Right Way Academy

Maths Grade 12
Unit 3 & 4

1. What is the equation of tangent line of $g(x) = -2x^2 + 3$ at $x = 1$?

A. $y = -4x + 3$
B. $y + x = 5$
C. $y = -4x + 5$
D. $y = -4x$

Answer: ~~X~~ C

equation of tangent line
$f(x) - f(a) = f'(a)(x - a)$
$f(1) = 1$ & $f'(1) = -4$
$f(x) - 1 = -4(x - 1)$
$f(x) = -4x + 5$

2. What is the derivative of $g(x) = 6(2x^5 - 1)^3$

A. $180x^4(2x^5 - 1)^2$
B. $18(2x^5 - 1)^2$
C. $180x^4$
D. $18(2x^5 - 1)^2 + 10x^4$

Answer: A

$g'(x) = (6(2x^5 - 1)^3)'(2x^5 - 1)'$
$= 18(2x^5 - 1)^2 (10x^4$
$= 180x^4(2x^5 - 1)^2$

3. What will the value of dy/dx in terms of x and y for the equation $x + xy^2 - y = 3$?

A. $\frac{2xy - 1}{-1 - y^2}$
B. $\frac{2xy + 1}{y^2 - 1}$
C. $\frac{-1 - y^2}{2xy - 1}$
D. $y^2 + 2xy - y$

Answer: C

$\frac{d}{dx}(x + xy^2 - y = 3)$
$\frac{dx}{dx} + \frac{d}{dx}(xy^2) - \frac{dy}{dx} = \frac{d}{dx}(3)$
$\frac{dx}{dx} + \left(\frac{d}{dx} \cdot x\right)y^2 + \left(\frac{d}{dx} \cdot y^2\right)x - \frac{dy}{dx} = 0$
$1 + y^2 + 2xy\frac{dy}{dx} - \frac{dy}{dx} = 0$
$\frac{dy}{dx}(2xy - 1) = -1 - y^2$

4. Let $f(x) = 5/2\,x^2 - e^x$. Find the value of x for which the second derivative $f''(x)$ equals zero.

A. $\ln 5$
B. $\ln 5e$
C. 0
D. e^5

Answer: A

$f(x) = 5/2\,x^2 - e^x$
$\Rightarrow f'(x) = 5x - e^x \ln e$
$\Rightarrow f''(x) = 5 - e^x \ln e$
$5 = e^x$, $\ln e^x = \ln 5$
$x = \ln 5$

5. What is the derivative of $f(x) = (1 + x^4 - 1/x)^{5/3}$

A. $20/3\,x^{17/3} + 5/3\,x^{8/3}$
B. $5/3\,(1 + x^4 + 1/x)^{2/3}$
C. $5/3\,(1 + x^4 - 1/x)^{2/3}(4x^3 + 1/x^2)$
D. $5/3\,x^{2/3}(4x^3 + 1/x^2)$

Answer: C

$f'(x) = \left(1 + x^4 - 5/x\right)^{5/3}\right)'(1 + x^4 - 1/x)'$
$= \frac{5}{3}(1 + x^4 - 5/x)^{5/3 - 1}(4x^3 + 1/x^2)$

5. Which of the following statements are true? (Since there may be more than one correct answer, determine all correct answers.)

A. If $\lim \frac{f(x)-f(a)}{x-a}$ exists, then f is differentiable at a.

B. If f is continous at a, then f is differantiable at a.

C. If $\lim_{x\to a} f(x)$ exists, then f is differantiable at a.

D. If f is differentiable at a, then $\lim_{x\to a} f(x) = f(a)$

Answer: A and D

6. If $F(x) = f(g(x))$, where $f(-2)=8$, $f'(-2)=4$, $f'(1)=3$, $g(1)=-2$, $g'(1)=6$, find $F'(1)$

A. 24
B. 8
C. 12
D. 20

Answer: A

$f'(-2) = \frac{f(x)-f(-2)}{x+2}$ $\quad g'(1) = \frac{g(x)-g(1)}{x-1}$ $\quad F'(x) = f \circ g(x)$

