MEKELLE UNIVERSITY.
FACULTY OF SCIENCE AND TECHNOLOGY
DEPARTMENT OF CIVIL ENGINEERING.

Class: Year IV.
Course Code: CEng. 412.
Credit hour: 3(2+3).
Semester: 2 (even).
Title: Design of Steel and Timber Structures.
Instructor Name: Mague Perez.

Course content:
Methods used in design of structural steel members: tension members, compression members, combined bending and compression members. Structural connection and joint. Introduction to plastic design of steel frames. Timber: grades of timbers, design of tension and compression members, beams connections.

Course outline:

1. **Introduction**: Steel structures, field of application. Merit and drawback of steel structures. Behavior of steel under tension. Design philosophies. LSM, specification of EBCS-3. Rolled steel sections.

2. **Tension members**: Introduction, types, members subjected to axial tension, effective area, slenderness ratio, members subjected to combined tension and bending.

3. **Compression members**: Introduction, column and struts, members subjected to axial compression, stability.

4. **Bending members**: Introduction, laterally restrained beams, laterally unrestrained beams, plastic behavior of steel beams.

5. **Beam-column members**: Members subjected to combined compression and bending, base plate.

6. **Connections**: Introduction, bolted/riveted and welding connections, tension splice connections, welded end plate to beam connection, bolted beam to column connections.

7. **Introduction to plastic design of steel frames**:

8. **Timber structures**: Grades of timber, design of tension and compression members, beams and connections.

References:

1. Arya Chanakya. “Design of Structural Elements”.
2. Ambrose. “Building Structures”.
3. Ramachandra. “Design of Steel Structures”.
4. EBCS-3. Design of Steel Structures”.

Evaluation criteria:

Assignment	10%.
Mid Examination	40%.
Final Examination	50%

Chapter 1. Introduction.
1.1- Field of application.
1.2- Merit and drawback of steel structures.
1.3- Mechanical properties of steel. Behavior of steel under tension.
1.4- Rolled steel sections and their main uses.
1.5- Design philosophies. LMD Method.
1.6- Classification of cross-sections.
References: Class notes.
EBCS-3. Design of Steel Structures.

1.1 – Field of application: Steel structural members can be used in several types of structures, as follow:
A – Framework or skeleton systems, having as their main element beams, girders, trusses and columns, such as:

1. The frameworks of industrial building and structures with their internal members such as crane, girders, platform, etc.
2. Railways, highways and urban large-span bridges.
3. Civic multistories buildings, pavilions for exhibition, domes, etc.
4. Special purpose buildings such as hangars, shipbuilding, etc.
5. Special structures like towers, mast, hydraulic engineering structures, cranes, etc.

B – Shell systems.

1. Gasholders and tanks for the storage and distribution of gases.
2. Tanks and reservoirs for the storage of liquids.
3. Bunkers for the storage of loose materials.

1.2. – Merits and drawbacks of steel structures.
Merits:

1. The ability to resist high loads, due to the high strength of steel. Because of the high strength of the material, steel members are small in size, which makes them convenient for transportation.
2. Gas tightness and water tightness, which is due to the high density of steel.
3. Have a long service live, determined by the high and homogeneous strength and density properties of steel.
4. The possibility of industrializing construction work, attained by the use of prefabricated members with mechanized erection thereof at the construction site.
5. The possibility of readily disassembling or replacing steel members, which makes it easier to reinforce or replace parts of structures.
6. The possibility of sending steel members to any parts of the country no matter the bad conditions of site.

Drawbacks.
The principal drawback of steel members is their susceptibility to corrosion, which necessitates their painting or the use of other methods for their protection, and less fire resistance.

1.3 – Mechanical properties of steel.

1. Strength; is determined by the resistance of the material to external loads and forces.
2. Elasticity; is the property of the material to restore its initial shape after removal of the external loads.
3. Plasticity; is the reverse of elasticity, i.e. the property of a material not to return to its initial dimensions after removal of the external loads or, in other words, the property of obtaining permanent sets.

Behavior of steel under tension.
The standard requires that the manufacturer shall carry out tension tests on specimens taken from each type of section rolled from cast steel to ensure that the material has specified properties. A typical test specimen is shown below. See Fig. 1.

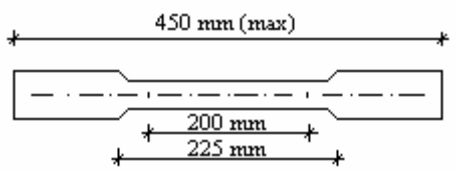


Fig.1. Typical tension specimen.

If a specimen of steel is subjected to tension by gradually increasing the load P, and the resulting elongation ΔL be measured, the results can be used to plot an experimental tension diagram of elongation Vs load. For convenience we plot stress Vs unit elongation. See Fig. 2.

Now; stress f = P/A; that is load/area. Units N/mm^2.

Strain $\varepsilon = \frac{\Delta L}{L} * 100$ where: f – Normal stress (N/mm^2).

A – cross-sectional area of the specimen. (mm^2).
ε – strain or unit elongation in percent.
L – gauge length or original length of the specimen.
ΔL – longitudinal elongation of the specimen.

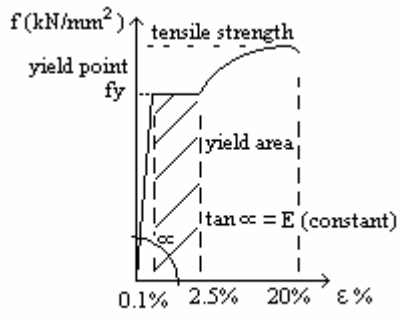


Fig. 2 f Vs ε diagram.

The relation between the stress and strain follows the Hook's Law; Robert Hook around 1678 stated his low by the following equation: **f = Eε.**
Note that the highest stress in a material, after which the relation between stress and strain no longer remains linear, is called **yield point**. After this point appears elongation without an increase in load, then, appear the **yield area**.
E – Modulus of elasticity.
For all types of steel E = 2.1*10^5 Mpa is accepted.

Types of steel. (according with EBCS – 3).

Nominal steel grades	Thickness t (mm)			
	t ≤ 40 mm		40 mm < t ≤ 100 mm	
	f_y (Mpa)	f_u (Mpa)	f_y (Mpa)	f_u (Mpa)
F_e 360	235	360	215	340
F_e 430	275	430	255	410
F_e 510	355	510	335	490

Materials Coefficients.

1. Modulus of elasticity E = 210 Gpa.
2. Shear Modulus G = 80 Gpa.
3. Unit mass $\rho = 7850\ kg/m^3$.
4. Poison's ratio $\nu = 0.3$.
5. Coefficient of linear expansion $\alpha = 12 \times 10^{-6}$ per ^oC.

1.4 – Rolled steel sections and their main use.

These sections are designed to achieve economy of material while maximizing strength, particularly in bending. Bending strength can be maximized by concentrating the metal at the extremities of the section, where it can sustain the tensile and compressive stress associated with bending. The most commonly used sections are universal beams (Ubs) and universal columns (Ucs). See Fig.3.

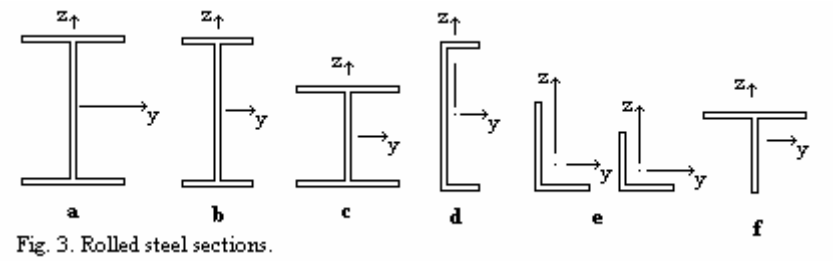


Fig. 3. Rolled steel sections.

a) W shapes. Wide flange sections. Are rolled with parallel flanges and are specified by their serial size and mass in kg per meter, e.g. W 310 x 202. It nominal depth is 310 mm and the mass is 202 kg per meter. May be used principally as columns and also may be used as beams too.
b) S shapes. Known as universal beams. It has $I_y >> I_z$, for this reason is recommended to be used as beams.
c) HP shapes. High Powered shape. Available on the USA codes. It has practically same depth compared with wide to diminish the difference between I_y and I_z. Is recommended for columns exclusively.
d) Standard Channels (C shapes). The difference between I_y and I_z is very significant. Are used as purlin in the roof of industrial buildings, as a light beam to resist bending and in built-up sections connected by batten plates.
e) Angles. Fabricated as equal legs angles and unequal legs angles. Are described by their nominal dimensions, first number is the large leg; second number is small dimension and third number the thickness of the section. Are used mainly as members of trusses, for ties in steel frames, etc.
f) T shapes. Available on the USA codes, is used as member in trusses and also in built-up beams with different types of steel.

1.5 – Design Philosophies.

During the history of the design of structures activities, have been used three design philosophies namely:

1. Permissible stress design method.
2. Load factor design method.
3. Limit state design method.

In permissible stress design method, the stress in the structure at working loads are not allowed to exceed a certain portion of the yield stress of the construction material, therefore, the working stress level is within the elastic range of the behavior of steel. The working stress is obtained by dividing the characteristic value by a unique factor of safety.

In load factor method all safety is attached to the acting load, then the acting load is obtained by multiplying the working loads by a load factor greater than the unity. The material supposes to work at the yield point, that is, at the characteristic value.

The limit state design method was formulated in the former Soviet Union in the 1930s and developed in Europe in the 1960s, this approach can perhaps be seen as a compromise between the permissible and load factor methods. It is in fact a more comprehensive approach, which take into account both methods in appropriate ways. The majorities of modern structural codes of practice are now based on the limit state design method.

Limit state design method.
A structure or part of the structure is considered unfit for use when it exceeds a particular state, called Limit State beyond which it infringes one of the criteria governing its performance for use. The Limit State can be placed in two categories:

1. The Ultimate Limit States are those associated with collapse, or with other forms of structural failure, which may endanger the safety of the people. States prior to structural collapse which, for simplicity, are considered in place of the collapse itself, are treated as ultimate limit states. Normally the ultimate limit state is concerning with the strength of the structure.
2. The Serviceability Limit States corresponds to states beyond which specified service requirements are no longer met, e.g. deformation or deflections which affect the appearance or effective use of the structure (including the malfunction of machines or services) or cause damage to finishes of non structural members; vibration which cause discomfort to people.

Characteristic and design values.
Characteristic loads are normally obtained from code practices. See EBCS-1. Chapter 2.
Design loads = characteristic loads x partial safety factor for the load (γ_f).
Design strength = characteristic strength / partial safety factor for strength (γ_m).
In general, the ultimate limit state design method is stated as follow:

Design action ≤ Design strength.

For partial safety factor for strength γ_m see 4.1. (2). EBCS-1.
For partial load factor and combination of actions see 2.8.2.2. EBCS-1.

1.6 – Classification of cross-sections.
When plastic global analysis is used, the members shall be capable of forming plastic hinges with sufficient rotation capacity to enable the required redistribution of moments to develop. When elastic global analysis is used, any class of cross-section may be used for the members, provided the design of members takes into account the possible limits of resistance of cross-section due to local buckling.
Four classes of cross-section are defined, as follow:

1. Class 1 or plastic cross-sections are those in which all elements subjected to compression comply with the values given in Table 4.1 of EBCS-3. Design of Steel Structures for plastic elements. A plastic hinge can be developed with sufficient rotation capacity to allow redistribution of moments within the structure. Only Class 1 section may be used for plastic design.
2. Class 2 or compact cross-sections are those in which all elements subject to compression comply with the values given in Table 4.1 for compact elements. The full plastic moments capacity can be developed but local buckling may prevent development of a plastic hinge with sufficient rotation capacity to permit plastic design. Class 2 sections can be used without restriction except for plastic design.
3. Class 3 or semi-compact sections are those in which the elements subject to compression comply with the values given in Table 4.1 for semi-compact elements. The stress at the extreme fibbers can reach the design strength but local buckling may prevent the development of the full plastic moment. Class 3 sections are subjected to limitations on their capacity.
4. Class 4 or thin-walled sections are those that contain thin-walled elements subjected to compression due to moment or axial force. Local buckling may prevent the stress in a thin-walled section from reaching the design strength. Design of Class 4 sections requires special attention.

Chapter 2. **Tension members**.
2.1 Introduction.
2.2 Design value of axial tension force.
2.3 Effective area.
2.4 Members subjected to combined tension and bending.
2.5 Slenderness ratio.
Reference: EBCS-3. Design of steel structures.

2.1 – Introduction.
Axially loaded tension members are used mainly as members of the roof truss, truss for bridges and as tie to take horizontal forces on industrial buildings.

2.2 – Design value of axial tension force.
The design value of the axial force is $N_{t,Sd} \le N_{t,Rd}$

1. The design plastic resistance of the gross section is $N_{pl,Rd} = \dfrac{Af_y}{\gamma_{M1}}$
2. The design ultimate resistance of the net section at the bolt hole is $N_{u,Rd} = \dfrac{0.9A_{eff}\, f_u}{\gamma_{M2}}$

Where: A – gross section area (area without reduction).
A_{eff} – effective area.
f_y – stress at the yield point of the steel.
f_u – ultimate tensile stress.

2.3 – Effective area.
The effective area is taken as **Net Area.** The net area of cross-section or element section shall be taken as it gross area less appropriate deductions for all holes and openings. When calculating net section properties, the deduction of a single hole shall be the gross cross sectional area of the hole in the plane of its axis. Provided that the fastener holes are not staggered, the total area to be deducted for fastener holes shall be the maximum sum of the sectional areas of the holes in any cross-section perpendicular to the member axis.
When the fastener holes are staggered, the total area to be deducted for fastener holes shall be the greater of:

1. The deduction for non-staggered holes.
2. The sum of the sectional area of all holes in any diagonal or zigzag line extending progressively across the member or part of member, less $s^2t/4p$ for each gauge space in the chain of holes. See Fig. 4.

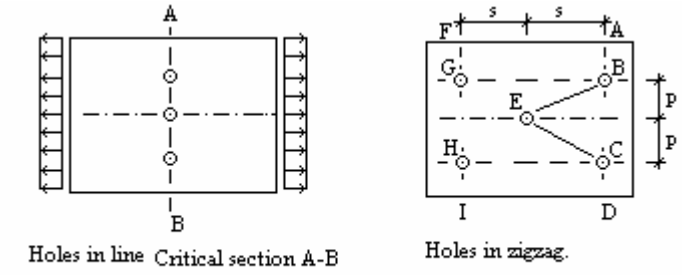


Fig. 4 Possible cases.

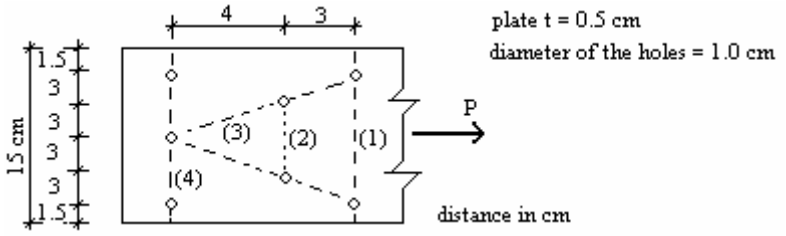
Therefore the net width d_n can be computed by using the following formula which is known as "the chain formula".

$$d_n = total\ width - nd + \frac{as^2}{4p}$$

where: n – number of holes in the chain of holes
a – number of diagonal space p in the chain
s – is the pitch, the spacing of the centers of two consecutive holes in the chain measured parallel to the member axis
p – is the spacing between the centers of the holes measured perpendicular to the ember axis
d – diameter of holes.

Finally the net area should be the net width x thickness of the plate: d x t.
Note: The diameter for holes is given in Table 6.1 of the EBCS-3.

Example N_r 1.
Calculate the net critical area for the bolt distribution shown below.



Solution:
Chain (1) $d_n = 15 - 2 \times 1 = 13$ cm.

Chain (2) s = 3; p = 3 $d_n = 15 - 4x1 + \frac{2x3^2}{4x3} = 12.5\ cm$

Chain (3) s = 4; p = 3 $d_n = 15 - 5x1 + \frac{2x3^2}{4x3} + \frac{2x4^2}{4x3} = 14.17\ cm$

Chain (4) $d_n = 15 - 3x1 = 12$ cm

Therefore the Net Critical Area = 12 x 0.5 = 6 cm^2.

Design example $N_r = 2$.
Calculate the maximum design load for the plate of the example N_r 1. Steel grade $F_e = 360$.
Solution:

1. The design plastic resistance of the gross section.

Gross area A = 15 x 0.5 = 7.5 cm^2
Yield strength $f_y = 23.5$ kN/cm^2 (Table 3.1, EBCS-3).
Partial safety factor $\gamma_{M1} = 1.1$ (Section 4.1.1, EBCS-3).

$$N_{pl,Rd} = \frac{7.5x23.5}{1.1} = 160.2\ kN$$

2. The design ultimate resistance of the section at the bolt holes.

Effective area $A_{eff} = 6$ cm^2.
Ultimate resistance $f_u = 36.0$ kN/cm^2 (Table 3.1 EBCS-3)
Partial safety factor $\gamma_{M2} = 1.25$

$$N_{u,Rd} = \frac{0.9x6x36}{1.25} = 155.52\ kN$$

Therefore, the design force is 155.52 kN.

2.4 Members subjected to combined tension and bending.

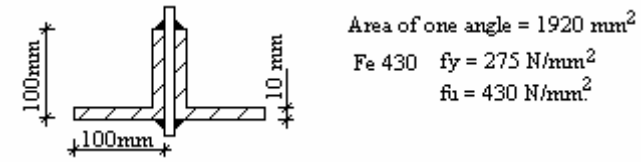
To check members under simultaneous action of tension and bending moment the following criterion may be used:

$$\frac{N_{Sd}}{N_{pl,Rd}} + \frac{M_{y,Sd}}{M_{pl,y,Rd}} + \frac{M_{z,Sd}}{M_{pl,y,Rd}} \leq 1$$

2.5 Slenderness ratio.
Even if the tension members are not under the action of reversal stress, to avoid damages during the transportation and erecting of the members, its slenderness ratio is limited to 350.

Example N_r 3.
Determine the design strength of two angles 100 x 100 x 10 in grade F_e 430 used as a welded bracing member.

Solution:

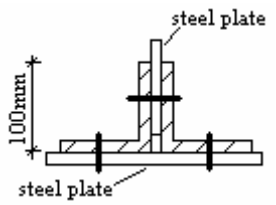


Because there is not holes (welded connection), only design plastic resistance must be checked.
The partial safety factor for the section $\gamma_{M1} = 1.1$.
The design plastic resistance is:

$$N_{pl,Rd} = \frac{Af_y}{\gamma_{M1}} = \frac{2x1920x275}{1.1} = 960000\ N = 960\ kN\ .$$

Example $N_r = 4$.

Determine the design strength for the two angles of the example N_r 3 if now are used as a bolted bracing member with single row of 16.5 mm holes at each leg of the angle.



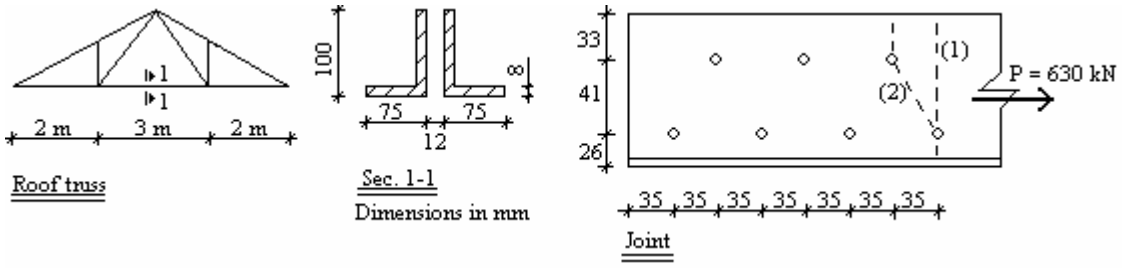
The partial safety factor is $\gamma_{M2} = 1.25$
The effective area is the net area. $A_{net} = 3840 - 4 \times 16.5 \times 10 = 3180\ mm^2$

$$N_{u,Rd} = \frac{0.9A_{eff}\, f_u}{\gamma_{M2}} = \frac{0.9x3180x430}{1.25} = 984528\ N = 484.528\ kN > 960\ kN$$

Therefore, the design strength of the bolted bracing member is controlled by the yield strength of the full section.
Thus, $N_{pl,Rd} = 960$ kN.

Example N_r 5.

Check the section used as a main tie of the roof truss shown in the Figure below. The section is formed with 2 unequal leg angle 100 x 75 x8 mm. Steel grade F_e 430 is used. The joint was made with 7 bolts diameter 20 mm as shown. The acting tensile force is 630 kN.



Steel Grade F_e 430 f_y = 275 Mpa = 27.5 kN/cm^2

f_u = 430 Mpa = 43.0 kN/cm^2

Gross area for one angle 100 x 75 x 8 = 13.49 cm^2

d_{hole} = d + 2 = 20 + 2 = 22 mm (Table 6.1, EBCS-3)

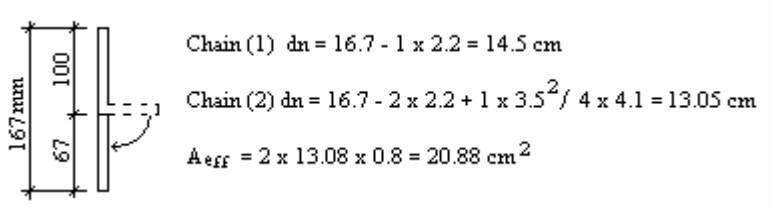
Solution:

1). Plastic resistance of the gross section:

$$N_{pl,Rd} = \frac{2x13.40x27.5}{1.1} = 670\ kN$$

2). Ultimate resistance of the net section at the bolt holes.

Calculation of the A_{eff}. S = 3.5 cm and p = 4.1 cm.



Calculation of $N_{u, Rd}$.

$$N_{u,Rd} = \frac{0.9x20.88x43}{1.25} = 646.44\ kN$$

Checking for the maximum slenderness ratio.

Slenderness ratio = L_{eff} / k_{min}; minimum radius of gyration k_{min} = 1.62 mm.

Slenderness ratio = 300 / 1.62 = 185 < 350 OK.

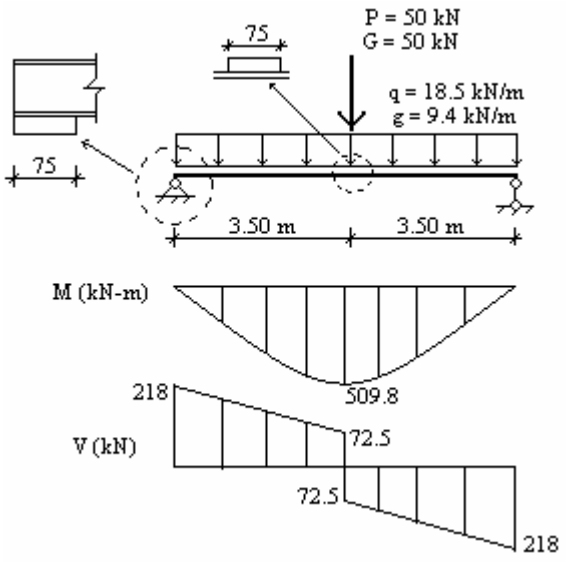
Answer:

The design tension resistance capacity of the cross-section is 646.44 kN, therefore, because 646.44 kN > 630 kN, the section 1-1 used for design is adequate.

Example Nr 2. Beams.

A simply supported beam 7.00 m span is laterally supported at the third points and carries un factored uniform loads of 18.5 kN/m and 9.4 kN/m permanent load. In addition the beam carries at mid span un factored concentrated load of 50 kN permanent load and 50 kN imposed load. Find a universal beam of grade Fe 430.

Solution:
Geometry, materials and loads.



Factored loads:

Imposed loads: q = 1.6 x 18.50 = 29.60 kN/m
Q = 1.6 x 50.00 = 80.00 kN.

Permanent loads: g = 1.3 x 9.40 = 12.20 kN/m
G = 1.3 x 50.00 = 65.00 kN.

Fe 430; $f_y = 275$ N/mm^2 (assume $t \leq 40$ mm)

Step 1. Maximum bending moment and shear force.

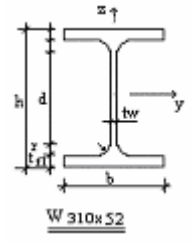
$$\max M = \frac{(29.6+12.2)x7^2}{8} + \frac{(80+65)x7}{4} = 509.8\ kN-m$$

$$\max V = \frac{(29.6+12.2)x7}{2} + \frac{(80+65)}{2} = 218.8\ kN$$

Step 2. Required plastic modulus.

$$W_{pl} = \frac{M}{f_y/\gamma_{M1}} = \frac{509.8x10^2\ (kN-cm)}{27.5\ (kN/cm^2)/1.1} = 2039\ cm^3$$. Tray 533 x 210 x 92 UB.

Step 3. Selection of the profile.



The relevant section properties are:

h = 533.1 mm
d = 476.5 mm
t_f = 15.6 mm
$_w$ = 10.2 mm
b = 209.3 mm

w = 0.92 kN/m
A = 118 cm^2
I_y = 55400 cm^4
I_z = 2390 cm^4
$W_{el,y}$ = 2080 cm^3
$W_{pl,y}$ = 2370 cm^3

I_t = 76.2 cm^4
I_w = 1.6 x 10^6 cm^6

Class of section. $\varepsilon = \sqrt{\frac{235}{275}} = 0.92$. $\frac{209.3/2}{15.6} = 6.70 < 11x0.92 \quad OK$ $\frac{476.5}{10.2} = 46.7 < 83x0.92 \quad OK$ The section is Class 2 at least.

Step 4. Resistant moment. (for class 2 section).

$$M_{pl,Rd} = \frac{W_{pl} f_y}{\gamma_{Mo}} = \frac{2370x275x10^{-3}}{1.1} = 592.5\ kN-m$$

Check of self-weight of the beam. (w = 0.92 kN/m)
Factored weight: 1.3 x 0.92 = 1.2 kN/m
Additional moment: (1.2 x 7^2)/8 = 7.35 kN-m.
Total moment: 509.8 + 7.35 = 517 kN-m < 592.5 kN-m OK.

Step 5: Check for shear.

Maximum shear force, V_{Sd} = 218.8 + (1.2 x 7)/2 = 223 kN.

Shear resistance of section.

$\frac{d}{t_w} = \frac{476.5}{10.2} = 46.7 < 69x0.92 = 63.5 \quad OK.$ Shear buckling resistance must not be verified.

$$V_{pl,Rd} = \frac{A_V\left(f_y/\sqrt{3}\right)}{\gamma_{Mo}} = \frac{1.04x533.1x10.2\left(275/\sqrt{3}\right)}{1.1}x10^{-3} = 816.4\ kN > 233\ kN \quad OK.$$

and $\frac{V_{Sd}}{V_{pl,Rd}} = \frac{233}{816.4} = 0.27 < 0.5$ Therefore, no reduction of design resistance moment is required.

Step 6: Check for deflection.

For uniformly distributed load: $\delta = \frac{5wL^4}{384EI_y}$

For concentrated load: $\delta = \frac{PL^3}{48EI_y}$ and $EI_y = 2.1x10^5 x55400x10^{14} = 1.1634x10^{14}\ N-mm^2$.

Dead load deflection: $\delta_{LL} = \frac{5x9.4x7000^4}{385x1.1634x10^{14}} + \frac{50x10^3 x7000^3}{48x1.1634x10^{14}} = 5.59\ mm.$

Imposed load deflection: $\delta_{IL} = \frac{5x18.5x7000^4}{384x1.1634x10^{14}} + \frac{50x10^3 x7000^3}{48x1.1634x10^{14}} = 8.04\ mm.$

Allowable deflection for imposed load $\delta = \frac{L}{350} = \frac{7000}{350} = 20\ mm. > 8.04\ mm \quad OK.$

Total deflection $\delta_{\max} = 5.59 + 8.04 = 13.63\ mm.$

Allowable total deflection $\delta = \frac{L}{250} = \frac{7000}{250} = 28\ mm > 13.63\ mm \quad OK.$

Step 7: Check for lateral – torsional buckling.

$$M_{b,Rd} = \frac{\chi_{LT}\beta_w W_{ol,y} f_y}{\gamma_{M1}}. \quad c/t_f = 209.3/15.6 = 6.70 < 10x0.92 = 9.2.$$

The section is Class 1 and $\beta_w = 1$.

Determination of M_{cr}.

Lateral support to the beam is provided at the ends at the third points. Therefore the effective buckling length is L = span/3 = 7000/3 = 2333 mm.

The critical moment for lateral-torsional buckling is:

$$M_{cr} = C_1 \frac{\pi^2 EI_z}{L^2}\sqrt{\frac{I_w}{I_z} + \frac{L^2 GI_t}{\pi^2 EI_z}}. \qquad C_1 = 1.132 \quad \text{for the worse condition; G = 80 Gpa}$$

and $\frac{G}{\pi^2 E} = \frac{80000}{\pi^2 x210000} = 0.039$

$$M_{cr} = \frac{1.132\pi^2 x2.1x10^5 x2390x10^4}{2333^2}\sqrt{\frac{1.6x10^{12}}{2390x10^4} + \frac{0.39x2333^2 x76.2x10^4}{2390x10^4}} = 2.67x10^9\ N-mm.$$

$$\lambda_{LT} = \sqrt{\frac{\pi^2 EW_{pl,y}}{M_{cr}}} = \sqrt{\frac{\pi^2 x2.1x10^5 x2370x10^3}{2.67x10^9}} = 42.9$$

$$\lambda_1 = 93.9\varepsilon = 93.9x0.92 = 86.8 \quad and \quad \bar{\lambda}_{LT} = \frac{42.9}{86.8} = 0.4942 > 0.4$$

For rolled section curve **a** is used. Therefore $\chi_{LT} = 0.9250$

$$M_{b,Rd} = \frac{0.9250x1x2370x10^3 x275}{1.1}x10^{-6} = 548\ kN-m > 517\ kN-m$$

Therefore, resistance of the member is adequate in bending.

Step 8.1. Check for web crushing (at the support).

$R_{y,Rd} = (S_s + S_y)t_w f_{yw}/\gamma_{M1}$; Where S_s = 75 mm; S_y – is the length over which the applied force is effectively distributed.

$$S_y = \frac{1}{2}\left[2t_f\sqrt{\left(\frac{b_f}{t_w}\right)\left(\frac{f_{yf}}{f_{yw}}\right)\left(1-\frac{\sigma_{f,Ed}}{f_{yf}}\right)^2}\right] \quad at\ the\ end\ S_y\ is\ half.$$

Check if $b_f < 25\ t_f$; 209.3 < 25 x 15.6 = 390 OK.

$\sigma_{f,Ed} = 0 \quad at\ the\ \text{sup}\ port.$

Then $S_y = \frac{1}{2}\left[2x15.6\sqrt{209.3/10.2}\right] = 70.66\ mm \quad and\ R_{y,Rd} = \frac{(75+70.66)10.2x275}{1.1}x10^{-3}$

$R_{y,Rd} = 371.3\ kN > 223\ kN \quad OK$

8.2 Check for web crippling.

Crippling resistance:

S_s = 75 mm < 0.2 d = 0.2 x 476.5 = 95.22 mm. OK.

$$R_{a,Rd} = 0.5t_w^2\sqrt{Ef_{yw}}\left[\sqrt{\frac{t_f}{t_w}} + 3\left(\frac{t_w}{t_f}\right)\left(\frac{S_s}{d}\right)\right]/\gamma_{M1}$$

$$R_{a,Rd} = \frac{0.5x10.2\ \sqrt{210000x275}\left[\sqrt{(15.6/10.2)} + 3(10.2/15.6)(75/476.5)\right]10^{-3}}{1.1} = 660.6\ kN > 233\ \text{kN}$$

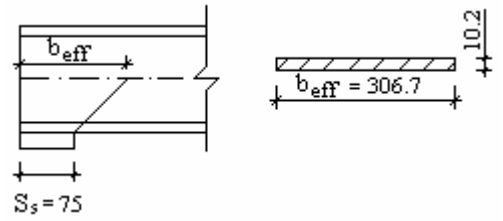
Interaction at mid-span. Where the member is also subjected to bending moments, the following criteria should be satisfied.

