

Projectile Motion

- The path that a moving object follows is called its *trajectory*.
 - An object thrown horizontally is accelerated downward under the influence of gravity.
 - Gravitational acceleration is only vertical, not horizontal.
 - The object's horizontal velocity is unchanged, if we can neglect air resistance.
- *Projectile motion* involves the trajectories and velocities of objects that have been launched, shot, or thrown.

The diagram shows the positions at 0.10-sec intervals of a ball moving left to right. Is the ball accelerated?



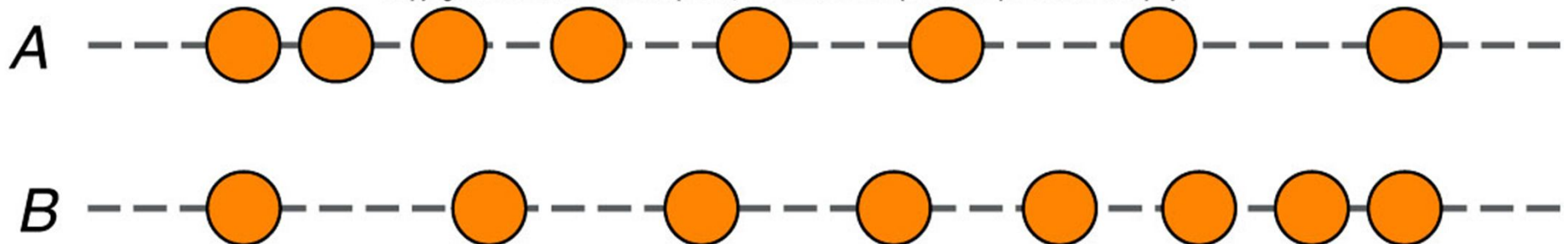
- a) Yes.
- b) No.
- c) Unable to determine.

The ball moves an equal distance during each 0.10-sec interval, so the speed does not change.

Thus, the ball is not accelerated.

The diagram shows the positions at 0.05-sec intervals of two balls moving left to right. Are either or both of these balls accelerated?

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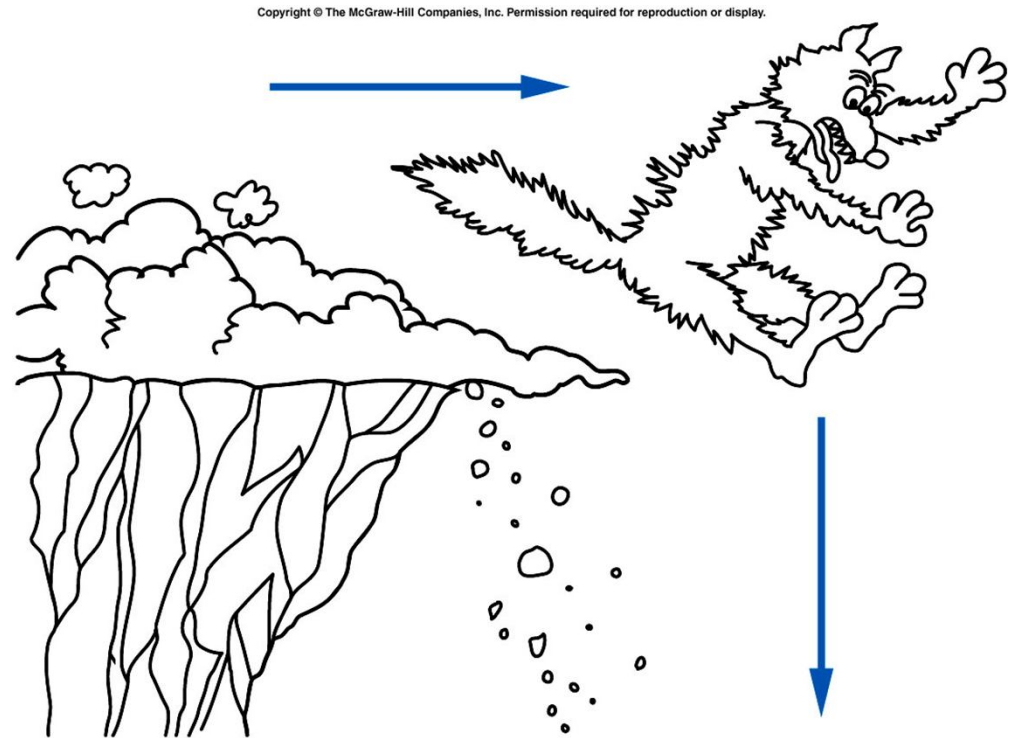
- a) Ball A is accelerated.
- b) Ball B is accelerated.
- c) Both balls are accelerated.
- d) Neither ball is accelerated.

Both balls are accelerated. Ball A covers an increasing distance in each 0.05-sec interval, so it is speeding up. Ball B is covering less and less distance with each interval, so it is slowing down. Both of these are accelerations.

Does this represent a realistic trajectory?

