

Electric Potential

- Chapter 17 (Giancoli)
- All sections except 17.6 (electric dipoles)

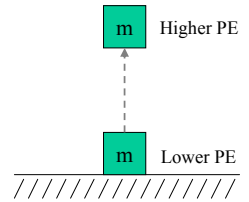
Electric Potential and Potential Energy

Work-energy theorem:

Change in potential energy = work done

Gravitational Potential Energy

It requires a certain amount of work to raise an object of mass m from the ground to some distance above the ground.

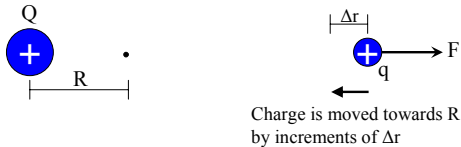


i.e. We have increased the potential energy of the object.

$$PE = W = \text{Force} \times \text{Displacement}$$

Electric Potential Energy

Find the work done in bringing a charge q from infinitely far away to a distance R from charge Q .



For a small displacement Δr , the work done is:

$$\Delta W = \text{force} \times \text{displacement} = -F \cdot \Delta r$$

(we have a negative sign as the direction of the force is opposite to the direction of the displacement)

$$\Delta W = -F \Delta r = -\left(\frac{kqQ}{r^2}\right) \cdot \Delta r$$

$$PE = \int_{\infty}^R \Delta W = \int_{\infty}^R -\frac{kqQ}{r^2} \Delta r$$

$$= kqQ \left[\frac{1}{r} \right]_{\infty}^R$$

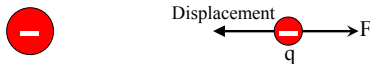
$$PE = \frac{kqQ}{R}$$

*Note: $PE = 0$ when $R \Rightarrow \infty$

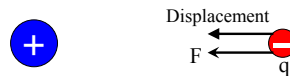
This is the PE of a charge q when it is a distance R from Q .

- PE is a **scalar** quantity
- The sign of the charges **must be kept** in all calculations
- Depending on the signs of Q and q , the PE could be **positive** or **negative**

If the PE is **positive** (when the charges are both positive or both negative), then work must be **done on** the charge q to bring it closer to Q , increasing its PE.



If the PE is **negative** (when the charges have opposite signs), then the work is **done by** the charge, decreasing its PE.



Potential Energy of a System of Charges :

The potential energy (PE) of a charge or a collection of charges is the amount of **work** needed in assembling the charges.

Example: What is the PE of a charge Q located at point P?



PE is the work done in bringing the charge from ∞ to point P.

Work done = force $\times$ displacement = 0, as there is no force acting on the charge. $\Rightarrow$ PE = 0

Electric Potential

Definition: Define the electric potential, V, at a point by the PE of a charge at that point divided by the value of the charge.

$$V = \frac{PE}{q} \quad \text{Units for V: J/C}$$

The potential, similar to PE, is a **scalar** quantity.
i.e. We must **keep** the sign of the charges in all calculations

For a point charge, Q'

$$V = \left(\frac{kqQ'}{r} \right) \frac{1}{q}$$

$$V = \frac{kQ'}{r}$$

This is the potential a distance r from the charge Q'.

* This property of the charge Q' is more fundamental than PE.

If we know the potential at a point in space, when we place a charge Q at that point, its PE can be directly give by:

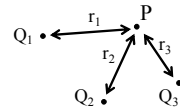
$$PE = QV$$

The potential V at a point can be due to the presence of a single charge or due to many charges.

For many charges, $Q_1, Q_2, Q_3, \dots$, the potential at a point P is given by the **superposition** principle:

$$V_p = V_1 + V_2 + V_3 + \dots$$

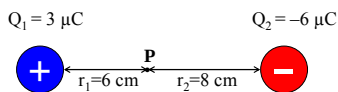
$$= \frac{kQ_1}{r_1} + \frac{kQ_2}{r_2} + \frac{kQ_3}{r_3} + \dots$$



* Remember to include the sign of the charges $Q_1, Q_2, Q_3, \dots$, when calculating V_p .