$\frac{F_{Sd}}{R_{a,Rd}} + \frac{M_{Sd}}{M_{c,Rd}} \le 1$; Substituting the value of $M_{c,Rd}$ = 592.5 kN-m

$$\frac{(80+65)}{660.6} + \frac{517}{592.5} = 1.09 < 1.5 \quad OK.$$

8.3 Check for web buckling (at the support). S_s = 75 mm.

$$b_{eff} = 0.5\sqrt{h^2 + S_s^2} + \frac{S_s}{2} = 0.5\sqrt{533.1^2 + 75^2} + \frac{75}{2} = 306.7\ mm$$



$$i = \sqrt{\frac{I}{A}} = \sqrt{\frac{b_{eff}t_w^3}{12b_{eff}h_w}} = \frac{t_w}{\sqrt{12}}$$

assuming that l = o.7 d = 0.7 x 476.5 = 333.6 mm.

Radius of gyration of the web is: $i = \frac{t_w}{\sqrt{12}} = \frac{10.2}{\sqrt{12}} = 2.94\ mm$

$$\lambda = \frac{333.6}{2.94} = 113.5; \quad \lambda_1 = 93.9\varepsilon = 93.9\sqrt{\frac{235}{275}} = 86.8; \quad and \quad \bar{\lambda} = \frac{113.5}{86.8} = 1.31$$

$$\frac{h}{b} = \frac{533.1}{208.7} = 2.55 > 1.2 \quad and \quad t_f = 15.6\ mm \le 40\ mm$$

Using buckling curve **c** for solid section χ = 0.3848.

The buckling resistance of the web is: $R_{b,Rd} = \chi\beta_A A f_y/\gamma_{M1}; \quad \beta_A = 1; \quad and \quad A = b_{eff}t_w$

$$A = 306.7x10.2 = 3124\ mm.$$

$$R_{b,Rd} = \frac{0.3848x1x3124x275}{1.1}x10^{-3} = 300.5\ kN > 223\ kN. \quad OK.$$

Step 9. Check for flange induced buckling.

$$\frac{d}{t_w} \le k\left(\frac{E}{f_{yf}}\right)\sqrt{\frac{A_w}{A_{fc}}}$$

$$\frac{c}{t_f} = \frac{209.3/2}{15.6} = 6.7 < 10\varepsilon; \quad \textit{therefore, the fange is class } 1;\ k = 0.3$$

$$\frac{d}{t_w} = \frac{476.5}{10.2} = 46.7 < 0.3\left(210000/275\right)\sqrt{\frac{533.1x10.2}{209.3x15.6}} = 296 \quad ok.$$

Step 10. Check for transverse force on the web.

In the absence of shear force the web of a member subject to transverse force in the plane of the web shall also satisfy the following condition:

$$\left[\frac{\sigma_{x,Ed}}{f_{yd}}\right]^2 + \left[\frac{\sigma_{z,Ed}}{f_{yd}}\right]^2 - \left[\frac{\sigma_{x,Ed}}{f_{yd}}\right]\left[\frac{\sigma_{z,Ed}}{f_{yd}}\right] \le 1$$

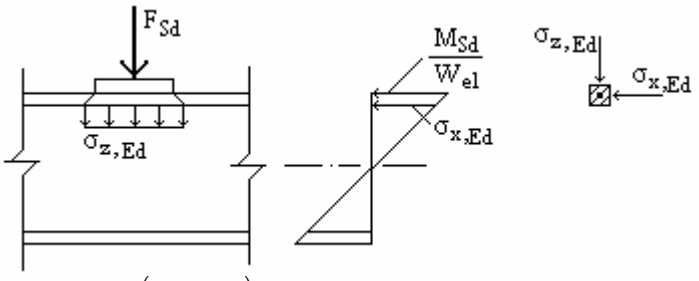
$\sigma_{x,Ed}$ – Is the design value of the local longitudinal stress due to moment and axial force at the point.

$\sigma_{z,Ed}$ – Is the design value of the stress at the same point due to the transverse force.

$$f_{y,d} = f_y / \gamma_{Mo}$$

$\sigma_{x,Ed}$ *and* $\sigma_{z,Ed}$ *shall be taken as positive for compression and negative for tension.*

The point to be considered is the joint between flange and web.



$$\sigma_{x,Ed} = \frac{M_{Sd}}{W_{el}}\frac{\left(h/2 - t_{fl}\right)}{h/2} = \frac{517x10^6}{2980x10^3}x\frac{533.1/2 - 15.6}{533.1/2} = 234\ N/mm^2$$

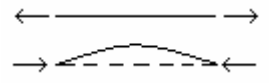
$$\sigma_{z,Ed} = \frac{F_{Sd}}{\left(S_s + t_{fl}\right)t_w} = \frac{(80+65)x10^3}{(75+15.6)x10.2} = 134\ N/mm^2;\ \textit{assume } S_s = 75\ mm$$

Therefore:

$$\left[\frac{234}{250}\right]^2 + \left[\frac{134}{250}\right]^2 - \left[\frac{234}{250}\right]\left[\frac{134}{250}\right] = 0.66 < 1 \quad OK.$$

Chapter 3. Compression Members and Stability Problem.

If a 6 mm diameter steel rod 1 m long is placed in a resting machine subjected to a pull, as shown in a Figure below, it will be found to carry a load of about 7 kn before failure occurs. If on the other hand this same rod had been subjected to compression, then the maximum load, which would have been carried, would be about 0.035 kN, a very big difference.

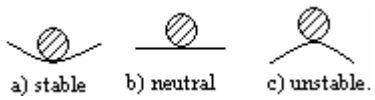


Failure in the first test occurs by the fracture of the member; in the second it is due to bending out of the line of action of the load, as indicated.

Since the load carrying capacity of a member in compression is very different from that of a similar member in tension, requires special treatment. It is seen that failure takes place by bending. This can not occurs unless a moment acts on a member and this moment results from a number of effects, which make an apparently axial load acts eccentrically. The causes are:

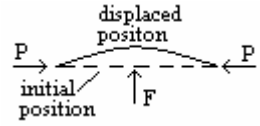
1. The fact that no member can be made perfectly straight.
2. Imperfection in manufacturing leaving some part of the member with slightly different mechanical properties from the remainder.
3. Inability to ensure that the load actually acts along the centre of area of the cross-section.

Types of Equilibrium.



a) Stable: The body returns to its initial position after disturbing its condition of equilibrium.
b) Neutral: The body remains in the same apparent equilibrium in its new position.
c) Unstable: The body loss its initial condition of equilibrium.

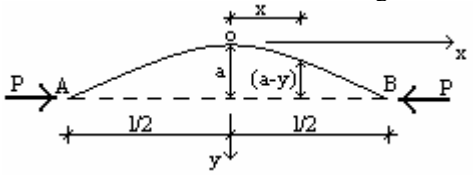
Now lets apply a disturbing force F at mid-height acting as shown in the figure.



If the strut returns to its position prior to the application of F, then it is in stable equilibrium.
If it remains in the deflected position, it is in neutral equilibrium.
If it continues to deflect, it is in unstable equilibrium and the strut loses its load carrying capacity and fails.

We can see that for low value of P the equilibrium is stable, but that as P is increased a load value is obtained which causes the strut to be in a state of neutral equilibrium. This load value is known as the **critical** or **buckling** load of a strut.

Critical load of a pin – ended strut. (Euler formula).
Lets consider the strut AB with length l as shown in the following figure.



The maximum deflection is **a** at mid span, and at distance x from the origin, the deflection is (a – y).
The differential equation of bending gives

$$EI\frac{d^2y}{dx^2} = M = P(a-y) \quad ; \quad writing \ \mu^2 = \frac{P}{EI};$$

$$\frac{d^2y}{dx^2} = \mu^2(a-y) \Rightarrow \frac{d^2y}{dx^2} - \mu^2(a-y) = 0 \quad or \quad \frac{d^2y}{dx^2} + \mu^2(y-a) = 0 \quad differential\ equation\ of\ second\ degree.$$

The solution for this equation is: $y = A\sin\mu x + B\cos\mu x + a$; where A and B are constants of integration. To evaluate A and B it is as follows:

1. When x = 0; y = 0 $\therefore 0 = A\sin 0 + B\cos 0 + a \Rightarrow B\cos 0 + a = 0 \Rightarrow B + a = 0\ and\ B = -a$

2. When x = 0; $\frac{dy}{dx} = 0$; (angle of rotation).

$\frac{dy}{dx} = A\cos\mu x.\mu - B\sin\mu x.x = 0$, therefore $A\mu\cos 0^o + a\mu\sin 0^o = 0$; is possible only if A = 0;

Finally the solution is: $y = -a\cos\mu x + a = a(1-\cos\mu x)$

Now, when x = ± l/2; y = a, and then $a = a(1-\cos\mu\frac{l}{2})$, from which $1 = 1-\cos\mu\frac{l}{2} \Rightarrow \cos\mu\frac{l}{2} = 0$,

Therefore $\mu l = \pi$, and $\sqrt{\frac{P}{EI}}l = \pi$; now squaring $\frac{Pl^2}{EI} = \pi^2$, and finally we obtain the formula to calculate the critical load, known as Euler Formula.

$P_E = \frac{\pi^2 EI}{l_e^2}$; where l_e = effective length.

Value for Ratio l_e / l for different end conditions. (Theoretically).

Ends / Ratio	Both sides built-in	Both sides pinned	One built-in other pinned	One built-in other free
l_e/l	0.5 l_e = 0.5 l	1 l_e = 1	0.7 l_e = 0.7 l	2 l_e = 2 l

To write the Euler formula in terms of stress, divide the critical load over the area.

$\sigma_E = \frac{P_E}{A} = \frac{\pi^2 EI}{l_e^2 A}$; $but\ r = \sqrt{\frac{I}{A}}\ therefore;\ r^2 = \frac{I}{A}$; is the least radius of gyration.

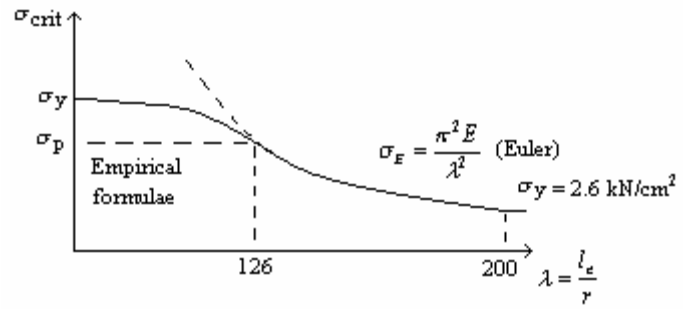
$\sigma_E = \frac{\pi^2 Er^2}{l_e^2} = \frac{\pi^2 E}{\left(\frac{l_e}{r}\right)^2}$. The relation $\frac{l_e}{r} = \lambda$ is the Slenderness ratio. And $\sigma_E = \frac{\pi^2 E}{\lambda^2}$

<u>Limitation of the Euler Formula</u>.

The formula show that σ_E depends only on the elastic modulus of the material and on the slenderness ratio, this value is true only for a constant modulus of elasticity; i.e. within elastic limits of the steel.

The steel behaves elastic only up to Proportional Limit σ_p. The Structural Stability Research Council (SSRC) of the USA accept for $\sigma_p = 0.5\ \sigma_y$, that is 0.5 the value for the yield limit to ensure perfectly elastic behaviour. Then for mild steel like A – 36, $\sigma_y = 24.82$ kN/cm^2 and E = 2 x 10^4 kN/cm^2:

$\sigma_E = \frac{\pi^2 E}{\left(\frac{l_e}{r}\right)^2} = 0.5\sigma_y \Rightarrow \left(\frac{l_e}{r}\right) \geq \frac{\pi^2 E}{0.5\sigma_y} = \frac{\pi^2 x2x10^4}{0.5x24.82} \therefore \left(\frac{l_e}{r}\right) \geq 126$. Therefore, for values of the slenderness ratio less than 126, Euler's formula is not valid, as shown in the following figure.



As we see, the Euler's buckling load can only represents column behaviour at higher values of the slenderness ratio. For lower values of the le / r, empirical formulae are used.

The SSRC of the USA recommended the following formula, which is accepted by the AISC (American Institute for Steel Construction) code.

$\sigma_{crit} = \left[1 + \frac{\left(\frac{l_e}{r}\right)^2}{2C_c^2}\right]\sigma_y$. In which $C_c = \sqrt{\frac{2\pi^2 E}{\sigma_y}}$, and for mild steel like A – 36 $C_c = 126$.

Design of Axially loaded Columns.

According with EBCS – 3. Design of Steel Structures, section 4.5.4,1; the compression resistance of cross section is as follows:

1. For member in axial compression, the design value of the compressive force $N_{com,Sd}$ at each cross-section shall satisfy:

   $N_{com,Sd} \le N_{com,Rd}$.

   Where $N_{com,Rd}$ is the design compression resistance of the cross-section, taken as the smaller of:

   a) The design plastic resistance of the gross section, $N_{pl,Rd} = \frac{Af_y}{\gamma_{Mo}}$, (for classes 1 – 3 cross-sections)

   b) The design local buckling resistance of the gross section, $N_{o,Rd} = \frac{A_{eff} f_y}{\gamma_{M1}}$ where A_{eff} is the effective area of the cross section (for class 4 section).

2. Buckling Resistance of Axially Loaded Compression Members ($N_{b,Rd}$).

   $N_{b,Rd} = \frac{\chi \beta_A A f_y}{\gamma_{M1}}$; Where $\beta_A = 1$ *for Class* 1, 2 *or* 3 *cross* – sections

   $\beta_A = \frac{A_{eff}}{A}$ *for Class* 4 *cross* – sections

   χ *is the reduction factor for the relevant buckling* mode.

   For the constant axial compression in members of constant cross-sections, the value of χ for the appropriate non-dimensional slenderness $\bar{\lambda}$, may be determined from:

   $$\chi = \frac{1}{\phi + \left[\phi^2 - \left(\bar{\lambda}\right)^2\right]^{0.5}} \quad but\ \chi \le 1.$$

Where:

$$\phi = 0.5\left[1+\alpha\left(\bar{\lambda}-0.2\right)+\left(\bar{\lambda}\right)^2\right]$$

α *is an imperfection factor.*

$$\bar{\lambda} = \left[\frac{\beta_A A f_y}{N_{cr}}\right]^{0.5} = \left(\frac{\lambda}{\lambda_1}\right)\left(\beta_A\right)^{0.5}$$

λ *is the slenderness ratio for the relevant buckling* mode.

$$\lambda_1 = \pi\left[\frac{E}{f_y}\right]^{0.5} = 93.9\varepsilon$$

$$\varepsilon = \left[\frac{235}{f_y}\right]^{0.5} \qquad \left(f_y \text{ in MPa}\right)$$

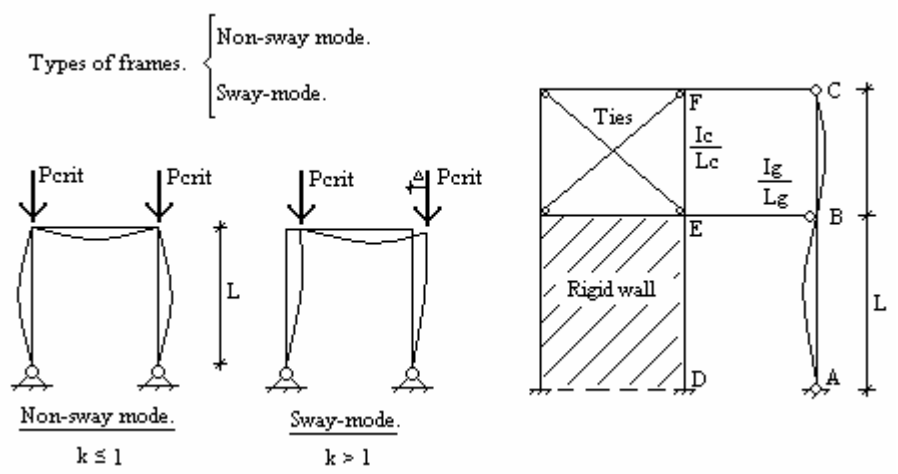
N_{cr} *is the elastic critical force for the relevant buckling* mode.

Notes:

- The imperfection factor α corresponding to the appropriate buckling curve shall be obtained from**Table 4.8**, page 21 of EBCS-3.
- The selections for a buckling curve for a cross-section shall be obtained from **Table 4.11**, page 24 of EBCS-3.
- Values for the reduction factor χ for the appropriate non-dimensional slenderness $\bar{\lambda}$ may be obtained from **Table 4.9**, page 21 of EBCS-3.

Buckling length of compression members.

For the basis about buckling length read 4.5.2.1, EBCS-3. When the column belong to a building frame, the procedure is as follow.



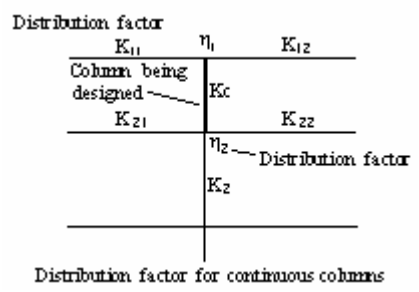
The frames are divided into 2 types, as shown in the figure above. The coefficient for buckling length ratio (k) depends of the type of frames; as shown, if sway is not allowed, k < 1, other case if sway occur then k > 1.
According to Appendix A of EBCS-3,

1 The buckling length l of a column in non-sway mode may be obtained from Fig. A.2.1.
2 The buckling length l of a column in a sway mode may be obtained from Fig. A.2.2.

The distribution factors at the ends of the member η_1 and η_2 are obtained from:

$$\eta = \frac{\Sigma K_{columns}}{\Sigma K_{columns} + \Sigma K_{beams}}$$

The symbol Σ includes only those members rigidly connected to the joint. For example:



The distribution factors are:

$$\eta_1 = \frac{K_c}{K_c + K_{11} + K_{22}} \quad and \quad \eta_2 = \frac{K_c + K_2}{K_c + K_2 + K_{21} + K_{22}}$$

K_c is the column stiffness coefficient $= I_{column}/L$

K_{ij} is the effective beam stiffness coefficient $= I_{beam}/L$

Finally, the slenderness ratio shall be taken as follows:

$\lambda = \frac{l}{r}$; Where r is the radius of gyration about relevant axis, determined using the properties of the gross cross-section.

The values of the slenderness ratio λ shall not exceed the following:

1	For members resisting loads other than wind loads	180
2	For members resisting self weight and wind loads only	250
3	For any member normally acting as a tie but subject to reversal of stress resulting from the action of wind	350

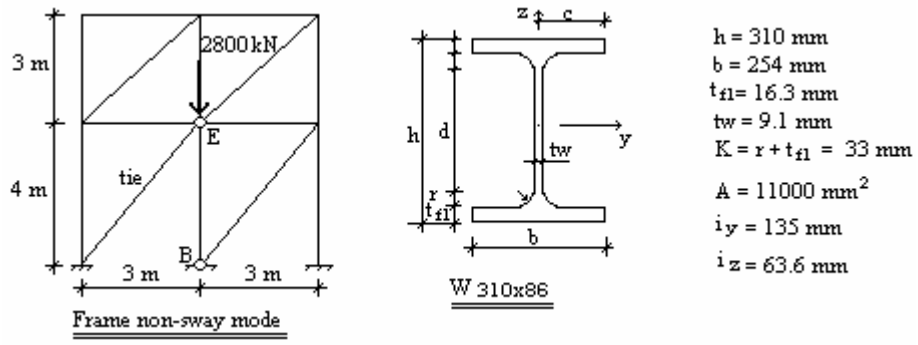
Design step for loading compression members:

1. Determine the axial load, N_{sd}.
2. Determine the buckling length, l, which is a function of the column length, L, and the statical system of the column.
3. Select a trial section (take into consideration economy, i.e. least weight per unit length).
4. Determine the Class of the section according to Section 4.3.2 and Table 4.1. If the cross-section is classified as Class 4, determine A_{eff} according to Section 4.3.4 and Table 4.4.
5. Determine the non-dimensional slenderness ratio $\overline{\lambda}$ (Section 4.5.4.3).
6. Using Table 4.11 determine the appropriate buckling curve.
7. Using Table 4.9 find the value of χ. Interpolation must be used to determine more exact values.
8. Calculate the design buckling resistance $N_{b,Rd}$ of the member. Buckling about both principal axes must be checked.
9. Check the computed buckling resistance against the applied load. If the calculate value is inadequate or is too high, select another section and go back to Step 4.

Chapter 3**. Columns**.

Example N_r1.

The column B – E on the Figure shown below is under the action of N_{Sd} = 2800 kN. Both sides are pinned. Check the resistance of the column. Steel grade F_e 430 is used.



Solution:

Step 1: Axial load N_{Sd} = 2800 kN.

Step 2: Buckling length L = 4000 mm (pinned end both sides. Frame non-sway mode).

Step 3: The section is given.

Step 4: Determine the class of the cross-section and check for a local buckling. The section is subjected to uniform compression. For the section to be classified as at least class 3, in order to avoid any modification to the full cross sectional area due to local buckling, the limiting width to thickness ratio for class 3 section are (See Table 4.1 EBCS-3).

Outstand element of compression flange: $c / t_f \le 15\,\varepsilon$.

Web subject to compression only: $d / t_w \le 39\,\varepsilon$.

For F_e 430 steel grade f_y = 275 N / mm^2. Thus $\varepsilon = \sqrt{235/275} = 0.92$

This gives the following limiting values:

Outstand element of compression flange: c / t_f = (254/2) / 16.3 = 7.78 < 15 x 0.92 = 13.8 OK.

Web subject to compression only: d / t_w = (310-2 (33)) / 9.1 = 26.8 < 39 x 0.92 = 35.88 OK.

Therefore, the section belongs to at least Class 3. Thus, $\beta_A = 1.0$

Step 5: Determine the non-dimensional slenderness ratio.

For F_e 430 steel grade, $\lambda_1 = 93.9\,\varepsilon = 93.9 \times 0.92 = 86.39$

Slenderness ratio about y-axis: $\lambda_y = L / i_y = 4000 / 135 = 29.63$

Slenderness ratio about z-axis: $\lambda_z = L / i_z = 4000/63.6 = 62.89$

Hence, the non-dimensional slenderness ratio is determined as:

$$\bar{\lambda}_y = \left(\lambda_y / \lambda_1\right)\sqrt{\beta_A} = 29.63/86.39\left(\sqrt{1}\right) = 0.34$$

$$\bar{\lambda}_z = \left(\lambda_z / \lambda_1\right)\sqrt{\beta_A} = 62.89/86.39\left(\sqrt{1}\right) = 0.73$$

Step 6: Determine the appropriate column curves (Table 4.11 EBCS-3).

$$h/b = 310/254 = 1.22 \quad and \quad t_f = 16.3\ mm < 40$$

Use curve **a** for buckling about y-axis and curve **b** for buckling about z-axis.

Step 7: Determine value of χ. Using Table 4.9 and interpolating:

For y-axis: curve a for $\bar{\lambda}_y = 0.34 \Rightarrow \chi_y = 0.97$

For z-axis: curve b for $\bar{\lambda}_z = 0.73 \Rightarrow \chi_z = 0.77$

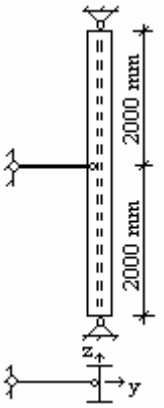
Therefore, buckling about the z-axis becomes critical.

Step 8: Calculate the design buckling resistance.

$$N_{b,Rd} = \frac{\chi \beta_A A f_y}{\gamma_{M1}} = \frac{0.77x1x11000x275}{1.1} = 2117500\ N = 2117.5\ kN$$

Step 9: Because 2800 kN > 2117.5 kN, the column do not resist.

Solution 1. Add an additional hinged support at mid-height to increase the resistance about the minor axis.



Go to Step 5.
Slenderness ratio about z-axis = 29.63 (don't varies)
Slenderness ratio about z-axis = 2000 / 63.6 = 31.45
Non dimensional slenderness ratio $\bar{\lambda}_y = o.34$ don't varies

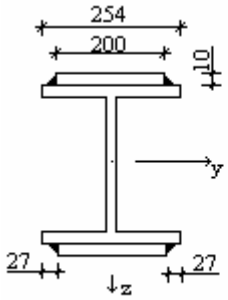
$$\bar{\lambda}_z = \frac{31.45}{86.39}(1) = 0.36$$

Values of χ:
y-axis: $\chi_y = 0.97$ don't varies
z-axis: Curve **b** for $\bar{\lambda}_z = 0.36 \Rightarrow \chi_z = 0.94$

Hence buckling about the z-axis becomes critical

$$N_{b,Rd} = \frac{0.94x11000x275}{1.1} = 2585\ kN < 2800\ kN. \quad don't\ resist$$

Solution 2: Add 2 plates 200 x 10 mm to reinforce the weak axis.



A = 11000 + 4000 = 15000 mm^2

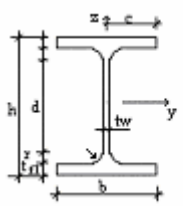
Now: $I_z = I_{zW} + 2\frac{10x200^3}{12} = 44.5x10^6 + 13.3x10^6 = 5.78x10^7\ mm^4$

$$i_z = \sqrt{\frac{I_z}{A}} = \sqrt{\frac{5.78x10^7}{1.5x10^4}} = 62\ mm\text{; then } \lambda_z = \frac{4000}{62} = 64.52 \quad and \quad \bar{\lambda}_z = \frac{64.52}{86.39} = 0.72$$

For column curve **b**; $\chi_z = 0.77$

And $N_{b,Rd} = \frac{0.77x1.0x15000x275}{1.1} = 2887.5\ kN > 2800\ kN \quad OK$

Example N_r 4.
Determine the design buckling resistance of a 457 x 152 x 52 UB used as a pin-ended column. The column is 3.00 m long and its steel grade is F_e 360.



h = 449.8 mm
b = 152.4 mm
t_f = 10.9 mm
t_w = 7.6 mm
r = 10.2 mm

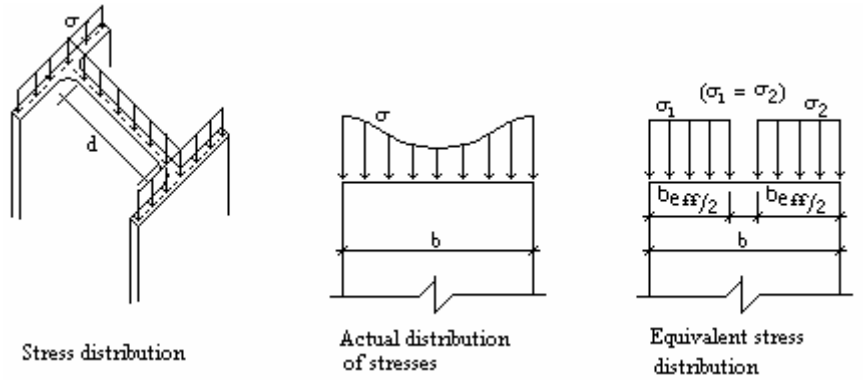
A = 66.5 cm^2
Iy = 21300 cm^4
Iz = 645 cm^4
iy = 17.9 cm
iz = 3.11 cm

Step 2: Buckling length = 3000 mm.
Step 3: The section is given.
Step 4: Determine the class of the cross-section and check for local buckling.

For F_e steel grade f_y = 235 N / mm^2. Thus, $\varepsilon = \sqrt{235/f_y} = 1$

These limiting values are:
Outstand element of compression flange: $c / t_f \le 15\ \varepsilon = 15$
Web subject to compression only: $d / t_w \le 39\ \varepsilon = 39$

For the 457 x 152 x 52 UB profile, the actual values are:
Outstand element of compression flange: $c / t_f = (152.4 / 2) / 10.9 = 7 < 15$ OK.
Web subject to compression only: $d / t_w = (449.8 - 2 \times 10.9 - 2 \times 10.2) / 7.6 = 53.60 > 39$
Therefore, the flange satisfies the Class 3 requirement, but the web is Class 4 section. Consequently, there must be a reduction in the strength of the section to allow for the load buckling which will take place in the web. Therefore, the effective area, A_{eff} must be determined for the web.
Explanation for the effect.



The effective width is b_{eff} = reduction factor x b = ρ x b.
The method to calculate the effective area (A_{eff}) is explained in section 4.3.4 of EBCS-3.
To calculate the reduction factor ρ is as follow.

a). ρ = 1; if $\bar{\lambda}_p \le 0.673$

b). $\rho = \left(\bar{\lambda}_p - 0.22\right) / \bar{\lambda}_p^{\,2}$ *if* $\bar{\lambda}_p > 0.673$

Where $\bar{\lambda}_p = \left(\bar{b}/t\right)\left(28.4\varepsilon\sqrt{k_\sigma}\right)$

In which **: t** is the relevant thickness.
$\mathbf{k_\sigma}$ is the buckling factor corresponding to the stress ratio ψ from Table 4.3 or 4.4 as appropriate.
$\bar{b} = d$ for webs.

In our example, since the column is axially loaded the stress distribution is uniform, i.e. $\sigma_1 = \sigma_2$. Table 4.3 is used to calculate the effective width.

Thus, $\sigma_1/\sigma_2 = 1$, and $k_\sigma = 4.0$ (see lower part of table 4.3)

$$\bar{b} = d = 407.6\ mm$$

$$\bar{b}/t_w = 407.6/7.6 = 53.6$$

$$\bar{\lambda}_p = 53.6/\left(28.4x1x\sqrt{4}\right) = 0.944 > 0.673$$

$$\therefore \rho = \left(\bar{\lambda}_p - 0.22\right)/\bar{\lambda}_p^2 = (0.944 - 0.22)/0.944^2 = 0.812$$

$$And \quad \bar{b}_{eff} = \rho\bar{b} = 0.812x407.6 = 331.2\ mm$$

Therefore the area that should be ignored at the center of the web is: $\Delta A = (407.7 - 331.2)x7.6 = 581.4\ mm^2$

And then $\beta_A = \frac{A_{eff}}{A} = (6650 - 581.4)/6650 = 0.913$

Step 5: Determine the non-dimensional slenderness ratio (axis-z govern).

$$\lambda_z = \frac{3000}{31.1} = 96.5$$

$$\lambda_1 = 93.9\varepsilon = 93.9$$

Hence the non dimensional slenderness ratio $\bar{\lambda}_z = \left(\lambda_z/\lambda_1\right)\sqrt{\beta_A} = (96.5)/93.9\sqrt{0.913} = 0.98$

Step 6: Appropriate column curve.

For h / b = 449.8 / 152.4 = 2.95 > 1.2; and t_f = 10.9 < 40 mm; use curve **b** for buckling about z-axis.

Step 7: Determine the value of χ.

Using Table 4.9 and interpolating, z-axis: curve **b** for $\bar{\lambda}_z = 0.98 \Rightarrow \chi_z = 0.6034$

Step 8: Calculate the design buckling resistance.

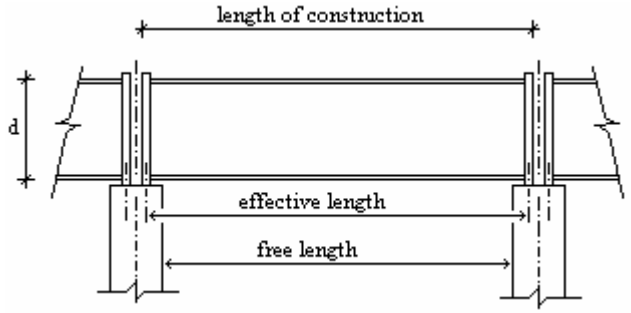
$$N_{b,Rd} = \frac{\chi\beta_A A f_y}{\gamma_{M1}} = \frac{0.6034x0.913x6650x235}{1.1} = 782660\ N$$

Answer: The design buckling resistance $N_{b,Rd} = 782.66\ kN$.

Chapter 4. Bending Members.

4.1 Introduction.
4.2 Plastic behaviour of steel beams.
4.3 Laterally restrained beams.
4.4 Laterally unrestrained beams.
4.5 Resistance of web to transverse forces.

