

UNIT - 3

WAVE OPTICS

Optics:- is branch of physics that studies about light and its properties.

Wave optics:- is study about light based on its wave nature.

Light exhibit different properties such as, reflection, refraction, diffraction, interference and polarization.

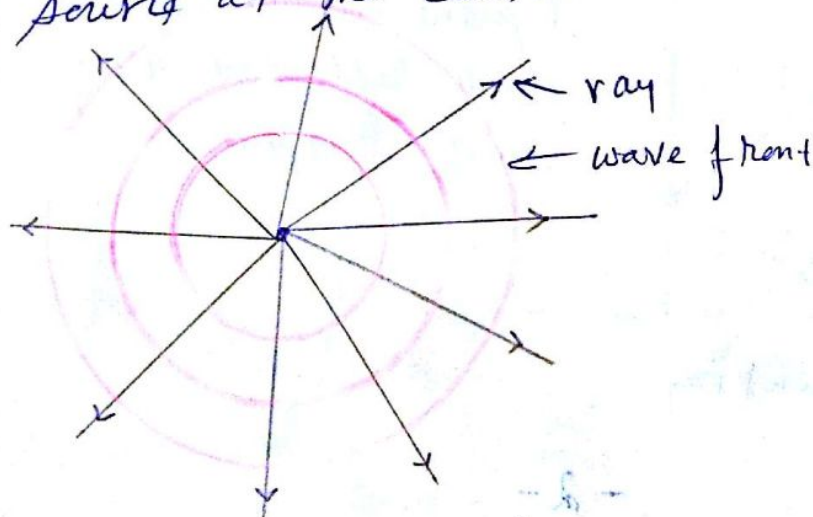
Rays and Wave fronts

Rays:- are the arrows drawn to show the direction of propagation of light (or direction to which the light energy is transferred)

Wave front is an imaginary line or surface where all the points have the same phase.

- it is a line or a surface of constant phase.
* Always rays are at 90° to the wave fronts.
The distance between two adjacent (or near by) wave fronts whose phase difference is 2π rad is equal to one wave length.

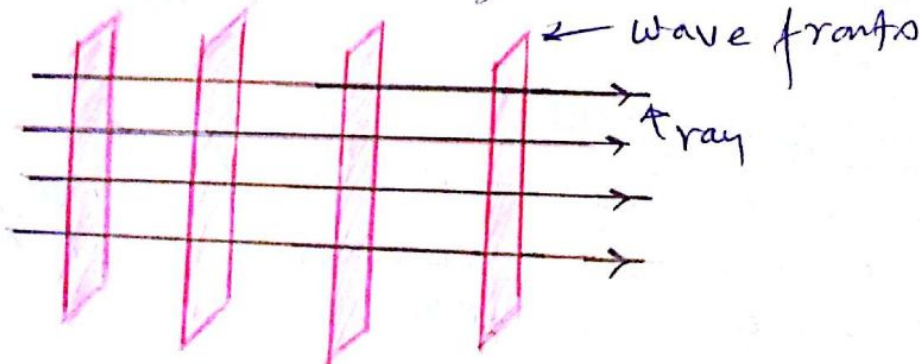
Spherical wave:- is a wave that spreads out from the source to all directions in space.
- its wave fronts are concentric spheres with the source at the center.



Plane wave is a wave that travels to a fixed direction.

- its rays are parallel to each other and the wave fronts are plane surfaces.

Spherical wave at a very large distance away from the source ~~the~~ its wave fronts are nearly plane surface. Thus it can be approximated to a plane wave at a very large distance away from the source.

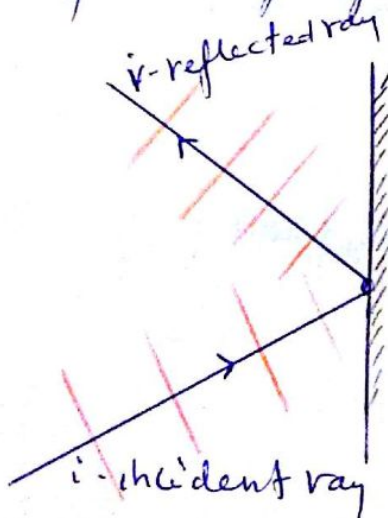


Reflection of light

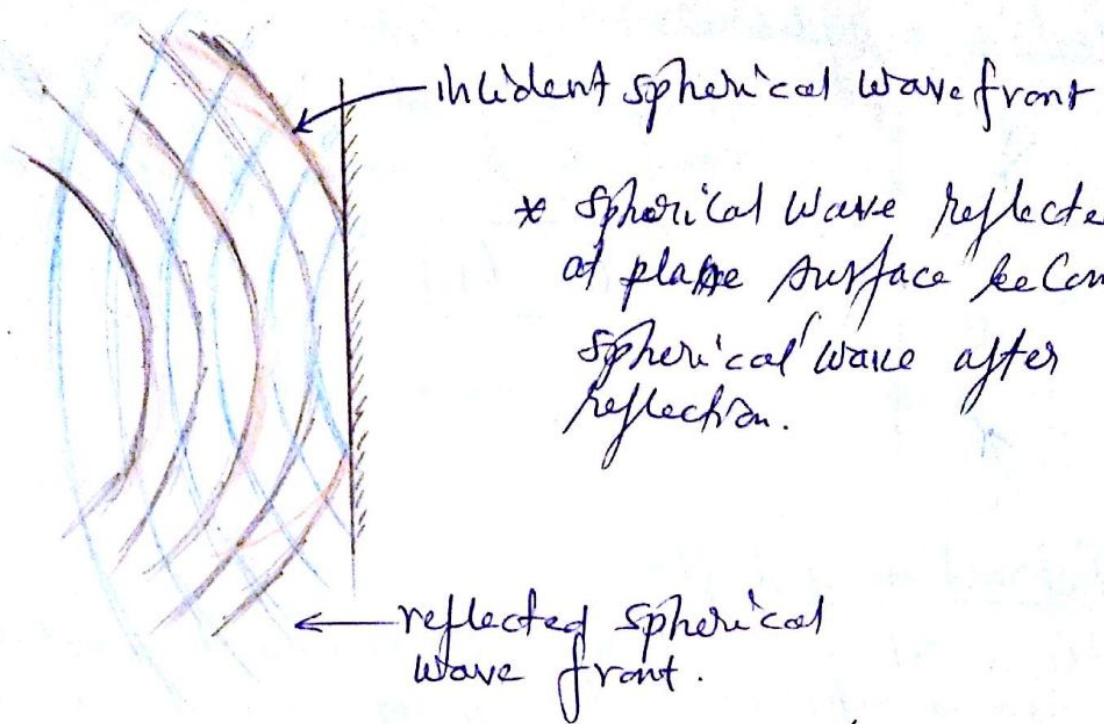
Reflection is the turning of a wave to the transmitting medium from a boundary surface when the surface is rigid surface the reflected wave

becomes inverted (or $180^\circ = \pi \text{ rad}$) out of phase.
* During reflection speed, wave length & frequency of the light remain the same.
* The shape of the reflected wave front is affected by the shape of the reflecting surface.

