

Hypothesis Testing

Chapter 8

Introduction

- Researchers interesting in many types of questions
 - Earth warming
 - Medication effect on blood pressure
 - Seat belts and accidents
- Are differences in means of populations and means of samples real, or by chance?

Introduction cont.

- **Hypothesis testing**
 - Decision-making process for evaluating claims about a population
- In hypothesis testing, researcher must:
 - Define population under study
 - State particular hypothesis to be investigated
 - Give significance level
 - Select sample from population
 - Collect data
 - Perform calculations required
 - Reach a conclusion

Two Tests & Three Methods

- Two specific statistical tests are used for hypotheses concerning means:
 - *z test*
 - *t test*
- Three methods used to test hypotheses:
 - Traditional method
 - P-value method
 - Confidence interval method

8.1 - Steps in Hypothesis Testing (Traditional Method)

- **Every hypothesis-testing situation begins with statement of hypothesis**
- **Statistical hypothesis**
 - Conjecture about a population parameter
 - Conjecture may or may not be true

Two Types of Hypotheses

- **Null hypothesis**
 - Symbolized by H_0
 - Statistical hypothesis that states there is no difference between a parameter and a specific value, or there is no difference between two parameters
- **Alternative hypothesis (research hypothesis)**
 - Symbolized by H_1
 - Statistical hypothesis that states existence of difference between a parameter and specific value, or states there is difference between two parameters

Situations

- Situation A
 - $H_0: \mu = 82$ *and* $H_1: \mu \neq 82$
- Situation B
 - $H_0: \mu = 36$ *and* $H_1: \mu > 36$
- Situation C
 - $H_0: \mu = \$78$ *and* $H_1: \mu < \$78$

Left, Right, Two-Tailed Tests

- Null and alternative hypotheses are stated together, and null hypothesis contains the equals sign (where k represents a specified number)

Two-Tailed Test	Right-tailed test	Left-tailed test
$H_0: \mu = k$	$H_0: \mu = k$	$H_0: \mu = k$
$H_1: \mu \neq k$	$H_1: \mu > k$	$H_1: \mu < k$

Support or Reject?

- Claim can be stated as either null or alternative hypothesis
 - Statistical evidence can only *support* claim if it is the alternative hypothesis
 - Statistical evidence can be used to *reject* claim if it is the null hypothesis
- Facts are important when stating conclusion of a statistical study

Example 8 - 1

- State the null and alternative hypotheses for each conjecture
 - a) A researcher thinks that if expectant mothers use vitamin pills, the birth weight of the babies will increase. The average birth weight of the population is 8.6 pounds
 - b) An engineer hypothesizes that the mean number of defects can be decreased in a manufacturing process of compact disks by using robots instead of humans for certain tasks. The mean number of defective disk per 1000 is 18
 - c) A psychologist feels that playing soft music during a test will change the results of the test. The psychologist is not sure whether the grades will be higher or lower. In the past, the mean of the scores was 73

Designing the Study

- After stating hypothesis, researcher designs study
- Selects correct statistical test, then chooses appropriate level of significance



Making Decisions

- Remember: distribution of sample means will be approximately normal when sample size is 30 or more
- Farther away sample mean is from population mean, more evidence exists for rejecting null hypothesis
- Where does researcher draw the line?
 - STATISTICS is the answer!

Test & Errors

- **Statistical test**
 - Uses data obtained from a sample to make a decision about whether null hypothesis should be rejected
- **Test value**
 - Numerical value obtained from a statistical test
- In this type of statistical test, mean is computed for data obtained from sample and is compared with population mean

Four Outcomes

- In hypothesis testing, there are four possible outcomes
 - Figure 8-2 page 404

Errors can occur...

- **Type I error**
 - Occurs if you reject null hypothesis when it is true
- **Type II error**
 - Occurs if you do not reject null hypothesis when it is false

Using Probability

- *The only way to prove anything statistically is to use entire population*
- Most cases not possible, so probabilities are used
- **Level of significance**
 - Maximum probability of committing a type I error
 - Symbolized by Greek letter α , $P(\text{type I error}) = \alpha$
 - Type II error symbolized by Greek letter β

Critical Values

- **Critical value**
 - Separates critical region from noncritical region
 - Symbol is C.V.
- **Critical (rejection) region**
 - Range of values of test value that indicates there is significant difference and null hypothesis should be rejected
- **Noncritical (nonrejection) region**
 - Range of values of test value that indicates difference was probably due to chance and null hypothesis should not be rejected

