MECHANICS I (Static)MID EXAM

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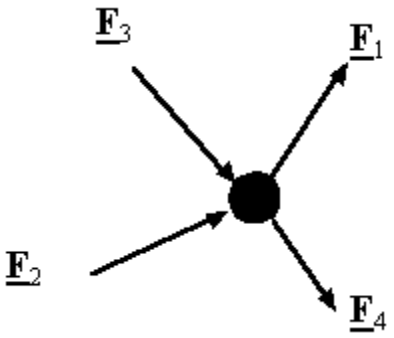
Ethio Exams

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Static Equilibrium for a Particle

A particle: An object with inertia (mass) but of negligible dimensions

Equilibrium equations for a particle: A particle is in equilibrium if the resultant of ALL forces acting on the particle is equal to zero



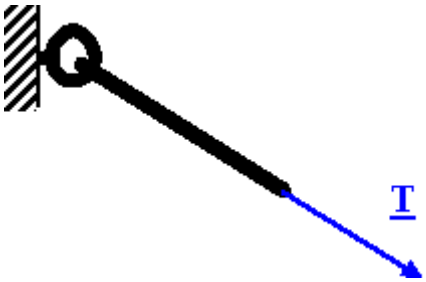
$$\sum \underline{\mathbf{F}} = 0$$

Equilibrium equations in component form: In a rectangular coordinate system the equilibrium equations can be represented by three scalar equations:

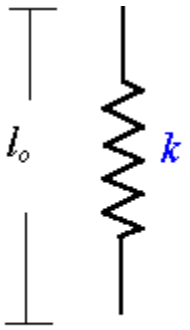
$$\sum F_x = 0$$
$$\sum F_y = 0$$
$$\sum F_z = 0$$

Free- Body diagram: A diagram showing the particle under consideration and all the forces acting on this particle. Each force in this diagram must labeled.

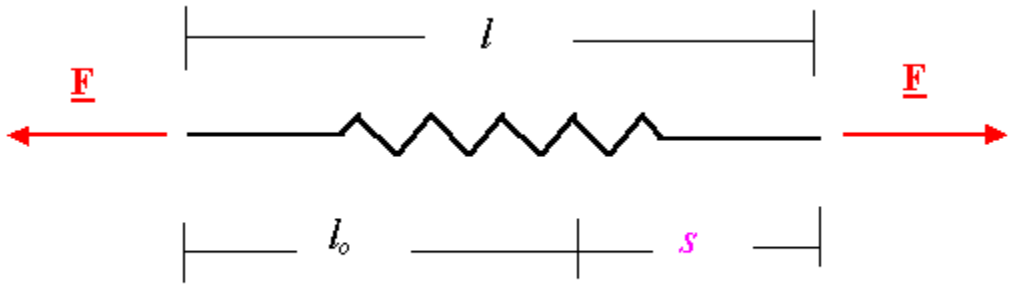
String or cable: A mechanical device that can only transmit a tensile force along itself.



Linear spring: A mechanical device which exerts a force along its line of action and proportional to its extension.