Reflection of light at a plane surface

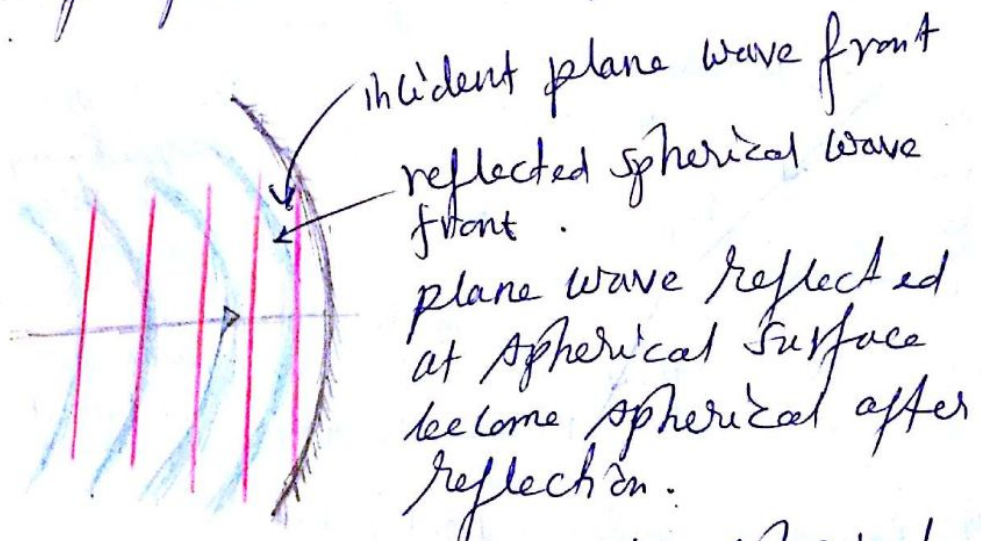


A plane incident wave becomes plane reflected wave at a plane surface.

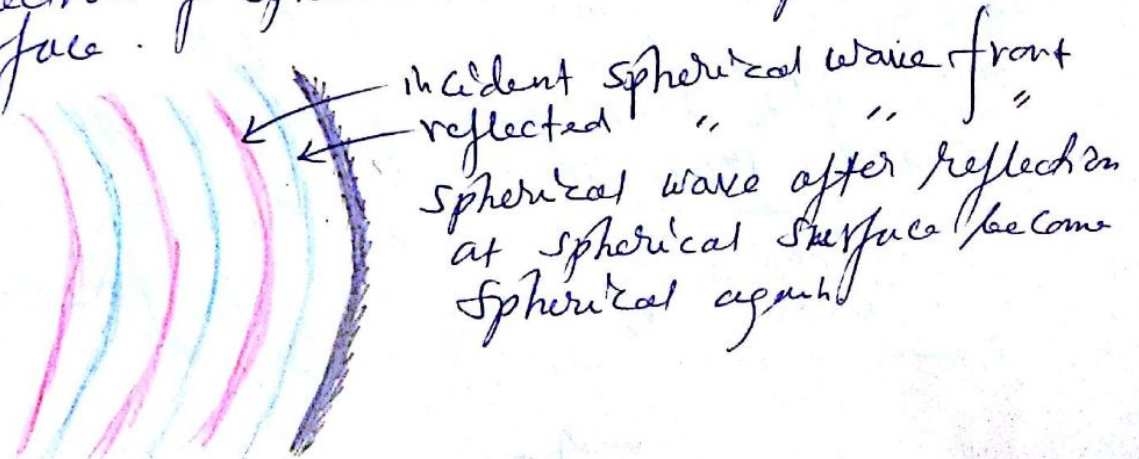


Reflection of light at a spherical surface

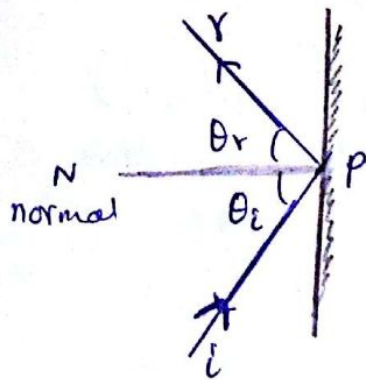
a) reflection of a plane wave at spherical (concave) surface.



b) reflection of spherical wave at a spherical surface.



The law of reflection

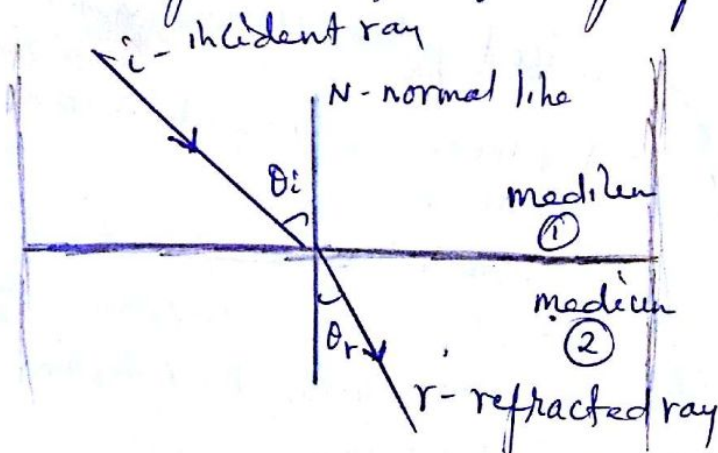


Always the angle of incidence θ_i is equal to the angle of reflection θ_r

$$\boxed{\theta_i = \theta_r}$$

Refraction of light

When light passes from one medium to another medium of different nature its speed and wave length and also direction of propagation are changed. This phenomenon is called refraction. During refraction of light frequency remains unchanged.