* If a charge Q is placed at point P its potential energy will be:
 $PE = Q V_p$

Example : a) Calculate the potential at point P in the figure;



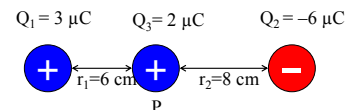
$$V_p = V_1 + V_2$$

$$= \frac{kQ_1}{r_1} + \frac{kQ_2}{r_2}$$

$$= \frac{(9 \times 10^9 \text{ N} \cdot \text{m}^2 / \text{C}^2)(3 \times 10^{-6} \text{ C})}{(0.06 \text{ m})} + \frac{(9 \times 10^9 \text{ N} \cdot \text{m}^2 / \text{C}^2)(-6 \times 10^{-6} \text{ C})}{(0.08 \text{ m})}$$

$$= -2.25 \times 10^5 \text{ J/C}$$

b) What is the **potential energy** of a charge $Q_3 = 2 \mu\text{C}$ when placed at P?



$$PE = Q V_p$$

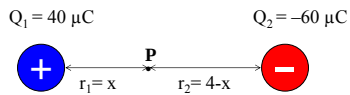
$$= (2 \times 10^{-6} \text{ C})(-2.25 \times 10^5 \text{ J/C})$$

$$= -0.45 \text{ J}$$

Potential is not affected by the presence of a charge at that point. (Charges cannot contribute to their own potential)

If free to move, an object will always try to lower its PE.

Example: Along a line joining the two charges, $Q_1 = 40 \mu\text{C}$ and $Q_2 = -60 \mu\text{C}$, separated by 4 m, where does V vanish?



Q_2 has more charge than Q_1 , therefore **P must be closer to Q_1**
($V \sim Q/r$)

$$V_p = V_1 + V_2 = 0$$

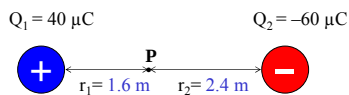
$$\frac{kQ_1}{r_1} + \frac{kQ_2}{r_2} = 0$$

$$\Rightarrow \frac{40 \times 10^{-6} \text{ N} \cdot \text{m}^2 / \text{C}}{(x)\text{m}} - \frac{60 \times 10^{-6} \text{ N} \cdot \text{m}^2 / \text{C}}{(4-x)\text{m}} = 0$$

$$\Rightarrow 40(4-x) = 60x$$

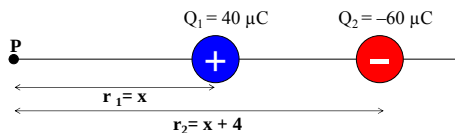
Solving for x , we get $x = 1.6 \text{ m}$

Therefore,



Are there **other** solutions?

➡ To the **left** of Q_1



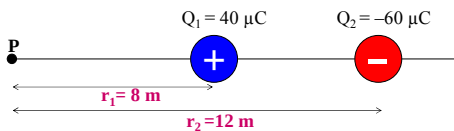
$$V_p = V_1 + V_2 = 0$$

$$\frac{kQ_1}{r_1} + \frac{kQ_2}{r_2} = 0$$

$$\Rightarrow \frac{40 \times 10^{-6} \text{ N} \cdot \text{m}^2 / \text{C}}{(x)\text{m}} - \frac{60 \times 10^{-6} \text{ N} \cdot \text{m}^2 / \text{C}}{(x+4)\text{m}} = 0$$

$$\Rightarrow 40(x+4) = 60x$$

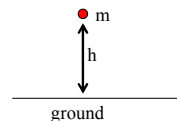
Solving for x , we get $x = 8 \text{ m}$



Also, $V=0$ at +ve infinity, and -ve infinity
Therefore, the potential energy of a charge Q at P is equal to $\text{PE} (\infty) = 0$.

Sometimes it is not necessary to know the absolute value of V or PE .

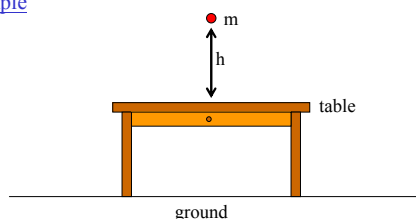
Example



For convenience, we can take the PE of the object of mass m to be zero when it is on the ground.

Then, at a height h above the ground, its PE , relative to the ground is mgh .

Example



In this example, we may choose to take the zero of PE at the top of the table. Then the PE of the object a distance h above the table is mgh .

* The zero of PE is chosen for convenience.

Similarly for the electric potential and potential energy, sometimes we are interested in the changes in V and PE.

$$\Delta V = \frac{\Delta PE}{q}$$

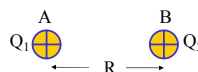
Where ΔV is the potential difference and ΔPE is the change in the potential energy, measured from a convenient reference point.