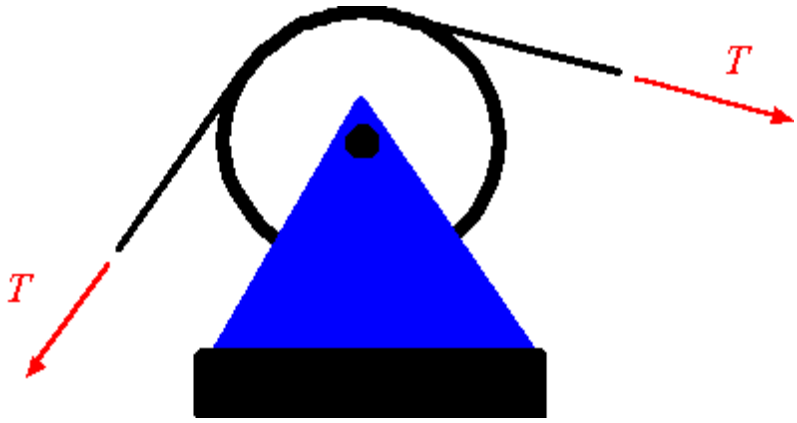


Free Spring



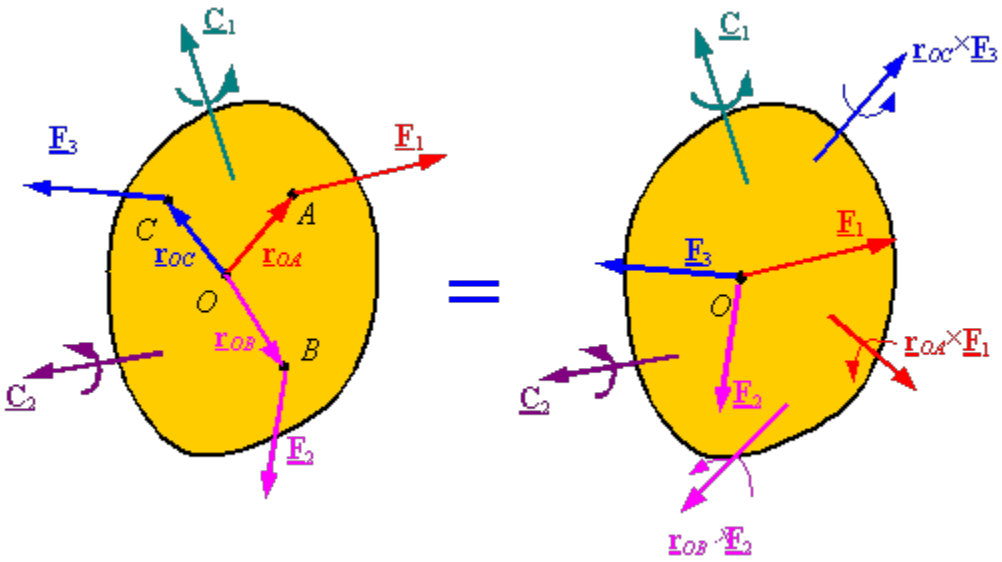
Stretched Spring

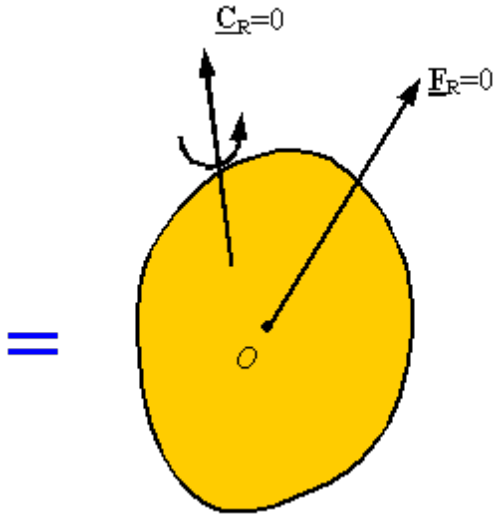
Frictionless pulleys: For a frictionless pulley in static equilibrium, the tension in the cable is the same on both sides of the pulley.



- ## Equilibrium of rigid bodies

- **Static equilibrium for a rigid body:** A body (or any part of it) which is currently stationary will remain stationary if the resultant force and resultant moment are zero for all the forces and couples applied on it.
- $$\begin{cases} \underline{\mathbf{F}}_R = \sum \underline{\mathbf{F}} = 0 \\ \underline{\mathbf{C}}_R = \sum \underline{\mathbf{M}}_O = \sum \underline{\mathbf{C}} + \sum \underline{\mathbf{r}} \times \underline{\mathbf{F}} = 0 \end{cases}$$





- **Newton's 3rd law:** Each action has a reaction equal in magnitude and opposite in direction. This law provides the method used for one body (or part of a body) to interact with another body (or another part of the body).
-
- **Free- Body Diagram:** A diagram of a body (or a part of it) which shows all the forces and couples applied on it, and which has all the forces and couples labeled for use in the solution of the problem is called a free- body diagram.

Follow these steps to draw a free- body diagram.

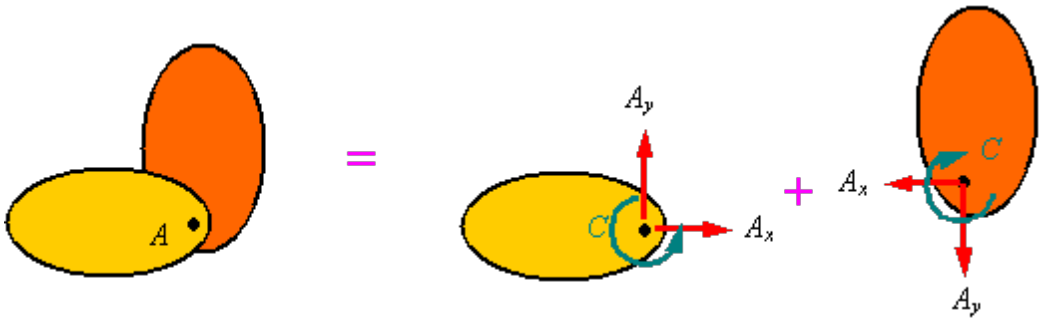
1. Select the body (or part of a body) that you want to analyze, and draw it.

2. Identify all the forces and couples that are applied onto the body and draw them on the body. Place each force and couple at the point that it is applied.

3. Label all the forces and couples with unique labels for use during the solution process.

4. Add any relevant dimensions onto your picture.

Composite bodies and internal forces: Forces and couples which are a result of interaction between one part of an object and another part of it will not appear in the free- body diagram of the whole object. This is due to Newton's 3rd law. The two bodies in the following example are welded at A. When the two parts are looked at as a single body, the internal forces and couples are added together, and as a result of Newton's third law will cancel.