$f(x) = 4x+16$ $\quad g(x) = 6x-32$ $\quad = (4(6x-32)+16)$

$= 24$

7. Find the domain of the function $f(x) = \sqrt{1-x}\ln x$.

A. $(0, +\infty)$
B. $(-\infty, 1)$
C. $(0, 1]$
D. $[0, 1]$

$\sqrt{1-x}$, $1-x \geq 0$ and $\ln x$, $x > 0$

$1 \geq x$ and $x > 0$

$0 < x \leq 1$ $\quad (0, 1]$

Answer: C

8. For what value of constant c is the function f(x) is continous on $(-\infty, \infty)$?

$f(x) = \begin{cases} cx^2+2x, & \text{if } x < 2 \\ 2x+4, & \text{if } x \geq 2 \end{cases}$

If f(x) is continous

$\lim_{x\to 2^-} cx^2+2x = \lim_{x\to 2^+} 2x+4$

$c(2)^2 = 4$

$c = 1$

A. 0
B. 1
C. 2
D. 4

Answer: B

9. For what value(s) of x does the graph of $f(x) = \frac{1}{3}x^3 - x^2 + 3$ have a horizontal tangent?

A. 0
B. 0 and 3
C. 2
D. 0 and 2

horizontal tangent $\Rightarrow f'(x) = x^2 - 2x$

$x^2 - 2x = 0$

$x(x-2)$ $\quad x = 0$ and $x = 2$

Answer: D

10. Given the function $f(x) = x^3 - 4x^2 + 5x$, find the open interval(s) where f is concave down.

A. $(4/3, \infty)$
B. $(-\infty, 4/3)$
C. $(-\infty, 4/3)$
D. $(-4/3, 8)$

concave down, $f''(x) < 0$

$f'(x) = 3x^2 - 8x$

$f''(x) = 6x - 8 < 0$

$x < 8/6 = \cdot (-\infty, 4/3)$

Answer: C

11. Find an equation of tangent line to the curve $y = 2x\sin x$ at the point $(\pi/2, \pi)$

A. $y = 2x + 2\pi$

B. $y = 2x$

C. $y = -2x + 2\pi$

D. $y = -2x$

Answer: B

$y(\pi/2) = 2(\pi/2)\sin(\pi/2) = \pi$

$y'(\pi/2) = (2x)'\sin x + (\sin x)'(2x)$

$= 2\sin \pi/2 + \cos x(2x)$

$y'(\pi/2) = 2$

tangent line; $f(x) - f(\pi/2) = f'(\pi/2)(x - \pi/2)$

17. Find $K'(s)$ if $K(s) = \dfrac{\ln s}{s^2}$

A. $1/2s^2$

B. $-2/s^4$

C. $1/s^3 + \dfrac{2\ln s}{s^3}$

D. $1/s^3 - \dfrac{2\ln s}{s^3}$

Answer: D

$K(s) = \dfrac{(\ln s)'(s^2) - (s^2)'(\ln s)}{(s^2)^2}$

$= \dfrac{1/s(s^2) - 2s(\ln s)}{s^4}$

$= s\left(\dfrac{1 - 2\ln s}{s^4}\right)$

D. Find the $\lim_{h\to 0} \dfrac{|h|}{h}$

A. ⌟

B. ¬

C. ∼

D. The limit does not exist

Answer: D

14. On what interval(s) is the function $f(x) = x^3 e^x$ increasing?

A. $(-3,0) \cup (0, +\infty)$

B. $(-3, \infty)$

C. $[-3, 0)$

D. $[-3, 0) \cup (0, +\infty)$

Answer: A

$f'(x) = (x^3)'e^x + (e^x)'x^3$

$= 3x^2e^x + e^x x^3$

$= e^x x^2(3 + x) > 0$, increasing

$x \neq 0 \quad x > -3$

$(-3, 0) \cup (0, \infty)$

16. Find $f'(t)$ where $f(t) = \dfrac{1 + t^2}{\sqrt{t}}$

A. $\dfrac{3t^2 - 1}{2t}$

B. $\dfrac{-t^{-3/2}}{2} + \dfrac{3t^{1/2}}{2}$

C. $\dfrac{-t^{-3/2}}{2} - \dfrac{3t^{1/2}}{2}$

D. $\dfrac{1}{2\sqrt{t}} + \dfrac{t^2}{2\sqrt{t}}$