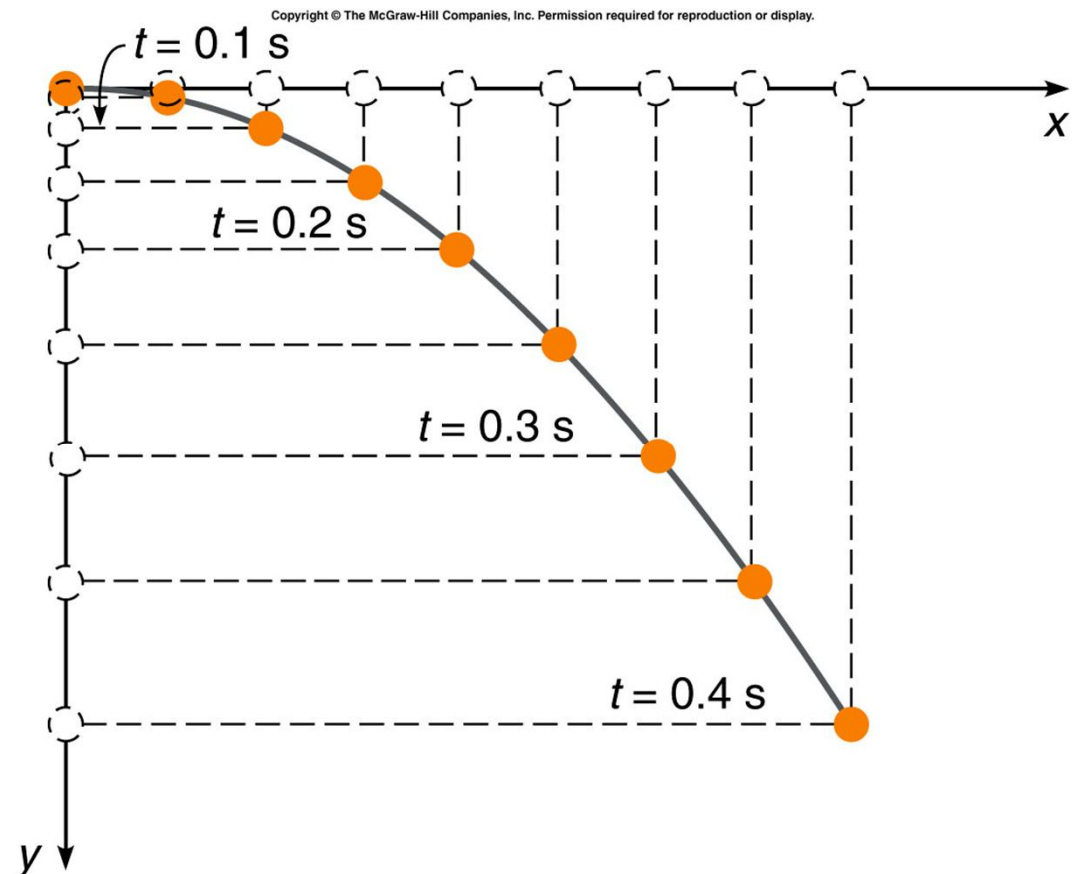
- a) Yes.
- b) No.
- c) Maybe.

The coyote would not go straight horizontally, pause, and then fall straight down. There are many examples in movies and on television of unrealistic trajectories. Can you think of any others?



What does the trajectory look like?

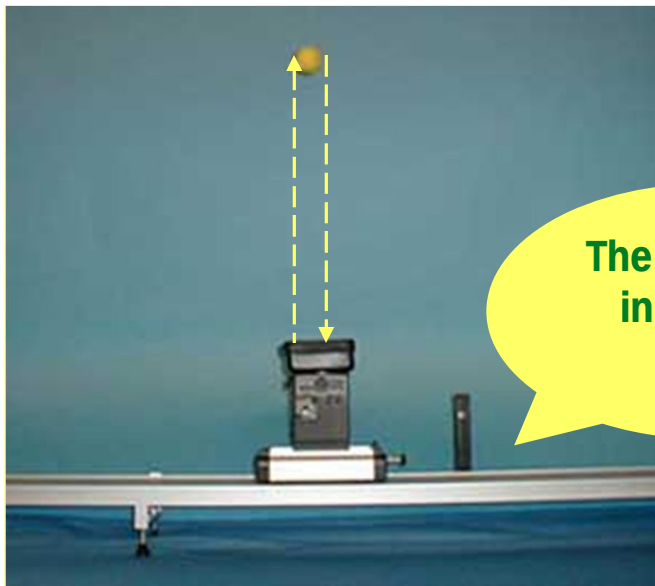
- **The acceleration of the horizontal motion is zero** (in the absence of air resistance).
 - The object moves with constant horizontal velocity.
 - It travels equal horizontal distances in equal time intervals.
- **The acceleration in the vertical direction is constant.**
 - Its vertical velocity increases downward just like the falling ball.
 - In each successive time interval, it falls a greater distance than in the previous time interval.