Forces and couples on a free- body diagram: Each force or couple you put on a free- body diagram represents a model of how the body in the free- body diagram is effected by its surroundings. In selecting the forces and couples that are to be applied on the free- body diagram follow these steps:

1. **Identify all the forces which come from the interaction of one body with another. Many of the common supports and their effects are shown in table… Remember that for each way in which a support restricts the free motion of the body, a force or a moment must be applied to the body to impose the restriction on the motion.**

2. **Apply the weight of the body to its center of gravity (if it is uniform, then apply it to the centroid).**

3. Remember that strings and cables can only pull on an object.

4. Remember that internal loads cancel out and should not be put on the free-body diagram.

5. Remember that if you have selected the direction of forces or couples of interaction on one body, then Newton's 3rd law states that you must apply the forces or couples in the opposite direction on the other body.

Solving for unknowns: You can write one set of equilibrium equations for each free- body diagram you draw. Things to remember are:

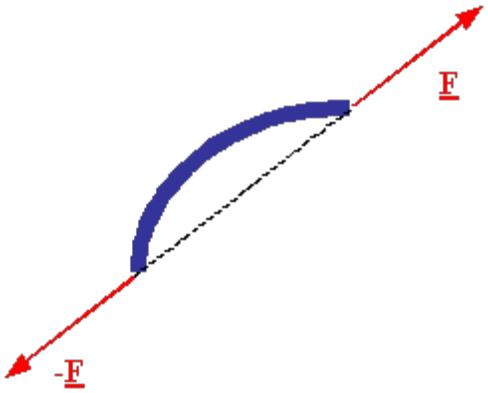
1. In 2- D problems the equilibrium equations result in three independent scalar equations (two components of force and one component of moment). Therefore, you can only solve for three scalar unknowns.

2. Try to select the point you take moments around such that the line of action of at least one unknown force passes through that point. This will eliminate one unknown from your moment equation and will result in simpler equations to work with. This step is not essential, but will significantly simplify the algebra involved in solving your system of equations.

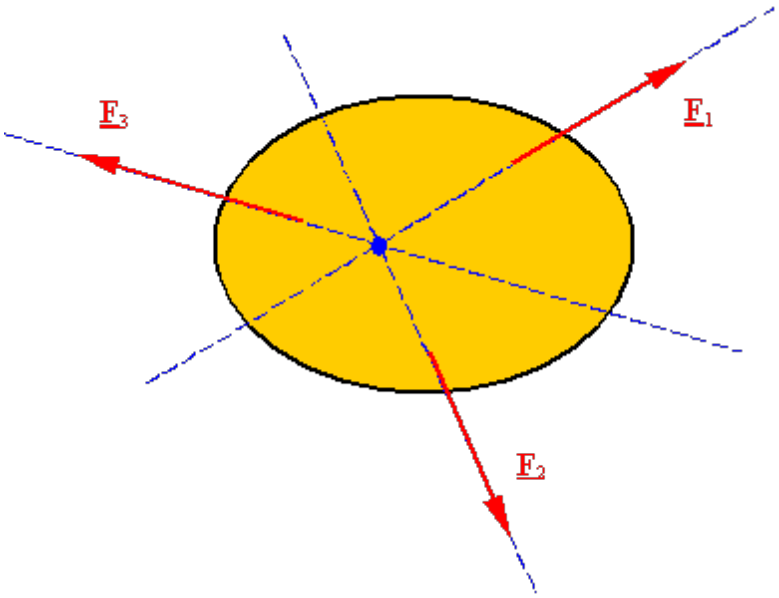
3. You can sometimes take moments about two or three different points in a problem. Select each point so that you eliminate one or more unknowns from the resulting moment equation. Remember that the additional equations you generate in this way are not independent of the original equations, and, therefore, you will still have only three independent equations in 2- D problems per free- body diagram and you can only solve for three unknowns per free- body diagram.

4. For a composite body, if you have drawn a free- body diagram and written the equilibrium equations for each of its subsections, you will gain no additional information if you draw the free- body diagram of the entire composite body and write its equilibrium equations.

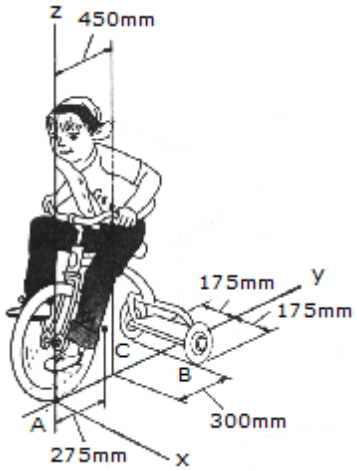
5. In 3- D problems the equilibrium equations result in six independent scalar equations (three components of force and three components of moment). Therefore, you can solve for up to six scalar unknowns per free- body diagram.

