

Engineering Mechanics – I (Statics): Final Examination

Allotted Time: 3 Hours

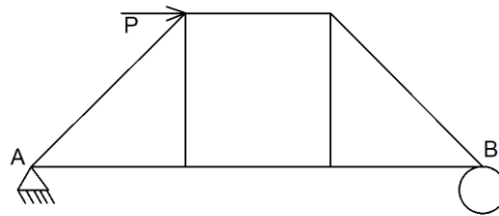
Part – I: Theoretical Questions. Give brief answers to the following questions.

1. State the differences between frame structures and truss structures. [5%]

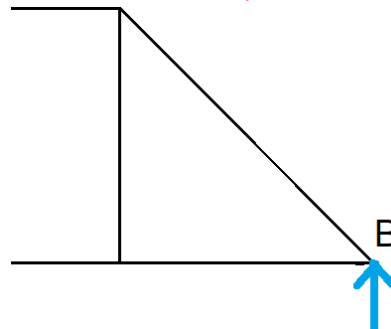
Answer:

- *Truss members are two-force members, while frame members aren't*
- *Truss members carry joint loads, hence bear no flexural loads*

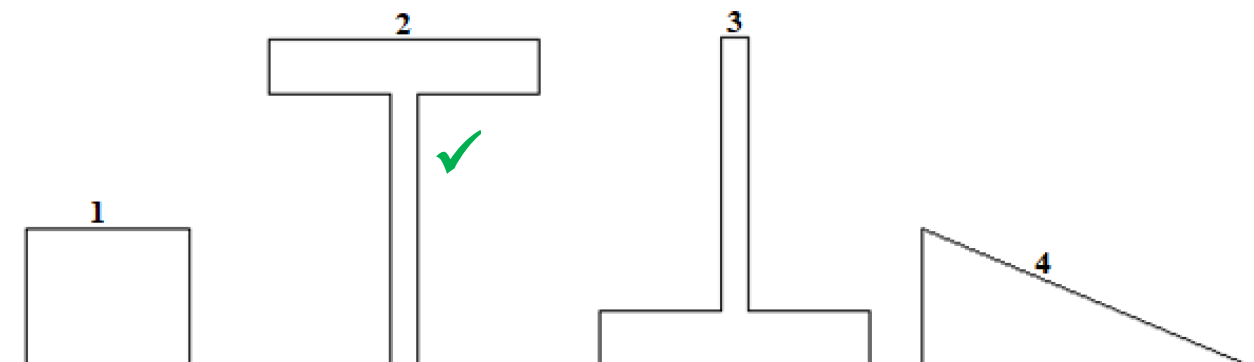
2. Using method of sections, show that the truss given below is unstable. [5%]



Answer: *The vertical reaction at B, has no internal force to resist it.*



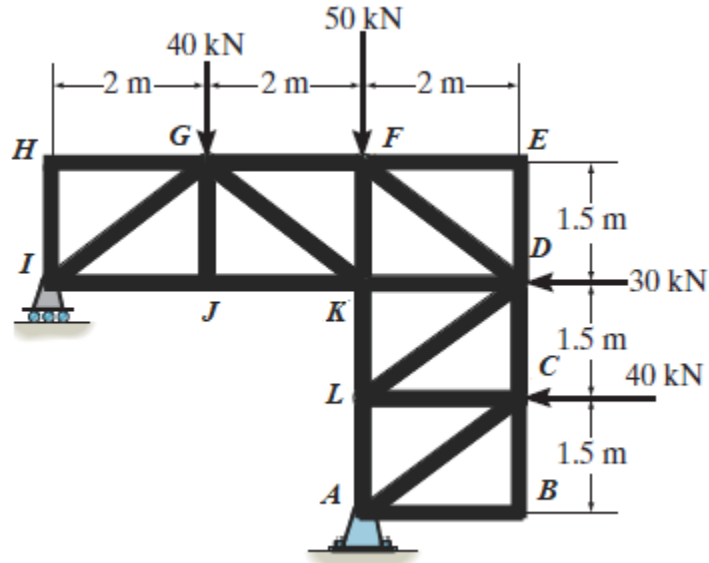
3. Describe the factors that affect the moment of inertia of a cross-section. Accordingly, if all the shapes in the figure below have the same area, which one is bound to have the largest moment of inertia about the base axis (bottom)? [10%]