Let v_1 and v_2 be respectively the speed of the wave in medium (1) and (2) and λ_1 and λ_2 be its wavelength respectively in medium (1) and (2)

The law of refraction (Snell's law)

The ratio of the sine of the angle of incidence θ_i to the sine of angle of refraction θ_r is equal to the ratio of the speeds of the light in the first medium (1) to the second medium (2)

$$\frac{\sin \theta_i}{\sin \theta_r} = \frac{v_1}{v_2}$$

$$\begin{aligned} \text{but } v_1 &= f \lambda_1 \\ v_2 &= f \lambda_2 \end{aligned}$$

$$\Rightarrow \frac{\sin \theta_i}{\sin \theta_r} = \frac{f \lambda_1}{f \lambda_2} = \frac{\lambda_1}{\lambda_2}$$

Refractive index (n) of a transparent medium is defined as the ratio of Speed of light in free space (c) to the speed of light in that medium (v)

$$\Rightarrow n = \frac{c}{v}$$

$$c = 3 \times 10^8 \text{ m/s}$$

the refractive index of the first medium $n_1 = \frac{c}{v_1}$
 " " " " " Second " $n_2 = \frac{c}{v_2}$

$$\Rightarrow \text{then } \frac{n_2}{n_1} = \frac{c/v_2}{c/v_1} = \frac{v_1}{v_2}$$

$$\text{then } \boxed{\frac{\sin \theta_i}{\sin \theta_r} = \frac{v_1}{v_2} = \frac{\lambda_1}{\lambda_2} = \frac{n_2}{n_1}}$$

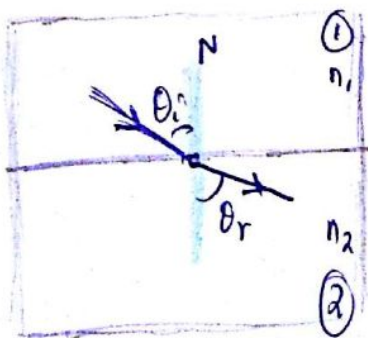
$\frac{n_2}{n_1} = n_{2,1}$ - is the relative refractive index of medium (2) to medium (1)

* A medium with higher refractive index is said to be optically denser and that with lesser refractive index is optically lighter medium.

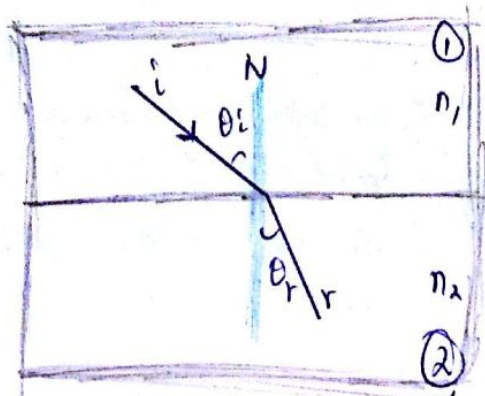
* When light passes from optically denser to the lighter medium it is bent away from the normal.

when $n_1 > n_2$

then $\theta_i < \theta_r$



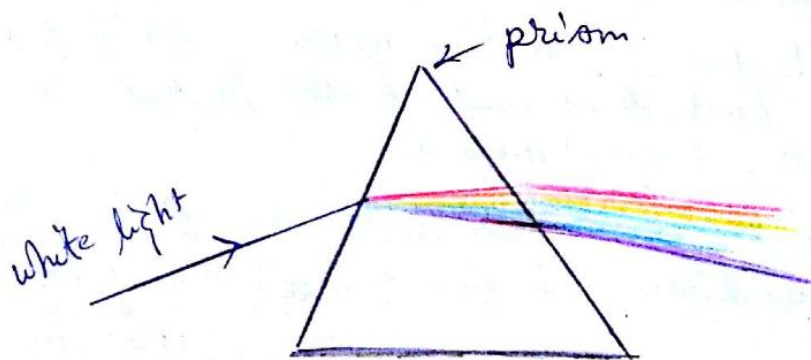
When light passes from optically lighter to denser medium it is bent towards the normal.



When $n_1 < n_2$
then $\theta_i > \theta_r$

* The angle of refraction of a given light ray depends on its wave length and frequency. When different light rays of different wave length pass from a medium to another medium they are refracted at different angles.

White light is composed of different (spectral) rays of different wave lengths. As white light passes through a transparent material such as prism the different colours (spectral rays) split apart this phenomenon is called dispersion of light.



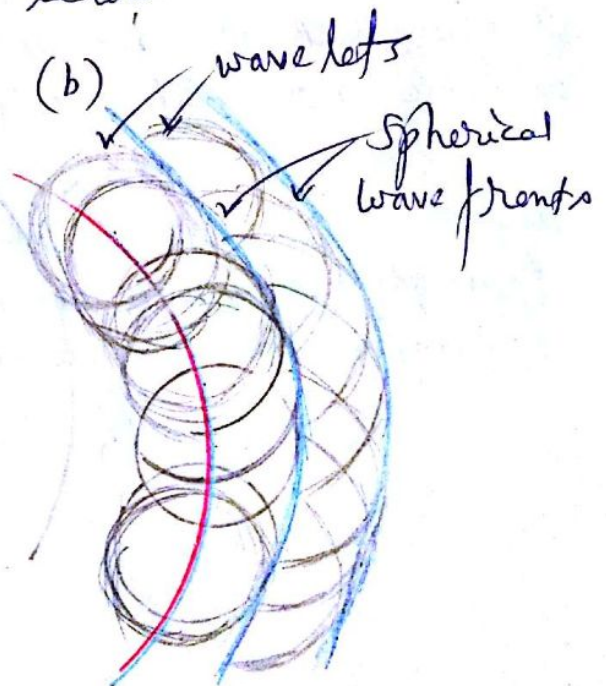
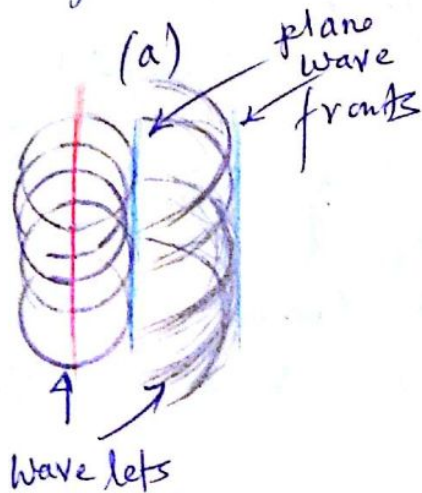
Huygen's principle

This principle describes about construction of wave fronts in a medium where wave is propagating and it enable us to visualize the reflection, refraction, diffraction and interference of light waves.