Experimentally, we always perform measurements in the laboratory (and not at infinity!), so the symbol ' Δ ' is not really used, i.e. $V \equiv \Delta V$.

Historically, the difference in potential was measured or calculated with the unit of volts [V] while the absolute potential was measured or calculated with the unit of joules per coulomb [J/C]. The two units are equivalent and are now used interchangeably.

$$1V = 1 J/C$$

Example: What is the PE of the two charges ?



Bring one charge at a time from ∞ , calculating the work done in each case. The PE of the system of the two charges is the total work done.

For the first charge, Q_1 , the work done is zero, as V_A is zero, $W_1 = 0$.

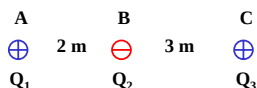
For the second charge, Q_2 , the potential at point B is not zero.

$$V_B = \frac{kQ_1}{R} \quad \Rightarrow \quad W_2 = V_B Q_2 = \left(\frac{kQ_1}{R} \right) Q_2 = \frac{kQ_1 Q_2}{R}$$

$$PE = W_1 + W_2 = \frac{kQ_1 Q_2}{R}$$

Note: This is also the PE of Q_1 or Q_2 .

Example : Consider three charges, $Q_1 = 2 \mu C$, $Q_2 = -1 \mu C$ and $Q_3 = 3 \mu C$ located at points A, B and C as shown in the diagram. Calculate the PE of the three charges.

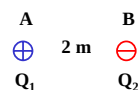


Solution : Assume that the three charges are infinitely far away. Bring the charges, one at a time, each time calculating the work done on the charge. The total work done is the PE of the system of the three charges (**work-energy theorem**).

1. Start with Q_1 . The work done, W_1 , in bringing the charge from ∞ to point A is given by:

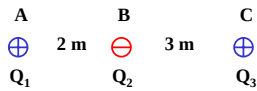
$$W_1 = Q_1 V_A = 0 \quad \{ \text{as } V_A = 0 \text{ (no charges are present)} \}$$

2. Next bring Q_2 . The work done is $W_2 = Q_2 V_B$. V_B is not zero as Q_1 is present at point A.



$$W_2 = Q_2 V_B = Q_2 (kQ_1/R) = -0.009 J$$

3. For the last charge, Q_3 , the work done is $W_3 = Q_3 V_C$, where both Q_1 and Q_2 contribute to the potential at point C.

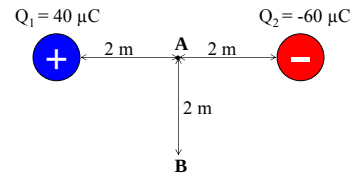


$$W_3 = Q_3 \{ (kQ_1/R_1) + (kQ_2/R_2) \} = 0.0018 \text{ J}$$

$$\text{Total work done} = W_1 + W_2 + W_3 = -0.0072 \text{ J}$$

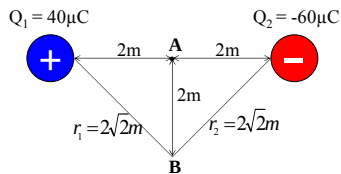
$$\therefore \text{PE} = -0.0072 \text{ J}$$

Example : Two charges, $Q_1=40 \mu\text{C}$ and $Q_2=-60 \mu\text{C}$ are 4 m apart. How much **work** is done in moving a charge $Q=50 \mu\text{C}$ from point A to point B?



Work done = change in potential energy

$$= QV_B - QV_A$$



$$V_B = \frac{kQ_1}{r_1} + \frac{kQ_2}{r_2}$$

$$= \frac{(9 \times 10^9 \text{ N} \cdot \text{m}^2 / \text{C}^2)(40 \times 10^{-6} \text{ C})}{(2\sqrt{2}) \text{ m}} - \frac{(9 \times 10^9 \text{ N} \cdot \text{m}^2 / \text{C}^2)(60 \times 10^{-6} \text{ C})}{(2\sqrt{2}) \text{ m}}$$

$$= -6.39 \times 10^4 \text{ J/C}$$

$$\text{Work done} = Q (V_B - V_A)$$

$$= 50 \times 10^{-6} \text{ C} (-6.39 \times 10^4 \text{ J/C} - (-9 \times 10^4 \text{ J/C}))$$

$$= 1.3 \text{ J}$$

Conservation of Energy

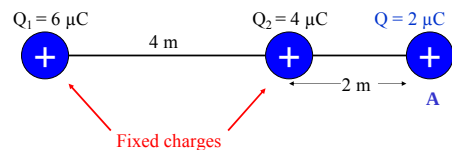
In many situations, it is very useful to use the conservation of energy principle.