Answer: *The moment of inertia of a cross section depends on the area of the shape and the proportion of the area that is found at a given distance from the axis about which the moment is calculated.*

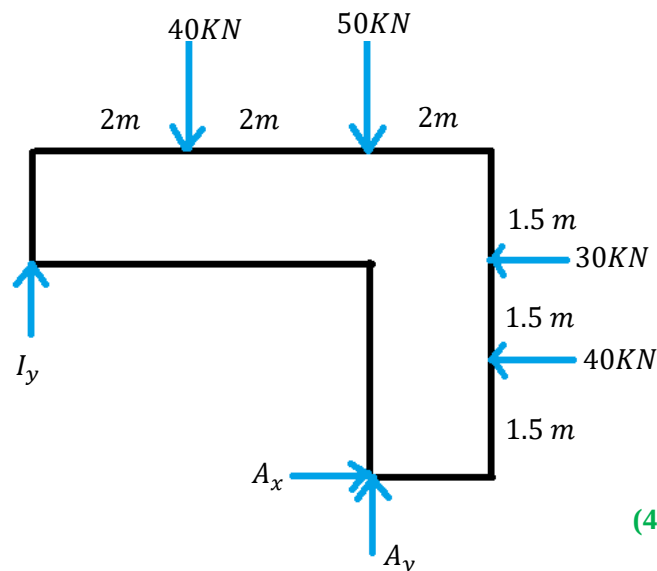
Part – II: Computational Questions. Solve the following problems and show your computational work clearly.

1. Determine the force in **members GF, GK, and JK** of the truss, and state if the members are in tension or compression. And also **identify zero force members**, if there are any. [20%]



Solution

Step-1: Compute the necessary support reactions



(4 Points)

$$\sum M_A = 0 \text{ (clockwise + ve)}$$

$$4I_y - 2 * 40 - 3 * 30 - 1.5 * 40 = 0$$

$$I_y = 57.5 \text{ kN}$$

(2 Points)

$$\sum F_y = 0$$

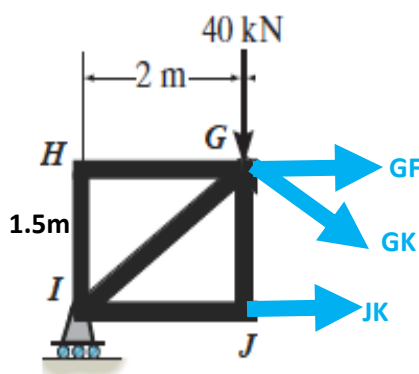
$$I_y + A_y - 40\text{KN} - 50\text{KN} = 0$$

$$A_y = 32.5\text{ KN} \quad (2\text{ Points})$$

$$\sum F_x = 0$$

$$A_x - 30 - 40 = 0 : A_x = 70\text{KN} \quad (2\text{ Points})$$

Step-2: Compute the required internal forces



$$\sum M_G = 0 \text{ (clockwise + ve)}$$

$$2I_y + 1.5JK = 0$$

$$JK = -76.67\text{ KN} ; \quad (2\text{ Points})$$

$$\sum M_K = 0 \text{ (clockwise + ve)}$$

$$4I_y + 1.5GF - 2 * 40 = 0$$

$$GF = -100\text{ KN} \quad (2\text{ Points})$$

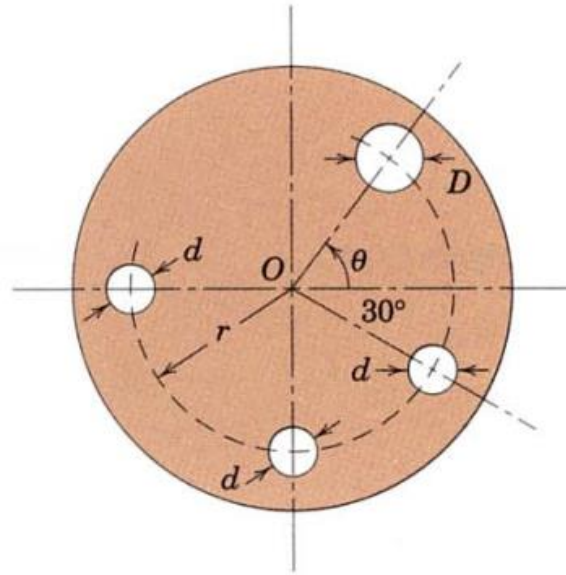
$$\sum F_x = 0$$

$$GF + GK \left(\frac{2^2}{1.5^2 + 2^2} \right) + JK = 0$$

$$Gk = 29.16\text{ KN} \quad (2\text{ Points})$$

Zero force members: AB, BC, ED, EF, JG, HG, HI (4 Points)