1D-21

Independence of Vertical and Horizontal Motions

A ball is projected vertically from a cart traveling horizontally



**The trajectory
in the cart
frame**

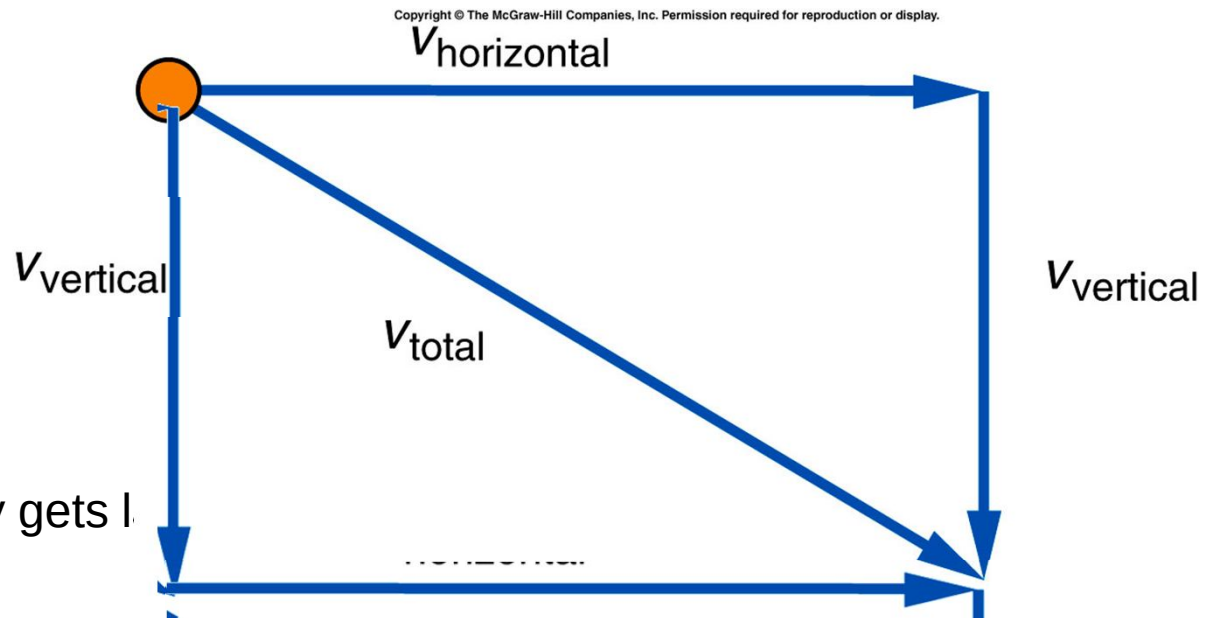


**The
trajectory in
the room
frame**

- THE HORIZONTAL MOTION OF THE BALL IS UNAFFECTED BY ITS VERTICAL MOTION.

What does the trajectory look like?

- The **total velocity at any point** is found by adding the vertical component of the velocity, at that point, to the horizontal component of the velocity at that point.
- The horizontal velocity remains constant, because there is no acceleration in that direction.
- The length of the horizontal velocity vector doesn't change.
- The downward (vertical) velocity gets larger due to the acceleration due to gravity.

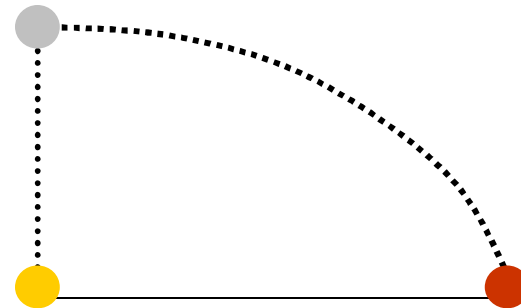
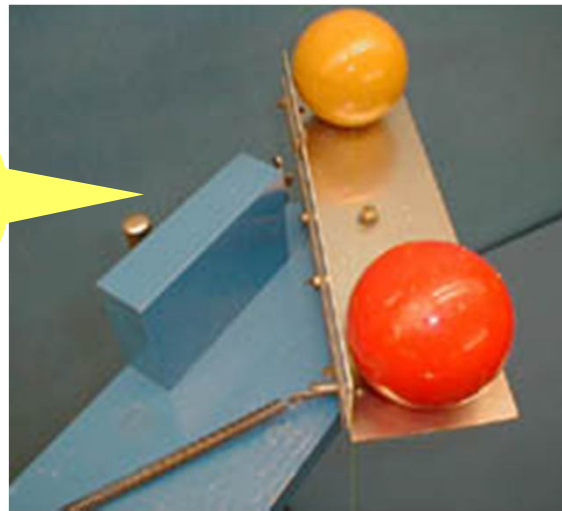


1D-20

Independence of Vertical & Horizontal Motions (Drop-Kick)

One ball drops from rest. The other ball is simultaneously projected horizontally

Which ball
will hit the
ground first ?

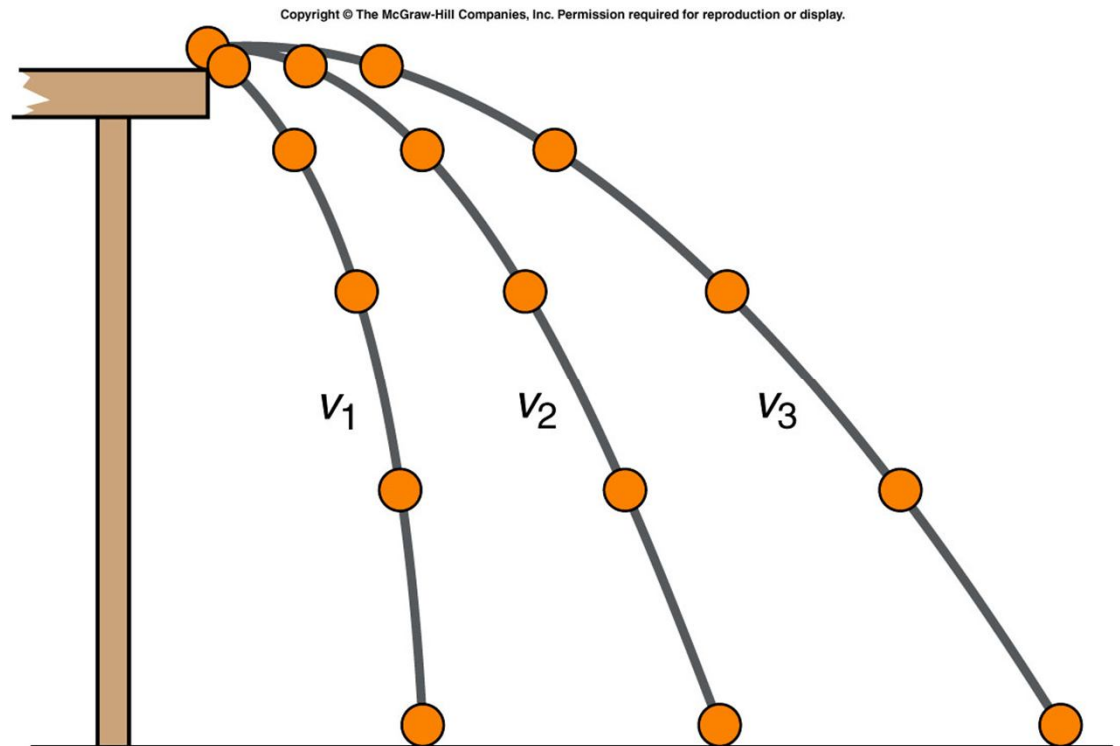


Listen to the **SOUND** when they hit the ground and when they bounce.