$$[\text{Total Energy at point A}] = [\text{Total Energy at point B}]$$

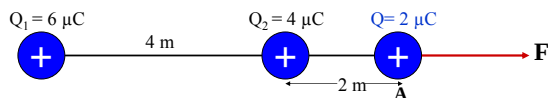
$$(\text{KE} + \text{PE})_A = (\text{KE} + \text{PE})_B$$

$$\frac{1}{2} mv_A^2 + QV_A = \frac{1}{2} mv_B^2 + QV_B$$

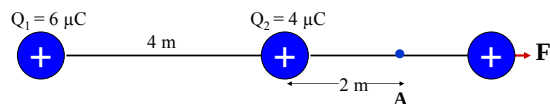
Example : Two fixed charges, $Q_1 = 6 \mu\text{C}$ and $Q_2 = 4 \mu\text{C}$ are 4 m apart. A charge of $2 \mu\text{C}$ and mass 2 g is placed at A. If this charge is released from rest where will it go and can one determine its final velocity (if yes, find the final velocity)?



The charge Q will move in the **direction** of the force acting on it. Therefore, the charge Q moves to the right.



The charge will move in the **direction** of the force acting on it. The charge moves to the right.



The charge moves to the right and reaches infinity.

$$(P.E. + K.E.)_A = (P.E. + K.E.)_{\infty}$$

$$QV_A + \cancel{\frac{1}{2}mv_A^2} = \cancel{QV_{\infty}} + \frac{1}{2}mv_{\infty}^2$$

Therefore,

$$QV_A = \frac{1}{2}mv_{\infty}^2 \quad \text{Need to evaluate } V_A$$

$$V_A = V_1 + V_2 = \frac{kQ_1}{r_1} + \frac{kQ_2}{r_2}$$

$$= \frac{(9 \times 10^9 \text{ Nm}^2/\text{C}^2)(6 \times 10^{-6} \text{ C})}{(4 + 2) \text{ m}} + \frac{(9 \times 10^9 \text{ Nm}^2/\text{C}^2)(4 \times 10^{-6} \text{ C})}{(2) \text{ m}}$$

$$= 2.7 \times 10^4 \text{ J/C}$$

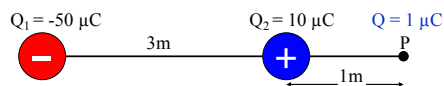
$$QV_A = \frac{1}{2}mv_{\infty}^2$$

$$(2 \times 10^{-6} \text{ C})(2.7 \times 10^4 \text{ J/C}) = \frac{1}{2}(2 \times 10^{-3} \text{ kg})(v_{\infty}^2)$$

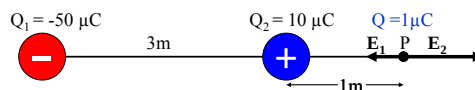
$$\longrightarrow v_{\infty} = 7.35 \text{ m/s}$$

Example : Two charges, $Q_1 = -50 \mu\text{C}$ and $Q_2 = 10 \mu\text{C}$ are 3 m apart. A particle of charge $Q = 1 \mu\text{C}$ and mass 5 g is placed at point P, 1 m to the right of Q_2 as shown. The particle is released from rest.

- What is the force acting on the particle?
- Can one determine the “final” velocity of the particle.