2. The circular disk rotates about an axis through its center O and has **three holes of diameter d** positioned as shown. A fourth hole is to be drilled in the disk at the same radius r so that the disk will be in balance (mass center at O). **Determine the required diameter D of the new hole (in terms of d), and its angular position.** [20%]



Solution

There are to be four cutout areas. Areas 1,2, and 3 are the small cutout circles of diameter, d. Area -4 is the larger cut-out circle of diameter, D. Considering a coordinate system with the origin at O, then after the final cutout, we need to satisfy: $\bar{x} = 0$ and $\bar{y} = 0$.

No.	Area	$\bar{x}$	$\bar{y}$	$A\bar{x}$	$A\bar{y}$
1	$\frac{\pi d^2}{4}$	$-r$	0	$-\frac{\pi d^2 r}{4}$	0
2	$\frac{\pi d^2}{4}$	0	$-r$	0	$-\frac{\pi d^2 r}{4}$
3	$\frac{\pi d^2}{4}$	$r \cos 30^\circ$	$-r \sin 30^\circ$	$\frac{\pi d^2 r \cos 30^\circ}{4}$	$-\frac{\pi d^2 r \sin 30^\circ}{4}$
4	$\frac{\pi D^2}{4}$ (2 Points)	$r \cos \theta$ (2 Points)	$r \sin \theta$ (2 Points)	$\frac{\pi D^2 r \cos \theta}{4}$ (2 Points)	$\frac{\pi D^2 r \sin \theta}{4}$ (2 Points)

$$\bar{x} = \frac{\sum A_i x_i}{\sum A_i} = 0; \sum A_i x_i = 0$$

$$-\frac{\pi d^2 r}{4} + 0 + \frac{\pi d^2 r \cos 30^\circ}{4} + \frac{\pi D^2 r \cos \theta}{4} = 0$$

$$D^2 \cos \theta = d^2 (1 - \cos 30^\circ) \quad (2 \text{ Points})$$

$$\bar{y} = \frac{\sum A_i y_i}{\sum A_i} = 0; \sum A_i y_i = 0$$

$$0 - \frac{\pi d^2 r}{4} - \frac{\pi d^2 r \sin 30^\circ}{4} + \frac{\pi D^2 r \sin \theta}{4} = 0$$

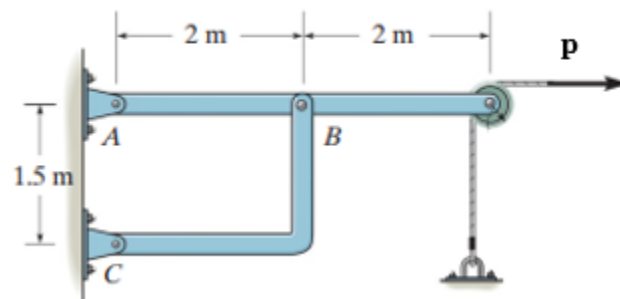
$$D^2 \sin \theta = d^2 (1 + \sin 30^\circ) \quad (2 \text{ Points})$$