- THE VERTICAL & HORIZONTAL MOTIONS ARE INDEPENDENT.
- THE HORIZONTAL VELOCITY DOES NOT AFFECT THE VERTICAL MOTION.
- THE VERTICAL FALL TIME IS THE SAME AS LONG AS THE BALLS DROP SIMULTANEOUSLY FROM THE SAME HEIGHT.

What does the trajectory look like?

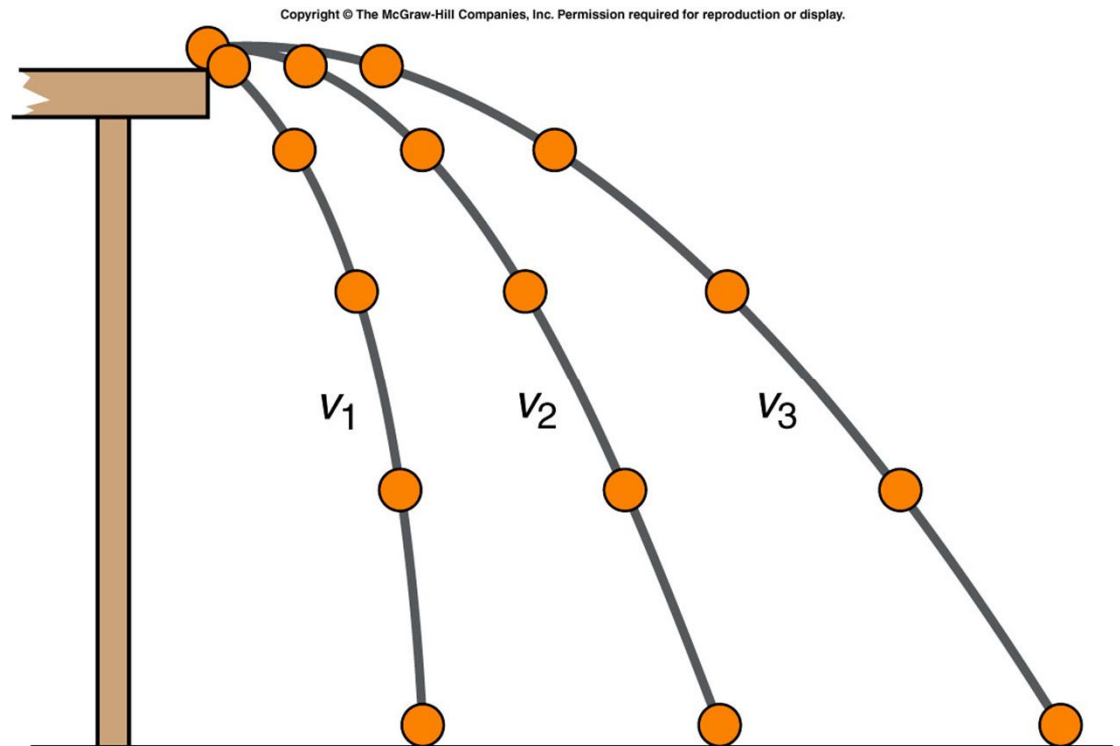
- Trajectories for different initial velocities of a ball rolling off a table:
 - The largest initial velocity is v_3 .
 - The smallest initial velocity is v_1 .
- The ball travels greater horizontal distances when projected with a larger initial horizontal velocity.



Quiz: Which of these three balls would hit the floor first if all three left the tabletop at the same time?

- a) The ball with initial velocity V_1 .
- b) The ball with initial velocity V_2 .
- c) The ball with initial velocity V_3 .
- d) They would all hit at the same time.

Since all three balls undergo the same downward acceleration, and they all start with a vertical velocity of zero, they would all **fall** the same distance in the same time!



Summary of Projectile Motion

- Treating the vertical motion independently of the horizontal motion, and then combining them to find the trajectory, is the secret.
 - A horizontal glide combines with a vertical plunge to produce a graceful curve.
 - The downward gravitational acceleration behaves the same as for any falling object.
 - There is no acceleration in the horizontal direction if air resistance can be ignored.
 - The projectile moves with constant horizontal velocity while it is accelerating downward.

