \cap \overline{W}) = (0.2)(0.1)$$

$$\therefore P(\text{neither}) = P(\overline{H} \cap \overline{B}) = 0.02$$

$$c. P(\text{at least one}) = 1 - P(\text{neither})$$

$$\Rightarrow P(\text{at least one}) = 1 - 0.02$$

$$\therefore P(\text{at least one}) = 0.98$$

Assignment-I

1. Show that the probability that exactly one of the events **A** or **B** occurs is given by:

$$P(A) + P(B) - 2P(A \cap B)$$

2. If **A** and **B** are mutually exclusive events and $P(A)=0.29$, $P(B)=0.43$, then find $P(A \cap \bar{B})$.
3. A box contains 3 double-headed coins, 2 double-tailed coins and 5 normal coins. A single coin is selected at random from the box and flipped.
 - a. What is the probability that it shows a head?
 - b. Given that a head is shown, what is the probability that it is the normal coin?

Assignment-I

4. The probability that a husband watches a certain television show is 0.4 and the probability that a wife watches the show is 0.5. The probability that a husband watches the show given that his wife does is 0.7. Find the probability that
 - a. both of them watch the show
 - b. a wife watches the show given that her husband does
 - c. at least one of them watch the show
5. In a shooting test, the probability of hitting the target is $\frac{1}{2}$ for **A**, $\frac{2}{3}$ for **B** and $\frac{3}{4}$ for **C**. If all of them fire at the target, find the probability that
 - a. none of them hits the target
 - b. at most two of them hit the target