$$\tan \theta = \frac{1 + \sin 30}{1 - \cos 30} \quad (2 \text{ Points})$$

$$\theta = 84.9^\circ \quad (2 \text{ Points})$$

$$D = 1.227d \quad (2 \text{ Points})$$

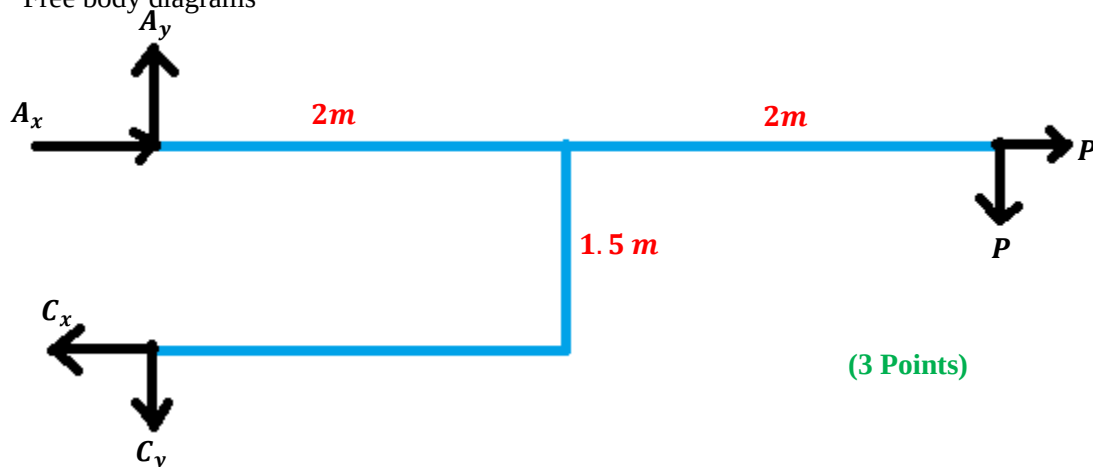
3. The frame shown below is subject to a tension load of P. [20%]

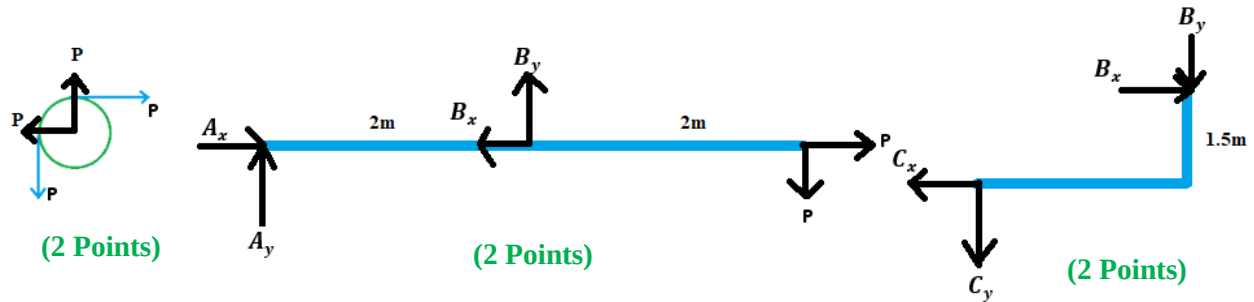


- Draw **three separate free body diagrams** for:
 - The entire frame
 - The two members of the frame
 - The pulley
- Find the resultant reaction forces at pins A, B and C in terms of P.
- If the resultant at **pin B** was found to be **200kN**. What are the resultant reactions forces at pins A and C.

Solution

- a. Free body diagrams





b. Reaction forces at pins

From member AB:

$$\sum F_x = A_x - B_x + P = 0$$

$$\sum F_y = A_y + B_y - P = 0$$

$$\sum M_A = 0 \text{ (clockwise + ve): } 2B_y - 4 * P = 0$$

$$B_y = 2P \quad (1 \text{ Point})$$

$$A_y = -P \quad (1 \text{ Point})$$

From member BC:

$$\sum F_x = B_x - C_x = 0$$

$$\sum F_y = -C_y - B_y = 0$$

$$C_y = 2P \quad (1 \text{ Point})$$

$$\sum M_C = 1.5B_x + 2B_y = 0$$

$$B_x = -2B_y = -\frac{8}{3}P \quad (1 \text{ Point})$$

$$C_x = -\frac{8}{3}P \quad (1 \text{ Point})$$

From member AB:

$$A_x = -\frac{11}{3}P \quad (1 \text{ Point})$$

- c. Reaction force values at A & C

For resultant reaction at B = 200KN,

$$B_x^2 + B_y^2 = (200KN)^2$$

$$\frac{100}{9}P^2 = (200KN)^2$$

$$P = 60KN \quad (2 \text{ Points})$$

the following are the pin reaction force values.

