

CHAPTER-4

ANALYSIS OF STRUCTURES

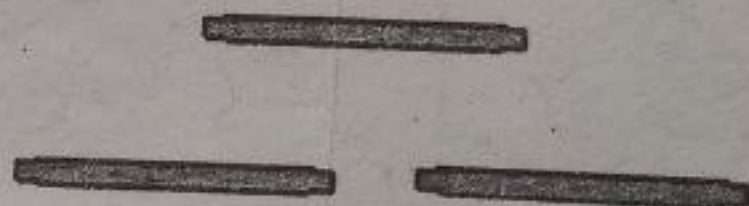
4.1 Introduction

Structure and Truss:

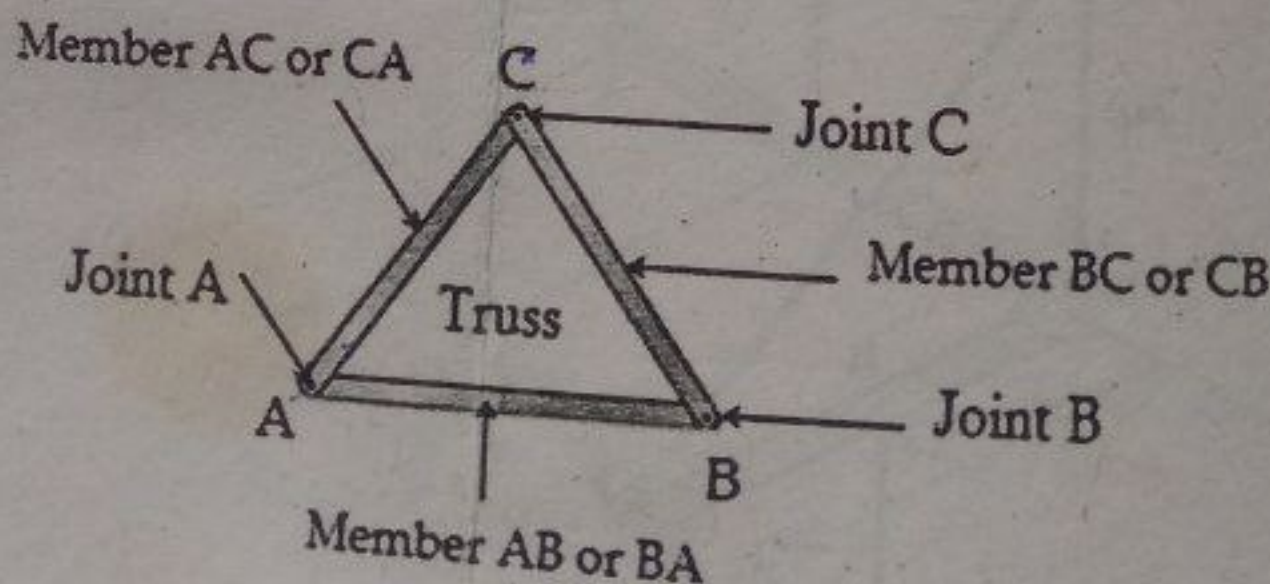
A systematic arrangement or configuration of different parts of a construction or a building framework (like bridge, roof, machines) is called structure. A structure constructed from slim or slender members that are joined together at their ends is called truss.

Truss members and joints: The slim or thin members that are joined together to form a truss are called members. The points at which two or more members are connected are called joints. The basic or the simplest form of a truss is a triangle. Then, more complicated trusses are constructed by adding a member that forms a new triangle.

Basic Elements of a plane Truss



These bars or rods are what we call members.



Types of Truss:

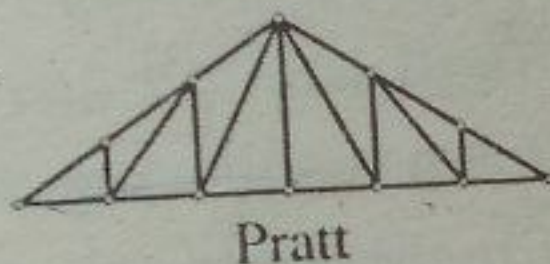
There are two basic types of truss: Plane Truss and Space Truss. The basic advantage of a truss is to support loads of a given construction so that to increase the sustainability of the construction. Here, under let's see plane trusses in detail.

4.2 Plane Truss

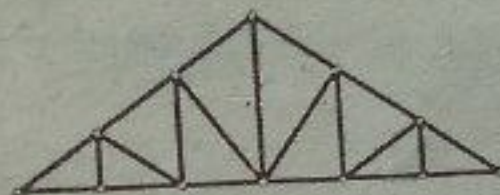
If all the triangles constituting the truss are found on the same plane, the truss is called plane truss.

Examples: Roofs and bridges are commonly supported by plane truss.

Examples of Roof truss

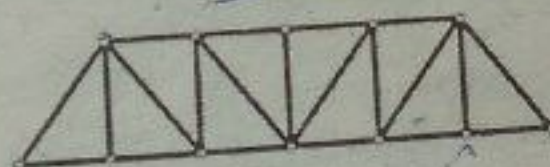


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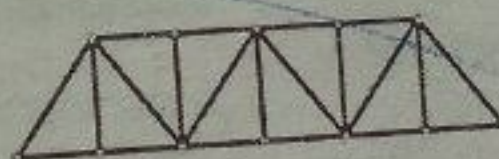


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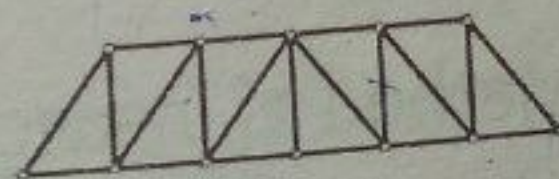
Examples of Bridge truss



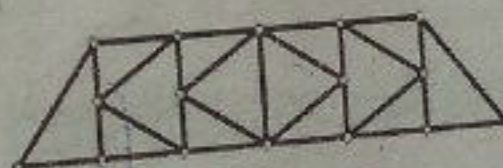
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K-Truss

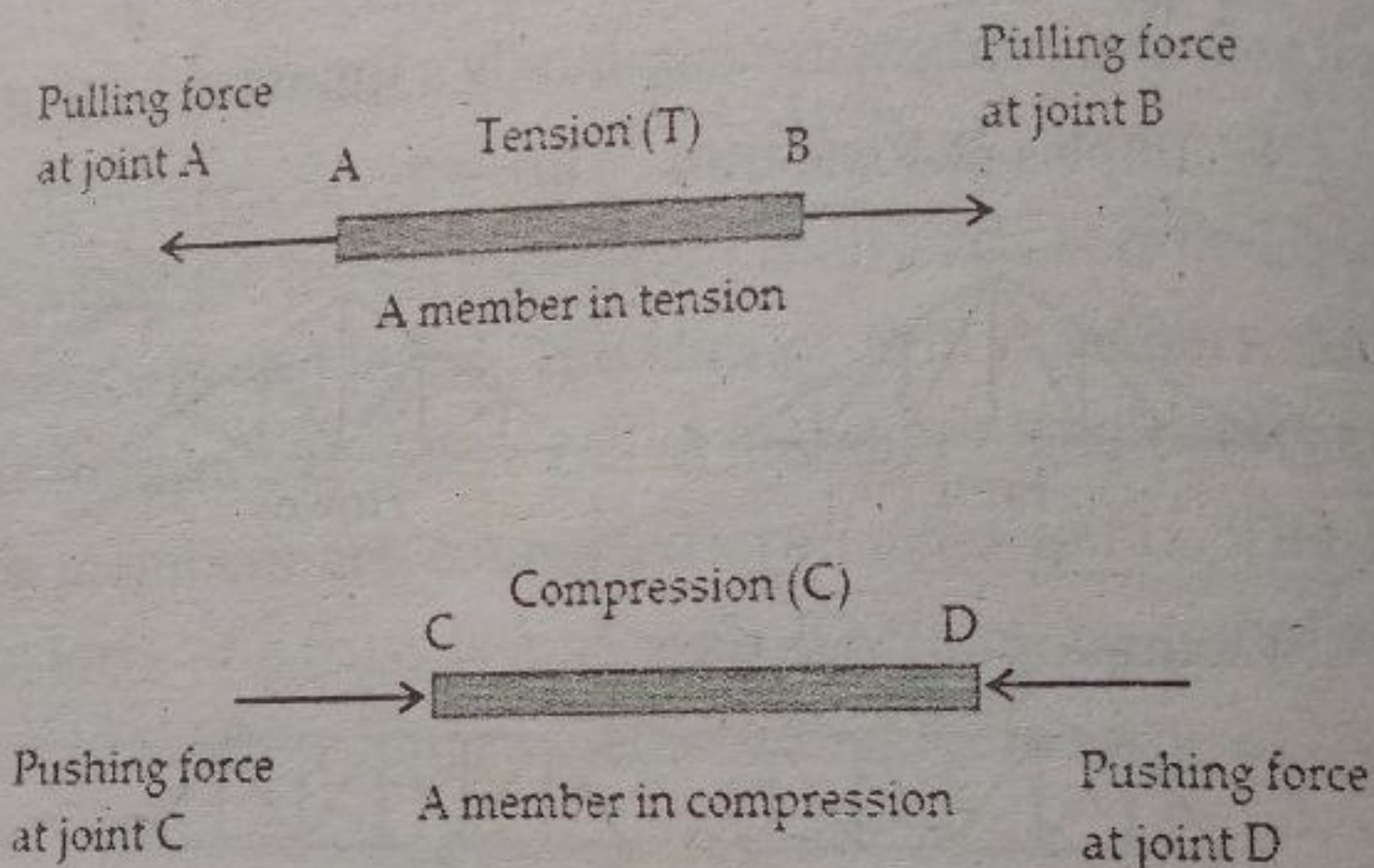
Basic Assumptions about Plane Trusses:

- All members must be connected by smooth pins at their joints
- All loads or Forces are applied at the joints only.
- The weight of each of the individual members is negligible as compared to the load or force it supports.
- All members are two force members (tension or compression)

Note: Tension or Compression

If the forces on the two ends of a member are pulling the member in the opposite direction (in the sense of enlarging the member), the member is said to be in tension and the force on that member is called tension (T). On the other hand, if the forces on the two ends of a member are pushing the member (in the sense of shortening it), the member is said to be in compression and the force on that member is called compression (C).

Forms of Tension and Compression Forces



4.3 Zero-Force Members and Method of Inspections

A truss member that supports no force or load is called a *zero-force member*. Trusses often contain many zero-force members.

Method of Inspections of joints

Case-I: When exactly two members are connected at a joint

If a joint connects only two members, then both of them are zero-force members whenever the two conditions are satisfied:

- There is no external force or load at the joint
- There is no support reaction at the joint

Examples:

A truss structure is shown with joints labeled A, B, C, D, E, F, G, H, and I. The truss is supported by a pin support at joint A and a roller support at joint H. A downward force $F = 20\text{ N}$ is applied at joint E. The truss consists of a bottom chord (A-C-E-G-H), a top chord (B-D-F), and various internal members including verticals (B-C, D-E, F-G), diagonals (A-B, C-D, E-F, G-H), and a vertical member (D-F). A diagonal member I is also shown connecting joint G to joint H.

inspection of Joint I), we can assume as there are three members meeting at joint G, GE, GH and GF. Then, GE and GH are collinear. Besides, there is no external force G. Thus, GF is also a zero-force member.

4.4 Methods of Truss Analysis

Mathematical determination of forces acting or being loaded on each member of a truss is generally termed as analysis.

In general, the main goal of analysis of a truss is determination of the support reactions for the truss and determination of the forces supported by the individual members of the truss.

The mathematical results are useful in designing a structure, in applying a load on different members of a truss based on their capacity. Analysis, in general, helps to engineers to prevent unexpected failure of a construction due to unbalanced application of loads. There are two basic types of method for truss analysis.

- 1) Method of Joints
- 2) Method of Sections

4.4.1 Method of Joints

The technique of determining member forces by applying equilibrium conditions to every joint of a given truss is known as *method of joints*. In the *method of joints*, a truss is analyzed by treating each joint as a particle.

The basis of this method: This method is based on the assumption that if the entire truss is in equilibrium, then every joint within the truss is also in equilibrium.

General Procedures in Method of Joints:

First: Determination of reaction forces at supports:

Draw the FBD of the whole truss (considering the entire truss). Then, to get the reaction forces, take moments at a joint with more number of reaction forces first and then take force summations.



Second: Determination of member forces

Joint Selection (Key-Step). Select a joint to start obtaining member forces. (በጣም ወሳኝ፡ ለምሆኑ እንደት ነጻ ጀይንት የሚመረጠው?)

Criteria of joint selection: The selection of a joint is based on the number of unknowns at that joint. Select a joint with at MOST TWO unknown forces and at LEAST ONE known forces.

NOTE: Considerations

1. **The first joint:** Be careful, you may not select a joint at which the member force is asked rather you may select far away joints from what you are asked. Then, you may proceed analysis to reach at the member whose force is asked. In short, to analyze a plane truss by the method of joints, we need to proceed the analysis from joint to joint until we reach the member whose force is required.

2. Direction of Forces: Tension or Compression

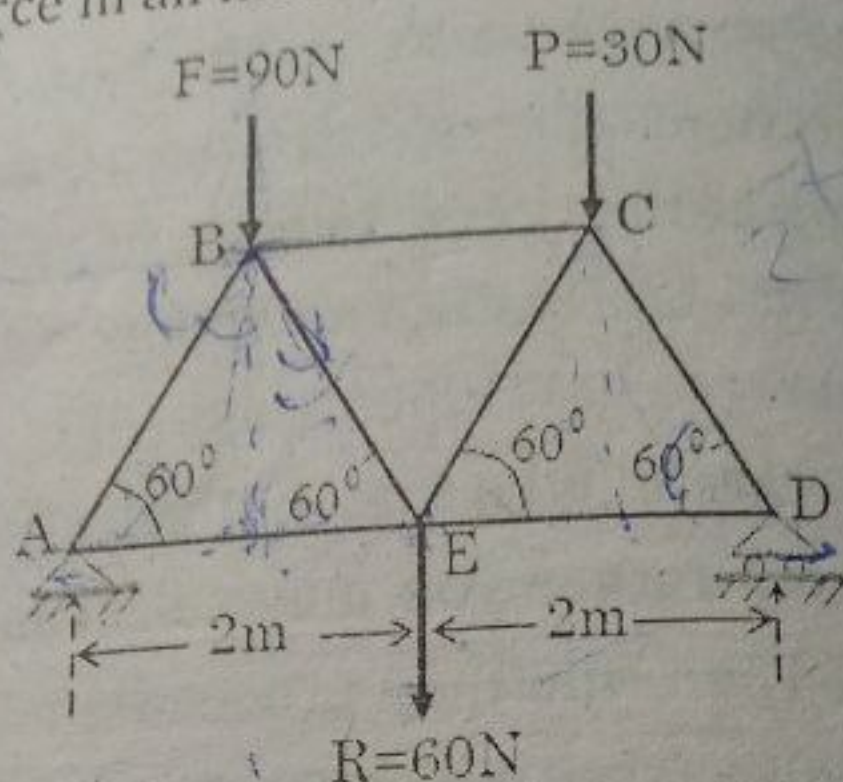
One of the basic assumptions of a plane truss is that all members of a truss are two force members. However, it is not always possible to assign the correct direction of a force on a member when the FBD of the first joint is drawn. For the first selected joint, you can assign the direction of the forces in any direction you want. Finally, the sign of the result will tell you the correct direction. If you get a positive result, the direction you assigned is correct and if you get a negative result, the direction is opposite to what you have assigned. Then, for the next analysis, CORRECT the direction of the calculated force and proceed. If a force on a member acts into the joint at one end of a member, it also acts into the joint at the other end of that member. Thus it is a compression.

3. **Roller supports:** Reaction at a roller support always acts into the joint. This is also a useful hint.

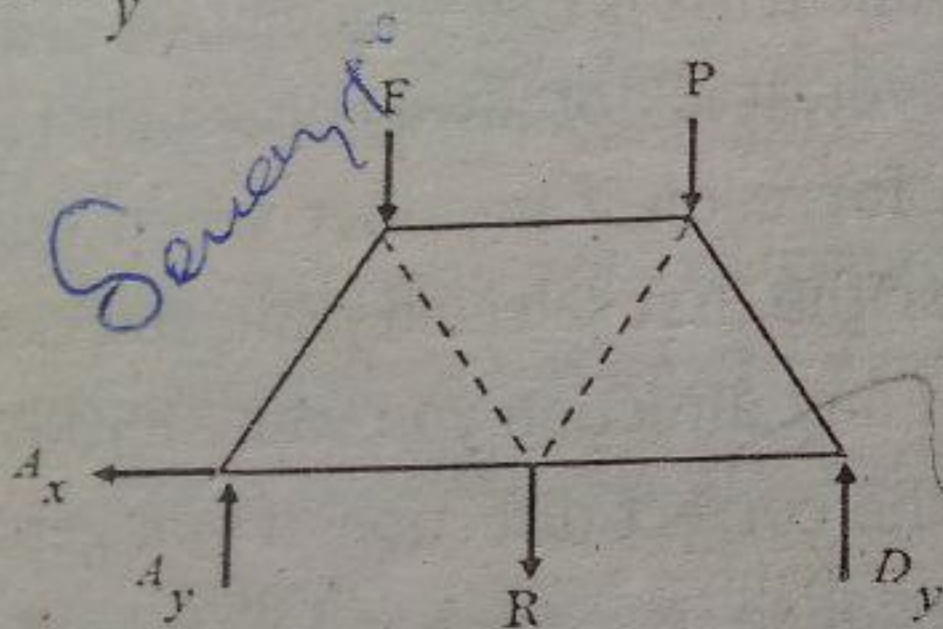
4. **Zero force members:** In a truss with large number of members, it is advisable first to identify zero force members.

Examples:

1. Determine the force in all members of the truss structure below.



Solution: First, determine the reactions at points A and D from the entire truss. Let the reactions at A be A_x , A_y and the reaction at D be D_y . Then, the FBD of the entire truss is as shown.



Using equilibrium conditions for the whole truss, we have

$$i) \sum M_A = 0 \Rightarrow D_y \cdot d_D - F \cdot d_F - R \cdot d_R - P \cdot d_P = 0$$

$$\Rightarrow 4D_y - 90N(1) - 60N(2) - 30N(3) = 0$$

$$\Rightarrow 4D_y = 300N \Rightarrow D_y = 75N$$

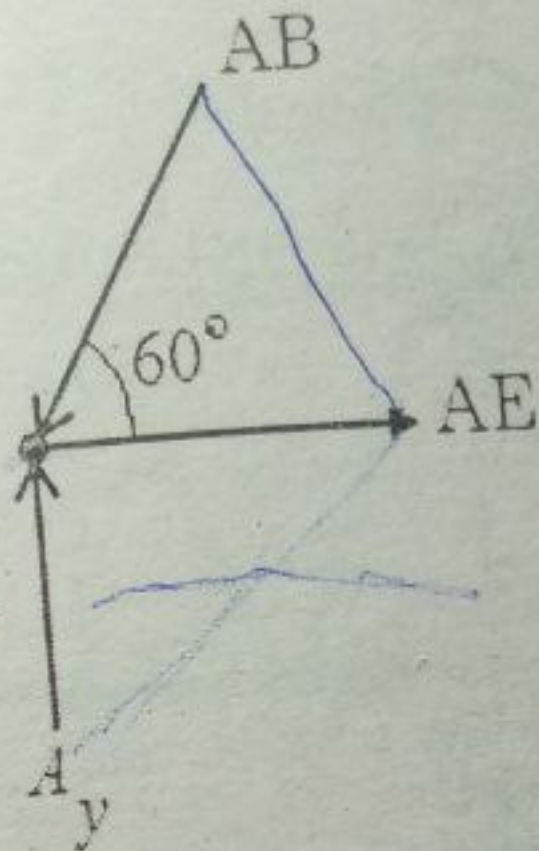
$$ii) \sum F_y = 0 \Rightarrow A_y + D_y - F - R - P = 0 \Rightarrow A_y + 75N - 180N = 0$$

$$\Rightarrow A_y = 105N$$

$$iii) \sum F_x = 0 \Rightarrow A_x = 0$$

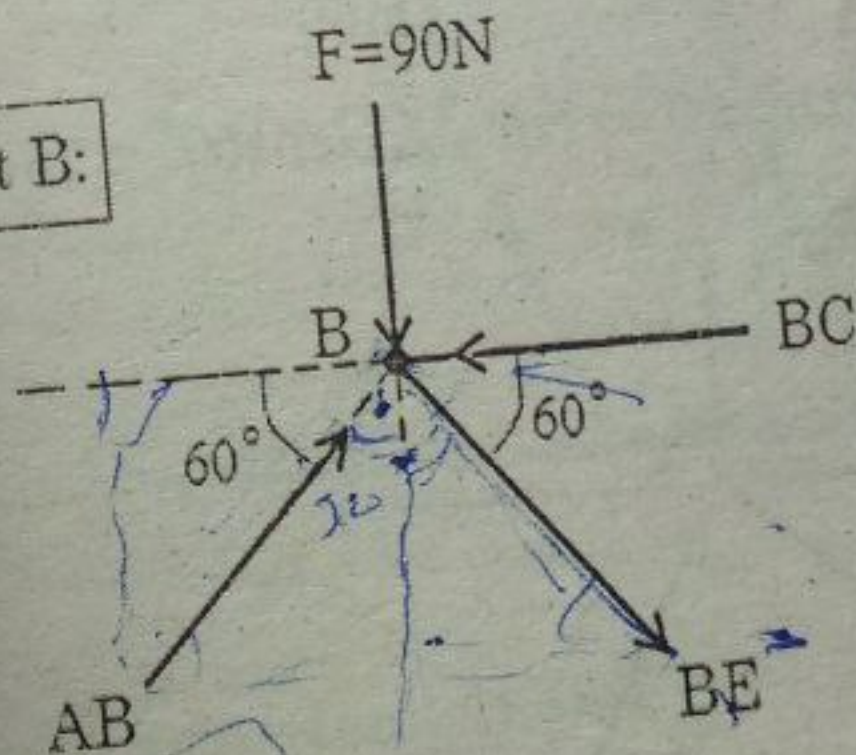
Second: Determination of member forces: By method of joints

1) At joint A:



$$\begin{cases} \sum F_y = 0 \Rightarrow A_y - AB \sin 60^\circ = 0 \Rightarrow AB = \frac{A_y}{\sin 60^\circ} = \frac{210\text{N}}{\sqrt{3}} = 70\sqrt{3}\text{N (C)} \\ \sum F_x = 0 \Rightarrow AE - AB \cos 60^\circ = 0 \Rightarrow AE = AB \cos 60^\circ = 35\sqrt{3}\text{ (T)} \end{cases}$$

2) At joint B:

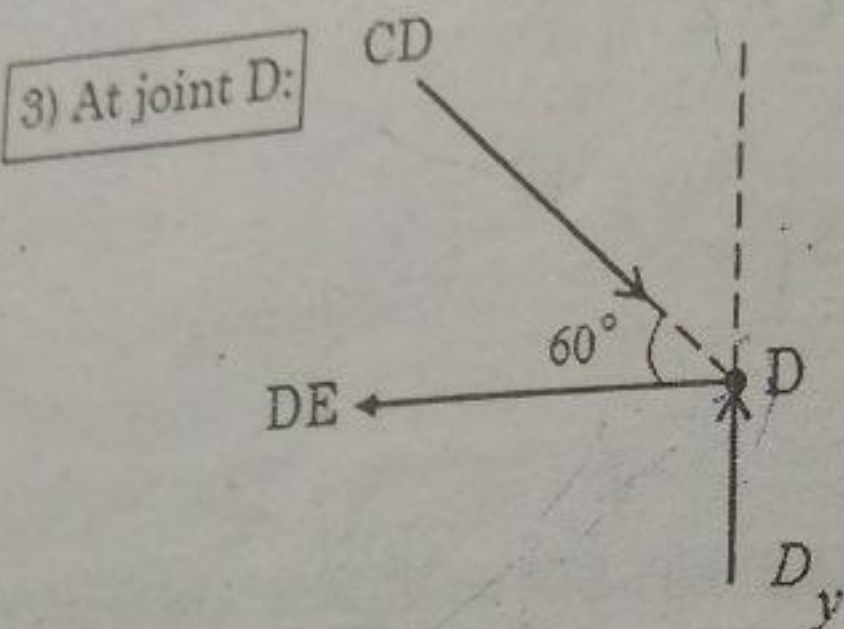


$$\begin{aligned} i) \sum F_y = 0 &\Rightarrow AB \sin 60^\circ - F - BE \sin 60^\circ = 0 \\ &\Rightarrow BE \sin 60^\circ = AB \sin 60^\circ - F \\ &\Rightarrow \frac{\sqrt{3}}{2} BE = \frac{\sqrt{3}}{2} (70\sqrt{3}\text{N}) - 90\text{N} = 15\text{N} \\ &\Rightarrow \frac{\sqrt{3}}{2} BE = 15\text{N} \Rightarrow BE = \frac{30}{\sqrt{3}}\text{N} = 10\sqrt{3}\text{N (T)} \end{aligned}$$

$$ii) \sum F_x = 0 \Rightarrow AB \cos 60^\circ + BE \cos 60^\circ - BC = 0$$

$$\Rightarrow BC = AB \cos 60^\circ + BE \cos 60^\circ$$

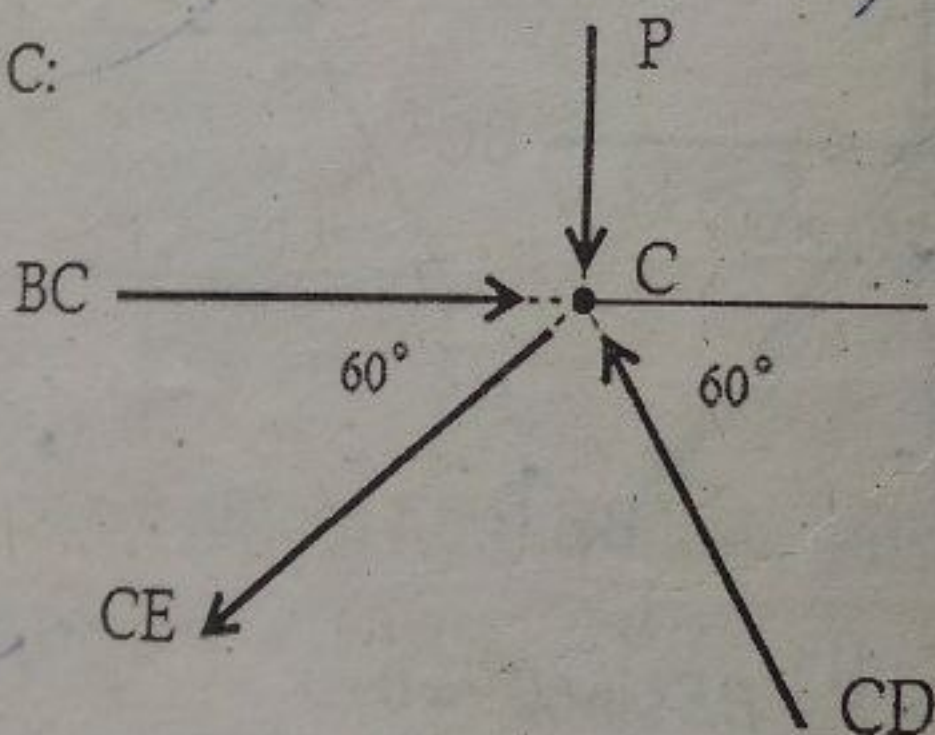
$$\Rightarrow BC = \frac{1}{2}(70\sqrt{3}N) + \frac{1}{2}(10\sqrt{3}N) = 40\sqrt{3}N (C)$$



$$i) \sum F_y = 0 \Rightarrow D_y - CD \sin 60^\circ = 0 \Rightarrow CD = \frac{D_y}{\sin 60^\circ} = 50\sqrt{3}N (C)$$

$$ii) \sum F_x = 0 \Rightarrow CD \cos 60^\circ - DE = 0 \Rightarrow DE = CD \cos 60^\circ = 25\sqrt{3}N (C)$$

