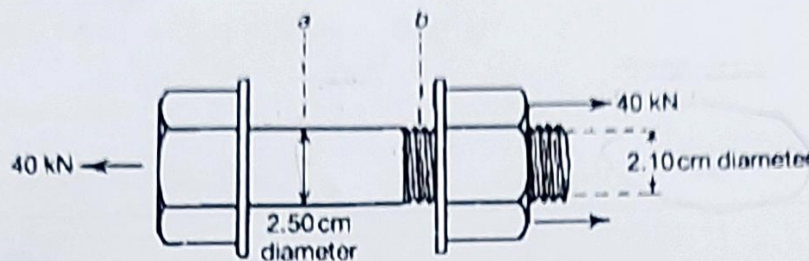


1. A steel bolt, 2.50 cm in diameter, carries a tensile load of 40 kN. Estimate the average tensile stress at the section *a* and at the screwed section *b*, where the diameter at the root of the thread is 2.10 cm.



Solution

The cross-sectional area of the bolt at the section *a* is

$$A_a = \frac{\pi}{4} (0.025)^2 = 0.491 \times 10^{-3} \text{ m}^2$$

The average tensile stress at *A* is then

$$\sigma_a = \frac{P}{A_a} = \frac{40 \times 10^3}{0.491 \times 10^{-3}} = 81.4 \text{ MN/m}^2$$

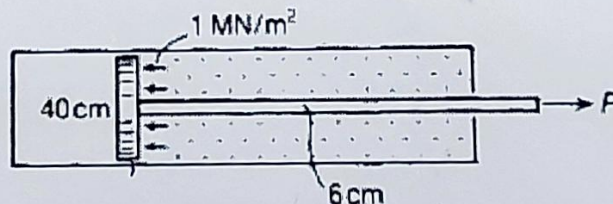
The cross-sectional area at the root of the thread, section *b*, is

$$A_b = \frac{\pi}{4} (0.021)^2 = 0.346 \times 10^{-3} \text{ m}^2$$

The average tensile stress over this section is

$$\sigma_b = \frac{P}{A_b} = \frac{40 \times 10^3}{0.346 \times 10^{-3}} = 115.6 \text{ MN/m}^2$$

2. The piston of a hydraulic ram is 40 cm diameter, and the piston rod 6 cm diameter. The water pressure is 1 MN/m^2 . Estimate the stress in the piston rod and the elongation of a length of 1 m of the rod when the piston is under pressure from the piston-rod side. Take Young's modulus as $E = 200 \text{ GN/m}^2$.



Solution

The pressure on the back of the piston acts on a net area

$$\frac{\pi}{4} [(0.40)^2 - (0.06)^2] = \frac{\pi}{4} (0.46) (0.34) = 0.123 \text{ m}^2$$

The load on the piston is then

$$P = (1) (0.123) = 0.123 \text{ MN}$$

Area of the piston rod is

$$A = \frac{\pi}{4} (0.060)^2 = 0.283 \times 10^{-2} \text{ m}^2$$

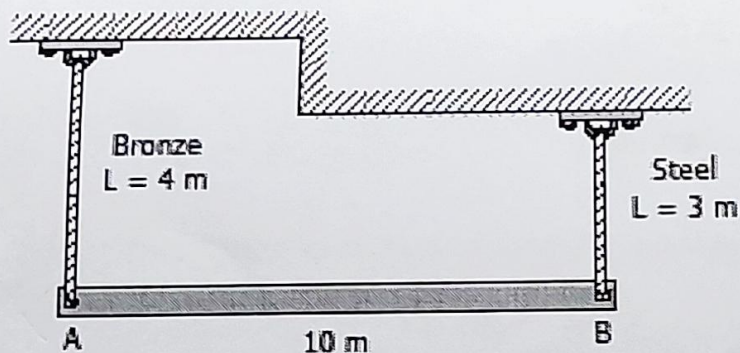
The average tensile stress in the rod is then

$$\sigma = \frac{P}{A} = \frac{0.123 \times 10^6}{0.283 \times 10^{-2}} = 43.5 \text{ MN/m}^2$$

From equation (1.6), the elongation of a length $L = 1 \text{ m}$ is

$$\begin{aligned} e &= \frac{PL}{EA} = \frac{P}{A} \left(\frac{L}{E} \right) = \frac{\sigma L}{E} \\ &= \frac{(43.5 \times 10^6) (1)}{200 \times 10^9} \\ &= 0.218 \times 10^{-3} \text{ m} \\ &= 0.0218 \text{ cm} \end{aligned}$$

3. A homogeneous 800 kg bar AB is supported at either end by a cable as shown. Calculate the smallest area of each cable if the stress is not to exceed 90MPa in bronze and 120MPa in steel.