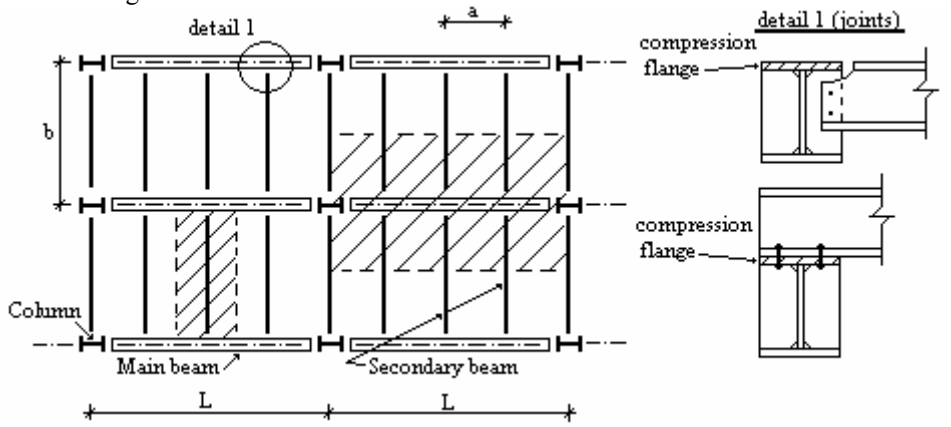
4.1 **Introduction**.
Beams work principally under the action of the vertical loads, which rise to bending of the beam. The principal dimensions are the length and the depth. There are 3 types of length as shown in the figure.



The minimum d recommended to avoid excessive deformation is as follows.

$\delta/L=1/r_0$	1/1000	1/750	1/600	1/500	1/400	1/250	1/200
d_{min}/L	1/6	1/8	1/10	1/12	1/15	1/25	1/30

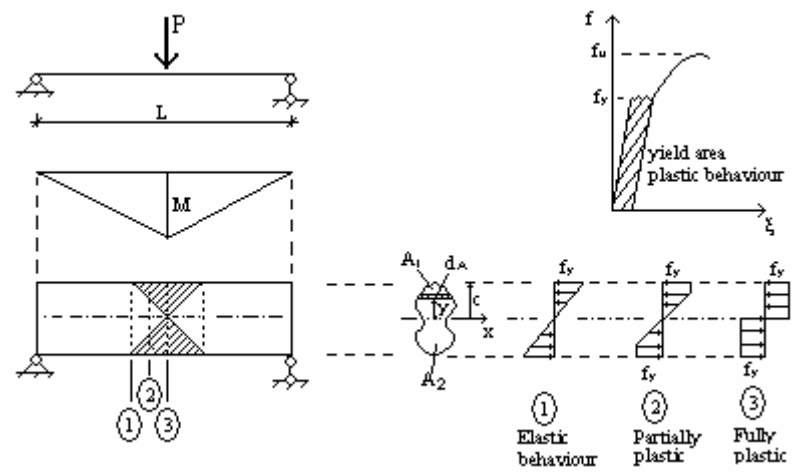
Beam arrangement.



Tributary area.
Secondary beam a x b
Main beam b x L
For column b x L
Secondary beams should be continuous for better structural behaviour as shown in the figure below.



4.2. Plastic behaviour of steel beams.
Let study a beam of any cross-section.



In stage (1) the beam behaves elastically, the extreme fibbers rich the yield point.

For elastic behaviour $f = \frac{M}{W} \le f_y \quad where\ W = \frac{Inertia}{c}\ is\ the\ elastic\ section\ modulus.$ And the maximum value for f is the yield limit f_y.

Stage (2) is partially plastic, yield stress go deep into the cross-section.
Stage (3) is fully plastic, the section rotate and **plastic hinge** is formed. The section is under the action of the **Plastic Moment M_P**

$$M_p = \int_A f_y dA \cdot y = \int_{A_1} f_y y dA + \int_{A_2} f_y y dA = f_y \left[\int_{A_1} y dA + \int_{A_2} y dA \right]$$

but $\int y dA = S$ is the First Moment of Area.

Therefore $M_p = f_y (S_1 + S_2)$ and for symmetric section $S_1 = S_2 = S$.

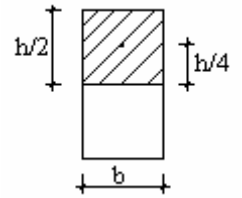
Hence $M_p = f_y 2S$; doing $W_p = 2S$ -- Plastic Modulus. **S** is the first moment of area for the half section.

Finally we can write (by similarity) $M_p = f_y W_p$.

Then;

Elastic behaviour	**Plastic behaviour**
$M = f_y W$	$M_P = f_y W_P$

To compare **M** with $\mathbf{M_P}$ let evaluate $\mathbf{W_e}$ and $\mathbf{W_P}$ for rectangular section.



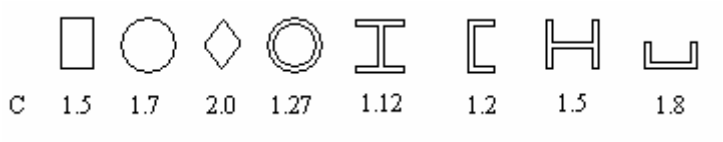
$$W = \frac{bh^2}{6} \quad and \quad W_P = 2S = 2b\frac{h}{2}\cdot\frac{h}{4} = \frac{bh^2}{4}$$

Now the ratio between Plastic Moment and Elastic one is M_P / M_e

$$\frac{M_P}{M_e} = \frac{f_y W_P}{f_y W_e} = \frac{bh^2/4}{bh^2/6} = \frac{6}{4} = 1.5$$

For the general cases $\frac{M_P}{M_e} = C$; Where C is the **Shape Coefficient** of the section.

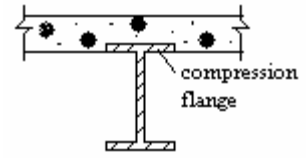
The most common values of the shape coefficient are as follow.



C	1.5	1.7	2.0	1.27	1.12	1.2	1.5	1.8

4.3 Laterally restrained beam.

A beam is prevented from moving side ways, by a floor resistance due to the use of bracing or insitu or precast floor construction.



Resistance to bending moment.

According to EBCS-3, for bending about one axis in the absence of shearing force, the design value of bending moment $M_{Sd} \le M_{c,Rd}$. The design moment resistance of a cross-section without holes for fasteners may be determined as follows:

a) Class 1 or 2 cross-sections: $M_{c,Rd} = \frac{W_{pl} f_y}{\gamma_{M0}}$

b) Class 3 cross-sections: $M_{c,Rd} = \frac{W_{el} f_y}{\gamma_{M0}}$

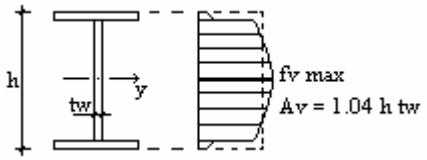
c) Class 4 cross-sections: $M_{c,Rd} = \frac{W_{eff} f_y}{\lambda_{M1}}$

Fastener holes in the tension flange need not be allowed for, provide that for the tension flange: $\left(\frac{0.9A_{f,net}}{A_f}\right) \ge \left(\frac{f_y}{f_u}\right)\left(\frac{\gamma_{M2}}{\gamma_{M1}}\right)$

Resistance to shear.

The design value of the shear force V_{Sd} at each cross-section shall satisfy: $V_{Sd} \le V_{pl,Rd}$

Where $V_{pl,Rd} = \frac{A_v\left(f_y/\sqrt{3}\right)}{\gamma_{M0}}$ is the plastic shear resistance. A_v is the shear area.



For simplicity, a rectangular distribution of shear stress is accepted and $A_v = 1.04\ h\ t_W$ for a rolled I, H or channel section, load parallel to web.

Resistance for bending and shear.

The theoretical plastic resistance moment of a cross-section is reduced by the presence of the shear. For small values of the shear force this reduction is not significant and may be neglected. However, when the shear force exceeds half of the plastic shear resistance, allowance shall be made for its effect on plastic resistance moment.

Hence, if the value of the shear force V_{Sd} does not exceed 50% of the design plastic shear resistance no reduction need be made in the resistance moments. When V_{Sd} exceeds 50% the design resistance moment of the cross-section should be reduced to $M_{v,Rd}$ obtained as follows:

a) For cross-sections with equal flanges, bending about the mayor axis

$$M_{v,Rd} = \left[W_{pl} - \frac{\rho A_v^2}{4t_w} \right] \frac{f_y}{\gamma_{M0}} \quad but\ M_{v,Rd} \le M_{c,Rd}; \quad \rho = \left(\frac{2V_{Sd}}{V_{pl,Rd}} - 1 \right)^2$$

b) For other cases $M_{v,Rd}$ should be taken as the design plastic resistance moment of the cross-section, calculated using a reduced strength $(1 - \rho)\, f_y$ for the shear area, but not more than $M_{c,Rd}$.

Deflections.

Deflection belongs to serviceability limit states; the loads used to calculate deflections are characteristic loads that are unfactored loads. For vertical deflection the value for the maximum deflection is calculated as follows:

$$\delta_{\max} = \delta_1 + \delta_2 - \delta_0$$

Where: δ_{max} – is the sagging in the final state relative to the straight line joining the supports

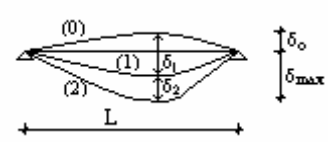
δ_0 – is the pre-camber of the beam in unloaded state, (state 0)

δ_1 – is the variation of the deflection of the beam due to the permanent loads immediately after load, (state 1)

δ_2 – is the variation of the deflection of the beam due to the variable loading plus any time dependent deformation due to the permanent load, (state 2).

Limiting values.

For buildings, the recommended limits values for vertical deflections are given in Table 5.1 of EBCS-3, in which L is the span of the beam. For cantilever beams, the length L to be considered is twice the projecting length of the cantilever. The vertical deflection to be considered is illustrated in the following Figure.



Conditions	Limits	
	δ_{max}	δ_2
Roof generally	L/200	L/250
Roof frequently carrying personnel other than for maintenance.	L/250	L/300
Floor generally	L/250	L/300
Floors and roofs supporting plaster or other brittle finish or non-flexible partitions.	L/250	L/350
Floors supporting columns (unless the deflection has been included in the global analysis for the ultimate limit state).	L/400	L/500
Where δ_{max} can impair the appearance of the building	L/250	

For horizontal deflection the recommended limits at the tops of the columns are:

1. Portal frames without gantry cranes: h/150
2. Other single storey building: h/300
3. In multi-storey building:
   (i) in each storey h/300
   (ii) on the structure as a whole h_0/500

Where h is the height of the column or of the storey

h_0 is the overall height of the structure.

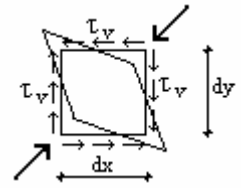
Lost of Stability

- Stability
  - General Stability (Lateral – torsional buckling)
  - Local Stability
    - Due by skear on the web.
    - Due by compressive stress on the flange.

During bending, part of the web and one flange at least is under compressive stress, therefore can be subjected to the loss of stability.

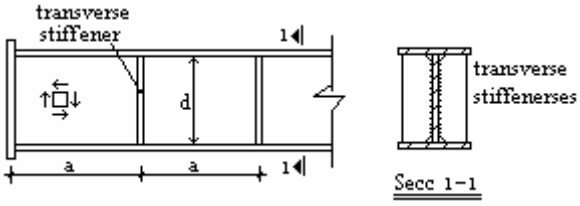
Let study first the problem of local stability.

1). Shear buckling resistance. Near the support, where there is a considerable acting shear force, the web of the beam can lost its stability as follows:



Because the action of the shearing stress, appear on the web folds of buckles.

This problem is prevented by putting in place transverse stiffness as shown in the figure belows.



The shear buckling resistance of the web depends on the depth – to thickness ratio d/t_w and the spacing of any intermediate web stiffeners. All webs with d/t_w greater than 69ε shall be provided with transverse stiffeners at the supports. Webs with d/t_w greater than 69ε for an unstiffened web, or $30\varepsilon\sqrt{k_\tau}$ for stiffened web, shall be checked for resistance to shear buckling.
Normally, $a/d > 3$ is used, for these beams the simple post critical method is recommended.
According with this method, the design shear buckling resistance $V_{be,Rd}$ should be obtained from:
$V_{ba,Rd} = dt_w\tau_{ba}/\gamma_{M1}$, Where τ_{ba} is the simple post-critical shear strength and should be determined as follows:

$$\tau_{ba} = \left(f_{yw}/\sqrt{3}\right) \quad if \quad \bar{\lambda} \le 0.8$$

$$\tau_{ba} = \left[1 - 0.625\left(\bar{\lambda} - 0.8\right)\right]\left(f_{yw}/\sqrt{3}\right) \quad if \quad 0.8 < \bar{\lambda} < 1.2$$

$$\tau_{ba} = \left(0.9/\bar{\lambda}\right)\left(f_{yw}/\sqrt{3}\right) \quad if \quad \bar{\lambda} \ge 1.2$$

in which $\bar{\lambda} = \dfrac{d/t_w}{37.4\varepsilon\sqrt{k_\tau}}$ is the web slenderness.

k_τ is the buckling factor for shear, is given by the following:

a). for webs with transverse stiffeners at the supports but no intermediate transverse stiffeners $k_\tau = 5.34$

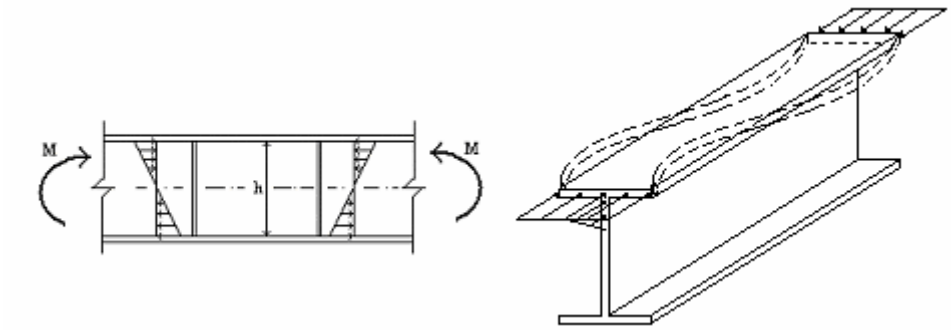
b). for webs with tranverse stiffenerss at the supports and intermediate transverse stiffeners

$$k_\tau = 4 + 5.34/(a/d)^2 \quad if \quad a/d < 1$$

$$k_\tau = 5.34 + 4/(a/d)^2 \quad if \quad a/d \geq 1$$

2). Flange induced buckling.

As we can see in the figure below, the upper flange is under the action of the compressive stress and may lose it local stability.



To prevent the possibility of the compression flange buckling in a plane of the web, the ratio d/t_w of the web shall satisfy the following criterion:

$d/t_w \leq k(E/f_{yf})\sqrt{(A_w/A_{fc})}$ Where A_w is the area of the web.

A_{fc} is the area of the compression flange

and f_{yf} is the yield strength of the compression flange.

The values of **k** shuold be taken as follows:

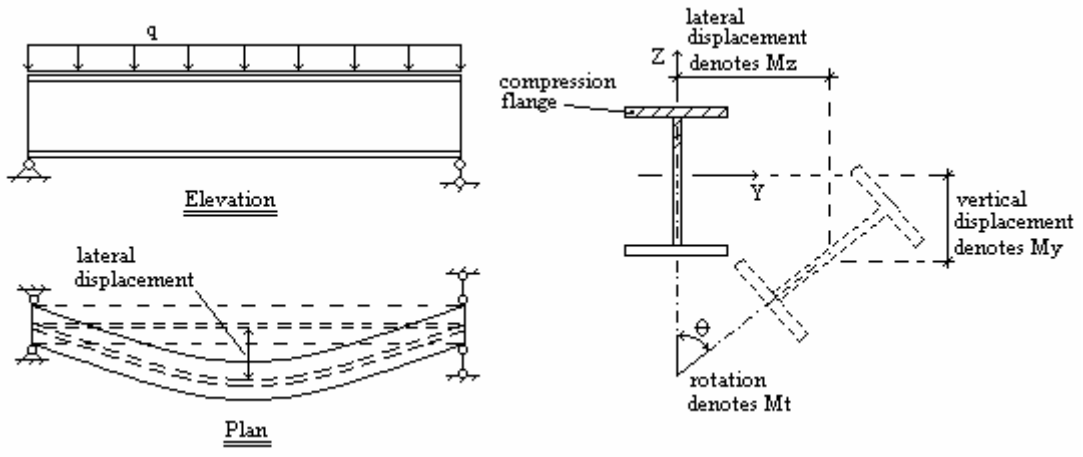
Class 1 flanges = 0.3

Class 2 flanges = 0.4

Class 3 or Class 4 flanges = 0.55

4.4 Laterally unrestrained beams. (Lateral – torsional buckling).

Lateral – torsional buckling should be present on laterally unrestrained beams. When the beam has a higher bending stiffness in the vertical plane compared with the horizontal plane, the beam can twist sideways under the action of the load as shown in the Figure belows:



Design Buckling Resistance Moment.
The design buckling resistance moment of laterally unrestrained bean shall be taken as:

$$M_{b,Rd} = \chi_{LT}\beta_w W_{pl,y} f_y / \gamma_{M1}$$

Where $\beta_w = 1$ for Class 1 or Class 2 cross-sections

$\beta_w = W_{el}/W_{pl,y}$ for Class 3 cross-sections

$\beta_w = W_{eff,y}/W_{pl,y}$ for Class 4 cross-sections.

And the value of χ_{LT} is the reduction factor for lateral-torsional buckling, is calculated as follows for appropriate non-dimensional slenderness $\overline{\lambda}_{LT}$.

$$\chi_{LT} = \frac{1}{\phi_{LT} + \left[\phi_{LT}^2 - \overline{\lambda}_{LT}^2\right]^{0.5}} \quad but\ \chi_{LT} \le 1$$

Where $\phi_{LT} = 0.5\left[1 - \alpha_{LT}\left(\overline{\lambda}_{LT} - 0.2\right) + \overline{\lambda}_{LT}^2\right]$

The value of the imperfection factor α_{LT} for lateral-torsional buckling should be taken as follows:

$\alpha_{LT} = 0.21$ for rolled sections.

$\alpha_{LT} = 0.49$ for welded sections.

Values of the reduction factor χ_{LT} for the appropriate non-dimensional slenderness $\overline{\lambda}_{LT}$ may be obtained from Table 4.9 with $\overline{\lambda} = \overline{\lambda}_{LT}$ *and* $\chi = \chi_{LT}$ using:

1. for rolled sections curve **a** (α = 0.21)
2. for welded sections curve **c** (α = o.49)

$$\overline{\lambda}_{LT} = \left[\frac{\lambda_{LT}}{\lambda_1}\right](\beta_w)^{0.5} \quad \text{where} \quad \lambda_1 = 93\varepsilon \quad \varepsilon = \sqrt{\frac{235}{f_y}} \quad and \quad f_y\ in\ MPa$$

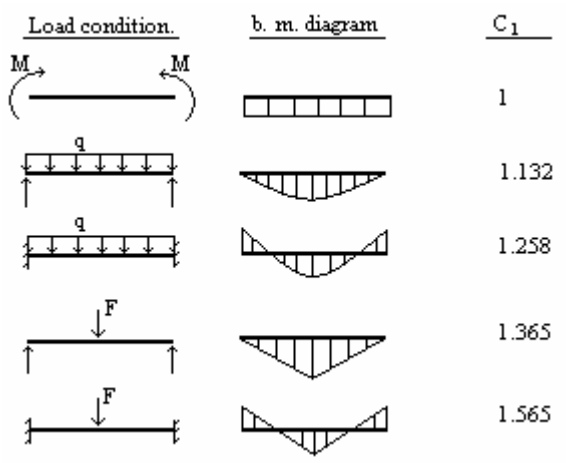
The geometrical slenderness ratio λ_{LT} for lateral-torsional buckling is given for all cases by:

$$\lambda_{LT} = \left[\frac{\pi^2 E W_{pl,y}}{M_{cr}} \right]^{0.5}$$

M_{cr} is the elastic critical moment for lateral-torsional buckling and for beam of uniform symmetrical cross-section with equal flanges, under standards conditions of restraint at each end, loaded trough its shear centre and subjected to uniform moment is calculated as follows:

$$M_{cr} = C_1 \frac{\pi^2 E I_z}{(kL)^2} \left[\frac{I_w}{I_z} + \frac{L^2 G I_t}{\pi^2 E I_z} \right]^{0.5}$$

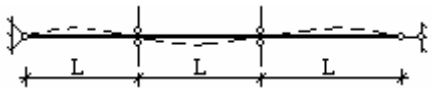
Factor C_1 depends on the loading conditions (See table 4.12 and 4.13 of EBCS-3)
The most common values for C_1 are as follows:



The rest of the terms are:

$$G = \frac{E}{2(1+\upsilon)} = 80\ GPa$$

I_t is the torsion constant.
I_w is the warping constant.
I_z is the second moment of area about the minor axis.
kL is the length of the beam between points which have lateral restraint.
k = 0.5 for full fixity.
k = 1.0 for no fixity.
k = 0.7 for one end fixed and one end free.

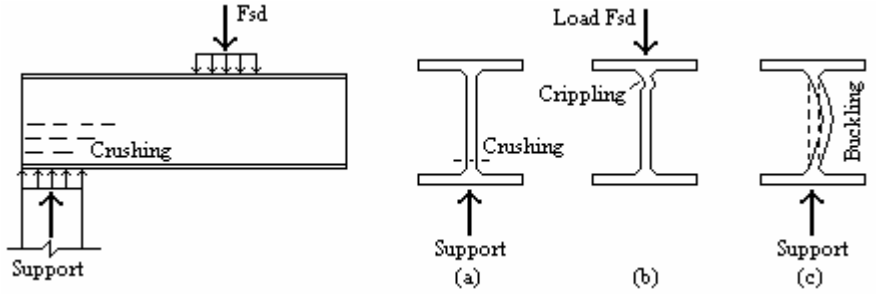


Notes:

1. A beam with full restraint does not need to be checked for lateral-torsional buckling.
2. Where the non-dimensional slenderness $\overline{\lambda} \le 0.4$ no allowance for lateral-torsional buckling is necessary.
3. The standard conditions of restraint at each end are:
   - Restrained against lateral movement.
   - Restrained against rotation about the longitudinal axis
   - Free to ratate in plan.

4.5 Resistance of web to transverse forces.

Due to high vertical stresses directly over a support or under concentrated load, the beam web may actually crush, or buckle as a result of these stresses, as illustrated in a figure below.



The resistance of an unstiffened web to transverse forces applied through a flange, is governed by one of the following modes of failure:

a) Crushing of the web close to the flange, accompanied by plastic deformation of the flange. See (a).
b) Crippling of the web in the form of localized buckling and crushing of the web close to the flange, accompanied by deformation of the flange. See (b).
c) Buckling of the web over most of the member. See (c).

A distinction is made between two types of load application, as follows:

1. Forces applied through one flange and resisted by shear in the web, See Fig (a). in this case the resistance of the web to transverse fosces should be taken as the smaller of:
   I) The crushing resistance.
   II) The crippling resistance.

2. Forces applied to one flange and transferred through the web directly to the other flange. See Fig (b). In this case the resistance of the web to transverse forces should be taken as the amaller of:
   I) The crushing resistance.
   II) The buckling resistance.

(I). **Crushing Resistance**.

The design crushing resistance $\mathbf{R_{y,Rd}}$ of the web of an I, H or U section should be obtained from:

$R_{y,Rd} = \dfrac{(S_s + S_y)t_w f_{yw}}{\gamma_{M1}}$, in which S_y is given by:

$S_y = 2t_f \sqrt{(b_f / t_w)(f_{yf} / f_{yw})\left[1 - (\sigma_{f,Ed} / f_{yf})^2\right]}$ but b_f should not be taken as more than 25 t_f and $\sigma_{f,Ed}$ is the longitudinal stress in the flange.

S_y represents the length over which the applied force is effectively distributed. At the end of the member S_y should be halved.

S_S is the length of the stiff bearing. See Fig. 4,28 and 4.29 of EBCS-3.

For wheel loads from cranes, transmitted through a crane rail bearing on a flange but not welded to it, the design crushing resistance of the web $R_{y,Rd}$ should be taken as:

$R_{y,Rd} = S_y t_w f_{yw} / \gamma_{M1}$, in which:

$$S_y = k_R \left(\frac{I_f + I_R}{t_w} \right)^{1/3} \sqrt{\left[1 - \left(\sigma_{f,Ed} / f_{yf} \right)^2 \right]} \text{ or more approximately } S_y = 2\left(h_R + t_f \right) \sqrt{\left[1 - \left(\sigma_{f,Ed} / f_{yf} \right)^2 \right]}$$

Where: h_R is the height of the crane rail.

I_f is the second moment of area of the flange about its horizontal centroidal axis.

I_R is the second moment of area of the crane rail about its horizontal centroidal axis.

k_R is a constant taken as follows:

a). When the crane rail is mounted directly on the flange, $k_R = 3.25$

b). When a suitable resilient pad not less than 5 mm thick is interposed between the crane rail and the beam flange. $K_R = 4.0$

II).Crippling Resistance.

The design crippling resistence $R_{a,Rd}$ of the web of an I, H or U section should be obtained from:

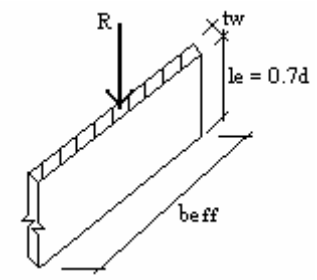
$R_{a,Rd} = 0.5 t_w^2 \sqrt{E f_{yw}} \left[\sqrt{t_f / t_w} + 3\left(t_w / t_f \right)\left(S_s / d \right) \right] / \gamma_{M1}$, but S_s/d should not be taken as more than 0.2

Where the member is also subjected to bending moments, the following criteria should be satisfied;

$$\frac{F_{Sd}}{R_{a,Rd}} + \frac{M_{Sd}}{M_{c,Rd}} \le 1.5$$

III). Buckling Resistance.

For the web the design buckling resistance should be obtained by considering the web as a virtual compression member with an effective breath $b_{eff.}$.



The buckling resistance should be determuned from Chapter 3 using buckling curve **c** and $\beta = 1$. The buckling length of the virtual compression member should be determined from the conditions of lateral and rotational restraint at the flanges at the point of load application, but not less than 0.75d

Transverse stiffeners.

End stiffeners and intermediate stiffeners at internal support normally be double sided and symmetric about the centerline of the web.

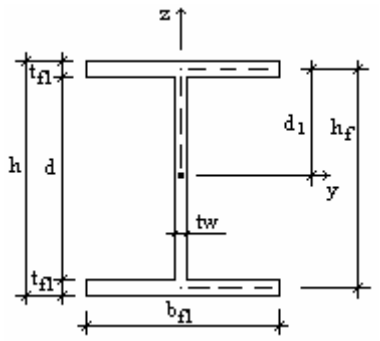
When checking the buckling resistance, the effective cross-section of a stiffener should be taken as including a width of the web plate equal to $30\varepsilon t_w$, arranged with $15\varepsilon t_w$ each side of the stiffener, as shown in Fig. 4.30. At the ends of the member (or openings in the web) the dimension of $15\varepsilon t_w$ should be limited to the actual dimension available.

In addition to checking the buckling resistance, the cross-section resistance of a load bearing stiffeners should also be checked adjacent to the loaded flange. The width of web plate included in the effective cross-section should be limited to S_y and allowance should be made for any opening cut in the stiffener to clear the web-to-flange welds. For intermediate transverse stiffeners it is only necessary to check the buckling resistance, provided that they are not subjected to external loads.

Chapter 4. Beams.

Built-up beams.

Dimensions of the section.



$$h = 1.2\sqrt{\frac{W_{req}}{t_w}} \quad where \quad W_{req} = \frac{M_{SD}}{f_y / \gamma_{Mo}} \quad and \quad t_{w,Min} = 8\ mm$$

Try that $\frac{h}{t_w} \le 69\varepsilon$

$b_{fl} = \frac{W_{req}}{h t_{fl}} - \frac{h t_w}{6 t_{fl}}$ *use* $t_w \le t_{fl} \le 3t_w; \quad t_{fl,Max} = 40\ mm$. Try that c/t_{fl} satisfies Class 1 or 2 conditions.

Now, with all the dimensions defined:

$$I_y = \frac{t_w d^3}{12} + 2 b_{fl} t_{fl} d_1^2$$

$$I_z = 2\frac{t_{fl} b_{fl}^3}{12}$$

$$W_{el,y} = \frac{2I_y}{h} \quad and \quad W_{pl,y} = 1.12 W_{el,y}$$

Warping Constant $I_w = \frac{I_z h_f^2}{4}$

Torsion Constant $I_t = \Sigma \frac{b_i t_i^3}{3} = \frac{d h_w^3}{3} + 2\frac{b_{fl} t_{fl}^3}{3}$

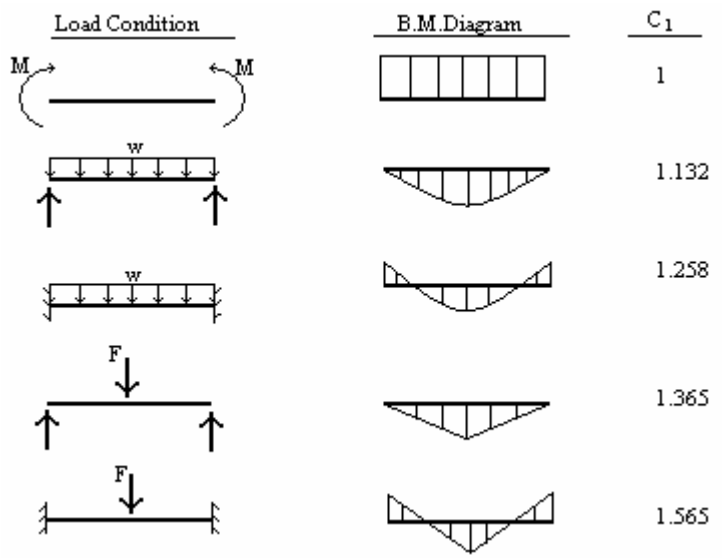
Thickness of the plates multiples of 2 mm.
Width of the plates multiples of 20 mm.

Values for M_{cr}.
For beams with doubly symmetric cross-sections and with no end fixity.

$$M_{cr} = C_1 \frac{\pi^2 EI_z}{L^2}\left[\frac{I_w}{I_z} + \frac{L^2 GI_t}{\pi^2 EI_z}\right]$$

Factor C_1 depends on the loading conditions (see table 4.12 and 4.13 of EBCS-3)

$C_1 = 1.88 - 1.44\psi + 0.52\psi^2 \le 2.7$, But the most commons values for C_1 are as follows:



Example Nr 1.

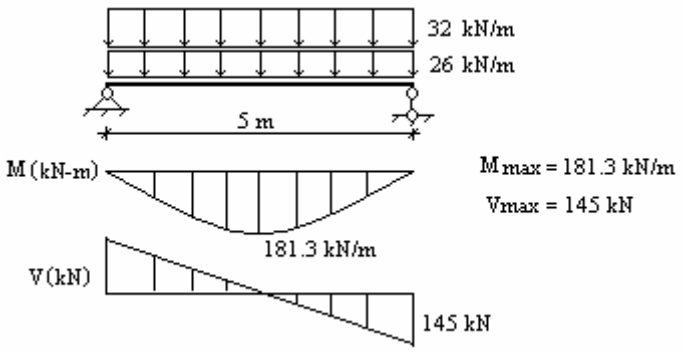
Design a simple supported beam of F_e 430 steel grade. The span of the beam is 5.00 m. The top flange is embedded in a reinforced concrete floor providing sufficient restraint against lateral-torsional buckling. The beam carries a uniformly distributed load of 20 kN/m permanent load and 20 kN/m imposed load.

Solution:

The factored loads are: Imposed load: 1.6 x 20 = 32 kN/m

Permanent load: 1.3 x 20 = 26 kN/m.

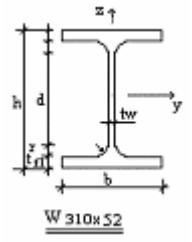
Step 1: Maximum bending moment.