4) At Joint C:



$$\sum F_y = 0 \Rightarrow DC \sin 60^\circ - CE \sin 60^\circ - P = 0$$

$$\Rightarrow CE \sin 60^\circ = DC \sin 60^\circ - P$$

$$\Rightarrow \frac{\sqrt{3}}{2} CE = 75N - 30N = 45N \Rightarrow CE = 30\sqrt{3}N (T)$$

Solution:
First: Deter
Draw the F

$$i) \sum M_A$$

$$ii) \sum F$$

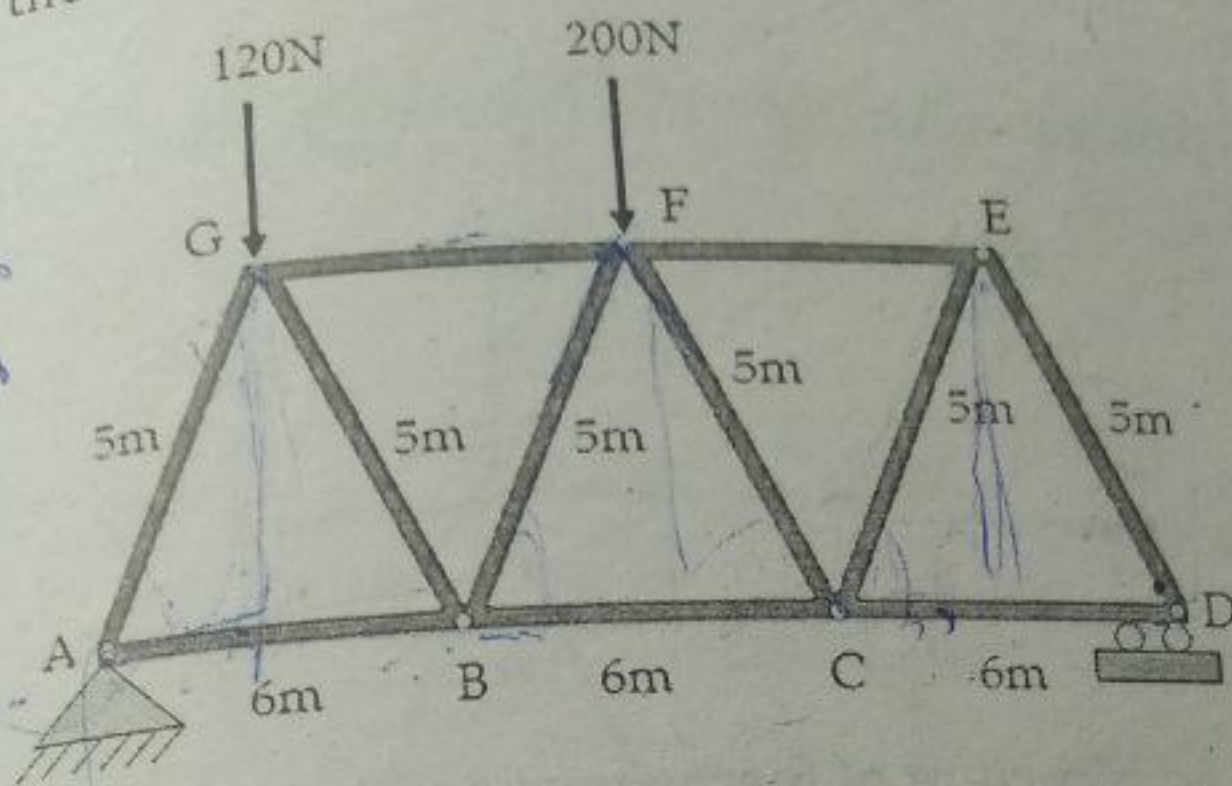
$$iii) \sum F$$

Second

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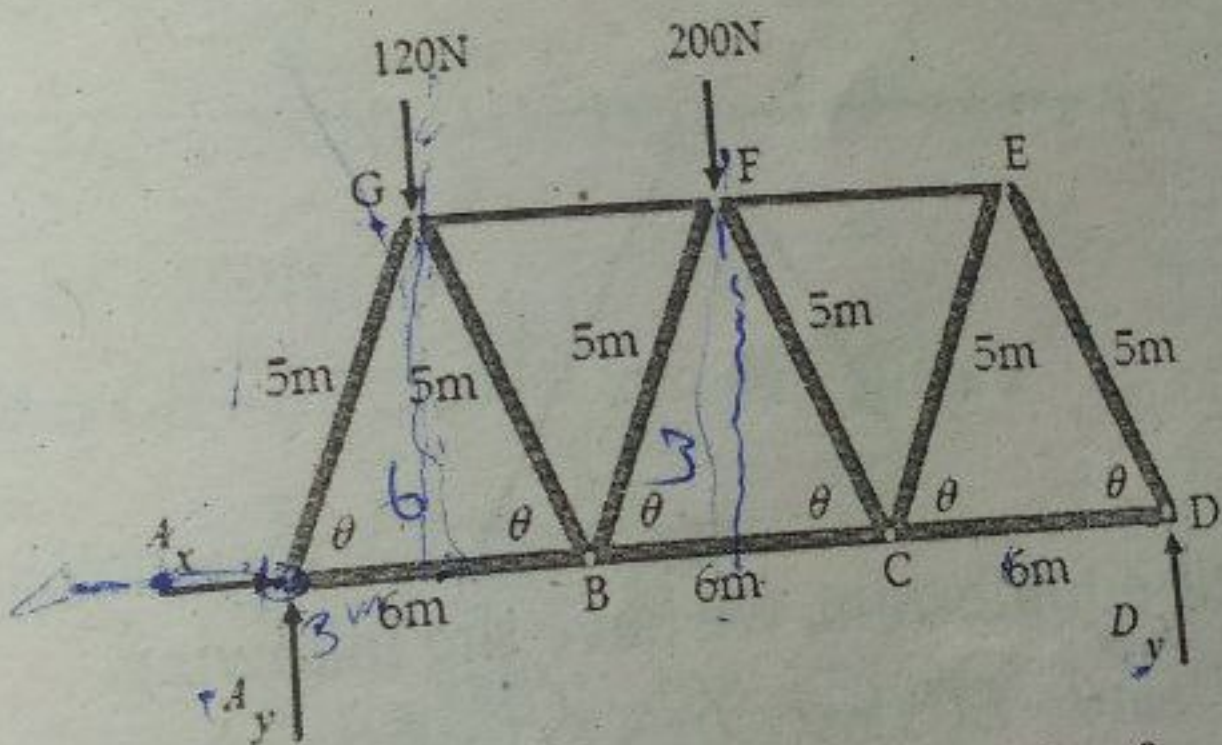
2. Determine the reactions and the forces in all members the truss.



Solution:

First: Determination of support reactions (Using the whole truss)

Draw the FBD of the entire truss and label all forces and reactions.



$$i) \sum M_A = 0 \Rightarrow D_y \cdot (18m) - 120N(3m) - 200N(9m) = 0$$

$$\Rightarrow 18D_y = 2160N \Rightarrow D_y = 120N$$

$$ii) \sum F_x = 0 \Rightarrow A_x = 0$$

$$iii) \sum F_y = 0 \Rightarrow A_y + D_y - 120N - 200N = 0 \Rightarrow A_y = 200N$$

Second: Determination of member forces

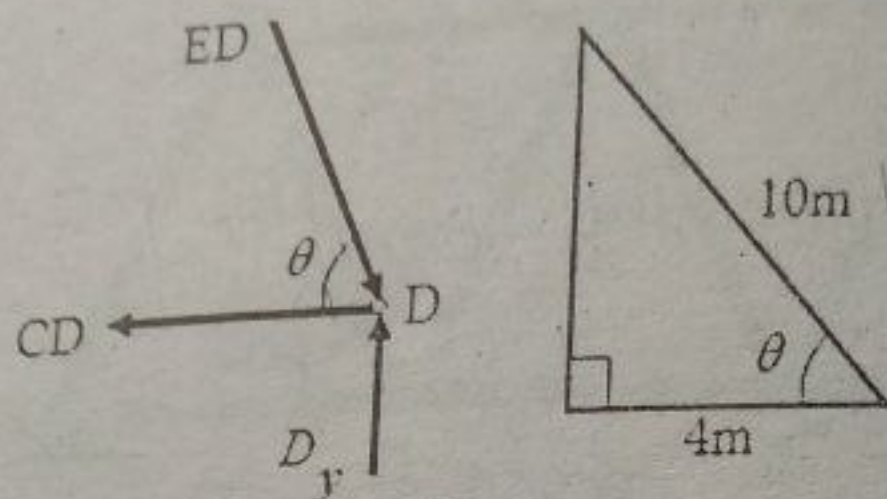
In all case, the angle is given by $\sin \theta = \frac{4}{5}$, $\cos \theta = \frac{3}{5}$

Let's start at joint A. Then, using equilibrium of forces, we have

$$i) \sum F_y = 0 \Rightarrow A_y - AG \sin \theta = 0 \Rightarrow \frac{4}{5} AG - 200 \text{ N} = 0 \Rightarrow AG = 250 \text{ N (C)}$$

$$ii) \sum F_x = 0 \Rightarrow AB - AG \cos \theta = 0 \Rightarrow AB - \frac{3}{5} (250 \text{ N}) = 0 \Rightarrow AB = 150 \text{ N (T)}$$

At Joint D:

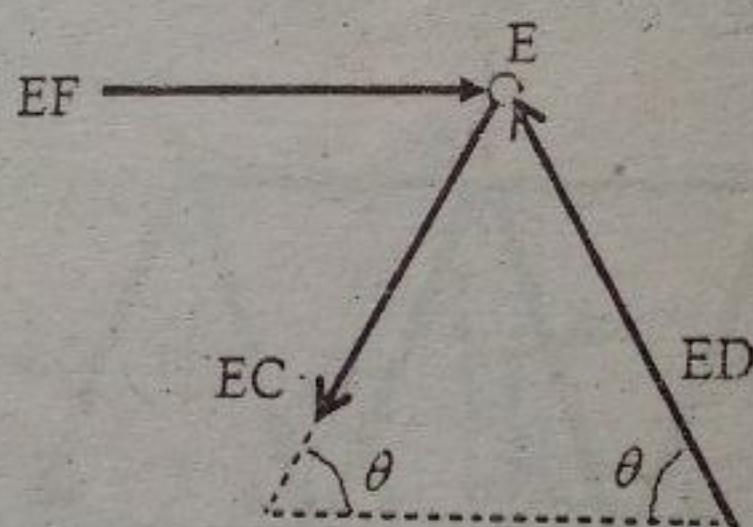


Then, using equilibrium of forces for a joint, we have

$$i) \sum F_y = 0 \Rightarrow D_y - ED \sin \theta = 0 \Rightarrow 120 \text{ N} - \frac{4}{5} ED = 0 \Rightarrow ED = 150 \text{ N (C)}$$

$$ii) \sum F_x = 0 \Rightarrow ED \cos \theta - CD = 0 \Rightarrow CD = 90 \text{ N (T)}$$

At Joint E:

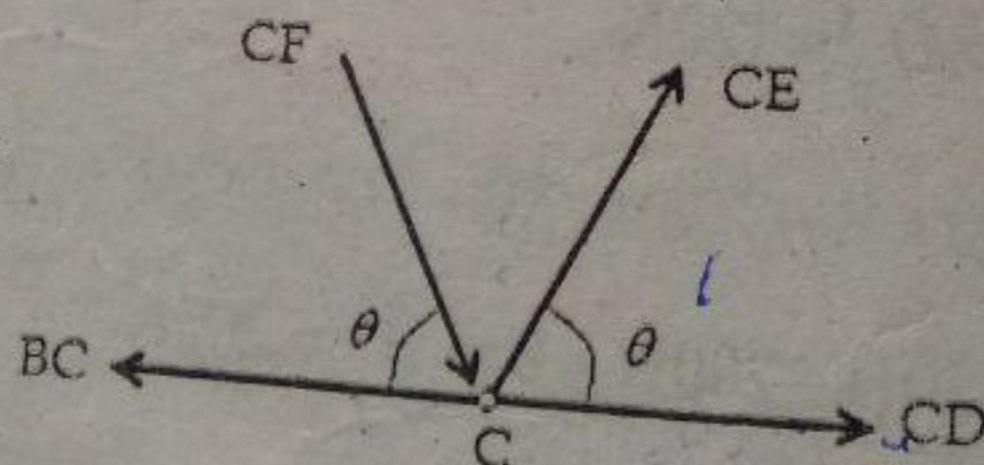


Then, using static equilibrium, we have the following.

$$i) \sum F_y = 0 \Rightarrow ED \sin \theta - EC \sin \theta = 0 \Rightarrow EC = ED \Rightarrow EC = 150 \text{ N (T)}$$

$$ii) \sum F_x = 0 \Rightarrow EF - ED \cos \theta - EC \cos \theta = 0 \Rightarrow EF = 180 \text{ N (C)}$$

At Joint C:



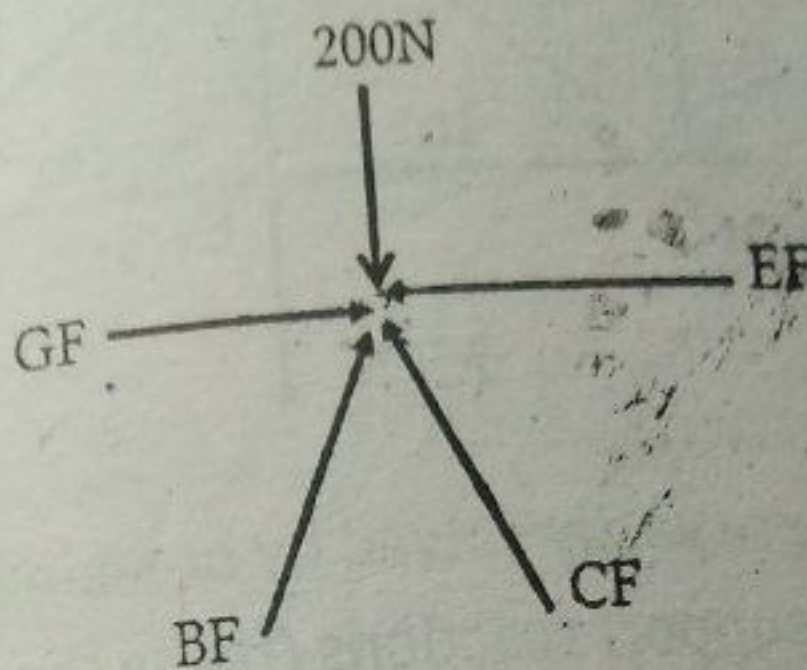
Then, using equilibrium equation of forces, we have

$$\sum F_y = 0 \Rightarrow CE \sin \theta - CF \sin \theta = 0 \Rightarrow CF = 150 \text{ N (C)}$$

$$\sum F_x = 0 \Rightarrow CD + CE \cos \theta + CF \cos \theta - BC = 0$$

$$\Rightarrow 90 \text{ N} + \frac{3}{5}(150 \text{ N}) + \frac{3}{5}(150 \text{ N}) - BC = 0 \Rightarrow BC = 270 \text{ N (T)}$$

At Joint F:



Then, using static equilibrium, we have the following.

$$\sum F_y = 0 \Rightarrow BF \sin \theta + CF \sin \theta - 200 \text{ N} = 0$$

$$\Rightarrow \frac{4}{5}BF = 200 \text{ N} - \frac{4}{5}(150 \text{ N}) \Rightarrow \frac{4}{5}BF = 80 \text{ N}$$

$$\Rightarrow BF = 100 \text{ N}$$

$$\sum F_x = 0 \Rightarrow GF + BF \cos \theta - CF \cos \theta - EF = 0$$

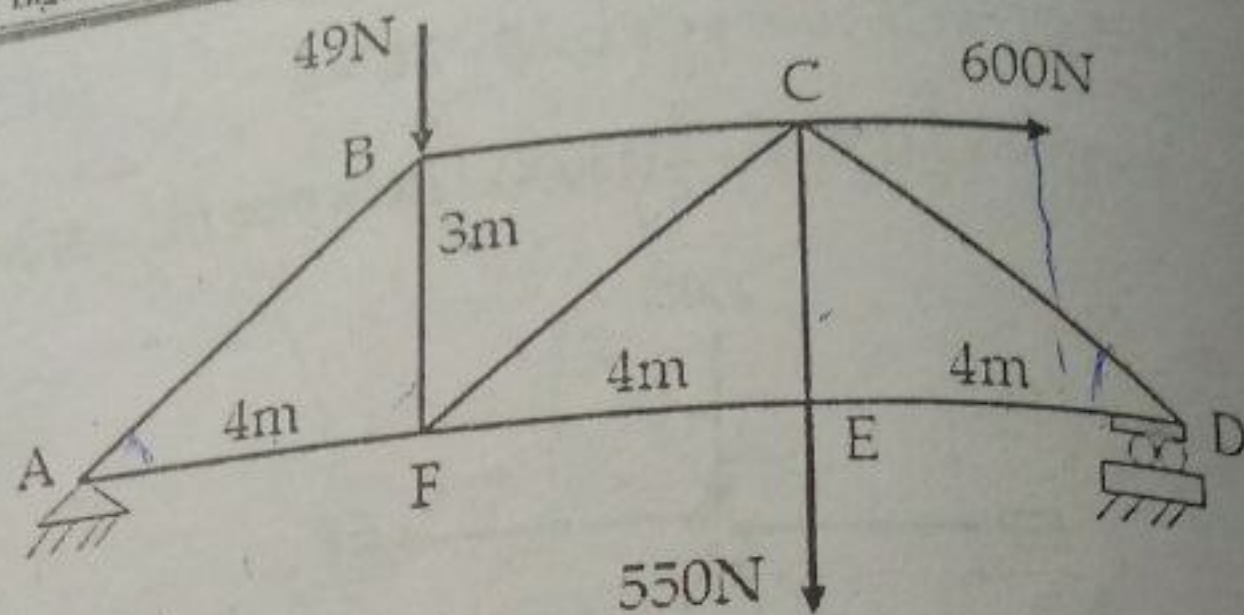
$$\Rightarrow GF = \frac{3}{5}CF + EF - \frac{3}{5}BF$$

$$\Rightarrow GF = 90 \text{ N} + 180 \text{ N} - 60 \text{ N} = 210 \text{ N}$$

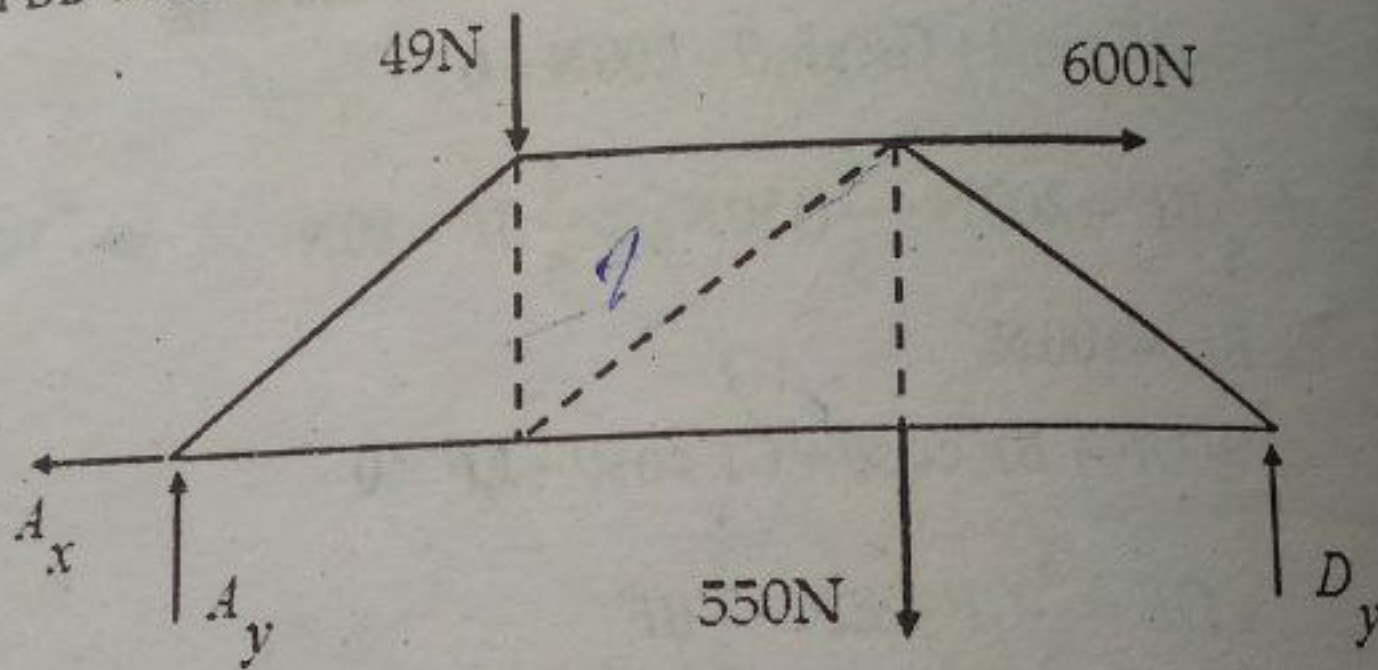
3) At joint B: Then, using equilibrium of forces,

$$\sum F_y = 0 \Rightarrow BG \sin \theta - BF \sin \theta = 0 \Rightarrow BG = BF = 100 \text{ N}$$

3. In the loaded truss structure below, determine the support reactions and the forces in the members BF, BC, EF.



Solution: Such problems are easily solved by breaking into two parts.
Step-1: Determination of support reactions (Using the whole truss)
Draw a FBD of the entire truss with all applied and reaction forces.



Using equilibrium equations, we have

$$i) \sum M_A = 0 \Rightarrow D_y \cdot (12m) - 49N(4m) - 550N(8m) - 600N(3m) = 0$$

$$\Rightarrow 12D_y = 6396N \Rightarrow D_y = 533N$$

$$i) \sum F_x = 0 \Rightarrow 600N - A_x = 0 \Rightarrow A_x = 600N$$

$$iii) \sum F_y = 0 \Rightarrow A_y + D_y - 49N - 550N = 0$$

$$\Rightarrow A_y - 66N = 0 \Rightarrow A_y = 66N$$

Second: Determination of member forces (Using selected joints)

Let's start at joint A. We choose directions for AB and AF arbitrarily

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1) At Joint A

Then, using

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$$ii) \sum F_x = 0$$

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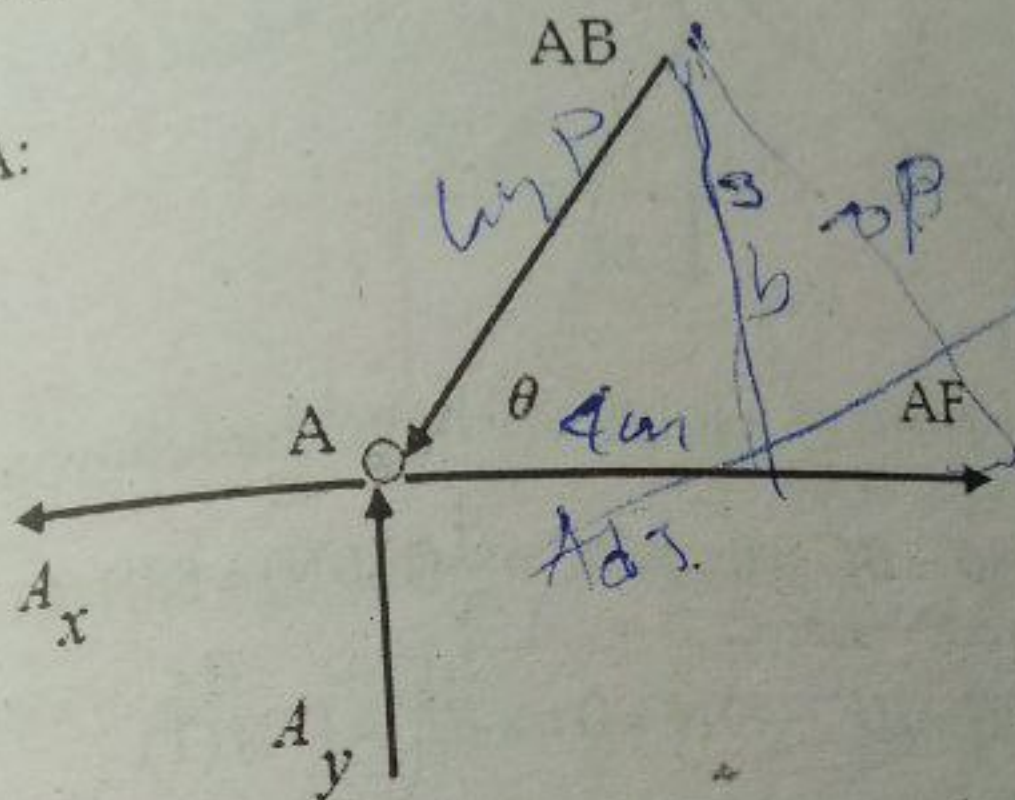
Now, as w

into joint

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because this is the first joint that we selected. When we solve AB and AF, if these values are negative, the chosen directions were incorrect and the forces act in the opposite direction.