By symmetry:

$$P_{br} = P_{st} = \frac{1}{2}(7848) \\ = 3924 \text{ N}$$

For bronze cable:

$$P_{br} = \sigma_{br} A_{br}$$

$$3924 = 90 A_{br}$$

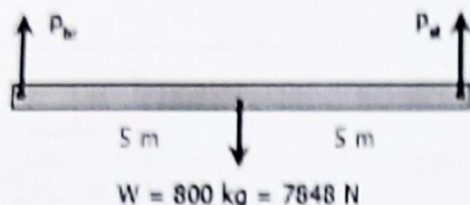
$$A_{br} = 43.6 \text{ mm}^2$$

For steel cable:

$$P_{st} = \sigma_{st} A_{st}$$

$$3924 = 120 A_{st}$$

$$A_{st} = 32.7 \text{ mm}^2$$



4. (a) A 25 mm diameter bar is subjected to an axial tensile load of 100 kN. Under the action of this load a 200 mm gauge length is found to extend $0.19 \times 10^{-3} \text{ mm}$. Determine the modulus of elasticity for the bar material.
- (b) If, in order to reduce weight whilst keeping the external diameter constant, the bar is bored axially to produce a cylinder of uniform thickness, what is the maximum diameter of bore possible given that the maximum allowable stress is 240 MN/m^2 ? The load can be assumed to remain constant at 100 kN.
- (c) What will be the change in the outside diameter of the bar under the limiting stress quoted in (b)? ($E = 210 \text{ GN/m}^2$ and $\nu = 0.3$).

Solution

(a) From eqn. (1.2),

$$\begin{aligned} \text{Young's modulus } E &= \frac{PL}{A \delta L} \\ &= \frac{100 \times 10^3 \times 200 \times 10^{-3}}{\frac{\pi(25 \times 10^{-3})^2}{4} \times 0.19 \times 10^{-3}} \\ &= 214 \text{ GN/m}^2 \end{aligned}$$

(b) Let the required bore diameter be $d \text{ mm}$; the cross-sectional area of the bar will then be reduced to

$$A = \left[\frac{\pi \times 25^2}{4} - \frac{\pi d^2}{4} \right] 10^{-6} = \frac{\pi}{4} (25^2 - d^2) 10^{-6} \text{ m}^2$$

$$\therefore \text{stress in bar} = \frac{P}{A} = \frac{4 \times 100 \times 10^3}{\pi(25^2 - d^2) 10^{-6}}$$

(b) Let the required bore diameter be d mm; the cross-sectional area of the bar will then be reduced to

$$A = \left[\frac{\pi \times 25^2}{4} - \frac{\pi d^2}{4} \right] 10^{-6} = \frac{\pi}{4} (25^2 - d^2) 10^{-6} \text{ m}^2$$

$$\therefore \text{stress in bar} = \frac{P}{A} = \frac{4 \times 100 \times 10^3}{\pi (25^2 - d^2) 10^{-6}}$$

But this stress is restricted to a maximum allowable value of 240 MN/m^2 .

$$\therefore 240 \times 10^6 = \frac{4 \times 100 \times 10^3}{\pi (25^2 - d^2) 10^{-6}}$$

$$\therefore 25^2 - d^2 = \frac{4 \times 100 \times 10^3}{240 \times 10^6 \times \pi \times 10^{-6}} = 530.5$$

$$\therefore d^2 = 94.48 \quad \text{and} \quad d = 9.72 \text{ mm}$$

The maximum bore possible is thus **9.72 mm**.