- Find the resultant E - field : $(F = QE)$
 $\vec{E}_p = \vec{E}_1 + \vec{E}_2$

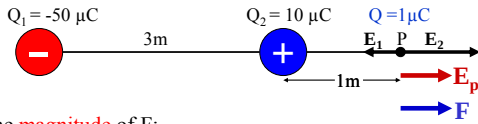


$$E_1 = \frac{kQ_1}{r_1^2} = \frac{(9 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}) (50 \times 10^{-6} \text{ C})}{(3\text{m} + 1\text{m})^2} = 2.81 \times 10^4 \text{ N/C}$$

$$E_2 = \frac{kQ_2}{r_2^2} = \frac{(9 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}) (10 \times 10^{-6} \text{ C})}{(1\text{m})^2} = 8.99 \times 10^4 \text{ N/C}$$

The **magnitude** of E_p : $E_p = E_2 - E_1$
 $= 6.18 \times 10^4 \text{ N/C}$

The **direction** of E_p :



The **magnitude** of F :

$$\begin{aligned}\vec{F} &= Q \vec{E}_p \\ &= (1 \times 10^{-6} \text{ C})(6.18 \times 10^4 \text{ N/C}) \\ &= 0.0618 \text{ N}\end{aligned}$$

The **direction** of F : Same as for E_p

→ The particle of charge Q moves to the right

b) It seems that the particle will reach infinity.

Using **conservation of energy**:

$$(PE + KE)_p = (PE + KE)_\infty$$

$$QV_p + \frac{1}{2}mv_p^2 = QV_\infty + \frac{1}{2}mv_\infty^2$$

$$QV_p = \frac{1}{2}mv_\infty^2$$

$$V_p = V_1 + V_2 = \frac{kQ_1}{r_1} + \frac{kQ_2}{r_2}$$

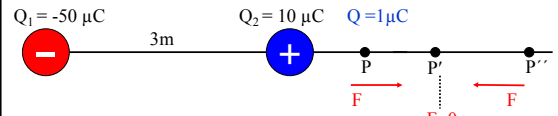
$$\begin{aligned}&= \frac{(9 \times 10^9 \text{ Nm}^2/\text{C}^2)(-50 \times 10^{-6} \text{ C})}{(4)\text{m}} + \frac{(9 \times 10^9 \text{ Nm}^2/\text{C}^2)(10 \times 10^{-6} \text{ C})}{(1)\text{m}} \\ &= -2.25 \times 10^4 \text{ V}\end{aligned}$$

$$v_\infty = \sqrt{\frac{2QV_p}{m}}$$

$$\Rightarrow v_\infty = \sqrt{\frac{2(1 \times 10^{-6} \text{ C})(-2.25 \times 10^4 \text{ V})}{(5 \times 10^{-3} \text{ kg})}}$$

The velocity has an imaginary value!!!

The particle cannot reach ∞



As the particle moves to the right, the net force acting on it starts to decrease until it reaches a point (P') where this force becomes zero and changes its sign (direction).

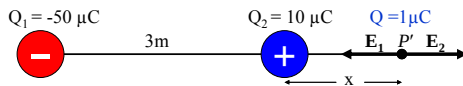
As a result, the particle starts to slow down, and at a certain point (P') it will stop ($v=0$) and reverse its direction of motion i.e. return.

Where is point P' ?

At point, P' , the force on the particle becomes zero and then changes its direction.

$F = 0$ when the E-field is zero.

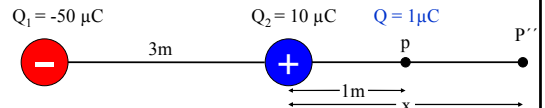
$$\therefore E_1 = E_2$$



$$\frac{kQ_1}{r_1^2} = \frac{kQ_2}{r_2^2} \Rightarrow \frac{(50 \times 10^{-6} \text{ C})}{(3m+x)^2} = \frac{(10 \times 10^{-6} \text{ C})}{x^2}$$

$$\Rightarrow \frac{(3m+x)^2}{x^2} = 5 \quad \text{Solving for } x, \text{ we get } x = 2.43 \text{ m}$$

Where will the particle stop and start to return? (i.e. where is point P' ?)



When velocity = 0; apply the conservation of energy:

$$(P.E. + K.E.)_p = (P.E. + K.E.)_{p'}$$

$$QV_p + \frac{1}{2}mv_p^2 = QV_{p'} + \frac{1}{2}mv_{p'}^2$$

$$QV_p = QV_{p^*}$$

$$\Rightarrow V_p = V_{p^*}$$

$$\Rightarrow \frac{kQ_1}{(3 \text{ m} + x)} + \frac{kQ_2}{x} = -2.25 \times 10^4 \text{ V}$$

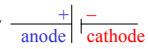


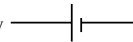
Solving for x; $x = 12 \text{ m}$

Electric Battery

An electric battery produces a voltage or potential difference across its terminals.