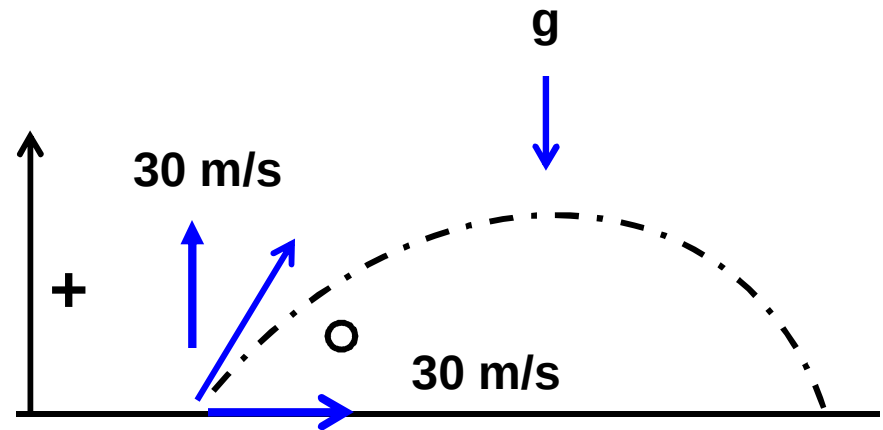
Ch 3 E16

$$V_{0v} = 30 \text{ m/s} \quad V_{0H} = 30 \text{ m/s}$$

$$g = -9.8 \text{ m/s}^2$$

a) What is time to top?

b) What is the range?



$$\text{a) } v = v_0 + at \quad t = 30/9.8 = 3.06\text{s} \quad t_R = 6.12\text{s}$$

$$\text{b) } d = 30 \times t_R = 183.6\text{m}$$

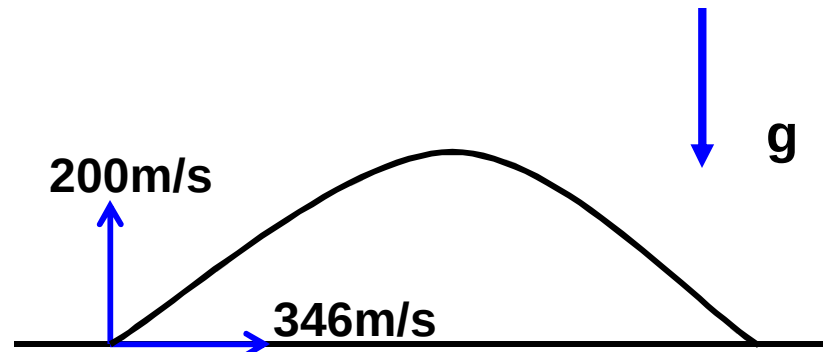
Ch 3 CP4

$$V_{0v} = 200\text{m/s } v_{0H} = 346\text{m/s}$$

a) How long in the air?

b) How far?

$$\text{c) } v_{0v} = 346 \quad v_{0H} = 200$$



$$\text{a) } v = v_0 + at \quad \text{time to top} = 200/9.8 = 20.4\text{s}$$

$$\text{time to range} = 400/9.8 = 40.8\text{s}$$

$$\text{b) } d = 346 \times 40.8 = 14120\text{m}$$

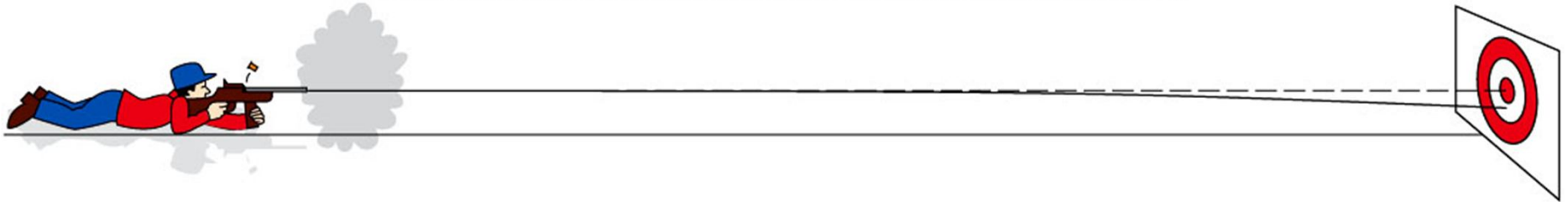
$$\text{c) } \uparrow 346 \rightarrow 200 \quad \text{time} = 692/9.8 = 70.6\text{s}$$

$$d = 200 \times 70.6 = 14120$$

Hitting a Target

- If the rifle is fired directly at the target in a horizontal direction, will the bullet hit the center of the target?
- Does the bullet fall during its flight?

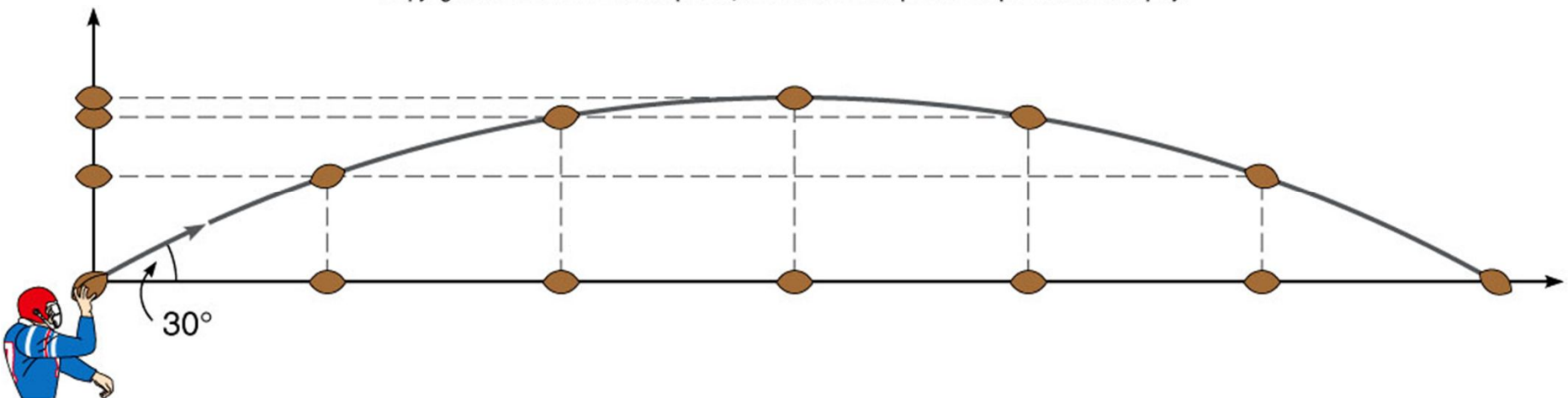
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Hitting a Target

- The trajectory depends on the initial velocity.
- The trajectory depends on the launch angle.