Answer: B

$f(t) = \dfrac{1 + t^2}{\sqrt{t}}$

$f'(t) = \dfrac{(1+t^2)'(\sqrt{t}) - (\sqrt{t})'(1+t^2)}{(\sqrt{t})^2}$

$= \dfrac{2t\sqrt{t} - \frac{1}{2\sqrt{t}}(1+t^2)}{t}$

$= \dfrac{4t^2 - 1 - t^2}{2t\sqrt{t}}$

$= \dfrac{3t^2 - 1}{2t\sqrt{t}}$

16. What is the value of $f'(\pi/3)$, where $f(x) = \frac{g(x)}{\sin x}$, and it is known that $g(\pi/3) = 1$ and $g'(\pi/3) = 0$.

A. $-2/3$
B. $2/3$
C. $-1/\sqrt{3}$
D. $\frac{1}{2}$

Answer: A

$$f'(\pi/3) = \frac{g'(x)(\sin x) - (\sin x)'(g(x)}{\sin^2 x}$$

$$= \frac{(0)\sin\pi/3 - \cos(\pi/3)\,1}{\sin^2 \pi/3}$$

$$= -1/2 \div (\sqrt{3}/2)^2 = \frac{-1}{2} \times \frac{4}{3} = -2/3$$

17. Find the derivative $g''(x)$, where $g(x) = x \ln x$

A. $\ln x + 1$
B. $\ln x$
C. $1/x$
D. $-1/x^2$

Answer: C

$g(x) = x \ln x$

$g'(x) = (x)'\ln x + (\ln x)'(x)$

$= \ln x \cdot 1$

$g''(x) = 1/x$

18. Given $y = 5e^{3x} + \sin x$, dy/dx is ___.

A. $5e^{3x} + \cos x$
D. $15e^{3x} + \cos x$
C. $15e^{3x} - \cos x$
D. $2.667e^{3x} - \cos x$

Answer: B

$\frac{d}{dx}(y = 5e^{3x} + \sin x)$

$\frac{dy}{dx} = \frac{d}{dx}(5e^{3x}) + \frac{d}{dx}(\sin x)$

$= 15e^{3x} + \cos x$

19. Given, $\sin 6° = 0.105$, $\cos 6° = 0.994$, $y = \sin 2x$, $\frac{dy}{dx}$ at $x = 3°$ is most nearly

A. 0.9600
B. 0.9945
C. 1.920
D. 1.989

Answer: D

for fun $y = \sin 2x$

$\Rightarrow \frac{dy}{dx} = \frac{d}{dx}\sin 2x$

$= (\cos 2x)\frac{d}{dx}(2x)$

$= 2\cos 2x$

$= 2\cos 6° = 1.989$

20. Find the value of dy/dx for the function $y = x^3 \ln x$,

A. $3x^2 \ln x$
B. $3x^2 \ln x + x^2$
C. x^2
D. $3x$

Answer: B

$\frac{dy}{dx} = \frac{d}{dx}(x^3)(\ln x) + \left(\frac{d}{dx} \cdot \ln x\right)x^3$

$= 3x^2 \ln x + \frac{1}{x} \cdot y^3$

$= 3x^2 \ln x + x^2$

21. If $x^2 + 2xy = y^2$, then $\frac{dy}{dx}$ is

A. $\frac{x+y}{y-x}$
B. $2x+2y$
C. $\frac{x+y}{y}$
D. $-x$

Answer: A

$\Rightarrow \frac{d}{dx}(x^2 + 2xy = y^2)$

$\Rightarrow \frac{d}{dx}(x^2) + \frac{d}{dx}(2xy) = \frac{d}{dx} \cdot y^2$

$\Rightarrow 2x\frac{dx}{dx} + \frac{d}{dx}(2x)y + \left(\frac{d}{dx} \cdot y\right)2x$

$= 2y\frac{dy}{dx}$

~~$2x + 2\frac{dy}{dx}y$~~

$2x + 2y + \frac{dy}{dx}(2x)$

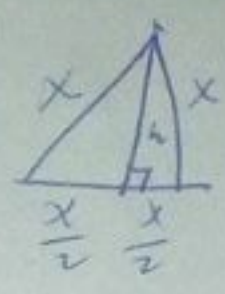
$= 2y\frac{dy}{dx}$

$2x + 2y = (2y - 2x)\frac{dy}{dx}$

27. The side of equilateral triangle is increasing at the rate of 2 cm/s. The rate at which area increases when the side is 10cm will be ____.

a. $10\ cm^2/s$
b. $\sqrt{3}\ cm^2/s$
C. $10\sqrt{3}\ cm^2/s$
D. $10/3\ cm^2/s$.

