

Pre-Publication



Chapter

7

Quadratic Functions and Equations

LEARNING GOALS

You will be able to develop your algebraic and graphical reasoning by

- Determining the characteristics of quadratic functions and using these characteristics to sketch their graphs
- Solving quadratic equations by graphing, by factoring, and by using the quadratic formula
- Solving problems that can be modelled with quadratic functions and equations

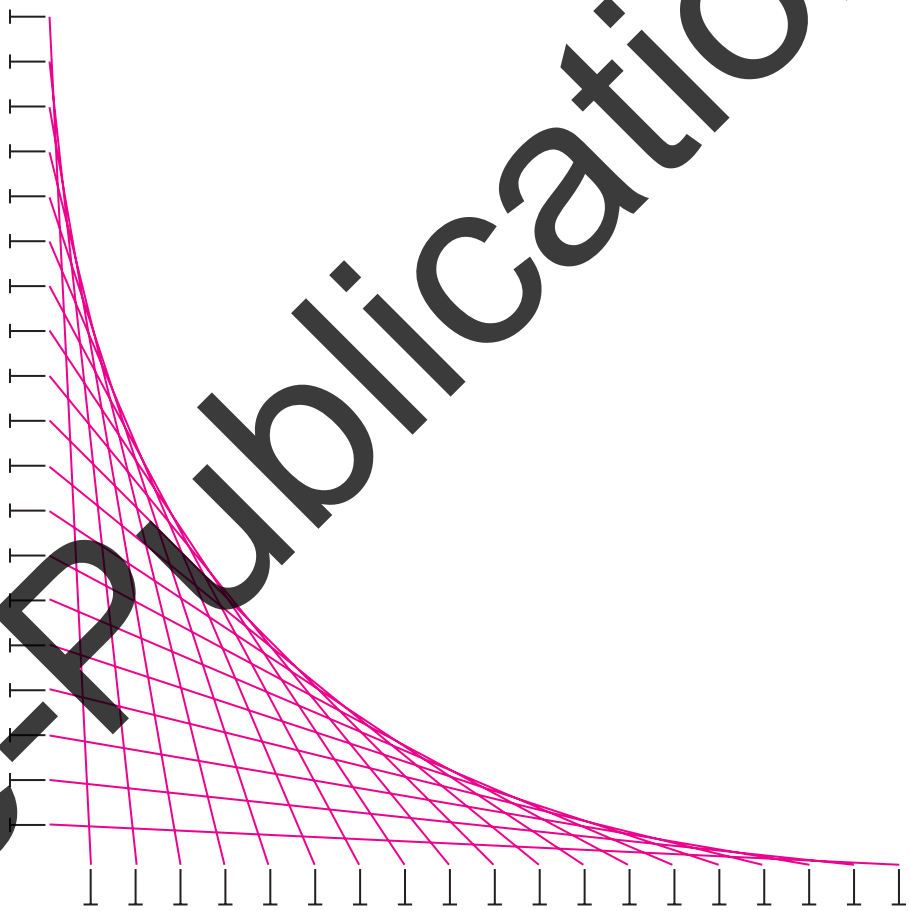
? If you drew a graph of height versus time for a javelin throw, what would it look like? How could you use your graph to determine how long the javelin was in the air?

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7 Getting Started

String Art

Robert's grandmother is teaching him how to make string art. In string art, nails are spaced evenly on a board and connected to each other by lines of string or yarn. The board that Robert is using has an array of 38 nails: 19 placed horizontally and 19 placed vertically as shown. The nails are 1 cm apart. Robert started to make the artwork and noticed that the lines of yarn were different lengths.



? Use a model to describe the relation between the position of each nail and the length of the yarn that connects it to another nail.

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A. Model Robert's art on a coordinate grid, using line segments to represent the pieces of string.

B. Determine the length of each line segment.

C. Create a table of values like the one shown to compare nail position with string length.

Nail Position, *x*	String Length, *y*
1	
2	
3	

D. Describe any patterns you see in your table of values.

E. Is the relation linear? Explain.

F. Determine the domain and range for the relation.

G. Graph the relation.

H. What conclusions about string length can you make from your models?

WHAT DO You Think?