The principle states that

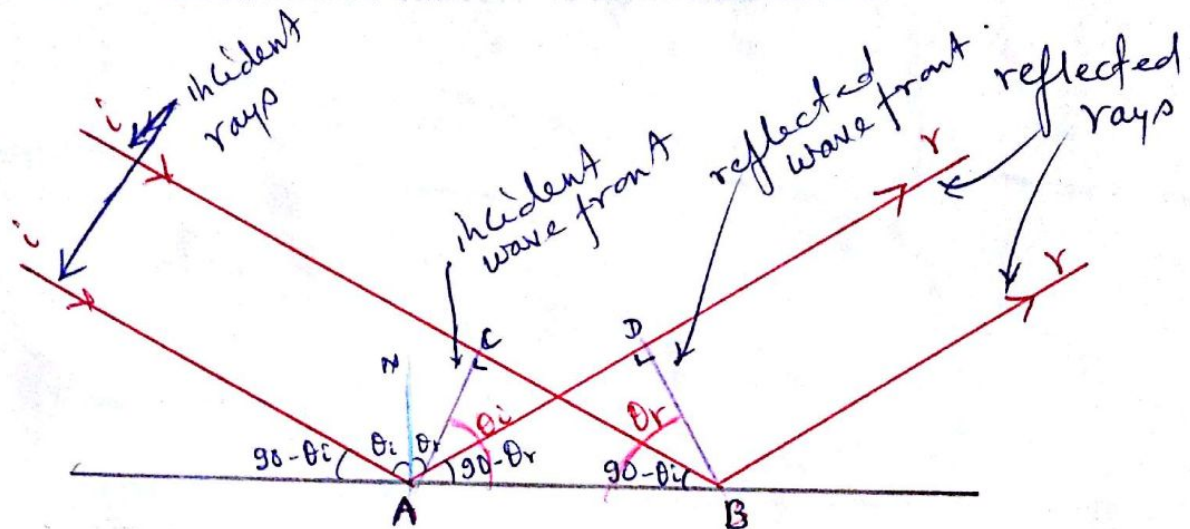
- Every point on a wave front is a source of new spherical waves (known as wavelets)
- The secondary wavelets spread out to all directions in the medium with the same speed and wave length as the original wave
- New wave front is formed when the secondary wavelets recombine (the tangent to the wavelets from all the points on the wave front is the new wave front)

Huygen's principle to a plane wave (a) and spherical wave (b) shown below



Proof of the law of reflection

Consider a plane wave incident (i) to plane reflecting surface and the reflected plane wave (r) from the surface as shown below.



Assume the incident wave front $\overline{AC}$ and the reflected wave front $\overline{BD}$ are consecutive (or successive) wave fronts which are out of phase by 2π rad. Thus the distances $\overline{CD}$ and $\overline{AD}$ are equal to λ (wave length)

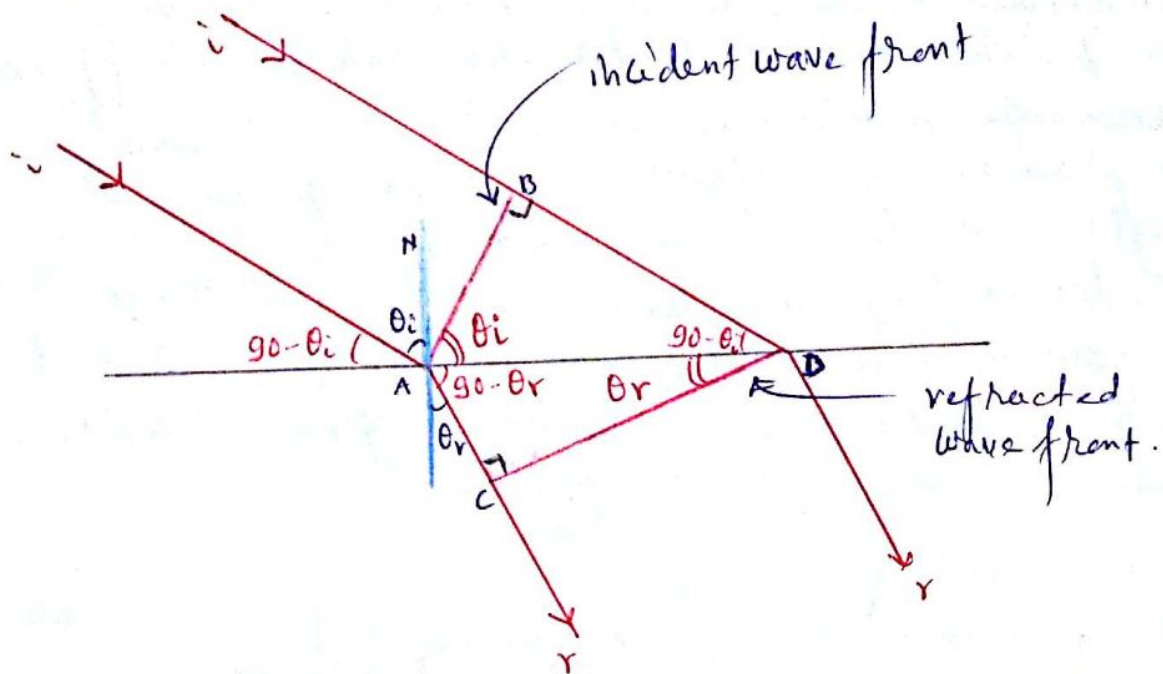
$$\overline{CD} = \overline{AD}$$

and the side $\overline{AB}$ is common side to the right triangles $\triangle ACB$ and $\triangle ADB$ thus the two triangles are congruent therefore hence

$$\theta_i = \theta_r$$

proof of the law of refraction

assume a plane wave transmitted from medium ① with refractive index n_1 to medium ② of refractive index n_2 . Its speed and wave length are changed from v_1, λ_1 to v_2 and λ_2 as shown in the figure below.



Assume that the wave fronts $\overline{AB}$ and $\overline{CD}$ are adjacent wave fronts which are out of phase by 2π rad then the distance $\overline{BD} = \lambda_1$ and the distance $\overline{AC} = \lambda_2$. Consider the two right triangles $\triangle ADB$ and $\triangle ADC$.
for the $\triangle ADB$ $\sin \theta_i = \frac{\overline{BD}}{\overline{AD}} = \frac{\lambda_1}{\overline{AD}}$

$$\text{for the } \triangle ADC \quad \sin \theta_r = \frac{\overline{AC}}{\overline{AD}} = \frac{\lambda_2}{\overline{AD}}$$

$$\Rightarrow \frac{\sin \theta_i}{\sin \theta_r} = \frac{\lambda_1 / \overline{AD}}{\lambda_2 / \overline{AD}} = \frac{\lambda_1}{\lambda_2}$$

$$\text{but } \lambda_1 = v_1 / f \quad \& \quad \lambda_2 = v_2 / f$$

$$\Rightarrow \frac{\sin \theta_i}{\sin \theta_r} = \frac{\lambda_1}{\lambda_2} = \frac{v_1}{v_2}$$

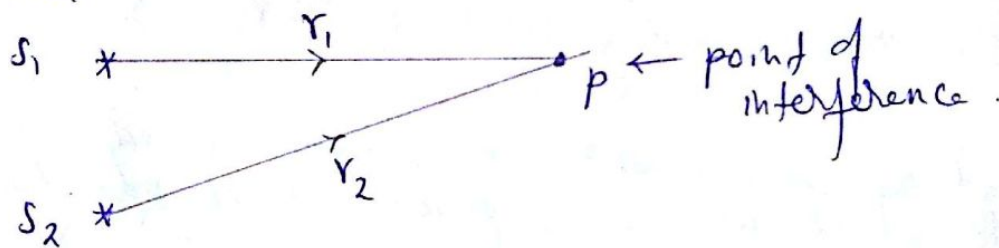
Interference and diffraction of light

Interference

Light rays which come from different sources (or points) can produce interference when they meet at a point. The light rays can produce a constructive interference of maximum intensity (that is bright band) or destructive interference of low intensity or minimum intensity (dark band). To produce consistent interference pattern the light rays have to be coherent rays.

