UNIT 1: LINEAR EQUATIONS AND THEIR INTERPRETATIVE APPLICATIONS IN BUSINESS

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1.0AIMS AND OBJECTIVES

After reading the chapter students must be able to:

- define algebraic expression, equation & linear equation
- explain the different ways of formulating or developing equations of a line
- understand the breakeven point and its application
- define the cost output relation ship
- explain the different cost elements

1.1 INTRODUCTION

Mathematics, old and newly created, coupled with innovative applications of the rapidly evolving electronic computer and directed toward management problems, resulted in a new field of study called quantitative methods, which has become part of the curriculum of colleges of business. The importance of quantitative approaches to management problems is now widely accepted and a course in

mathematics, with management applications is included in the core of subjects studied by almost all management students. This manual develops mathematics in the applied context required for an understanding of the quantitative approach to management problems.

1.2 Linear Equations

***Equation*:** - A mathematical statement which indicates two algebraic expressions are

equal

Example: Y = 2X + 3

Algebraic expressions: - A mathematical statement indicating that numerical quantities

are linked by mathematical operations.

Example: X + 2

Linear equations: - are equations with a variable & a constant with degree one.

- Are equations whose terms (the parts separated by +, -, = signs)
- Are a constant, or a constant times one variable to the first power

Example: 2X – 3Y = 7

- the degree (the power) of the variables is 1
- the constant or the fixed value is 7
- the terms of the equation are 2X and 3Y separated by – sign

However 2X + 3XY = 7 isn't a linear equation, because 3XY is a constant times the product of 2 variables.

* No X^2 terms, No X/Y terms, and no XY terms are allowed.

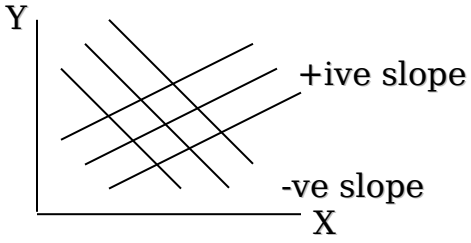
- Linear equations are equations whose slope is constant throughout the line.

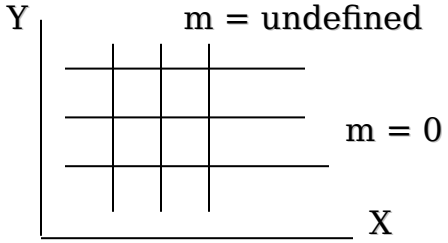
- The general notion of a linear equation is expressed in a form Y = mx + b where m = slope, b = the Y- intercept, Y = dependent variable and X = independent variable.

If Y represents Total Cost, the cost is increased by the rate of the amount of the slope m.

$$\text{Slope (m)} = \frac{\Delta Y}{\Delta X} = \frac{rise(fall)}{run} = \frac{Y_2 - Y_1}{X_2 - X_1} \text{ if } X_1 \neq X_2$$

Slope measures the steepness of a line. The larger the slope the more steep (steeper) the line is, both in value and in absolute value.





- A line that is parallel to the X-axis is the gentlest of all lines i.e. m = 0
- A line that is parallel to the Y-axis is the steepest of all lines i.e. m = undefined or infinite.

The slope of a line is defined as the change-taking place along the vertical axis relative to the corresponding change taking place along the horizontal axis, or the change in the value of Y relative to a one-unit change in the value of X.

1.2.1 Developing equation of a line

There are at least three ways of developing the equation of a line. These are:

1. The slope-intercept form
2. The slope-point form
3. Two-point form

1. The slope-intercept form

This way of developing the equation of a line involves the use of the slope & the intercept to formulate the equation.

Often the slope & the Y-intercept for a specific linear function are obtained directly from the description of the situation we wish to model.

Example # 1

Given Slope = 10

Y-intercept = +20, then

Slope-intercept form: the equation of a line with slope = m and Y-intercept b is

$Y = mx + b$

$Y = 10X + 20$

Interpretative Exercises

#2 Suppose the Fixed cost (setup cost) for producing product X be br. 2000. After setup it costs br. 10 per X produced. If the total cost is represented by Y:

1. Write the equation of this relationship in slope-intercept form.
2. State the slope of the line & interpret the number
3. State the Y-intercept of the line & interpret the number

#3. A sales man has a fixed salary of br. 200 a week In addition; he receives a sales commission that is 20% of his total volume of sales. State the relationship between the sales man's total weekly salary & his sales for the week.