Decide whether you agree or disagree with each statement. Explain your decision.

1. Graphs of functions are straight lines.
2. Functions are not symmetrical.
3. If the domain of a function is the set of real numbers, then its range will also be the set of real numbers.

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7.1 Exploring Quadratic Relations

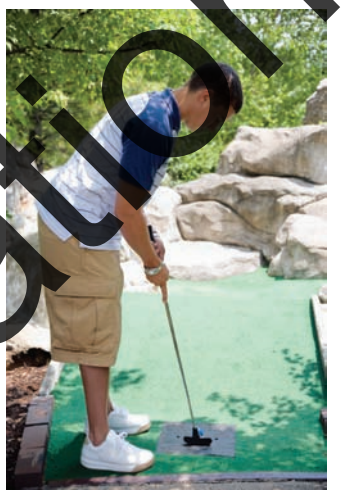
YOU WILL NEED

- graphing technology

GOAL

Determine the characteristics of quadratic relations.

EXPLORE the Math



A moving object that is influenced by the force of gravity can often be modelled by a **quadratic relation** (assuming that there is no friction). For example, on one hole of a mini-golf course, the ball rolls up an incline after it is hit, slowing all the way due to gravity. If the ball misses the hole, it rolls back down the incline, accelerating all the way. If the initial speed of the ball is 6 m/s, the distance of the ball from its starting point in metres, y, can be modelled by the quadratic relation

$$y = -2.5x^2 + 6x$$

where x is the time in seconds after the ball leaves the putter.

quadratic relation
A relation that can be written in the standard form $y = ax^2 + bx + c$, where $a \neq 0$; for example, $y = 4x^2 + 2x + 1$

? How does changing the coefficients and constant in a relation that is written in the form $y = ax^2 + bx + c$ affect the graph of the relation?

Reflecting

parabola
The shape of the graph of any quadratic relation.

A. Describe the common characteristics of each of the **parabolas** you graphed.

B. Describe any symmetry in your graphs.

C. Are the quadratic relations that you graphed functions? Justify your decision.

D. What effects do the following changes have on a graph of a quadratic relation?

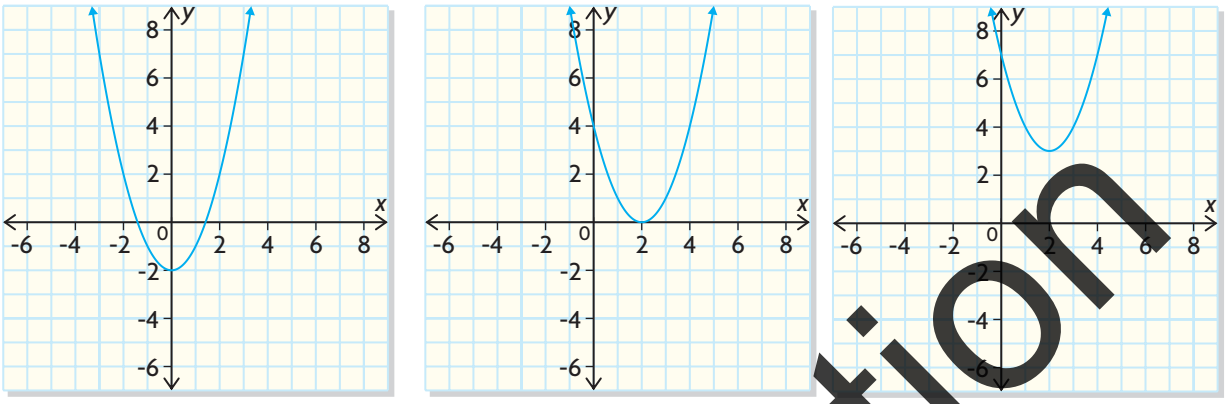
i) The value of a is changed, but b and c are left constant.

ii) The value of b is changed, but a and c are left constant.

iii) The value of c is changed, but a and b are left constant.

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E. The graphs of three quadratic relations are shown. Predict possible values of a, b, and c in the equation for each graph.