Tests

- Critical value can be on right side of mean or on left side of mean for one-tailed test
- **One-tailed test**
 - Indicates null hypothesis should be rejected when test value is in critical region on one side of the mean (either right-tailed or left-tailed) depending on direction of inequality of alternative hypothesis
- **Two-tailed test**
 - Null hypothesis should be rejected when test value is in either of the two critical regions

Common Critical Values

Left-tailed: $H_0: \mu = k, H_1: \mu < k$

$$\begin{cases} a = 0.10, C.V. = -1.28 \\ a = 0.05, C.V. = -1.65 \\ a = 0.01, C.V. = -2.33 \end{cases}$$

Right-tailed: $H_0: \mu = k, H_1: \mu > k$

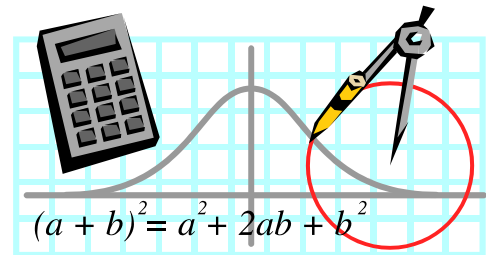
$$\begin{cases} a = 0.10, C.V. = +1.28 \\ a = 0.05, C.V. = +1.65 \\ a = 0.01, C.V. = +2.33 \end{cases}$$

Two-tailed: $H_0: \mu = k, H_1: \mu \neq k$

$$\begin{cases} a = 0.10, C.V. = \pm 1.65 \\ a = 0.05, C.V. = \pm 1.96 \\ a = 0.01, C.V. = \pm 2.58 \end{cases}$$

Example 8 - 2

- Using Table E, find critical value(s) for each situation and draw the appropriate figure, showing the critical region
 - a. Left-tailed test with $\alpha = 0.10$
 - b. Two-tailed test with $\alpha = 0.02$
 - c. Right-tailed test with $\alpha = 0.005$

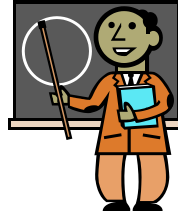


8.2 - z Test for a Mean

- Two statistical tests are used
 - Z test when σ is known
 - T test when σ is unknown
- **Z test**
 - Statistical test for the mean of a population
 - Used when $n \geq 30$, or when population is normally distributed and σ is *known*

$$z = \frac{\bar{X} - \mu}{\sigma / \sqrt{n}}$$

Examples



- 8 – 3
 - A researcher reports that the average salary of assistant professors is more than \$42,000. A sample of 30 assistant professors has a mean salary of \$43,260. At $\alpha = 0.05$, test the claim that assistant professors earn more than \$42,000 per year. The standard deviation of the population is \$5230.

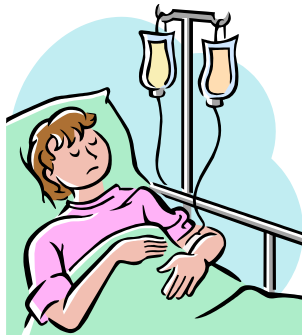
Example 8 - 4

- A researcher claims that the average cost of men's athletic shoes is less than \$80. He selects a random sample of 36 pairs of shoes from a catalog and finds the following costs (in dollars). Is there enough evidence to support the researcher's claim at $\alpha = 0.10$? Assume $\sigma = 19.2$

60	50	120	110	75	110	70
40	90	65	60	85	75	80
75	80	90	45	55	70	85
85	90	90	80	50	80	85
60	70	55	95	60	45	95
70						

Example 8 - 5

- The Medical Rehabilitation Education Foundation reports that the average cost of rehabilitation for stroke victims is \$24,672. To see if the average cost of rehabilitation is different at a particular hospital, a researcher selects a random sample of 35 stroke victims at the hospital and finds that the average cost of their rehabilitation is \$25,226. The standard deviation of the population is \$3251. At $\alpha=0.01$, can it be concluded that the average cost of stroke rehabilitation at a particular hospital is different from \$24,672?