Answer: C



$\left(\frac{x}{2}\right)^2 + h^2 = x^2$ --- pythagorean theorem

$h^2 = x^2 - \frac{x^2}{4} = \frac{3x^2}{4}$ $h = \frac{\sqrt{3}x}{2}$

$A = \frac{1}{2} x \left(\frac{\sqrt{3}}{2}\right) x$ $\frac{A}{dt} = \frac{\sqrt{3}}{2} x\, dx/dt$

$A = \frac{\sqrt{3}}{4} x^2$ $= \frac{\sqrt{3}}{2}(10cm)(2\ cm/s)$

25. The point(s) on the curve $y = x^2$, at which y-coordinate is changing six times as fast as x-coordinate is/are ____.

A. (2,4)
B. (3,9)
C. (3,9), (9,3)
D. (6,2)

Answer: B

$y = x^2$

$\frac{dy}{dt} = 2x \frac{dx}{dt}$

$\therefore 6 \frac{dx}{dt} = 2x \frac{dx}{dt}$

$x = 3$

24. The absolute maximum value of $y = x^3 - 3x + 2$ in $0 \le x \le 2$ is

A. 4
B. 6
C. 2
D. 0

Answer: A

$y' = 3x^2 - 3 = 3(x^2 - 1) = 3(x+1)(x-1)$

$y'(1) = 1^3 - 3 + 2 = 0$

$y(-1) = (-1 - 3(-1) + 2) = 4$

$y(0) = 2$

25. If $f(x) = (4 - x^2)^{2/3}$, then f has a:

A. local maxima at x=0
B. local maxima at x=2
C. local maxima at x=-2
D. none of these

$f(0) = \sqrt[3]{(4)^2} = \sqrt[3]{16}$ --- max

$f(2) = \sqrt[3]{(0)^2} = 0$ --- min

$f(-2) = \sqrt[3]{0^2} = 0$ --- min

Answer: A

26. Let $f(x) = x^3 - 6x^2 + 9x + 18$, then f(x) is strictly decreasing in

A. $(-\infty, 1]$
B. $[3, \infty)$
C. $(-\infty, 1] \cup [3, \infty)$
D. $[1, 3]$

$f(x) = x^3 - 6x^2 + 9x + 18$

$f'(x) = 3x^2 - 12x + 9$

$= 3(x^2 - 4x + 3)$

$= 3(x-3)(x-1)$

		1		3	
$(x-3)$	- - -	0	- - -	0	+ + +
$x-1$	- - -	0	+ + +	+	+ + +
$3(x-3)(x-1)$	+ + +	0	- - -	0	+ + +

f(x) is increasing $(-\infty, 1] \cup [3, \infty)$

f(x) is decreasing $[1, 3]$

Answer: D

27. The radius of a circle is increasing at rate of 4 cm/min what is rate of change of area when $r = 10$ cm.

A. $80\pi \text{ cm}^2/\text{min}$
B. $40\pi \text{ cm}^2/\text{min}$
C. $80 \text{ cm}^2/\text{min}$
D. $40 \text{ cm}^2/\text{min}$

Answer: A

$$\frac{dA}{dt} = \pi r^2 \left(\frac{d}{dt}\right)$$

$$\frac{A}{dt} = 2\pi r \frac{d}{dt}$$

$$= 2\pi (10\text{cm})(4 \text{ cm/min})$$

$$= 80\pi \text{ cm}^2/\text{min}$$

28. On which one of the following intervals does $f(x) = x^4 + 4x$ increase? (CUEE)

A. $(-\infty, -1]$
B. $(-\infty, 0]$
C. $[-1, \infty)$
D. $(-\infty, \infty)$

Answer: C

$f(x) = x^4 + 4x$

$f'(x) = 4x^3 + 4 \geq 0$, increasing

$f'(x) \geq 0 \quad 4x^3 > -4$

$\frac{4}{4} - \frac{4}{4}$

$x \geq \sqrt[3]{-1}$

$x \geq -1$

$[-1, \infty]$

29. The position of a particle moving along a horizontal line at any time is given by $s(t) = 3t^2 - 4t - 5$. For what value(s) of t is the particle is not moving