Coherent rays are those rays of the same frequency and wave length and have constant phase relationship (or constant phase difference) when they meet at the point.

Sources



The path difference between the two rays is $\Delta x = |r_2 - r_1|$.

The condition for the constructive interference bright bands (maxima) is that

$$\Delta \phi = 2n\pi$$

where $n = 0, 1, 2, 3, \dots$

and

$$\Delta x = \frac{\lambda}{2\pi} \Delta \phi = \frac{\lambda}{2\pi} (2n\pi)$$

$$\Delta x = n\lambda$$

The condition for the destructive interference dark bands (minima) is that

$$\Delta\phi = (2n+1)\pi$$

$$n = 0, 1, 2, 3, \dots$$

$$\text{and } \Delta x = \frac{\lambda}{2\pi} \Delta\phi = \frac{\lambda}{2\pi} (2n+1)\pi$$

$$\Delta x = \left(n + \frac{1}{2}\right) \lambda$$

Diffraction of light

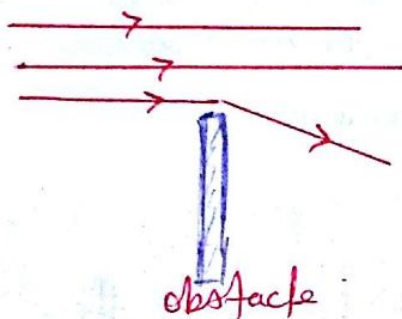
Diffraction is the bending of a wave around the edge of an obstacle.

or it is the spread out of a wave as it passes through a narrow opening (hole)

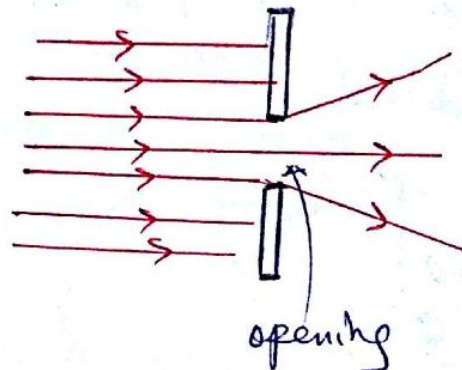
Significant (or noticeable) diffraction is observed when the wave length of the light ray is comparable to the size of the opening.

* Diffraction of light is noticed to a significant amount when the size of the hole is very small that is comparable to the wave length of the light ray.

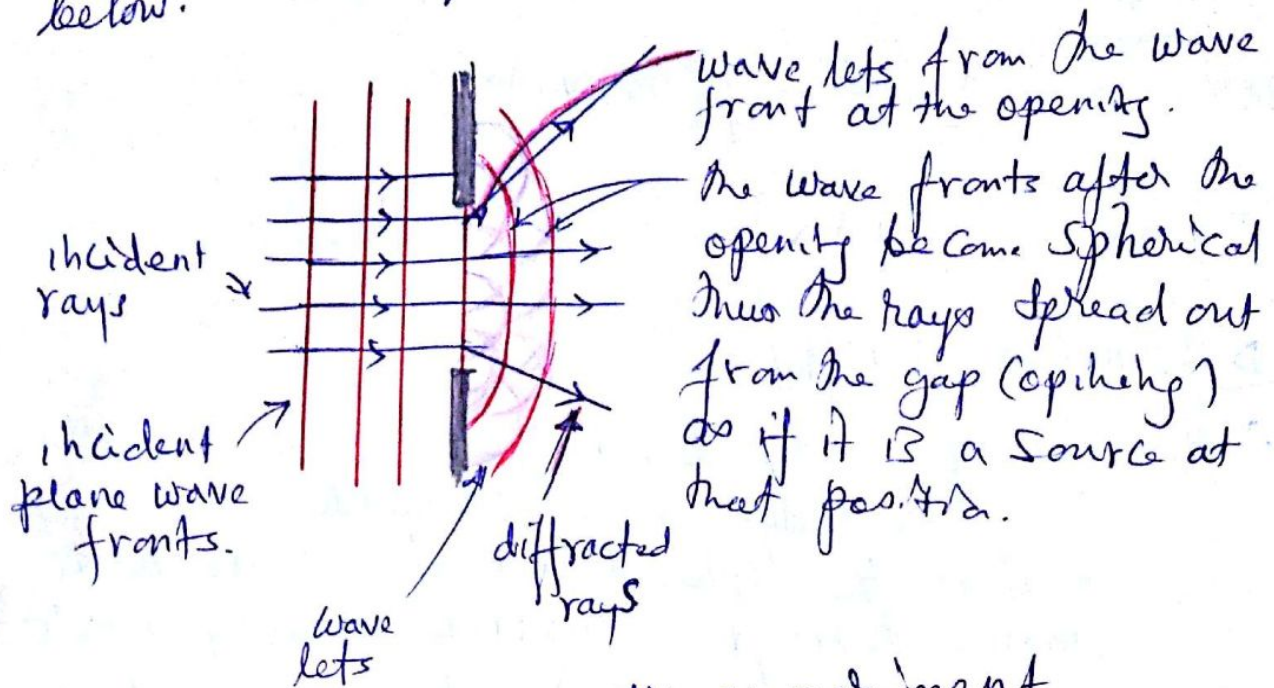
diffraction around an edge.



diffraction through a hole

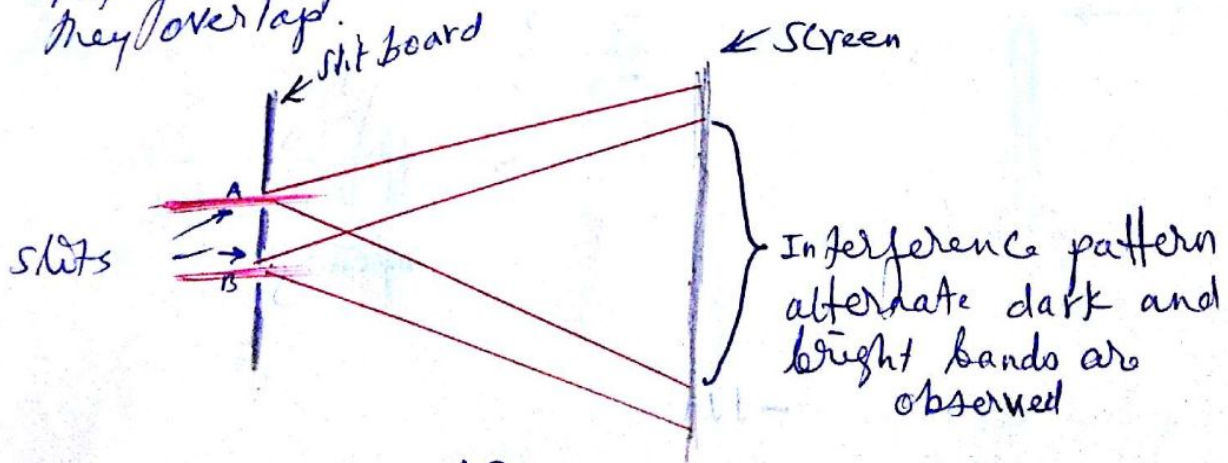


Diffraction of light through opening is expilustrated using Huggen's principle as shown in the figure below.