In Summary

Key Ideas

- The degree of all quadratic functions is 2.
- The standard form of a quadratic function is

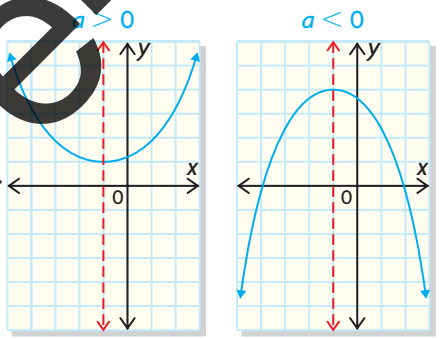
$$y = ax^2 + bx + c$$

where $a \neq 0$.

- The graph of any quadratic function is a parabola with a single vertical line of symmetry.

Need to Know

- A quadratic function that is written in standard form, $y = ax^2 + bx + c$, has the following characteristics:
  - The highest or lowest point on the graph of the quadratic function lies on its vertical line of symmetry.
  - If a is positive, the parabola opens up. If a is negative, the parabola opens down.



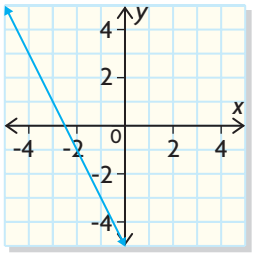
  - Changing the value of b changes the location of the parabola's line of symmetry.
  - The constant term, c, is the value of the parabola's y-intercept.

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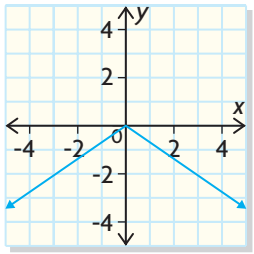
FURTHER Your Understanding

1. Which graphs appear to represent quadratic relations? Explain.

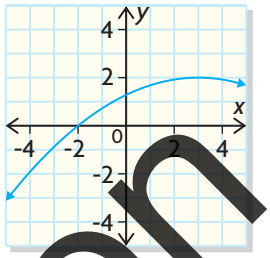
a)



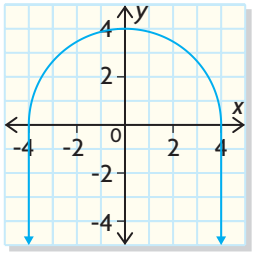
c)



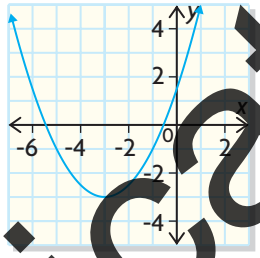
e)



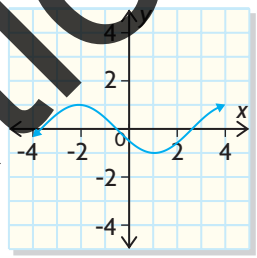
b)



d)



f)



2. Which of the following relations are quadratic? Explain.

a) $y = 2x - 7$

b) $y = 2x(x + 3)$

c) $y = (x + 4)^2 + 1$

d) $y = x^2 - 5x - 6$

e) $y = 4x^3 + x^2 - x$

f) $y = x(x + 1)^2 - 7$

3. State the y-intercept for each quadratic relation in question 2.

4. Explain why the condition $a \neq 0$ must be stated when defining the standard form, $y = ax^2 + bx + c$.

5. Each of the following quadratic functions can be represented by a parabola. Does the parabola open up or down? Explain how you know.

a) $y = x^2 - 4$

b) $y = -2x^2 + 6x$

c) $y = 9 - x + 3x^2$

d) $y = -\frac{2}{3}x^2 - 6x + 1$

6. Each table of values lists points in a quadratic relation. Decide, without graphing, the direction in which the parabola opens.

a)

x	−4	−3	−2	−1	0	1
y	12	5	0	−3	−4	−3

b)

x	0	1	2	3	4	5
y	−13	−3	3	5	3	−3

c)

x	−5	−4	−3	−2	−1	0
y	3.0	−0.5	−3.0	−4.5	−5.0	−4.5

d)

x	0	1	2	3	4	5
y	−4	19	40	59	76	91

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7.2 Properties of Graphs of Quadratic Functions

GOAL

Identify the characteristics of graphs of quadratic functions, and use the graphs to solve problems.