1) At Joint A:



Then, using equilibrium of forces for a joint, we have the following.

$$i) \sum F_y = 0 \Rightarrow A_y - AB \sin \theta = 0 \Rightarrow \frac{3}{5} AB = 66 \text{ N} \Rightarrow AB = 110 \text{ N (C)}$$

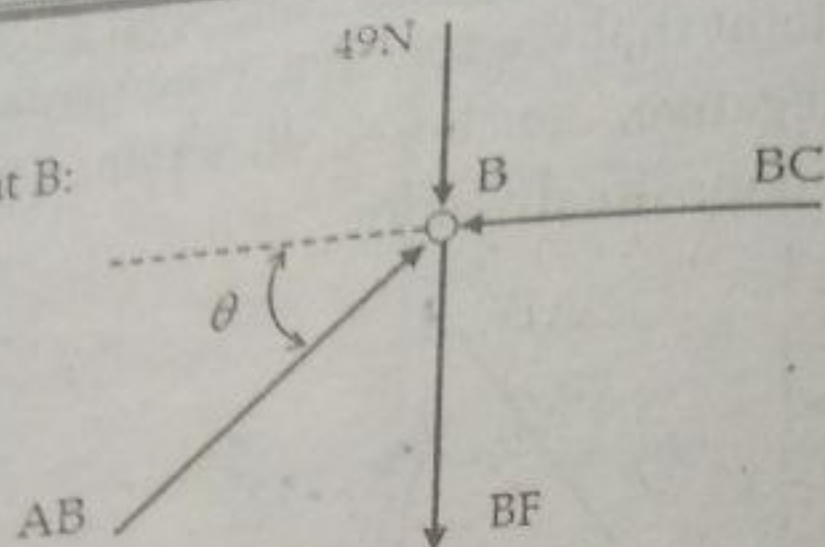
$$ii) \sum F_x = 0 \Rightarrow AF - AB \cos \theta - A = 0$$

$$\Rightarrow AF - \frac{4}{5}(110 \text{ N}) - 600 \text{ N} = 0$$

$$\Rightarrow AF - 88 \text{ N} - 600 \text{ N} = 0 \Rightarrow AF = 688 \text{ N (T)}$$

Basic Question: Let me ask you one basic concept. We are asked to find the forces on the members BF, BC, EF but not on AB and AF. So, why do we start from joint A? Why not we directly start at joint B? The main reason is that if we start at joint B, we have three unknown forces on members AB, BC, and BF but only two equations. (This is not solvable). From this explanation, we can see that to get what we are asked, sometimes we may start from what is not asked. Now, as we have the forces in members AB and AF, we can move into joint B and find two of the unknowns we are looking for.

2) At Joint B:

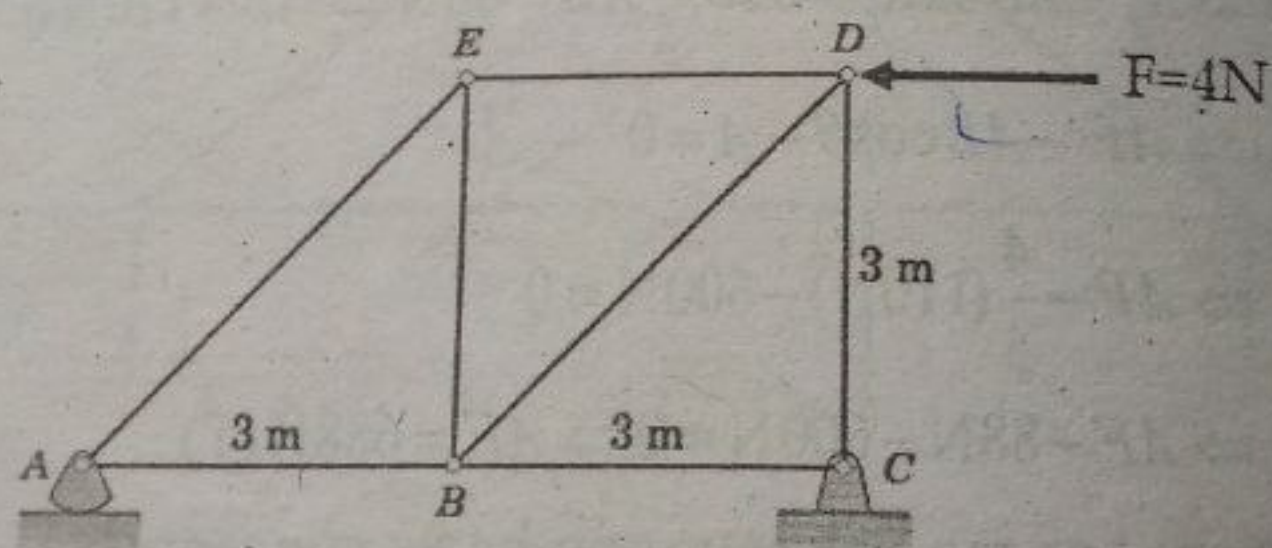


Then, using equilibrium of a joint, we have the following.

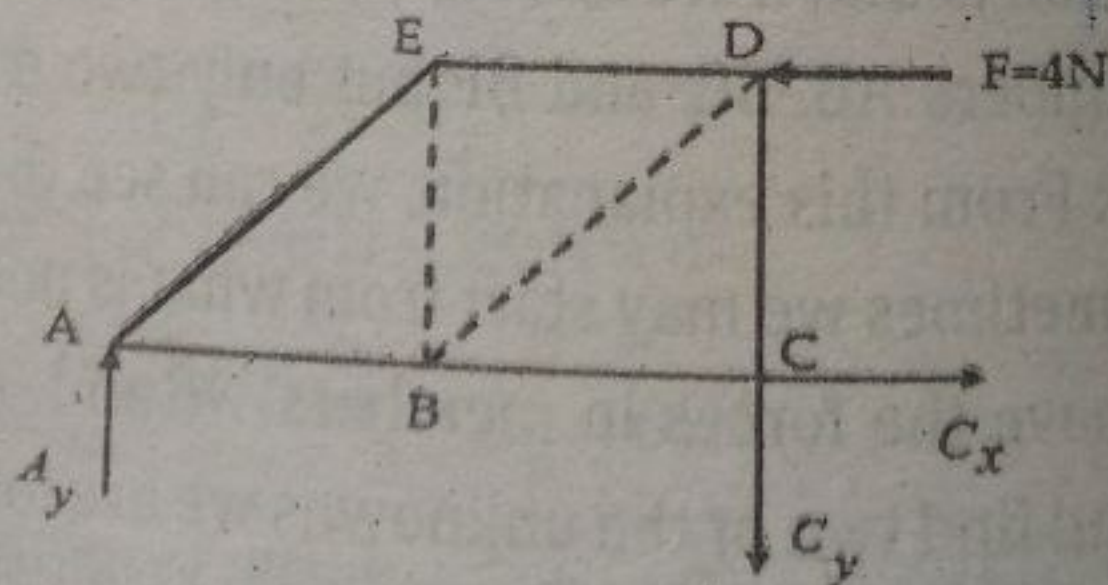
$$\begin{cases} \sum F_x = 0 \Rightarrow AB \cos \theta - BC = 0 \Rightarrow BC = \frac{4}{5}(110\text{N}) = 88\text{N (C)} \\ \sum F_y = 0 \Rightarrow AB \sin \theta - BF - 49\text{N} = 0 \Rightarrow BF = 17\text{N (T)} \end{cases}$$

Following similar procedures, complete the solutions.

4. Determine the force in each member of the loaded truss below.



Solution: First find the reaction forces A_x, A_y at pint A and the reaction forces C_x, C_y at pin C. Consider the FBD of the whole truss.



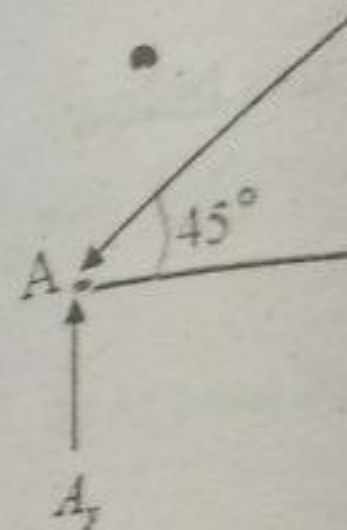
i) $\sum M_c = 0 \Rightarrow$

ii) $\sum F_x = 0 \Rightarrow$

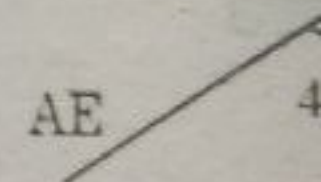
iii) $\sum F_y = 0 \Rightarrow$

Now, let's apply

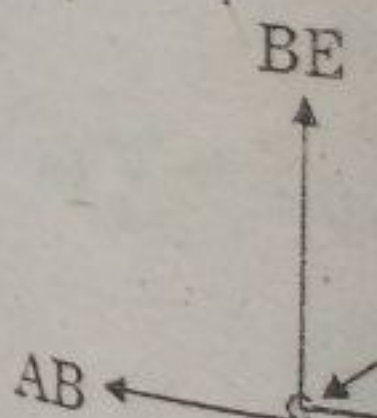
1) At Joint A:



2) At Joint B:



3) At Joint B:



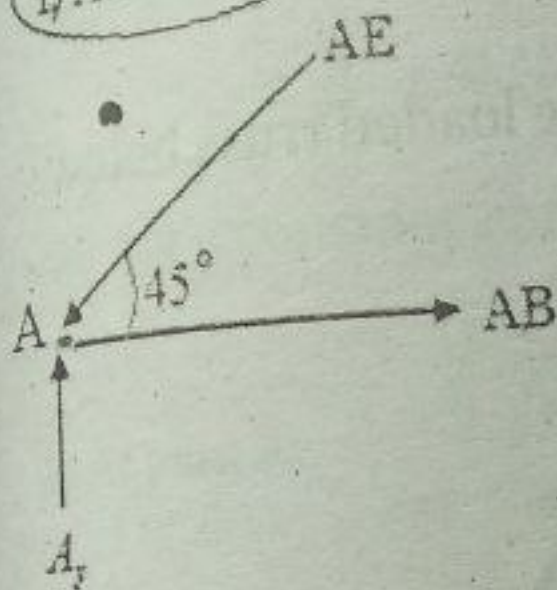
$$i) \sum M_C = 0 \Rightarrow F \cdot (3m) - A_y \cdot d_A = 0 \Rightarrow 6A_y - 12N = 0 \Rightarrow A_y = 2N$$

$$ii) \sum F_x = 0 \Rightarrow C_x - F = 0 \Rightarrow C_x = F = 4N$$

$$iii) \sum F_y = 0 \Rightarrow A_y - C_y = 0 \Rightarrow C_y = A_y = 2N$$

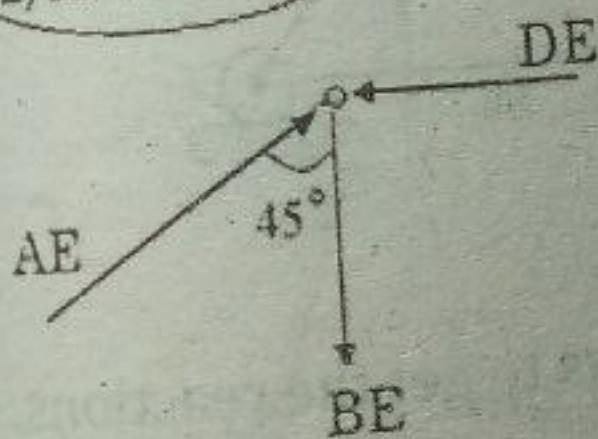
Now, let's apply the Method of Joints.

1) At Joint A:



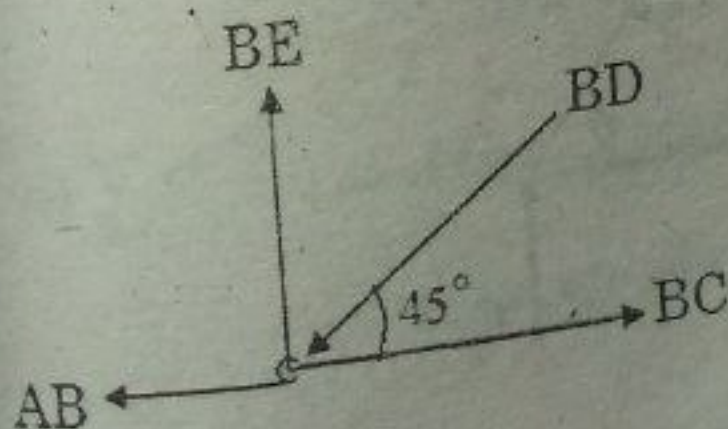
$$\left\{ \begin{aligned} \sum \vec{F}_y = \vec{0} &\Rightarrow A_y - \overline{AE} \sin 45^\circ = \vec{0} \\ &\Rightarrow 2N - \frac{1}{\sqrt{2}} \overline{AE} = \vec{0} \\ &\Rightarrow \overline{AE} = 2\sqrt{2}N \\ \sum \vec{F}_x = \vec{0} &\Rightarrow \overline{AB} - \overline{AE} \cos 45^\circ = \vec{0} \\ &\Rightarrow \overline{AB} = \frac{1}{\sqrt{2}} (2\sqrt{2}N) = 2N \end{aligned} \right.$$

2) At Joint E:



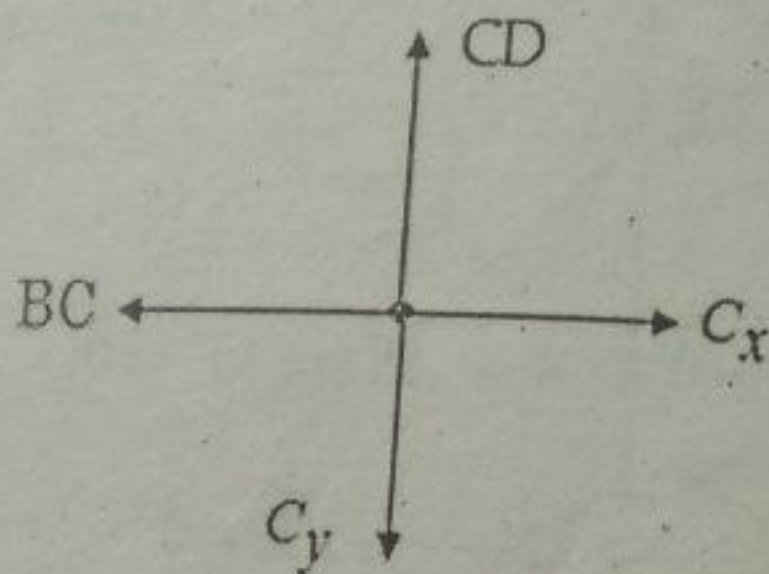
$$\left\{ \begin{aligned} \sum \vec{F}_x = \vec{0} &\Rightarrow \overline{AE} \sin 45^\circ - \overline{DE} = \vec{0} \\ &\Rightarrow \overline{DE} = \frac{1}{\sqrt{2}} (2\sqrt{2}N) = 2N \\ \sum \vec{F}_y = \vec{0} &\Rightarrow \overline{AE} \cos 45^\circ - \overline{BE} = \vec{0} \\ &\Rightarrow \overline{BE} = \frac{1}{\sqrt{2}} \overline{AE} = 2N \end{aligned} \right.$$

3) At Joint B:



$$\left\{ \begin{aligned} \sum \vec{F}_y = \vec{0} &\Rightarrow \overline{BE} - \overline{BD} \sin 45^\circ = \vec{0} \\ &\Rightarrow \frac{1}{\sqrt{2}} \overline{BD} = \overline{BE} \Rightarrow \overline{BD} = 2\sqrt{2}N \\ \sum \vec{F}_x = \vec{0} &\Rightarrow \overline{BC} - \overline{BD} \cos 45^\circ - \overline{AB} = \vec{0} \\ &\Rightarrow \overline{BC} = \overline{BD} \cos 45^\circ + \overline{AB} = 4N \end{aligned} \right.$$

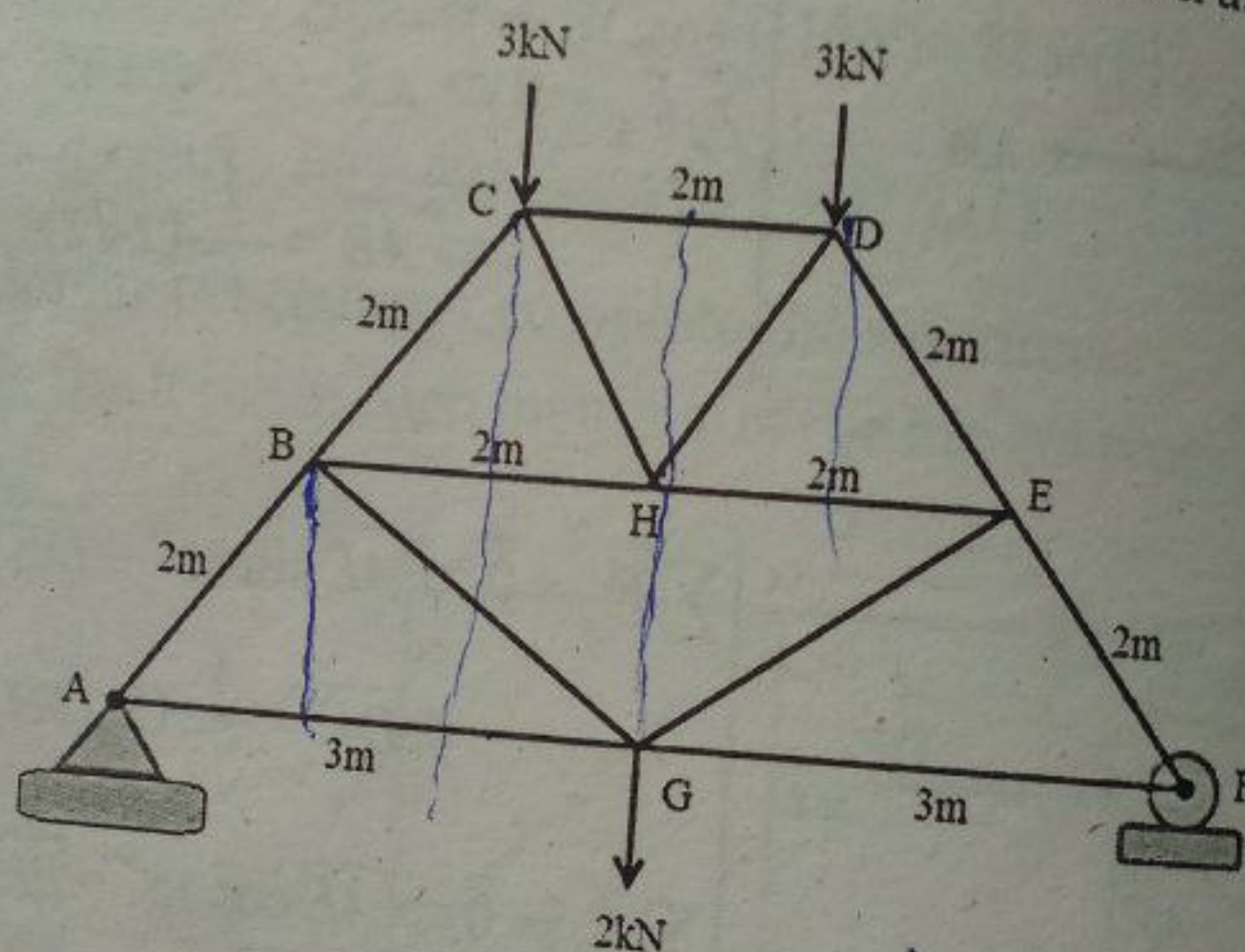
4) At Joint C:



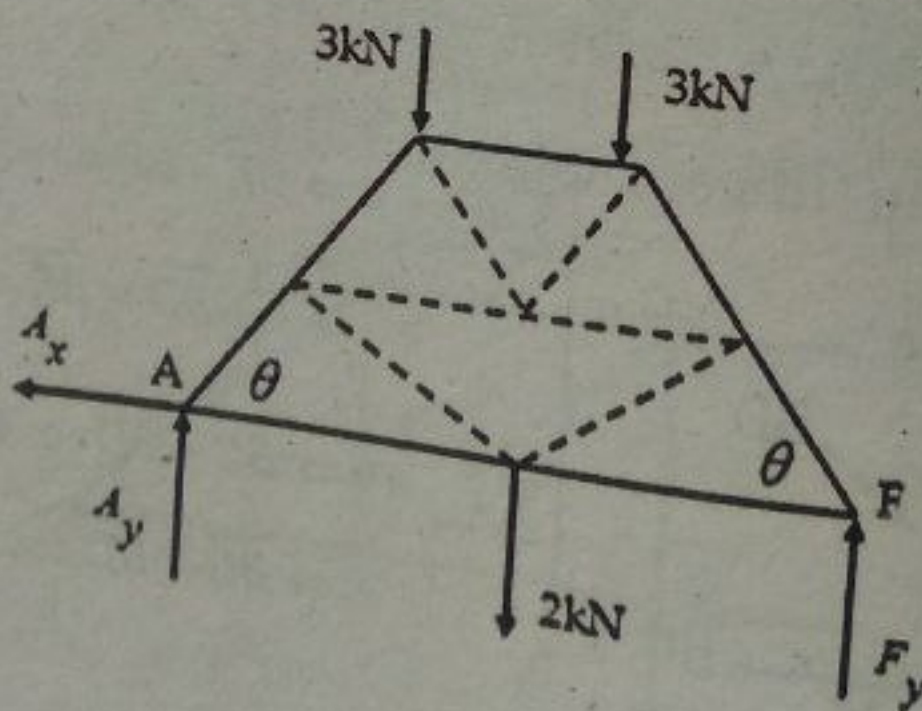
$$\sum \bar{F}_y = 0 \Rightarrow \bar{CD} - C_y = 0$$

$$\Rightarrow \bar{CD} = C_y = 2N$$

4. Determine the force in each member of the loaded truss below.



Solution: Consider the FBD of the entire truss to get the reactions.



Using moment equilibrium equations at A, we have

$$\sum M_A = 0 \Rightarrow F_y(6m) - 2kN(3m) - 3kN(2m) - 3kN(4m) = 0$$

$$\Rightarrow 6F_y - 24kN = 0 \Rightarrow F_y = 4kN$$

Again, using equilibrium equations for the forces, we have

$$\sum F_x = 0 \Rightarrow A_x = 0$$

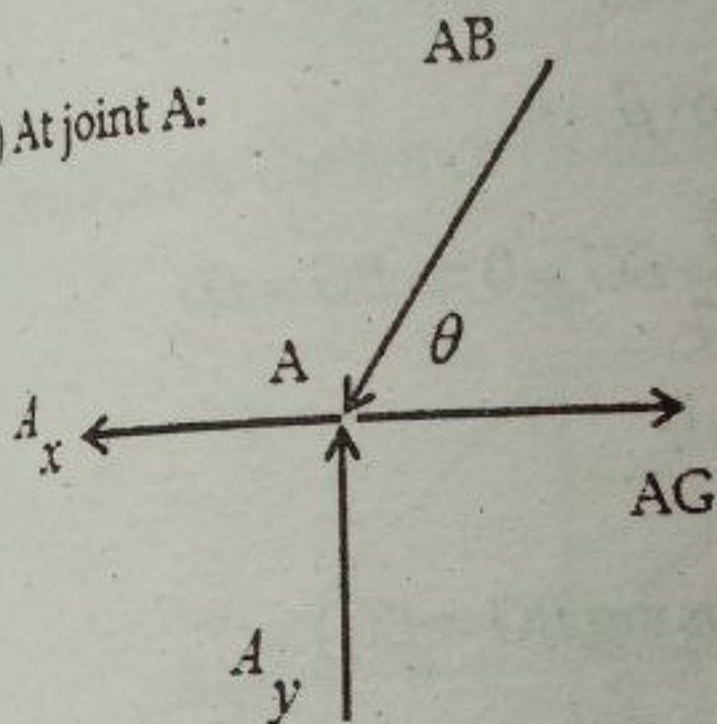
$$\sum F_y = 0 \Rightarrow A_y + F_y - 2kN - 3kN - 3kN = 0$$

$$\Rightarrow A_y + 4kN - 8kN = 0 \Rightarrow A_y = 4kN$$

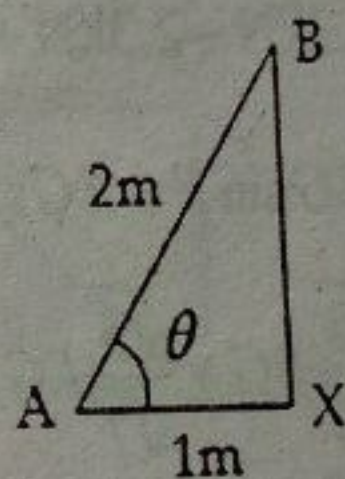
below.

Now, let's consider each joint for the member forces.

1) At joint A:



Get the angle from this triangle



tions.