Step 2. Required plastic modulus.

$$W_{pl} = \frac{M}{f_y / \gamma_{Mo}} = \frac{181.3x10^6\ N-mm}{275/1.1} = 7.25x10^5\ mm^3 = 725\ cm^3$$

Step3: Selection of the profile: Try 310 x 52 W Shape.



h = 317 Mass = 0.52 kN/m

b = 167 $W_{y,el}$ = 747 cm^3

t_{fl} = 13.2 mm $W_{y,pl}$ = 837 cm^3

t_w = 7.6 mm I_y = 11800 cm^4

d = 257 mm

Class of the section: $\varepsilon = \sqrt{\frac{235}{275}} = 0.92 \quad \frac{167/2}{13.2} = 6.33 < 9.5\varepsilon \quad and \quad \frac{257}{7.6} = 33.82 < 83\varepsilon \quad OK$

Satisfies conditions for Class 2.

Step 4. Resistance moment.

$$M_{pl,Rd} = \frac{W_{pl} f_y}{\gamma_{Mo}} = \frac{837x10^3 x275}{1.1} = 209.25\ kN-m > 181.3\ kN-m \quad OK.$$

Check of the self-weight of the beam.

w = 0.52 kN/m; additional factored dead weight moment = 1.3 x 0.52 x 5^2 / 8 = 2.11 kN-m

Total moment: M_{Sd} = 181.3 + 2.11 = 183.41 < $M_{pl,Rd}$ OK.

Step 5: Check for shear.

Maximum shear force is 145 kn + 1.3 x 5/2 = 146.69 kN.

Shear resistance of section. Un stiffened web: shear buckling resistance must not be verified if $d/t_w \le 69\varepsilon$.

d/t_w = 257/7.6 = 33.82 < 69 x 0.92

Therefore: $V_{pl,Rd} = A_v\left(f_y/\sqrt{3}\Big/\gamma_{Mo}\right)$ *where* $A_v = 1.04h_w t_w$

$A_v = 1.04x317x7.6 = 2506\ mm^2$ *then* $V_{pl,Rd} = \dfrac{2506\left(275/\sqrt{3}\right)}{1.1} = 361.7\ kN$

Effect of shear force on the resistance moment.

$\dfrac{V_{Sd}}{V_{pl,Rd}} = \dfrac{146.69}{1.1} = 0.41 < 0.5$ Therefore no reduction of design resistance moment required.

Step 6. Check for deflection.

For a simple span uniformly loaded beam $\delta = \dfrac{5}{384}\dfrac{wL^4}{EI}$

Deflection due to un factored load $w = 20 + 20 + \dfrac{0.52}{5} = 40.10$

$\delta = \dfrac{5x40.10x10^3 x5000^4}{384x118x10^6 x2.1x10^5} = 13.17\ mm$

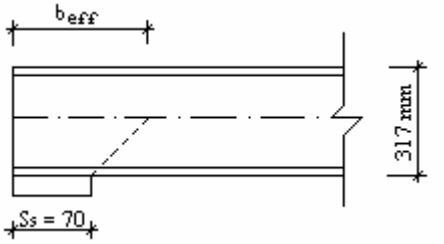
Assumption: - beam is not pre-cambered.
-beam is carrying a reinforced concrete floor.

Limiting values for vertical deflections (see Table 5:1 EBCS-3)

1. Total deflection of the span: $\delta_{\max} = L/250 = 5000/250 = 20\ mm > 13.17$
2. Live load deflection at span (δ_2): $\delta_{\max} = L/300 = 5000/350 = 14.3 > 12\ mm$

Step 7. Check for lateral-torsional buckling.
Since the reinforced concrete floor provides sufficient restraint against lateral movement, the beam is not checked for lateral-torsional buckling.

Step 8. Check for resistance of web to transverse forces.
The beam must be checked at the support. Suppose the support as follows:



8.1 Check for web crushing: $R_{y,Rd} = \dfrac{(S_s + S_y)t_w f_{y,w}}{\gamma_{M1}}$ where S_s = 70 mm (length of stiff bearing) and

$S_y = \dfrac{1}{2}\left[2t_f\sqrt{\left(\dfrac{b_f}{t_w}\right)\left(\dfrac{f_{yf}}{f_{yw}}\right)\left(1-\dfrac{\sigma_{f,Ed}}{f_{yf}}\right)^2}\right]$ *at the end Ss is half.*

b_f = 167 mm < 25 x 13.2 = 330 mm OK
t_w = 7.6 mm
$f_{yf} = f_{yw}$ = 275 N/mm^2
$\sigma_{f,Ed}$ = 0 (at the support there is no moment)

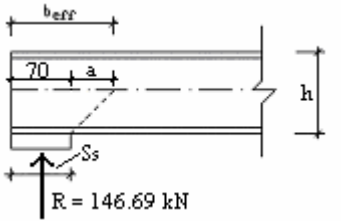
$$S_y = \frac{1}{2}\left[2x13.2\sqrt{\left(\frac{167}{7.6}\right)(1)(1)}\right] = 62\ mm \Rightarrow \quad then\ R_{y,Rd} = \frac{(70+62)x7.6x275}{1.1} = 250\ kN > 146.69\ OK$$

8.2 Check for web crippling.

$$R_{a,Rd} = 0.5t_w^2\sqrt{Ef_{yw}}\left[\sqrt{\frac{t_f}{t_w}} + 3\left(\frac{t_w}{t_f}\right)\left(\frac{S_s}{d}\right)\right]/\gamma_{M1}$$ but $S_s / d = 70 / 257 = 0.27$ therefore take $S_s / d = 0.2$

$$R_{a,Rd} = 0.5x7.6^2\sqrt{2.1x10^5 x275}\left[\sqrt{\frac{13.2}{7.6}} + 3\left(\frac{7.6}{13.2}\right)(0.2)\right]/1.1 = 332\ kN > 146.69 \quad OK$$

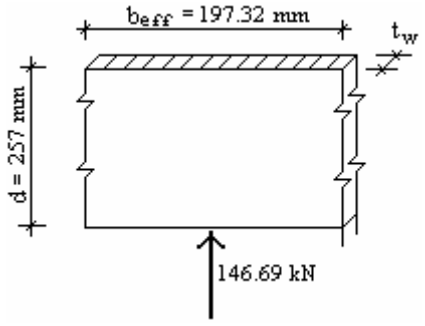
8.3 Check the web buckling: Un stiffened web at the point where concentrated load (reaction acts).



$$a = \left[\frac{\sqrt{h^2 + S_s^2} - 70}{2}\right] = \left[\frac{\sqrt{317^2 + 70^2} - 70}{2}\right] = 127.32$$

$$b_{eff} = 70 + 127.32 = 197.32\ mm.$$

Therefore; the virtual compression member for the web is:



Assumptions: Web is partially fixed at top and bottom; thus the effective length is taken no less than 0.75 d
l = 0.75 x 257 mm = 193 mm.

Radius of gyration of the web is $i = \sqrt{\frac{I}{A}} = \sqrt{\frac{b_{eff}t_w^3}{12b_{eff}t_w}} = \sqrt{\frac{t_w^2}{12}} = \frac{t_w}{\sqrt{12}} = \frac{7.6}{\sqrt{12}} = 2.19\ mm$

Web slenderness ratio = 193/2.19 = 88.19 and $\lambda_1 = 93.9\ \varepsilon = 93.9$ x 0.92 = 86.39

Relative slenderness ratio $\bar{\lambda} = 88.19/86.39 = 1.02$

Using curve c of table 4.9 of EBCS-3; the reduction factor $\chi = 0.48$

Hence; the buckling resistance of the web is:

$$R_{b,Rd} = \frac{\chi \beta_A A f_y}{\gamma_{M1}};\ \beta_A = 1;\ A = b_{eff} t_w = 197.32x7.6 = 1499.6\ mm^2;\ f_y = 275\ N/mm^2\ and\ \gamma_{M1} = 1.1$$

$$R_{b,Rd} = \frac{0.48x1x1499.6x275}{1.1} = 180.2\ kN > 146.69\ kN \quad OK$$

Step 9: Check for flange induced buckling.

The ratio $d/t_w \le k\left(E/f_{yf}\right)\sqrt{A_w/A_{fc}}$

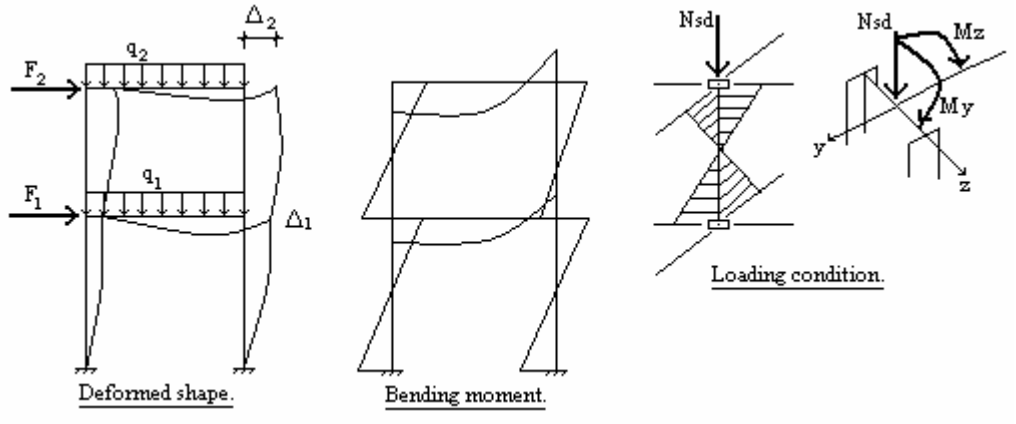
Since $c/t_f = \frac{167/2}{13.2} = 6.33 < 8.5\varepsilon = 8.5x0.92 = 7.82$, the flange is Class 1; therefore, k = 0.3

$$d/t_w = \frac{257}{7.6} = 33.81 \quad ? \quad 0.3\left(2.1x10^5/275\right)\sqrt{(257x7.6)/167x13.2}$$

$$33.81 < 215.6 \quad OK$$

Chapter 5. **Beam – column members**.

In the foregoing consideration has been given to a columns carrying axial loads only. Loads are rarely concentrically applied in practice and the effect of eccentricicy of loading must be taken into account.



Buckling Resistance of compression members with moments.
The total stress due by the combined action of axial force and bending moment is:

$\sigma_M + \sigma_{M,y} + \sigma_{M,z} \le \frac{f_y}{\gamma_{M1}}$, then we can write $\frac{\sigma_N}{f_y/\gamma_{M1}} + \frac{\sigma_{M,y}}{f_y/\gamma_{M1}} + \frac{\sigma_{Mz}}{f_y/\gamma_{M1}} \le 1$

and finally: $\frac{N}{A\cdot f_y/\gamma_{M1}} + \frac{M_y}{W_y\cdot f_y/\gamma_{M1}} + \frac{M_z}{W_z\cdot f_y/\gamma_{M1}} \le 1$. Now, taking into account the problem of the loss of stability, the design according with EBCS-3 is as follows:

a). When lateral-torsional buckling is not a potential failure mode, for Class 1 and 2 cross-sections.

$$\frac{N_{Sd}}{\chi_{\min} A\cdot f_y/\gamma_{M1}} + \frac{k_y M_{y,Sd}}{W_{pl,y}\cdot f_y/\gamma_{M1}} + \frac{k_z M_{z,Sd}}{W_{pl,z}\cdot f_y/\gamma_{M1}} \le 1.0$$

for Class 3 sections: $W_{pl,y} = W_{el,y}$
for Class 4 sections: $W_{pl,y} = W_{eff,y}$ and $A = A_{eff}$
χ_{min} is the lesser of χ_y and χ_z (reduction factor)

$$k_y = 1 - \frac{\mu_y N_{Sd}}{\chi_y A f_y} \le 1.5 \quad \text{and} \quad k_z = 1 - \frac{\mu_z N_{Sd}}{\chi_z A f_y} \le 1.5$$

$$\mu_y = \bar{\lambda}_y\left(2\beta_{My} - 4\right) + \frac{W_{pl,y} - W_{el,y}}{W_{el,y}} \le 0.90 \quad \text{and} \quad \mu_z = \bar{\lambda}_z\left(2\beta_{Mz} - 4\right) + \frac{W_{pl,z} - W_{el,z}}{W_{el,z}} \le 0.90$$

$\frac{W_{pl} - W_{el}}{W_{el}} = 0$ for Class 3 nd 4 sections.

β_{My} *and* β_{Mz} are equivalent uniform factors to be obtained from the following Table according to the shape of the bending moment diagram between the relevant braced points as follows:

Factor	Moment about axis	Points braced in direction
β_{My}	y-y	z-z
β_{Mz}	z-z	y-y
β_{MLT}	y-y	y-y

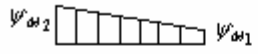
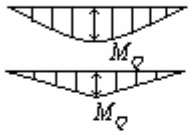
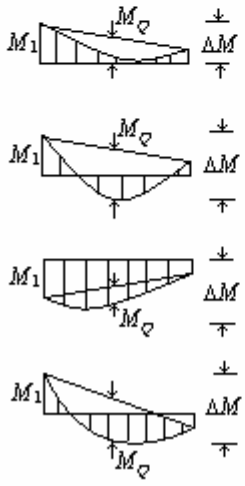
When Lateral – torsional buckling is a potential failure mode (When $\bar{\lambda} > 0.4$), these members shall also satisfied:

$$\frac{N_{Sd}}{\chi_z A \cdot f_y/\gamma_{M1}} + \frac{k_{LT} M_{y,Sd}}{\chi_{LT} W_{pl,y} \cdot f_y/\gamma_{M1}} + \frac{k_z M_{z,Sd}}{W_{pl,z} \cdot f_y/\gamma_{M1}} \le 1.0$$

$$k_{LT} = 1 - \frac{\mu_{LT} N_{Sd}}{\chi_z A f_y} \quad but\ k_{LT} \le 1$$

$$\mu_{LT} = 0.15\bar{\lambda}_z \beta_{M,LT} - 0.15 \quad but\ \mu_{LT} \le 0.90$$

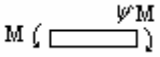
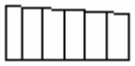
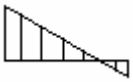
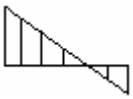
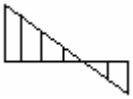
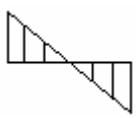
Equivalent Uniform Moment Factors.

Moment diagram	Equivalent uniform moment factor β_M
end moments  $-1 \le \psi \le 1$	$\beta_{M,\psi} = 1.8 - 0.7\psi$
moments due to in-plane lateral loads 	$\beta_{M,Q} = 1.3$ $\beta_{M,Q} = 1.4$
moments due to in-plane lateral loads plus end moments 	β_M must be $\beta_{M,}$ $\beta_{M,c}$ $\beta_{M,LT}$ $\beta_M = \beta_{M,\psi} + \frac{M_Q}{\Delta M}(\beta_{M,Q} - \beta_{M,\psi})$ $M_Q = \lvert\max M\rvert$ due to lateral load only $\Delta M = \begin{cases} \lvert\max M\rvert & \text{for moment diagram without change of sign.} \\ \lvert\max M\rvert + \lvert\min M\rvert & \text{for moment diagram with change of sign.} \end{cases}$

The values of Factors C_1, C_2, and C_3 corresponding to values of factor k are obtained from the following Table.
Notes:

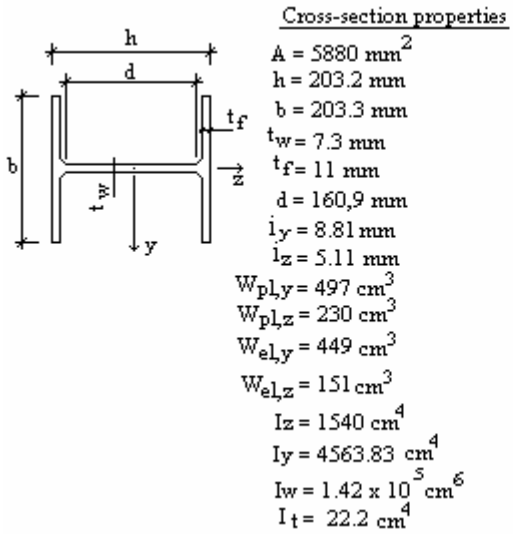
k = 1 – For no fixity at the ends.
k = 0,7 – for one end fixed and one end free.
k = 0.5 – For full fixity at both ends.

Values of factors C_1, C_2, and C_3

Loading and support conditions	Bending moment diagram	Value of K	Values of factors		
			C_1	C_2	C_3
	 $\psi = +1$	1.0 0.7 0.5	1.000 1.000 1.000	-	1.000 1.113 1.144
	 $\psi = + 3/4$	1.0 0.7 0.5	1.141 1.270 1.305	-	0.998 1.565 2.293
	 $\psi = + 1/2$	1.0 0.7 0.5	1.323 1.473 1.514	-	0.992 1.556 2.271
	 $\psi = + 1/4$	1.0 0.7 0.5	1.563 1.739 1.788	-	0.977 1.531 2.235
	 $\psi = 0$	1.0 0.7 0.5	1.879 2.092 2.150	-	0.939 1.473 2.150
	 $\psi = - 1/4$	1.0 0.7 0.5	2.281 2.538 2.609	-	0.855 1.340 1.957
	 $\psi = - 1/2$	1.0 0.7 0.5	2.704 3.009 3.090	-	0.676 1.059 1.546
	 $\psi = - 3/4$	1.0 0.7 0.5	2.927 3.009 3.093	-	0.366 0.575 0.837
	 $\psi = - 1$	1.0 0.7 0.5	2.752 3.063 3.149	-	0.000 0.000 0.000

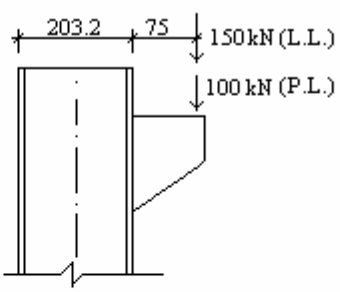
Chapter 5. Example 1.
A 4.00 m pin-ended column supports a beam with a reaction of 100 kN permanent load and 150 imposed load. Assuming the beam reaction to be applied 75 mm from the face of the flange. Check the adequacy of a 203 x 203 x 46 UC grade 430 steel profile.

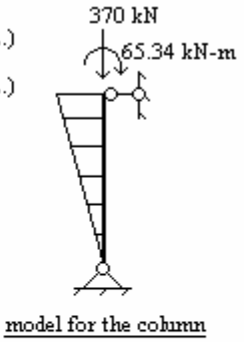
Solution:



Cross-section properties

$A = 5880\ mm^2$
$h = 203.2\ mm$
$b = 203.3\ mm$
$t_w = 7.3\ mm$
$t_f = 11\ mm$
$d = 160{,}9\ mm$
$i_y = 8.81\ mm$
$i_z = 5.11\ mm$
$W_{pl,y} = 497\ cm^3$
$W_{pl,z} = 230\ cm^3$
$W_{el,y} = 449\ cm^3$
$W_{el,z} = 151\ cm^3$
$I_z = 1540\ cm^4$
$I_y = 4563.83\ cm^4$
$I_w = 1.42 \times 10^5\ cm^6$
$I_t = 22.2\ cm^4$





model for the column

Step 1: Applied axial forces and moments.

		Partial safety factor	Factored load
Permanent load	100 kN	1.30	130 kN
Imposed load	150 kN	1.60	240 kN

Moment (beam is connected to the flange of the column).

$$M_{y,Sd} = 370(0.2032/2 + 0.075) = 65.34\ kN - m$$

Step 2: Effective length of column.

Since both column ends are hinged, the effective length is l = 4.00 m.

Step 3: Select the trial section. (Section are given).
Step 4: Check the classification of the cross-section; check the section local buckling. If necessary determine the effective cross-section and its properties.

a) Flanges. $\dfrac{c}{t_f} = \dfrac{203.2/2}{11.00} = 9.24 < 11\varepsilon = 11\sqrt{235/275} = 10.2$ (Limit for Class 2.)

b) Web (assuming α = 1) (web generally). $\dfrac{d}{t_w} = \dfrac{160.9}{7.4} = 22 < \dfrac{66\varepsilon}{0.4 + 0.6} = 60.70$

(Web is Class 1).

The whole cross-section is Class 2.

Step 5: Check the first condition:

$\frac{N_{Sd}}{\chi_{\min} A f_y / \gamma_{M1}} + \frac{k_y M_{y,Sd}}{W_{pl,y} f_y / \gamma_{M1}} + \frac{k_z M_{z,Sd}}{W_{pl,z} f_y / \gamma_{M1}} \leq 1$. Since there is no bending about the minor axis, $M_{z,Sd} = 0$.

Determination of the parameters:

5.1 – Determonation of χ_{min}

5.1.1 – Buckling about y-y axis.

Slenderness ratio: $\lambda_y = \frac{400}{88.1} = 45.4$

Relative slenderness: $\bar{\lambda}_y = \frac{\lambda_y}{\lambda_1} = \frac{45.4}{93.9\sqrt{\frac{235}{275}}} = 0.523$

$\frac{h}{b} = \frac{203.2}{203.3} = 1 < 1.2$; Thus, buckling curve **b** has to be used (see Table 4.11) and $\chi_y = 0.8763$.

5.1.2 – Buckling about z-z axis.

Slenderness ratio: $\lambda_z = \frac{400}{51.1} = 78.3$

Relative slenderness: $\bar{\lambda}_z = \frac{78.3}{93.9x0.92} = 0.906$

$\frac{h}{b} = 1 < 1.2$; Thus, buckling curve **c** has to be used and $\chi_z = 0.5962$ (is the critical value for this case).

Therefore $\chi_{\min} = 0.5962$

5.2 – Determination of k_y.

$$k_y = 1 - \frac{\mu_y N_{Sd}}{\chi_y A f_y} \leq 1.5$$

$$\mu_y = \bar{\lambda}_y (2\beta_{My} - 4) + \frac{W_{pl,y} - W_{el,y}}{W_{el,y}} \leq 0.90$$

$$\beta_{M,y} = \beta_{M,\psi} + \frac{M_Q}{\Delta M}(\beta_{M,Q} - \beta_{M,\psi})$$

Where:

ψ is the ratio of the end moments (is = 0 in our case).

$\beta_{M,\psi} = 1.8 - 0.7\psi$

M_Q is the maximum moment from the lateral load.

$\beta_{M,Q} = 1.3$ for uniformly distribuited lateral load and

= 1.4 for a central lateral point load.

Δ M is the maximum span moment, to which the maximum end moment is added if the sign of the diagram changes.

Thus:

$\psi = 0$ *and* $\beta_{M,\psi} = 1.8$

$M_Q = 0$ (no lateral load between top and botton of the column).

$\beta_{M,y} = 1.8$

$\bar{\lambda}_y = 0.523$ (refer to step 5.1.1)

$$\therefore \quad \mu_y = 0.523(2x1.8-4) + {(497-449)}/{449} = -0.1023 < 0.9 \quad OK$$

$$k_y = 1 - \frac{-0.1023x370x10^3}{0.8763x5880x275} = 1.026 < 1.5$$

5.3. Substituting into the interaction equation for first condition:

$$\frac{370x10^3}{0.5962x5880x275/1.1} + \frac{1.025x65.3x10^6}{497x10^3 x275/1.1} = 0.420 + 0.540 = 0.96 < 1 \quad OK$$

Thus, the section is adequate to carry the combined compression and bending.

Step 6: Check for the second condition. (If $\bar{\lambda}_{LT} > 0.4$).

6.1 - $\lambda_{LT} = \sqrt{\frac{\pi^2 E W_{pl,y}}{M_{cr.}}}$ for all the cases.

$$M_{cr} = \frac{C_1 \pi^2 E I_z}{L^2}\sqrt{\frac{I_w}{I_z} + \frac{L^2 G I_t}{\pi^2 E I_z}}$$

Value of C_1 – See Table 4.2; for $\psi = 0$ and $k = 1$; $C_1 = 1.879$

$$M_{cr} = \frac{1.879 * \pi^2 * 210000x1540x10^4}{4000^2}\sqrt{\frac{1.42x10^5}{1540} + \frac{4000^2 x0.39x22.3x10^4}{1540x10^4}}$$

$$M_{cr} = 5.07x10^8 N - mm$$

$$\lambda_{LT} = \sqrt{\frac{\pi^2 x210000x497x10^3}{5.07x10^8}} = 45.07$$

$$\lambda_1 = 93.9\xi = 93.9\sqrt{\frac{235}{275}} = 86.8$$

$\bar{\lambda}_{LT} = \frac{45.07}{86.8} = 0.519 > 0.4$. Then, check the second condition is valid.

6.2 – The interaction equation for Class 2 section is:

$$\frac{N_{Sd}}{\chi_z A f_y / \gamma_{M1}} + \frac{k_{LT} M_{y,Sd}}{\lambda_{LT} W_{pl,y} f_y / \gamma_{M1}} \le 1$$

$\mu_{LT} = 0.15\bar{\lambda}_z \beta_{M,LT} - 0.15 \quad \beta_{M,LT} = 1.8$ *as in step* 5.2

$\mu_{LT} = 0.15x0.906x1.8 - 0.15 = 0.0946$

$$k_{LT} = 1 - \frac{\mu_{LT} N_{Sd}}{\chi_z A f_y} = 1 - \frac{0.0946x370x10^3}{0.5962x5880x275} = 0.964 < 1 \quad OK$$

Value for χ_{LT}:

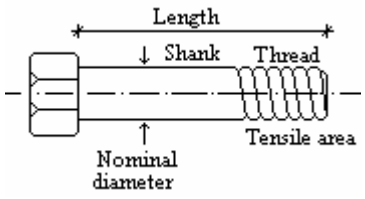
For rolled sections, curve **a** is used; the corresponding value for $\bar{\lambda}_{LT} = 0.519$ *is* $\chi_{LT} = 0.9178$(*Table* 4.9).

Finally: $\frac{370000}{0.5x5880x275/1.1} + \frac{0.964x65.34x10^6}{0.9178x497x10^3 x275/1.1} = 0.422 + 0.552 = 0.974 < 1$

Therefore the section is satisfactory in respect of lateral-torsional buckling and axial compression.

Bolted Connections

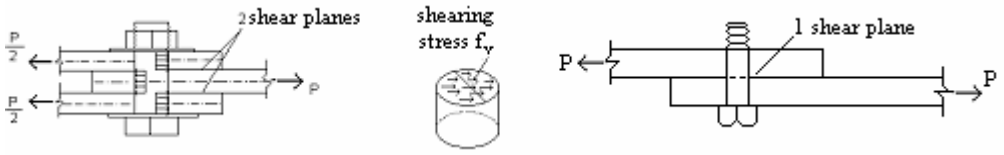
Bolted connections are employed mainly in structures subjected under reversed and vibration loads, over all in members with heavy conditions.
The black hexagon bolt shown in the Figure below with nut and washer is the most commonly used structural fastener.



Behaviour of bolts in joints.

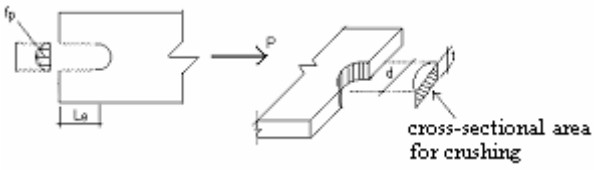
Three types of behaviour appear for bolted connection as follows:

1. Shear due to shearing of their shank.

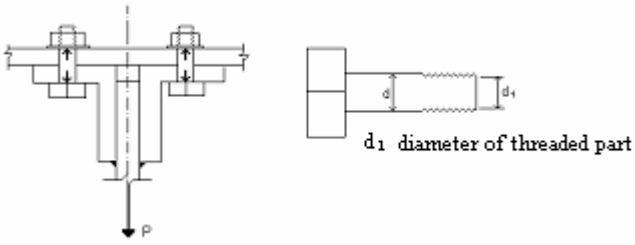


If the force P is large enough, the bolt could fail in shear; breaking by sliding of its fibres along the shear planes. The area of the steel bolt resisting the failure is the circular area of the bolt shank. The resisting force depends upon the number of **shear planes.**

2. Bearing due to bearing of steel plates or bolt material.



3. Tension in the direction of the acting force along the shank of the bolt.



Shear and bearing should be present at the same time in the joint. It will be seen that bolt may be designed on the basis of their strength in shear or their strength in bearing.
In actual design the lesser of these two values will have to use.
When designing of this type of connection, the following questions should be asked:

1. Is the connection in single or double shear?
2. What is the safe appropriate shear load on one bolt?
3. What is the safe bearing load on one bolt?

Effective area of bolts.

Since threads can occur in the shear plane, the area A_s for resisting shear should normally be taken at the bottom of the threads. When threads do not occur in the plane A_s may be taken as the shank area.
Tensile stress area for bolts as determined by ISO Standards shank and tensile areas area tabulated below.

Bolt diameter (mm)	Tensile area (mm^2)	Shank area (mm^2)
12	84	113
16	157	201
20	245	314
22	303	380
24	353	452
27	459	572
30	561	707

Shear capacity
Provided that no reductions are required for long joints the shear capacity for shear plane $F_{v,Rd}$ of a bolt shall be taken as:

$F_{v,Rd} = f_{v,d} A_s$ Where the design shear strength $f_{v,d} = \dfrac{0.6\, f_{ub}}{\gamma_M} \quad but \leq \dfrac{0.87\, f_{yb}}{\gamma_M}$

Bearing capacity.

The effective capacity of a bolt in bearing on any ply shall be taken as the lesser of the bearing capacity of the bolt and the bearing capacity of the connected ply.

The bearing capacity of the bolt. $F_{bb,Rd} = dtf_{bb,d}$

Where d is the nominal diameter of the bolt
t is the thickness of the ply (the minimum thickness on one part of the joint)
$f_{bb,d}$ is the design bearing strength of the bolt.

The bearing capacity of the connected ply. $F_{bp,Rd} = dtf_{bp,d} \quad but \leq 1/2 e_1 tf_{bp,d}$

Where $f_{bp,d}$ is the design bearing strength of the connected parts.
e_1 is the edge distance.

$$f_{bb,d} = \frac{0.9\left(f_{ub} + f_{yb}\right)}{\gamma_M} \quad and \quad f_{bp,d} = \frac{0.8\left(f_u + f_y\right)}{\gamma_{M2}}$$

Where f_{yb} is the specified minimum yield strength of the fastener
f_{ub} is the specified minimum ultimate tensile strength of the fastener
γ_M is the partial safety factor γ_{Mr} or γ_{Mb}; as the case may be
f_y is the specified minimum yield strength
f_u is the specified minimum ultimate strength.

Bolt subjected to tension.

The tension capacity of a bolt $F_{t,Rd} = f_{t,d} A_s$

Where the design tension capacity strength $f_{t,d} = \dfrac{0.7\, f_{ub}}{\gamma_M} \quad but \leq \dfrac{1.0\, f_{yb}}{\gamma_M}$

The partial safety factor for all the cases are $\gamma_M = 1.25$

Combined shear and tension.

When bolts are subjected to both shear and tension then in addition to the conditions studied before the following relationship shall be satisfied:

$$\frac{F_{v,Sd}}{F_{v.Rd}} + \frac{F_{t,Sd}}{F_{t,Rd}} \leq 1.4$$

Where $F_{v,Sd}$ is the design shear force per bolt for the ultimate limit state
$F_{t,Sd}$ is the design tensile force per bolt for the ultimate limit state
$F_{v,Rd}$ is the shear capacity per bolt
$F_{t,Rd}$ is the tension capacity per bolt.

Notes:

1. The size of the holes are given in Table 6.1
2. The edge distances and spacing of holes for fasteners are given in Table 6.2

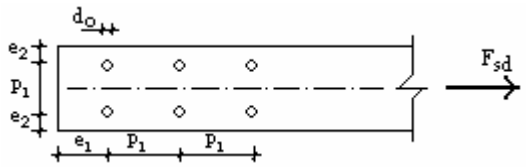


Table 6.1 Maximum dimensions of holes

Bolt shank diameter (mm)	Clearance hole diameter (mm)	Oversize hole diameter (mm)	Short slotted hole dimensions (mm)		Long slotted hole dimensions (mm)	
≤14	d + 1	d + 4	d + 1	d + 4	d + 1	2.5d
14≤d≤22	d + 2	d + 5	d + 2	d + 6	d + 2	2.5d
24	d + 2	d + 6	d + 2	d + 8	d + 2	2.5d
≥27	d + 3	d + 8	d + 3	d + 10	d + 3	2.5d

Table 6.2 Edge distances and spacing of holes for fasteners.