YOU WILL NEED

- ruler
- graph paper

LEARN ABOUT the Math



Nicolina plays on her school's volleyball team. At a recent match, her Nonno, Marko, took some time-lapse photographs while she warmed up. He set his camera to take pictures every 0.25 s. He started his camera at the moment the ball left her arms during a bump and stopped the camera at the moment that the ball hit the floor. Marko wanted to capture a photo of the ball at its greatest height. However, after looking at the photographs, he could not be sure that he had done so. He decided to place the information from his photographs in a table of values.

Time (s)	Height (ft)
0.00	2
0.25	6
0.50	8
0.75	8
1.00	6
1.25	2

From his photographs, Marko observed that Nicolina struck the ball at a height of 2 ft above the ground. He also observed that it took about 1.25 s for the ball to reach the same height on the way down.

? When did the volleyball reach its greatest height?

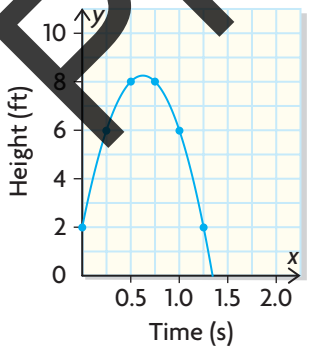
EXPLORE...

- Parabolic skis are marketed as performing better than traditional straight-edge skis. Parabolic skis are narrower in the middle than on the ends. Design one side of a parabolic ski on a coordinate grid. In groups, discuss where any lines of symmetry occur and how the parabolic shape works in your design.



EXAMPLE 1 Using symmetry to estimate the coordinates of the vertex

Marko's Solution

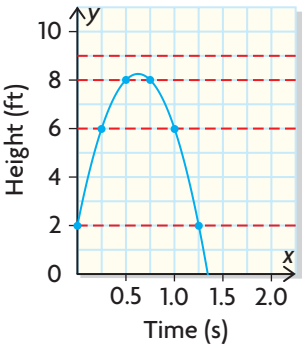


I plotted the points from my table, and then I sketched a graph that passed through all the points.

The graph looked like a parabola, so I concluded that the relation is probably quadratic.

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vertex
The point at which the quadratic function reaches its maximum or minimum value.



I knew that I could draw horizontal lines that would intersect the parabola at two points, except at the **vertex**, where a horizontal line would intersect the parabola at only one point.

Using a ruler, I drew horizontal lines and estimated that the coordinates of the vertex are around (0.6, 8.2).

This means that the ball reached maximum height at just over 8 ft, about 0.6 s after it was launched.

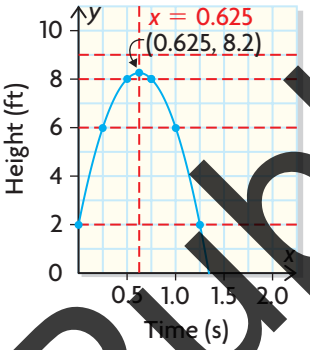
Equation of the axis of symmetry:

$$x = \frac{0 + 1.25}{2}$$

$$x = 0.625$$

I used points that have the same *y*-value, (0, 2) and (1.25, 2), to determine the equation of the **axis of symmetry**. I knew that the axis of symmetry must be the same distance from each of these points.

axis of symmetry
A line that separates a 2-D figure into two identical parts. For example, a parabola has a vertical axis of symmetry passing through its vertex.



From the equation, the *x*-coordinate of the vertex is 0.625. From the graph, the *y*-coordinate of the vertex is close to 8.2.

Therefore, 0.625 s after the volleyball was struck, it reached its maximum height of approximately 8 ft 2 in.

I revised my estimate of the coordinates of the vertex.