First, notice that using trigonometric, $\sin \theta = \frac{\sqrt{3}}{2}$, $\cos \theta = \frac{1}{2}$

$$\sum F_y = 0 \Rightarrow A_y - AB \sin \theta = 0 \Rightarrow AB \sin \theta = A_y$$

$$\Rightarrow \frac{\sqrt{3}}{2} AB = 4kN \Rightarrow AB = \frac{8}{\sqrt{3}} kN = 4.6kN$$

$$\sum F_x = 0 \Rightarrow AG - AB \cos \theta = 0$$

$$\Rightarrow AG = AB \cos \theta = \frac{1}{2} \left(\frac{8}{\sqrt{3}} kN \right) = \frac{1}{2} (4.6kN) = 2.3kN$$

At Joint C:

$$i) \sum F_y = 0 \Rightarrow BC \sin 60^\circ - CH \sin \theta - 3\text{kN} = 0$$

$$\Rightarrow \frac{\sqrt{3}}{2} CH = \frac{\sqrt{3}}{2} (2\sqrt{3}\text{kN}) - 3\text{kN} \Rightarrow CH = 0$$

$$ii) \sum F_x = 0 \Rightarrow BC \cos 60^\circ - CD = 0 \Rightarrow CD = BC \cos 60^\circ = \sqrt{3}\text{kN}$$

At Joint H:

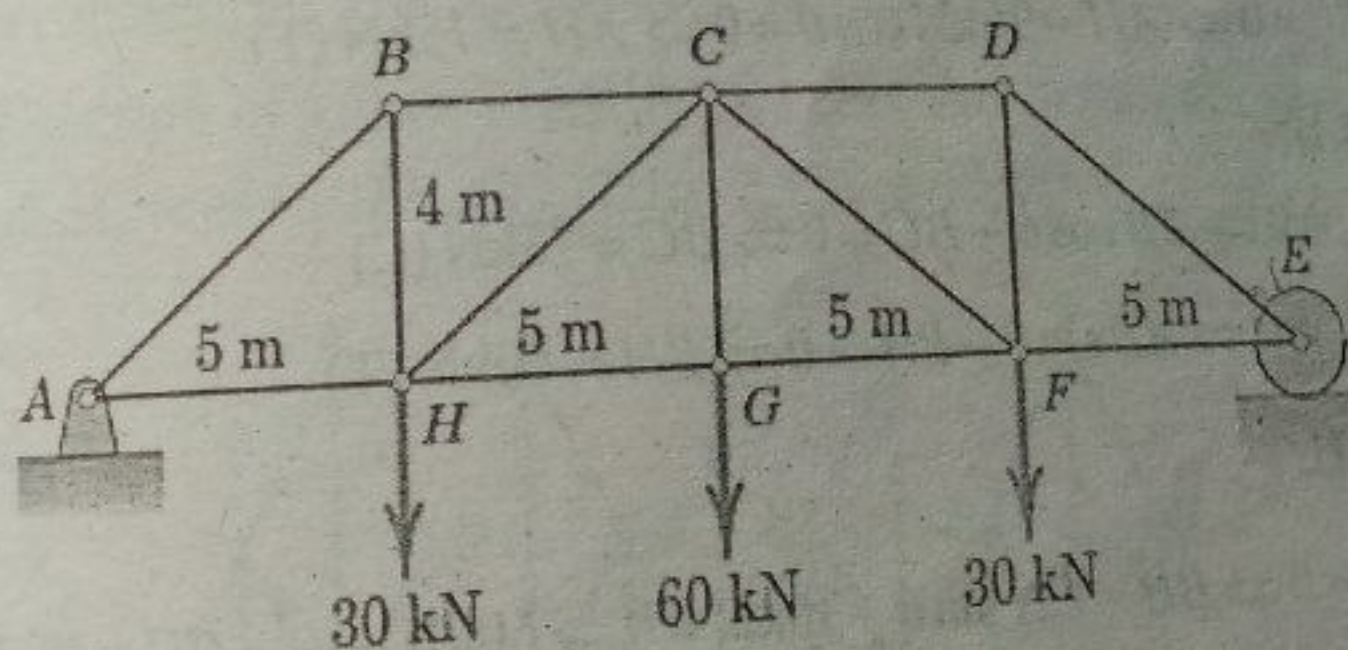
At this joint, since $CH = 0$, we must also have $HD = 0$.

$$\text{Then, } BH - HE = 0 \Rightarrow HE = BH = \sqrt{3}\text{kN}$$

At Joint D:

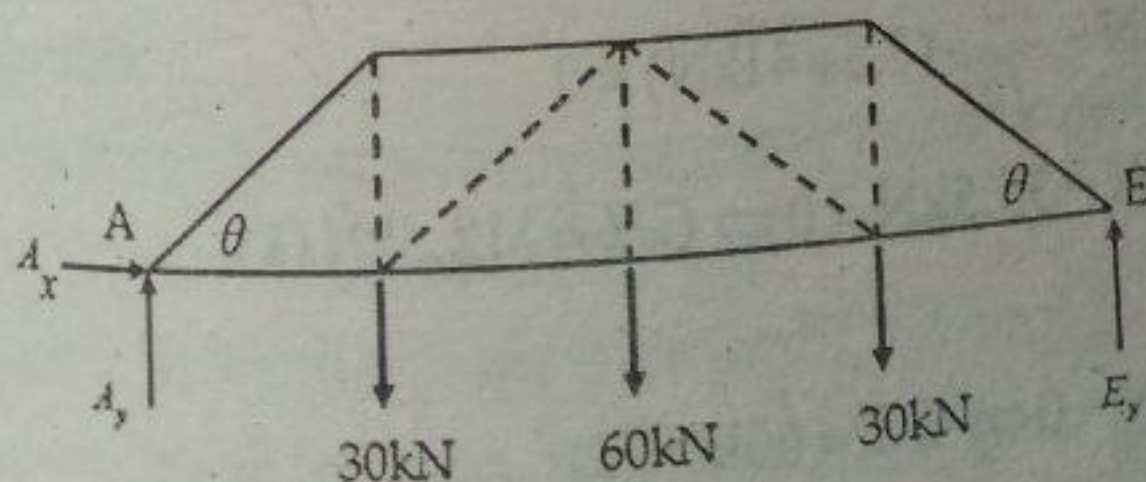
$$i) \sum F_y = 0 \Rightarrow DE \sin 60^\circ - 3\text{kN} = 0 \Rightarrow \frac{\sqrt{3}}{2} DE = 3\text{kN} \Rightarrow DE = 2\sqrt{3}\text{kN}$$

5. Determine the force in each member of the loaded truss below.

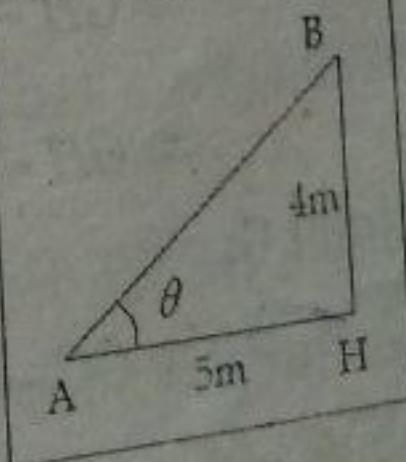


$$\frac{6}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{6\sqrt{3}}{3} = 2\sqrt{3}$$

Solution: Consider the FBD of the entire truss to get the reactions.



Get the angle from this triangle



Using moment equilibrium equations at A, we have

$$\begin{aligned}\sum M_A = 0 &\Rightarrow E_y(20\text{m}) - 30\text{kN}(15\text{m}) - 60\text{kN}(10\text{m}) - 30\text{kN}(5\text{m}) = 0 \\ &\Rightarrow 20E_y - 1200\text{kN} = 0 \Rightarrow E_y = 60\text{kN}\end{aligned}$$

Again, using equilibrium equations for the forces, we have

$$\text{i) } \sum F_x = 0 \Rightarrow A_x = 0$$

$$\begin{aligned}\text{ii) } \sum F_y = 0 &\Rightarrow A_y + E_y - 30\text{kN} - 60\text{kN} - 30\text{kN} = 0 \\ &\Rightarrow A_y + 60\text{kN} - 120\text{kN} = 0 \Rightarrow A_y = 60\text{kN}\end{aligned}$$

Now, let's consider each joint for the member forces.

At Joint A: From the above diagram, $\sin \theta = \frac{4}{\sqrt{41}}$, $\cos \theta = \frac{5}{\sqrt{41}}$

$$\text{i) } \sum F_y = 0 \Rightarrow A_y - AB \sin \theta = 0 \Rightarrow 60\text{kN} - AB \sin \theta = 0 \Rightarrow AB = 96\text{kN (C)}$$

$$\text{ii) } \sum F_x = 0 \Rightarrow AH - 96\text{kN} \cos \theta = 0 \Rightarrow AH = 75\text{kN (T)}$$

At Joint B:

$$\text{i) } \sum F_x = 0 \Rightarrow AB \cos \theta - BC = 0 \Rightarrow BC = 75\text{kN (C)}$$

$$\text{ii) } \sum F_y = 0 \Rightarrow AB \sin \theta - BH = 0 \Rightarrow BH = 60\text{kN (T)}$$

At Joint H:

$$\text{i) } \sum F_y = 0 \Rightarrow BH - CH \sin \theta - 30\text{kN} = 0 \Rightarrow 60\text{kN} - \frac{4}{\sqrt{41}} CH - 30\text{kN} = 0$$

$$\Rightarrow CH = 7.5\sqrt{41}\text{kN} = 48\text{kN (C)}$$

$$\text{ii) } \sum F_x = 0 \Rightarrow GH - AH - CH \cos \theta = 0$$

$$\Rightarrow GH - 75\text{kN} - \frac{5}{\sqrt{41}}(7.5\sqrt{41}\text{kN}) = 0$$

$$\Rightarrow GH - 75\text{kN} - 35.5\text{kN} = 0 \Rightarrow GH = 112.5\text{kN (T)}$$

At Joint G:

$$\text{i) } \sum F_y = 0 \Rightarrow GC - 60\text{kN} = 0 \Rightarrow GC = 60\text{kN (T)}$$

$$\text{ii) } \sum F_x = 0 \Rightarrow GF - GH = 0 \Rightarrow GF = GH = 112.5\text{kN (T)}$$



At Joint E:

$$i) \sum F_y = 0 \Rightarrow E_y - DE \sin \theta = 0 \Rightarrow \frac{4}{\sqrt{41}} DE = E_y \Rightarrow DE = 96 \text{ kN (C)}$$

$$ii) \sum F_x = 0 \Rightarrow DE \cos \theta - EF = 0 \Rightarrow EF = \frac{5}{\sqrt{41}} DE = 75 \text{ kN (T)}$$

At Joint D:

$$i) \sum F_y = 0 \Rightarrow DE \sin \theta - DF = 0 \Rightarrow DF = \frac{4}{\sqrt{41}} DE = 60 \text{ kN} \Rightarrow DE = 96 \text{ kN (T)}$$

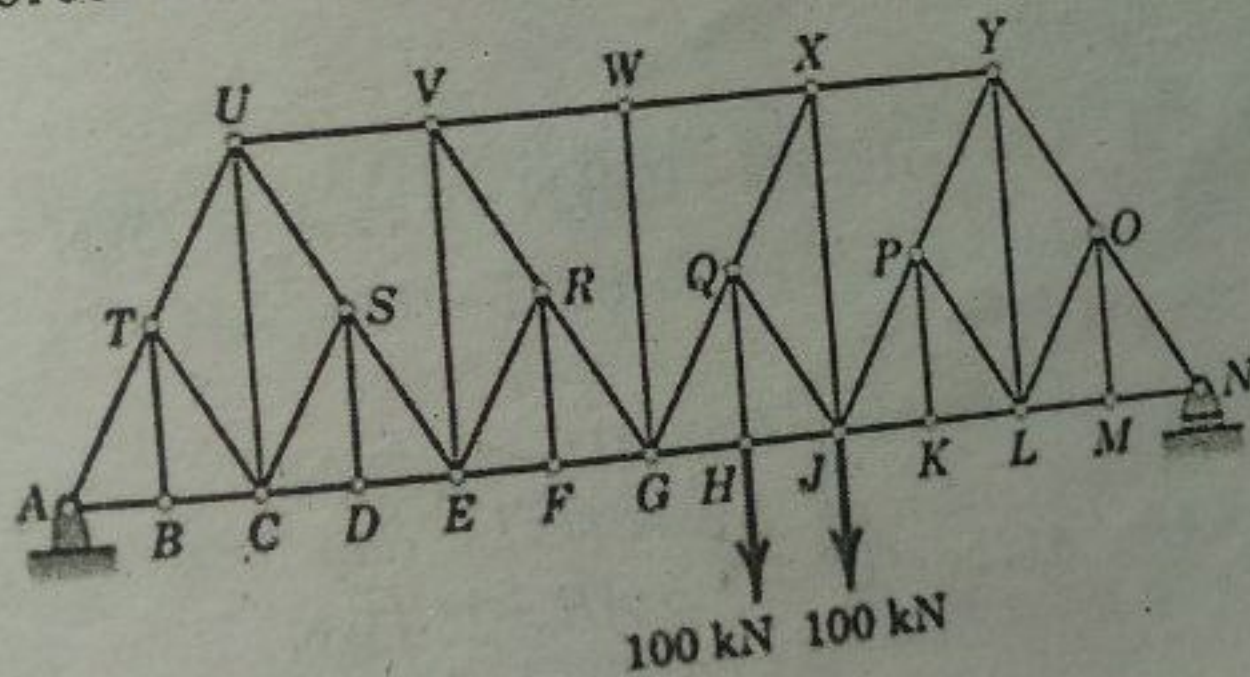
$$ii) \sum F_x = 0 \Rightarrow CD - DE \cos \theta = 0 \Rightarrow CD = \frac{5}{\sqrt{41}} DE = 75 \text{ kN (C)}$$

At Joint F:

$$i) \sum F_y = 0 \Rightarrow DF - CF \sin \theta - 30 \text{ kN} = 0 \Rightarrow CF = 48 \text{ kN}$$

6. Given the Baltimore truss where all the angles in any triangle that could be considered are $30^\circ, 60^\circ, 90^\circ$ or 120° .

- By inspection of joints, identify the zero-force members.
- Find the forces in WX, GQ, GH, HJ, QX and JQ



Solution:

- To identify zero-force members of the given truss, always examine each joint with three members.

At joint B: At this joint members AB and BC are collinear and BT is non-collinear to them. Besides, there is no connection or applied load at this joint. So, member BT is a zero force member.

At joint T: Since BT is zero-force member, we can assume as it is not there and thus we have only the collinear members AT, TU and TC. But AT and TU are collinear and no load is applied there. So, we get TC is also a zero force member.

At joint D: Similarly, DS, ST and CU are zero force members.

At joint F: Members FR and RE are zero force members.

At joint W: Member WG is a zero force member.

At joint K: Members KP and PL are zero force members.

At joint M and O: Members MO and OL are zero force members.

At joint L: Member LY is a zero force member. Identification of zero force members facilitates the calculation by minimizing the number of unknown force members and the number of joints to be analyzed.

b) Analyze the force on the remaining members.

Analysis-I: By Method of joints

First, determine reaction forces using the FBD of the whole truss.

$$i) \sum M_A = 0 \Rightarrow N_y \cdot (12D) - 100\text{kN} \cdot (7D) - 100\text{kN} \cdot (8D) = 0 \Rightarrow N_y = 125\text{kN}$$

$$ii) \sum F_x = 0 \Rightarrow A_x = 0$$

$$iii) \sum F_y = 0 \Rightarrow A_y + N_y - 100\text{kN} - 100\text{kN} = 0 \Rightarrow A_y = 75\text{kN}$$

At Joint A:

$$i) \sum F_y = 0 \Rightarrow A_y - AU \sin 60^\circ = 0 \Rightarrow AU = 50\sqrt{3}\text{kN}$$

$$ii) \sum F_x = 0 \Rightarrow AE - AU \cos 60^\circ = 0 \Rightarrow AE = 25\sqrt{3}\text{kN}$$

At Joint U:

$$i) \sum F_y = 0 \Rightarrow AU \sin 60^\circ - UE \sin 60^\circ = 0 \Rightarrow UE = AU = 50\sqrt{3}\text{kN}$$

$$ii) \sum F_x = 0 \Rightarrow AU \cos 60^\circ + UE \cos 60^\circ - UV = 0 \Rightarrow UV = 50\sqrt{3}\text{kN}$$

At Joint E:

$$i) \sum F_x = 0 \Rightarrow EG - UE \cos 60^\circ - AE = 0 \Rightarrow EG = 50\sqrt{3}\text{kN}$$

$$ii) \sum F_y = 0 \Rightarrow UE \sin 60^\circ - VE = 0 \Rightarrow VE = UE \sin 60^\circ = 75\text{kN}$$



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At Joint V:
i) $\sum F_y = 0 \Rightarrow VE - VG \sin 60^\circ = 0 \Rightarrow \frac{\sqrt{3}}{2} VG = 75 \text{ kN} \Rightarrow VG = 50\sqrt{3} \text{ kN}$

ii) $\sum F_x = 0 \Rightarrow UV + VG \cos 60^\circ - VW = 0 \Rightarrow VW = 75\sqrt{3} \text{ kN}$

At Joint G:
i) $\sum F_y = 0 \Rightarrow VG \sin 60^\circ - GQ \sin 60^\circ = 0 \Rightarrow GQ = VG = 50\sqrt{3} \text{ kN}$
ii) $\sum F_x = 0 \Rightarrow GH - VG \cos 60^\circ - GQ \cos 60^\circ - EG = 0$
 $\Rightarrow GH = VG \cos 60^\circ + GQ \cos 60^\circ + EG = 100\sqrt{3} \text{ kN}$

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At Joint H:
i) $\sum F_x = 0 \Rightarrow HJ - GH = 0 \Rightarrow HJ = GH = 100\sqrt{3} \text{ kN}$
ii) $\sum F_y = 0 \Rightarrow HQ - 100 \text{ kN} = 0 \Rightarrow HQ = 100 \text{ kN}$

SS.
5 kN

At Joint N:
i) $\sum F_y = 0 \Rightarrow N_y - YN \sin 60^\circ = 0 \Rightarrow YN = 144.4 \text{ kN}$
ii) $\sum F_x = 0 \Rightarrow YN \cos 60^\circ - JN = 0 \Rightarrow JN = 72.2 \text{ kN}$

At Joint Y:
i) $\sum F_y = 0 \Rightarrow YN \sin 60^\circ - YJ \sin 60^\circ = 0 \Rightarrow YJ = YN = 144.4 \text{ kN}$
ii) $\sum F_x = 0 \Rightarrow XY - YN \cos 60^\circ - YJ \cos 60^\circ = 0 \Rightarrow XY = 144.4 \text{ kN}$

At Joint J:
i) $\sum F_x = 0 \Rightarrow JQ \cos 60^\circ + JN + YJ \cos 60^\circ - HJ = 0$
 $\Rightarrow \frac{1}{2} JQ = 100\sqrt{3} \text{ kN} - 72.2 \text{ kN} - \frac{1}{2} (144.4 \text{ kN}) \Rightarrow JQ = 57.6 \text{ kN}$
ii) $\sum F_y = 0 \Rightarrow JX + YJ \sin 60^\circ - JQ \sin 60^\circ - 100 \text{ kN} = 0 \Rightarrow JX = 25 \text{ kN}$

At Joint X:
i) $\sum F_y = 0 \Rightarrow QX \sin 60^\circ - JX = 0 \Rightarrow QX \sin 60^\circ - JX = 0 \Rightarrow QX = 28.9 \text{ kN}$
ii) $\sum F_x = 0 \Rightarrow WX + QX \cos 60^\circ - XY = 0 \Rightarrow WX = 129.9 \text{ kN}$

4.4.2 Method of Sections

The technique of analyzing a member force by dividing the whole truss into two parts such that by taking one of the parts is known as Method of Section. Each part is called section.

The *method of sections* is a useful approach for determining the forces supported by individual members of a truss. It is effectively applicable especially when forces in only a portion of the members are needed.

The basic advantage of the *method of sections* is that in many cases the force in a selected member can be found directly from analysis of a section which has cut that member.

Direction of forces: Tension or Compression

2. Direction of Forces: Tension or Compression

In the method of sections, if the member force acts away from the section, it is named as tension and if the member force acts into or toward the section, it is named as compression.

However, it is not always possible to assign the proper direction of a force on a given member when the free-body diagram of a section is initially drawn.

Generally, assign all unknown forces arbitrarily as positive (away from the section). Then, the sign of the answer will tell you whether the direction is correct or not. The plus sign tells you the direction is correct (the force is tension) and the minus sign tells you the direction is in the opposite direction (the force is compression).

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Procedures to apply Method of Sections:

First: Determination of reaction forces (If necessary)

In many situations, it may be necessary to determine the reactions for the truss as a whole before applying the method of section. Of course in a truss, if both of the supports are on one side of a section, you can determine the member forces by taking the section found on the side where there are no supports.

Second: Formation and selection of a section

Formation: To form sections, select a member whose force is asked and take a cut in which NOT MORE THAN THREE UNKNOWN member are cut. This divide the truss into two sections.

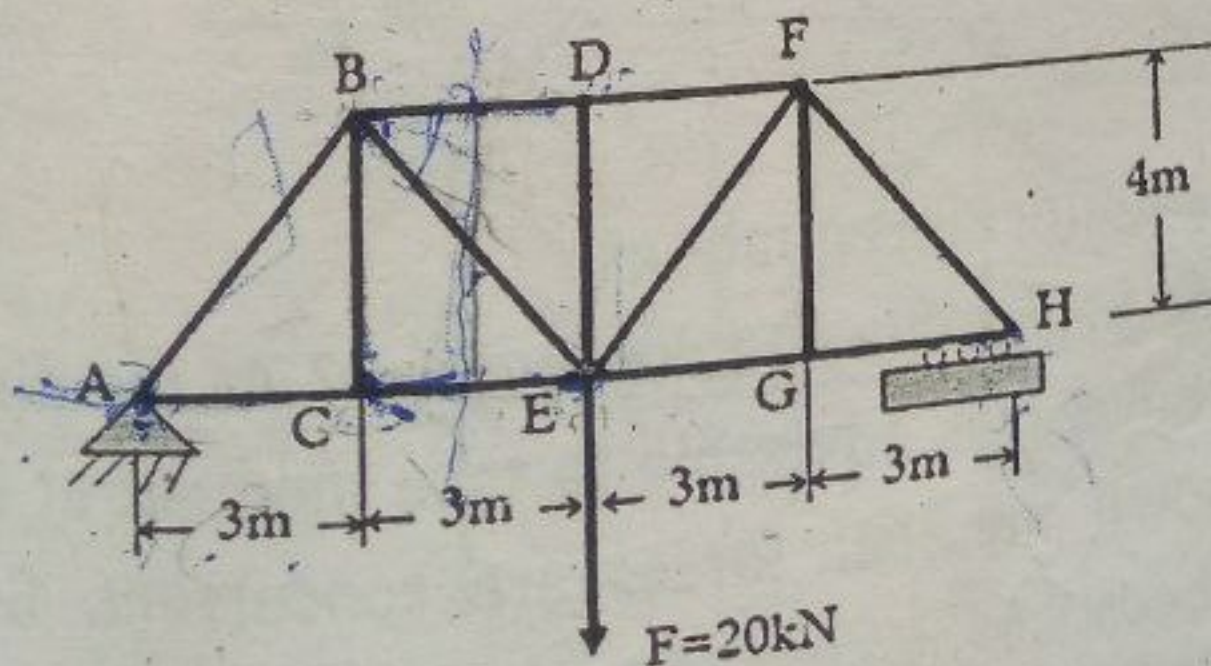
Selection and labeling:

የብረት ለምሳሌ ከሁለቱ ሴክሽኖች የትኛው ነው የሚመለከተው? From the two sections, take the one with less unknown forces and assume the selected section as a new system in equilibrium. Then, draw its FBD and label applied forces, reactions and unknowns member forces on that section.

Third: Apply equilibrium assumptions: The basic assumption is that if the whole is in equilibrium, then both sections of the truss are also in equilibrium. To simplify the calculation, first take a moment at a point where two of the unknown members pass.

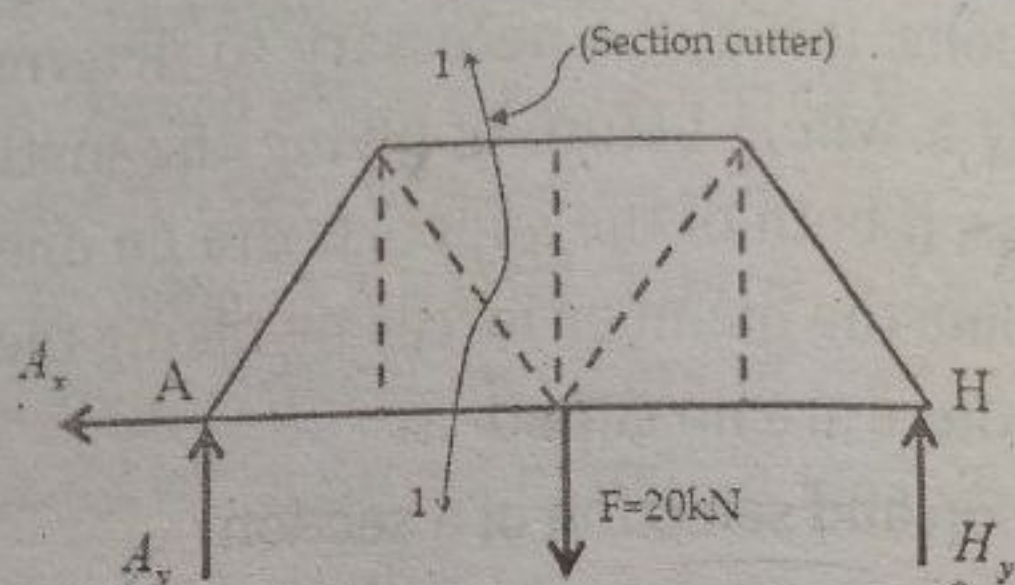
Examples:

1. Determine the force supported by the members BD , BE , CE



Solution:

First: Determination of reaction forces: Draw the FBD of the whole truss and label the reactions A_x, A_y at A, the reaction H_y at H.



From equilibrium conditions,

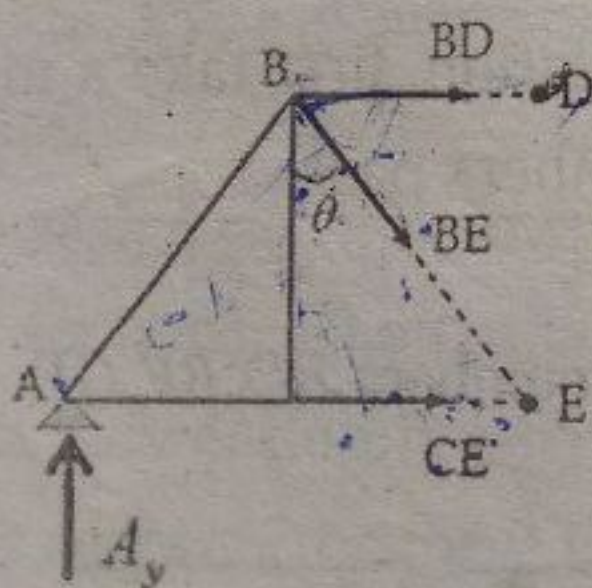
$$i) \sum M_A = 0 \Rightarrow H_y \cdot d_H - F \cdot d_F = 0 \Rightarrow 12H_y - 20 \text{ kN}(6) = 0 \Rightarrow H_y = 10 \text{ kN}$$

$$ii) \sum F_x = 0 \Rightarrow A_x = 0$$

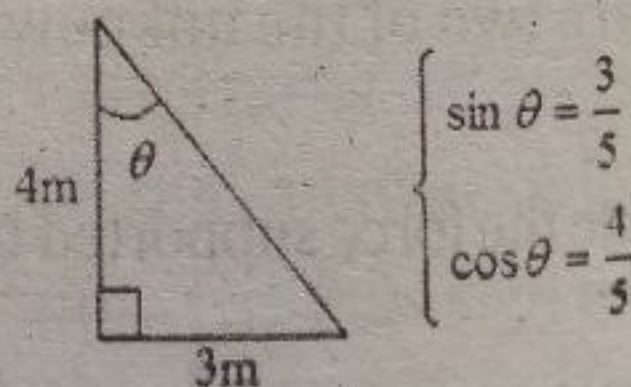
$$iii) \sum F_y = 0 \Rightarrow A_y + H_y - 20 \text{ kN} = 0 \Rightarrow A_y + 10 \text{ kN} - 20 \text{ kN} = 0 \Rightarrow A_y = 10 \text{ kN}$$

Second: Formation and selection of a section

Consider a section cut through the members BD, CE and BE as shown in the above FBD. Then, select the left section of section line 1-1. For this section, we have the following FBD.



