

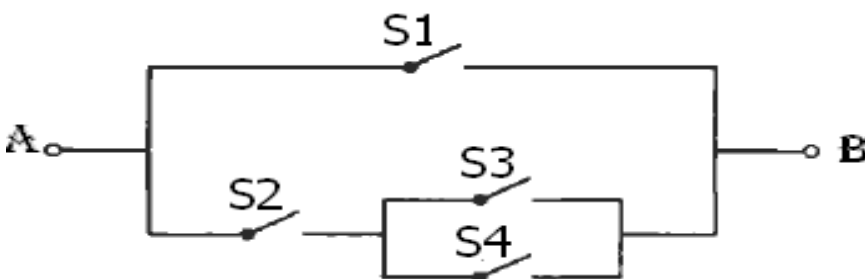
ADAMA SCIENCE & TECHNOLOGY UNIVERSITY
SCHOOL OF ELECTRICAL ENGINEERING & COMPUTING
DEPARTMENT OF ELECTRONICS & COMMUNICATION ENGINEERING

Probability and Random Processes (ECE3103)

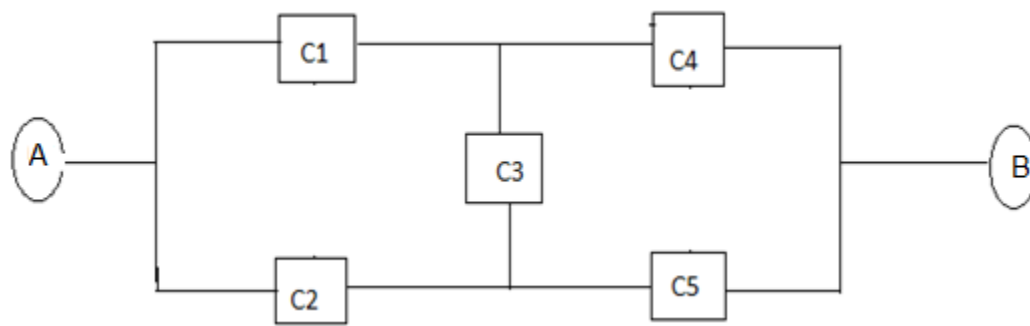
Assignment #1 {1, 3, 4, 5, 8, 12, 18}

Be in group (five per group)

1. A cell-phone tower has a circular coverage area of radius 10 km. We observe the source locations of calls received by the tower.
 - a) Suggest a sample space to model all possible source locations of calls that the tower can receive.
 - b) Using your sample space from part (a), what is the event that the source location of a call is between 2 and 5 km from the tower?
2. A lot of 100 semiconductor chips contain 20 that are defective. Two chips are selected at random (one @ a time) without replacement, from the lot.
 - a) What is the probability that the first one selected is defective?
 - b) What is the probability that the second one selected is defective given that the first one was defective?
 - c) What is the probability that both are defective?
 - d) The probability @ least one is defective
3. Suppose that a person chooses a letter at random from RESERVE and then chooses one at random from VERTICAL. What is the probability the same letter is chosen?
4. Consider the switching network shown in Fig. 1. It is equally likely that a switch will or will not work. Find the probability that a closed path will exist between terminals A and B. The switches fail independently.



a



b.

Fig . 1

5. A company producing electric relays has three manufacturing plants producing 50, 30, and 20 percent, respectively, of its product. Suppose that the probabilities that a relay manufactured by these plants is defective are 0.02, 0.05, and 0.01, respectively.
 - a) If a relay is selected at random from the output of the company, what is the probability that it is defective?
 - b) If a relay selected at random is found to be defective, what is the probability that it was manufactured by plant 2?
6. Four computer companies A, B, C, D supply microprocessors to a company. From previous experience it is known that the probability of the processor being bad if it comes from A is 4%, B is 2%, C is 5%, and D is 1%. The probability of picking supplier A is 20%, B is 30%, C is 10%, and D is 40%. Find the probability that a processor chosen at random is bad.
7. Consider the binary communication channel shown in Fig. 2. The channel input symbol X may assume the state 0 or the state 1, and, similarly, the channel output symbol Y may assume either the state 0 or the state 1. Because of the channel noise, an input 0 may convert to an output 1 and vice versa. The channel is characterized by the channel transition probabilities p_0 , q_0 , p_1 , and q_1 , defined by:

$$p_0 = P(y_1/x_0) \text{ and } p_1 = P(y_0/x_1)$$

$$q_0 = P(y_0/x_0) \text{ and } q_1 = P(y_1/x_1)$$

Where x_0 and x_1 denote the events $(X = 0)$ and $(X = 1)$, respectively, and y_0 and y_1 denote the events $(Y = 0)$ and $(Y = 1)$, respectively. Note that $p_0 + q_0 = 1 = p_1 + q_1$. Let

$$P(x_0) = 0.5, p_0 = 0.1, \text{ and } p_1 = 0.2.$$

- a) Find $P(y_0)$ and $P(y_1)$.
- b) If a 0 was observed at the output, what is the probability that a 0 was the input state?
- c) If a 1 was observed at the output, what is the probability that a 1 was the input state?
- d) Calculate the probability of error P_e .

