

Addis Ababa Science and Technology University
College of Natural and Social Sciences, Department of Mathematics
Mathematics for Natural Science (Math1007)
Worksheet I

1. State the following terms (Phrases).

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| (a) Proposition | (d) Set and Empty(Null) set |
| (b) Open proposition | (e) Subset and Power set of a set |
| (c) Argument form and validity of an argument | (f) Disjoint sets and complement of a set |

2. Which of the following are propositions? Determine the truth value whenever possible.

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| (a) Thank you for your cooperation. | (e) It is amazing! |
| (b) $6 < 5$ or there are infinitely many prime numbers. | (f) $x^2 = y^2$ whenever $x = y$. |
| (c) $x^2 + y^2 = 1$ for all $x, y \in \mathbb{R}$. | (g) The function is differentiable. |
| (d) The first human being lived in Africa. | (h) This proposition is false. |

3. Determine the truth value of the proposition p if

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|---|---|
| (a) $(q \Rightarrow \neg p) \vee r$ is false. | (c) $(p \Rightarrow p) \Rightarrow (r \Rightarrow q)$ is false. |
| (b) $\neg q$ and $p \Rightarrow \neg q$ are true. | (d) $(p \vee \neg q) \Rightarrow (\neg r \vee p)$ is false. |

4. Construct a truth table and determine whether they are tautology, contradiction or contingency.

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| (a) $p \Rightarrow (p \vee q)$. | (d) $((p \Rightarrow q) \wedge (q \Rightarrow r)) \Rightarrow (p \Rightarrow r)$. |
| (b) $[p \wedge (p \Rightarrow q)] \Rightarrow q$. | (e) $(p \Rightarrow q) \Leftrightarrow (\neg p \vee q)$. |
| (c) $(p \Rightarrow q) \wedge \neg(\neg p \Rightarrow \neg p)$. | (f) $(p \wedge q) \Leftrightarrow (p \Rightarrow \neg q)$. |

5. Express the following using "if..., then..."

- (a) The polynomial has at most two roots if it is a quadratic.
- (b) The data is widely spread only if the standard deviation is large.
- (c) The function being constant implies that its derivative is null.
- (d) The system of equations is consistent is necessary for it to have a solution.
- (e) It never rains when it is cold.
- (f) Whenever it rains it is cold.

6. Write the truth value for each proposition.

- (a) A system of equations always has a solution, or a quadratic equation always has a real solution.
- (b) Every integer is even, or every even natural number is an integer.
- (c) Any real number is negative or positive, but not both.

7. Write the converse, inverse and contra-positive of the following statements.

- (a) If you study hard, then you will pass the exam.
- (b) x is an even integer whenever x^2 is an even integer.

8. Using quantifiers write each of the following statements in symbolic form.

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| (a) For every real number $x < 0$, $\sqrt{x^2} \geq 0$. | (f) Someone loved by everyone. |
| (b) There exists some real number $x < 0$ such that $\sqrt{x^2} \geq 0$. | (g) Someone loves someone. |
| (c) The square of any real number x is positive. | (h) Everything is identical to something. |
| (d) For each rational x , the number $\frac{1}{x}$ is rational. | (i) There is something such that everything is identical to that thing. |
| (e) Everyone loves someone. | |

9. If $\neg[(\neg r) \Rightarrow (p \wedge q)]$ is true, then find the truth value of $[(p \Leftrightarrow r) \vee q] \Leftrightarrow (\neg p \wedge r)$.

10. Find the truth values and the negation of the following quantified statements.

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| (a) $(\forall x \in \mathbb{R})(x^2 + x \geq 0)$. | (g) $(\exists y \in \mathbb{N})(\forall x \in \mathbb{N})(x + y = 6)$. |
| (b) $(\forall x \in \mathbb{R})(x + 2x^2 = x^2 + 4x + 4)$. | (h) $(\forall x \in \mathbb{R})(x < 2 \Rightarrow x^2 < 4)$. |
| (c) $(\exists x \in \mathbb{Z})(\sqrt{x} - 1 = 0)$. | (i) $(\exists y \in \mathbb{R})(y < 5 \wedge y > 5)$. |
| (d) $(\forall x \in \mathbb{R})(\sqrt{x^2} = x)$. | (j) $(\exists x \in \mathbb{R})(\forall y \in \mathbb{R})(x^2 \leq y^2)$. |
| (e) $(\exists x \in \mathbb{R})(\sqrt{x^2} = x)$. | (k) $(\forall x \in \mathbb{R})(\exists y \in \mathbb{R})(x^2 < y^2)$. |
| (f) $(\forall x \in \mathbb{N})(\exists y \in \mathbb{N})(x + y = 6)$. | (l) $(\forall x \in \mathbb{R})(\exists y \in \mathbb{R})(x^2 - y^2 = (x - y)(x + y))$. |

11. Consider $P(x) : x$ is an integer greater than 7.

$Q(x) : x$ is a natural number. Then, determine the truth value of the following compound propositions.

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| (a) $P(3) \wedge Q(-1)$ | (e) $P(8) \Leftrightarrow Q(-1)$ |
| (b) $P(8) \vee Q(0)$ | (f) $[P(1) \Rightarrow Q(2)] \vee Q(8)$ |
| (c) $P(9) \wedge Q(2)$ | (g) $[P(6) \vee Q(7)] \wedge P(0)$ |
| (d) $P(1) \Rightarrow Q(1)$ | (h) $(\forall x)(p(x) \Rightarrow Q(x))$ |

12. Let $P(x) : x$ is a prime number

$Q(x) : x$ is an even number

$R(x) : x$ is an odd number

$S(x) : x$ is an integer number

Write a sentence, which corresponds to each of the following.

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| (a) $S(x) \Rightarrow Q(x) \vee R(x)$ | (c) $P(x) \Leftrightarrow Q(x) \vee R(x)$ |
| (b) $R(x) \vee Q(x) \Rightarrow P(x)$ | (d) $P(x) \wedge Q(x)$ |

13. Determine whether each of the following argument is valid or not.

- (a) $\neg q \Rightarrow \neg p, r \Rightarrow p, \neg q \vdash r$.
- (b) $\neg r \vee \neg s, (\neg s \Rightarrow p) \Rightarrow r \vdash p \Rightarrow q$.
- (c) If the rain does not come, then the crop is ruined and the people will starve.
The crop is not ruined or the people will not starve.
Therefore, the rain comes.

14. Let $U = \{1, 2, 3, 4, 5\}$, $A = \{1, 2, 3\}$, $B = \{2, 5\}$. Then find

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| (a) A' | (d) $A' \cap B'$ | (g) $A \Delta B$ |
| (b) $(A \cup B)'$ | (e) $A' \cup B'$ | (h) $(B')'$ |
| (c) $(A \cap B)'$ | (f) $A - B$ | |

15. Let the set $A = \{x : x \text{ is an English alphabet letters in the word "AASTU"}\}$. Find the power set of A .