You can get the angle from the right triangle below.



Third: Apply equilibrium equations

Which point is preferable to take moment? As we discussed in the procedures above, it is advisable to take moment at a point where two of unknown member forces are concurrent. Good! So, take moment at B because the members BD and BE are concurrent at B.

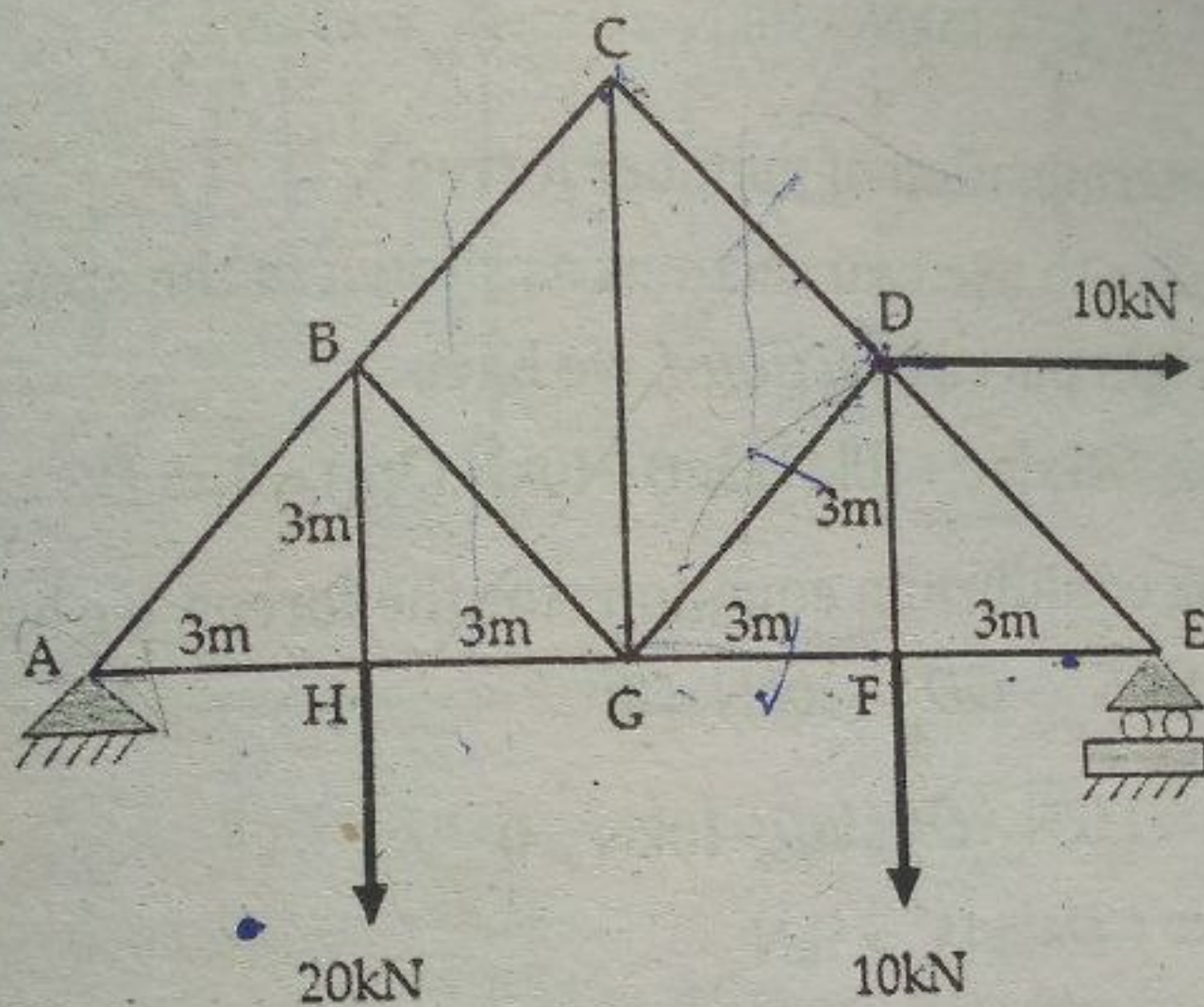
Of course you can also take a moment at E because BE and CE are concurrent at E. Thus, by applying moment and force equations,

i) $\sum M_B = 0 \Rightarrow CE \cdot (4\text{m}) - A_y \cdot (3\text{m}) = 0 \Rightarrow 4CE - 10\text{ kN} \cdot (3\text{m}) = 0$
 $\Rightarrow 4CE = 30\text{ kN} \Rightarrow CE = 7.5\text{ kN (T)}$

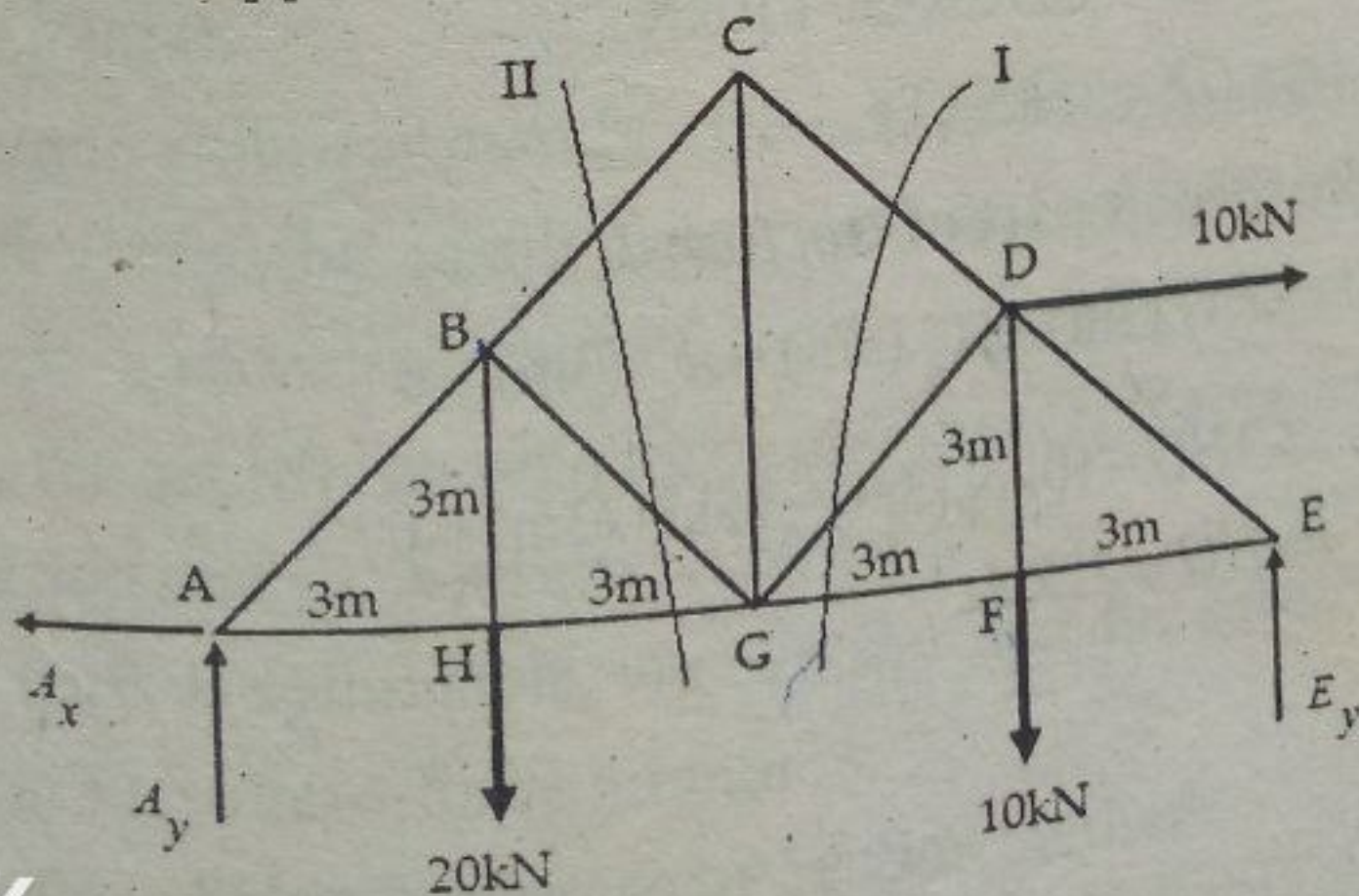
$$ii) \sum F_y = 0 \Rightarrow A_y - BE \cos \theta = 0 \Rightarrow BE \cos \theta = A_y \Rightarrow BE 12.5 \text{ kN (T)}$$

ii) $\sum F_y = 0 \Rightarrow CE + BE \sin \theta - BD = 0 \Rightarrow BD = CE + BE \sin \theta = 15 \text{ kN (C)}$

2. Using Method of Sections, find the forces in CD, DG, GF and HG .



Solution: Such problems are easily solved by taking two sections.
First: Determine support reactions (Using the whole truss)



Using moment equilibrium equations at D , we have

$$\begin{aligned}\sum M_A = 0 &\Rightarrow E_y(12\text{m}) - 10\text{N}(3\text{m}) - 10\text{N}(9\text{m}) - 20\text{N}(3\text{m}) = 0 \\ &\Rightarrow 12E_y - 180\text{N} = 0 \Rightarrow E_y = 15\text{kN}\end{aligned}$$

Again, using equilibrium equations for the forces, we have

$$\text{i) } \sum F_x = 0 \Rightarrow 10\text{kN} - A_x = 0 \Rightarrow A_x = 10\text{kN}$$

$$\begin{aligned}\text{ii) } \sum F_y = 0 &\Rightarrow A_y + E_y - 20\text{kN} - 10\text{kN} = 0 \\ &\Rightarrow A_y + 15\text{kN} - 30\text{kN} = 0 \Rightarrow A_y = 15\text{kN}\end{aligned}$$

Second: Determination of member forces

Here, we have to use two sections as shown in the above FBD.

Using the right part of *section-I*, we have

$$\sum M_E = 0 \Rightarrow DG \sin \theta + 10\text{kN}(3\text{m}) - 10\text{kN}(3\text{m}) = 0 \Rightarrow DG = 0$$

Again, using equilibrium equations for the forces, we have

$$\begin{aligned}\text{i) } \sum F_y = 0 &\Rightarrow E_y - CD_y - 10\text{kN} = 0 \\ &\Rightarrow 15\text{kN} - CD \sin \theta - 10\text{kN} = 0\end{aligned}$$

$$\Rightarrow CD \sin 45^\circ = 5\text{kN}$$

$$\Rightarrow CD = 5\sqrt{2}\text{kN (C)}$$

$$\text{i) } \sum F_x = 0 \Rightarrow CD_x - GF + 10\text{kN} = 0$$

$$\Rightarrow GF = CD \cos 45^\circ + 10\text{kN}$$

$$\Rightarrow GF = 15\text{kN (T)}$$

Again, take the left part of *section-II*.

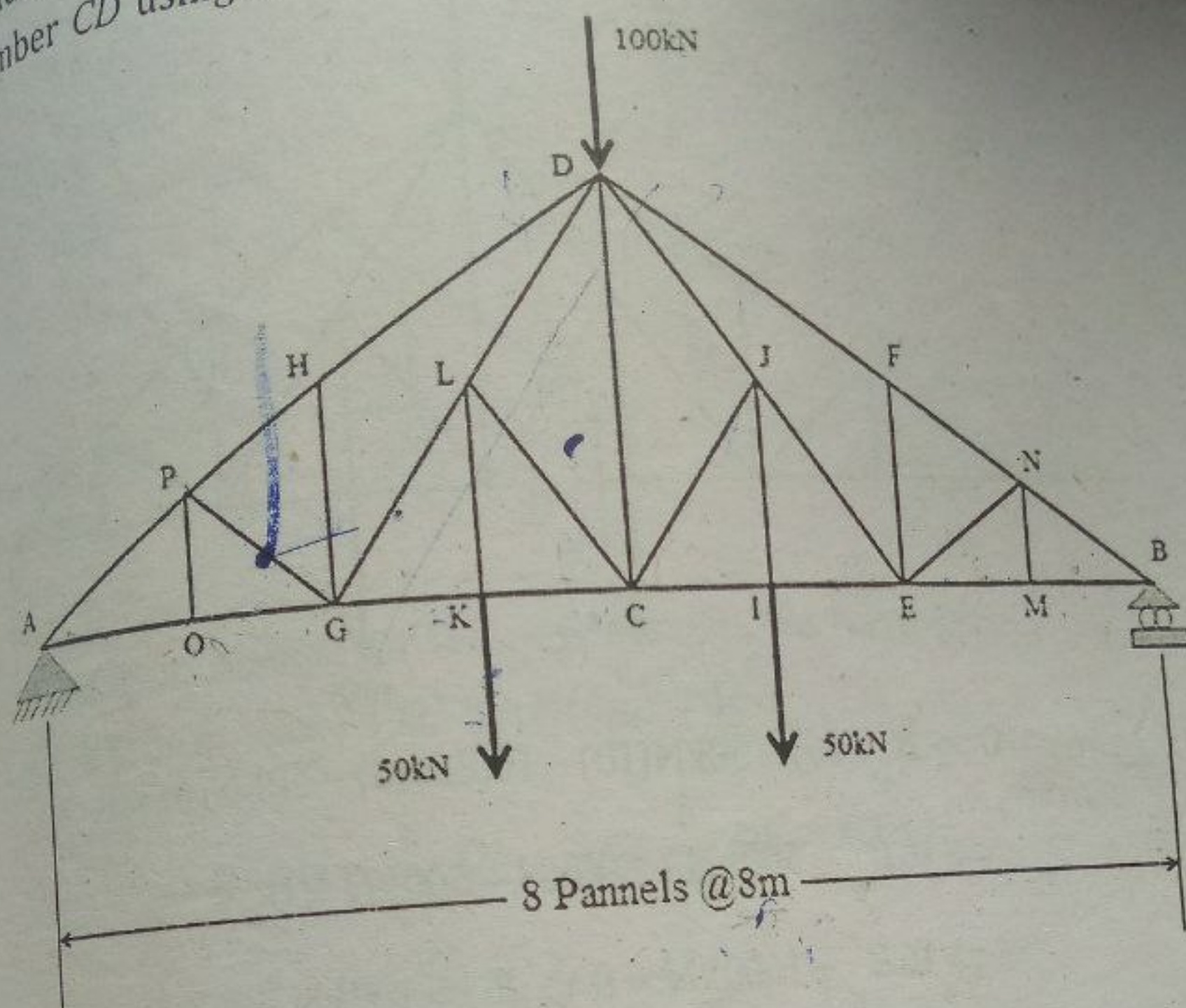
$$\sum M_B = 0 \Rightarrow HG(3\text{m}) - A_x(3\text{m}) - A_y(3\text{m}) = 0$$

$$\Rightarrow 3HG - 10\text{kN}(3\text{m}) - 15\text{kN}(3\text{m}) = 0$$

$$\Rightarrow HG = 25\text{kN}$$

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3*. Identify all the zero force members and determine the force in member CD using Section Method.



Solution:

Firs: Let's identify the zero-force members bi inspection of joints:

At joint O: At this joint three members AO, OG and OP are connected with AO and OG being collinear. Besides, there is no applied force or support at joint O.

Hence, the non-collinear member OP is zero-force member.

At joint P: Using the result from joint O that OP is a zero-force member, we can conclude that PG is also a zero force member.

At joint H: We get HG is a zero force member. This result also leads to the conclusion that GL is a zero force member.

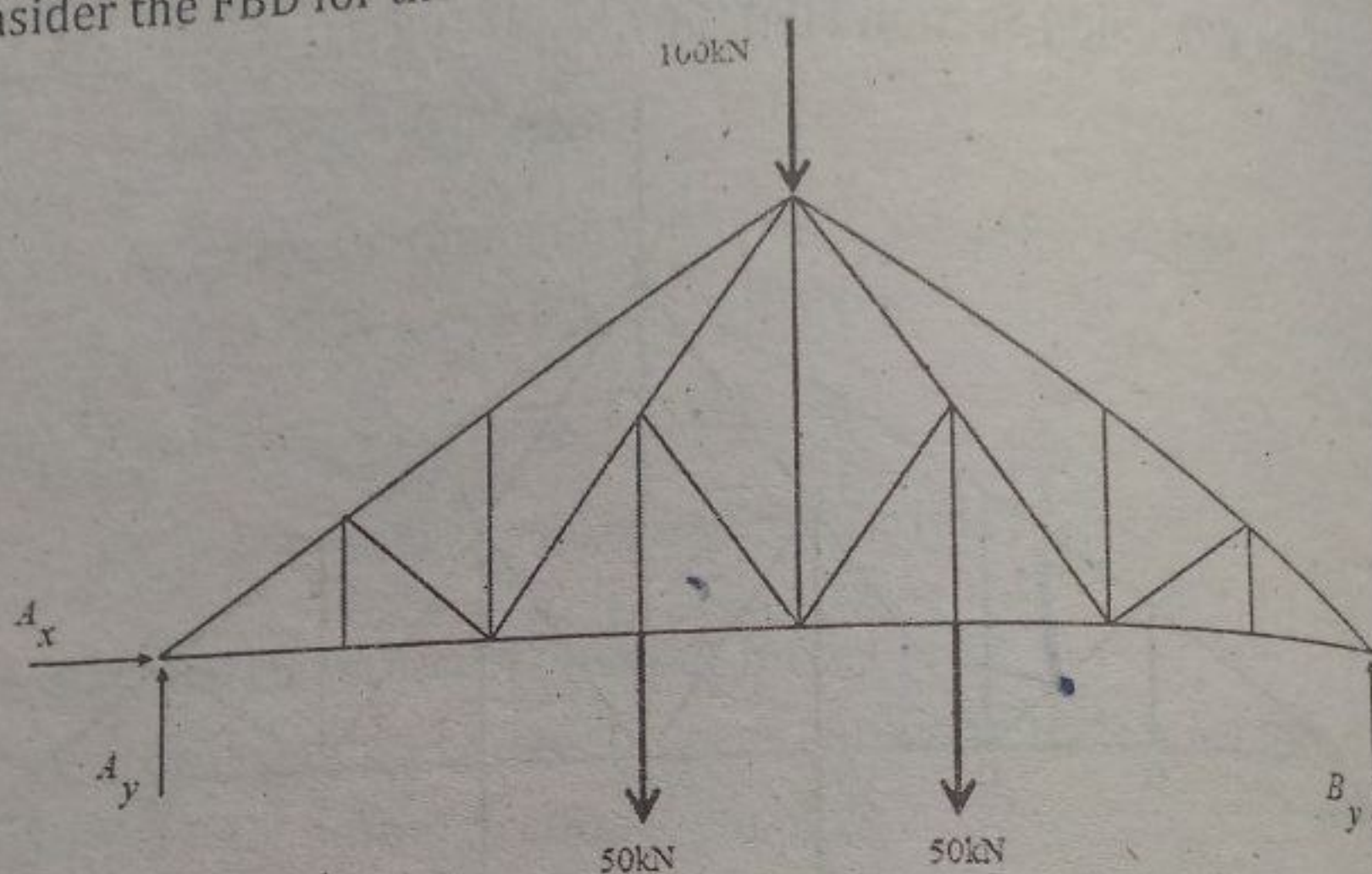
Since the truss is symmetric, we can say that the members MN, NE, FE and JE are also zero-force members.

Determination of Member forces by Section Method:

Determination of support reactions

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Consider the FBD for the whole truss as shown below.



$$i) \sum M_A = 0 \Rightarrow B_y (16) - 50\text{kN}(10) - 100\text{kN}(8) - 50\text{kN}(6) = 0$$

$$\Rightarrow 16B_y - 500\text{kN} - 800\text{kN} - 300\text{kN} = 0$$

$$\Rightarrow 16B_y - 1600\text{kN} = 0 \Rightarrow B_y = 100\text{kN}$$

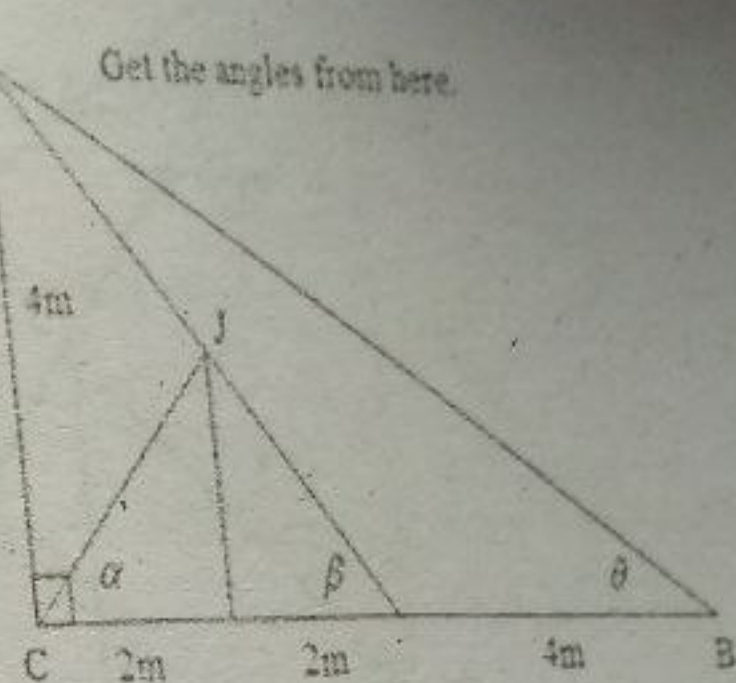
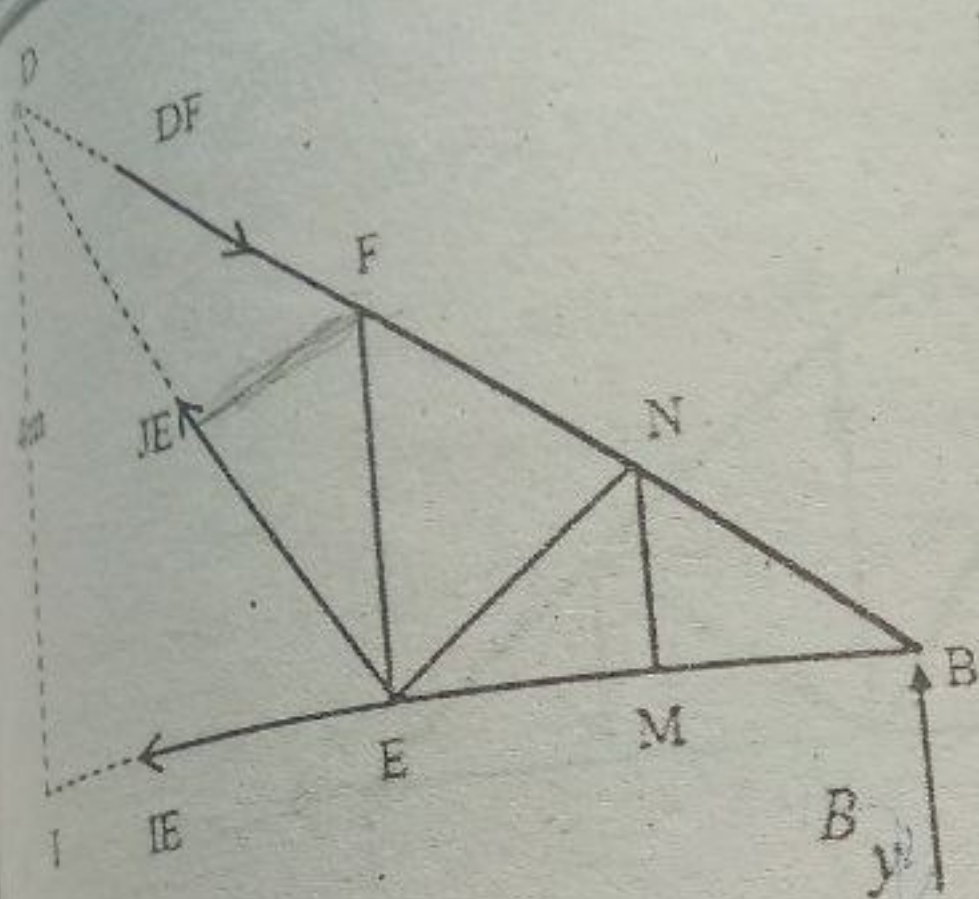
$$ii) \sum F_x = 0 \Rightarrow A_x = 0$$

$$iii) \sum F_y = 0 \Rightarrow A_y + B_y - 50\text{kN} - 50\text{kN} - 100\text{kN} = 0$$

$$\Rightarrow A_y + 100\text{kN} - 200\text{kN} = 0 \Rightarrow A_y = 100\text{kN}$$

Step-2: Determination of member forces (Using Section Method)

Now, take a section cutting the members IE, JE, DF and analyze the right part of this section.