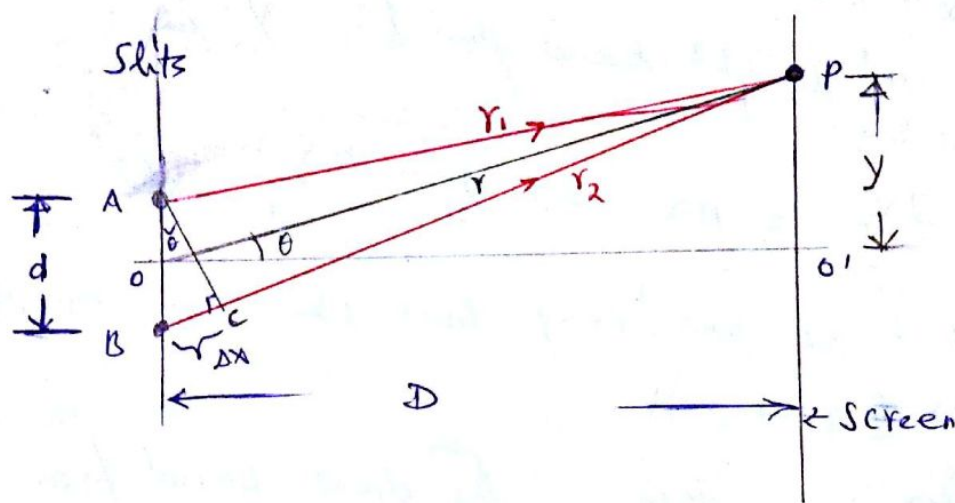
■ young's Double Slit experiment

In 1801, Thomas Young performed a double-slit experiment this interference experiment not only demonstrated the wave nature of light but also allowed the first measurement of the wave length of light. As shown in the figure below coherent light of wave length λ illuminates a board in which there are two narrow parallel slits which are closer to each other. as the light passes through the slits they are diffracted apart and produce interference pattern on the screen behind where they overlap.



Mathematical description of the young's double slit experiment.

Let d - be the distance b/n the slits (slit separation)
 D - " " " " " the slits and the screen.



Let $BC = AX = r_2 - r_1$ the path difference of the two rays coming from the slits A and when they meet at the position P (on the screen)

Since $D \gg d$ (the separation b/n the screen and the slits is much much larger than the separation b/n the slits) then the lines $r_1 = AP$, $r_2 = BP$ and $r = OP$ are nearly parallel to each other. Thus the line AC intersect these lines at 90° .

and then $\angle BAC = \theta$

In the right triangle $\triangle ABC$

$$\sin \theta = \frac{AX}{d} \Rightarrow \underline{AX = d \sin \theta} \quad (1)$$

Since the angle θ is very small $\Rightarrow \sin \theta \approx \tan \theta$
 for $\triangle O'P$ $\tan \theta = \frac{y}{D}$

$$\Rightarrow \sin \theta = \frac{y}{D}$$

$$\Delta x = \frac{dy}{D}$$

The position (point P) be position of bright band when $\Delta x = n\lambda$

$\Rightarrow$ the n^{th} bright band position y_n is given by

$$\frac{dy_n}{D} = n\lambda \Rightarrow y_n = \frac{n\lambda D}{d} \quad (2)$$

The point P be position of dark band (minimum) when $\Delta x = (n + \frac{1}{2})\lambda$

$\Rightarrow$ position the distance of the dark band from the center ($O O'$) is

$$\frac{dy_{\frac{1}{2}}}{D} = (n + \frac{1}{2})\lambda \Rightarrow y_{\frac{1}{2}} = (n + \frac{1}{2}) \frac{\lambda D}{d} \quad (3)$$

Locations of maxima (bright bands) from the center ($O O'$) are

$n=0 \Rightarrow y_0 = 0$ Central maximum (bright band)

$n=1 \Rightarrow y_1 = \frac{\lambda D}{d}$ 1st bright band (maximum)

$n=2 \Rightarrow y_2 = \frac{2\lambda D}{d}$ 2nd " " "

$n=3 \Rightarrow y_3 = \frac{3\lambda D}{d}$ 3rd " " "

The positions of minima from the center

$n=0 \Rightarrow y = \frac{\lambda D}{2d}$ - 1st dark band (minimum)

$n=1 \Rightarrow y = \frac{3\lambda D}{2d}$ - 2nd " " "

$n=2 \Rightarrow y = \frac{5\lambda D}{2d}$ - 3rd " " "

fringe spacing (or fringe width) (ω)

is the distance b/w two nearby bright bands or between dark bands is given by

$$\omega = y_{n+1} - y_n$$

$$y_{n+1} = \frac{(n+1)\lambda D}{d}$$

$$y_n = \frac{n\lambda D}{d}$$

$$\omega = \frac{(n+1)\lambda D}{d} - \frac{n\lambda D}{d}$$

$$\boxed{\omega = \frac{\lambda D}{d}} \quad (4)$$

from eqn. (4)

$$\Rightarrow \boxed{\lambda = \frac{d\omega}{D}}$$

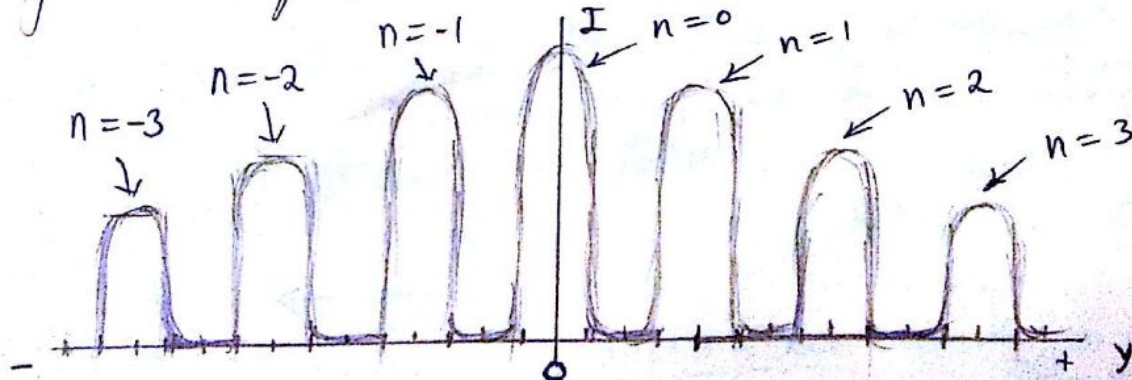
measuring d , ω and D enable us to compute the wave length of the light ray.