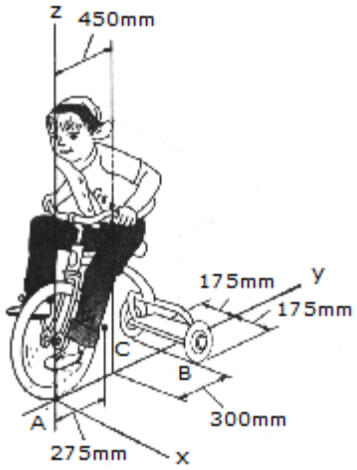


- **A two- force member:** A body which has forces applied onto it at only two points, and no couples applied onto it at all, is called a two- force member. A two- force member can only be in equilibrium if the line of action of the resultant of the forces at each point passes through the other point, and each resultant force is equal in magnitude but opposite in direction to the resultant of the forces applied to the other point.
- **A three- force member:** A body which has forces applied onto it at only three points, and no couples applied onto it at all, is called a three- force member. A three- force member can only be in equilibrium if the lines of action for the resultants of the forces at each point intersect at a single point.



1)



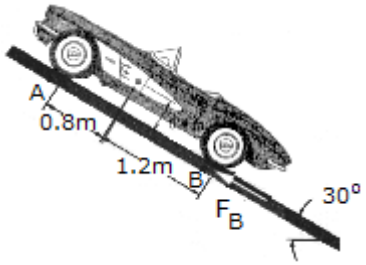


The girl has a mass of 17kg and mass center at G_g, and the tricycle has a mass of 10kg and mass center at G_t. Determine the normal reactions at each wheel for equilibrium.

A. $N_A = 14.77$ N, $N_B = N_C = 6.12$ N
$N_A = 128.8$ N, $N_B = N_C = 68.0$
B. N

C. $N_A = 144.9$ N, $N_B = N_C = 60.0$ N
D. $N_A = 13.15$ N, $N_B = N_C = 6.93$ N

2.

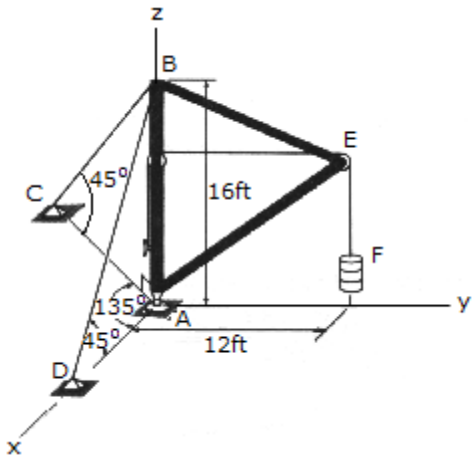


The sports car has a mass of 1.5 Mg and mass center at *G*. If the front two springs each have a stiffness of k_A= 58 kN/m and the rear two springs each have a stiffness of k_B = 65 kN/m, determine their compression when the car is parked on the 30° incline. Also, what frictional force F_B must be applied to each of the rear wheels to hold the car in equilibrium?

A. x_A = 16.1 mm, x_B = 42.2 mm, F_B = 6.37 kN
B. x_A = 65.9 mm, x_B = 39.2 mm, F_B = 6.37 kN
x_A = 53.2 mm, x_B = 50.5 mm, F_B = 3.68
C. kN

D. x_A = 76.1 mm, x_B = 45.3 mm, F_B = 3.68 kN

3.



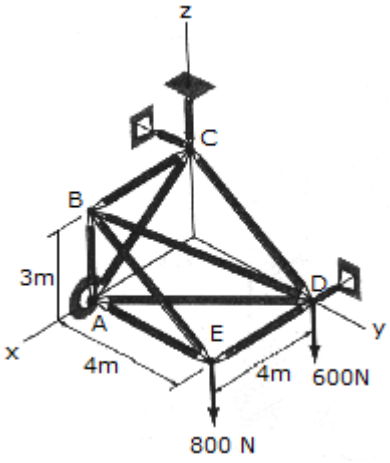
Determine the tension in the supporting cables *BC* and *BD and the components of reaction at the ball- and- socket joint A of the boom. The boom supports a drum having a weight of 200 lb. at F. Points C and D lie in the x—y plane.*

A_x = 0, A_y = 150 lb, A_z = 562 lb, T_{BC} = 300 lb, T_{BD} = 212
A. lb

B. A_x = 0, A_y = 150 lb, A_z = 456 lb, T_{BC} = 150 lb, T_{BD} = 212 lb

C. $A_x = 0$, $A_y = 267$ lb, $A_z = 843$ lb, $T_{BC} = 533$ lb, $T_{BD} =$ lb
D. $A_x = 0$, $A_y = 150$ lb, $A_z = 500$ lb, $T_{BC} = 212$ lb, $T_{BD} = 212$ lb

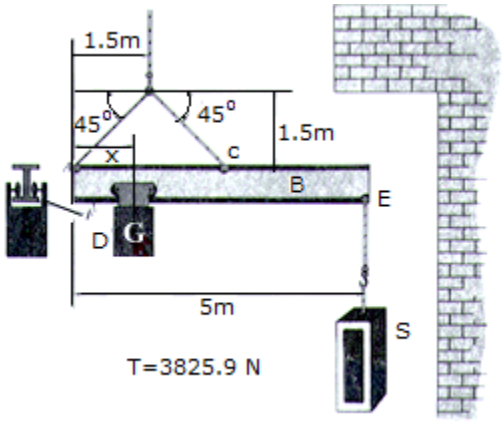
4.