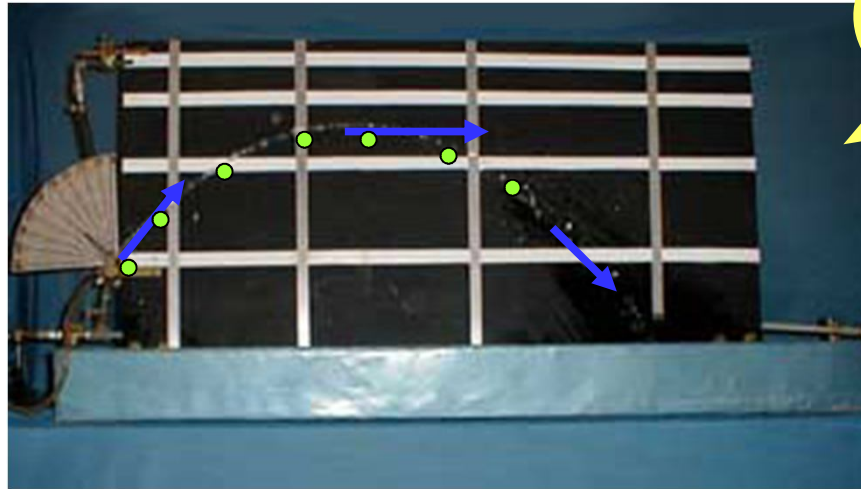
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1D-22 Water Jets & Projectile Motion

PROJECTILE MOTION OF A WATER JET

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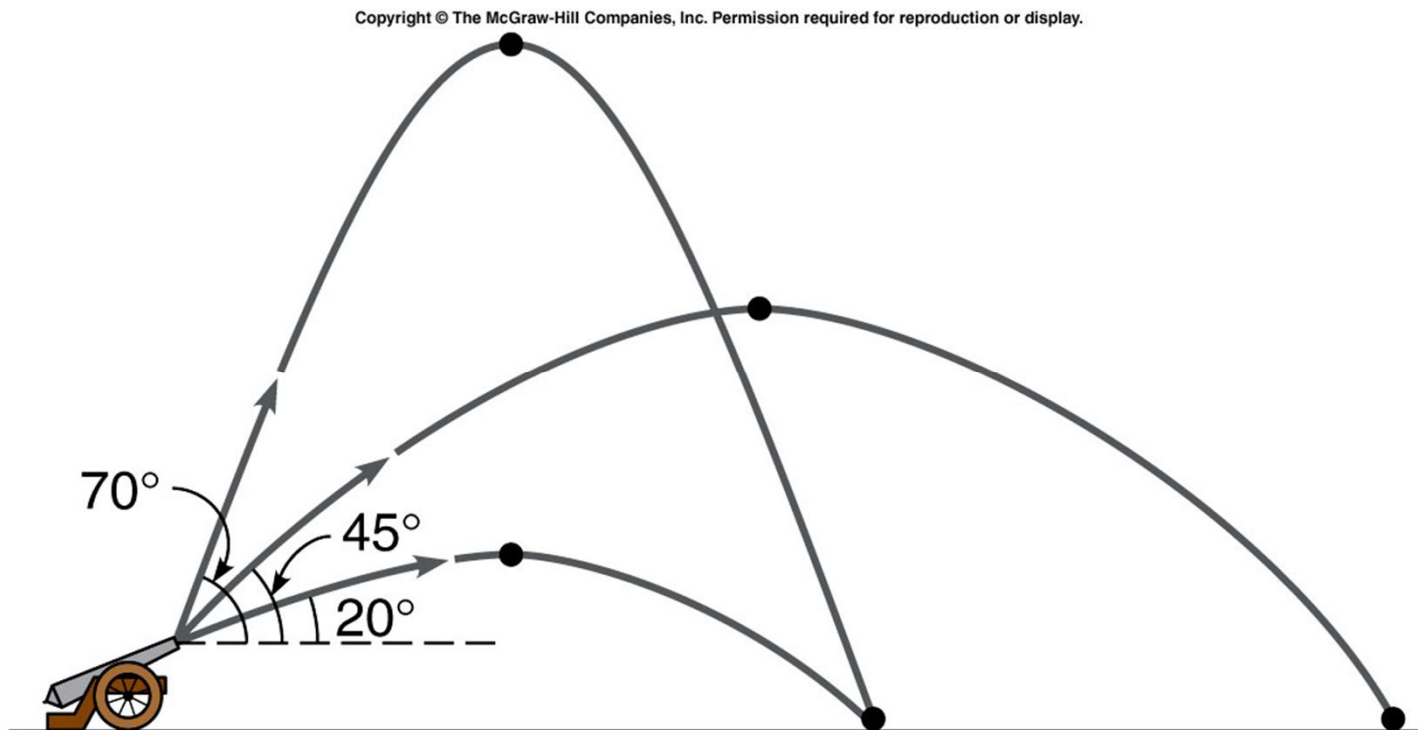


What angle gives
the maximum
range?