	1	2	3	4	5
1	Edge distances			Hole distances	
2	Minimum edge distance	For a rolled, machine Flame cut, sawn or Planned edge e_1 e_2	1.25 d_o	Minimum hole distance p_1	2.5 d_0
3		For sheared or hand flame cut edge and any end e_1 e_2	1.4 d_o		
4	Maximum Edge distance	e_1 e_2	12 t or 150 mm	Maximum Hole distance p_1 in unstiffeded plates	14 t or 200 mm

t is the thickness of the thinner outside ply
d_o is the diameter of hole

Where the members are exposed to corrosive influences the maximum distances shall not exceed:

(a) for edge distances: 40 mm + 4t
(b) for hole distances: 16t or 200 mm.

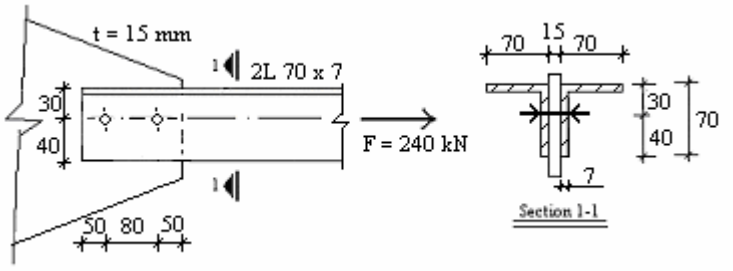
Bolt grades.

The grade of the bolt is given by two figures separated by a point. The first figure is 1% of the minimum ultimate strength in N/mm^2 and the second is $1/10^{th}$ of the percentage ratio of the minimum yield strength. Thus 5.6 grade means that the minimum ultimate strength is 500 N/mm^2 and the yield strength is 60% of this strength which is 300 N/Mm^2. the nominal values of the yield strength f_{yb} and the ultimate strength f_{ub} to be adopted as characteristic values in calculations are given below.

Bolt grade	4.6	4.8	5.6	5.8	6.8	8.8	10.9
f_{yb} (N/mm^2)	240	320	300	400	480	640	900
f_u (N/mm^2)	400	400	500	500	600	800	1000

Bolted connections.

Example Nr 1. The connection shown in the Figure below is subjected to a design tensile force of 240 kN. The steel Grade is F_e 430, the bolt Grade 8.8 and its diameter is 20 mm. Check that the connection is adequate.



Check for the geometry.
Bolts M 20; Grade 8.8 f_{yb} = 640 N/mm^2 , f_{ub} = 800 N/mm^2
Diameter of the holes: (see Table 6.1). The hole diameter shall be d_0 = d + 2 mm = 20 + 2 = 22 mm.

$$Minimum\ edge\ distance\ e_1 = 1.25d_0 = 1.25x22 = 27.5\ mm < 30\ ok$$

$$Minimum\ hole\ distance\ p_1 = 2.5d_0 = 2.5x22 = 55\ mm < 50\ ok$$

$$Maximum\ edge\ distance\ e_1 = 12t = 12x7 = 84\ mm > 50\ ok$$

$$Maximum\ hole\ distance\ p_1 = 14t = 14x7 = 98\ mm > 80\ ok$$

Shear capacity of bolts.
Assumptions: - There are two shear planes per bolts.
- Threads are in the shear plane i.e; A_s = 245 mm^2

Shear capacity of bolt.

$$F_{v,Rd} = f_{vd}A_s = \frac{2x0.6\,f_{ub}A_s}{\gamma_{Mb}} \le \frac{2x0.87\,f_{yb}A_s}{\gamma_{Mb}}$$

$$= \frac{2x0.6x800x245x10^{-3}}{1.25} = 188\ kN \quad (Governs\ the\ design)$$

$$and \quad = \frac{2x0.87x640x45x10^{-3}}{1.25} = 218\ kN$$

Therefore, because there are 2 bolts: $188 > \frac{240}{2}$ *is OK*

Bearing capacity of members and bolts.

The bearing capacity of the bolts is:

$F_{bb,Rd} = d\,t\,f_{bb,d}$; where t = 14 mm (the gusset plate is not the critical member since t = 15 mm > 2 x 7 = 14 mm)

$$F_{bb,Rd} = \frac{dt\left[0.9\left(f_{ub} + f_{yb}\right)\right]}{\gamma_{Mb}} = \frac{20x14x0.9(800+640)x10^{-3}}{1.25} = 290.3\ kN\ (per\ bolt) > \frac{240}{2}\ OK$$

The bearing capacity of the gusset plate is:

$$F_{bp,Rd} = d\,t\,f_{bp,d} \le \frac{1}{2} e_1\,t\,f_{bb,d} \;\; (per\ bolt)$$

$$F_{bp,Rd} = \frac{dt\left[0.8\left(f_u + f_y\right)\right]}{\gamma_{M2}}$$

$$F_{bp,Rd} = \frac{20x15x0.8(430+275)x10^{-3}}{1.25} = 135.36 > \frac{240}{2} = 120\ kN. \quad Ok \quad (governs\ design)$$

$$and \quad \frac{1}{2}x\frac{50x15x0.8(430+275)x10^{-3}}{1.25} = 169.2 > 135.36 \quad OK$$

The bearing capacity of one angle is:

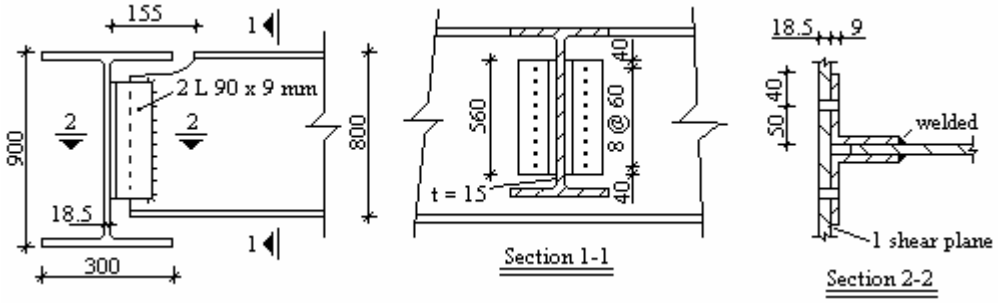
$$F_{bp,Rd} = \frac{d\,t\left[0.8\left(f_u + f_t\right)\right]}{\gamma_M} \le \frac{1}{2} e_1 t\,f_{bp,d}$$

$$= \frac{20x7\left[0.8(430+275)\right]x10^{-3}}{1.25} = 63.2\ kN > \frac{240}{2x2} = 60\ kN\ (per\ angle)$$

$$and \quad \frac{1}{2}x\frac{50x7x0.8(430+275)x10^{-3}}{1.25} 79.0 > 63.2 \quad OK$$

Example Nr 2.

Check that the secondary girder to primary girder connection by means of angles shown in the figure below is adequate. All data required are provided in the figure.



Main girder, Secondary girder and Angles L 90 x 9 with Steel Grade F_e 430, f_u = 275 N/mm^2.
Bolts Grade 8.8, f_{yb} = 640 N/mm^2, f_{ub} = 800 N/mm^2; Diameter 22 mm.
Bolt area at the bottom of the thread: A_s = 303 mm^2.
Applied load: Shear force V = 890 kN (at the centreline of the web of the main girder).

Solution:

Diameter of holes $d_0 = d + 2 = 22 + 2 = 24$ mm.
Minimum edge distance, $e_1 = 1.25\ d_0 = 1.25 \times 24 = 30$ mm < 40 mm. OK.
Minimum hole distance, $p_1 = 2.50\ d_0 = 2.50 \times 24 = 60$ mm = 60 mm OK.
Maximum edge distance, $e_1 = 12\ t = 12 \times 9 = 108$ mm > 40 mm OK.
Maximum hole distance, $p_1 = 14\ t = 14 \times 9 = 126$ mm > 60 mm OK.

Shear Capacity of bolts.

Assumptions: - one shear area per bolt.

- threads area in the shear plane.

Shear capacity of a bolt:

$$F_{v,Rd} = f_{vd} = \frac{0.6 f_{ub} A_s}{\gamma_{Mb}} \le \frac{0.87 f_{yb} A_s}{\gamma_{Mb}}$$

$$= \frac{0.6x800x303x10^{-3}}{1.25} = 116.4 > \frac{890}{2x9} = 49.4\ kN$$

$$and\ \frac{0.87x640x303x10^{-3}}{1.25} = 135 > 116.4\ kN \quad OK$$

Capacity of connection main girder and connection angle.

Bearing capacity of bolts.

Since the web thickness of the beam t_w = 18.5 mm is grater than the angle leg thickness t_a = 9 mm, the angle is the critical member.

$$F_{bb,Rd} = d\,t\,f_{bb,d} = \frac{dt\left[0.9\left(f_{ub} + f_{yb}\right)\right]}{\gamma_{Mb}} = \frac{22x9x0.9x(800+640)x10^{-3}}{1.25} = 205.3\ kN > 49.4\ kN \quad OK$$

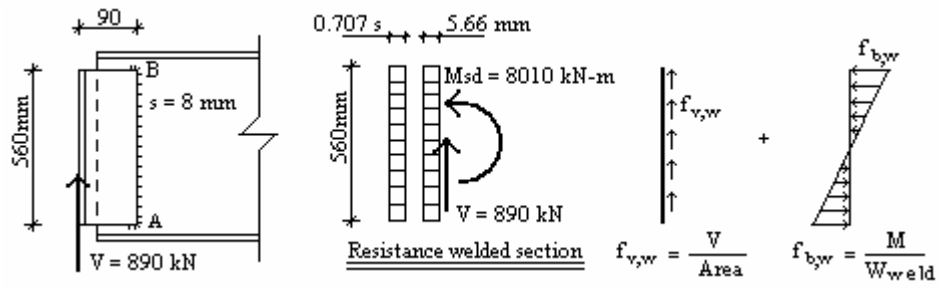
Bearing Capacity of angle.

$$F_{bp,Rd} = \frac{dt\left[0.8\left(f_u + f_y\right)\right]}{\gamma_{Mb}} \le \frac{1}{2} e_1\,t\,f_{bp,d}$$

$$= \frac{22x9x0.8(430+275)x10^{-3}}{1.25} = 89.3\ kN > 49.4\ OK$$

$$and\ = \frac{1}{2}x\frac{40x9x0.8(430+275)x10^{-3}}{1.25} = 81.2 < 89.3 \quad but > 49.4\ kN \quad OK.$$

Capacity of connection Secondary Girder and connection Angle (welded).



Design Moment M_{sd} = V * e = 890 x 9 = 8010 kN-cm.

Resistance condition: $f_{R,w} \le f_{vw,d} = 0.63\frac{f_{ye}}{\gamma_{Mw}} \quad but \quad \le 0.65\frac{f_u}{\gamma_{Mw}}$

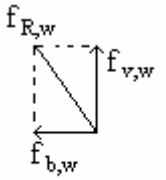
Area of welded section = 2 x 0.566 cm x 56 cm = 63.9 cm^2

$$\therefore\ f_{v,w} = \frac{890\ kN}{63.9\ cm^2} = 14.04\ kN/cm^2$$

Section modulus of the weld section $W_{weld} = \frac{2x0.566x56^2}{6} = 591.66\ cm^3$

$\therefore\ f_{b,w} = \frac{8010\ kN - cm}{591.66\ cm^3} = 13.54\ kN/cm^2$

Finally, for point B.



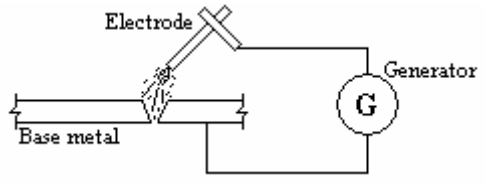
Point B

$f_{R,w} = \sqrt{f_{b,w}^2 + f_{v,w}^2}$

$\therefore\ f_{R,w} = \sqrt{13.54^2 + 14.04^2} = 19.51 < 0.65\frac{43}{1.25} = 22.36\ kN/cm^2$

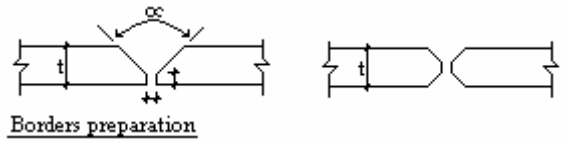
Welded Connections.

Electric welding is the most widespread method of connecting the elements of steel members. The welding process is shown in the following figure.

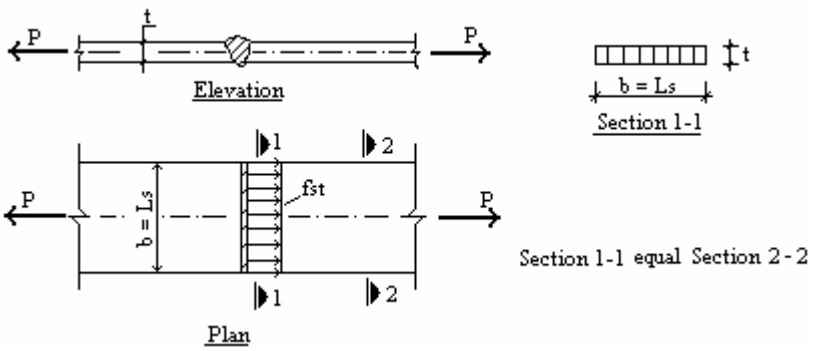


Types of weld.
The commom types of weld are illustrated in Table 6.3. To study the behaviour of the joints they are divided mainly into 2 types, Butt weld and Fillet ones.

Butt welds. This type is used mostly to weld steel plates of same or similar thickness. You can use it also in welding of beams with sections I or C. Their disadvantage consists in to achieve complete penetration. For foils thickness bigger than 10 mm it is necessary to prepare the borders appropriately, that wich requires of special cares and appropriate facilities. This work is carried out in shops where the welding process can be controlled with quality.

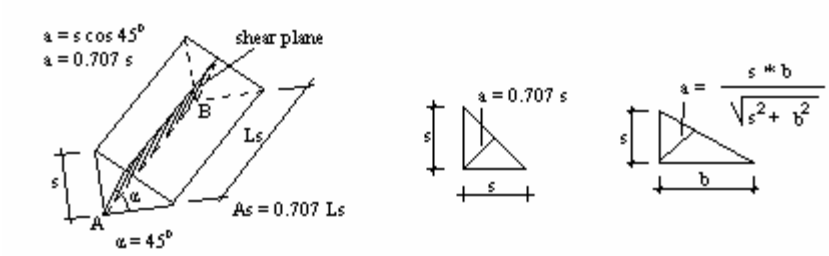


Behaviour of the butt joint.



Then, the tensile stress due to the axial force P on the welding section 1 – 1 is: $f_{st} = \frac{P}{L_s t} = f_t = \frac{P}{bt}$, it is similar to the tension that take place in a section 2 – 2 for the base metal. Usually if the resistance of the material of contribution of the electrode is bigger than that of the base netal, the resistance of the joint is guaranteed and it is not necessary further calculation.

Fillet welds.
Fillet welds may be used for connecting parts where the fusion faces form an angle of between 60^0 and 120^0. Smaller angles are also permitted. However, in such a cases the weld shall be considered to be partial penetratrion butt weld.
Fillet welds terminating at the ends or sides of parts should be returned continuously around the corners for a distance of not less than twice the length **s** of the weld unless access or the configuration renders this impracticable. This detail is particularly important for filled welds on the tension side of parts carrying a bending load.
In lap joints the minimum lap shall be no less than 4t where t is the thickness of the thinner part joined. Single fillet welds should only be used where the parts are restrained to prevent opening of the joint.



As it is observed in the figure, the tensions that appear in the welding chord are of **shear**, being the points of the ends (A and B) the most loaded for what reach the yiend point first. Then the interior points go reaching the yield point gradually and in the moment of the failure, all the points of the welding chord will be working contributing the maximum resistance evenly. Numerous researsh works show that the failure really happens for the half plane of the cord, that which defines the efective area as the product of multiplying the effective with of the throat of the cord (**a**) for the longitude of the chord.

Throat thickness.

The effective throat size **a** of a fillet weld shall be taken as the perpendicular distance from the root of the weld to a straight line joining the fusion faces wich lies within the cross-section of the weld. It is not, however, be taken as greater than 0.707 times the effective leg with **s.**

The throat thickness of a fillet weld should not be less than 3 mm.

Design Strength.

The codes usually use they calculate the maximum force that resists the unit of longitude.

The design strength $F_{w,Rd}$ of a fillet weld per unit of length shall be obtained from:

$F_{w,Rd} = f_{vw,d}\,a$; where $f_{vw,d}$ is the design shear strength of the weld and shall be determined from:

$$f_{vw,d} = \frac{0.63\,f_{ye}}{\gamma_{Mw}} \quad but \quad f_{v,w} \le \frac{0.65\,f_u}{\gamma_{Mw}}$$

where f_{ye} is the minimum tensile strength of the electrodes.

f_u is the specified minimum ultimate tensile strength of the weaker part joined.

$\gamma_{Mw} = 1.25$.

Long joints.

In lap joints the design resistance of a fillet weld shall be reduced by multipliying it by a reduction factor β_{Lw} to allow for the effects of non-uniform distribution of the stresses along its length.

This provision is not apply when the stress distribution along the weld corresponds to the stress distribution in the adjacent base metal, as for esample, in the case of weld connecting the flange and the web of a plate girder.

Generally in lap joints longer than 150**a** reduction factor β_{Lw} should be taken as $\beta_{Lw,1}$ given by:

$\beta_{Lw,1} = 1.2 - 0.2L_j/(150a) \quad but \quad \beta_{Lw.1} \le 1.0$; where L_j is the overall length of the lap in the direction of the force transfer.

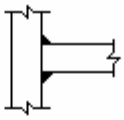
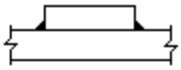
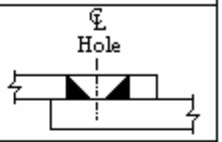
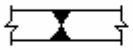
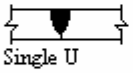
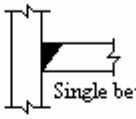
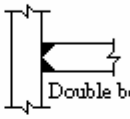
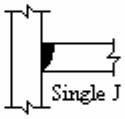
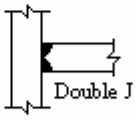
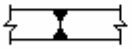
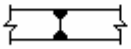
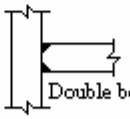
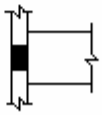
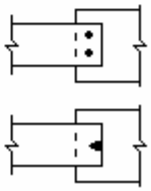
For fillet welds longer than 1.7 meters connecting transverse stiffeners in plated members, the reduction factor β_{Lw} may be taken as $\beta_{Lw.2}$ given by:

$\beta_{Lw,2} = 1.1 - L_w/17 \quad but \quad 0.6 < \beta_{Lw,2} \le 1.0$; where L_w is the length of the weld in meter.

Types of elctrodes.

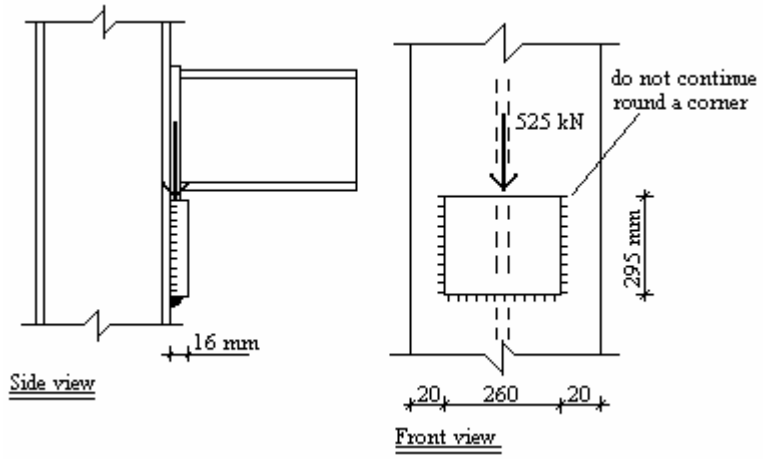
For a common structural steel, the AWS (American Welding Society) recommends electrodes types E 60 XX and E 70 XX. E – denotes electrode, the first 2 numbers represent the tensile strength of the electrode in Ksi (kilopound per square inches); then for the electrodes abobe the tensile strength are 60 Ksi (414 Mpa) and 70 Ksi (483 Mpa) respectively.

Table 6.3 Common types of welded joints.

Types of weld	Type of joint		
	Butt joint	Tee - butt joint	Lap joint
Fillet weld			
Slot weld			
Full penetration butt weld *	 Single V  Double V  Single U  Double U	 Single bevel  Double bevel  Single J  Double J	
Partial penetration butt weld	 Double V  Double U	 Double bevel	
Plug weld			
* Butt welds can sometimes be formed without cutting any weld preparation			

Welded Connections.

Example Nr 1. (Checking problem).
In the beam to column connection shown in the figure, the steel plate is supporting a support factored reaction of 525 kN from the beam. If the size of the weld is 8 mm and steel Grade Fe 360, check if the connection is adequate.



Geometry, materials and loading.

Plate t = 16 mm. Fe 360, f_y = 235 N/mm^2 and f_u = 360 N/mm^2.
Size of the welds s = 8 mm.
Throat thickness: a = 0.707 s = 0.707 x 8 = 5.66 mm.
Length of the weld l = 2 x 295 + 260 – 4 x 8 = 818 mm.

1). Check the dimension of the chords:
Minimum weld length l = 40 mm or 6 x a = 6 x 5.66 = 34 mm < 260 mm. OK.
Maximum weld length l = 150 a = 150 x 5.66 = 849 mm > 295 OK.

2). The design strength per unit length is $F_{w,Rd} = f_{vw,d}a$

where $f_{vw,d} = \dfrac{0.63\, f_{ye}}{\lambda_{Mw}} \leq \dfrac{0.65\, f_u}{\gamma_{Mw}}$

In most practical cases, the quality of the electrodes are greater than that of the base metal and then the ultimate tensile strength of the weaker part joined, that is, the base metal govern the design. Therefore the second statement of the equation is checked.

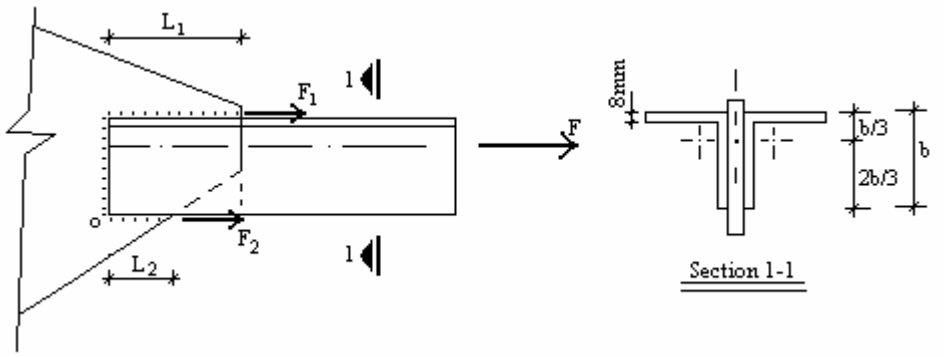
$$F_{w,Rd} = \frac{0.65x360}{1.25} x5.66 = 1059.55 N/mm$$

And the total resistance force $F = 1059.55\ N/mmx818\ mmx10^{-3} = 866\ kN > 525\ kN \quad OK$.

Example Nr 2. (Design problem).

Special case (Eccentrically loaded joint. Unsymmetrical section).

When securing an unsymmetrical section, for example two angles to a plate, attention is paid to uneven distribution of the load between the welds transmitting the force field from the angles to the plate.



The Force F is discomposed into F_1 and F_2

Taken moment with respect to point o.

$$F\frac{2}{3}b = F_1 b \;\Rightarrow\; F_1 = \frac{2}{3}F \;;\; Thus\; F_2 = \frac{F}{3}$$

therefore; F_1 is taken by two chord (filled weld) length L_1each and F_2 is taken by the lower two fillet weld length L_2 each.

Example: Compute the welds required for connecting two angles 75x75x8 mm to a gusset plate with a thickness 10 mm. The factored tensile force in the angle is 450 kN. The material is steel Grade 430.

Geometry, Materials.

Plate and angles Grade 430 steel, $f_y = 275\ N/mm^2$; $f_u = 430\ N/mm^2$

Size of the welds ≤ 8 mm; take s = 6 mm.

Throat thickness a = 0.707 x 6 = 4.24 mm.

1). Acting Force F_1 and F_2.

$$F_1 = \frac{2}{3}F_1 = \frac{2}{3}x450\ kN = 300\ kN$$

$$F_2 = \frac{450}{3} = 150\ kN$$

2). Design strength per unit length. (Suppose the weaker part is the base steel).

$$F_{w,Rd} = \frac{0.65x430}{1.25}x4.24 = 948N/mm.$$

3). Length of fillets:

$$L_1 = \frac{F_1}{2_{(\ fillet\ welds)}\ F_{w,Rd}} = \frac{300x10^3\ N}{2x948\ N/mm} = 158.2\ mm$$

Actual length if no round a corner $L_1 = 158.2\ mm + 2s = 158.2 + 2x6 = 170\ mm\ each\ side.$

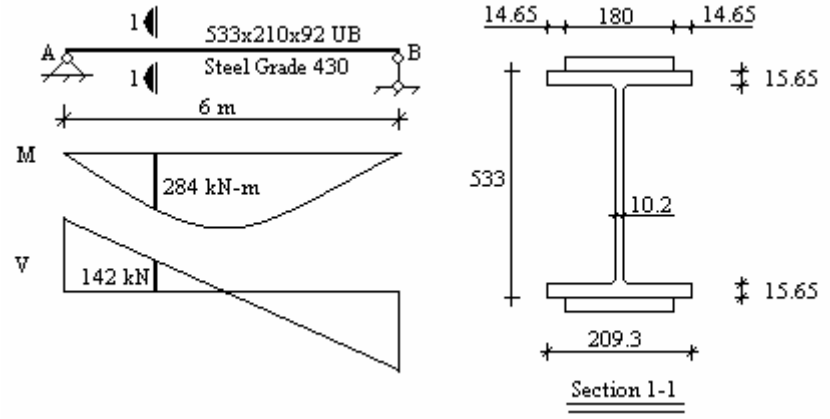
And $L_2 = 158.2 + 2x6 = 91.1\ mm \quad (take\ 92\ mm)$

4). Check the dimension of the fillets.

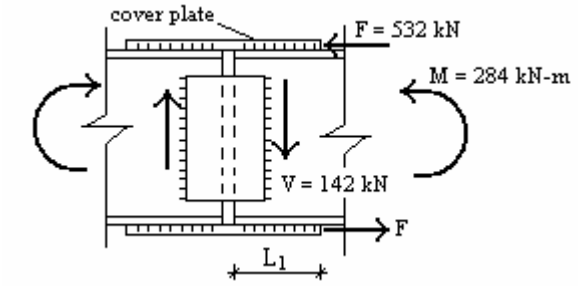
$6x4.24 < 92 \quad and \quad 170 < 150x4.24 \quad OK$

- Joints in beams under the action of bending moment and shear force.

Suppose a beam such that:



Principle: The flanges take the acting bending moment and the web takes shear force.



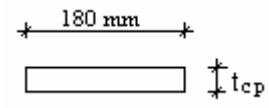
M is discomposed into a couple of forces F.

F = M/h acting on the flange levels.

Then $F = \dfrac{284\ kN - m}{0.533\ m} = 532\ kN.$

Design of cover plates.

The design plastic resistance of the gross section $N_{pl,Rd} = \dfrac{Af_y}{\gamma_{Mo}} = 532\ kN$



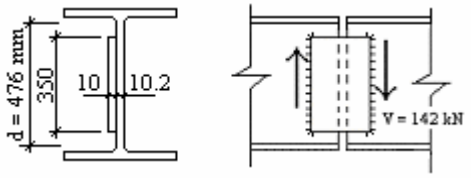
$$A = 180t_{c,p} = \frac{\gamma_{Mo} x532x10^3}{f_y} = \frac{1.1x532x10^3}{275} = 2128\ mm^2$$

$$t_{c,p} = \frac{2128\ mm^2}{180\ mm} = 11.82\ mm \quad take\ 12\ mm.$$

- suppose size of the weld 8 mm < 12 mm.
- throat thickness $a = 0.707x8\ mm = 5.66\ mm$
- strength per unit length $F_{w,Rd} = \dfrac{0.65x430}{1.25} x5.66 = 1266\ N/mm$

The length of the filled weld $L_1 = \dfrac{F}{2F_{w,Rd}} = \dfrac{532x10^3\ N}{2x1266\ N/mm} = 210\,mm\ each\ sides + (round\ 2x6\ mm)$

Design of the central plate for shear.



size of the weld (assume 6 mm)
throat thickness = 0.707 x 6 = 4.24 mm

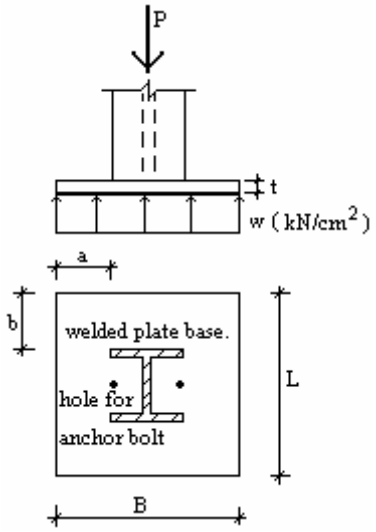
Strength per unit length $F_{w,Rd} = \frac{0.65x430}{1.25} x4.24 = 948N/mm$

The total resistance force $= 948N/mm\,x\,350\ mm\ x\ 10^{-3} = 331.8\ kN > 142\ kN \quad OK.$

Analysis and design of Bases.

The base of a column is designed to distribute the concentrated column load over a certain definite foundation area and to ensure connection of the lower column end to the foundation.
Two basic types of bases are distinguished, namely **pinned and rigid ones.**

Pinned bases. Used to transmit axial load only.



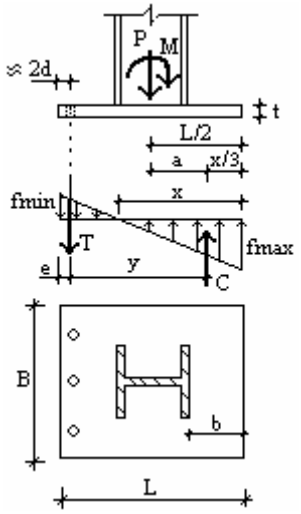
The minimum thickness is: $t = \sqrt{\frac{2.5}{f_{yp,d}} w\left(a^2 - 0.35b^2\right)} \geq t_f$, where:

a is the greater projection of the plate beyond the column
b is the lesser projection of the plate beyond the column
w is pressure of underside of the plate assuming an uniform distribution
Check that $w \leq 0.4\ f_{cu}$
$f_{yp,d}$ is the design strength of the plate. See Table 6.4 bolow
t_f is the flange thickness of the column.

Thickness less than or equal (mm)	Design strength $f_{yp,d}$ (Mpa)
16	170
40	265
63	255
100	245

Rigid base.
In these case both axial load and bending moment are present. Two cases are to be considered:

1. The combined effect of axial load and bending moment produce a uniformly varying presure (compression) over the entire underside of the base (use 4 anchor bolts).
2. The combined action produces a zone of compression and a zone of tension (use anchor bolts to take the tension force in this zone).



C is the resultant of compressive zone
T is total force in all the anchor bolts located at one side of the footing

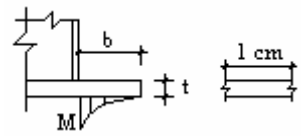
$f_{\max} = \frac{P}{BL} + \frac{6M}{BL^2} \leq 0.4\, f_{cu}$ (design compressive stress of concrete)

$f_{\min} = \frac{P}{BL} - \frac{6M}{BL^2}$

Taking moment with respect to C. $T = M - Pa - Ty = 0 \therefore T = \frac{M - Pa}{y}$

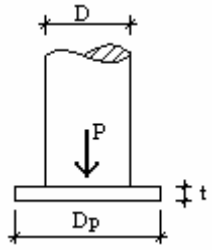
Where: $a = \frac{L}{2} - \frac{x}{3}; \quad x = \frac{f_{\max}}{f_{\max} + f_{\min}} L; \quad and \quad y = L - \frac{x}{3} - e$

To calculate the thickness of the plate t.



$f = \frac{M}{W} = \frac{6M}{1xt} \leq f_{yp,d}$ *from which* $t \geq \sqrt{\frac{6M}{f_{yp,d}}}$

For round columns.



$$t = \sqrt{\frac{w}{2.4} D_p \left(D_p - 0.9D\right)}$$

D is diameter of the column.