Reflecting

A. How could Marko conclude that the graph was a quadratic function?

B. If a horizontal line intersects a parabola at two points, can one of the points be the vertex? Explain.

C. Explain how Marko was able to use symmetry to determine the time at which the volleyball reached its maximum height.

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APPLY the Math

EXAMPLE 2 | Reasoning about the maximum value of a quadratic function

Some children are playing at the local splash pad. The water jets spray water from ground level. The path of water from one of these jets forms an arch that can be defined by the function

$$f(x) = -0.12x^2 + 3x$$

where x represents the horizontal distance from the opening in the ground in feet and $f(x)$ is the height of the sprayed water, also measured in feet. What is the maximum height of the arch of water, and how far from the opening in the ground can the water reach?



Manuel's Solution

$f(x) = -0.12x^2 + 3x$

I knew that the degree of the function is 2, so the function is quadratic. The arch must be a parabola.

I also knew that the coefficient of x^2, a, is negative, so the parabola opens down. This means that the function has a **maximum value**, associated with the y-coordinate of the vertex.

$f(0) = 0$

$f(1) = -0.12(1)^2 + 3(1)$
$f(1) = -0.12 + 3$
$f(1) = 2.88$

I started to create a table of values by determining the y-intercept. I knew that the constant, zero, is the y-coordinate of the y-intercept. This confirms that the stream of water shoots from ground level.

I continued to increase x by intervals of 1 until I noticed a repeat in my values. A height of 18.72 ft occurs at horizontal distances of 12 ft and 13 ft.

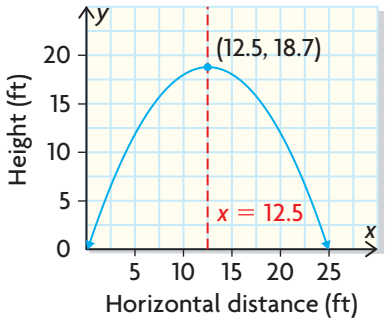
x	0	1	2	...	12	13
$f(x)$	0	2.88	5.52	...	18.72	18.72

Based on symmetry and the table of values, the maximum value of $f(x)$ will occur halfway between (12, 18.72) and (13, 18.72).

The arch of water will reach a maximum height between 12 ft and 13 ft from the opening in the ground.

maximum value
The greatest value of the dependent variable in a relation.

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I used my table of values to sketch the graph. I extended the graph to the x-axis. I knew that my sketch represented only part of the function, since I am only looking at the water when it is above the ground.

$$x = \frac{12 + 13}{2}$$
$$x = 12.5$$

I used two points with the same y-value, (12, 18.72) and (13, 18.72), to determine the equation of the axis of symmetry.

Equation of the axis of symmetry:
$x = 12.5$

Height at the vertex:

$$f(x) = -0.12x^2 + 3x$$
$$f(12.5) = -0.12(12.5)^2 + 3(12.5)$$
$$f(12.5) = -0.12(156.25) + 37.5$$
$$f(12.5) = -18.75 + 37.5$$
$$f(12.5) = 18.75$$

I knew that the x-coordinate of the vertex is 12.5, so I substituted 12.5 into the equation to determine the height of the water at this horizontal distance.

The water reaches a maximum height of 18.75 ft when it is 12.5 ft from the opening in the ground.

The water can reach a maximum horizontal distance of 25 ft from the opening in the ground.

Due to symmetry, the opening in the ground must be the same horizontal distance from the axis of symmetry as the point on the ground where the water lands. I simply multiplied the horizontal distance to the axis of symmetry by 2.

The domain of this function is $0 \leq x \leq 25$, where $x \in \mathrm{R}$.

Your Turn

Another water arch at the splash pad is defined by the following quadratic function:

$$f(x) = -0.15x^2 + 3x$$

a) Graph the function, and state its domain for this context.
b) State the range for this context.
c) Explain why the original function describes the path of the water being sprayed, whereas the function in *Example 1* does not describe the path of the volleyball.

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EXAMPLE 3 Graphing a quadratic function using a table of values

Sketch the graph of the function:

$$y = x^2 + x - 2$$

Determine the y-intercept, any x-intercepts, the equation of the axis of symmetry, the coordinates of the vertex, and the domain and range of the function.

Anthony's Solution

$y = x^2 + x - 2$

The function is a quadratic function in the form

$y = ax^2 + bx + c$
$a = 1$
$b = 1$
$c = -2$

The degree of the given equation is 2, so the graph will be a parabola.