Answer $Y = 0.20X + 200$

2. The slope point form

The equation of a non-vertical line, L, of slope, m, that passes through the point (X_1, Y_1) is: defined by the formula $Y - Y_1 = m (X - X_1)$

$Y - Y_1 = m (X - X_1)$

Example #1 ⇒ $Y - 2 = 4 (X - 1)$

Given, slop = 4 and $Y - 2 = 4X - 4$

Point = (1, 2) $Y = 4X - 2$

#2 A sales man earns a weekly basic salary plus a sales commission of 20% of his total sales. When his total weekly sales total br. 1000, his total salary for the week is 400. derive the formula describing the relationship between total salary and sales.

Answer $Y = 0.2X + 200$

#3 If the relationship between Total Cost and the number of units made is linear, & if costs increases by br. 7.00 for each additional unit made, and if the Total Cost of 10 units is br. 180.00. Find the equation of the relationship between Total Cost (Y) & number of units made (X)

Answer: $Y = 7X + 110$

3. Two-point form

Two points completely determine a straight line & of course, they determine the slope of the line. Hence we can first compute the slope, then use this value of m together with either point in the point-slope form $Y - Y_1 = m (X - X_1)$ to generate the equation of a line.

⇒ By having two coordinate of a line we can determine the equation of the line.

Two point form of linear equation: $(Y - Y_1) = \left(\frac{Y_2 - Y_1}{X_2 - X_1}\right)(X - X_1)$

Example #1 given (1, 10) & (6, 0)

First slope = $\frac{0 - 10}{6 - 1} = \frac{-10}{5} = -2$, then

$Y - Y_1 = m (X - X_1)$ ⇒ $Y - 10 = -2 (X - 1)$

$Y - 10 = -2X + 2$

$$Y = -2X + 12$$

#2 A salesman has a basic salary &, in addition, receives a commission which is a fixed percentage of his sales volume. When his weekly sales are Br. 1000, his total salary is br. 400. When his weekly sales are 500.00, his total salary is br. 300. Determine his basic salary & his commission percentage & express the relationship between sales & salary in equation form.

Answer: $Y = 0.2X + 200$

#3 A printer costs a price of birr 1,400 for printing 100 copies of a report & br. 3000 for printing 500 copies. Assuming a linear relationship what would be the price for printing 300 copies?

Answer: $Y = 4X + 1000$

Cost = 4.0 (300) + 1000 = br. 2200

1.2.2 Special formats

a) Horizontal & vertical lines

When the equation of a line is to be determined from two given points, it is a good idea to compare corresponding coordinates because if the Y values are the same the line is horizontal & if the X values are the same the line is vertical

Example: 1 Given the points (3, 6) & (8, 6) the line through them is horizontal because

both Y-coordinates are the same i.e. 6

The equation of the line becomes Y = 6,which is different from the form Y = mx + b If the X-coordinates of the two different points are equal

Example (5, 2) & (5, 12) the line through them is vertical, & its equations is X = 5 i.e. X is equal to a constant. If we proceed to apply the point slope procedure, we would obtain.

Slope (M) = $\frac{12-10}{5-5} = \frac{2}{0} = \infty$ & if m = ∞ infinite the line is vertical & the form of the equation is: X = constant

b) Parallel & perpendicular lines (// and ⊥)

Two lines are parallel if the two lines have the same slope, & two lines are perpendicular if the product of their slope is –1 or the slope of one is the negative reciprocal of the slope of the other. However, for vertical & horizontal lines. (They are perpendicular to each other), this rule of M_1 (the first slope) times M_2 (the second slope) equals –1 doesn't hold true. i.e. $M_1 \times M_2 \neq -1$

Example: Y = 2X – 10 & Y = 2X + 14 are parallel because their slope are equal i.e. 2