P-Value Method for Hypothesis Testing

- **P-value (probability value)**
 - Probability of getting a sample statistic (such as the mean) or a more extreme sample statistic in the direction of the alternative hypothesis when the null hypothesis is true
 - If P-value is less than α , then reject null hypothesis
 - If P-value is greater than α , then do not reject null hypothesis
- P-values for the z test can be found by using table E

Procedure for P-value Method

- Solving Hypothesis-Testing Problems (P-value)
 - 1) State the hypothesis and identify the claim
 - 2) Compute the test value
 - 3) Find the P-value
 - 4) Make the decision
 - 5) Summarize the results

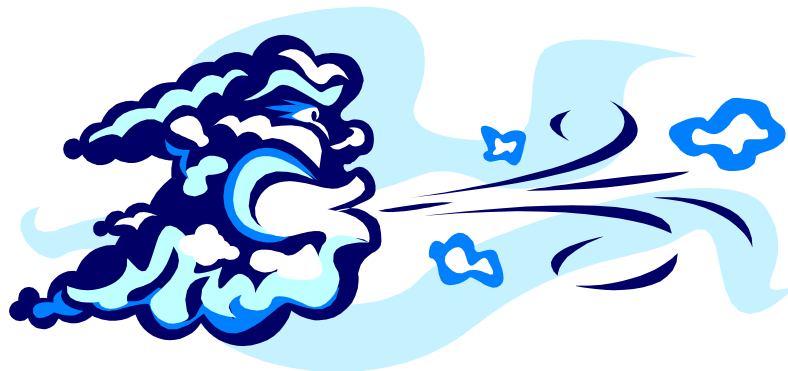
Example 8 - 6

- A researcher wishes to test the claim that the average cost of tuition and fees at a four-year public college is greater than \$5700. She selects a random sample of 36 four-year public colleges and finds the mean to be \$5950. The population standard deviation is \$659. Is there evidence to support the claim at $\alpha = 0.05$? Use the P-value method.



Example 8 - 7

- A researcher claims that the average wind speed in a certain city is 8 miles per hour. A sample of 32 days has an average wind speed of 8.2 miles per hour. The standard deviation of the population is 0.6 miles per hour. At $\alpha = 0.05$, is there enough evidence to reject the claim? Use the P-value method



Decision Rule & Guidelines for P-values

- If $P\text{-value} \leq \alpha$, reject null hypothesis
- If $P\text{-value} > \alpha$, do not reject null hypothesis
- **Guidelines for P-values**
 - If $P\text{-value} \leq 0.01$, reject null hypothesis. Difference is highly significant
 - If $P\text{-value} > 0.01$ but ≤ 0.05 , reject null hypothesis. Difference is significant
 - If $P\text{-value} > 0.05$ but ≤ 0.10 , consider consequences of type I error before rejecting null hypothesis
 - If $P\text{-value} > 0.10$, do not reject null hypothesis. Difference is not significant
- **Additional Note:**
 - Researcher should distinguish between *statistical significance* and *practical significance*

8.3 - t Test for a Mean

- When population standard deviation is unknown, z test is not used: t test is used instead
- Ways t distribution is similar to normal distribution
 1. Bell-shaped
 2. Symmetric about the mean
 3. Mean, median, mode are equal to 0 and located at center of distribution
 4. Curve never touches x-axis
- Ways t distribution differs from normal distribution
 1. Variance is greater than 1
 2. Family of curves based on degrees of freedom
 3. As sample size increases, t distribution approaches normal distribution

t Test

- **t test**
 - Statistical test for mean of a population
 - Used when population is normally or approximately normally distributed
 - σ is unknown (population standard deviation)

$$t = \frac{\bar{X} - \mu}{s / \sqrt{n}}$$

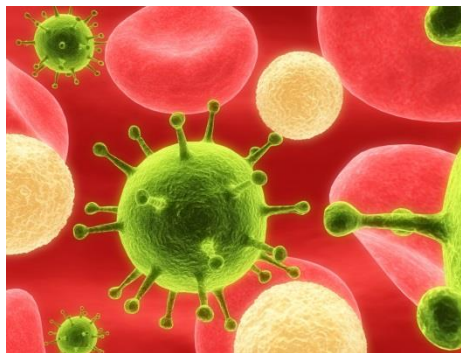
(degrees of freedom are d.f. = $n - 1$)

Finding Critical t Values

- Example 8 – 8
 - Find critical t value for $\alpha = 0.05$ with d.f. = 16
 - Right-tailed t test
- Example 8 – 9
 - Find critical t value for $\alpha = 0.01$ with d.f. = 22
 - Left-tailed t test
- Example 8 – 10
 - Find critical values for $\alpha = 0.10$ with d.f. = 18
 - Two-tailed t test
- Example 8 – 11
 - Find critical value for $\alpha = 0.05$ with d.f. = 28
 - Right-tailed t test

Example 8 - 12

- A medical investigation claims that the average number of infections per week at a hospital in southwestern Pennsylvania is 16.3. A random sample of 10 weeks had a mean number of 17.7 infections. The sample standard deviation is 1.8. Is there enough evidence to reject the investigator's claim at $\alpha = 0.05$?