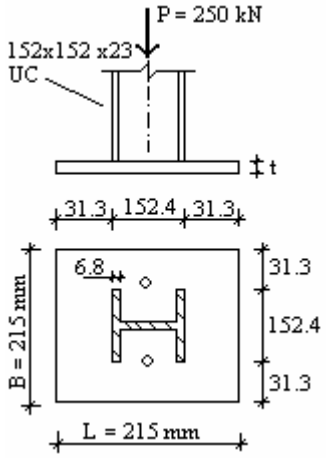
D_p is the length of the side or diameter of the cup of the base plate, but not less than 1.5(D+75) mm.

Notes.

1. The design resistance of the holding down bolts shoud be determined from section 6.2.4 of EBCS – 3.
2. The anchorage length shoud be such as to prevent bond failure before yielding of the bolt.

Example Nr 1.

Find the general dimension for the base plate for the following column. Consider f_c at 28 days = 20 Mpa.



$$w = \frac{P}{BL} = 0.4x2.0kN/mm^2 \Rightarrow BL = \frac{250}{0.8} = 312.5\ cm^2$$

Using square plate

$$B = L = \sqrt{312.5} = 17.18\ cm\ less\ than\ 15.24\ cm$$

Therefore use base plate 30 mm greater than each side to allow welding.

$$B = L = 215\ mm.$$

$$a = b = \frac{215 - 152.4}{2} = 31.3\ mm$$

$$w = \frac{250}{21.5x21.5} = 0.54\ kN/cm^2 < 2.0\ kN/cm^2 \quad OK$$

$$\therefore t = \sqrt{\frac{2.5x0.54}{0.4x2}\left(3.13^2 - 0.3x3.13^2\right)} = 3.4\ cm$$

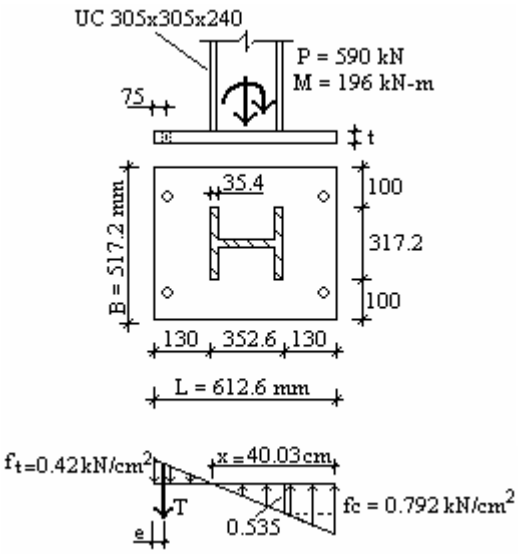
$$t = 34\ mm > 6.8\ mm \quad OK.$$

Example Nr 2

For the base of the crane column shown below, calculate the thickness of the base plate and the tensile force for anchor bolts. The caracteristic concrete steength at 28 days of the foundation is 20 MPa.

Factored axial force is 590 kN.

Factored acting bending moment is 196 kN-m.



Solution:

1. The stress distribution.

$$f_c = \frac{P}{BL} + \frac{6M}{BL^2} = \frac{590}{51.72x61.26} + \frac{6x196x100}{51.72x61.26^2}$$

$$f_c = 0.186 + 0.606 = 0.792\ kN/cm^2 < 0.4x2\ OK$$

$$f_t = 0.186 - 0.606 = -0.42\ kN/cm^2\ \textit{tensile stress.}$$

2. Thickness of the plate.

$$M = \frac{0.535x13^2}{2} + \frac{1}{2}(0.792 - 0.535)(13)\left(\frac{2}{3}x13\right)$$

$$M = 59.69\ kN - cm$$

suppouse $16 \le t \le 40$ mm, then $f_{yp,d} = 26.5$ kN/cm^2

$t = \sqrt{\frac{6x59.69}{26.5}} = 3.67\ cm$, take 3.8 cm = 38 mm > 35.4 mm OK.

3. Tensile force: T

$$T = \frac{M - Pa}{y}$$

$$y = L - \frac{x}{3} - e; \quad e = 75mm, \quad x = \frac{f_c}{f_c + f_t}L = \frac{0.792}{0.792 + 0.42}61.26 = 40.3\ cm.$$

$$y = L - \frac{40.3}{3} - 7.5 = 40.42\ cm$$

$$a = \frac{L}{2} - \frac{x}{3} = \frac{61.26}{2} - \frac{40.03}{3} = 17.29\ cm \quad \textit{then finally}:$$

$$T = \frac{196x10^2 - 590x17.29}{40.42} = 235.53kN.$$

$$\textit{For 2 bolts}: \quad T_{1bolt} = \frac{235.53}{2} = 116.26\ kN\ \textit{each bolt}.$$

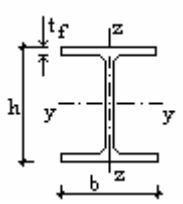
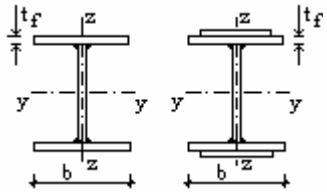
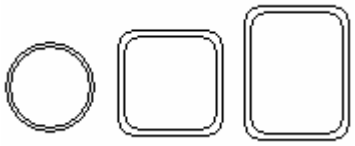
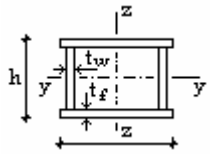
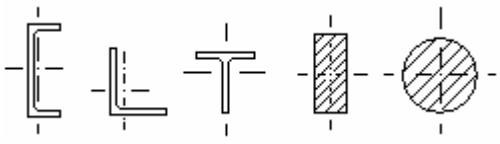
Table 4.9 Imperfection Factors.

Buckling Curve	a	b	c	d
Imperfection factor α	0.21	0.34	0.49	0.76

Table 4.9 Reduction Factors.

$\overline{\lambda}$	Buckling curve			
	a	b	c	d
0.2	1.0000	1.0000	1.0000	1.0000
0.3	0.9795	0.9641	0.9491	0.9235
0.4	0.9258	0.9261	0.8973	0.8504
0.5	0.9243	0.8842	0.8430	0.7793
0.6	0.8900	0.8371	0.7854	0.7100
0.7	0.8477	0.7837	0.7247	0.6431
0.8	0.7957	0.7245	0.6622	0.5797
0.9	0.7339	0.6612	0.5998	0.5208
1.0	0.6656	0.5970	0.5399	0.4671
1.1	0.5960	0.5352	0.4842	0.4189
1.2	0.5300	0.4781	0.4338	0.3762
1.3	0.4703	0.4269	0.3888	0.3385
1.4	0.4179	0.3817	0.3492	0.3055
1.5	0.3724	0.3422	0.3145	0.2766
1.6	0.3332	0.3079	0.2842	0.2512
1.7	0.2994	0.2781	0.2577	0.2289
1.8	0.2702	0.2521	0.2345	0.2093
1.9	0.2449	0.2294	0.2141	0.1920
2.0	0.2229	0.2095	0.1962	0.1766
2.1	0.2036	0.1920	0.1803	0.1630
2.2	0.1867	0.1765	0.1662	0.1508
2.3	0.1717	0.1628	0.1537	0.1399
2.4	0.1585	0.1506	0.1425	0.1302
2.5	0.1467	0.1397	0.1325	0.1214
2.6	0.1362	0.1299	0.1234	0.1134
2.7	0.1267	0.1211	0.1153	0.1062
2.8	0.1182	0.1132	0.1079	0.0997
2.9	0.1105	0.1060	0.1012	0.0937
3.0	0.1036	0.0994	0.0951	0.0882

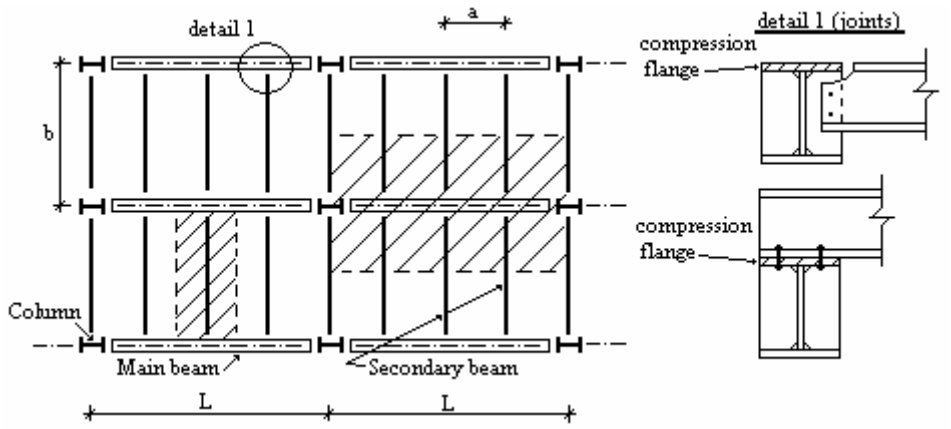
Table 4.11 Selection of Buckling Curve for a Cross-section.

Cross-section	Limits	Buckling about axis	Buckling curve
Rolled I – sections 	h/b > 1.2: $t_f \leq 40$ mm	y – y z – z	a b
	40 mm < $t_f \leq 100$ mm	y – y z – z	b c
	h/b ≤ 1.2: $t_f \leq 100$ mm	y – y z – z	b c
	$t_f > 100$ mm	y – y z – z	d d
Welded I – sections 	$t_f \leq 40$ mm	y – y z – z	b c
	$t_f > 40$ mm	y – y z – z	c d
Hollow section 	Hot rolled	any	a
	Cold formed -using f_{yb}	any	b
	Cold formed -using f_{ya}	any	c
Welded box sections 	Generally (except as below)	any	b
	Thick welds and $b/t_f < 30$ $h/t_w < 30$	y – y z - z	c c
U, L, and Solid Sections 		any	c

Mekelle University.
Department of Civil Engineering.

Assignment N^{r} 1. Steel and Timber Structures.

For the floor system shown below, design each of the elements of the system. The floor is composed by a Reinforced Concrete slab thickness 10 cm with screed 5 cm and finishing 2 cm. The floor carries in addition to its own weight, loads from partitions of magnitude 3 kN/m^2 and live load of **w** kN/m^2.



Special notes:

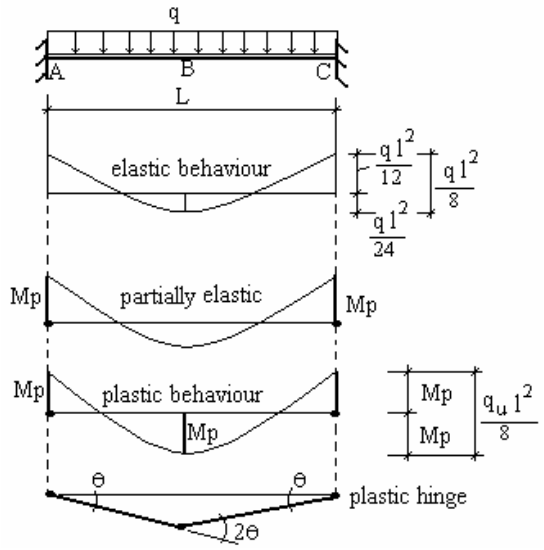
1. Steel Grade is indicated for each student.
2. Secondary beams rest over compression flange of main beams.
3. Columns are fixed at the foundation level and the frames are non-sway mode. The heights of the columns are 5.00 m.

Required:

1. Design of secondary beams.
2. Design of main beams.
3. Design of the columns with its base.
4. Design of the main beam to column connection by using bolted connection.
5. Represent the design of each member and details of joints on sketch with adequate scale.

Introduction to Plastic Analysis and Design.
Reference: Ram Chandra. "Design of Steel Structures".

Behaviour of a statically indeterminate structure.



In elastic behaviour

$$M_A = M_C = \frac{ql^2}{12} > M_B = \frac{ql^2}{24}.$$

Therefore if q increases; the first plastic hinge will be formed at A and C; at point B ; $M_B < M_P$

The beam is transformed from statically indeterminate structure in to simply supported beam.

Now, with a further increase of external load q up to q_u, the 3th plastic hinge at point B is formed and the beam is transformed into a mechanism with three plastic hinge lie on the same straight line.

Therefore, $M_P + M_P = \frac{q_u l^2}{8}$

$$2M_P = \frac{q_u l^2}{8} \quad \Rightarrow \quad M_P = \frac{q_u l^2}{16}$$

And the collapse load $q_u = \frac{16 M_P}{l^2}$

Fundamental conditions for plastic analysis.

In the elastic analysis, the equilibrium condition, continuity and limiting stress conditions are satisfied. According to the first condition, the structure subjected to any system of loading should be in equilibrium. According to the second condition the distortion or deformation of the various fibres should be compatible with those of adjacent fibres. According to third condition, the maximum stress at any section, in any fibre should be less than the yield stresses i.e. the bending moment should be less than the yield moment.
Similarly, in the plastic analysis, the following fundamental conditions should be satisfied:

1. **Mechanism condition.** The ultimate or collapse load is reached when a mechanism is formed. The number of plastic hinges developed should be just sufficient to form a mechanism.
2. **Equilibrium condition.** The algebraic sum of all vertical forces acting over a structure should be zero. The algebraic sum of horizontal forces acting over the structure should be zero. The moment of all the forces acting over a structure about any point should be zero.
3. **Plastic moment condition.** The bending moment at any section of the structure should not be more than the fully plastic moment of the section.

MECHANISM.

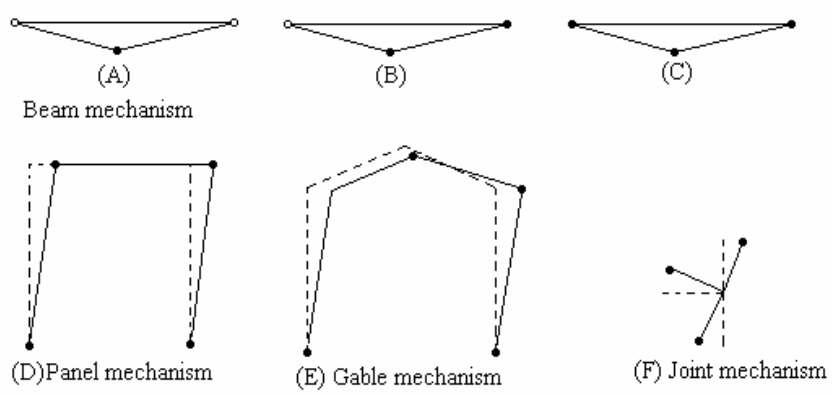
When any elastic body is subjected to a system of loads and deformation takes place and the resistance is set up against the deformation, then the elastic body is known as **structure.** In contradiction to this, if no resistance is set up in the body against the deformation, then it is known as a **mechanical mechanism.**
When a statically indeterminate structure is subjected to loads and the bending moment diagram is drawn, it is seen that is two or more points of peak moments. As the loads are gradually increased, the cross-section having the

maximum of these peak moments will reach yield. A plastic hinge is formed at such a cross-section and the value of bending moment becomes $\mathbf{M_P}$ at the section. As further load is added, this value of bending moment, M_P remains constant and this cross-section having the plastic hinge rotates, and other sections having the bending moment smaller than M_P maintain the equilibrium with the increased load by a proportionate increase in moment. These sections having peak point in the bending moment diagram reach yielding and the plastic hinges are formed in succession. The values of bending moments at these peak points become M_P in succession. The process of moment transfer known as the redistribution of moment due to successive formation of plastic of plastic hinges continues. After the formation of sufficient number of plastic hinges, the structure reduces to a **plastic mechanism**, and the failure or collapse of the structure occurs.

If the degree of indeterminacy of the structure is r, then the indeterminate structure becomes a determinate structure on the formation of r number of the plastic hinges. If one additional hinge is formed, after the structure has become a determinate one, then a mechanism is formed. Thus, the (r + 1) number of plastic hinges are necessary to convert a structure into a mechanism. A simply supported beam is determinate structure. If one plastic hinge is formed, then this structure reduces to a mechanism. The structure fails or collapses as soon as a mechanism is formed. The load corresponding to the formation of first plastic hinge is known as yield load, the load corresponding to the formation of a mechanism is known as collapse or ultimate load.

The formation of plastic hinges results in modest increase of load. The redistribution of moment further increases the load carrying capacity (reserve strength) of indeterminate structures loaded beyond the elastic limit.

Types of independent mechanism.



Methods of Plastic Analysis.

1. **Static method.** Based on equilibrium equation of bending moments.
   See "Design of steel structures" by Ram Chandra. Page 632.

2. **Kinematic Method.** (Or mechanism method of analysis). Based on the Principle of virtual work.

   The principle of virtual work states that if deformable elastic body is in equilibrium under a system of external loads and remains in equilibrium while it is subjected to a small virtual deformation, the external virtual work done by the external forces acting on the body is equal to the internal virtual work of deformation done by the internal stress.

Therefore; ***The work done by external force is equal to the work absorbed by the plastic hinges at collapse.***

The Project and Construction of Industrial Building.

1. Project of industrial building.
2. Column layout, expansion joints. Façade.
3. Physical model, crane loads.
4. Bracing system of Industrial Buildings
5. Plastic design of steel frames.
6. Joints by using high strength bolts.
7. Quality control measurements for welded connections.

References:

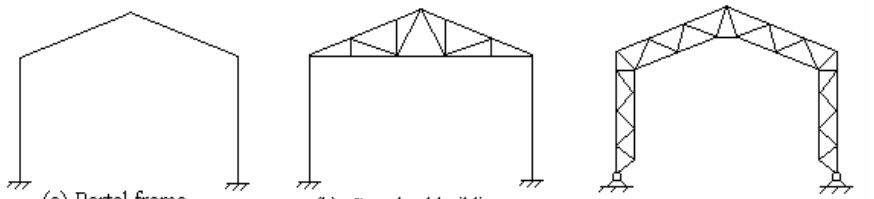
1. Ram Chandra. “Design of Steel Structures”.
2. Mac Ginley TJ.& Ang T. “Structural Steelwork”.
3. Beddle L S. “Plastic Design of Steel Frames”.

1. Introduction.

The industrial building are designed and constructed to store the raw materials or manufactured products of the industry or to support and house a manufacturing process for the industry. Normally are simple roof structures on open frames. These buildings require large and clear areas unobstructed by the columns. These large floor are provides sufficient flexibility and facility for later change in the production layout without major building alterations. The industrial buildings are constructed with adequate headroom for the use of an overhead traveling crane.

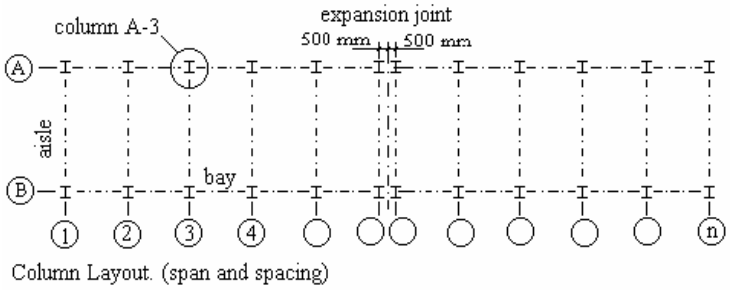
The function of industrial building dictates the degree of sophistication. For the proper design of the industrial building, the designer should have knowledge of the manufacturing process and the purpose for which the industrial building is installed. From the external architectural point of view, the industrial building should be properly related aesthetically to the community and its natural environment. The internal architecture of the industrial building should provide for the development of design so as to satisfy the requirements of the occupants. The total area and volume requirements are determined for proper planning and layout of the industrial building. The designer then develops the exterior dimensions for the structure surrounding the manufacturing process

The most common types of frames used are as shown in the figure below.



The distance between to adjacent transverse columns is known as **bay length** or simply bay and the distance between two columns line as **aisle**. The industrial building are generally single-aisle structures. However, the industrial building with two, three, four or more aisles are also built.

2. Column Layout.

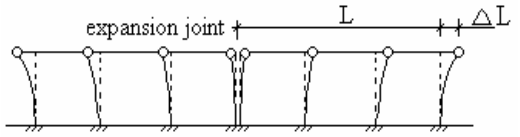


Column Layout. (span and spacing)

Normally the span (aisle) is taken multiples of 3 m, and the spacing (bay) 6 m or 12 m as required.

Expansion joints.

If a building has a considerable length or width, a significant deformation of its separate elements appears due to temperature changes.



$\Delta L = \alpha L \Delta t$ where $\alpha = 0.000012^{o}C^{-1}$ coefficient of linear expansion of steel.

L length of the block

Δ t difference of temperature

Maximum permissible dimension of expansion section of building and structures

Category of building or structure.	Maximum distance from end of section to centerline of nearest vertical brace.	Maximum length of section along building.	Maximum width of building.
Heated buildings	90	230	150
Unheated buildings and hot shops.	75	200	120

Note: when precast reinforced concrete column are used, the expansion joints are spaced not more than 60 m apart.

Wall construction.

The permanent wall construction is used except where low cost is essential. The type of wall construction used for the industrial buildings has adequate resistance and insulating properties. The type of wall construction used for industrial building should be durable, and easily maintained. The bearing walls are made strong enough to support, in addition to their self-weight, the loads from floors, the roof which frame into them. The solid masonry, hollow masonry, plain cement concrete and reinforced concrete are used for wall construction.

The corrugated cement asbestos sheets, trafford cement-asbestos sheets, galvanized-iron corrugated sheets are also used for wall construction. The glass enclosures are also becoming common.

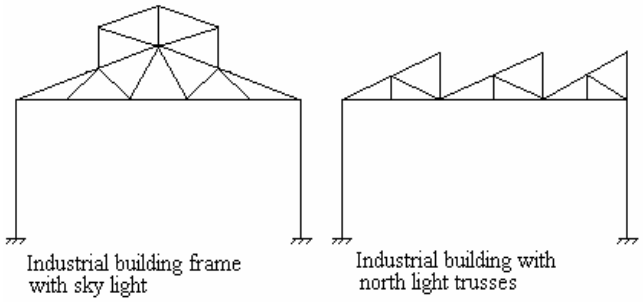
Roof system
The roof trusses supported on columns provide structural roof system for the industrial building. The type of roof covering, its insulating value, acoustical properties, the appearance from inner side, the weight and the maintenance are the various factors, which are given consideration while designing the roof system. The asbestos corrugated and trafford cement sheets, and the galvanized-iron corrugated sheets are used as the roof covering materials.

Floor construction.
The industrial building are mostly one-storey structures. The part of the industrial building used as office may consist of two or more storeys. The cement concrete solid slabs are most frequently used. The cement concrete slabs are made with 100 mm as minimum thickness. These slabs are supported on the intermediate rolled steel joists. The intermediate rolled steel joists are supported on columns spacing into two or more panels.

Partition walls.
The partition walls used for industrial building should have pleasing appearance, acoustical properties, less weight and ease in erection. The partition walls carry their self-weight. The clay tiles, cinder block, and gypsum tiles are commonly used for the partition walls of the industrial buildings. The hollow partitions and lightweight partitions are also becoming popular. The asbestos cement boards, and metal, and wire glass are also used for the partition wall of industrial buildings.

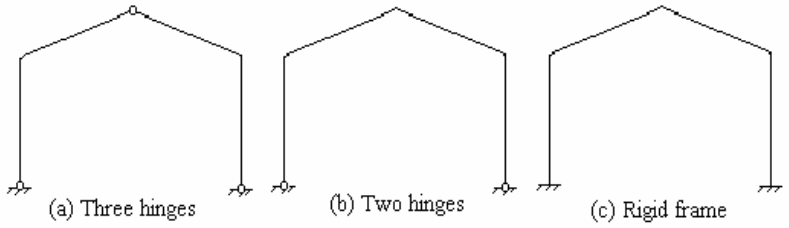
Staircases.
The staircases provided in industrial buildings should allow safe, ease and comfortable passage from one floor to the other. The staircases provided should have sufficient width. The landing should be provided between the flights of staircases. The proportions of tread and rise should be suitable. The size of area to be served and the number of persons to be accommodated govern the number of staircases and its width.

Lighting.
As far as possible, attempt is made to use day light most satisfactorily. The industrial building are made with monitor as shows the figure below to allow the sky light. The monitors in the industrial buildings are provided lengthwise. The north light roof trusses or saw tooth roof truss as shows in the proper figure are also used in the industrial building. The north light roof trusses provide uniform day light throughout the day.



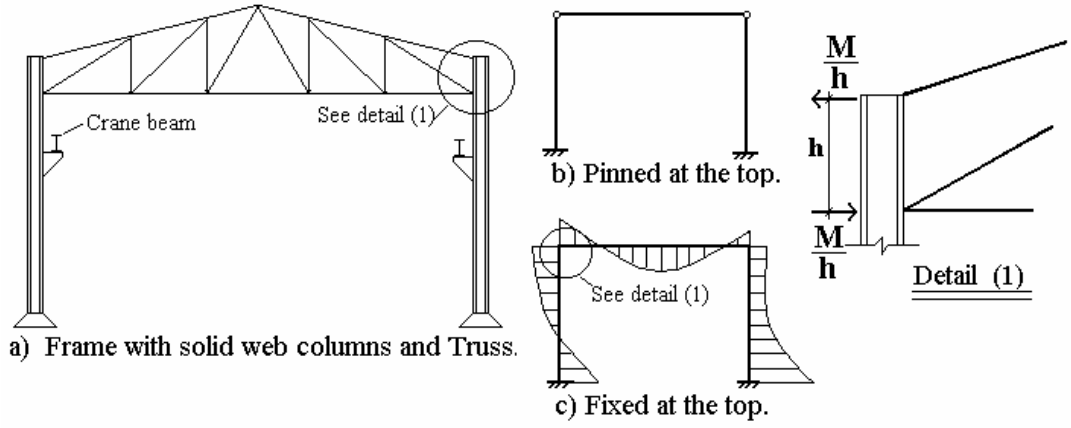
3. **Physical model, crane loads**.

For the portal frame solid web there are three models to be used as shown in the figure below.



Frames (a) and (b) are recommended for places with poor soil conditions. Frame (c) for well soil conditions.

Other models for frames with roof trusses are shown in the following figure.



The squeme for the analysis is defined by following the centre of gravity for the columns and the centre of gravity of the lower chord of the truss.

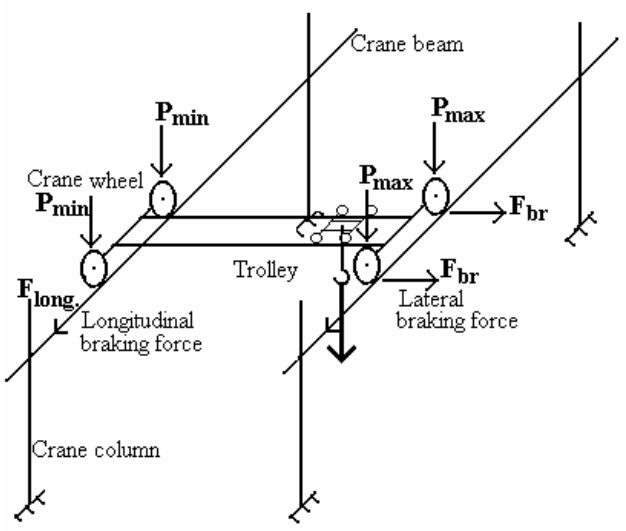
When a rigid frame as shown in (c) is used, the acting bending moment M is taken by a pair of forces values M/h acting at the upper and lower joints as shown in Detail (1).

Crane loads.

The overhead travelling cranes are used in the industrial buildings to lift the heavy materials equipment, and to carry them from one place to the other over an entire floor area of one or more bays between any two lines of building columns. These cranes are either **hands operated** or **electrical operated.** These cranes include hoist trolley and a crane bridge on a roller track. The bridges as whole moves longitudinally on rails provided at the ends. The rails on either side of the bridge are supported on the girders. The load being handled as well as the weight of the crane and the trolley is transmitted to the crane girders through the crane wheels, and the crane girder finally transmit the total load to the frame by the support reaction on the bracket supports. A general squeme about crane loads is shown in the figure below.

Depending upon the location of the trolley the crane wheel load may have a maximum or minimum value. The maximum working loads of crane wheels as well as the arrangement of the wheels with respect to each other are indicated in the standards relating to cranes.

Owing to braking of the trolley, lateral horizontal braking force appears. As a result, a lateral braking force is transmitted to the Crane Bridge.



This lateral braking force is given by the standard or can be calculated from the equation:

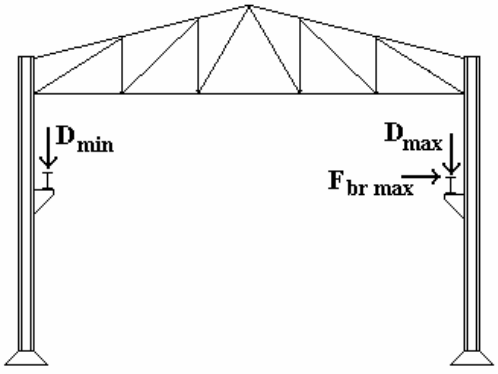
$$F_{bro} = f_r\left(Q+g\right)\frac{2}{4} = 0.1\left(Q+g\right)\frac{2}{4} = \frac{Q+g}{20}$$

F_{bro} – Lateral braking force.
F_r – Coefficient of friction. = 0.1
Q – Lifting capacity of crane.
g – weight of trolley, taken in accordance with the standards for cranes; if not data are available it may be assumed that g = 0.3 Q.
2/4 – fraction whose numerator indicates the number of brake wheels and denominator the total number of trolley wheels (since the force of friction appears only under those trolley wheels which are outfitted with brakes).

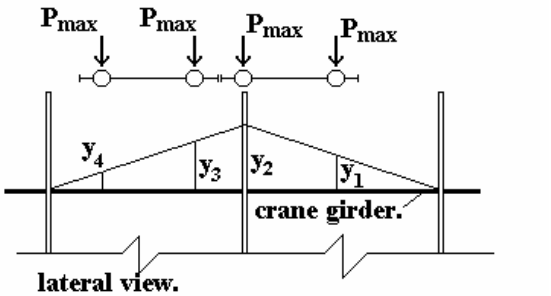
The braking force F_{br} is transmitted to one crane girder and is distributed uniformly between the wheels of the crane.

$$F_{br} = \frac{F_{bro}}{2}$$

The maximum value for the acting forces each side of the frames is obtained by using the influence line diagrams as shown in the figure below:



Crane forces action on the frame.



Influence line for the support reaction of the crane girder.

Some codes practices recommend preparing the structure to withstand the action of two cranes acting at the same times. The diagram of the influence line for the support reaction of the crane girder both sides of the frame being studied is shown in the figure above. The values for the forces are:

$$D_{max} = \Sigma P_{max} y_i + P_{cg}$$

Where P_{cg} is the weight of the crane girder.

$$D_{min} = \Sigma P_{min} y_i + P_{cg}$$

For the braking force the same influence line diagram is used because both forces; vertical forces and horizontal one have the same point of application.

Other types of loads like Live Load, Wind loads etc are established on EBCS-1 "Basic of Design and Actions on Structures". To estimate in first the weight of steel elements on industrial buildings in kN/m^2 the following table may be used.