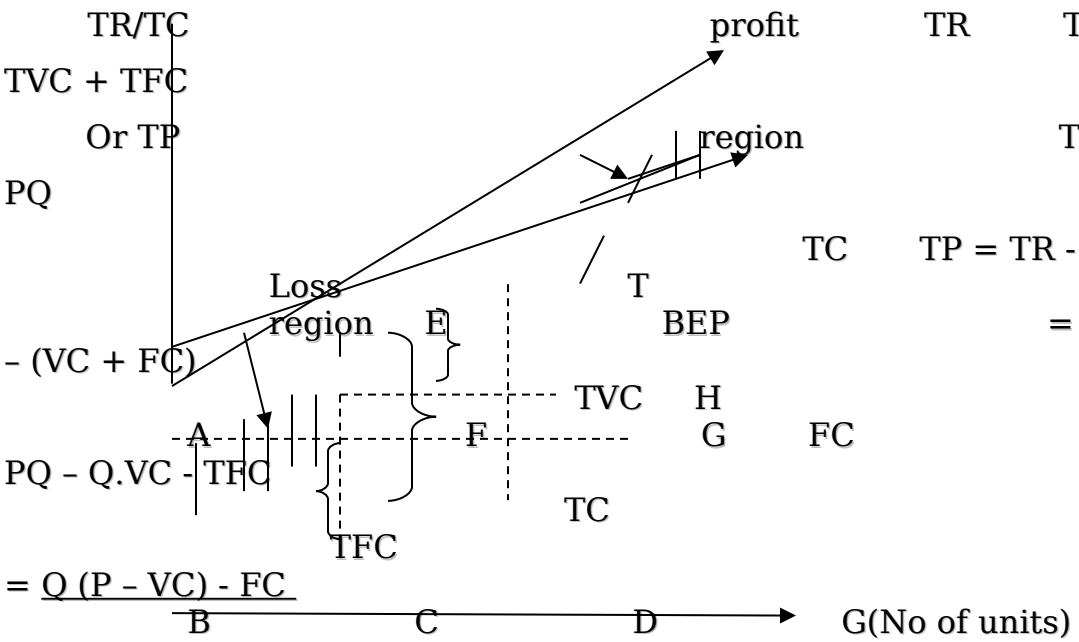
Y = 3/2X + 10 & Y = -2/3X + 100 are perpendicular to each other because the multiplication result of the two slope are –1 i.e. $\frac{3}{2} x \frac{-2}{3} = -1$

c) Lines through the origin

Any equation in the variables X & Y that has no constant term other than zero will have a graph that passes through the origin. Or, a line which passes through the origin has an X-intercept of (0, 0) i.e. both X and Y intercepts are zero.

1.3 Application of linear equations

1.3.1 Linear cost output relationships – VC, FC, TC, AC, MC, TR, Π:



Where Q = units

Product & units

sold in revenue

TC = Total Cost

FC = Fixed Cost

VC = Unit variable Cost

Interpretation of the graph:

1. The vertical distance between AB, FC, GD is the same because Fixed Cost is the same at any levels of output.

2. There is no revenue without sales (because Total Revenue function passes through the origin), but there is cost without production (because of Fixed Cost) & the TC function starts from A & doesn't pass through the origin

3. Up to point T, Total Cost is greater than Total Revenue ⇒ results in loss. While at point T, (Total Revenue = Total Cost) i.e. Breakeven. (0 profit), & above point T, TR > TC ⇒ +ve profit.
4. TFC remains constant regardless of the number of units produced. Given that there is no any difference in scale of production.

5. As production increases, Total Variable Cost increases at the same rate and Marginal cost is equal with Unit Variable Cost (MC = VC) only in linear equations.

6. As production increases TC increases by the rate equal to the AVC = MC (average cost equal to marginal cost)

7. AVC is the same through out any level of production, however Average Fixed Cost (AFC) decreases when Quantity increases & ultimately ATC decreases when Q increases because of the effect of the decrease in AFC.

8. As Quantity increases TR increases at a rate of P. and average revenue remains constant.

$$AR = \frac{TR}{Q} = \frac{P.Q}{Q} = P \Rightarrow AR = P \text{ in linear functions}$$

1.2.2 Breakeven Analysis

Break-even point is the point at which there is no loss or profit to the company. It can be expressed as either in terms of production quantity or revenue level depending on how the company states its cost equation.