Due to chemical reactions, positive charges accumulate on the positive terminal (**anode**) and negative charges accumulate on the negative terminal (**cathode**).

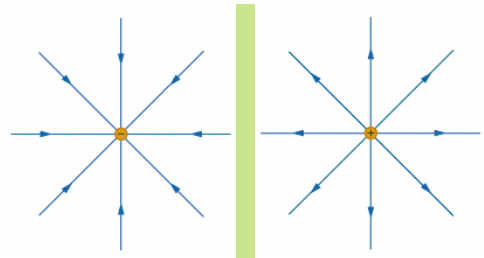
Symbol for the battery 

or simply 

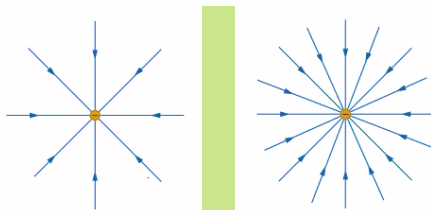
Electric Field Lines

- A graphical representation of electric fields.
- **Direction:** E-field lines start from a positive charge and end on a negative charge.
- **Magnitude:** The density of the electric field lines per unit area is proportional to the magnitude of the E-field.

Electric field lines of single charges



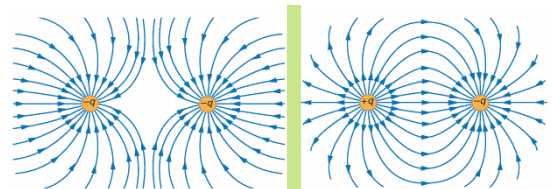
A weak and a stronger electric field



Charge = $-Q$
(Week field)

Charge = $-2Q$
(Strong field)

Electric field lines for two equal charges

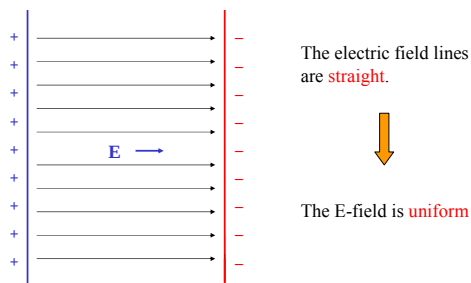


Charges have the **same** sign

Charges have **opposite** sign

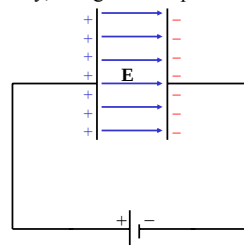
(**Electric Dipole**)

Field inside two oppositely charged parallel plates



Uniform E-fields

If a pair of conducting parallel plates is connected to the terminals of a battery, charges are deposited on the plates.



A **Uniform E-field** exists between the two plates.

Work done in moving a positive charge q from the negative plate to the positive plate is:

Work done = Force $\times$ displacement

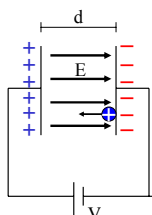
For a small displacement, Δr ,

$$\Delta W = F \Delta r \\ = (qE) \Delta r$$

Inside the two plates, the E-field is **uniform** (i.e. No need for integration)

$$\therefore W = qEd$$

Where d is the separation between the plates.



$$W = \text{change in PE} = qV$$

$$\therefore qEd = qV$$

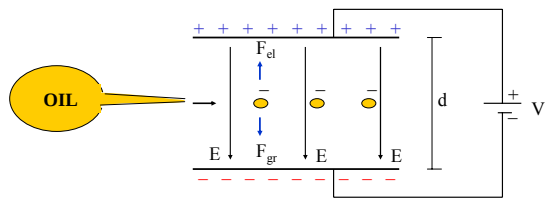
$$E = V/d$$

Alternate unit for E : [V/m] instead of N/C

Determination of the electronic charge, e

(Milikan's oil drop experiment 1906-1913)

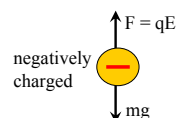
- Oil droplets emitted from a narrow nozzle, due to friction, acquire **negative** charges.
- The droplets are kept in a **uniform** electric field.



For a **stationary** droplet :

$$mg = qE = q \frac{V}{d}$$

$$q = \frac{mgd}{V}$$



The mass of the droplets is easily calculated from the oil's radius (hence its volume) and its density.

The results on several hundreds of droplets show that $q = Ne$, i.e. a multiple of the electronic charge.