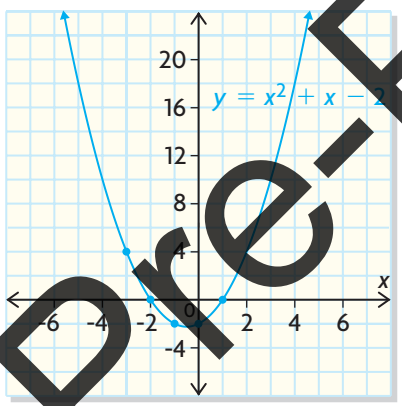
Since the coefficient of x^2 is positive, the parabola opens up.

Since the y-coordinate of the y-intercept is less than zero and the parabola opens up, there must be two x-intercepts and a **minimum value**.

x	-3	-2	-1	0	1
y	4	0	-2	-2	0

I made a table of values. I included the y-intercept, $(0, -2)$, and determined some other points by substituting values of x into the equation.

I stopped determining points after I had identified both x-intercepts, because I knew that I had enough information to sketch an accurate graph.



I graphed each coordinate pair and then drew a parabola that passed through all the points.

Equation of the axis of symmetry:

$$x = \frac{-2 + 1}{2}$$

$$x = \frac{-1}{2}$$

$$x = -0.5$$

I used the x-intercepts to determine the equation of the axis of symmetry.

minimum value

The least value of the dependent variable in a relation.

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y-coordinate of the vertex:

$y = (-0.5)^2 + (-0.5) - 2$
$y = 0.25 - 0.5 - 2$
$y = -2.25$
The vertex is $(-0.5, -2.25)$.

I knew that the vertex is a point on the axis of symmetry. The x-coordinate of the vertex must be -0.5. To determine the y-coordinate of the vertex, I substituted -0.5 for x in the given equation.

The y-intercept is -2.
The x-intercepts are -2 and 1.
The equation of the axis of symmetry is $x = -0.5$
The vertex is $(-0.5, -2.25)$.

The vertex, $(-0.5, -2.25)$, defines the minimum value of y.

Domain and range:
$\{(x, y) \mid x \in \mathrm{R}, y \geq -2.25, y \in \mathrm{R}\}$

No restrictions were given for x, so the domain is all real numbers.

Your Turn

Explain how you could decide if the graph of the function $y = -x^2 + x + 2$ has x-intercepts.

EXAMPLE 4 Locating a vertex using technology

A skier's jump was recorded in frame-by-frame analysis and placed in one picture, as shown.



The skier's coach used the picture to determine the quadratic function that relates the skier's height above the ground, y, measured in metres, to the time, x, in seconds that the skier was in the air:

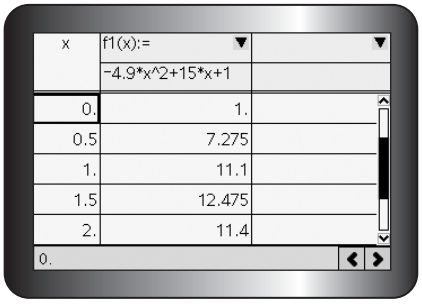
$$y = -4.9x^2 + 15x + 1$$

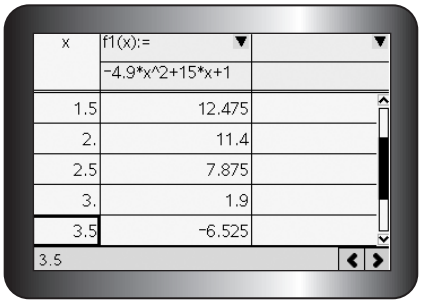
Graph the function. Then determine the skier's maximum height, to the nearest tenth of a metre, and state the range of the function for this context.

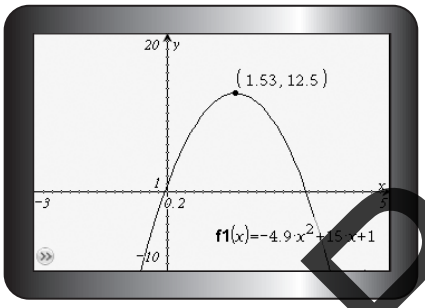
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Isidro's Solution

$y = -4.9x^2 + 15x + 1$







I entered the equation into my calculator.