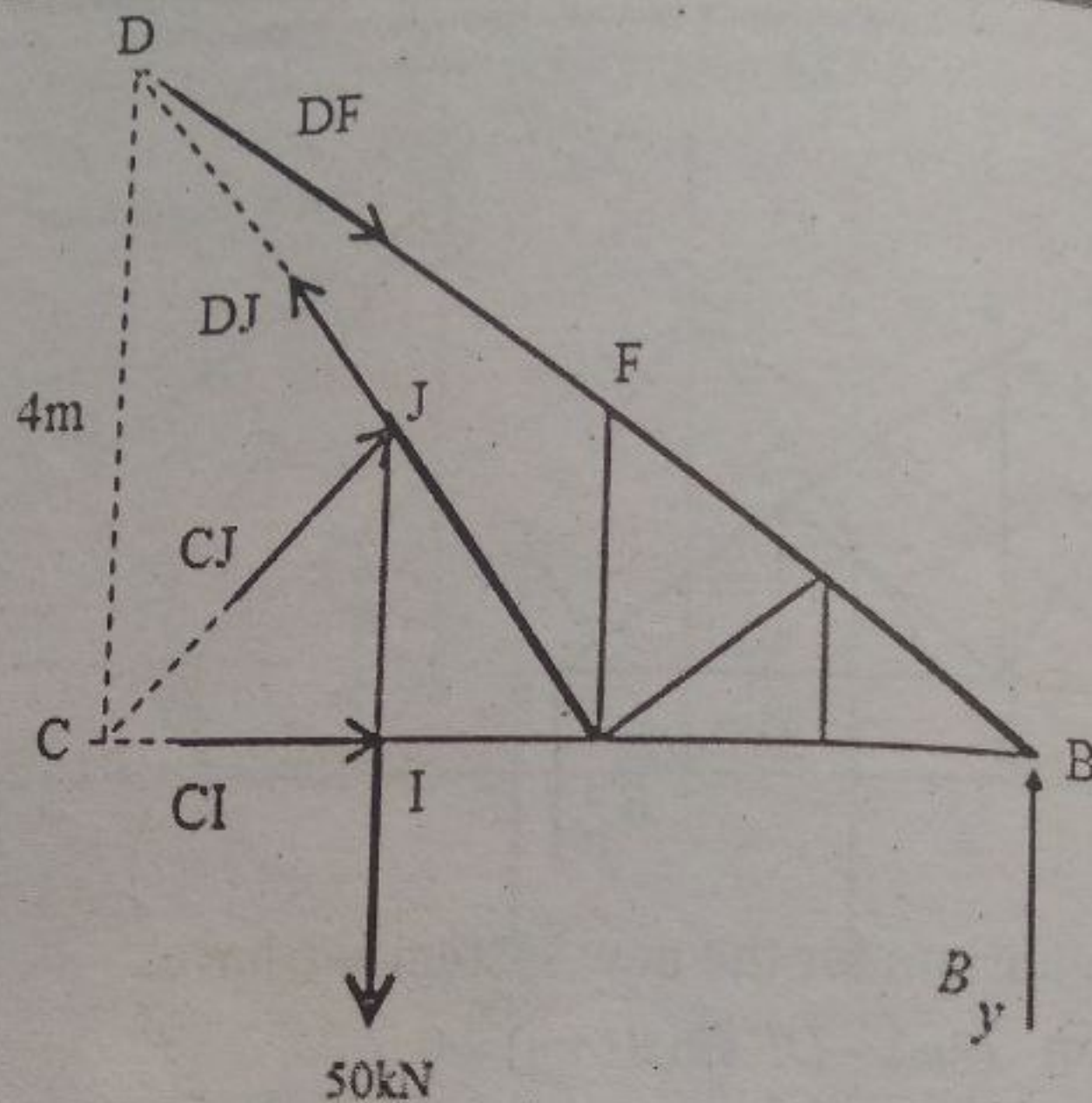
Then, using equilibrium for the new system, we have

$$\sum M_c = 0 \Rightarrow B_y (8m) - DF \sin \theta (4m) = 0$$

$$\Rightarrow 800kN - \frac{8}{\sqrt{5}} DF = 0 \Rightarrow \frac{8}{\sqrt{5}} DF = 800kN$$

$$\Rightarrow DF = 100\sqrt{5}kN$$

Again, take a section cutting the members CI, CJ, DJ, DF and analyze the right part of this section.



Then, using equilibrium for the new system, we have

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$$\sum M_c = 0 \Rightarrow B_y \cdot (8) - DF \sin \theta \cdot (4) - 50 \text{ kN} \cdot (2) + DJ \sin \beta \cdot (4) = 0$$

$$\Rightarrow 800 \text{ kN} - \frac{8}{\sqrt{5}} (100\sqrt{5} \text{ kN}) - 100 \text{ kN} + \frac{4\sqrt{2}}{2} DJ = 0$$

$$\Rightarrow 2\sqrt{2}DJ + 800 \text{ kN} - 800 \text{ kN} - 100 \text{ kN} = 0$$

$$\Rightarrow 2\sqrt{2}DJ = 100 \text{ kN} \Rightarrow DJ = 25\sqrt{2} \text{ kN}$$

$$\sum F_y = 0 \Rightarrow CJ \sin \alpha + DJ \cos \beta - DF \cos \theta - 50 \text{ kN} + B_y = 0$$

$$\Rightarrow \frac{\sqrt{2}}{2} CJ + \frac{\sqrt{2}}{2} (25\sqrt{2} \text{ kN}) - \frac{1}{\sqrt{5}} (100\sqrt{5} \text{ kN}) - 50 \text{ kN} + 100 \text{ kN} = 0$$

$$\Rightarrow \frac{\sqrt{2}}{2} CJ + 25 \text{ kN} - 100 \text{ kN} - 50 \text{ kN} + 100 \text{ kN} = 0$$

$$\Rightarrow \frac{\sqrt{2}}{2} CJ = 25 \text{ kN} \Rightarrow CJ = 25\sqrt{2} \text{ kN}$$

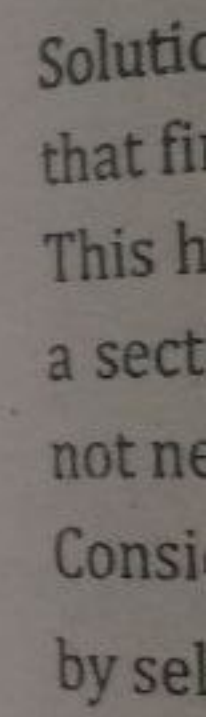
$$\sum F_x = 0 \Rightarrow CJ \cos \alpha - DJ \sin \beta + DF \sin \theta + CI = 0$$

$$\Rightarrow \frac{\sqrt{2}}{2} (25\sqrt{2} \text{ kN}) - \frac{\sqrt{2}}{2} (25\sqrt{2} \text{ kN}) - \frac{2}{\sqrt{5}} (100\sqrt{5} \text{ kN}) + CI = 0$$

$$\Rightarrow 25 \text{ kN} - 25 \text{ kN} - 200 \text{ kN} + CI = 0$$

$$\Rightarrow CI = 200 \text{ kN}$$

Finally, take a section cutting the members CK, CL, CD, DJ, DF and analyze the right part of this section.



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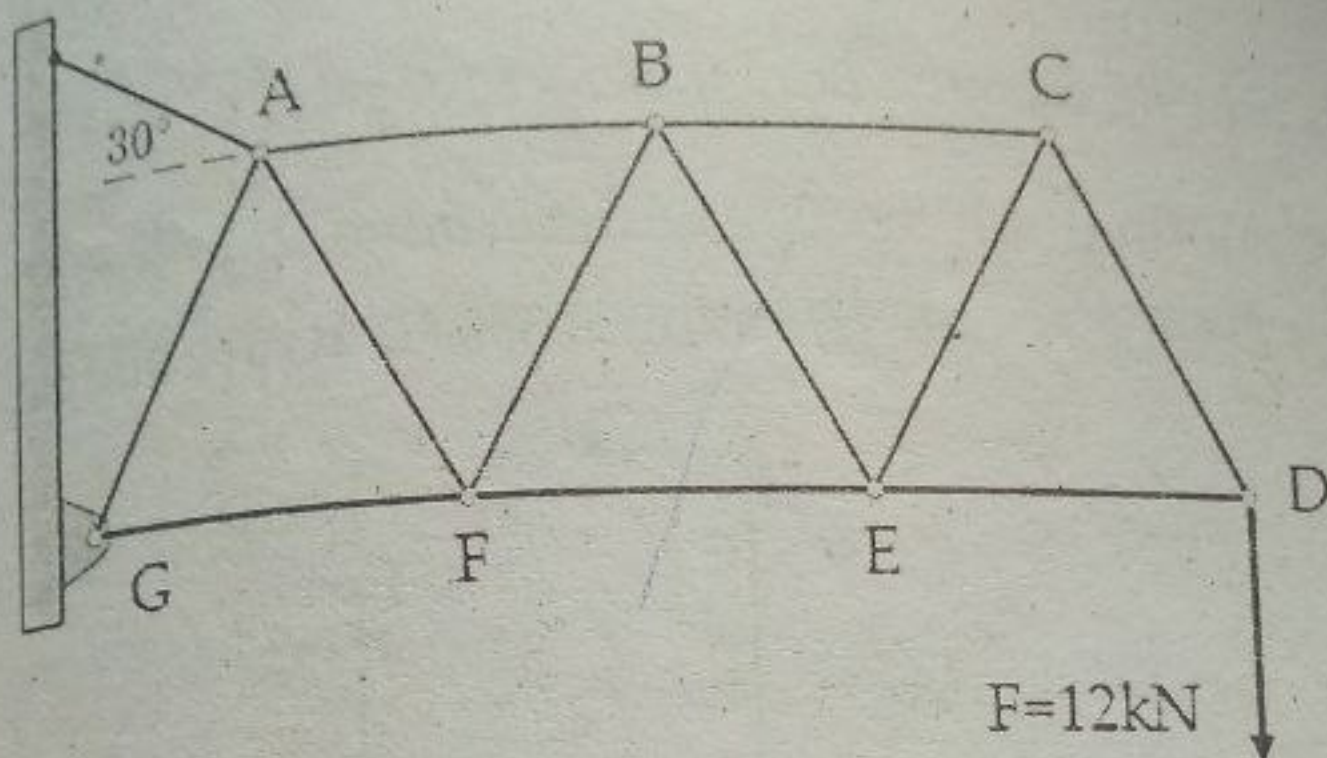
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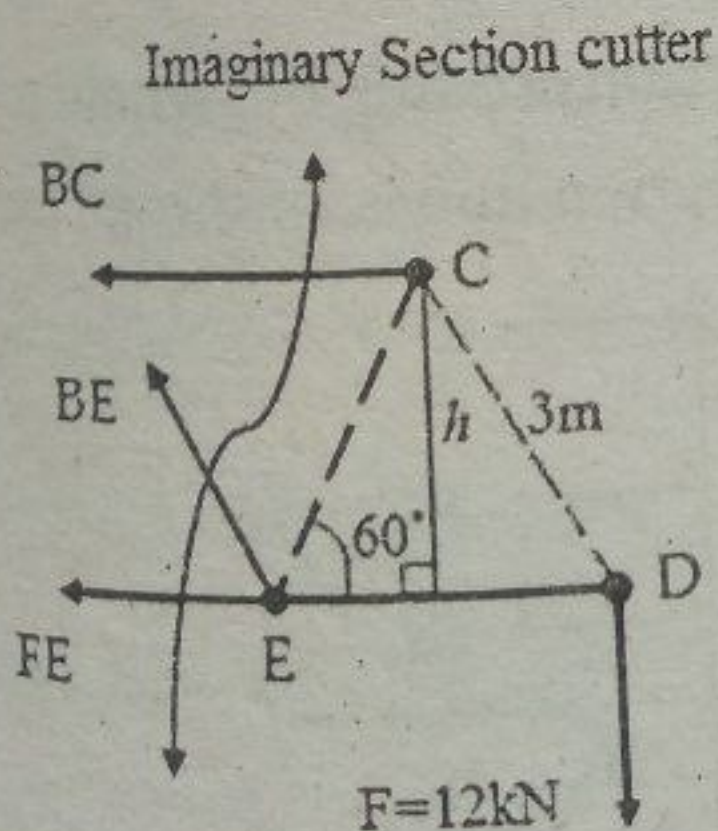
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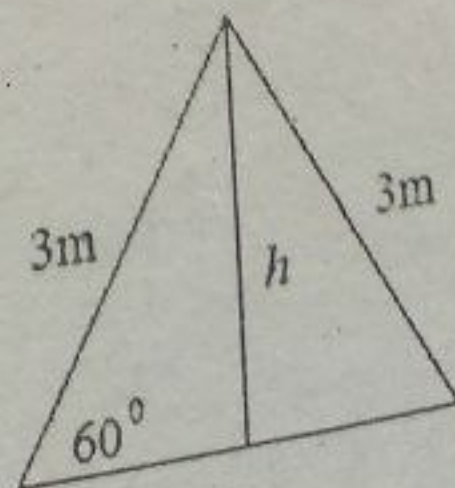
3. Using the methods of sections, on the truss structure below, determine the forces on the members BC , BE and FE . Assume all the triangles are equilateral triangles with sides $3m$.



Solution: In the explanation about method of sections, we have said that finding reaction forces may not be necessary. This happens especially when both supports are on the same side of a section we selected. This is an example where reaction forces are not needed to determine member forces. Consider a section cut of the members BC , BE and FE . For this cut, by selecting the right hand section, we have the following FBD.



From the equilateral triangle, we have

$$h = 3m \sin 60^\circ = \frac{3\sqrt{3}}{2}m$$


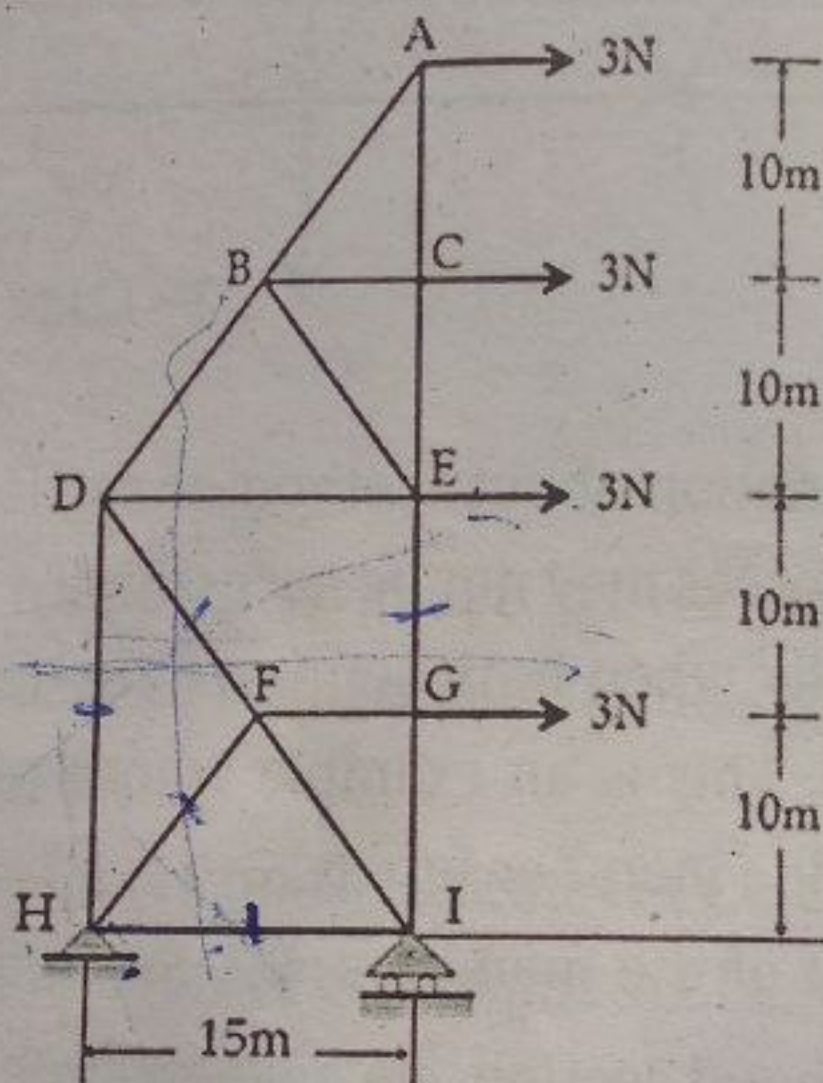
Hence, we have

$$i) \sum M_E = 0 \Rightarrow BC \cdot d_C - F \cdot d_F = 0 \Rightarrow BC \cdot h - 3F = 0 \Rightarrow BC = 8\sqrt{3} \text{ kN (T)}$$

$$ii) \sum F_y = 0 \Rightarrow BE \sin 60^\circ - F = 0 \Rightarrow \frac{\sqrt{3}}{2} BE - 12 \text{ kN} = 0 \Rightarrow BE = 8\sqrt{3} \text{ kN (T)}$$

$$iii) \sum F_x = 0 \Rightarrow FE - BE \cos 60^\circ - BC = 0 \Rightarrow FE = 12\sqrt{3} \text{ kN (C)}$$

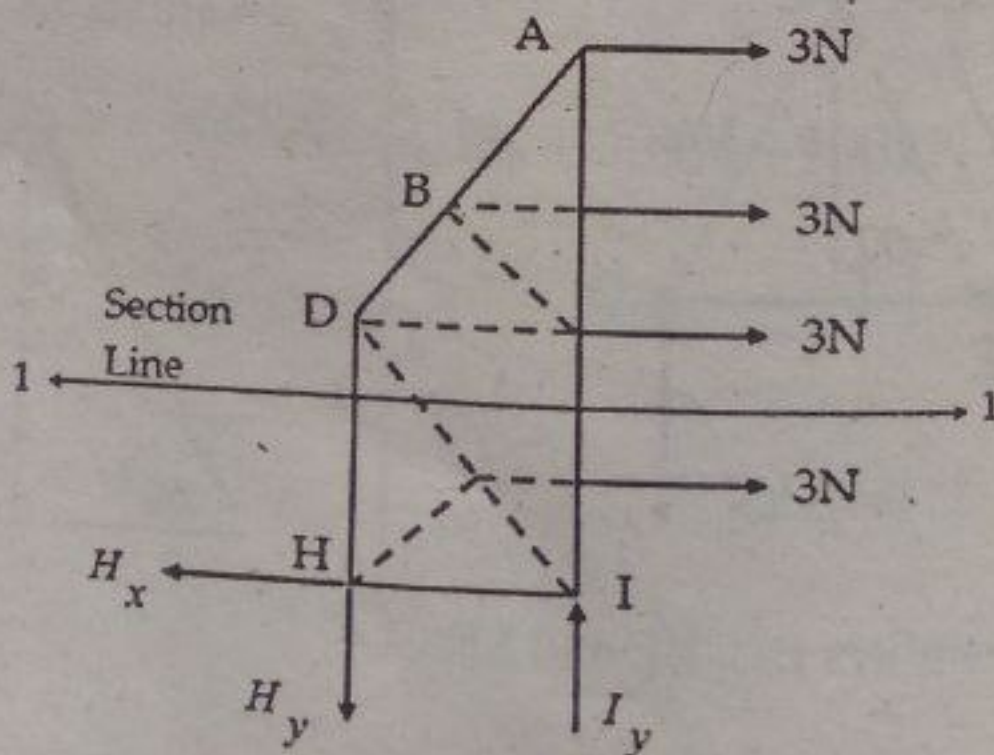
4*. Using the Method of Sections, determine the forces in the members HD, FD, GE, FH and HI in the truss structure shown below.



Solution:

Step-1: Determination of support reactions (Using the whole truss)

The FBD for the whole truss with a section line is as shown below.



$$\sum M_i = 0 \Rightarrow I_y \cdot (15m) - 3N \cdot (40m) - 3N \cdot (30m) - 3N \cdot (20m) - 3N \cdot (10m) = 0$$

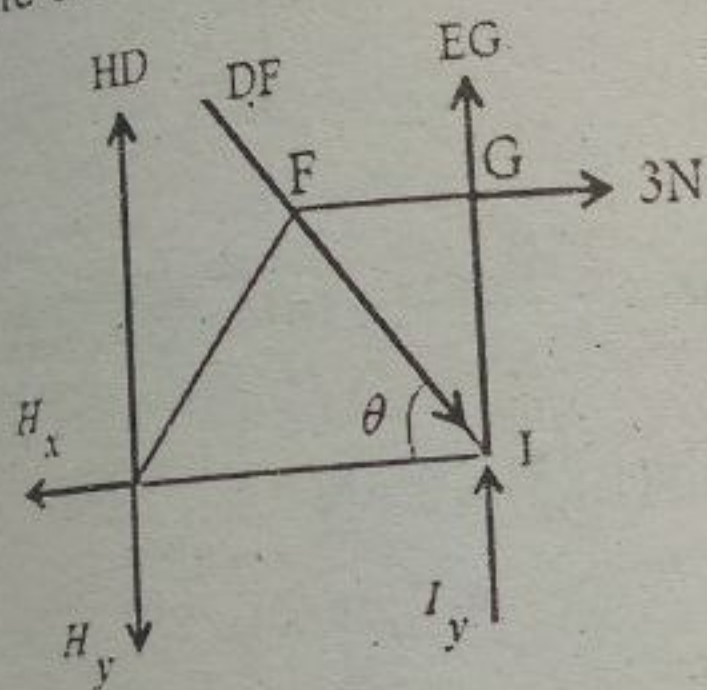
$$\Rightarrow 15I_y - 300N = 0 \Rightarrow I_y = 20N$$

$$\sum F_x = 0 \Rightarrow 3N + 3N + 3N + 3N - H_x = 0 \Rightarrow H_x = 12N$$

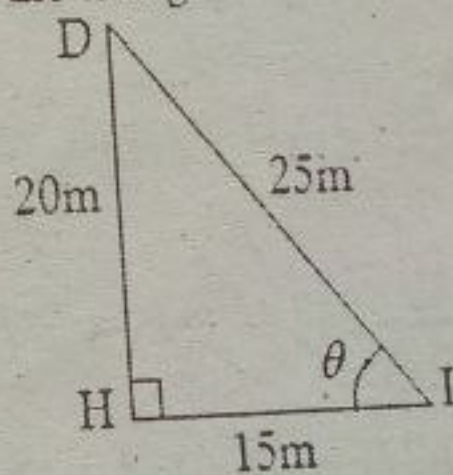
$$\sum F_y = 0 \Rightarrow I_y - H_y = 0 \Rightarrow H_y = I_y = 20N$$

Step 2: Determination of member forces

To find the forces in the members HD, FD, GE, FH and HI by method of sections, first take a section line horizontally with a line passing through the members HD, FD , and GE as shown below. Consider the section of the truss below the section line 1-1.



Get the angle from the triangle below



Then, using equilibrium for the new system, we have the following.

$$\sum M_i = 0 \Rightarrow H_y \cdot (15m) - HD(15m) - 3N(10m) = 0$$

$$\Rightarrow 20N(15m) - 15HD - 30N = 0$$

$$\Rightarrow 270N - 15HD = 0 \Rightarrow HD = 18N (T)$$

$$\sum F_x = 0 \Rightarrow FD \cos \theta - H_x + 3N = 0$$

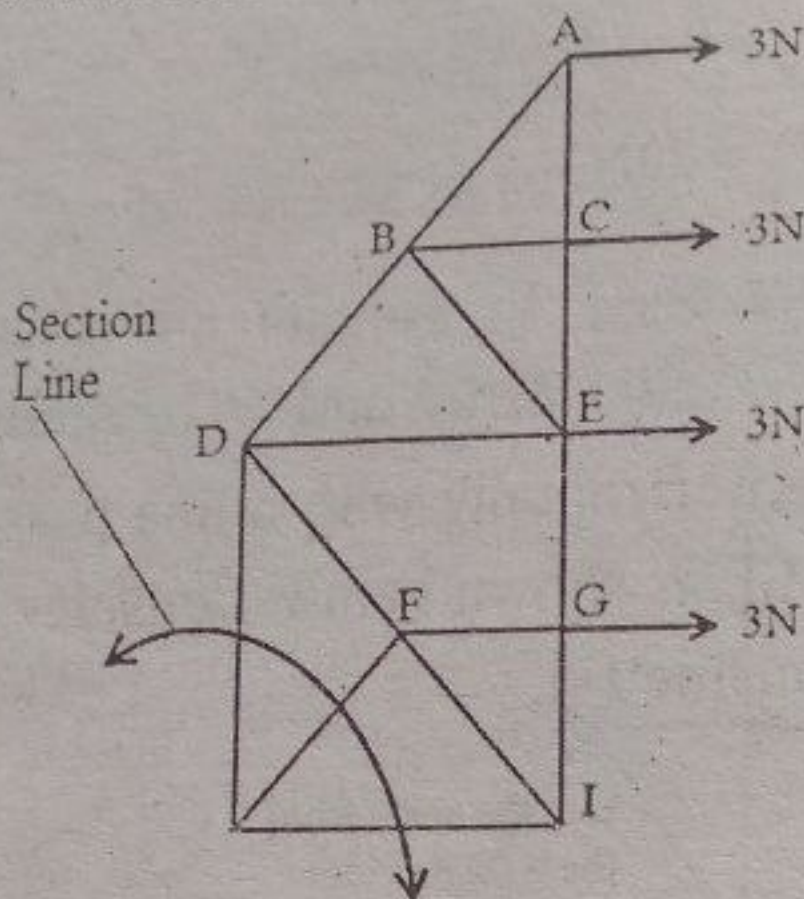
$$\Rightarrow \frac{3}{5}FD - 9N = 0 \Rightarrow FD = 15N (C)$$

$$\sum F_y = 0 \Rightarrow HD - FD \sin \theta + GE + I_y - H_y = 0$$

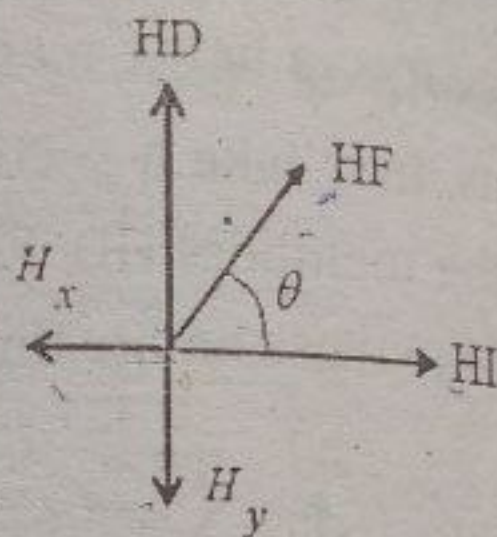
$$\Rightarrow 18N - \frac{4}{5}(15N) + GE + 20N - 20N = 0$$

$$\Rightarrow 18N + GE - 12N = 0 \Rightarrow GE = 6N (C)$$

To find the forces in the member FH and HI by method of sections, first take a curved section line and take the lower section of the section line as shown in the diagram on the left below.