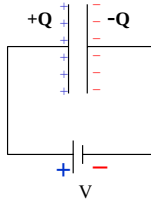
From his measurements, Milikan concluded that:

$$e = 1.6 \times 10^{-19} \text{ C}$$

The Capacitor

A pair of conducting surfaces separated by a gap constitute a capacitor.

If a capacitor is connected to a battery, after a very short time each plate acquires a charge.



The charge on the capacitor is defined to be the charge on its positively charged plate.

The amount of charge is directly proportional to the potential difference of the battery.

$$Q \sim V$$

$$Q = CV$$

Where C is the capacitance of the capacitor.

Capacitance refers to the “capacity to accumulate charges”

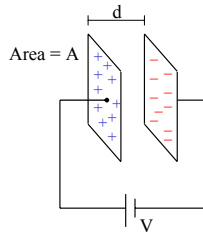
If C is large, so is Q for a given V .

Units for Capacitance: $C/V \equiv \text{Farad [F]}$

The capacitance C depends on the construction of the capacitor.

- The larger the area of the plates, the more charges can be put on the plate. i.e. $Q \propto A$

- The smaller the separation, d , between the plates, the greater the attraction between the oppositely charged plates. This will allow more charges to be deposited on each plate. i.e. $Q \propto 1/d$



$$C = \epsilon_0 \frac{A}{d}$$

Where the proportionality constant, ϵ_0 , is the permittivity of vacuum.

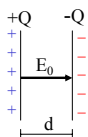
$$\epsilon_0 = 8.85 \times 10^{-12} \text{ C}^2/\text{Nm}^2$$

Dielectrics

What happens if the space between the capacitor plates is filled with a dielectric?

A dielectric is an insulator (to maintain the charges $\pm Q$ on the plates) composed of polarized molecules $\rightarrow$

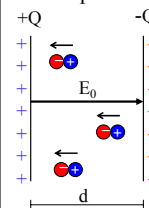
Charge the capacitor by connecting its plates to a battery with a potential difference of V . Then, disconnect the battery. The charge on the capacitor plates will not change.



$$Q = C_0 V = C_0 E_0 d$$

Where C_0 is the capacitance while in vacuum and E_0 is the E-field inside the capacitor.

Fill the space between the plates with a dielectric material.



$$Q = CEd$$

Where C is the capacitance with the dielectric material inserted between the two plates.

The E-field inside the capacitor decreases as the small field of each molecule is opposite in direction to E_0 . i.e. $E < E_0$

Since the amount of charge on each plate remains constant,

$$C_0 E_0 d = CE d$$

$$\text{or } \frac{E_0}{E} = \frac{C}{C_0} \geq 1$$

i.e. The capacitance increases when a dielectric material is inserted between the capacitor plates.

$$\frac{C}{C_0} = \kappa$$

κ is called the dielectric constant of the material or the relative permittivity. The permittivity of the dielectric is $\epsilon = \kappa \epsilon_0$.

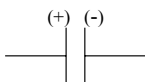
In general,

$$C = \epsilon_0 \kappa \frac{A}{d}$$

Material	κ
Vacuum	1
Air	1.0005
Glass	5-9
Paper	3-7
Water	80
TiO ₂	100

Symbol for the capacitor:



or  Electrolytic capacitor

Example : Construct a 20 pF capacitor using 2 plates with dimensions 4 cm x 6 cm.

a) What should be the separation between the plates?

$$C = \kappa \epsilon_0 \frac{A}{d} \quad \kappa = 1$$

$$d = \frac{\epsilon_0 A}{C} = \frac{(8.85 \times 10^{-12} \text{ C}^2 / \text{N} \cdot \text{m}^2)(0.04 \text{ m})(0.06 \text{ m})}{(20 \times 10^{-12} \text{ F})}$$

$$= 1.06 \times 10^{-3} \text{ m}$$

b) If the capacitor is charged to 0.33 μC . What is the E-field between the plates?

$$Q = CV = CE d$$

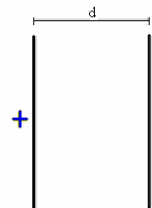
$$\Rightarrow E = \frac{Q}{Cd} = \frac{(0.33 \times 10^{-6} \text{ C})}{(20 \times 10^{-12} \text{ F})(1.06 \times 10^{-3} \text{ m})}$$

$$= 1.55 \times 10^7 \text{ V/m}$$

Energy stored in a capacitor

Calculate the work done in moving a charge from one plate to the other.