Example 8 - 13

- An educator claims that the average salary of substitute teachers in school districts in Allegheny County, Pennsylvania is less than \$60 per day. A random sample of eight school districts is selected, and the daily salaries are shown. Is there enough evidence to support the educator's claim at $\alpha = 0.10$?

60 56 60 55 70 55 60 55



Using P-values

- P-values for the t test can be found by using Table F, but specific P-values cannot be found so only intervals can be found for P-values
- Example 8 – 14
 - Find P-value when t test is 2.056, sample size is 11, and test is right-tailed
- Example 8 – 15
 - find P-value when t test value is 2.983, sample size is 6, and test is two-tailed

Example 8 - 16

- A physician claims that joggers' maximal volume oxygen uptake is greater than the average of all adults. A sample of 15 joggers has a mean of 40.6 milliliters per kilogram (ml/kg) and a standard deviation of 6 ml/kg. If the average of all adults is 36.7 ml/kg, is there enough evidence to support the physician's claim at $\alpha = 0.05$?



8.4 - z Test for a Proportion

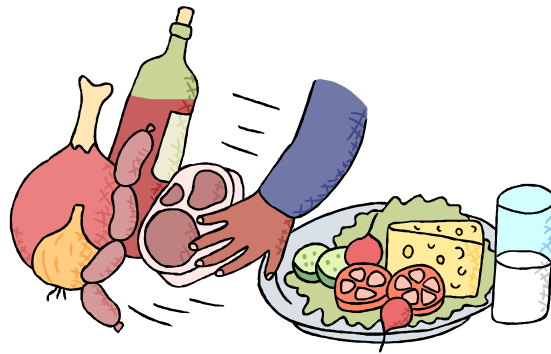
- Many hypothesis-testing situations involve proportions
- Hypothesis test involving a proportion can be considered as a binomial experiment when there are only two outcomes & probability of success does not change from trial to trial
- When $np \geq 5$ and $nq \geq 5$, standard normal distribution can be used to test hypotheses for proportions

Formula for z Test for Proportions

- $z = \frac{\hat{p} - p}{\sqrt{pq/n}}$ where
 - $\hat{p} = \frac{X}{n}$ (sample proportion)
 - $p = \text{population proportion}$
 - $n = \text{sample size}$
- *Sometimes it is necessary to find $\hat{p}$, and sometimes $\hat{p}$ is given in the exercise*

Example 8 - 17

- A dietician claims that 60% of people are trying to avoid trans fats in their diets. She randomly selected 200 people and found that 128 people stated that they were trying to avoid trans fats in their diets. At $\alpha = 0.05$, is there enough evidence to reject the dietician's claim?



Example 8 - 18

- A telephone company representative estimates that 40% of its customers have call-waiting service. To test this hypothesis, she selected a sample of 100 customers and found that 37% had call waiting. At $\alpha = 0.01$, is there enough evidence to reject the claim?



Example 8 - 19

- A statistician read that at least 77% of the population oppose replacing \$1 bills with \$1 coins. To see if this claim is valid, the statistician selected a sample of 80 people and found that 55 were opposed to the idea. At $\alpha = 0.01$, test the claim that at least 77% of the population are opposed to the change.



Example 8 - 20

- An attorney claims that more than 25% of all lawyers advertise. A sample of 200 lawyers in a certain city showed that 63 had used some form of advertising. At $\alpha = 0.05$, is there enough evidence to support the attorney's claim? Use the P-value method to solve.



8.5 - χ^2 Test for Variance or Standard Deviation

- Chi-square distribution was used to construct confidence interval for a single variance or standard deviation in chapter 7
- Chi-square distribution also used to test a claim about a single variance or standard deviation
- Use table G to find area under chi-square distribution
- **Three cases to consider:**
 1. Finding the chi-square critical value for a specific α when hypothesis test is right-tailed
 2. “...” is left-tailed
 3. “...” is two-tailed

Finding Critical Chi-Square Values

- 8 – 21
 - Find critical chi-square value for 15 degrees of freedom when $\alpha = 0.05$
 - Test is right-tailed
- 8 – 22
 - Find critical chi-square value for 10 degrees of freedom when $\alpha = 0.05$
 - Test is left-tailed
- 8 – 23
 - Find critical chi-square value for 22 degrees of freedom when $\alpha = 0.05$
 - Test is two-tailed