Approximate weight of elements of Steel Industrial building framework in KN/m^2

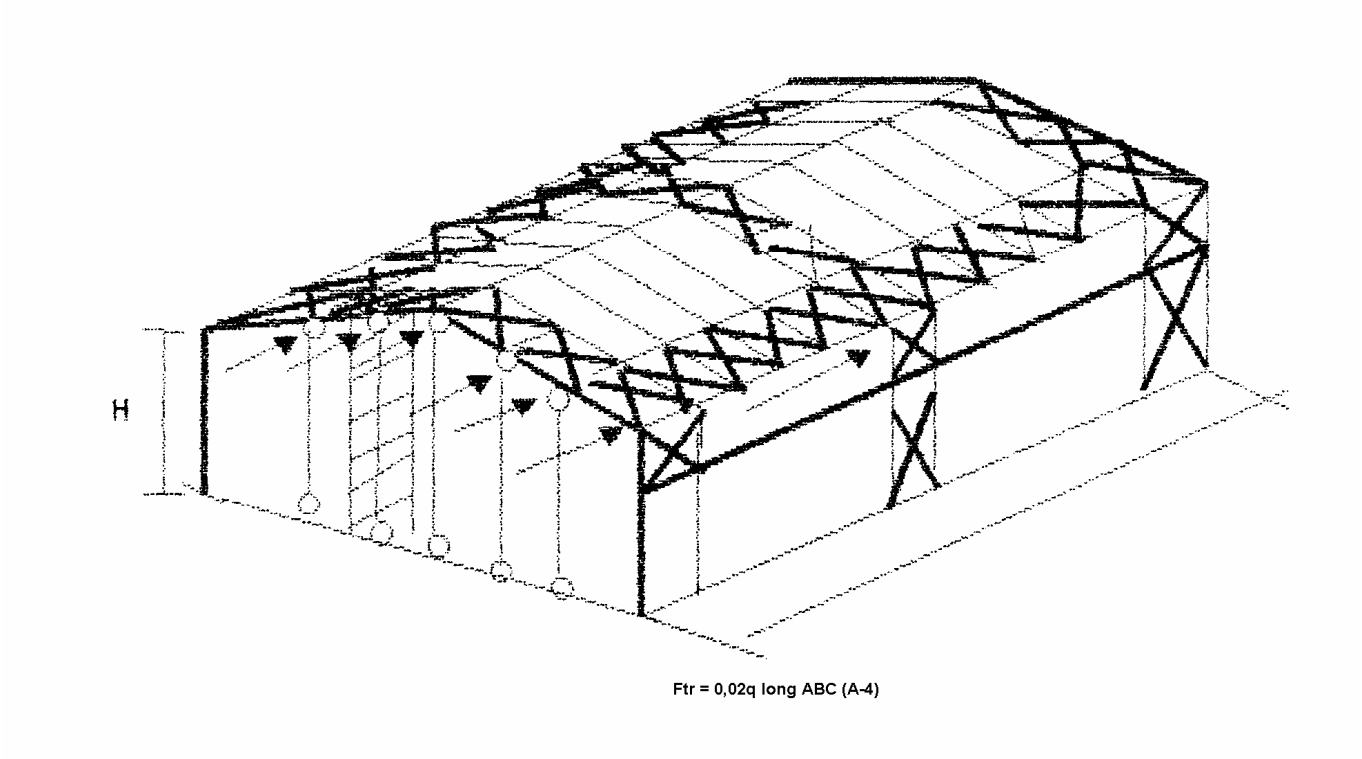
Elements of steel Framework.	Groups of Shops.		
	Light	Medium	Heavy
Roof trusses	0.16 – 0.25	0.18 – 0.30	0.20 – 0.40
Secondary trusses	0.00 – 0.06	0.04 – 0.07	0.08 – 0.20
Purlins	0.10 – 0.12	0.12 – 0.18	0.12 – 0.20
Skylights	0.00 – 0.10	0.08 – 0.12	0.08 – 0.12
Ties	0.03 – 0.04	0.03 – 0.05	0.08 – 0.15
Columns with ties and platforms	0.10 – 0.18	0.18 – 0.40	0.7 – 1.20
Crane girders with repair platforms	0.00 – 0.14	0.14 – 0.40	0.50 – 1.50
Wallframework	0.00 – 0.03	0.05 – 0.14	0.12 – 0.20

4. Bracing system of Industrial Buildings

All industrial buildings are thoroughly braced to prevent the deformation of the structures due to the action of wind, earthquake forces, and effects of moving equipment such as cranes. In braced industrial buildings, the roof trusses rest on columns with hinged type of connections. The function of bracing system is to transmit the lateral and longitudinal forces efficiently to the foundations of the buildings. The wind and earthquake loads may act in the direction parallel or normal to the plane of transverse bents of the industrial buildings. The longitudinal braking force of the Crane when it move along the building is also taken by the braced system. Therefore, the industrial buildings are thoroughly braced in the following three mutually perpendicular planes:

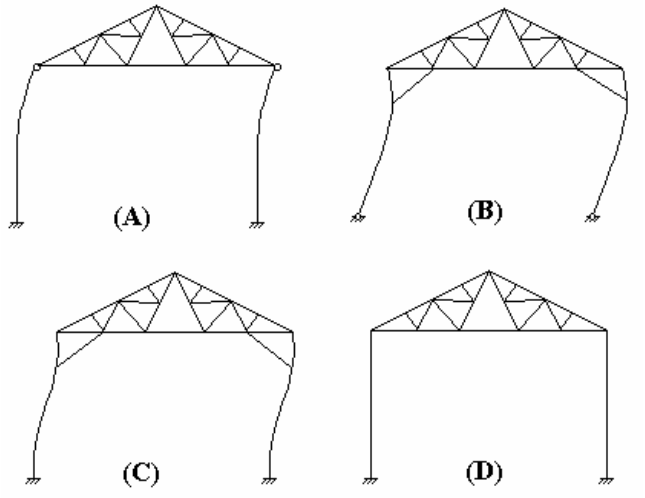
1. Transverse plane (horizontal plane at tie level of the roof truss).
2. Longitudinal vertical plane (vertical plane along the longitudinal section of columns).
3. Vertical planes in the end cross-sections (usually at the gable ends).

A general view of Bracing system is shown in the figure below.



Bracing of industrial buildings in transverse direction.

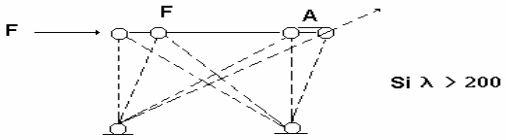
The industrial buildings bents are braced against transverse forces independently of the others. As a result of this, on completion of the erection, each industrial building bent remains stable transversely. The nature of soil under foundation, the fixity of columns at the base, and the rigidity of connections between the steel trusses and columns influence the structural stability in the transverse direction. Four methods of bracing an industrial building bent are shown in the following figure.



The suitability of each method of bracing depends on the height of the industrial building and the span of the truss.

The method of bracing a bent as shown in figure (A), i.e., by fixing the column at the base, and providing the mechanical hinges at the top, is suitable when the vertical column load and the size of the foundation are large, and the overturning moment is small. When the span of truss is large, then the vertical loads and consequently the size of the foundation are also large. When the height of building is small, then the overturning moment is also small. Hence, the method of fixing columns at the base is suitable for low and wide industrial building.
The overturning moment may be reduced by providing the **knee braces** between trusses and the columns. The method of bracing the bent by fixing the columns at both the ends is suitable for tall buildings. These methods of bracing the bents by providing knee braces as shown in fig. (B) and (C), are suitable when the spans of the trusses are small.
In case when the columns are hinged, then there is no bending moment on the foundation. The construction of foundation becomes cheap when the columns are hinged. The reduced moment is transferred to the column at the junction of knee-braces with the columns. In the case the columns are fixed, then the moments are further reduced. The knee-braces reduce the clear height of the buildings.
Some times it becomes necessary to have maximum headroom around the walls of industrial buildings. In such places, the knee braces cannot be provided. The method of bracing the bent without knee-braces as shown in fig. (D) is adopted. In this method, the column and truss connections are made rigid. The columns are fixed at the bases, and the foundations are made large.

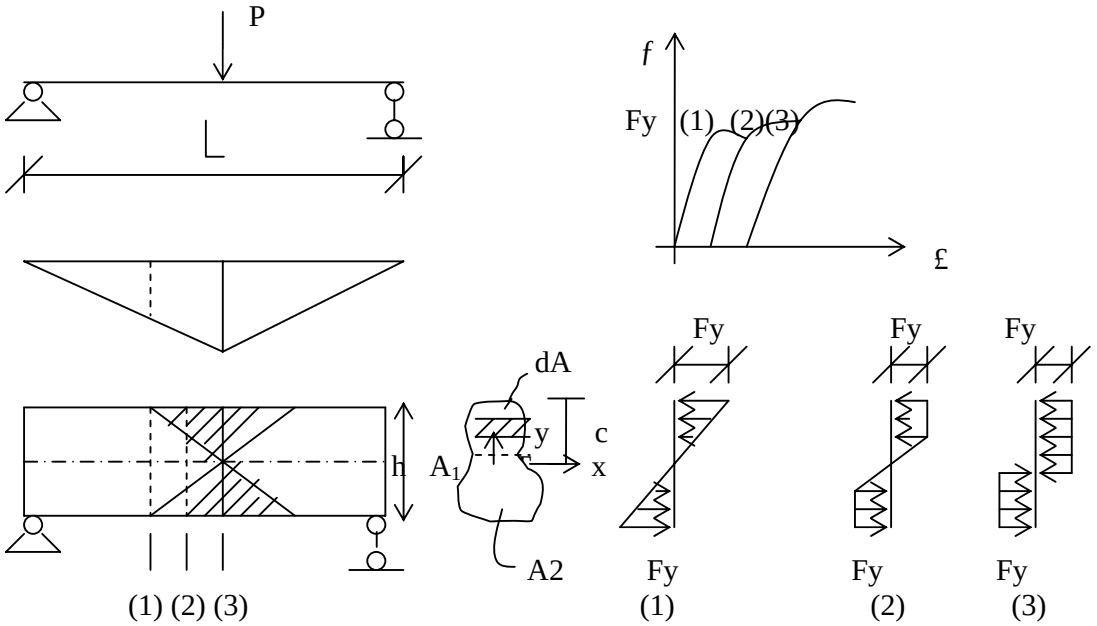
Bracing of industrial building in longitudinal direction.
When the wind acts in the direction normal to the plane of industrial building bents, that is, in the longitudinal direction, then it becomes essential to brace it to another to provide sufficient stability against wind or other longitudinal forces. The industrial building bents are braced in pairs. When two bents are braced together, then they form brace bay. A braced bay acts like a space tower under the action of wind. Such braced bents are provided at every third or fourth bay of the industrial building. It is essential to provide at least two braced bays in each industrial building. The bents are braced by providing trusses in a plane of top chord and in the plane of bottom chord of the roof trusses. These trusses maintain the rectangular shape of the building and prevent the collapse of the structure from twisting action of the diagonal wind. The horizontal truss bracing transmit the longitudinal forces to the columns at the sides. The columns are braced in the longitudinal vertical planes. These bracing transmit these loads to the foundations.
For a complete information about bracing on one practical example, see Reference “Design of Steel Structures” by Ram Chandra, Page 457 to 460.
The braces normally are constructed in cross-form as shows the figure below. In this form,



only the tension one is effectively taking the loads, the other one, which is under compression, because it large slenderness ratio, bent under the action of the force.

Plastic design of steel frames.

Let study a beam of any cross section.



In (1) the beam behaves elastically, the extreme fibbers rich the yield point.

For elastic behaviour $f_b = \frac{M}{W}$; where: $W = \frac{I}{c} \rightarrow$ section modulus.

For bending the condition with respect to X, the strength condition is:

$f_b = \frac{M_x}{W_x} \leq F_y \quad \rightarrow F_y$, stress at the yield point.

Stage (2) is partially plastic, yield stress go deep into the section.

Stage (3) is fully plastic, the section rotate and a **plastic hinge** is formed, the section is under the action of the **Plastic Moment Mp.**

$$M_p = \int_A F_Y \, y \, dA = \int_{A1} F_Y \, y \, dA + \int_{A2} F_Y \, y \, da = F_Y \left[\int_{A1} y \, dA + \int_{A2} y \, dA \right], \text{ but}$$

$\int y \, d A = S$, static moment or first moment of area

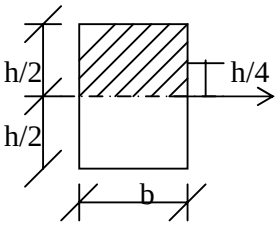
Therefore $M_p = F_Y \, (S_1 + S_2)$

For a symmetric section $S_1 = S_2 = S$ Therefore:

$M_p = F_Y \, . 2S$; doing $W_p = 2S$ **Plastic Modulus**: $M_p = F_Y \, W_p$.

	Elastic behaviour	Plastic behaviour
Hence:	$M = F_Y \, W_e$	$M_p = F_Y \, W_p$

Lets obtain both values Wp and We for a rectangular section.



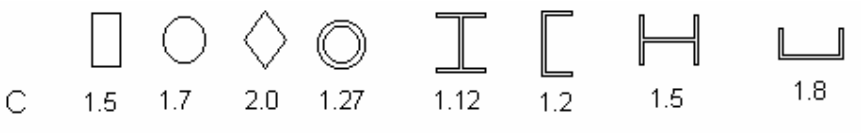
$W_E = \frac{bh^2}{6}$ $\qquad$ $W_P = 2\,S = 2\,b\,\frac{h}{2}\frac{h}{4} = \frac{bh^2}{4}$

What is the ratio between Plastic Moment and elastic one?

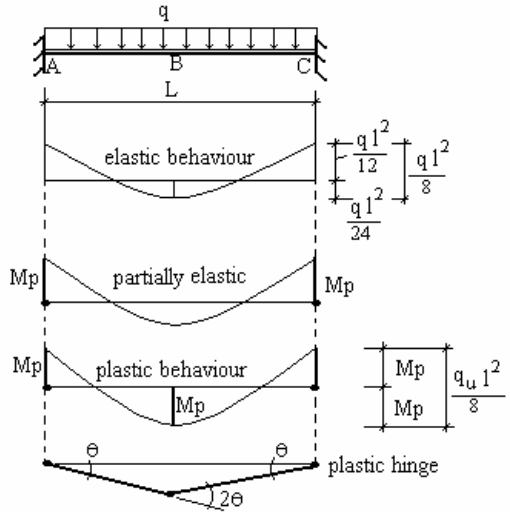
$$\frac{M_p}{M} = \frac{W_P F_Y}{W_E} = \frac{W_P}{W_E} = \frac{bh^2}{4} * \frac{6}{bh^2} = 1.5$$

The ratio $\frac{M_p}{M} = C$, is known as **Shape Coefficient** of the section .

Values for C.



Behaviour of a statically indeterminate structure.



In elastic behaviour

$$M_A = M_C = \frac{ql^2}{12} > M_B = \frac{ql^2}{24}.$$

Therefore if q increases; the first plastic hinge will be formed at A and C; at point B ; $M_B < M_P$

The beam is transformed from statically indeterminate structure in to simply supported beam.

Now, with a further increase of external load q up to q_u, the 3th plastic hinge at point B is formed and the beam is transformed into a mechanism with three plastic hinge lie on the same straight line.

Therefore, $M_P + M_P = \frac{q_u l^2}{8}$

$$2M_P = \frac{q_u l^2}{8} \quad \Rightarrow \quad M_P = \frac{q_u l^2}{16}$$

And the collapse load $q_u = \frac{16 M_P}{l^2}$

Fundamental conditions for plastic analysis.

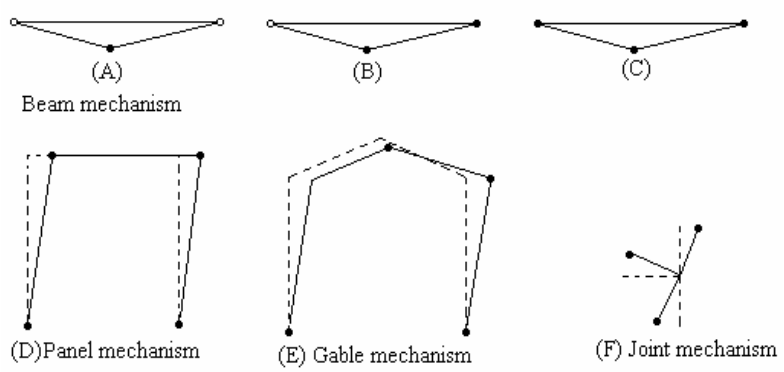
In the elastic analysis, the equilibrium condition, continuity and limiting stress conditions are satisfied. According to the first condition, the structure subjected to any system of loading should be in equilibrium. According to the second condition the distortion or deformation of the various fibres should be compatible with those of adjacent fibres. According to third condition, the maximum stress at any section, in any fibre should be less than the yield stresses i.e. the bending moment should be less than the yield moment.
Similarly, in the plastic analysis, the following fundamental conditions should be satisfied:

1. **Mechanism condition.** The ultimate or collapse load is reached when a mechanism is formed. The number of plastic hinges developed should be just sufficient to form a mechanism.
2. **Equilibrium condition.** The algebraic sum of all vertical forces acting over a structure should be zero. The algebraic sum of horizontal forces acting over the structure should be zero. The moment of all the forces acting over a structure about any point should be zero.
3. **Plastic moment condition.** The bending moment at any section of the structure should not be more than the fully plastic moment of the section.

MECHANISM.

When any elastic body is subjected to a system of loads and deformation takes place and the resistance is set up against the deformation, then the elastic body is known as **structure.** In contradiction to this, if no resistance is set up in the body against the deformation, then it is known as a **mechanical mechanism.**
When a statically indeterminate structure is subjected to loads and the bending moment diagram is drawn, it is seen that is two or more points of peak moments. As the loads are gradually increased, the cross-section having the maximum of these peak moments will reach yield. A plastic hinge is formed at such a cross-section and the value of bending moment becomes $\mathbf{M_P}$ at the section. As further load is added, this value of bending moment, M_P remains constant and this cross-section having the plastic hinge rotates, and other sections having the bending moment smaller than M_P maintain the equilibrium with the increased load by a proportionate increase in moment. These sections having peak point in the bending moment diagram reach yielding and the plastic hinges are formed in succession. The values of bending moments at these peak points become M_P in succession. The process of moment transfer known as the redistribution of moment due to successive formation of plastic of plastic hinges continues. After the formation of sufficient number of plastic hinges, the structure reduces to a **plastic mechanism**, and the failure or collapse of the structure occurs.
If the degree of indeterminacy of the structure is r, then the indeterminate structure becomes a determinate structure on the formation of r number of the plastic hinges. If one additional hinge is formed, after the structure has become a determinate one, then a mechanism is formed. Thus, the (r + 1) number of plastic hinges are necessary to convert a structure into a mechanism. A simply supported beam is determinate structure. If one plastic hinge is formed, then this structure reduces to a mechanism. The structure fails or collapses as soon as a mechanism is formed. The load corresponding to the formation of first plastic hinge is known as yield load, the load corresponding to the formation of a mechanism is known as collapse or ultimate load.
The formation of plastic hinges results in modest increase of load. The redistribution of moment further increases the load carrying capacity (reserve strength) of indeterminate structures loaded beyond the elastic limit.

Types of independent mechanism.



Methods of Plastic Analysis.

1. **Static method.** Based on equilibrium equation of bending moments.
   See "Design of steel structures" by Ram Chandra. Page 632.

2. **Kinematic Method.** (Or mechanism method of analysis). Based on the Principle of virtual work.

   The principle of virtual work states that if deformable elastic body is in equilibrium under a system of external loads and remains in equilibrium while it is subjected to a small virtual deformation, the external virtual work done by the external forces acting on the body is equal to the internal virtual work of deformation done by the internal stress.

Therefore; ***The work done by external force is equal to the work absorbed by the plastic hinges at collapse.***

Examples.

1. Simply beams with different ends conditions under the action of concentrated and distributed loads systems.
2. Continues beams.
3. Portal frames.

Plastic Design.

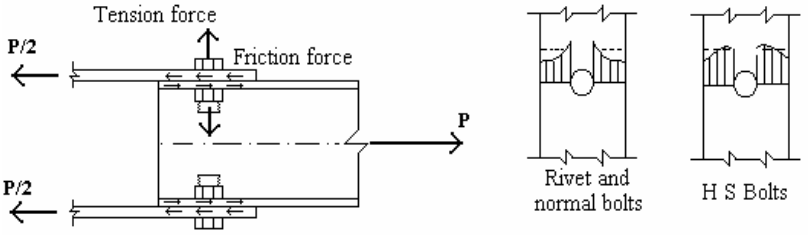
Plastic design is included by EBCS-3, but only sections that satisfy requirements for **Class 1** (Plastics) on Table 4.1 shall be used.

Fasteners of high elastic limit.

The meetings with fasteners of high elastic limit follow the principle of working for the friction forces that arise among the surfaces in contact product of the traction's that arise in the fasteners. On these fasteners the first studies began in E. U. and Germany in 1938 and already in 1956 they are accepted definitively when arising the first specifications on their use.

Already in the construction with the technique of the one riveted in hot was known of the existence of the forces of friction that appear in the joint, it is to take advantage of this forces using a much more advanced technology with the following advantages with regard to the riveted meetings:

- Forge no-necessity with the rising saving in compressors, scaffolds and the entire necessary one for the one riveted.
- Smaller effort manual.
- A distribution of efforts is achieved but to regulate in the borders of the holes.



The magnitude of the friction forces depends of:

1. Intensity of the traction force in the body of the screw.
2. The friction coefficient among the surfaces in contact.

Characteristic of the screws

Diameter d ≥ 12 mm.

Diameter of the hole, $d_a = d + (1 \div 2)\ mm$ in E.U. $d_a = d + 1.5\ mm$

Strength of the material. $R_u \geq 800\ Mpa$ in general it is accepted $(800 \leq R_u \leq 1200)\ Mpa$

Minimum yield point. = 640 Mpa.

Washers: thickness minimum 4 mm; there is adjustable washers for inclination of wings in some laminate profiles.

Strength of the material $(600 \leq R_u \leq 1000)\ Mpa.$

The screws TAR should crowd together until getting in its spike a tension under the simultaneous action of the traction and of the torsion moment to that it is subjected by virtue of the compression of the same nut, as minimum to 80% of its yield limit. This is achieved by means of the moment of final torsion applied to the nut by a gauged special key.

The torsion moment to be applied determines for the following formula: $M_t = k.d.N_0$

k - Coefficient of the key that varies between 0.15 and 0.19, recommended 0.18

d - nominal diameter of the screw.

No - Is an effort axial of pretension that must get in the spike of the screw. For a 80%;. It formula is: $N_0 = 0.8\ F_y\ A_n$

Fy - tension in the yield limit.

An - net area for the threaded area.

There are keys that automatically, after graduate, themselves indicates that it has been arrived to the torsion moment.

Note: To guarantee the one it presses wanted, to apply a superior moment to that of calculation in 10%.

Friction coefficient. (μ)

The surfaces in contact should be cleaned appropriately with jets of sand (sand blasting) or with chemical agents.

For steels structural degree, A - 36, S - 37, Ct - 3 and other similar ones, they intend the following values for the coefficient of friction.

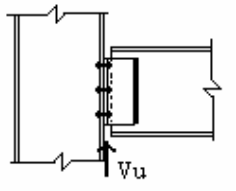
United States. $\mu = 0.2 \div 0.6$ one usually works with 0.45

Calculation of the unions.

If the traction force on the offspring of the fastener, $N_0 = 0.8F_yA_n$, this it will be the normal force among the surfaces in contact, then the force of friction of exhaustion for a fastener will be: $T_u = 1.07\ N_0\ \mu\ n$, here n is the number of planes of friction (similar to that of planes of cut), and 1.07 are an experimental coefficient.

Possible cases:

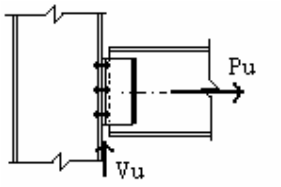
Subjected union to Shear Force.



The resistant force for a fastener will be:
$T_u = 1.07 N_0 \mu n$

The total Number gives necessary fasteners it will be then: $N = \frac{V_u}{T_u}$

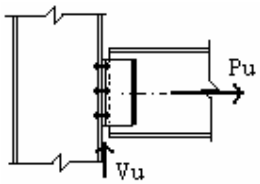
Subjected union to shear force and force of parallel traction to the axis of the screw.



Due to the tensile force Pu, each fastener this subjected one to a tensile force $N^* = \frac{P_u}{N}$; then the net tensile force that produces friction will be (No - N *). Then the resistant force gives a fastener it will be:

$T_u = 1.07(N_0 - N^*)\mu n, \quad and \quad N^* < N_0$

Subjected union to the simultaneous action of shaer force and bending moment



Due to the bending moment, a force appears in each screw gives friction Pi whose maximum value will be Pm in the line but far away the center of the bolts.

- Calculation of the maximum force of traction Pm.

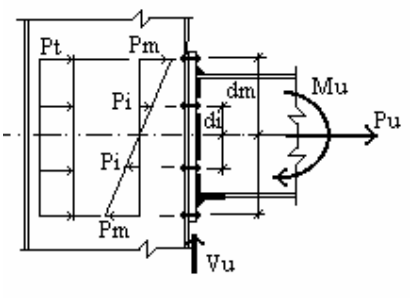
$P_m = \dfrac{M_u \; d_m}{\sum_{i=1}^{m} d_i^2}$ or $P_m = \dfrac{M_u \; d_m \; A_m}{I}$ where $I = \sum_{i=1}^{m} A_i \; d_i^2$. This second formula is more general, but it stops fasteners of same diameter both they coincide.

To verify:

a) La Pm ≤ No. To guarantee compression.

b) That the shear force on each fastener, $\dfrac{V_u}{N} \leq 1.07 \; N_0 \; \mu \; n$ that is to say, to verify the sharp one it is not considered the effect of the moment, since the ΣPi = 0.(Produced by the bending moment).

Subjected union to the simultaneous action of shear force, bending moment and parallel force to the axis of the screw.



To verify:

a) The maximum traction on a screw N * due to the combination of Pu and Mu; that is to say N * = Pm + Pt, be inferior to No. Then N * = Pm + Pt < No.

b) The maximum force resisted by a fastener to shear will be:

$T_u = 1.07 \left(N_0 - N^*\right) \mu \; n$; being N * = Pt that is to say only of the axial effort.

7). **Quality control measurement for Welded Connections.**

All welded fabrication must be checked, tested and approved before being accepted.

1. Visual inspection for uniformity of weld.
2. Surface tests for cracks using dyes or magnetic particles.
3. X – rays and ultrasonic tests to check for defects inside the weld.

Only visual and surface tests can be used on fillet welds. Butt welds can be checked internally, and such tests should be applied to important butt welds in tension.

Crack can occurs in welds and adjacent parts of the members being joined. The main types are shown in Figure 7.1 below.

Faulty welding procedure can lead to te following defects in the welds, all of which reduce the strength.

1. Over reinforcement and undercutting.
2. Incomplete penetration and lack of side-wall fusion.
3. Slag inclusions and porosity

When the weld metal cools and solidifies it contracts and sets residual stresses in members. It is not economic to relieve these stresses by heat treatment after fabrication, so allowance is made in design for residual stresses.

Welding also causes distortion, and special precautions have to be taken to ensure that fabricated members are square and free of twisting. Good detailing and using correct welding procedure can minimize distortion effects. Presetting, prebending and preheating are used to offset distortion.

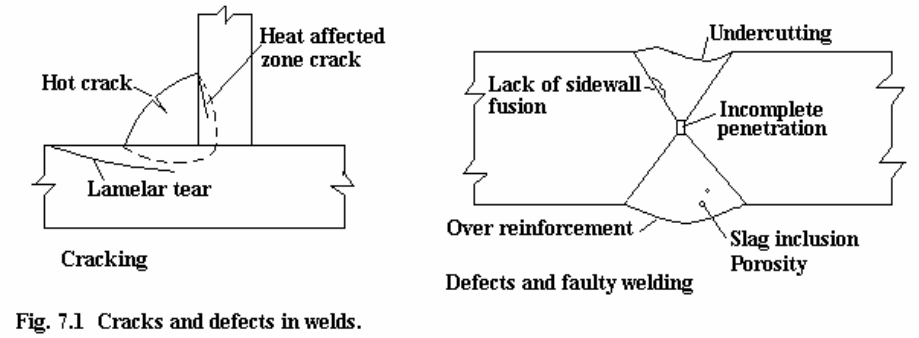
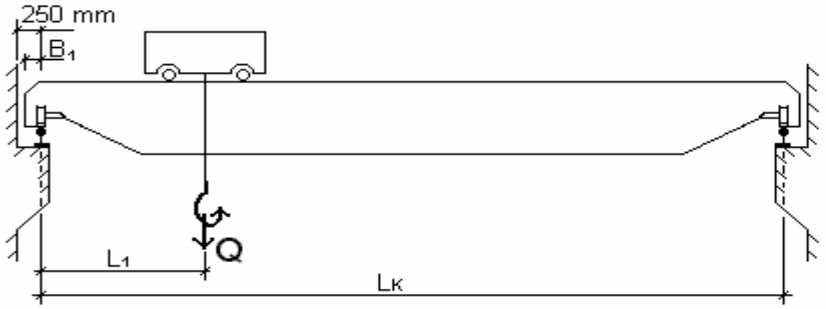
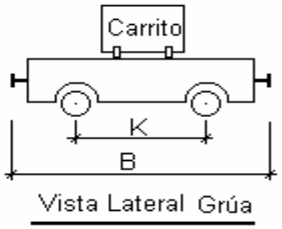


Fig. 7.1 Cracks and defects in welds.

Puentes grúas eléctricos para uso general con capacidad desde 5t hasta 15t y un gancho para regímenes mediano y pesado de trabajo (GOST 3332-54).

Q(t)	Lk (m)	Dimensiones Principales (mm)					Posiciones extremas del gancho			CARRIL	Presión por Rueda (t).			T_{long}	Peso en (t)		Lk (m)
		B	K	H	B_1	F	h	L_1	L_2		$P_{máx}$	$P_{mín}$	T_{trans}		Carrito.	Grúa Total	
5	11	5000	3500	1650	230	250	500	1100	800	KP70 Ó P38	7,0	2,3	2,2	0,70	2,2	13,6	11
	14					250					7,5	2,7		0,75		15,4	14
	17					250					8,2	3,4		0,82		18,1	17
	20					350					8,9	4,0		0,89		20,8	20
	23	6500	5000			450					10,1	4,9		1,01		25,0	23
	26					550					10,7	5,8		1,07		28,0	26
	29					650					11,5	6,6		1,15		31,2	29
	32					750					12,2	7,0		1,22		33,3	32
10	11	6300	4400	1900	260	250	500	1200	1100	KP70 Ó P38	11,5	2,2	4,0	1,15	4,0	17,5	11
	14					250					12,0	2.3		1,20		19,5	14
	17					300					12,5	3,0		1,25		21,0	17
	20					300					13,5	3,5		1,35		24,0	20
	23					600					14,5	4,0		1,45		27,0	23
	26					600					15,5	4,5		1,55		30,0	26
	29		5000			900					17,5	5,4		1,70		34,8	29
	32					900					18,0	7,0		1,80		40,0	32
15	11	6300	4400	2100	260	250	600	1300	1100	KP70 Ó P43	14,5	3,0	5,3	1,45	5,3	20,0	11
	14					250					15,5	3,0		1,55		22,2	14
	17					250					16,5	3,5		1,65		25,0	17
	20					250					17,5	4,0		1,75		28,0	20
	23					450					18,5	4,5		1,85		31,0	23
	26					450					19,5	5,0		1,95		34,0	26
	29		5000			750					21,0	7,0		2,10		41,0	29
	32					750					22,0	8,0		2,20		45,0	32





Chapter # 7 Timber Structures.

1. Introduction. Basis for design.
2. Design of compression members.
3. Design of flexural members.
4. Columns subject to bending and axial compression.

Reference: Chanakya Arya. "Design of structural elements". Page 328.

1. Introduction.

In Ethiopia around 100 different varieties of trees types are used in timber structures.

Advantages of timber structures:

1. Easy availability.
2. Easy to work on even with simple tools.
3. It has acceptable strength in compression, tension and bending.
4. It is a lightweight material.
5. Has a good resistance to acid and salts
6. Non conductor of electricity.
7. Temperature expansion/contraction is negligible.

Disadvantages:

1. It is inflammable.
2. Insect and piants, termits, fungus and worms deteriorate it.
3. The drying, sawing and other processes on wood to get timber as construction material is time consuming.
4. Moisture reduces the strength and volume of timber.
5. Organic structure changes the quality and volume of timber.
6. Joinary and use of fastenery need due attention and skilled working.

Basis of design.

Limit state principles: The terms ultimate and serviceability limit states apply in the same way as is understood in other limit state codes. Thus ultimate limit states are those associated with collapse, while serviceability limit states correspond to states beyond which specific service criteria are no longer met.

Actions:

G – Permanent actions.

Q – Imposed load. Wind, earthquake, snow loads.

The design values of actions, F_d, are obtained by multiplying the characteristic actions, F_k, by the appropriate partial safety factor γ_F

$$F_d = \gamma_F F_k$$

The partial safety factors for permanent actions, γ_G, and variable actions, γ_Q, states on EBCS 1 shall be used.