For the very first charge, q_1 , $V = 0$, so $W_1 = 0$. As more charges are put on the plates the potential increases and more work must be done to move charges.



For the very last charge, $W_N = q_N V$

$$\text{Average potential} = \frac{0 + V}{2} = \frac{V}{2}$$

If the total charge is $Q = q_1 + q_2 + \dots + q_N$

then the total work done $= Q\left(\frac{V}{2}\right) = \frac{1}{2}QV$

which is the potential energy stored in a charged capacitor.

$$PE = \frac{1}{2} QV$$

Other forms for the energy stored in a capacitor

$$\begin{aligned} PE &= \frac{1}{2} QV \\ &= \frac{1}{2} (CV)V = \frac{1}{2} CV^2 \\ PE &= \frac{1}{2} QV = \frac{1}{2} Q\left(\frac{Q}{C}\right) \\ PE &= \frac{1}{2} \frac{Q^2}{C} \end{aligned}$$

This stored PE can be released by electrically connecting the two plates.

Electric energy density

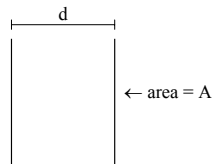
$$PE = \frac{1}{2} CV^2 = \frac{1}{2} \left(\kappa \epsilon_0 \frac{A}{d} \right) (Ed)^2$$

$$PE = \frac{1}{2} \kappa \epsilon_0 E^2 Ad$$

Ad = volume of the capacitor

$$\text{Energy density} = \frac{PE}{Ad} = \frac{1}{2} \kappa \epsilon_0 E^2$$

This is a **general** result. If we know the E-field in a region, we can easily calculate the electric energy density in that region.



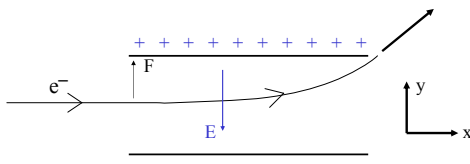
Example : Calculate the energy stored in a $600 \mu\text{F}$ capacitor that is charged to a voltage of 400 V .

$$\begin{aligned} \text{Energy} &= \frac{1}{2} CV^2 \\ &= \frac{1}{2} (600 \times 10^{-6} \text{ F})(400 \text{ V})^2 \\ &= 48 \text{ J} \end{aligned}$$

A car battery $\approx 10^6 \text{ J}$

Cathode Ray Oscilloscope “CRO”

Principle of CRO: Deflection of an electron beam by an electric field.



The electric field E is downwards. (negative y -direction)

The electric force on the electron is upwards. (y -direction)

Magnitude of the electric force: $F_y = eE = ma_y$

$$a_y = \frac{eE}{m}$$

E is constant between the two charged plates
 $\Rightarrow$ **constant** acceleration

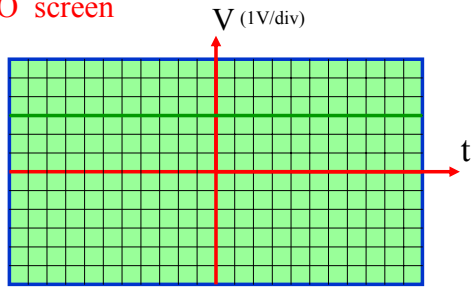
From kinematics, the upward deflection of the electron is

$$y = v_{oy}t + \frac{1}{2}a_yt^2 = \frac{1}{2}\frac{eE}{m}t^2 \quad \Rightarrow \quad y \propto E$$

The E -field is produced by the potential difference, V , applied to the plates.

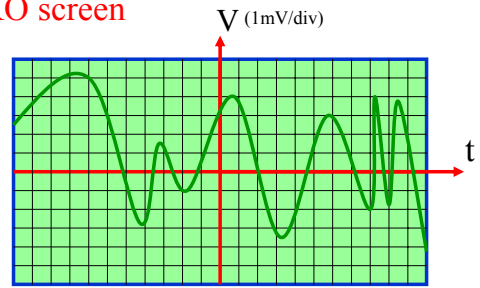
$\Rightarrow y \propto V$ i.e. Using the CRO, one can measure potential differences.

CRO screen



Screen shows a constant voltage of 3 V

CRO screen



Bio-signals