Chi-square Test

- When exact degrees of freedom sought are not specified in the table, use the closest smaller value (example: given d.f. = 36, use d.f. = 30)
- When testing a claim about a single variance using **chi-square test**, there are three possible test situations:
 - Right-tailed, left-tailed, & two-tailed

Formula for Chi-Square Test for a Single Variance

- $\chi^2 = \frac{(n-1)s^2}{\sigma^2}$

with degrees of freedom equal to $n - 1$ and where:

$n = \text{sample size}$

$s^2 = \text{sample variance}$

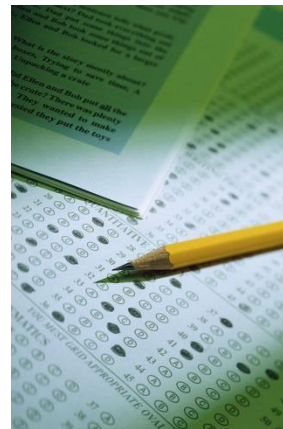
$\sigma^2 = \text{population variance}$

Assumptions for Chi-Square Test for a Single Variance

- Three assumptions are made:
 1. Sample must be randomly selected from the population
 2. Population must be normally distributed for the variable under study
 3. Observation must be independent of one another

Example 8 - 24

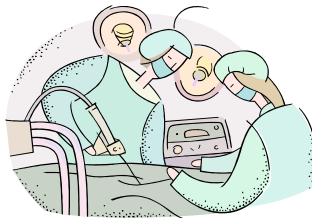
- An instructor wishes to see whether the variation in scores of the 23 students in her class is less than the variance of the population. The variance of the class is 198. Is there enough evidence to support the claim that the variation of the students is less than the population variance ($\sigma^2 = 225$) at $\alpha = 0.05$? Assume that the scores are normally distributed.



Example 8 - 25

- A hospital administrator believes that the standard deviation of the number of people using outpatient surgery per day is greater than 8. A random sample of 15 days is selected. The data are shown. At $\alpha = 0.10$, is there enough evidence to support the administrator's claim? Assume the variable is normally distributed.

25	30	5	15	18	42	16	9
10	12	12	38	8	14	27	



Example 8 - 26

- A cigarette manufacturer wishes to test the claim that the variance of the nicotine content of its cigarettes is 0.644. Nicotine content is measured in milligrams, and assume that it is normally distributed. A sample of 20 cigarettes has a standard deviation of 1.00 milligram. At $\alpha = 0.05$, is there enough evidence to reject the manufacturer's claim?

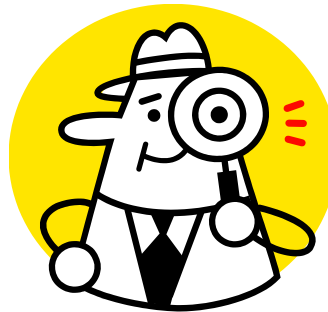


P-values for Chi-Square Test

- Approximate P-values for chi-square test can be found using table G
- Intervals will be determined for these tests
- Example 8 – 27
 - Find P-value when $\chi^2 = 19.274$ and $n = 8$
 - Test is right-tailed
- Example 8 – 28
 - Find P-value when $\chi^2 = 3.823$ and $n = 13$
 - Test is left-tailed

Two-tailed Test

- When chi-square test is two-tailed, both interval values must be doubled
- Example 8 – 29
 - A researcher knows from past studies that the standard deviation of the time it takes to inspect a car is 16.8 minutes. A sample of 24 cars is selected and inspected. The standard deviation is 12.5 minutes. At $\alpha = 0.05$, can it be concluded that the standard deviation has changed? Use the P-value method.



8.6 - Additional Topics Regarding Hypothesis Testing

- There is a relationship between confidence intervals and hypothesis testing
- When null hypothesis is rejected
 - Confidence interval for mean using same level of significance will not contain hypothesized mean
- When null hypothesis is not rejected
 - Confidence interval computed using same level of significance will contain hypothesized mean

Example 8 - 30

- Sugar is packed in 5-pound bags. An inspector suspects the bags may not contain 5 pounds. A sample of 50 bags produces a mean of 4.6 pounds and a standard deviation of 0.7 pound. Is there enough evidence to conclude that the bags do not contain 5 pounds as stated at $\alpha = 0.05$? Also, find the 95% confidence interval of the true mean.



Example 8 - 31

- A researcher claims that adult hogs fed a special diet will have an average weight of 200 pounds. A sample of 10 hogs has an average weight of 198.2 pounds and a standard deviation of 3.3 pounds. At $\alpha=0.05$, can the claim be rejected? Also, find the 95% confidence interval of the true mean.