For permanent actions $\gamma_G = 1.30$ (unfavourable effect)

$\gamma_G = 1.00$ (favourable effect)

For variable actions $\gamma_Q = 1.60$

Material properties. (strength classes).

European code specifies 15 strength classes. The typical characteristic strength and stiffness values and densities for each are given in table 11.3.
The characteristic strength values given in Table 11.3 are related to a depth in bending and witdth in tension of solid timber of 150 mm. For depth in bending or widths in tension of solid members, h les than 150 mm the characteristic strength may be increased by the factor k_h which is given by:

$$k_h = \left(\frac{150}{h}\right)^{0.2}$$

The characteristic strength, X_k, are converted to design values, X_d, by dividing them by a partial coefficient for material properties, γ_m, and multiplying by a factor k_{mod}. Both factor as follows

$$X_d = k_{mod}\,\frac{X_k}{\gamma_m} \quad \text{(Equation 11.3)}$$

Values for γ_m.

- Ultimate limit states. γ_m
  - Fundamentals combinations: timber – and wood – based materials 1.3
  - Steel used in joints 1.1
  - Accidental combinations 1.0
- Serviceability limits states 1.0

Values of $k_{mod.}$

Loads duration class	service class			order of duration	examples.
	1	2	3		
Permanent	0.60	0.60	0.50	> 10 years	self weight
Long term	0.70	0.70	0.55	6 month – 10 years	imposed storage.
Medium term	0.80	0.80	0.70	1 week – 6 month	imposed occupational loads
Short term	0.90	0.90	0.70	< 1 week	wind, snow.
Instantaneous	1.10	1.10	0.90	accidental impact.	

k_{mod} takes into account the effect on the strength parameters of duration of loading and climatic conditions.

The service classes are:

Service class	moisture contents	typical service conditions
1	≤ 12 %	20^0 C, 65 % R H
2	≤ 20 %	20^0 C, 85 % R H
3	> 20 %	Climatic conditions leading to a higher moisture content than in service class 2.

2. Design of Compression Members.

Members subject to axial compression only should be designed according to the following expression provided there is no tendency for buckling to occur.

$\sigma_{c,o\,d} \le f_{c,o,d}$ where $\sigma_{c,o\,d}$ = N/A; N – Axial factored load and A – cross-sectional area.

$f_{c,o,d}$ is the design compressive strength parallel to the grain obtained from Eq. 11.3.

3. Design of flexural members.

It involves principally:
1. Bending.
2. Deflection.
3. Shear.
4. Bearing.
5. Vibration.
6. Lateral buckling.

Description of methods.

3.1 Bending

If member is not to fail in bending, the following conditions should be satisfied:

$$k_m \frac{\sigma_{m,y,d}}{f_{m,y,d}} + \frac{\sigma_{m,z,d}}{f_{m,z,d}} \le 1$$

$$\frac{\sigma_{m,y,d}}{f_{m,y,d}} + k_m \frac{\sigma_{m,z,d}}{f_{m,z,d}} \le 1$$

Where $\sigma_{m,y,d}$ and $\sigma_{m,z,d}$ are the design bending stresses about axes y-y and z-z.
$f_{m,y,d}$ and $f_{m,z,d}$ are the design strengths from equation 11.3 and k_m the bending factor as follows: For rectangular sections $k_m = 0.7$
For other cross – sections $k_m = 1.0$

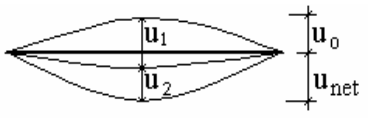
For a beam whit rectangular cross-section:

$$\sigma_{m,y,d} = \frac{M_y}{Z_y} = \frac{M_y}{bh^2/6} \quad and \quad \sigma_{m,z,d} = \frac{M_z}{Z_z} = \frac{M_z}{hb^2/6}$$

M_y and M_z are the design bending moments about axes y-y and z-z and Z_y and Z_z the moduli of elasticity about axes y-y and z-z.

3.2. Deflection.

The components of the deflection are:



u_0 – Precamber (if applied)
u_1 – deflection due to permanent loads.
u_2 – deflection due to variable loads.

Limiting values.

1. Instantaneous deflection due to variable load,
   $u_{2,inst}$, should not exceed:
   $u_{2,inst} \leq 1/300$ x span.
   $u_{2,inst} \leq 1/150$ x span (for cantilever)

2. Final deflection due to variable load only
   $u_{2,fin}$, should not exceed:
   $u_{2,fin} \leq 1/200$ x span.
   $u_{2,fin} \leq 1/100$ x span (for cantilever)

3. Final deflection due to all the loads and any precamber, $u_{net,\,fin}$
   $u_{2,net,fin} \leq 1/200$ x span.
   $u_{2,net,fin} \leq 1/100$ x span (for cantilever).

The instantaneous deflection due to the variable loads, $u_{2,inst,}$ and the final deflection due to the total load, $u_{2,net,fin}$, can be calculated using the formulae given in Table 6.9 and should be based on $E_{0,mean}$ or $E_{90,mean}$. The final deflection due to variable loading, $u_{2,fin}$, is derived from the instantaneous deflection using the following expression:

$$u_{fin} = u_{inst}\left(1 + k_{def}\right)$$

Where k_{def} is the deformation factor which takes into account the increase in deformation with time due to the combined effect of creep and moisture. Values of k_{def} are given as follow.

Load duration class.	Service class		
	1	2	3
Permanent	0.80	0.80	2.00
Long term	0.50	0.50	1.50
Medium term	0.25	0.25	0.75
Short term	0.00	0.00	0.00

3.3 Shear.

In flexural members are not to fail in shear, the following condition should be satisfied:

$$\tau_d \le f_{v,d}$$

where τ_d is the design shear stress and $f_{v,d}$ the design shear strength.
For beam with a rectangular cross-section, the design shear stress occurs at the neutral axis and is given by:

$\tau_d = \frac{3V_d}{2A}$; where V_d is the design shear force and A the cross-sectional area.

$f_{v,d} = \frac{k_{mod} f_{v,k}}{\gamma_m}$; where $f_{v,k}$ is the characteristic shear strength.

For beam notched at the ends as shown in Fig. Below, the following condition should be checked:

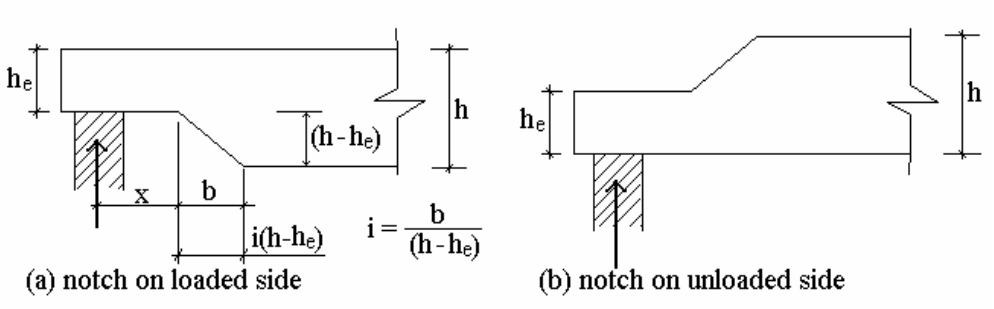
$\tau_d \le k_v f_{v,d}$; where k_v is the shear factor which may attain the following values:

a). For beams notched on the unloaded side $k_v = 1$

b). For beams of solid timber notched on the loaded side k_v is taken as the lesser of $k_v = 1$ and

$$k_v = \frac{5\left(1 + \frac{1.1 i^{1.5}}{\sqrt{h}}\right)}{\sqrt{h}\left[\sqrt{\alpha}(1-\alpha) + 0.8\frac{x}{h}\sqrt{\left(\frac{1}{\alpha} - \alpha^2\right)}\right]},$$

where $\alpha = h_e/h$ and **x** is the distance from line of action to the corner.



3.4 Bearing. (Compression perpendicular to grain).

For compression perpendicular to grain the following condition should be satisfied:

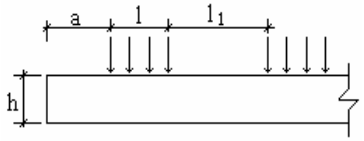
$$\sigma_{c,90,d} \le k_{c,90} f_{c,90,d}$$

where: $\sigma_{c,90,d}$ is the design compressive stress perpendicular to grain

$f_{c,90,d}$ is the design compressive strength perpendicular to grain from equation 11.3

$k_{c,90}$ is the compressive strength factor.

Here $k_{c,90}$ takes into account that the load can increased if the loaded length, l in Fig.belows is short.



Values for $k_{c,90}$ for various combination of a, l and l_1 are given in the following table.

	$l_1 \le 150$ mm	$l_1 > 150$ mm $a \ge 100$ mm	$a < 100$ mm
L $l \ge 150$ mm	1	1	1
$150 > l \ge 15$ mm	1	$1 + (150 - l)/170$	$1 + a(150 - l)/ 17000$
15 mm > l	1	1.8	$1 + a/125$

3.5 **Vibration.** (Applied to residential floors).

The method given in E.C. # 5 assumes that the floor is supported on four edges.
The fundamental frequency of vibration of a rectangular residential floor supported on four edges, f_I can be estimated using:

$f_1 = \frac{\pi}{2l^2} = \sqrt{\frac{(EI)l}{m}}$; where m is the mass equal to the self-weight of the floor and other permanent actions per unit area(kN/m^2)
l is the floor span (m).
(EI) l is the equivalent bending stiffness in the beam direction.
Unit (Nm^2/m).

For residential floors with a fundamental frequency greater than 8 Hz the following conditions should be satisfied:

$u/F \le 1.5 mm/kN \qquad and \qquad \upsilon \le 100^{(f_1 \xi - 1)}$

where ξ is the damping coefficient, normally taken as 0.01
u is the maximum vertical deflection caused by a concentrated static force F = 1 kN
and ν is the unit impulse velocity.
The transverse distribution of load can be taken as 50%, i.e., 0.5 kN on the loaded joist and 25 % on the adjacent ones.
The value of the unit impulse velocity ν may be estimated from:

$\nu = 4(0.4 + 0.6 n_{40}) / (mbl + 200) mN^{-1} S^{-2}$; where b is the floor width (m) and n_{40} the number of first – order modes with natural frequencies below 40 Hz given by.

$n_{40} = \left\{ \left[\left(\frac{40}{f_1} \right)^2 - 1 \right] \left(\frac{b}{l} \right)^4 \frac{(EI)l}{(EI)b} \right\}$; where (EI)l is the equivalent plate bending stiffness parallel to the beam.

3.6. Lateral buckling.

The following condition shall be satisfied.

$\sigma_{m,d} \le k_{inst} f_{m,d}$

where $\sigma_{m,d}$ is the design bending stress
$f_{m,d}$ is the design bending strength
k_{inst} is the instability factor, given by:

$k_{inst} = 1$ *for* $\lambda_{rel,m} \le 0.75$

$k_{inst} = 1.56 - 0.75\lambda_{rel,m}$ *for* $0.75 < \lambda_{rel,m} \le 1.4$

$k_{inst} = 1/\lambda^2_{rel,m}$ *for* $1.4 < \lambda_{rel,m}$

where $\lambda_{rel,m}$ is the relative slenderness ratio for bending.

For beams with rectangular cross-section, $\lambda_{rel,m}$ can be calculated from the following expression:

$\lambda_{rel,m} = \sqrt{\left[\frac{l_{ef}}{\pi b^2}\frac{hf_{m,k}}{E_{0,k05}}\sqrt{\frac{E_{0,mean}}{G_{mean}}}\right]}$ where l_{ef} is the effective length of the beam and is obtained from the figure below.

b is the width of the beam

h is the depth of the beam

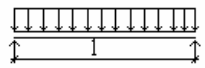
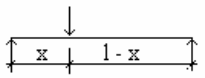
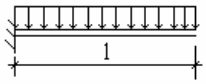
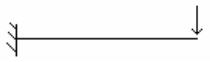
$f_{m,k}$ is the characteristic bending strength (table 11.3)

$E_{0,k05}$ is the characteristic modulus of elasticity parallel to the grain (Table 11.3)

$E_{0,mean}$ is the mean modulus of elasticity parallel to the grain (Table 11.3)

G_{mean} is the mean shear modulus = $E_{0,mean}/16$.

Values of ratio l_{ef}/l

	The load is acting at the		
	Top	mid-depth	botton
	0.95	0.90	0.85
	0.8	0.75α $\alpha = 1.35 - 1.4\frac{x}{l}\frac{l-x}{l}$	0.7α
		1	
		0.6	
		0.85	

4. Column subjects to bending and compression.

Eurocodes gives two sets of conditions for designing columns resisting combined bending and axial compression. Provided that the relative slenderness ratios about both the y-y and z-z axes of the column; $\lambda_{rel,y}$ and $\lambda_{rel,z}$ respectively, are not greater than 0.5, i.e. $\lambda_{rel,y} \le 0.5$ and $\lambda_{rel,z} \le 0.5$. The suitability of the design can be assumed using the more stringent of the following condition:

$$\left(\frac{\sigma_{c,0,d}}{f_{c,0,d}}\right)^2 + \frac{\sigma_{m,y,d}}{f_{m,y,d}} + k_m \frac{\sigma_{m,z,d}}{f_{m,z,d}} \le 1$$

$$\left(\frac{\sigma_{c,0,d}}{f_{c,0,d}}\right)^2 + k_m \frac{\sigma_{m,y,d}}{f_{m,y,d}} + \frac{\sigma_{m,z,d}}{f_{m,z,d}} \le 1$$

In all other cases the stress should satisfy the more stringent of the following conditions:

$$\left(\frac{\sigma_{c,0,d}}{k_{c,y} f_{c,0,d}}\right) + \frac{\sigma_{m,y,d}}{f_{m,y,d}} + k_m \frac{\sigma_{m,z,d}}{f_{m,z,d}} \le 1$$

$$\left(\frac{\sigma_{c,0,d}}{k_{c,z} f_{c,0,d}}\right) + k_m \frac{\sigma_{m,y,d}}{f_{m,y,d}} + \frac{\sigma_{m,z,d}}{f_{m,z,d}} \le 1$$

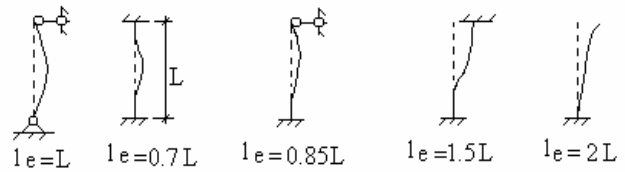
Where $\sigma_{c,0,d}$ is the design compressive stress = N/A

$f_{c,0,d}$ is the the design compressive strength (Equation 11.3)

k_m = 0.7 for rectangular sections and 1.0 for other cross-sections.

$$\lambda_{rel,y} = \sqrt{\left(\frac{f_{c,0,k}}{\sigma_{c,crit,y}}\right)} \quad and \quad \lambda_{rel,z} = \sqrt{\left(\frac{f_{c,0,k}}{\sigma_{c,crit,z}}\right)} \quad where \quad \sigma_{c,crit} = \frac{\pi^2 E_{0.05}}{\lambda^2} \quad and \quad \lambda = \frac{l_{ef}}{i}$$

Values of l_{ef}



σ_m is the bending stress due to any lateral or eccentric loads.

$$k_c = \frac{1}{k + \sqrt{k^2 - \lambda_{rel}^2}} \quad where \quad k = 0.5\left(1 + \beta_c\left(\lambda_{rel} - 0.5\right) + \lambda_{rel}^2\right)$$ and β_c = 0.2 for solid timber.

Chapter # 7 Timber Structures.

1. Introduction. Basis for design.
2. Design of compression members.
3. Design of flexural members.
4. Columns subject to bending and axial compression.

Reference: Chanakya Arya. "Design of structural elements". Page 328.

1. Introduction.

In Ethiopia around 100 different varieties of trees types are used in timber structures.

Advantages of timber structures:

1. Easy availability.
2. Easy to work on even with simple tools.
3. It has acceptable strength in compression, tension and bending.
4. It is a lightweight material.
5. Has a good resistance to acid and salts
6. Non conductor of electricity.
7. Temperature expansion/contraction is negligible.

Disadvantages:

1. It is inflammable.
2. Insect and piants, termits, fungus and worms deteriorate it.
3. The drying, sawing and other processes on wood to get timber as construction material is time consuming.
4. Moisture reduces the strength and volume of timber.
5. Organic structure changes the quality and volume of timber.
6. Joinary and use of fastenery need due attention and skilled working.

Basis of design.

Limit state principles: The terms ultimate and serviceability limit states apply in the same way as is understood in other limit state codes. Thus ultimate limit states are those associated with collapse, while serviceability limit states correspond to states beyond which specific service criteria are no longer met.

Actions:

G – Permanent actions.

Q – Imposed load. Wind, earthquake, snow loads.

The design values of actions, F_d, are obtained by multiplying the characteristic actions, F_k, by the appropriate partial safety factor γ_F

$$F_d = \gamma_F F_k$$

The partial safety factors for permanent actions, γ_G, and variable actions, γ_Q, states on EBCS 1 shall be used.

For permanent actions $\gamma_G = 1.30$ (unfavourable effect)

$\gamma_G = 1.00$ (favourable effect)

For variable actions $\gamma_Q = 1.60$

Material properties. (strength classes).

European code specifies 15 strength classes. The typical characteristic strength and stiffness values and densities for each are given in table 11.3.
The characteristic strength values given in Table 11.3 are related to a depth in bending and witdth in tension of solid timber of 150 mm. For depth in bending or widths in tension of solid members, h les than 150 mm the characteristic strength may be increased by the factor k_h which is given by:

$$k_h = \left(\frac{150}{h}\right)^{0.2}$$

The characteristic strength, X_k, are converted to design values, X_d, by dividing them by a partial coefficient for material properties, γ_m, and multiplying by a factor k_{mod}. Both factor as follows

$$X_d = k_{mod}\,\frac{X_k}{\gamma_m} \quad \text{(Equation 11.3)}$$

Values for γ_m.

- Ultimate limit states. γ_m
  - Fundamentals combinations: timber – and wood – based materials 1.3
  - Steel used in joints 1.1
  - Accidental combinations 1.0
- Serviceability limits states 1.0

Values of $k_{mod.}$

Loads duration class	service class			order of duration	examples.
	1	2	3		
Permanent	0.60	0.60	0.50	> 10 years	self weight
Long term	0.70	0.70	0.55	6 month – 10 years	imposed storage.
Medium term	0.80	0.80	0.70	1 week – 6 month	imposed occupational loads
Short term	0.90	0.90	0.70	< 1 week	wind, snow.
Instantaneous	1.10	1.10	0.90	accidental impact.	

k_{mod} takes into account the effect on the strength parameters of duration of loading and climatic conditions.

The service classes are:

Service class	moisture contents	typical service conditions
1	≤ 12 %	20^0 C, 65 % R H
2	≤ 20 %	20^0 C, 85 % R H
3	> 20 %	Climatic conditions leading to a higher moisture content than in service class 2.

2. Design of Compression Members.

Members subject to axial compression only should be designed according to the following expression provided there is no tendency for buckling to occur.

$\sigma_{c,o\,d} \le f_{c,o,d}$ where $\sigma_{c,o\,d}$ = N/A; N – Axial factored load and A – cross-sectional area.

$f_{c,o,d}$ is the design compressive strength parallel to the grain obtained from Eq. 11.3.

3. Design of flexural members.

It involves principally:
1. Bending.
2. Deflection.
3. Shear.
4. Bearing.
5. Vibration.
6. Lateral buckling.

Description of methods.

3.1 Bending

If member is not to fail in bending, the following conditions should be satisfied:

$$k_m \frac{\sigma_{m,y,d}}{f_{m,y,d}} + \frac{\sigma_{m,z,d}}{f_{m,z,d}} \le 1$$

$$\frac{\sigma_{m,y,d}}{f_{m,y,d}} + k_m \frac{\sigma_{m,z,d}}{f_{m,z,d}} \le 1$$

Where $\sigma_{m,y,d}$ and $\sigma_{m,z,d}$ are the design bending stresses about axes y-y and z-z.
$f_{m,y,d}$ and $f_{m,z,d}$ are the design strengths from equation 11.3 and k_m the bending factor as follows: For rectangular sections $k_m = 0.7$
For other cross – sections $k_m = 1.0$

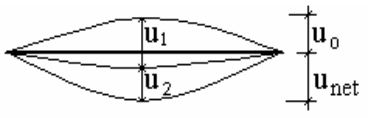
For a beam whit rectangular cross-section:

$$\sigma_{m,y,d} = \frac{M_y}{Z_y} = \frac{M_y}{bh^2/6} \quad and \quad \sigma_{m,z,d} = \frac{M_z}{Z_z} = \frac{M_z}{hb^2/6}$$

M_y and M_z are the design bending moments about axes y-y and z-z and Z_y and Z_z the moduli of elasticity about axes y-y and z-z.

3.2. Deflection.

The components of the deflection are:



u_0 – Precamber (if applied)
u_1 – deflection due to permanent loads.
u_2 – deflection due to variable loads.

Limiting values.

1. Instantaneous deflection due to variable load,
   $u_{2,inst}$, should not exceed:
   $u_{2,inst} \leq 1/300$ x span.
   $u_{2,inst} \leq 1/150$ x span (for cantilever)

2. Final deflection due to variable load only
   $u_{2,fin}$, should not exceed:
   $u_{2,fin} \leq 1/200$ x span.
   $u_{2,fin} \leq 1/100$ x span (for cantilever)

3. Final deflection due to all the loads and any precamber, $u_{net,\,fin}$
   $u_{2,net,fin} \leq 1/200$ x span.
   $u_{2,net,fin} \leq 1/100$ x span (for cantilever).

The instantaneous deflection due to the variable loads, $u_{2,inst,}$ and the final deflection due to the total load, $u_{2,net,fin}$, can be calculated using the formulae given in Table 6.9 and should be based on $E_{0,mean}$ or $E_{90,mean}$. The final deflection due to variable loading, $u_{2,fin}$, is derived from the instantaneous deflection using the following expression:

$$u_{fin} = u_{inst}\left(1 + k_{def}\right)$$

Where k_{def} is the deformation factor which takes into account the increase in deformation with time due to the combined effect of creep and moisture. Values of k_{def} are given as follow.

Load duration class.	Service class		
	1	2	3
Permanent	0.80	0.80	2.00
Long term	0.50	0.50	1.50
Medium term	0.25	0.25	0.75
Short term	0.00	0.00	0.00

3.3 Shear.

In flexural members are not to fail in shear, the following condition should be satisfied:

$$\tau_d \le f_{v,d}$$

where τ_d is the design shear stress and $f_{v,d}$ the design shear strength.
For beam with a rectangular cross-section, the design shear stress occurs at the neutral axis and is given by:

$\tau_d = \frac{3V_d}{2A}$; where V_d is the design shear force and A the cross-sectional area.

$f_{v,d} = \frac{k_{mod} f_{v,k}}{\gamma_m}$; where $f_{v,k}$ is the characteristic shear strength.

For beam notched at the ends as shown in Fig. Below, the following condition should be checked:

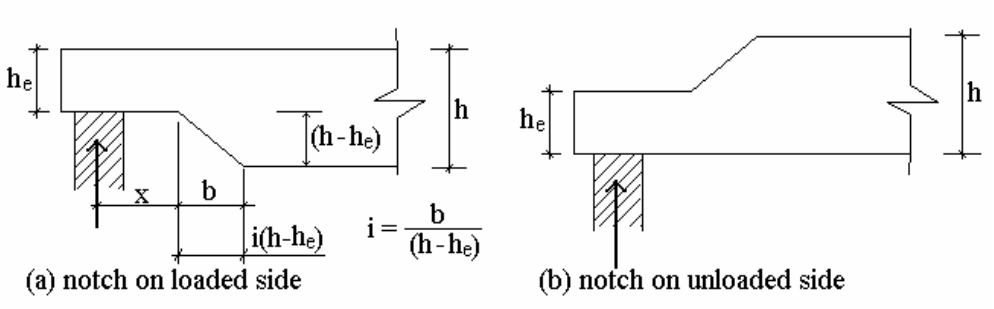
$\tau_d \le k_v f_{v,d}$; where k_v is the shear factor which may attain the following values:

a). For beams notched on the unloaded side $k_v = 1$

b). For beams of solid timber notched on the loaded side k_v is taken as the lesser of $k_v = 1$ and

$$k_v = \frac{5\left(1 + \frac{1.1 i^{1.5}}{\sqrt{h}}\right)}{\sqrt{h}\left[\sqrt{\alpha}(1-\alpha) + 0.8\frac{x}{h}\sqrt{\left(\frac{1}{\alpha} - \alpha^2\right)}\right]},$$

where $\alpha = h_e/h$ and **x** is the distance from line of action to the corner.



3.4 Bearing. (Compression perpendicular to grain).

For compression perpendicular to grain the following condition should be satisfied:

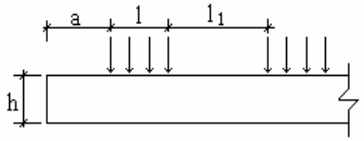
$$\sigma_{c,90,d} \le k_{c,90} f_{c,90,d}$$

where: $\sigma_{c,90,d}$ is the design compressive stress perpendicular to grain

$f_{c,90,d}$ is the design compressive strength perpendicular to grain from equation 11.3

$k_{c,90}$ is the compressive strength factor.

Here $k_{c,90}$ takes into account that the load can increased if the loaded length, l in Fig.belows is short.



Values for $k_{c,90}$ for various combination of a, l and l_1 are given in the following table.

	$l_1 \le 150$ mm	$l_1 > 150$ mm $a \ge 100$ mm	$a < 100$ mm
L $l \ge 150$ mm	1	1	1
$150 > l \ge 15$ mm	1	$1 + (150 - l)/170$	$1 + a(150 - l)/ 17000$
15 mm > l	1	1.8	$1 + a/125$

3.5 **Vibration.** (Applied to residential floors).

The method given in E.C. # 5 assumes that the floor is supported on four edges.
The fundamental frequency of vibration of a rectangular residential floor supported on four edges, f_I can be estimated using:

$f_1 = \frac{\pi}{2l^2} = \sqrt{\frac{(EI)l}{m}}$; where m is the mass equal to the self-weight of the floor and other permanent actions per unit area(kN/m^2)
l is the floor span (m).
(EI) l is the equivalent bending stiffness in the beam direction.
Unit (Nm^2/m).

For residential floors with a fundamental frequency greater than 8 Hz the following conditions should be satisfied:

$u/F \le 1.5 mm/kN \qquad and \qquad \upsilon \le 100^{(f_1 \xi - 1)}$

where ξ is the damping coefficient, normally taken as 0.01
u is the maximum vertical deflection caused by a concentrated static force F = 1 kN
and ν is the unit impulse velocity.
The transverse distribution of load can be taken as 50%, i.e., 0.5 kN on the loaded joist and 25 % on the adjacent ones.
The value of the unit impulse velocity ν may be estimated from:

$\nu = 4(0.4 + 0.6 n_{40}) / (mbl + 200) mN^{-1} S^{-2}$; where b is the floor width (m) and n_{40} the number of first – order modes with natural frequencies below 40 Hz given by.

$n_{40} = \left\{ \left[\left(\frac{40}{f_1} \right)^2 - 1 \right] \left(\frac{b}{l} \right)^4 \frac{(EI)l}{(EI)b} \right\}$; where (EI)l is the equivalent plate bending stiffness parallel to the beam.

3.6. Lateral buckling.

The following condition shall be satisfied.

$\sigma_{m,d} \le k_{inst} f_{m,d}$

where $\sigma_{m,d}$ is the design bending stress
$f_{m,d}$ is the design bending strength
k_{inst} is the instability factor, given by:

$k_{inst} = 1$ *for* $\lambda_{rel,m} \le 0.75$

$k_{inst} = 1.56 - 0.75\lambda_{rel,m}$ *for* $0.75 < \lambda_{rel,m} \le 1.4$

$k_{inst} = 1/\lambda^2_{rel,m}$ *for* $1.4 < \lambda_{rel,m}$

where $\lambda_{rel,m}$ is the relative slenderness ratio for bending.

For beams with rectangular cross-section, $\lambda_{rel,m}$ can be calculated from the following expression:

$\lambda_{rel,m} = \sqrt{\left[\frac{l_{ef}}{\pi b^2}\frac{hf_{m,k}}{E_{0,k05}}\sqrt{\frac{E_{0,mean}}{G_{mean}}}\right]}$ where l_{ef} is the effective length of the beam and is obtained from the figure below.

b is the width of the beam

h is the depth of the beam

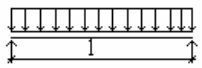
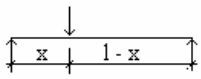
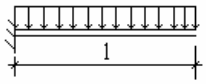
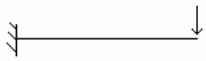
$f_{m,k}$ is the characteristic bending strength (table 11.3)

$E_{0,k05}$ is the characteristic modulus of elasticity parallel to the grain (Table 11.3)

$E_{0,mean}$ is the mean modulus of elasticity parallel to the grain (Table 11.3)

G_{mean} is the mean shear modulus = $E_{0,mean}/16$.

Values of ratio l_{ef}/l

	The load is acting at the		
	Top	mid-depth	botton
	0.95	0.90	0.85
	0.8	0.75α $\alpha = 1.35 - 1.4\frac{x}{l}\frac{l-x}{l}$	0.7α
		1	
		0.6	
		0.85	

4. Column subjects to bending and compression.

Eurocodes gives two sets of conditions for designing columns resisting combined bending and axial compression. Provided that the relative slenderness ratios about both the y-y and z-z axes of the column; $\lambda_{rel,y}$ and $\lambda_{rel,z}$ respectively, are not greater than 0.5, i.e. $\lambda_{rel,y} \le 0.5$ and $\lambda_{rel,z} \le 0.5$. The suitability of the design can be assumed using the more stringent of the following condition:

$$\left(\frac{\sigma_{c,0,d}}{f_{c,0,d}}\right)^2 + \frac{\sigma_{m,y,d}}{f_{m,y,d}} + k_m \frac{\sigma_{m,z,d}}{f_{m,z,d}} \le 1$$

$$\left(\frac{\sigma_{c,0,d}}{f_{c,0,d}}\right)^2 + k_m \frac{\sigma_{m,y,d}}{f_{m,y,d}} + \frac{\sigma_{m,z,d}}{f_{m,z,d}} \le 1$$

In all other cases the stress should satisfy the more stringent of the following conditions:

$$\left(\frac{\sigma_{c,0,d}}{k_{c,y} f_{c,0,d}}\right) + \frac{\sigma_{m,y,d}}{f_{m,y,d}} + k_m \frac{\sigma_{m,z,d}}{f_{m,z,d}} \le 1$$

$$\left(\frac{\sigma_{c,0,d}}{k_{c,z} f_{c,0,d}}\right) + k_m \frac{\sigma_{m,y,d}}{f_{m,y,d}} + \frac{\sigma_{m,z,d}}{f_{m,z,d}} \le 1$$

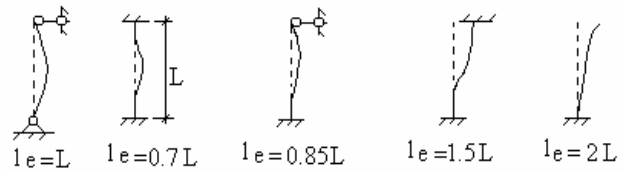
Where $\sigma_{c,0,d}$ is the design compressive stress = N/A

$f_{c,0,d}$ is the the design compressive strength (Equation 11.3)

k_m = 0.7 for rectangular sections and 1.0 for other cross-sections.

$$\lambda_{rel,y} = \sqrt{\left(\frac{f_{c,0,k}}{\sigma_{c,crit,y}}\right)} \quad and \quad \lambda_{rel,z} = \sqrt{\left(\frac{f_{c,0,k}}{\sigma_{c,crit,z}}\right)} \quad where \quad \sigma_{c,crit} = \frac{\pi^2 E_{0.05}}{\lambda^2} \quad and \quad \lambda = \frac{l_{ef}}{i}$$

Values of l_{ef}



σ_m is the bending stress due to any lateral or eccentric loads.

$$k_c = \frac{1}{k + \sqrt{k^2 - \lambda_{rel}^2}} \quad where \quad k = 0.5\left(1 + \beta_c\left(\lambda_{rel} - 0.5\right) + \lambda_{rel}^2\right)$$ and β_c = 0.2 for solid timber.