(c) The change in the outside diameter of the bar will be obtained from the lateral strain,

i.e. $\text{lateral strain} = \frac{\delta d}{d}$

But $\text{Poisson's ratio } \nu = \frac{\text{lateral strain}}{\text{longitudinal strain}}$

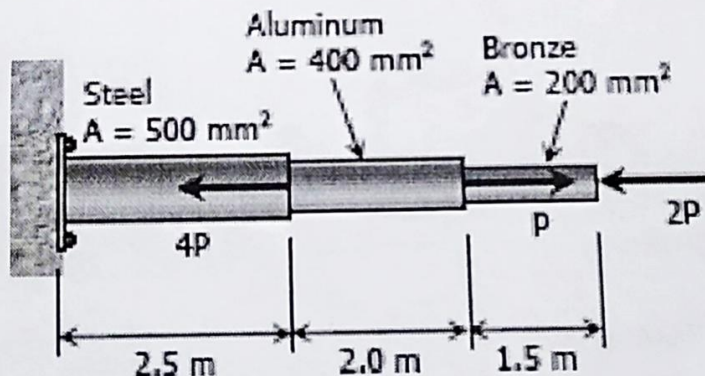
and $\text{longitudinal strain} = \frac{\sigma}{E} = \frac{240 \times 10^6}{210 \times 10^9}$

$$\therefore \frac{\delta d}{d} = -\nu \frac{\sigma}{E} = -\frac{0.3 \times 240 \times 10^6}{210 \times 10^9}$$

$$\therefore \text{change in outside diameter} = -\frac{0.3 \times 240 \times 10^6}{210 \times 10^9} \times 25 \times 10^{-3}$$

$$= -8.57 \times 10^{-6} \text{ m (a reduction)}$$

5. An aluminum rod is rigidly attached between a steel rod and a bronze rod as shown in Fig. Axial loads are applied at the positions indicated. Find the maximum value of P that will not exceed a stress in steel of 140 MPa , in aluminum of 90 MPa , or in bronze of 100 MPa .



Solution

For bronze:

$$\sigma_{br} A_{br} = 2P$$

$$100(200) = 2P$$

$$P = 10\,000\text{ N}$$

For aluminum:

$$\sigma_{al} A_{al} = P$$

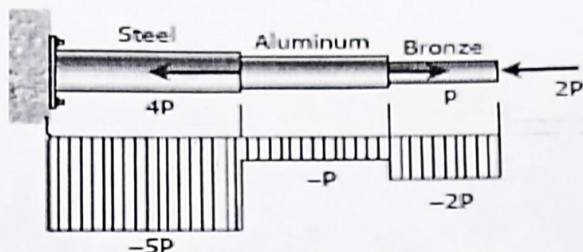
$$90(400) = P$$

$$P = 36\,000\text{ N}$$

For Steel:

$$\sigma_{st} A_{st} = 5P$$

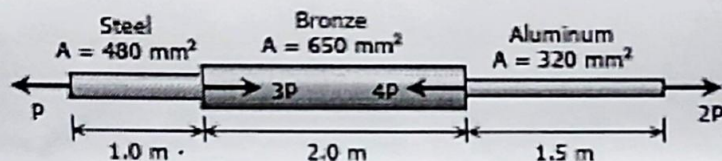
$$P = 14\,000\text{ N}$$



Negative (-) means compression

For safe P , use $P = 10\,000\text{ N} = 10\text{ kN}$

6. A bronze bar is fastened between a steel bar and an aluminum bar as shown in Fig. below. Axial loads are applied at the positions indicated. Find the largest value of P that will not exceed an overall deformation of 3.0 mm, or the following stresses: 140 MPa in the steel, 120 MPa in the bronze, and 80 MPa in the aluminum. Assume that the assembly is suitably braced to prevent buckling. Use $E_{st} = 200\text{ GPa}$, $E_{al} = 70\text{ GPa}$, and $E_{br} = 83\text{ GPa}$.



Solution

Based on allowable stresses:

Steel:

$$P_{st} = \sigma_{st} A_{st}$$

$$P = 140(480) = 67\,200\text{ N}$$

$$P = 67.2\text{ kN}$$

Bronze:

$$P_{br} = \sigma_{br} A_{br}$$

$$2P = 120(650) = 78\,000$$

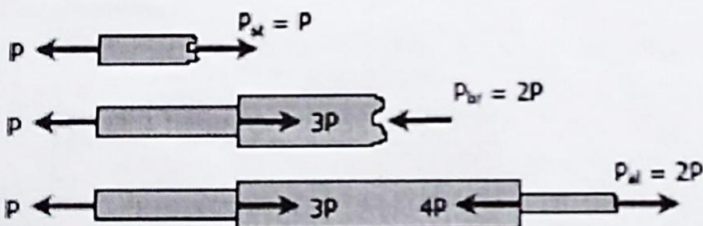
$$P = 39\,000\text{ N} = 39\text{ kN}$$

Aluminum:

$$P_{al} = \sigma_{al} A_{al}$$

$$2P = 80(320) = 25\,600\text{ N}$$

$$P = 12\,800\text{ N} = 12.8\text{ kN}$$



Based on allowable deformation:

(steel and aluminum lengthens, bronze shortens)

$$\delta = \delta_{st} - \delta_{br} + \delta_{al}$$

$$3 = \frac{P(1000)}{480(200\,000)} - \frac{2P(2000)}{650(70\,000)} + \frac{2P(1500)}{320(83\,000)}$$

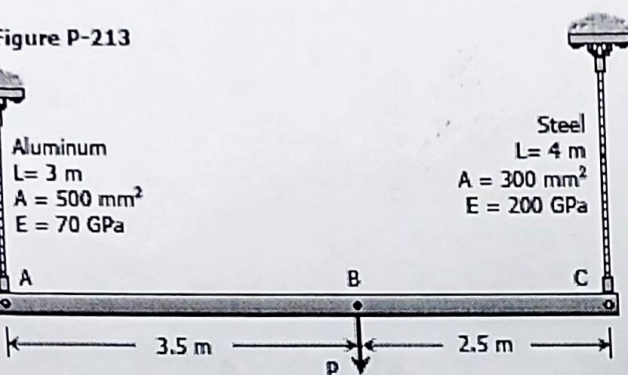
$$3 = \left(\frac{1}{96\,000} - \frac{1}{115\,75} + \frac{3}{26\,560} \right) P$$

$$P = 84\,610.99 \text{ N} = 84.61 \text{ kN}$$

Use the smallest value of P , $P = 12.8 \text{ kN}$

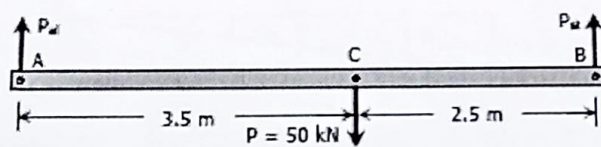
The rigid bar AB, attached to two vertical rods as shown in Fig. below, is horizontal before the load P is applied. Determine the vertical movement of P if its magnitude is 50 kN.

Figure P-213



solution

Free body diagram:



For aluminum:

$$[\Sigma M_B = 0]$$

$$6P_{al} = 2.5(50)$$

$$P_{al} = 20.83 \text{ kN}$$

$$\left[\delta = \frac{PL}{AE} \right]_{al}$$

$$\delta_{al} = \frac{20.83(3)1000^2}{500(70\,000)}$$

$$\delta_{al} = 1.78 \text{ mm}$$

For steel:

$$[\Sigma M_A = 0]$$

$$6P_{st} = 3.5(50)$$

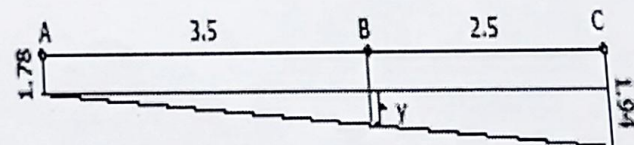
$$P_{st} = 29.17 \text{ kN}$$

$$\left[\delta = \frac{PL}{AE} \right]_{st}$$

$$\delta_{st} = \frac{29.17(4)1000^2}{300(200\,000)}$$

$$\delta_{st} = 1.94 \text{ mm}$$

Movement diagram:



$$\frac{y}{3.5} = \frac{1.94 - 1.78}{6}$$

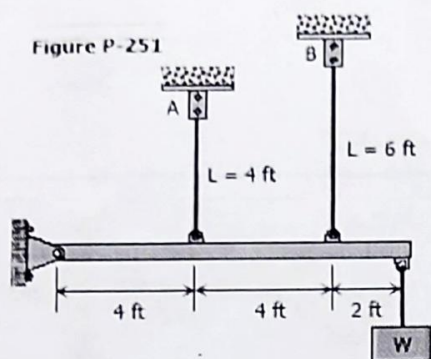
$$y = 0.09 \text{ mm}$$

δ_B = vertical movement of P

$$\delta_B = 1.78 + y = 1.78 + 0.09$$

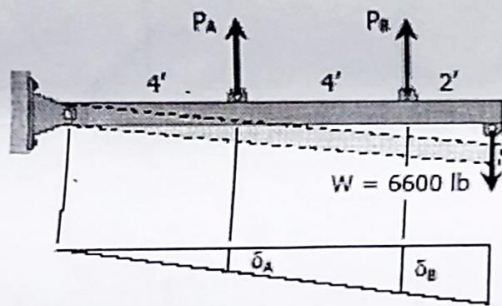
$$\delta_B = 1.87 \text{ mm}$$

8. The two vertical rods attached to the light rigid bar in Fig. below are identical except for length. Before the load W was attached, the bar was horizontal and the rods were stress-free. Determine the load in each rod if $W = 6600$ lb.



Solution

$$\begin{aligned}\sum M_{\text{pin support}} &= 0 \\ 4P_A + 8P_B &= 10(6600) \\ P_A + 2P_B &= 16500 \quad \rightarrow (1)\end{aligned}$$



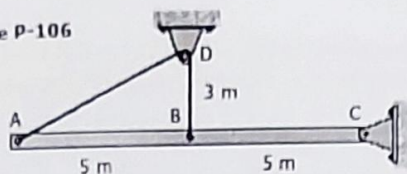
$$\begin{aligned}\frac{\delta_A}{4} &= \frac{\delta_B}{8} \\ \delta_A &= 0.5\delta_B \\ \left(\frac{PL}{AE}\right)_A &= 0.5\left(\frac{PL}{AE}\right)_B \\ \frac{P_A(4)}{AE} &= \frac{0.5P_B(6)}{AE} \\ P_A &= 0.75P_B\end{aligned}$$

$$\begin{aligned}\text{From equation (1)} \\ 0.75P_B + 2P_B &= 16500 \\ P_B &= 6000 \text{ lb}\end{aligned}$$

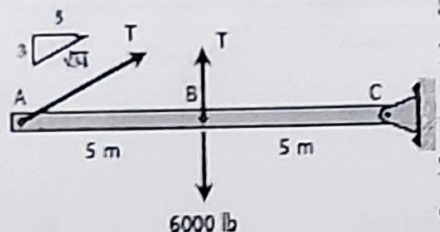
$$\begin{aligned}P_A &= 0.75(6000) \\ P_A &= 4500 \text{ lb}\end{aligned}$$

9. The homogeneous bar shown in Fig. below is supported by a smooth pin at C and a cable that runs from A to B around the smooth peg at D. Find the stress in the cable if its diameter is 0.6 inch and the bar weighs 6000 lb.

Figure P-106



Solution



$$\sum M_C = 0$$

$$5T + 10\left(\frac{3}{5}T\right) = 5(6000)$$

$$T = 2957.13 \text{ lb}$$

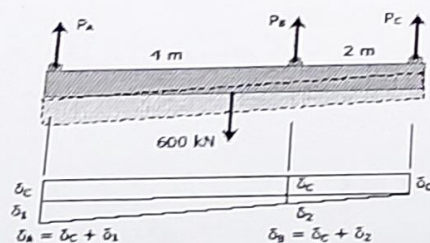
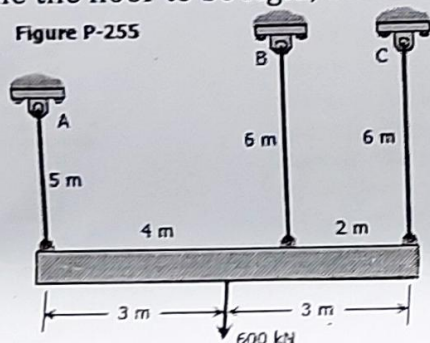
$$T = \sigma A$$

$$2957.13 = \sigma \left[\frac{1}{4} \pi (0.6^2) \right]$$

$$\sigma = 10458.72 \text{ psi}$$

10. Shown in Fig. below is a section through a balcony. The total uniform load of 600 kN is supported by three rods of the same area and material. Compute the load in each rod. Assume the floor to be rigid, but note that it does not necessarily remain horizontal.