The space truss is supported by a ball- and- socket joint at *A* and short links, two at *C* and one at *D*. Determine the *x, y, z* components of reaction at *A* and the force in each link.

A. $A_x = -1.050$ kN, $A_y = 1.050$ kN, $A_z = 0.800$ kN, $C_y = -1.050$ kN, $C_z = 0.600$ kN, $D_x = 1.050$ kN
B. $A_x = -1.400$ kN, $A_y = 1.400$ kN, $A_z = 0.800$ kN, $C_y = -1.400$ kN, $C_z = 0.600$ kN, $D_x = 1.400$ kN
C. $A_x = -2.49$ kN, $A_y = 1.867$ kN, $A_z = 0.800$ kN, $C_y = -2.49$ kN, $C_z = 0.600$ kN, $D_x = 2.49$ kN
D. $A_x = -1.867$ kN, $A_y = 1.867$ kN, $A_z = 0.800$ kN, $C_y = -1.867$ kN, $C_z = 0.600$ kN, $D_x = 1.867$ kN

5.



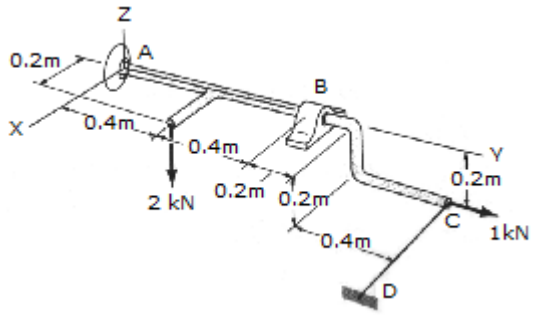
The *flying boom B* is used with a crane to position construction materials in coves and

underhangs. The horizontal "balance" of the boom is controlled by a 250- kg block *D*, which has a center of gravity at *G* and moves by internal sensing devices along the bottom flange *F* of the beam. Determine the position *x* of the block when the boom is used to lift the stone *S*, which has a mass of 60 kg. The boom is uniform and has a mass of 80 kg.

A. x = 2.500 m
x = 0.340
B. m

C. x = 1.180 m
D. x = 0.600 m

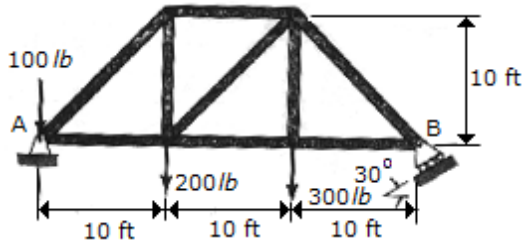
6.



There is a ball and socket connection at A. At point B there is a connection that opposes motion in the x and z directions only. Determine the unknown force components at A and B. Use a scalar analysis.

A. A_x = 5.5 kN, A_y = - 1.0 kN, A_z = 2.75 kN, B_x = 3.5 kN, B_z = 1.25 kN, F_{DC} = 2.0 kN
B. A_x = 1.5 kN, A_y = - 1.0 kN, A_z = 0.75 kN, B_x = - 3.5 kN, B_z = 1.25 kN, F_{DC} = 2.0 kN
C. A_x = 5.5 kN, A_y = - 1.0 kN, A_z = 2.75 kN, B_x = 3.5 kN, B_z = - 0.75 kN, F_{DC} = 2.0 kN
A_x = 1.5 kN, A_y = - 1.0 kN, A_z = 1.25 kN, B_x = - 3.5 kN, B_z = 0.75 kN, F_{DC} =
D. 2.0 kN

7.

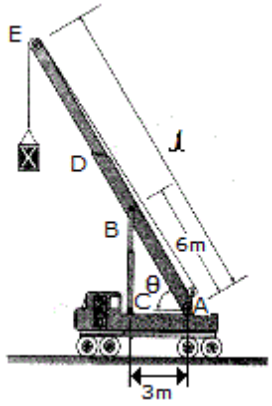


Determine the horizontal and vertical components of reaction at the pin *A* and the

reaction at the roller support *B* required for equilibrium of the truss.

A. $A_x = 0$, $A_y = 333$ lb, $N_B = 267$ lb
B. $A_x = 462$ lb, $A_y = 66.7$ lb, $N_B = 533$ lb
C. $A_x = 267$ lb, $A_y = 223$ lb, $N_B = 377$ lb
$A_x = 154.0$ lb, $A_y = 333$ lb, $N_B = 308$
D. lb

8.