Manufacturing companies usually state their cost equation in terms of quantity (because they produce and sell) where as retail business state their cost equation in terms of revenue (because they purchase and sell)

Case 1: Manufacturing Companies

Consider a Company with equation

TC = VC + FC / Total cost = Variable cost + Fixed cost

TR = PQ/ Total Revenue = Price x Quantity

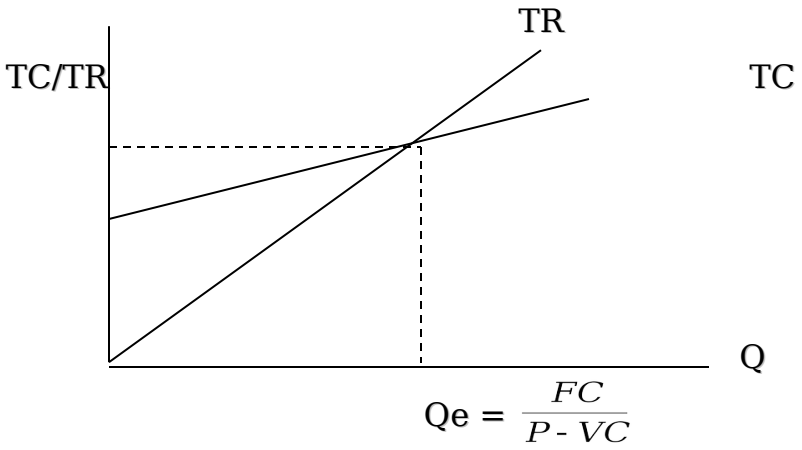
At Break-even point, TR = TC i.e TR – TC = 0

PQ = VC + FC where Qe = Breakeven Quantity

PQ – VC.Q = FC FC = Fixed cost

Q (P – VC) = FC P = unit selling price

$$Qe = \frac{FC}{P - VC}$$ VC = unit variable cost



Example #1 A manufacturing Co. has a Total Fixed Cost of Br. 10,000 & a Unit Variable Cost of Br. 5. if the co. can sell .What it produces at a price of Br. 10,

a) Write the Revenue, cost & Profit functions
b) Find the breakeven point in terms of quantity and sales volume
c) Show diagrammatically the Total Revenue, Total Cost, Total Profit, Fixed Cost and Variable Costs.
d) Interpret the results

Answer

a) TC = VC + FC TR = PQ Profit(Π) = TR – TC

TC = 5Q + 10,000 TR = 10Q = 10Q – (5Q + 10,000)

= 5Q – 10,000

b) At break even point TR = TC

$10Q = 5Q + 10{,}000$

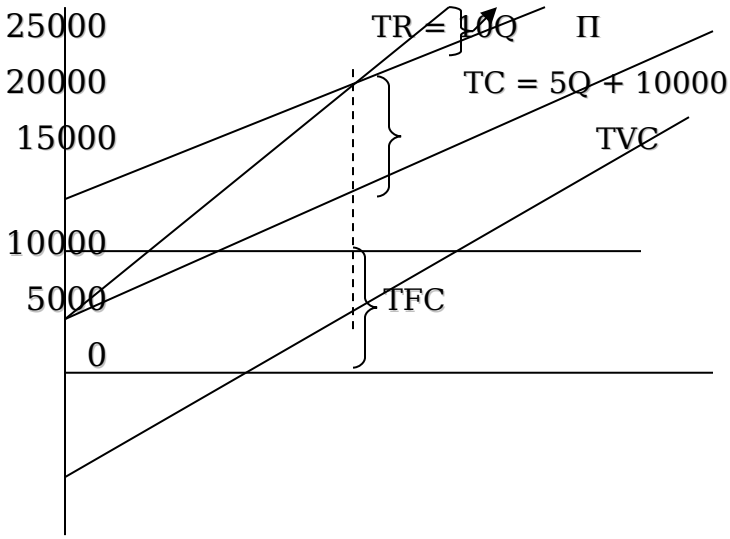
$5Q - 10{,}000 = 0$

$Qe = \frac{10{,}000}{5}$

Qe = 2000 units

i.e. Breakeven Quantity is 2000 units

Sales volume = 2000 X 10 = 20,000 br.



1000 2000 3000 4000 5000 Q (no. of units produced & sold)

-5000 -

-10000 -

Interpretation:

When a co. produces & sells 2000 units of output, there will not be any loss or gain (no profit, no loss)

⇒ The effect of changing one variable keeping other constant

Case 1 - Fixed cost

Assume for the above problem FC is decreased by Br. 5000, Citrus Paribus (other things being constant)

TC = 5Q + 5000 ⇒ Qe = $\frac{5000}{5}$ = 1000 units

TR = 10Q

Therefore, FC ↓ → Qe ↓ FC & Qe have direct relationship

FC ↑ → Qe ↑

Case 2-Unit variable cost

Assume for the above problem UVC decreased by 1 br. Citrus Paribus (keeping other thing constant)