Figure P-255



$$\delta_B = \delta_C + \delta_2$$

$$\delta_2 = \delta_B - \delta_C$$

$$\frac{\delta_1}{6} = \frac{\delta_2}{2}; \delta_1 = 3\delta_2$$

$$\delta_A = \delta_C + \delta_1 = \delta_C + 3\delta_2$$

$$\delta_A = \delta_C + 3(\delta_B - \delta_C)$$

$$\delta_A = 3\delta_B - 2\delta_C$$

$$\left(\frac{PL}{AE}\right)_A = 3\left(\frac{PL}{AE}\right)_B - 2\left(\frac{PL}{AE}\right)_C$$

$$\frac{P_A(5)}{AE} = \frac{3P_B(6)}{AE} - \frac{2P_C(6)}{AE}$$

$$P_A = 3.6P_B - 2.4P_C \rightarrow (1)$$

$$[\sum F_V = 0]$$

$$P_A + P_B + P_C = 600$$

$$(3.6P_B - 2.4P_C) + P_B + P_C = 600$$

$$4.6P_B - 1.4P_C = 600 \rightarrow (2)$$

$$[\sum M_A = 0]$$

$$4P_B + 6P_C = 3(600)$$

$$P_B = 450 - 1.5P_C \rightarrow (3)$$

Substitute $P_B = 450 - 1.5P_C$ to (2)

$$4.6(450 - 1.5P_C) - 1.4P_C = 600$$

$$8.3P_C = 1470$$

$$P_C = 177.11 \text{ kN}$$

From (3)

$$P_B = 450 - 1.5(177.11)$$

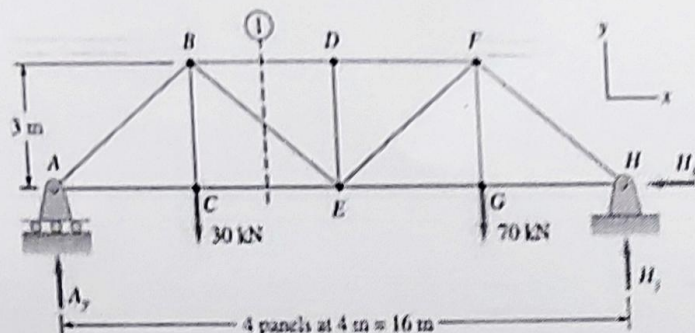
$$P_B = 184.34 \text{ kN}$$

From (1)

$$P_A = 3.6(184.34) - 2.4(177.11)$$

$$P_A = 238.56 \text{ kN}$$

11. For the truss shown in Fig. (a), calculate the normal stresses in (1) member AC ; and (2) member BD . The cross-sectional area of each member is 900 mm^2 .



Solution

Equilibrium analysis using the FBD of the entire truss in Fig. (a) gives the following values for the external reactions: $A_y = 40 \text{ kN}$, $H_y = 60 \text{ kN}$, and $H_x = 0$.

Part 1

Recall that according to the assumptions used in truss analysis, each member of the truss is an axially loaded bar. To find the force in member AC , we draw the FBD of pin A , as shown in Fig. (b). In this (FBD), P_{AB} and P_{AC} are the forces in members AB and AC , respectively. Note that we have assumed both of these forces to be tensile. Because the force system is concurrent and coplanar, there are two independent equilibrium equations. From the FBD in Fig. (b), we get

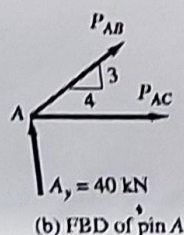
$$\sum F_y = 0 \quad +\uparrow \quad 40 + \frac{3}{5}P_{AB} = 0$$

$$\sum F_x = 0 \quad +\rightarrow \quad P_{AC} + \frac{4}{5}P_{AB} = 0$$

Solving the equations gives $P_{AC} = 53.33 \text{ kN}$ (tension). Thus, the normal stress in member AC is

$$\begin{aligned} \sigma_{AC} &= \frac{P_{AC}}{A_{AC}} = \frac{53.33 \text{ kN}}{900 \text{ mm}^2} = \frac{53.33 \times 10^3 \text{ N}}{900 \times 10^{-6} \text{ m}^2} \\ &= 59.3 \times 10^6 \text{ N/m}^2 = 59.3 \text{ MPa (T)} \end{aligned}$$

Answer



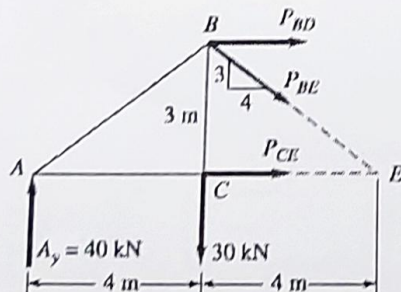
(b) FBD of pin A

Part 2

To determine the force in member BD , we see that section (1) in Fig. (a) cuts through members BD , BE , and CE . Because three equilibrium equations are available for a portion of the truss separated by this section, we can find the forces in all three members, if needed.