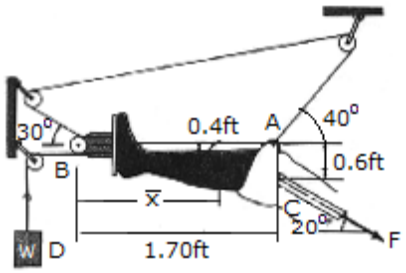


The crane provides a long- reach capacity by using the telescopic boom segment *DE*. The entire boom is supported by a pin at *A* and by the telescopic hydraulic cylinder *BC*, which can be considered as a two- force member. The rated load capacity of the crane is measured by a maximum force developed in the hydraulic cylinder. If this maximum force is developed when the boom supports a mass m = 6 Mg and its length is l = 40 and $\theta = 60°$, determine the greatest mass that can be supported when the boom length is extended to l = 50 m and $\theta = 45°$. Neglect the weight of the boom and the size of the pulley at E. Assume the crane does not overturn. Note: when $\theta = 60°$ BC is vertical; however, when $\theta = 45°$ this is not the case.

A. m = 7.50 Mg
B. m = 4.80 Mg
m = 3.26
C. Mg

D. m = 4.61 Mg

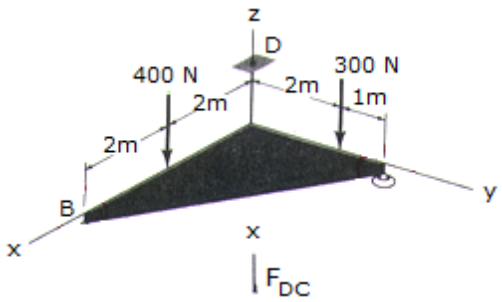
9.



A *Russell's traction* is used for immobilizing femoral fractures *C*. If the lower leg has a weight of 8 lb, determine the weight *W* that must be suspended at *D* in order for the leg to be held in the position shown. Also, what is the tension force *F* in the femur and the distance $\bar{x}$ which locates the center of gravity *G* of the lower leg? Neglect the size of the pulley at *B*.

x = 1.44 ft, w = 10.8 lb, F =
A. 12.61 lb

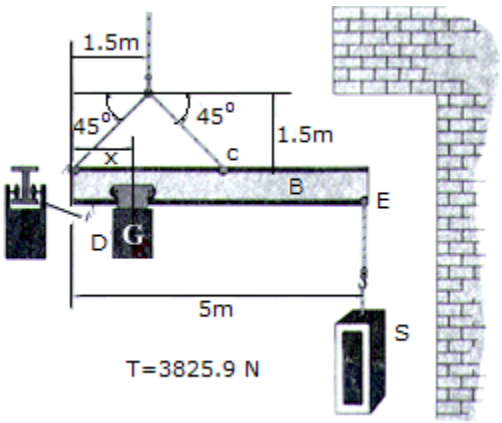
B. x = 1.33 ft, w = 15.76 lb, F = 20.0 lb
C. x = 1.56 ft, w = 9.75 lb, F = 12.69 lb
D. x = 0.869 ft, w = 6.44 lb, F = 5.03 lb
10.



There is a ball and socket connection at *A*. At *B* there is a roller that prevents motion in the −z direction. Corner *C* is tied to *D* by a rope. The triangle is weightless. Determine the unknown force components acting at *A*, *B*, and *C*. Use a scalar analysis.

A. $A_x = 0$, $A_y = 0$, $A_z = 267$ N, $B_z = 150$ N, $F_{DC} = 283$ N
B. $A_x = 0$, $A_y = 0$, $A_z = 450$ N, $B_z = 800$ N, $F_{DC} = 550$ N
$A_x = 0$, $A_y = 0$, $A_z = 200$ N, $B_z = 200$ N, $F_{DC} = 300$
C. N

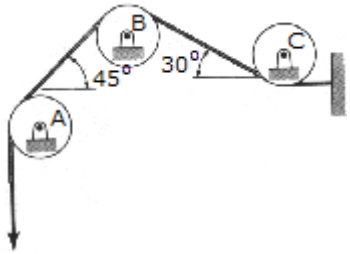
D. $A_x = 0$, $A_y = 0$, $A_z = 467$ N, $B_z = 350$ N, $F_{DC} = 117.1$ N
11.



If the boom in the previous problem is to remain horizontal when the stone S is removed, what is x?

x = 1.180
A. m

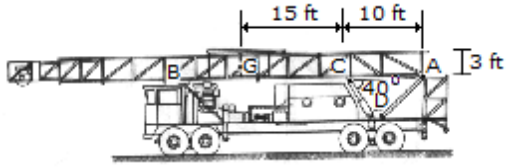
B. x = 2.500 m
C. x = 0.340 m
D. x = 0.660 m
12.



Compute the horizontal and vertical components of force at pin *B*. The belt is subjected to a tension of *T*=100 N and passes over each of the three pulleys.