TC = 4Q + 10000 ⇒ Qe = $\frac{10000}{6} = \underline{\underline{1,667 unit.}}$

TR = 10Q

Therefore, VC ↓ → Qe ↓ VC & Qe have direct relationship

VC ↑ → Qe ↑

Case 3- Selling price

Assume for the above problem selling price is decreased by br. 1, Citrus Paribus,

TC = 5Q + 10,000 $\Rightarrow$ Qe = $\frac{10000}{4} = \underline{\underline{2500 units}}$

TR = 9Q

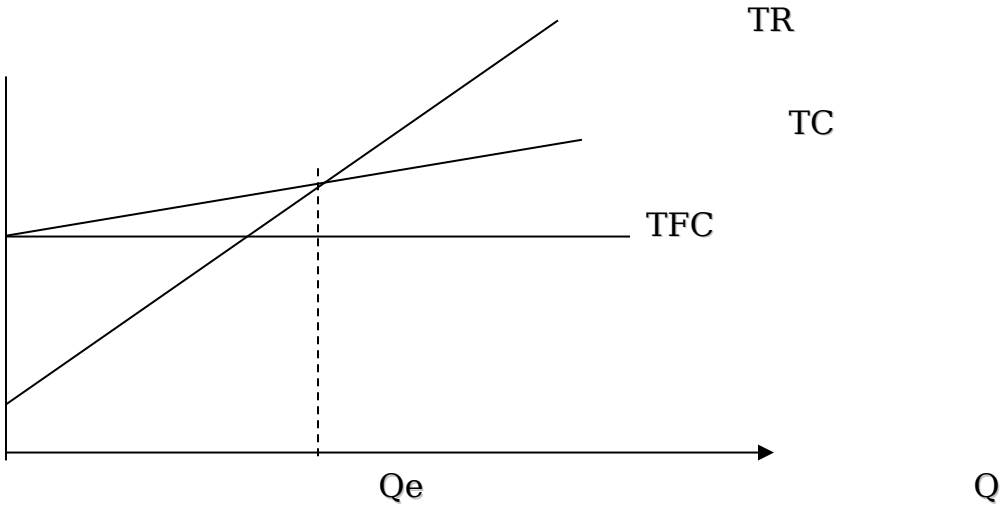
Therefore P ↓ → Qe ↑ Price and breakeven point have indirect relationship

P ↑ → Qe ↓

In the above example a company has the following options (to minimize its breakeven point and maximize profit).

- decreasing FC
- decreasing unit VC
- increasing the unit selling price

And if the organization is between option 2 & 3, it is preferable to decrease the unit variable cost because if we increase the selling price, the organization May loose its customers & also decreasing the FC is preferable.



Finding the quantity level which involves profit or loss

BEP = $\frac{FC+0}{P-V}$, any Q is related to the cost , profit----

$\Pi = TR - Tc$ Where: BEP = breakeven point

$\Pi = PQ - (VC.Q + FC)$ Π = Profit

$\Pi = (P.Q - VC.Q) - FC$ TR = Total revenue

$\Pi = Q(P - VC) - FC$ TC = Total cost

$\frac{\pm\Pi + FC}{P - VC} = for\ any\ qty\ level\ Q = \frac{FC \pm \Pi}{P - VC}$ Q = Quantity

∀C = Unit variable cost

Example #1 For the above manufacturing co. if it wants to make a profit of 25000 br. What should be the quantity level?

TR = 10Q $Q = \frac{FC \pm \Pi}{P - V}$ when there is Π, the quantity produced & sold have to be greater than the Breakeven quantity

TC = 5Q + 10,000 $= \frac{10{,}000 + 25000}{10 - 5}$

Π = 25,000 = 7000 units

Q = ?

If it expects a loss of br. 5000 what will be the quantity level.

$Q = \frac{FC \pm \overline{VC}}{P - VC} = \frac{10{,}000 - 5000}{10 - 5}$ * when there is loss, the qty produced & sold should be less than the BEQ

Case 2 Merchandising /Retail Business

Breakeven Revenue = BEQ X P

Assume a bus. Firm with product A has the following cost & revenue items.

Variable cost of A = 100 br.

Selling price = 150 br.

Markup = Selling price – Variable cost = 150 – 100 = 50

i. as a function of cost, the markup is 50/100 = 50%

ii. as a function of retail price, the markup is 50/150 = 33.3 % it is also called margin.