The FBD of the portion of the truss lying to the left of section (1) is shown in Fig. (c) (the portion lying to the right could also be used). We have again assumed that the forces in the members are tensile. To calculate the force in member BD , we use the equilibrium equation

$$\sum M_E = 0 \quad +\circlearrowleft \quad -40(8) + 30(4) - P_{BD}(3) = 0$$



(c) FBD

which yields

$$P_{BD} = -66.67 \text{ kN} = 66.67 \text{ kN (C)}$$

Therefore, the normal stress in member BD is

$$\begin{aligned}\sigma_{BD} &= \frac{P_{BD}}{A_{BD}} = \frac{-66.67 \text{ kN}}{900 \text{ mm}^2} = \frac{-66.67 \times 10^3 \text{ N}}{900 \times 10^{-6} \text{ m}^2} \\ &= -74.1 \times 10^6 \text{ N/m}^2 = 74.1 \text{ MPa (C)}\end{aligned}$$

Answer

12. Figure (a) shows a two-member truss supporting a block of weight W . The cross-sectional areas of the members are 800 mm^2 for AB and 400 mm^2 for AC . Determine the maximum safe value of W if the working stresses are 110 MPa for AB and 120 MPa for AC .

Solution

Being members of a truss, AB and AC can be considered to be axially loaded bars. The forces in the bars can be obtained by analyzing the FBD of pin A in Fig. (b). The equilibrium equations are

$$\begin{aligned}\sum F_x &= 0 \quad \rightarrow \quad P_{AC} \cos 60^\circ - P_{AB} \cos 40^\circ = 0 \\ \sum F_y &= 0 \quad \uparrow \quad P_{AC} \sin 60^\circ + P_{AB} \sin 40^\circ - W = 0\end{aligned}$$

Solving simultaneously, we get

$$P_{AB} = 0.5077W \quad P_{AC} = 0.7779W$$

Design for Normal Stress in Bar AB

The value of W that will cause the normal stress in bar AB to equal its working stress is given by

$$\begin{aligned}P_{AB} &= (\sigma_w)_{AB} A_{AB} \\ 0.5077W &= (110 \times 10^6 \text{ N/m}^2)(800 \times 10^{-6} \text{ m}^2) \\ W &= 173.3 \times 10^3 \text{ N} = 173.3 \text{ kN}\end{aligned}$$

Design for Normal Stress in Bar AC

The value of W that will cause the normal stress in bar AC to equal its working stress is found from

$$\begin{aligned}P_{AC} &= (\sigma_w)_{AC} A_{AC} \\ 0.7779W &= (120 \times 10^6 \text{ N/m}^2)(400 \times 10^{-6} \text{ m}^2) \\ W &= 61.7 \times 10^3 \text{ N} = 61.7 \text{ kN}\end{aligned}$$

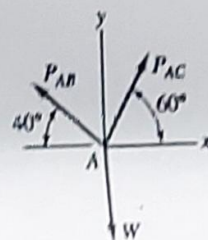
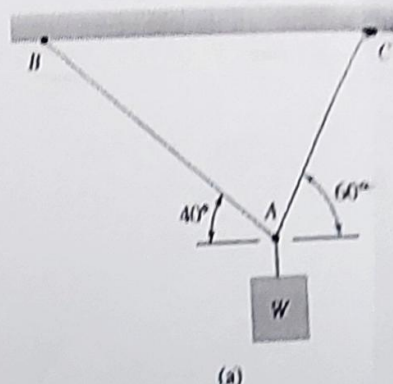
Choose the Correct Answer

The maximum safe value of W is the smaller of the preceding two values—namely,

$$W = 61.7 \text{ kN}$$

Answer

We see that the stress in bar AC determines the safe value of W . The other “solution,” $W = 173.3 \text{ kN}$, must be discarded because it would cause the stress in AC to exceed its working stress of 120 MPa .



(b) FBD of pin A