- NEGLECTING FRICTION THE RANGE IS A MAXIMUM AT 45° .
- TWO DIFFERENT ANGLES CAN GIVE THE SAME RANGE (ANGLES SYMMETRIC ABOUT 45°).
- A LARGER ANGLE MEANS A LONGER TIME OF FLIGHT, BUT LESS HORIZONTAL VELOCITY. A SMALLER ANGLE MEANS A LARGER HORIZONTAL VELOCITY, AND LESS FLIGHT TIME.
- THE TRAJECTORY IS SYMMETRIC.
-

Hitting a Target

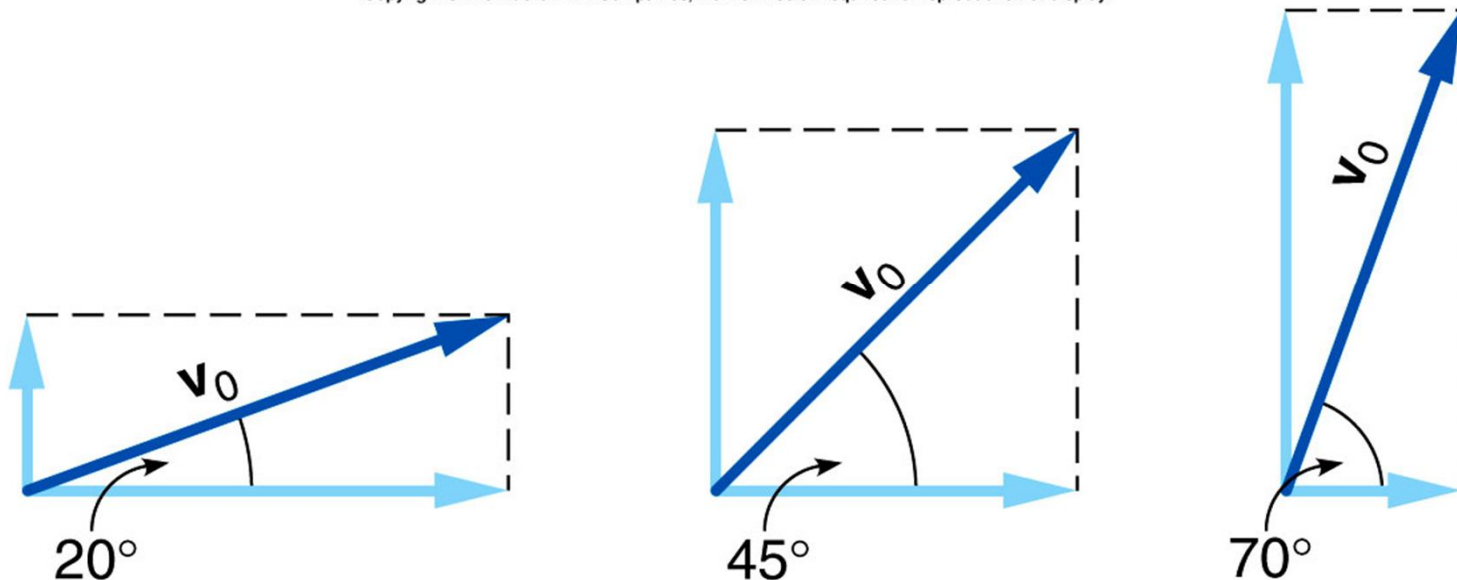
- The greatest distance is achieved using an angle close to 45° if the effects of air resistance are negligible.



Hitting a Target

- For the lowest angle, the horizontal velocity is much greater than the initial vertical velocity.
- The ball does not go very high, so its time of flight is short.

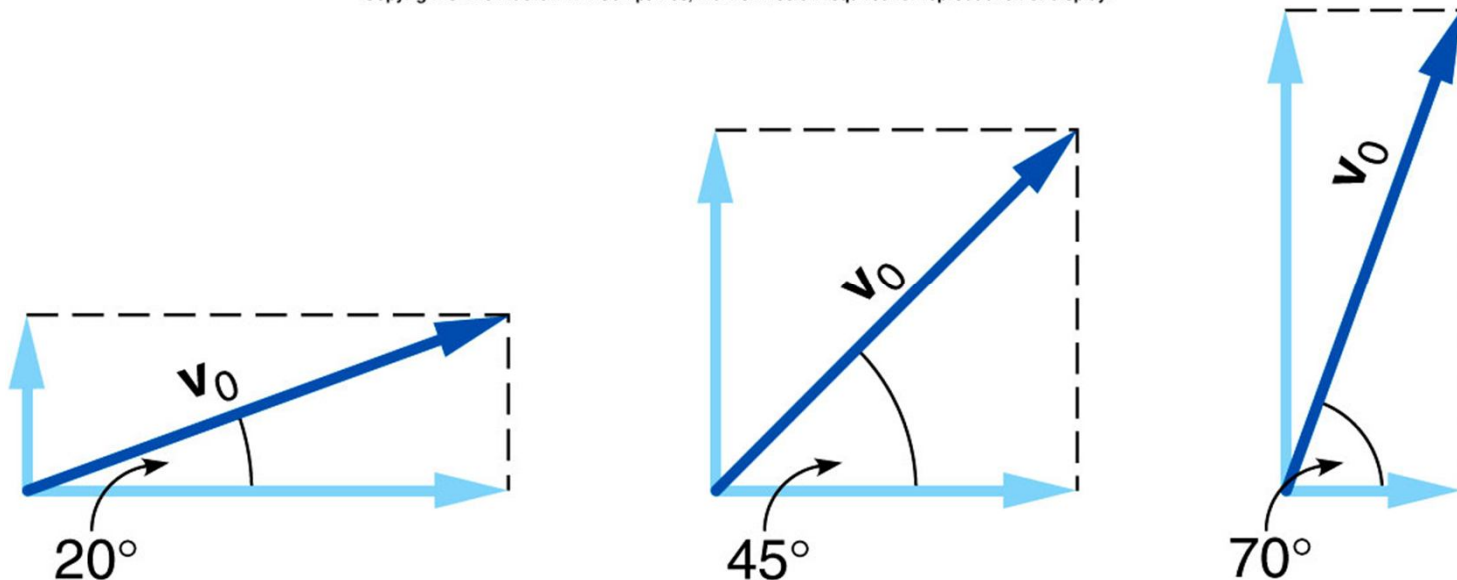
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Hitting a Target