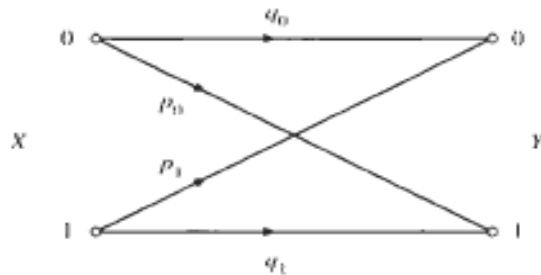


Fig-2

8. Consider the communication channel shown in Fig-3. The symbols transmitted are 0 and 1. However, three possible symbols can be received: 0, 1, and E. Thus, we define the input symbol set as $X = \{0, 1\}$ and the output symbol set as $Y = \{0, 1, E\}$. The transition (or conditional) probability $P[Y|X]$ is the probability that Y is received, given that X was transmitted. In particular, $P[0|0] = 0.8$, $P[1|0] = 0.1$ and $P[E|0] = 0.1$. Similarly, $P[0|1] = 0.2$, $P[1|1] = 0.7$ and $P[E|1] = 0.1$. If $P[X=0] = P[X=1] = 0.5$, determine the following:

- $P[Y=0]$, $P[Y=1]$ and $P[Y=E]$
- If 0 is received, what is the probability that 0 was transmitted?
- If E is received, what is the probability that 1 was transmitted?
- If 1 is received, what is the probability that 1 was transmitted?

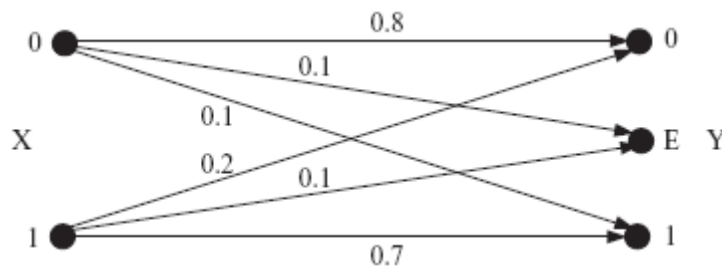


Figure -3

9. A communication receiver sends numbers 1, 2, 3. As a result of inherent noise in the channel, the conditional probabilities of receiving 1,2,3 under the condition that 1,2,3 were sent are shown in Fig.4. A priori probabilities are: $P(1_S) = 0.2$, $P(2_S) = 0.4$, $P(3_S) = 0.4$

- What is the probability that 2 was received at receiver?
- Find $P(1_S|2_R)$, $P(2_S|2_R)$ & $P(3_S|2_R)$.

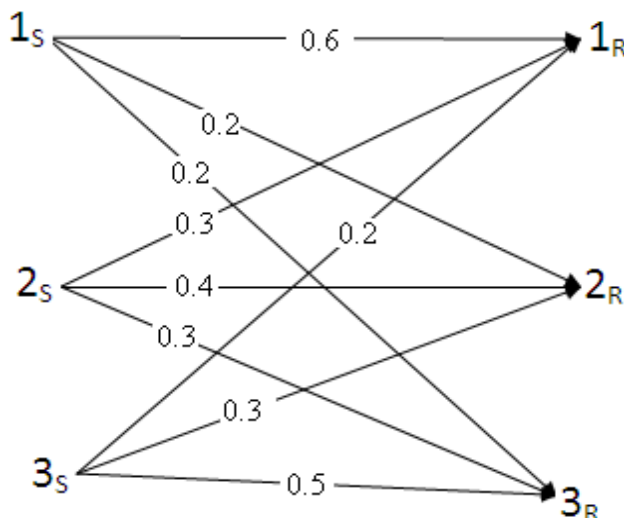


Figure -4

10. An information source generates symbols at random from a four-letter alphabet $\{a,b,c,d\}$ with probabilities $P(a) = \frac{1}{2}$, $P(b) = \frac{1}{4}$, and $P(c) = P(d) = \frac{1}{8}$ a coding scheme encodes these symbols into binary codes as follows:

a	0	c	110
b	10	d	111

Let X be the RV denoting the length of the code, that is, the number of binary symbols (bits).

- What is the range of X ?
- Assuming that the generations of symbols are independent, find the probabilities $P(X = 1)$, $P(X = 2)$, $P(X = 3)$, and $P(X > 3)$.
- Sketch the CDF $F_X(x)$ of X and specify the type of X .
- Find (i) $P(X \leq 1)$, (ii) $P(1 < X \leq 2)$, (iii) $P(X > 1)$, and (iv) $P(1 \leq X \leq 2)$.