~~Ex~~ * Since the bright and dark bands are formed above and below the central position (OO') then considering both directions we express

$$n = 0, \pm 1, \pm 2, \pm 3, \dots$$

+ve integers represent to those above OO' and -ve ones are to those below OO' .

In double slit experiment the bright and dark fringes have the same width and the intensity of ~~fringe~~ (I) as a function of position y is shown below.



The central maximum is the most intense and intensity of bright bands decreases to both sides. as shown in the figure.

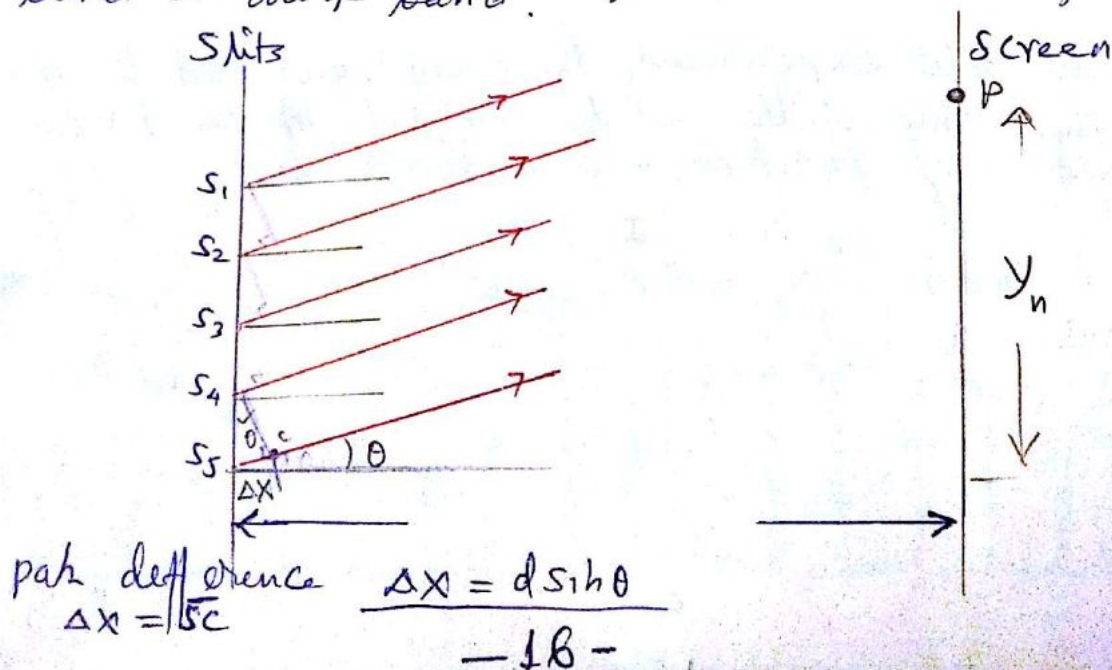
EX. 1. Two narrow parallel slits are separated by 0.3 mm and the slits are 2 m away from the screen. Light rays with unknown wave length which pass through the slits form interference pattern of bright and dark bands on the screen. The fringe width be 3 mm then

- what is the wave length of the light ray
- the distance of the 12^{th} bright band from the center.

Diffraction gratings

To measure the wave length of light rays with much accurate precision, a large number of parallel, equally spaced narrow slits is used. Such a device is called diffraction grating.

When the coherent rays coming through the slits meet on the screen they produce either bright band or dark band.



Let d - be distance b/w near by slits (slit spacing)
 D - screen and slits separation.

Since $D \gg d$ the all the light rays coming from the slits to the point of interference P are ^{approximately} parallel.

Let y_n - be the position of the n^{th} maximum and
the angular position of this maximum be θ_n

$$\text{then } \Delta x = n\lambda \Rightarrow \Delta x = d \sin \theta_n$$

$$\Rightarrow d \sin \theta_n = n\lambda \quad \left[\sin \theta_n = \frac{n\lambda}{d} \right]$$

$$\text{but for small angle } \theta_n \quad \sin \theta_n \approx \tan \theta_n = \frac{y_n}{D}$$

$$\Rightarrow \frac{dy_n}{D} = n\lambda$$

$$\boxed{y_n = \frac{n\lambda D}{d}}$$

where $n = 0, \pm 1, \pm 2, \pm 3, \pm 4, \dots$

If there are N - no of slits in the grating in a length of $L = Nd$, then the number of slits per unit length (N_e) is

$$N_e = \frac{N}{L} = \frac{N}{Nd} \Rightarrow \boxed{N_e = \frac{1}{d}}$$

$$\Rightarrow \sin \theta_n = n\lambda N_e$$

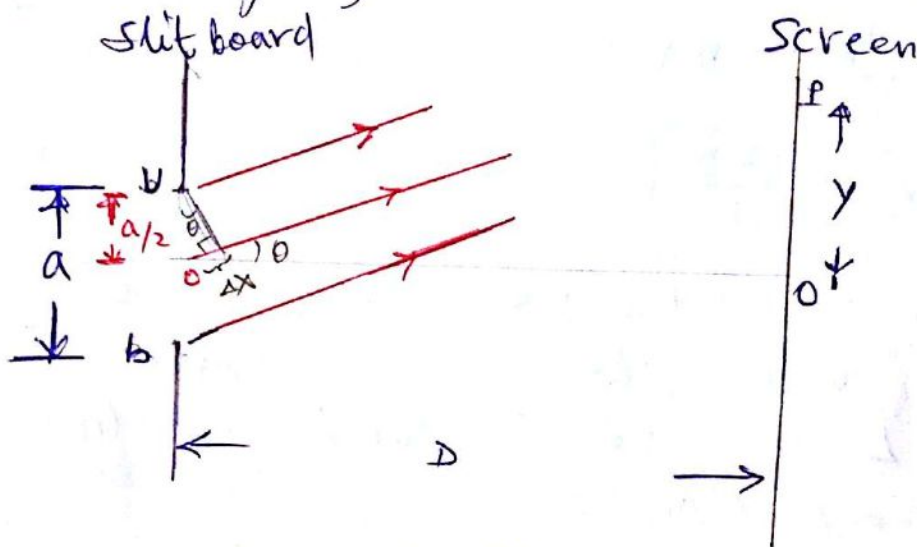
$$\text{and } y_n = n\lambda D N_e$$

EX-2. A light ray of wave length 600nm passes through diffraction gratings of 50 slits per centimeter. That is 600cm away from the screen. What is the position of the 2nd bright band from the central maximum?

Diffraction by single slit and the interference pattern.

As a plane wave passes through a narrow slit every point on the wave front spreads out spherical wavelets so that the rays are diffracted according to the Huygen's principle.

The diffracted rays coming from different points at the slit produce interference pattern on the screen when they meet on the screen. So bright and dark fringes are observed on the screen.