- For the highest angle, the initial vertical velocity is much greater than the horizontal velocity.
- The ball goes higher, so its time of flight is longer, but it does not travel very far horizontally.

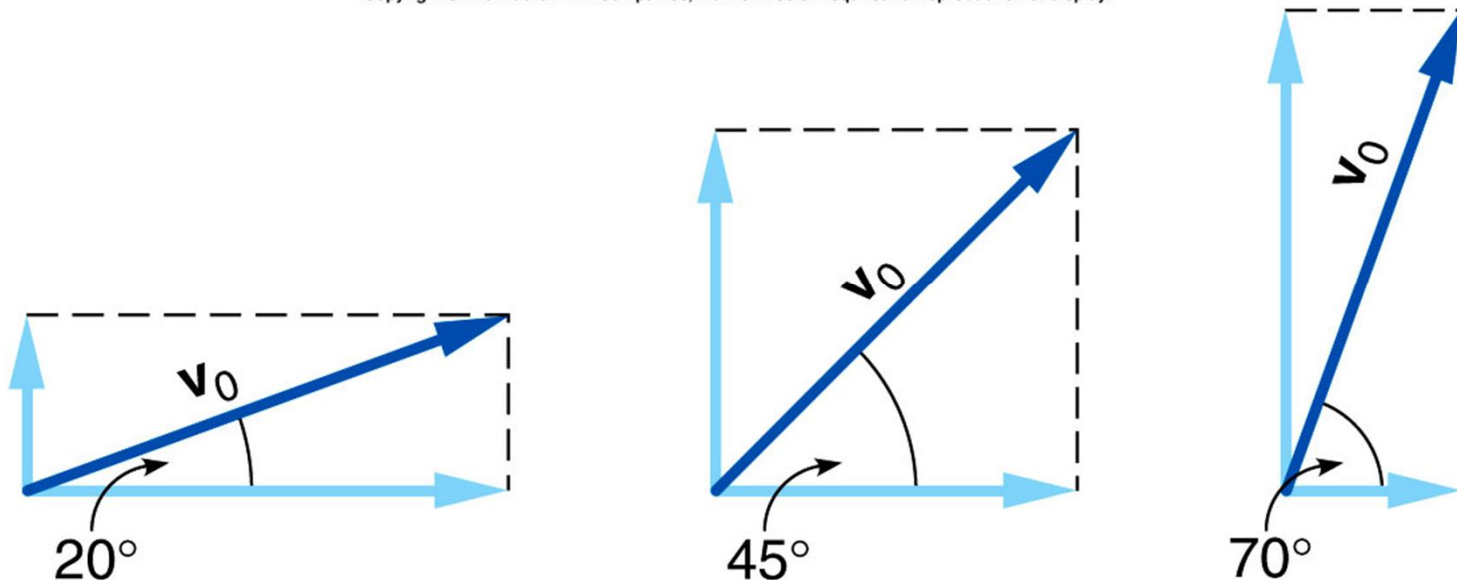
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Hitting a Target

- The intermediate angle of 45° divides the initial velocity equally between the vertical and the horizontal.
- The ball stays in the air longer than at low angles, but also travels with a greater horizontal velocity than at high angles.

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Where to aim in order to Hit the Falling object?

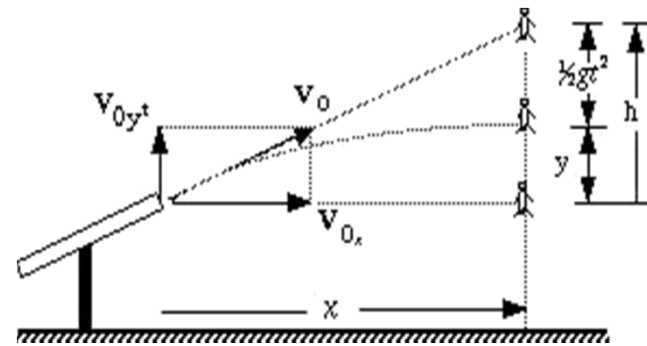
<http://www.mhhe.com/physsci/physical/giambattista/proj/projectile.html>

1D-23 Shoot the Monkey

The monkey falls out of the tree at the instant the gun is fired



WHERE SHOULD
ONE AIM, ABOVE,
BELOW OR AT?



Ignoring friction $y = v_{0y}t - \frac{1}{2}gt^2$ $t = x/v_{0x}$, $v_{0y}/v_{0x} = h/d$
at $x = d$ $y = h - \frac{1}{2}gt^2$ In the same time the monkey falls $\frac{1}{2}gt^2$
So the bullet always hits the monkey no matter what the value of v_0

- THE VERTICAL MOTION IS INDEPENDENT OF THE HORIZONTAL MOTION
- THE EFFECT OF FRICTION IS MINIMIZED BY USING A LARGE TARGET