Using the curved section line we get this section



$$i) \sum F_y = 0 \Rightarrow FH \sin \theta + HD - H_y = 0 \Rightarrow \frac{10}{12.5} FH + 18N - 20N = 0$$

$$\Rightarrow \frac{10}{12.5} FH = 2N \Rightarrow FH = 2N(1.25) = 2.5N$$

$$ii) \sum F_x = 0 \Rightarrow FH \cos \theta + HI - H_x = 0$$

$$\Rightarrow HI = H_x - FH \cos \theta$$

$$\Rightarrow HI = 12N - \frac{7.5}{12.5}(2.5N) = 2N$$

$$\Rightarrow HI = 12N - 1.5N = 10.5N$$

Solution:

Step-1: Det

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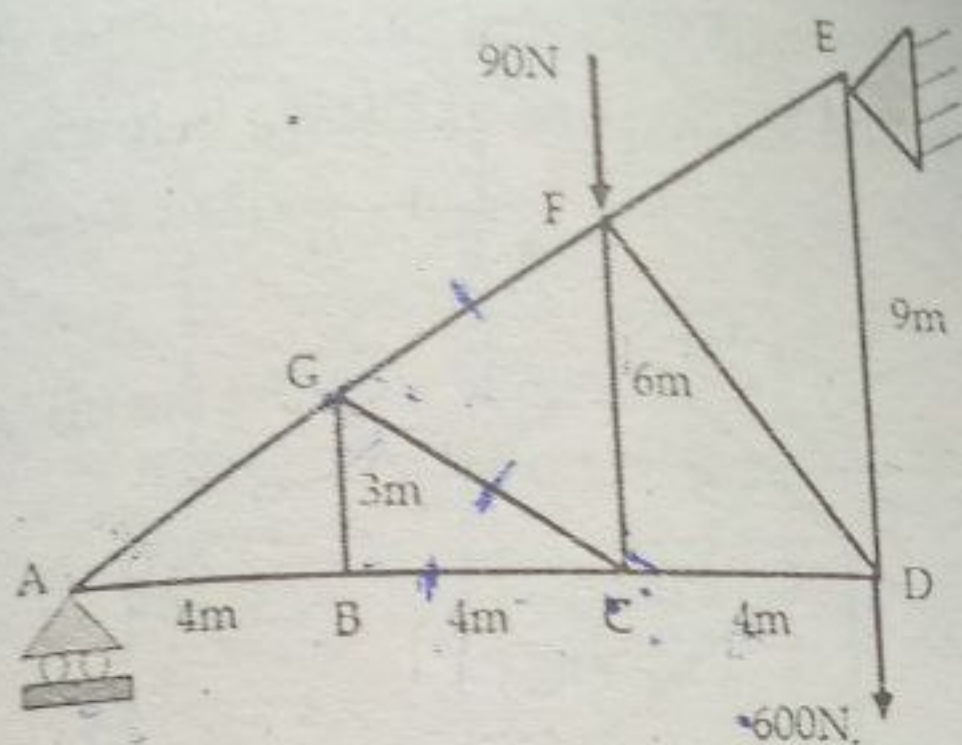
Using equi

$$i) \sum M_E =$$

$$ii) \sum F_x =$$

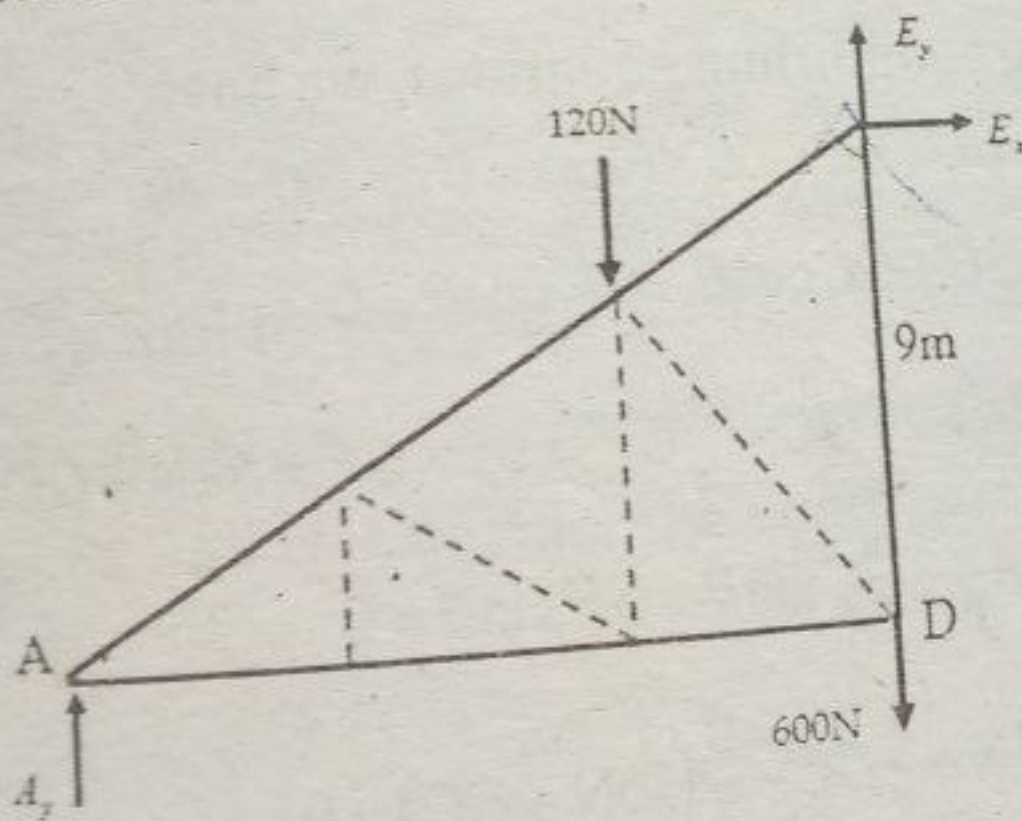
$$iii) \sum F_y =$$

For the truss structure below, determine the support forces acting on the structure and the force in members GF, GC, BC.



Solution:

Step-1: Determination of support reactions (Using the whole truss)
The FBD for the whole truss is as shown below.



Using equilibrium equations, we have

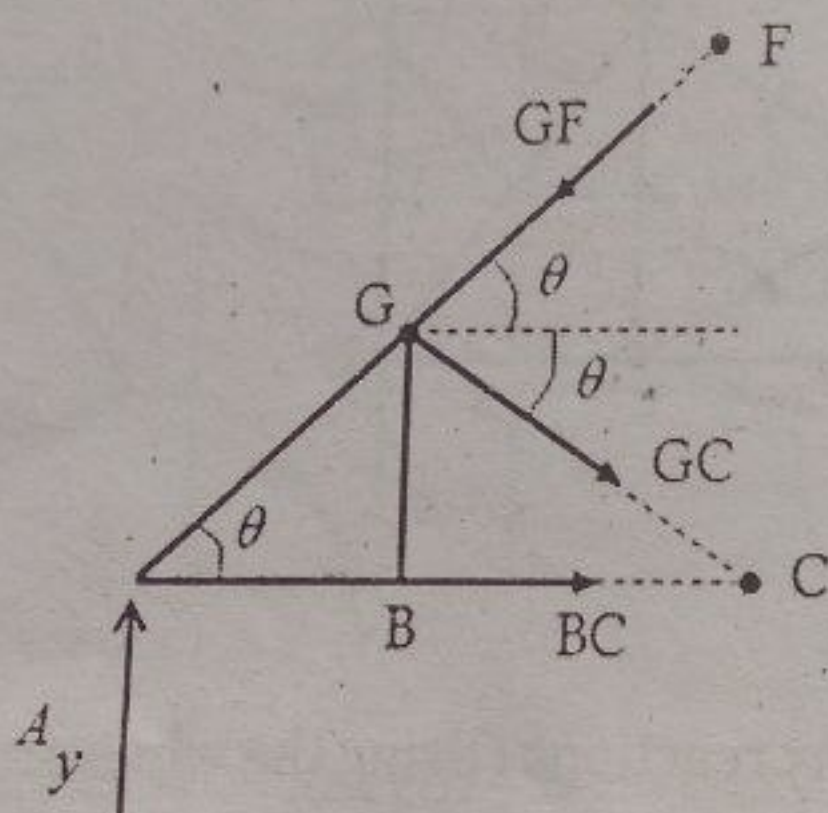
$$i) \sum M_E = 0 \Rightarrow 90N \cdot (4) - A_y \cdot (12) = 0 \Rightarrow 12A_y = 360N \Rightarrow A_y = 30N$$

$$ii) \sum F_x = 0 \Rightarrow E_x = 0$$

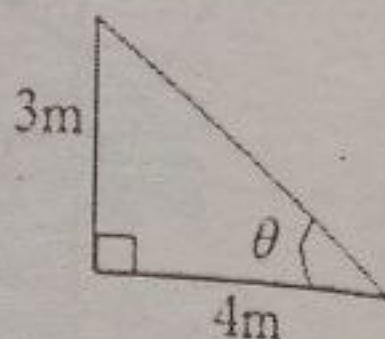
$$iii) \sum F_y = 0 \Rightarrow A_y + E_y - 90N - 600N = 0 \Rightarrow E_y = 660N$$

Step-2: Determination of member forces

To find internal forces in the members GF, GC, and BC by method of sections, first cut the truss vertically with a section line passing through the members GF, GC, and BC. Consider the section of the truss to the left of the section line as shown below.



Use this triangle to get θ



Then, applying equilibrium equations, we have

$$\text{i) } \sum M_G = 0 \Rightarrow BC.(3\text{m}) - 30\text{N}(4\text{m}) = 0 \Rightarrow BC = 40$$

$$\text{ii) } \sum F_x = 0 \Rightarrow BC + GC \cos \theta - GF \cos \theta = 0$$

$$\Rightarrow 40\text{N} + \frac{4}{5}GC - \frac{4}{5}GF = 0 \Rightarrow GF = 50\text{N} + GC$$

$$\text{iii) } \sum F_y = 0 \Rightarrow A_y - GC \sin \theta - GF \sin \theta = 0$$

$$\Rightarrow 30\text{N} - \frac{3}{5}GC - \frac{3}{5}(50\text{N} + GC) = 0$$

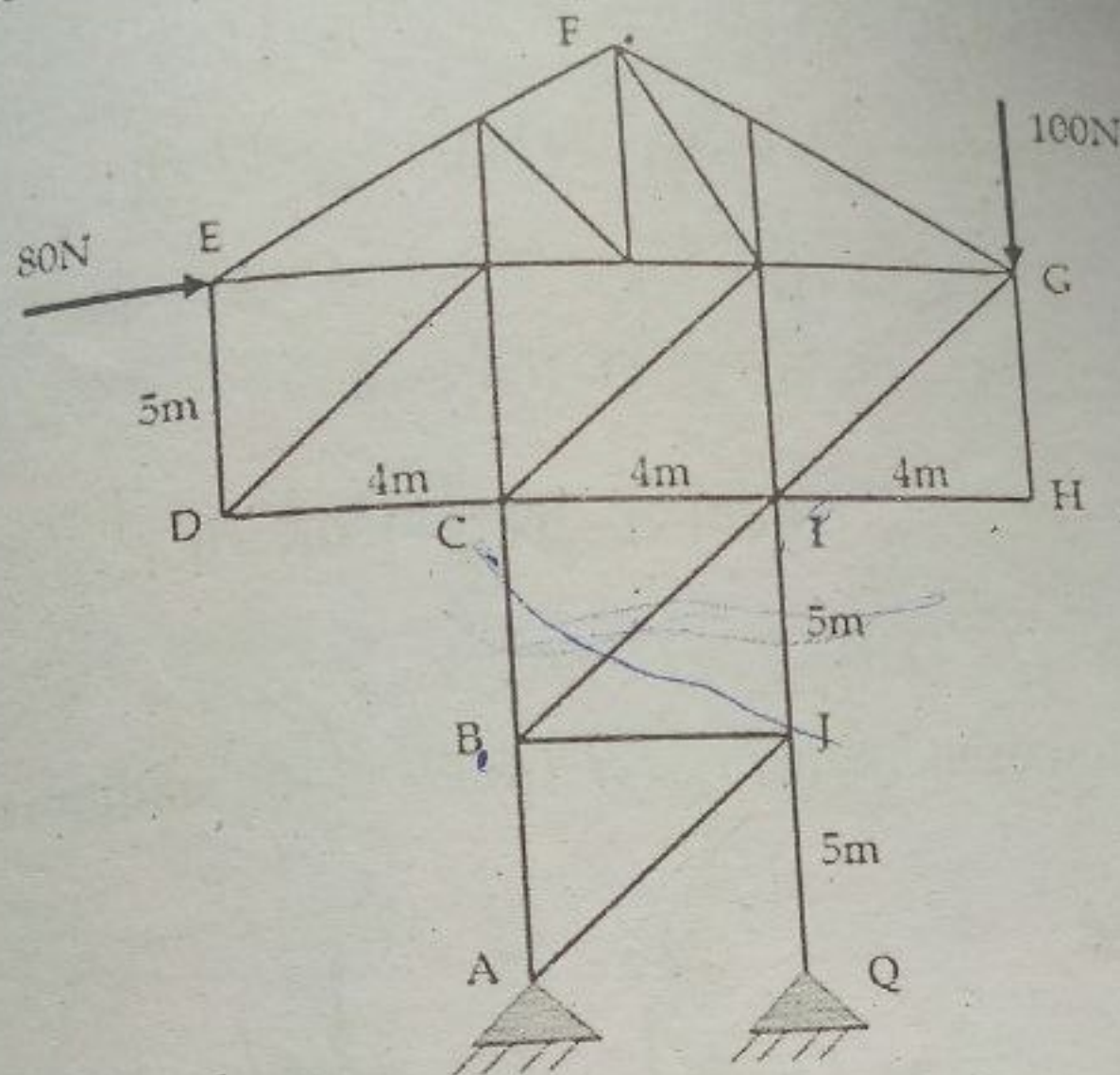
$$\Rightarrow 30\text{N} - \frac{3}{5}GC - 30\text{N} - \frac{3}{5}GC = 0$$

$$\Rightarrow -\frac{6}{5}GC = 0 \Rightarrow GC = 0$$

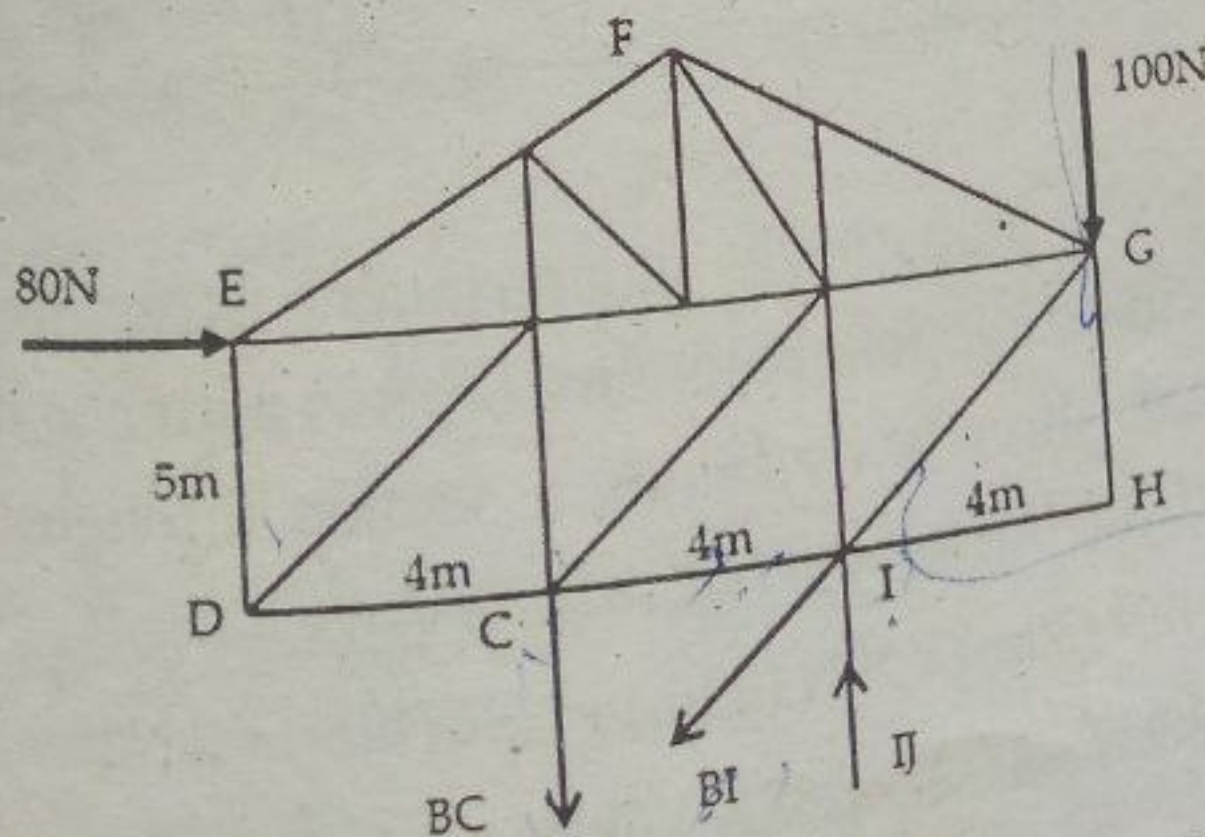
$$\text{But } GF = 50\text{N} + GC \Rightarrow GF = 50\text{N}$$

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7. For the truss structure below, determine the support forces and the force in members BC , BI and IJ .



Solution: Such problems can never be solved by method of joints. The basic advantage of method of sections is to solve such type of problems for structures whose support reactions are statically indeterminate. To find forces in the members BC , BI and IJ method of sections, first cut the truss horizontally with a section line passing through the BC , BI and IJ . Consider the upper section of the truss.



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Then, using equilibrium for the new system, we have the following.

$$i) \sum M_i = 0 \Rightarrow BC.(4m) - 80N.(5m) - 100N.(4m) = 0$$

$$\Rightarrow 4BC - 800N = 0 \Rightarrow BC = 200N$$

$$ii) \sum F_x = 0 \Rightarrow 80N - BI \cos \beta = 0 \Rightarrow 80N - \frac{4}{\sqrt{41}} BI = 0$$

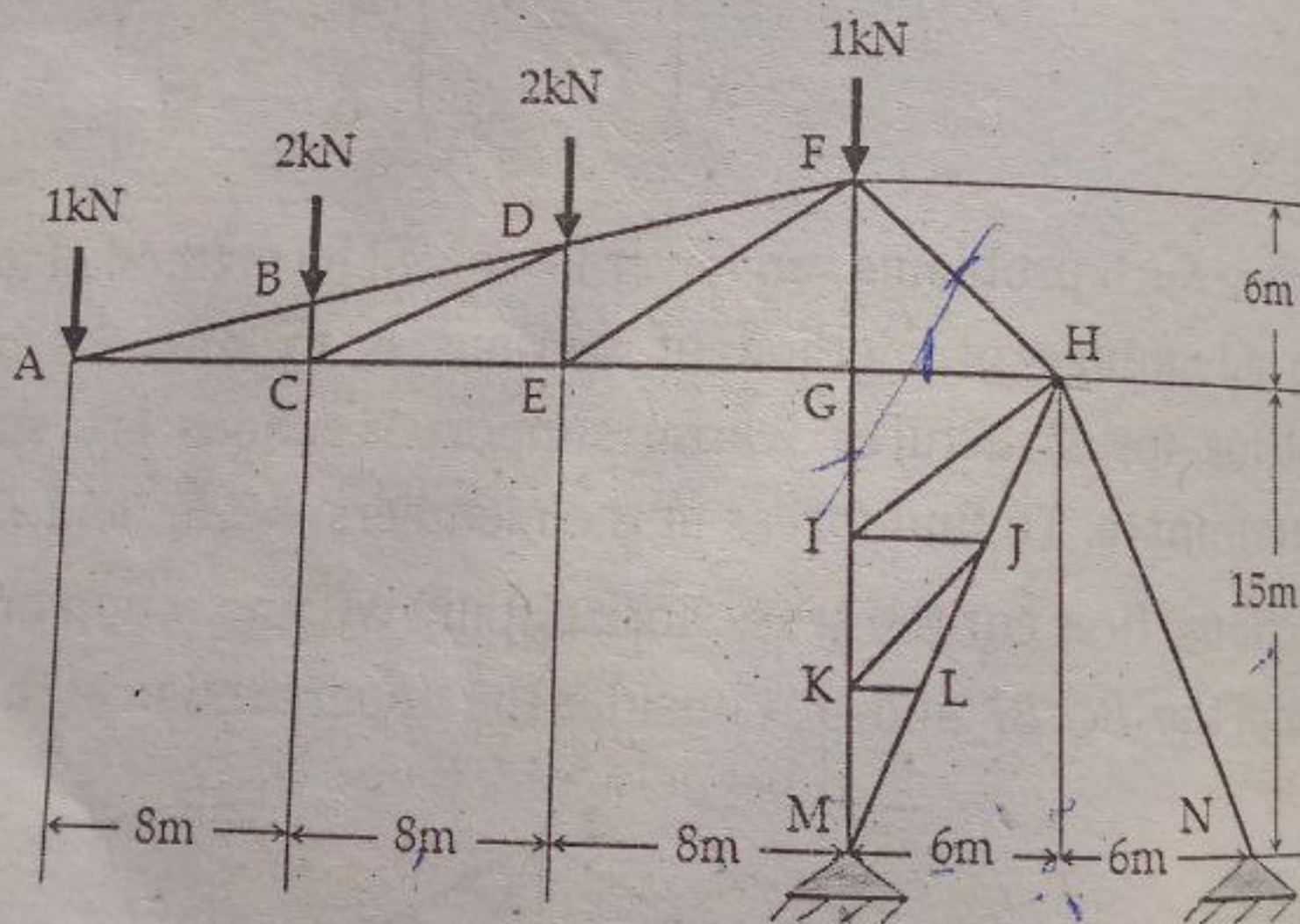
$$\Rightarrow BI = 20\sqrt{41}N = 128N$$

$$iii) \sum F_y = 0 \Rightarrow IJ - BI \sin \beta - BC - 100N = 0$$

$$\Rightarrow IJ - \frac{5}{\sqrt{41}} (20\sqrt{41}N) - 200N - 100N = 0$$

$$\Rightarrow IJ - 400N = 0 \Rightarrow IJ = 400N$$

8. Find the forces in members FH, GH, GI of the stadium truss shown.



Solution: This is statistically indeterminate structure because we cannot determine support reactions. So, it is better to determine the member forces directly by taking a section through the members FH, GH, GI . Using the left part of *section-I*, we have

$$\sum M_H = 0 \Rightarrow 1kN.(30m) + 2kN.(22m) + 2kN.(14m) + 1kN.(6m) - GI.(6m) = 0$$

$$\Rightarrow 30kN + 44kN + 28kN + 6kN - 6GI = 0 \Rightarrow GI = 18kN$$



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Again, using equilibrium equations for the forces, we have

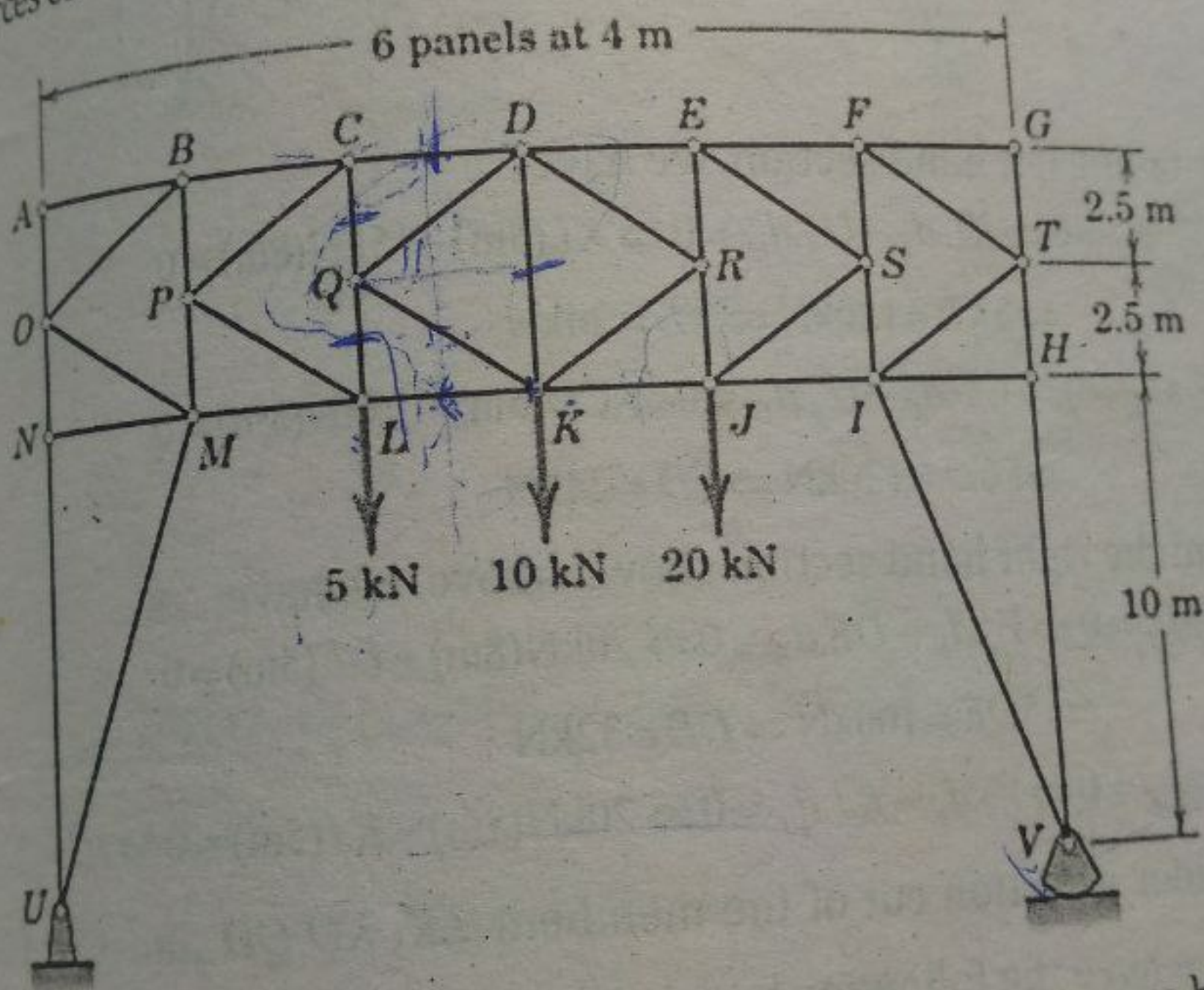
$$\sum F_y = 0 \Rightarrow GJ - FH_y - 1kN - 2kN - 2kN - 1kN = 0$$

$$\Rightarrow 18kN - FH \sin 45^\circ - 6kN = 0$$

$$\Rightarrow FH \sin 45^\circ = 12kN \Rightarrow FH = 12\sqrt{2}kN$$

$$\sum F_x = 0 \Rightarrow FH_x - GH = 0 \Rightarrow GH = FH \cos 45^\circ = 12kN$$

20. Using the methods of sections, on the truss structure below, determine the forces on the members LQ , QC , CD and DK .