To make sure that the graph models the situation, I set up a table of values. The skier's jump will start being timed at 0 s, and the skier will be in the air for only a few seconds, so I set the table to start at an x-value of zero and to increase in increments of 0.5.

I decided to set the minimum height at 0 m—it doesn't make sense to extend the function below the x-axis, because the skier cannot go below the ground. I checked the table and noticed that the greatest y-value is only 12.475... m, and that y is negative at 3.5 s. I used these values to set an appropriate viewing window for the graph.

I graphed the function and used the calculator to locate the maximum value of the function.

The skier achieved a maximum height of 12.5 m above the ground 1.5 s into the jump.

The range of the function is $\{y \mid 0 \leq y \leq 12.5, y \in \mathbb{R}\}$.

In this situation, the height of the skier varies between 0 m and 12.5 m.

Your Turn

On the next day of training, the coach asked the skier to increase his speed before taking the same jump. At the end of the day, the coach analyzed the results and determined the equation that models the skier's best jump:

$$y = -4.9x^2 + 20x + 1$$

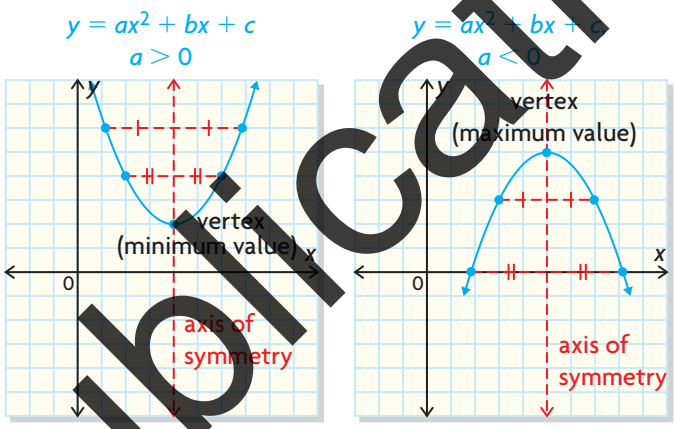
How much higher did the skier go on this jump?

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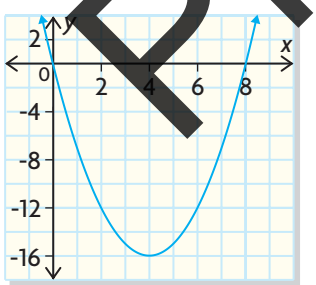
Key Idea

- A parabola that is defined by the equation $y = ax^2 + bx + c$ has the following characteristics:
  - If the parabola opens down ($a < 0$), the vertex of the parabola is the point with the greatest y-coordinate. The y-coordinate of the vertex is the maximum value of the function.
  - If the parabola opens up ($a > 0$), the vertex of the parabola is the point with the least y-coordinate. The y-coordinate of the vertex is the minimum value of the function.
  - The parabola is symmetrical about a vertical line, the axis of symmetry, through its vertex.



Need to Know

- For all quadratic functions, the domain is the set of real numbers, and the range is a subset of real numbers.
- When a problem can be modelled by a quadratic function, the domain and range of the function may need to be restricted to values that have meaning in the context of the problem.



CHECK Your Understanding

1. **a)** Determine the equation of the axis of symmetry for the parabola.
b) Determine the coordinates of the vertex of the parabola.
c) State the domain and range of the function.

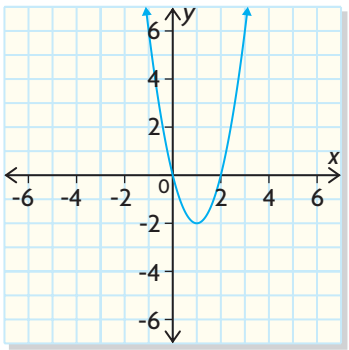
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2. State the coordinates of the y-intercept and two additional ordered pairs for each function.