$$A_x = -\frac{11}{3}P = -220 \text{ KN} \quad (0.5 \text{ Point})$$

$$A_y = -P = -60 \text{ KN} \quad (0.5 \text{ Point})$$

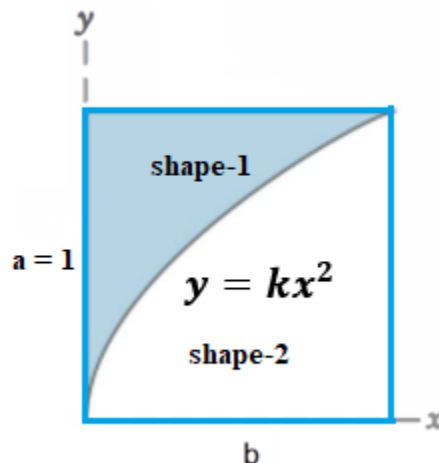
$$C_x = -\frac{8}{3}P = -160 \text{ KN} \quad (0.5 \text{ Point})$$

$$C_y = -2P = -120 \text{ KN} \quad (0.5 \text{ Point})$$

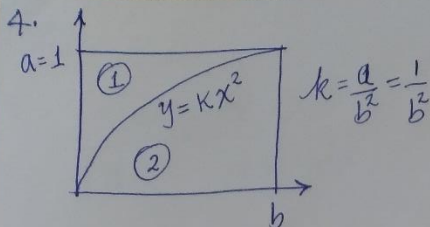
$$\text{Resultant reaction; point A: } R_A := \sqrt{A_x^2 + A_y^2} = 228.04 \text{ KN} \quad (0.5 \text{ Point})$$

$$\text{Resultant reaction; point C: } R_C := \sqrt{C_x^2 + C_y^2} = 200 \text{ KN} \quad (0.5 \text{ Point})$$

4. The figure below shows a rectangle of base, **b** and height **a**. The function $y = kx^2$ divides the rectangle into shape-1 (top) and shape-2 (bottom). What must be the function such **that the moment of inertia of shape-1 about its centroidal x-axis** is the same as **the moment of inertia of shape-2 about its centroidal y-axis**. [20%]



Solution

4. 

Step-1: find centroids

Shape-1: $\bar{y}$

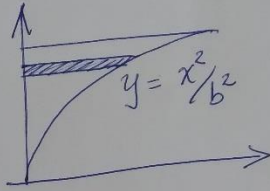
$$\bar{y} = \frac{\int y_c dA}{\int dA}; \quad y_c = y, \quad dA = x dy$$

$$\bar{y} = \frac{\int xy dy}{\int x dy} = \frac{\int_0^a by^{3/2} dy}{\int_0^a by^{1/2} dy}$$

$$\bar{y} = \frac{\frac{2}{5} y^{5/2} \Big|_0^a}{\frac{2}{3} y^{3/2} \Big|_0^a}; \quad a=1$$

$\bar{y} = \frac{3}{5} = 0.6$

(4 points)