Margin ↑ Cost of goods sold ↗

The cost of goods sold = 100% - 33.3 % = 66.6% ≈ 67%

Selling price ↓ ↘ CGS

Given other selling expense = 1%of the selling price i.e. 0.01X

So, the TC equation becomes:

$Y = 0.68X + FC$

Where: X is sales revenue

Y is total cost

Out of 100% selling price 68% is the variable cost of goods purchased & sold

Example Suppose a retail business sale its commodities at a margin of 25% on all items purchased & sold. Moreover the company uses 5% commission as selling expense & br. 12000 as a Fixed Cost.

Find the Breakeven revenue for the retail business after developing the equation

Solution Selling price 100% Let X represents selling price

Margin	25%	Y = total cost
CGS	75%	FC = 12000

Comm. Exp. 5% Xe = Breakeven revenue

Total VC 80%

$\therefore Y = 0.8X + 12000$

Break even revenue is obtained by making sales revenue & cost equals

At breakeven point TC = TR ↓ Y = mx + b

i.e. Y = X then, unit variable cost

$0.8X + 12000 = X \Rightarrow$

$\frac{FC}{1- m}$ or $\frac{FC \pm \Pi}{1- m}$ where $m = \frac{VC}{P} = \frac{TVC}{TR}$

$-0.2X = -12000$

$X = 60{,}000$ br. ⇒ When the co. receives br. 60,000 as sales revenue, there will be no loss or profit.

The Breakeven revenue (BER = $\frac{FC \pm \Pi}{1- m}$) method is useful, because we can use a single formula for different goods so far as the company uses the same amount of profit margin for all goods. However, in Breakeven quantity method or BEQ = $\frac{FC \pm \Pi}{P- V}$ it is not possible and hence we have to use different formula for different items.

Example #1 It is estimated that sales in the coming period will be br. 6000 & that FC will be br. 1000 & variable costs br. 3600, develop the total cost equation & the breakeven revenue.

Answer: $Y = \frac{3600}{6000}X + 1000 = 0.6X + 1000$

Where Y = Total Cost

X = Total revenue

$$BER = Xe = \frac{1000}{1-0.6} = \frac{1000}{0.4} = \underline{\underline{2500 br.}}$$

At the sales volume of br. 2500, the company breaks even.

* When the breakeven revenue equation is for more than one item it is impossible to find the breakeven quantity. It is only possible for one item by Qe = Xe/P

Where Xe = Break even revenue

P = selling price

Qe = Breakeven quantity

To change the breakeven revenue equation in to Breakeven quantity . We have to multiple price by the coefficient of X. likewise, to change in to breakeven revenue from Break even quantity, we have to divide the unit VC by price.

1.4 Model examination questions

1. XYZ company's cost function for the next four months is

   C = 500,000 + 5Q

   a) Find the BE dollar volume of sales if the selling price is br. 6 / unit
   b) What would be the company's cost if it decides to shutdown operations for the next four months
   c) If, because of strike, the most the company can produce is br. 100,000 units, should it shutdown? Why or why not?

2. In its first year, "Abol Buna Co" had the following experience

   Sales = 25,000 units Selling price = br. 100

   TVC = br. 1,500,000 TFC = br. 350,000

Required:

1. Develop Revenue, cost & profit functions for the co. in terms of quantity.

2. Find the Breakeven point in terms of quantity
3. Convert the cost equation in terms of quantity in to a cost equation in terms of revenue
4. Find the Breakeven revenue
5. If profit had been br. 500,000 what would have been the sales volume (revenue) & the quantity of sales
6. What would have been the profit if sales are br. 2,000,000.

3. A small home business set up with an investment of $ 10,000 for equipment. The business manufactures a product at a cost of br. 0.64 per unit. If the product sales for Br. 1.20 per unit how many units must be sold before the business breaks even?
4. A retail co. plans to work on a margin of 44% of retail price & to incur other Variable Cost of 4%. If is expected Fixed cost of Br. 20,000.
    i. Find the equation relating Total Cost to sales
    ii. Find the profit if sales are Br. 60,000
    iii. Find the breakeven revenue
    iv. If profit is Br. 15,000 what should be the revenue level?
    v. If you have any one item at a price of Br. 15/unit how do you convert the cost equation in terms of revenue in to a cost equation in terms of quantity?