A. B_x = 0 N, B_y = 141.4 N
B_x = - 15.89 N, B_y = 120.7
B. N

C. B_x = 20.7 N, B_y = 157.3 N
D. B_x = 0, B_y = 100.0 N
13.



The oil rig is supported on the trailer by the pin or axle at *A* and the frame at *B*. If the rig has a weight of 115,000 lb and the center of gravity at *G*, determine the force *F* that must de developed along the hydraulic cylinder CD in order to start lifting the rig (slowly) off *B* toward the vertical. Also compute the horizontal and vertical components of reaction at the pin *A*.

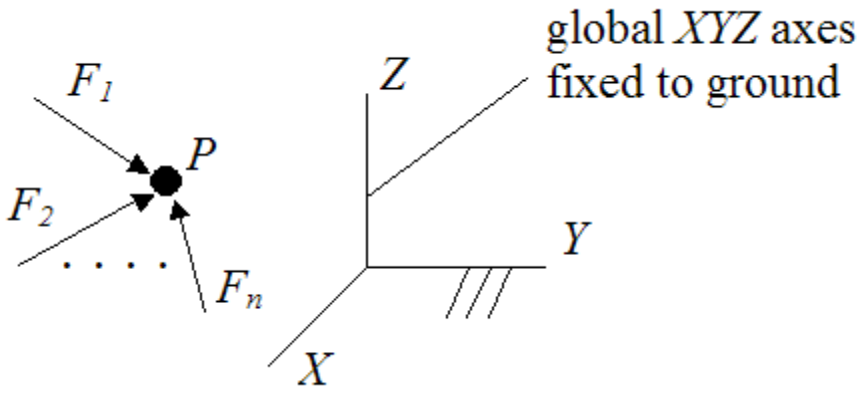
A_x = 343 kip, A_y = - 172.5 kip, FCD = 447
A. kip

B. A_x = 205 kip, A_y = - 57.5 kip, FCD = 268 kip
C. A_x = 241 kip, A_y = - 172.5 kip, FCD = 375 kip
D. A_x = 220 kip, A_y = - 70.1 kip, FCD = 288 kip

Equilibrium

Equilibrium For A Particle

A particle is defined as being in equilibrium if it is stationary (at rest), or moving at constant velocity, with respect to ground (an inertial reference frame). This means that the vector sum of all the forces acting on it is equal to zero. See figure below.



where F_1, F_2,..., F_n are the external forces acting on the particle *P*, and *XYZ* is an inertial reference frame fixed to ground.

By Newton's Second Law,

$$\sum \vec{F} = (\vec{F}_1 + \vec{F}_2 + \ldots + \vec{F}_n) = m\vec{a}$$

where ΣF is the vector sum of the forces, m is the mass of the particle, and a is the acceleration of the particle with respect to ground.

The particle P is in equilibrium if

$$\vec{a} = 0$$

Therefore,

$$\sum \vec{F} = (\vec{F}_1 + \vec{F}_2 + \ldots + \vec{F}_n) = 0$$

As a result, we can write the following scalar equations for the component directions of the sum of forces (along the X, Y, Z directions):

$$\sum F_X = 0 \quad (1)$$

$$\sum F_Y = 0 \quad (2)$$

$$\sum F_Z = 0 \quad (3)$$

Where:

ΣF_X is the sum of the forces in the X- direction

ΣF_Y is the sum of the forces in the Y- direction

ΣF_Z is the sum of the forces in the Z- direction

More specifically,

$$\sum F_X = F_{1X} + F_{2X} + .. + F_{nX}$$

$$\sum F_Y = F_{1Y} + F_{2Y} + .. + F_{nY}$$

$$\sum F_Z = F_{1Z} + F_{2Z} + .. + F_{nZ}$$

Where:

F_{1X}, F_{2X},..., F_{nX} are the *X*- components of the external forces acting on the particle *P*

F_{1Y}, F_{2Y},..., F_{nY} are the *Y*- components of the external forces acting on the particle *P*

F_{1Z}, F_{2Z},..., F_{nZ} are the *Z*- components of the external forces acting on the particle *P*

Equations (1)- (3) are all the equations that are necessary to solve for the forces on a particle in equilibrium.