A. $2/3$
B. $2, -2/3$
C. 2
D. $3/4$

Answer: A

is not moving $\Rightarrow V(t) = 0$.

for $s(t) = 3t^2 - 4t - 5$

$V(t) = 6t - 4 = 0$

$\Rightarrow 6t = 4$

$t = 2/3$

EUEE (Unit 3 & Unit 4 in hybrid)

30. At what point does the equation of a line tangent to the graph of $f(x) = e^{2x}$ at $x=0$ intersects x axis (..UEE)

A. $(\frac{1}{2}, 0)$
B. $(-2, 0)$
C. $(\frac{1}{2}, 0)$
D. $(\sqrt{e}, 0)$

Answer: A

$f(0) = 1$
$f'(0) = 2e^{2(0)} = 2$
$f(x) - f(0) = f'(0)(x-0)$
$f(x) = 2(x) + 1$

x axis, $y=0$
$2x+1=0$
$x = -1/2$

31. Let $f(x) = \begin{cases} 3x-1, & x \leq 2 \\ x^2+1, & x>2 \end{cases}$ then the derivative of $f(x)$ at $x=2$ is ___.

A. 3
B. 4
C. 5
D. Doesn't exist.

$\lim_{x\to 2^+} \frac{x^2+1 - f(2)}{x-2} \neq \lim_{x\to 2^-} \frac{3x-1-f(2)}{x-2}$

$\therefore$ DNE Answer: D

32. If $f(x) = \pi^{\pi x}$ then $f'''(0) =$ ___

A. π^3
B. $(\pi \ln \pi)^3$
C. $(\ln 5)^3$
D. $\pi(\ln 5)^3$

Answer: B

$f(x) = \pi^{\pi x}$
$f'(x) = \pi^{\pi x}(\pi \ln \pi)$
$f''(x) = \pi^{\pi x}(\pi \ln \pi)^2$
$f'''(x) = \pi^{\pi x}(\pi \ln 5)^3$

$f'''(0) = \pi^{\pi(0)}(\pi \ln \pi)^3$
$= (\pi \ln \pi)^3$

33. The equation of the line tangent to the graph of $f(x) = 2x - \sin(1-x)$ at $(1, f(1))$

A. $y = 3x - 5$
B. $y = 3x - 1$
C. $y = x + 1$
D. $y = x - 2$

Answer: C

34. What is the maximum area of a triangle which is inscribed in a circle of radius 2 units and one of its passing from the center of the circle?

A. 2 sq. unit
B. 4π sq. unit
C. 8 sq. unit
D. 4 sq. unit

$A_\Delta = \frac{1}{2} b \cdot h$, $b = 2(2 \text{ unit}) = 4 \text{ units}$
$= \frac{1}{2} b \sqrt{(4 \text{ units})^2 - b^2}$ ... max
$= \frac{1}{2} \sqrt{4^2 b^2 - b^4}$

To maximize derivate the radicand

$16b^2 - b^4$
$32b - 4b^3$
$4b(8 - b^2) = 0$ $b = \sqrt{8}$, $h = \sqrt{8}$
$A = \frac{1}{2}(\sqrt{8})(\sqrt{8}) = 4 \text{ unit}^2$

Answer: B

35. An open top square base rectangular box container is to have a volume of $32\,cm^3$. What should be the height of the box to obtain the minimum possible surface area.

A. $h = 2\,cm$
B. $h = 8\,cm$
C. $h = 4\,cm$
D. $h = 16\,cm$

$V = x^2 h = 32\,cm^3$

$h = \frac{32\,cm^3}{x^2}$

Surface area $= x^2 + 4xh$

$= x^2 + 4x\left(\frac{32}{x^2}\right)$

$x = 4$

$h = \frac{32\,cm^3}{(4\,cm)^2} = \underline{\underline{2\,cm}}$

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$y = x^2$

$\frac{dy}{dx} = \frac{d}{dx} x^2$

$= 2x \frac{d}{dx}$

$6 \frac{dy}{dx} = 2x \frac{d}{dx}$

$x = 6/2$

$\sqrt{(4-x^2}$