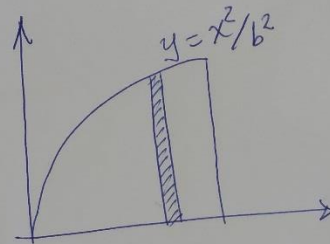
Shape-2: $\bar{x}$

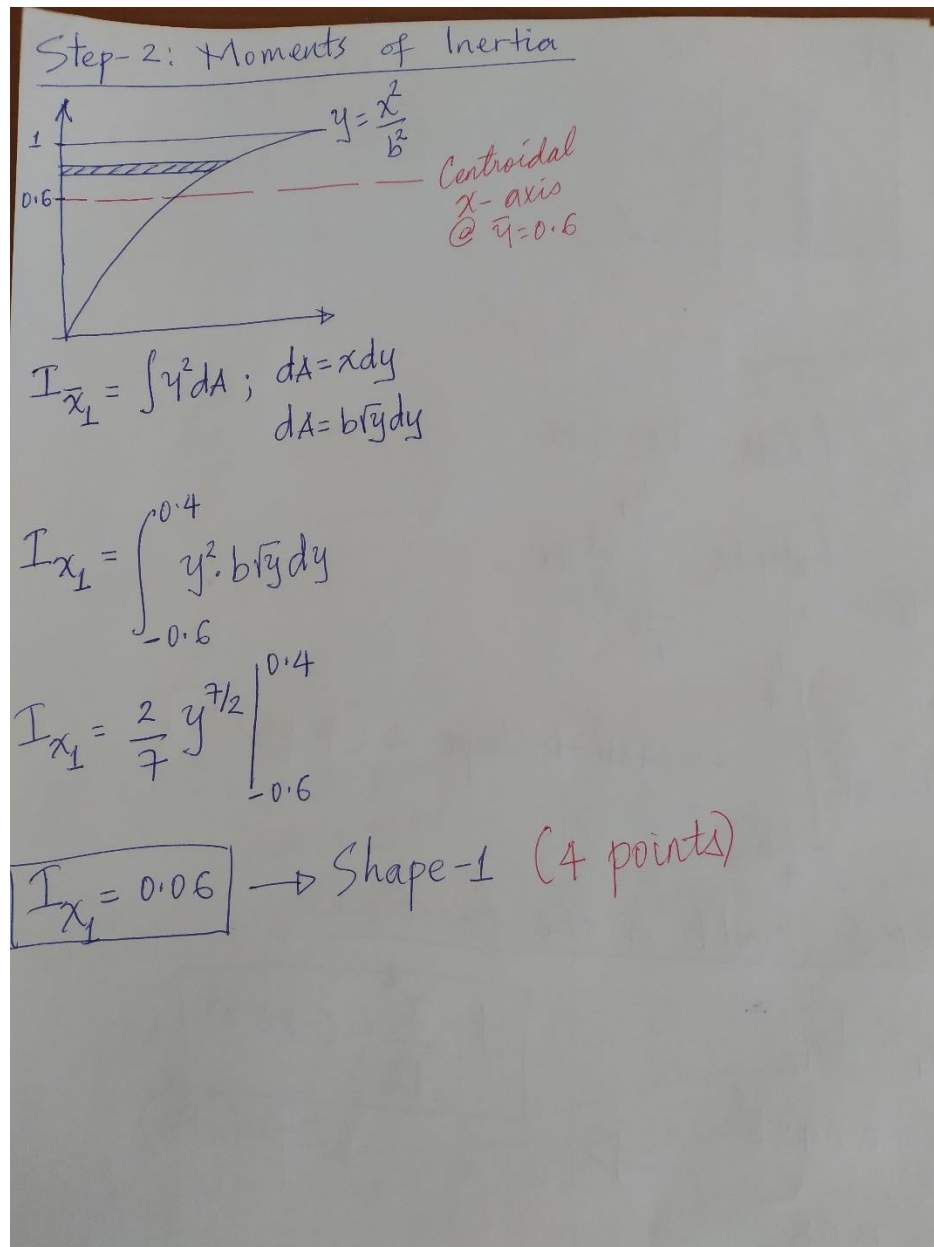
$$\bar{x} = \frac{\int x_c dA}{\int dA}; \quad x_c = x, \quad dA = y dx$$

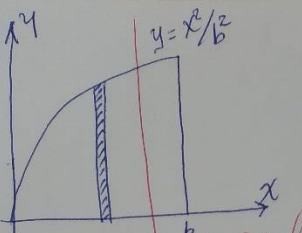
$$\bar{x} = \frac{\int xy dx}{\int y dx} = \frac{\int_0^b \frac{x^3}{b^2} dx}{\int_0^b \frac{x^2}{b^2} dx}$$

$$\bar{x} = \frac{\frac{x^4}{4b^2} \Big|_0^b}{\frac{x^3}{3b^2} \Big|_0^b} = \frac{b^2/4}{b/3}$$

$$\boxed{\bar{x} = \frac{3b}{4}} \quad \boxed{4 \text{ points}}$$







Centroidal
y-axis @ $x = \frac{3b}{4}$

$$I_{y_2} = \int x^2 dA ; dA = y dx$$

$$I_{y_2} = \int x^2 y dA = \int_{-\frac{3b}{4}}^{\frac{b}{4}} \frac{x^4}{b^2} dx$$

$$I_{y_2} = \frac{x^5}{5b^2} \Big|_{-\frac{3b}{4}}^{\frac{b}{4}} = 0.048b^3 \rightarrow \text{Shape-2 (4 points)}$$

Step:3 Find b & the function

$$I_{x_1} = I_{y_2}$$

$$0.06 = 0.048b^3 \Rightarrow$$

$y = \frac{x^2}{b^2} = 0.86x^2$

(2 points)

$$b = 1.08$$

(2 points)