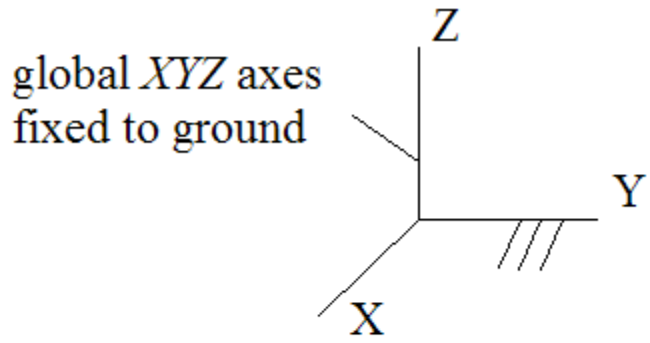
Equilibrium For A Rigid Body

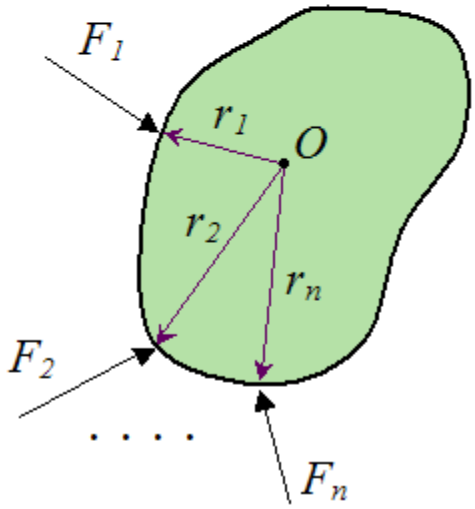
A rigid body is defined as being in equilibrium if:

• It is stationary (at rest), with respect to ground (an inertial reference frame).

• It is moving at constant velocity, with no rotation, with respect to ground.

These two conditions mean that the vector sum of all the forces acting on the rigid body is equal to zero. See figure below.





Where:

XYZ is an inertial reference frame fixed to ground

O is an arbitrary point, which can be located anywhere (on or off the rigid body)

F_1, F_2,..., F_n are the external forces acting on the rigid body

r_1, r_2,..., r_n are the position vectors from point *O* to the locations on the rigid body where the external forces F_1, F_2,..., F_n are acting

By Newton's Second Law,

$$\sum \vec{F} = (\vec{F}_1 + \vec{F}_2 + \ldots + \vec{F}_n) = m\vec{a}_G$$

where ΣF is the vector sum of the forces, m is the mass of the rigid body, and a_G is the acceleration of the center of mass G of the rigid body, with respect to ground.

The rigid body is in equilibrium if

$$\vec{a}_G = 0$$

Therefore,

$$\sum \vec{F} = (\vec{F}_1 + \vec{F}_2 + \ldots + \vec{F}_n) = 0$$

As a result, we can write the following scalar equations for the component directions of the sum of forces (along the X, Y, Z directions):

$$\sum F_X = 0 \quad (4)$$

$$\sum F_Y = 0 \quad (5)$$

$$\sum F_Z = 0 \quad (6)$$

Where:

ΣF_X is the sum of the forces in the X- direction

ΣF_Y is the sum of the forces in the Y- direction

ΣF_Z is the sum of the forces in the Z- direction

More specifically,

$$\sum F_X = F_{1X} + F_{2X} + .. + F_{nX}$$

$$\sum F_Y = F_{1Y} + F_{2Y} + .. + F_{nY}$$

$$\sum F_Z = F_{1Z} + F_{2Z} + .. + F_{nZ}$$

Where:

F_{1X}, F_{2X},..., F_{nX} are the *X*- components of the external forces acting on the rigid body

F_{1Y}, F_{2Y},..., F_{nY} are the *Y*- components of the external forces acting on the rigid body

F_{1Z}, F_{2Z},..., F_{nZ} are the *Z*- components of the external forces acting on the rigid body

The rigid body must also be in rotational equilibrium. This means that the sum of the moments about point *O* must equal zero. Thus, we can write the following scalar equations:

$$\sum M_{OX} = 0 \quad (7)$$

$$\sum M_{OY} = 0 \quad (8)$$

$$\sum M_{OZ} = 0 \quad (9)$$

Where:

ΣM_{OX} is the sum of the moments acting on the rigid body, taken about point *O* and about the *X*- direction

ΣM_{OY} is the sum of the moments acting on the rigid body, taken about point *O* and about the *Y*- direction

ΣM_{OZ} is the sum of the moments acting on the rigid body, taken about point *O* and about the *Z*- direction

Now,

$$\sum \vec{M}_O = \vec{r}_1 \times \vec{F}_1 + \vec{r}_2 \times \vec{F}_2 + \ldots + \vec{r}_n \times \vec{F}_n$$

where ΣM_O is the sum- of- moments vector

We can express ΣM_O as its vector components:

$$\sum \vec{M}_O = (\sum M_{OX}, \sum M_{OY}, \sum M_{OZ})$$

where ΣM_{OX}, ΣM_{OY}, ΣM_{OZ} are the terms on the left side of the (scalar) equations (7)- (9).

Equations (4)- (9) are all the equations that are necessary to solve for the forces on a rigid body in equilibrium.

Once again, point *O* can be chosen arbitrarily. However, it is generally easiest to place *O* at a location where one of the external forces is acting (for example, F_1), since this will result in the corresponding position vector (r_1) equaling zero. This will eliminate the force F_1 from the set of equations, thus reducing the number of unknowns to solve for in equations (4)–(9).

For planar problems (two- dimensional) you can place *O* at the intersection of the lines of action of any two unknown external forces. This will eliminate those two forces from the equations, since the moment of those forces will be zero (i.e. $r \times F = 0$, since the position vector *r* and the force vector *F* are parallel, for these two forces).

For more information on moments see moment of a force.

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