Let the slit width $bu = a$

If the point p be the position of the 1st dark band all rays cancel each other at the point. the paired rays coming between points o to u and b to o cancel each other.

the path difference for the 1st dark band is

$$\Delta x = \frac{a}{2} \sin \theta = \frac{\lambda}{2} \Rightarrow a \sin \theta = \lambda$$

for the 2nd dark band is

$$\Delta x = \frac{a}{2} \sin \theta = \frac{\lambda}{2} \Rightarrow a \sin \theta = 2\lambda$$

for the 3rd dark band is

$$\Delta x = \frac{a}{2} \sin \theta = \frac{\lambda}{2} \Rightarrow a \sin \theta = 3\lambda$$

In general in a single slit experiment the position of the m -dark band (minima) is given by

$$a \sin \theta = m \lambda$$

where $m = 1, 2, 3, 4, \dots$

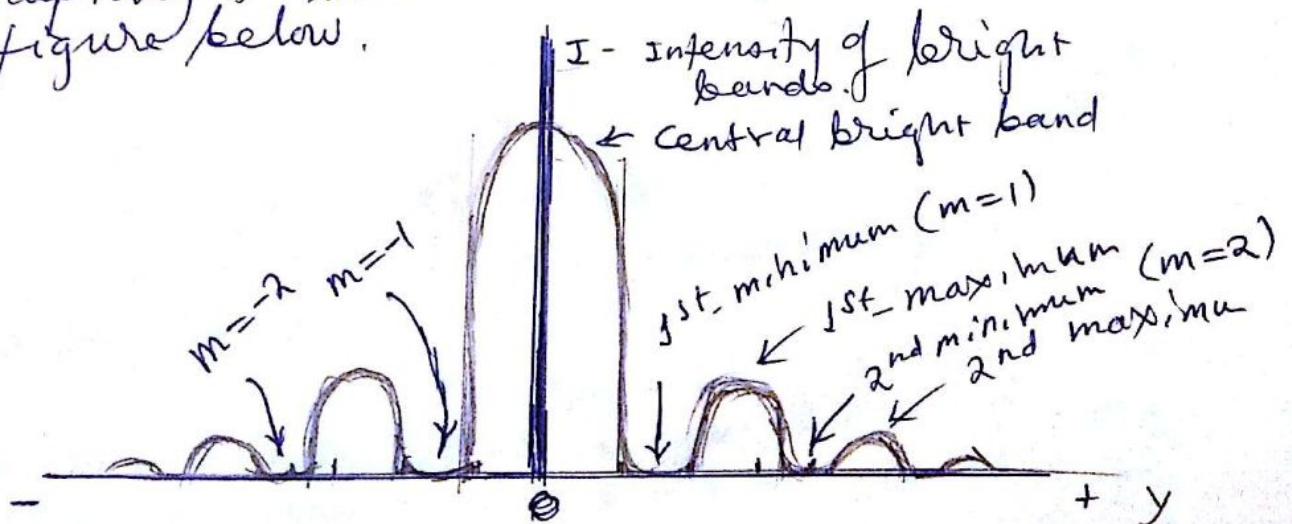
and the position (distance) of m^{th} -dark band to be y_m

$$\sin \theta \approx \tan \theta = \frac{y_m}{D}$$

$$\Rightarrow y_m = D \sin \theta$$

$$y_m = \frac{m \lambda D}{a}$$

In single slit experiment interference pattern the fringes are not spaced equally, and the central bright band is the most intense and the intensity of dark bright bands decreased rapidly to both directions as shown in the figure below.

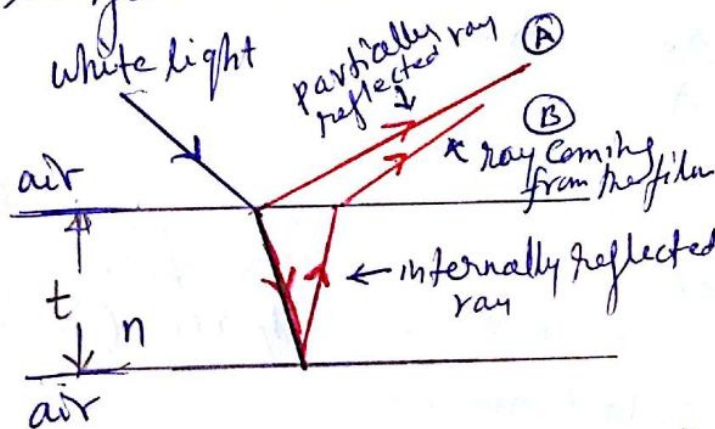


As we see the width of the central bright band is twice wider than that of the 1st bright band.

Thin film interference pattern

Soap bubbles and thin layers of oil floating on water surface are common examples of thin films. When white light falls on the surface part of it is reflected from the upper surface and the other part is refracted in to the film this part is dispersed in to the different colours in the film and some rays are reflected with in the film at the lower surface and transmitted back to the air as shown in the figure below.

Due to the interference of light rays reflected at the upper surface and those reflected from the lower surface we observe coloured patches on the surface.



Let t - be thickness of the film

n - refractive index of the film.

$n_{air} = 1$ refractive index of air

λ - wave length in the film

λ_0 wave length in the air (free space)

for a very thin film (t) and nearly normal incidence of the incident ray the path difference between the two rays A and B is

$$\Delta x = 2t$$

The ray A reflected at the upper surface is out of phase by π rad than the ray B reflected with in the film. Then additional path difference due to this is

$$\pi \equiv \frac{\lambda}{2}$$

$\Rightarrow$ total path difference is

$$\Delta x_t = 2t + \frac{\lambda}{2}$$

for constructive interference $\Delta x_t = m\lambda$

$$\Rightarrow m\lambda = 2t - \frac{\lambda}{2} \quad m = 0, 1, 2, 3, \dots$$

$$\boxed{2t = m\lambda + \frac{\lambda}{2}} \quad 2t = (m + \frac{1}{2})\lambda$$

for destructive interference $\Delta x_t = (m + \frac{1}{2})\lambda$

$$(m + \frac{1}{2})\lambda = 2t - \frac{\lambda}{2}$$

$$\boxed{2t = m\lambda}$$

$$\text{but } n = \frac{c}{v} = \frac{f\lambda_0}{f\lambda} = \frac{\lambda_0}{\lambda}$$

$$\Rightarrow \lambda = \frac{\lambda_0}{n}$$

In terms of the refractive index n of the film and wave length of the light ray in the free space (λ_0) we can express the relations as

for constructive interference

$$2t = (m + \frac{1}{2}) \frac{\lambda_0}{n} \Rightarrow \boxed{2nt = (m + \frac{1}{2})\lambda_0}$$

for destructive interference

$$2t = m\lambda \Rightarrow 2t = \frac{m\lambda_0}{n}$$

$$\Rightarrow \boxed{2nt = m\lambda_0}$$