11. A certain random variable has a probability density function of the form

- $f_X(x) = ce^{-2x}u(x)$.
- $f_X(x) = \frac{c}{x^2+4}$
- $f_X(x) = \frac{c}{\sqrt{25-x^2}}$ where $-5 < x < 5$, $\int \frac{1}{\sqrt{a^2-x^2}} dx = \sin^{-1}\left(\frac{x}{a}\right) + c$

Find the following:

- the constant c ,
 - $\Pr(X > 2)$,
 - $\Pr(X < 3)$,
 - $\Pr(X < 3 | X > 2)$.
12. The voltage of communication signal S is measured. However, the measurement procedure is corrupted by noise resulting in a random measurement with the PDF shown in the accompanying diagram fig 1. Find the probability that for any particular measurement, the error will exceed ± 0.8 percent of the correct value if this correct value is 10 volts.

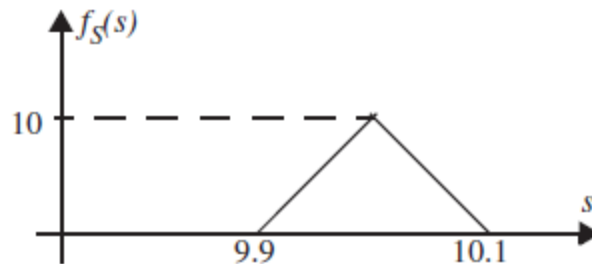


Fig . 5

13. Consider a function;

$$F_{XY}(x, y) = \begin{cases} 1 - e^{-(x+y)}, & 0 \leq x < \infty; 0 \leq y < \infty \\ 0, & \text{otherwise} \end{cases}$$

Can this function be a joint CDF of a bivariate r.v. (X, Y) ?

14. A Gaussian random variable has a probability density function of the form

$$f_X(x) = c \exp(-(2x^2 + 3x + 1)).$$

- Find the value of the constant c .
- Find the values of the parameters m and σ for this Gaussian random variable.

15. The joint PDF of a bivariate r.v. (X, Y) is given by:

$$a) f_{XY}(x, y) = \begin{cases} k(x+y), & 0 < x < 2, \\ & 0 < y < 2 \\ 0, & \text{otherwise} \end{cases}$$

$$d) f_{XY}(x, y) = \begin{cases} ke^{-(ax+by)}, & x > 0, y > 0 \\ 0, & \text{otherwise} \end{cases}$$

(a&b are constants)

$$b) f_{XY}(x, y) = \begin{cases} kxy, & 0 < x < 1, 0 < y < 1 \\ 0, & \text{otherwise} \end{cases}$$

$$e) f_{XY}(x, y) = \begin{cases} k, & 0 < y \leq x < 1 \\ 0, & \text{otherwise} \end{cases}$$

$$c) f_{XY}(x, y) = \begin{cases} kxy, & 0 < x < y < 1 \\ 0, & \text{otherwise} \end{cases}$$

Where k is constant.

- i. Find the value of k.
- ii. Find the marginal PDF's of X and Y.
- iii. Are X and Y independent?
- iv. (For (e)) Find $P(0 < X < 1/2, 0 < Y < 1/2)$.
- v. Find the conditional PDF's $f_{Y|X}(y|x)$ and $f_{X|Y}(x|y)$

16. A manufacturer has been using two different manufacturing processes to make computer memory chips. Let (X, Y) be a bivariate r.v., where X denotes the time to failure of chips made by process A and Y denotes the time to failure of chips made by process B. Assuming that the joint pdf of (X, Y) is

$$f_{XY}(x, y) = \begin{cases} abe^{-(ax+by)}, & x > 0, y > 0 \\ 0, & \text{otherwise} \end{cases}$$

where $a = 10^{-4}$ and $b = 1.2(10^{-4})$, determine $P(X > Y)$.

17. A Gaussian random variable has a PDF of the form

$$f_X(x) = \frac{1}{\sqrt{200\pi}} \exp\left(-\frac{(x-5)^2}{200}\right)$$

Write each of the following probabilities in terms of Q-functions (with positive arguments).

- | | |
|------------------|------------------------|
| a) $\Pr(X > 17)$ | e) $\Pr(X - 10 > 7)$ |
| b) $\Pr(X > 4)$ | f) $\Pr(X - 10 < 3)$ |
| c) $\Pr(X < 15)$ | g) $\Pr(X - 7 > 5)$ |
| d) $\Pr(X < -2)$ | h) $\Pr(X - 4 < 7)$ |

18. For positive constants a, b, c , and positive integer n , a pair of random variables has a joint PDF specified by

$$f_{X,Y}(x, y) = \frac{d}{(ax+by+c)^n} u(x)u(y)$$

- (a) Find the constant d in terms of a, b, c , and n .
- (b) Find the marginal PDFs, $f_X(x)$ and $f_Y(y)$.
- (c) Find $\Pr(X > Y)$.