Solution: Using equilibrium equations for the whole truss, we have

$$\sum M_A = 0 \Rightarrow V_y \cdot (24m) - 5kN(8m) - 10kN(12m) - 20kN(16m) = 0$$

$$\Rightarrow 24V_y - 480kN = 0 \Rightarrow V_y = 20kN$$

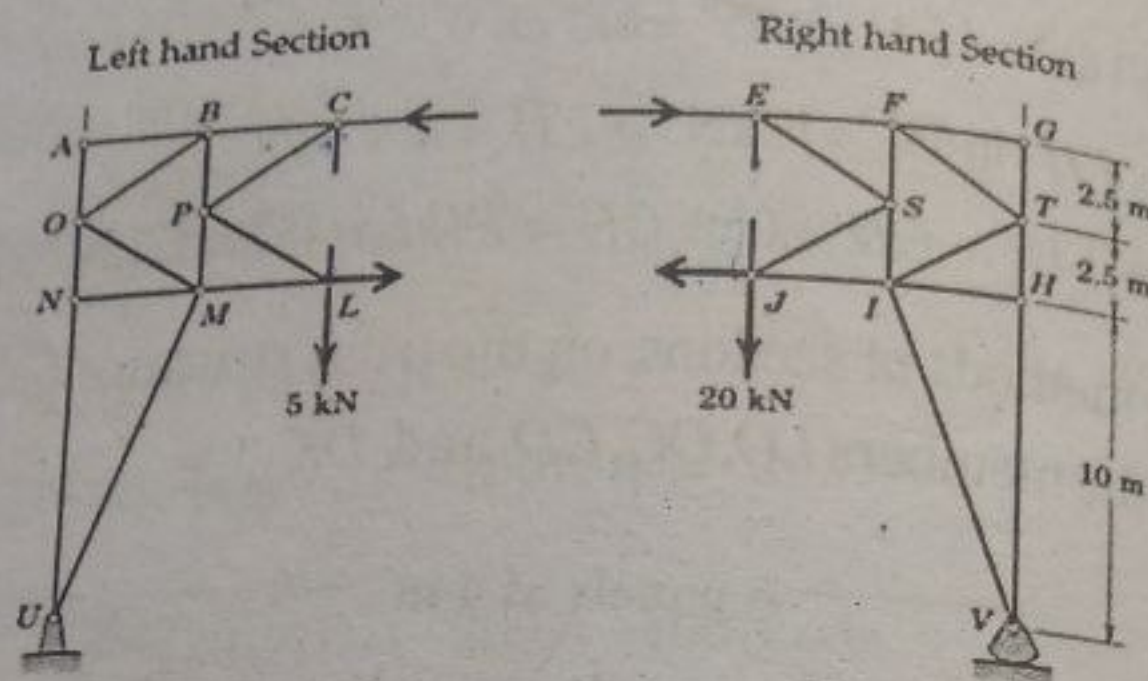
Again, using equilibrium equations for the forces, we have

$$i) \sum F_x = 0 \Rightarrow U_x = 0$$

$$ii) \sum F_y = 0 \Rightarrow U_y + V_y - 5kN - 10kN - 20kN = 0 \Rightarrow U_y = 15kN$$

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Consider a zigzag section cut of the members KL, LQ, QC and CD. For this cut, we have the following free-body diagram.



By taking left hand section, we have

$$i) \sum M_C = 0 \Rightarrow KL \cdot d_L - U_y \cdot d_U = 0 \Rightarrow KL(5m) - 15kN(8m) = 0$$

$$\Rightarrow 5KL = 120kN \Rightarrow KL = 24kN$$

$$i) \sum M_L = 0 \Rightarrow CD \cdot d_C - U_y \cdot d_U = 0 \Rightarrow CD(5m) - 15kN(8m) = 0$$

$$\Rightarrow 5CD = 120kN \Rightarrow CD = 24kN$$

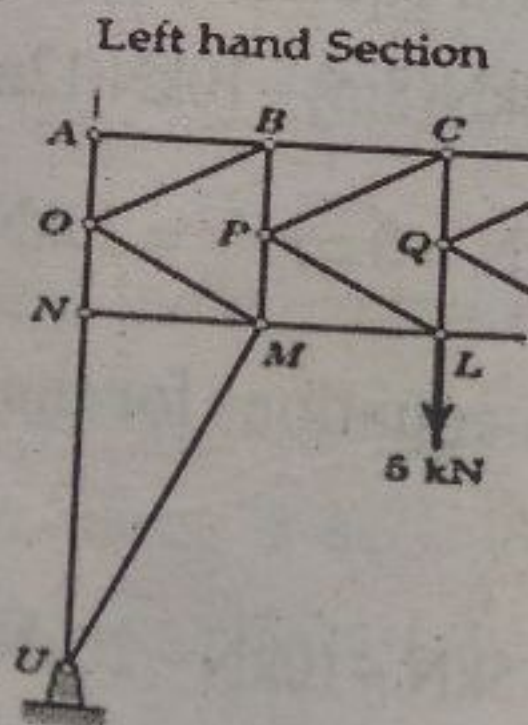
From the right hand section shown above, we have

$$i) \sum M_J = 0 \Rightarrow V_y \cdot d_V - DE \cdot d_E = 0 \Rightarrow 20kN(8m) - DE(5m) = 0$$

$$\Rightarrow 5DE = 160kN \Rightarrow DE = 32kN$$

$$ii) \sum M_E = 0 \Rightarrow V_y \cdot d_V - KJ \cdot d_J = 0 \Rightarrow 20kN(8m) - KJ(5m) = 0 \Rightarrow KJ = 32kN$$

Consider a section cut of the members LK, KQ, QD and CD. For this cut, we have the following free-body diagram.



From the right hand section shown above, we have

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$$i) \sum M_D = 0 \Rightarrow V_y \cdot d_V - KQ \cdot d_Q = 0$$

$$\Rightarrow \frac{20}{4} = \dots$$

Again, consider
For this cut, we

$$i) \sum F_x = 0 \Rightarrow \dots$$

$$ii) \sum F_y = 0 \Rightarrow \dots$$

Note: In some cases, a section cut can be made through at least one member.

11. Consider the following truss structure.

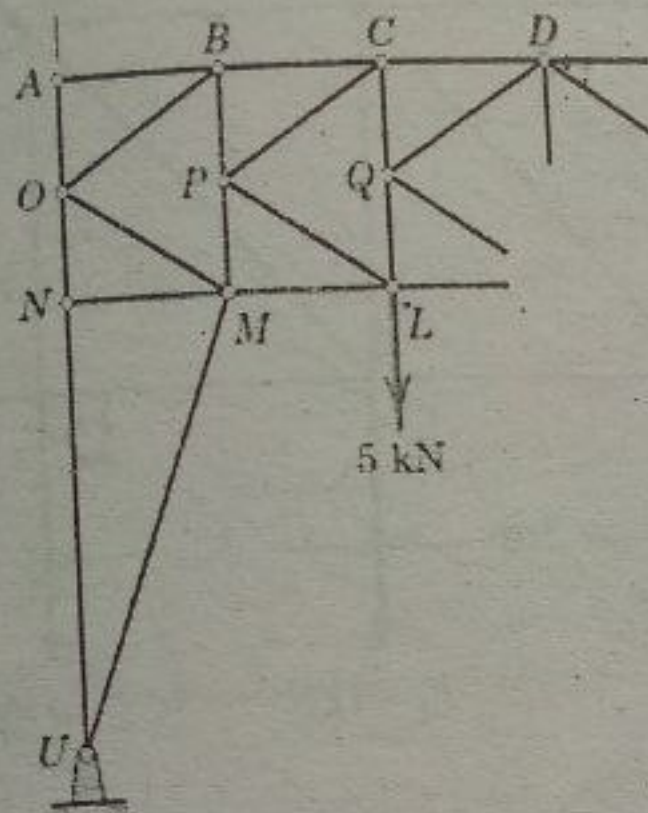
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$$\sum M_D = 0 \Rightarrow V_y \cdot d_y - KQ_x \cdot d_D - KL \cdot (5m) - 20kN(4m) = 0$$

$$\Rightarrow KQ \cos \theta \cdot (5m) = (20kN(12m) - 24kN(5m) - 20kN(4m))$$

$$\Rightarrow \frac{20}{4.7} KQ = 40kN \Rightarrow KQ = 9.4kN$$

Again, consider a section cut of the members LK, KQ, DK and DR .
For this cut, we have the following free-body diagram.



$$\sum F_x = 0 \Rightarrow KL + KQ \cos \theta - DR \cos \theta - DE = 0$$

$$\Rightarrow 24kN + \frac{4}{4.7} (9.4kN) - \frac{4}{4.7} DR - 32kN = 0$$

$$\Rightarrow DR = 0$$

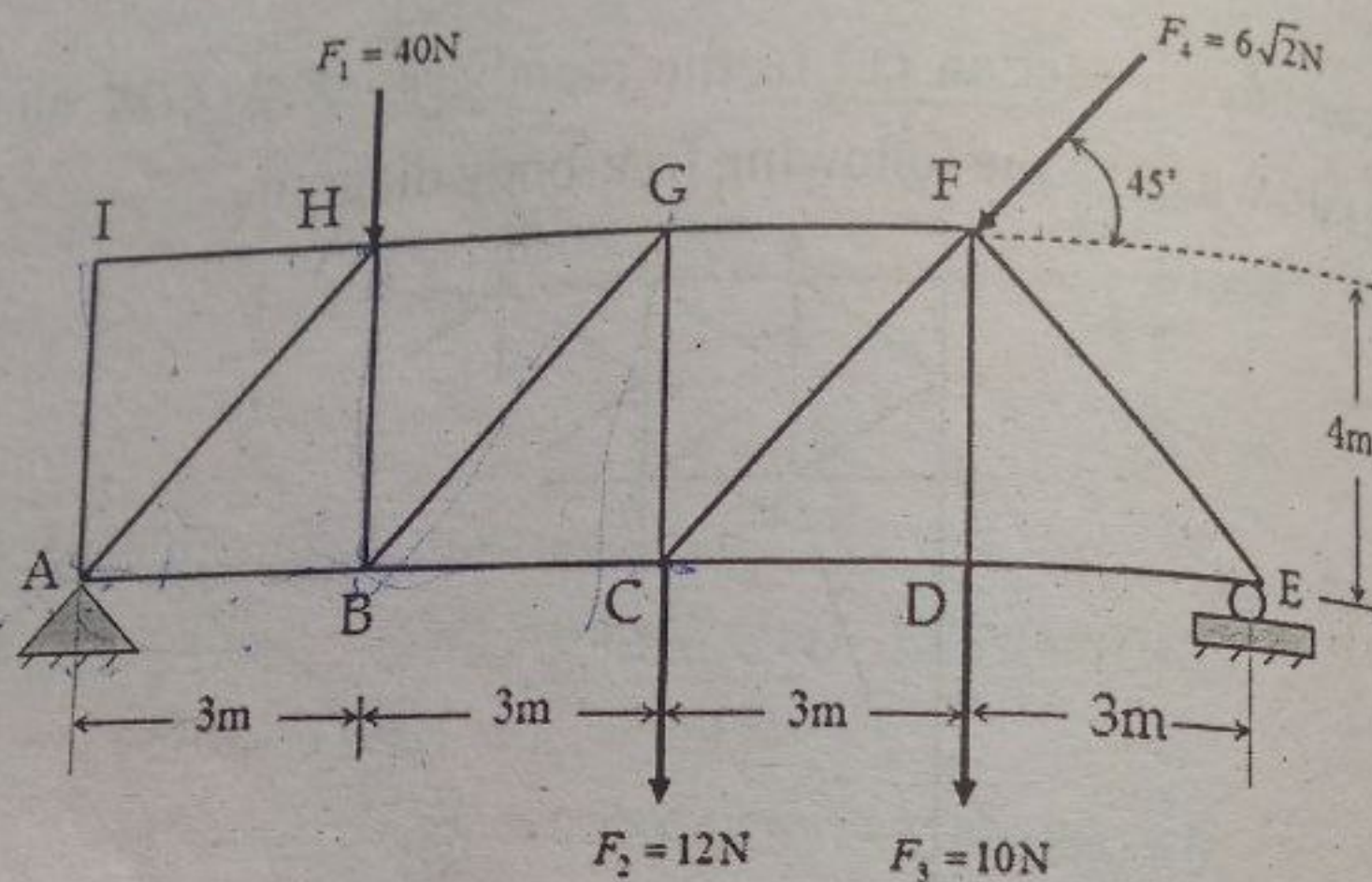
$$\sum F_y = 0 \Rightarrow U_y - DK - KQ \sin \theta - 5kN = 0$$

$$\Rightarrow 15kN - DK - \frac{2.5}{4.7} (9.4kN) - 5kN = 0$$

$$\Rightarrow 15kN - DK - 5kN - 5kN = 0 \Rightarrow DK = 5kN$$

Note: In some cases both the methods of sections and joints can be combined for an efficient solution. This occurs when it is not possible to pass a section through a member without passing through at least four unknown members.

- Identify the zero-force members
- Using both the methods, determine the forces in all members and state whether the members are in tension or compression.



Solution:

First determine support reactions:

$$\begin{aligned}\sum M_A = 0 &\Rightarrow 12E_y - F_1 d_1 - F_2 d_2 - F_3 d_3 + F_{4x} d_{4x} - F_{4y} d_{4y} = 0 \\ &\Rightarrow 12E_y - 40(3) - 12(6) - 10(9) + 6\sqrt{2} \cos 45^\circ (4) - 6\sqrt{2} \sin 45^\circ (9) = 0 \\ &\Rightarrow 12E_y - 120\text{N} - 72\text{N} - 90\text{N} + 24\text{N} - 54\text{N} = 0 \\ &\Rightarrow 12E_y - 360\text{N} = 0 \Rightarrow 12E_y = 360\text{N} \Rightarrow E_y = 30\text{N}\end{aligned}$$

Again, from force equilibrium, we have

$$i) \sum F_x = 0 \Rightarrow A_x - F_{4x} = 0 \Rightarrow A_x - F_4 \cos 45^\circ = 0$$

$$\Rightarrow A_x = F_4 \cos 45^\circ = \frac{\sqrt{2}}{2} (6\sqrt{2}\text{N}) = 6\text{N}$$

$$ii) \sum F_y = 0 \Rightarrow A_y + E_y - F_1 - F_2 - F_3 - F_{4y} = 0$$

$$\Rightarrow A_y + 30\text{N} - 40\text{N} - 12\text{N} - 10\text{N} - 6\text{N} = 0$$

$$\Rightarrow A_y + 30\text{N} - 68\text{N} = 0 \Rightarrow A_y = 38\text{N}$$

- First, identify zero-force members.

My the Method of inspection of joints, we get that the members AI, IH, BH, CG are zero force members.

b) Now, let's determine the member forces.

1) At joint A:

$$i) \sum F_y = 0 \Rightarrow A_y - AH \sin \theta = 0 \Rightarrow AH = 52.5N$$

$$ii) \sum F_x = 0 \Rightarrow A_x + AB + AH \cos \theta = 0 \Rightarrow AB = 25.5N$$

2) At joint B:

i) Since HB is a zero member $\sum F_y = 0$.

$$ii) \sum F_x = 0 \Rightarrow AB - BC = 0 \Rightarrow BC = AB = 25.5N$$

3) At joint H:

$$i) \sum F_y = 0 \Rightarrow AH \sin \theta - F_1 - HC \sin \theta = 0$$

$$\Rightarrow \frac{4}{5} HC = \frac{4}{5} AH - F_1 \Rightarrow \frac{4}{5} HC = 2N \Rightarrow HC = 2.5N$$

$$ii) \sum F_x = 0 \Rightarrow HC \cos \theta + AH \cos \theta - HG = 0$$

$$\Rightarrow \frac{3}{5}(2.5N) + \frac{3}{5}(52.5N) - HG = 0 \Rightarrow HG = 33N$$

4) At joint C:

$$i) \sum F_y = 0 \Rightarrow HC \sin \theta - CF \sin \theta - F_2 = 0 \Rightarrow \frac{4}{5} CF = F_2 - \frac{4}{5} HC$$

$$\Rightarrow \frac{4}{5} CF = 12N - \frac{4}{5}(2.5N)$$

$$\Rightarrow \frac{4}{5} CF = 12N - 2N \Rightarrow CF = 12.5N$$

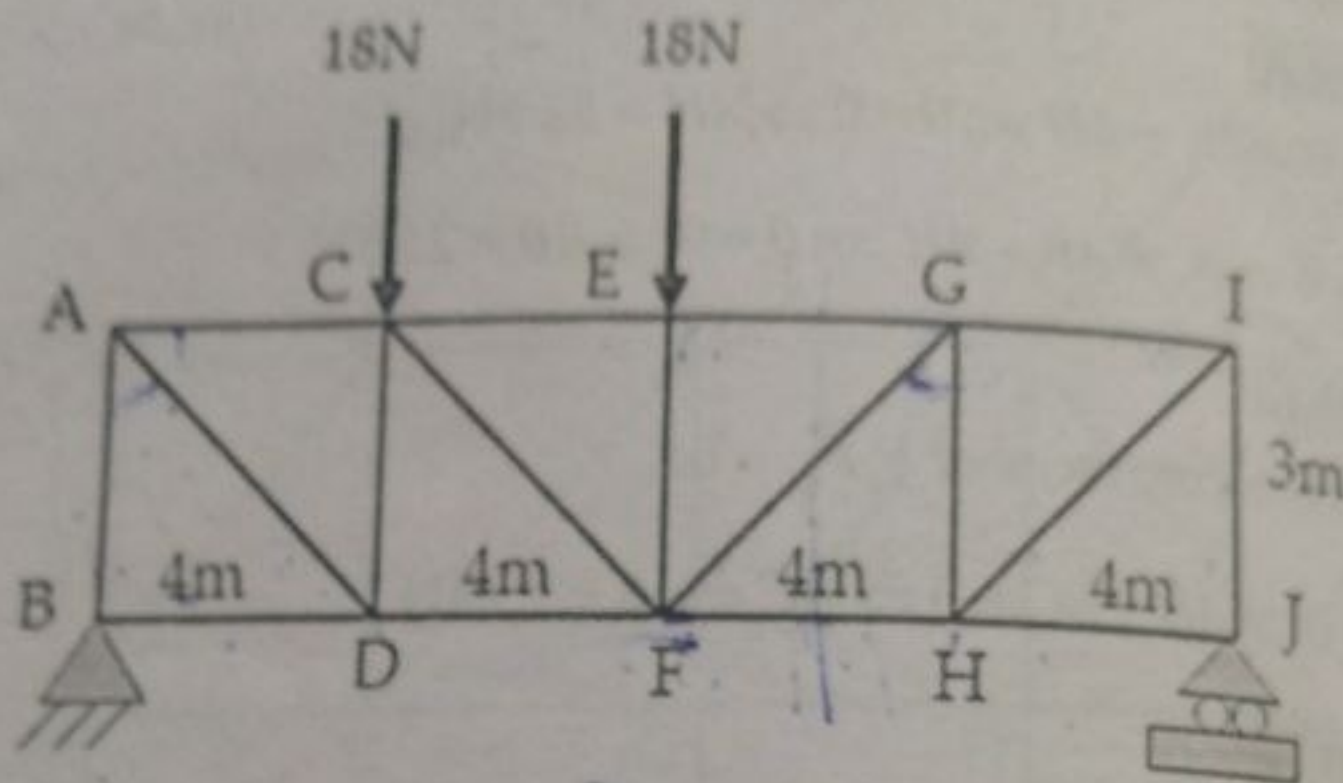
$$ii) \sum F_x = 0 \Rightarrow CD - BC - HC \cos \theta + CF \cos \theta = 0$$

$$\Rightarrow CD = 25.5N + \frac{3}{5}(2.5N) - \frac{3}{5}(12.5N) = 0 \Rightarrow CD = 19.5N$$

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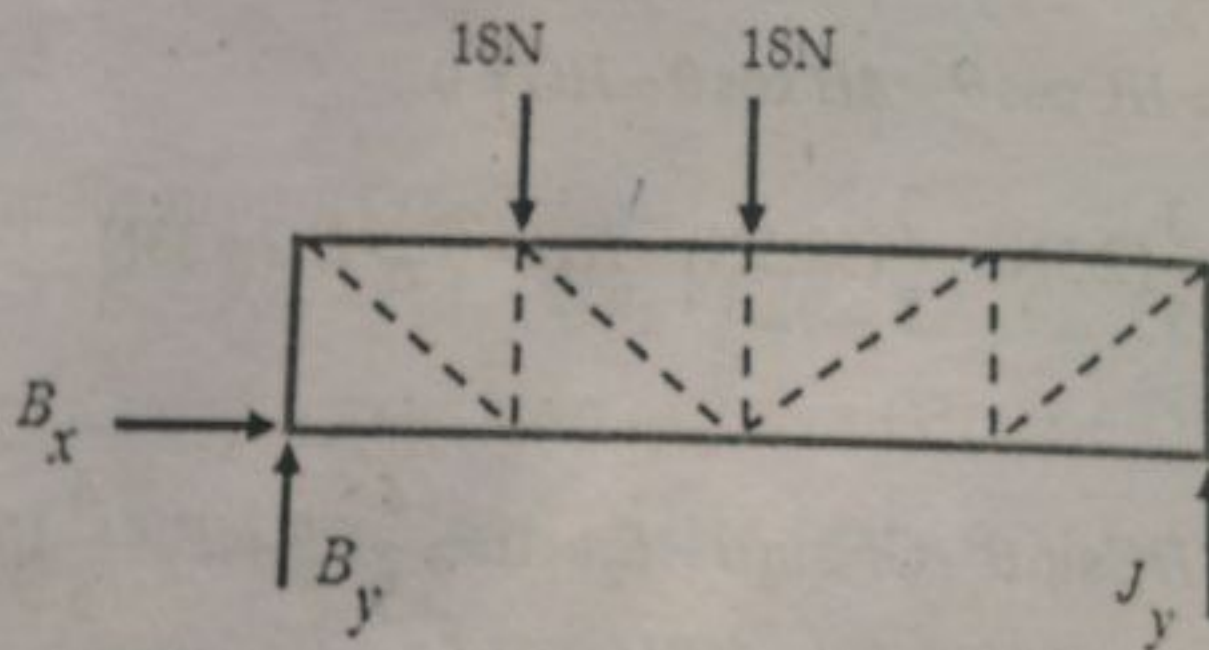
a) Using the Method of Joints, find the forces in members AB, AD, AC

b) By the Method of Sections, find the forces in members EG, FG, FH



Solution: Such problems are easily solved by breaking into two parts.

Step-1: Determination of support reactions (Using the whole truss)



Using equilibrium equations, we have

$$i) \sum M_B = 0 \Rightarrow J_y \cdot (16m) - 18N(4m) - 18N(8m) = 0$$

$$\Rightarrow 16J_y - 216N = 0 \Rightarrow J_y = 13.5N$$

$$ii) \sum F_x = 0 \Rightarrow B_x = 0$$

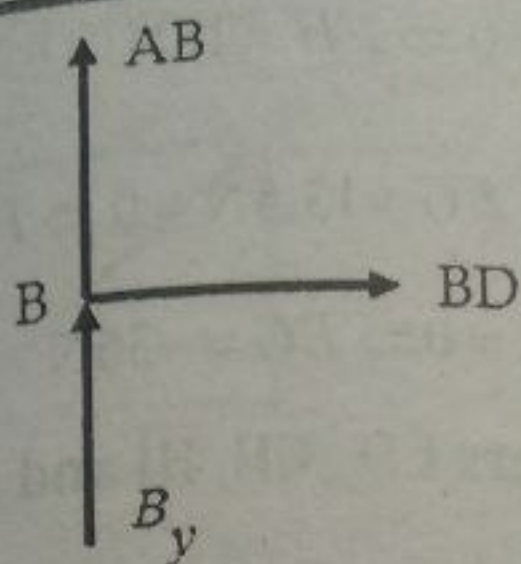
$$iii) \sum F_y = 0 \Rightarrow B_y + J_y - 18N - 18N = 0$$

$$\Rightarrow B_y + 13.5N - 36N = 0 \Rightarrow B_y = 22.5N$$

Step-2: Determination of member forces (Using selected joints)

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AD, AC
EG, FH

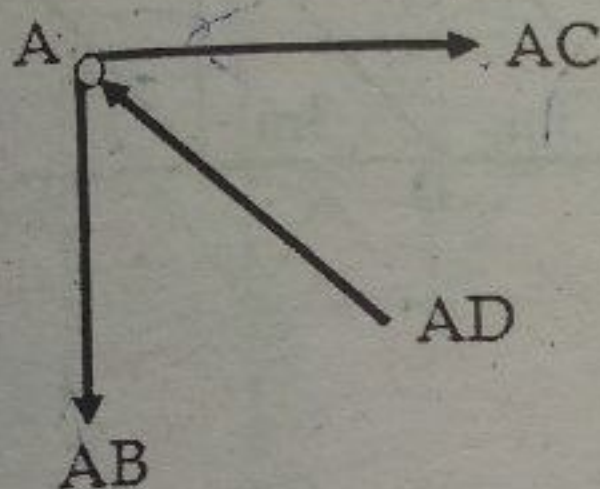
At Joint B:



$$\sum F_x = 0 \Rightarrow BD = 0$$

$$\sum F_y = 0 \Rightarrow AB + B_y = 0 \Rightarrow AB = -B_y = -22.5\text{N}$$

At Joint A:



$$\sum F_y = 0 \Rightarrow AD \cos \theta - AB = 0 \Rightarrow AD = \frac{5}{3}(22.5\text{N}) = 37.5\text{N}$$

$$\sum F_x = 0 \Rightarrow AC - AD \sin \theta = 0 \Rightarrow AC = \frac{4}{5}(37.5\text{N}) = 30\text{N}$$

b) Take a section through EG, FG, FH and apply equilibrium equations on the section to the right.