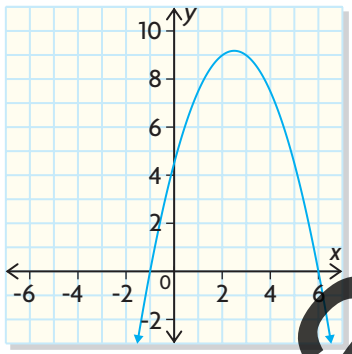
a) $f(x) = 2x^2 + 8x + 8$ **b)** $f(x) = 4x - x^2$

3. For each function, identify the x- and y-intercepts, determine the equation of the axis of symmetry and the coordinates of the vertex, and state the domain and range.

a)



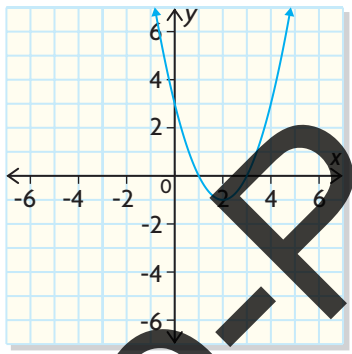
b)



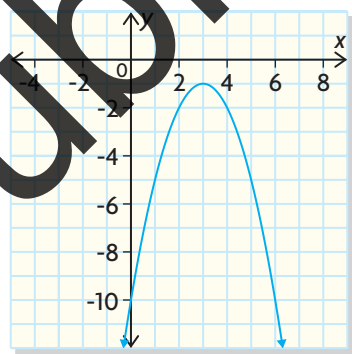
PRACTISING

4. For each function, identify the equation of the axis of symmetry, determine the coordinates of the vertex, and state the domain and range.

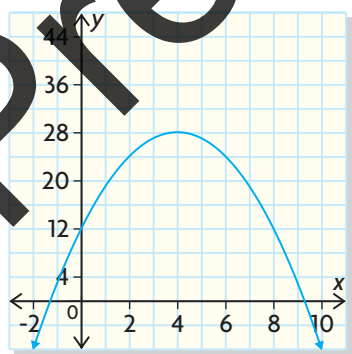
a)



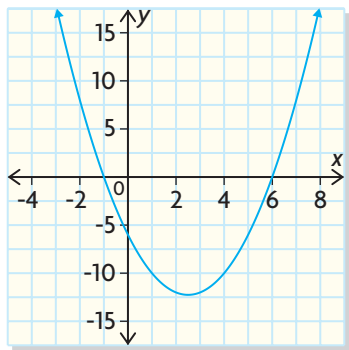
c)



b)



d)



Pre-Publication

5. Each parabola in question 4 is defined by one of the functions below.

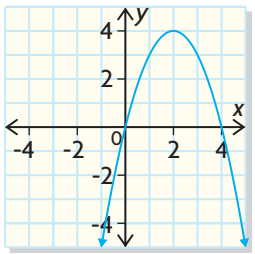
a) $f(x) = x^2 - 5x - 6$ **c)** $f(x) = -x^2 + 6x - 10$

b) $f(x) = -x^2 + 8x + 12$ **d)** $f(x) = x^2 - 4x + 3$

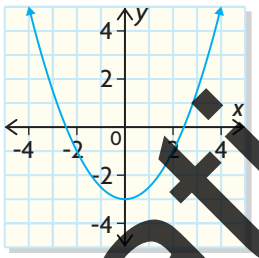
Identify the function that defines each graph. Then verify the coordinates of the vertex that you determined in question 4.

6. State whether each parabola has a minimum or maximum value, and then determine this value.

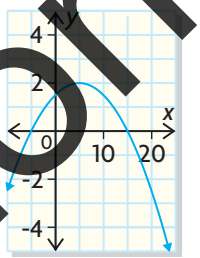
a)



b)



c)



x	−4	−2	0	2	4
y					

7. a) Complete the table of values shown for each of the following functions.

i) $y = -\frac{1}{2}x^2 + 5$ **ii)** $y = \frac{3}{2}x^2 - 2$

b) Graph the points in your table of values.

c) State the domain and range of the function.

8. a) Graph the functions $y = 2x^2$ and $y = -2x^2$.

b) How are the graphs the same? How are the graphs different?

c) Suppose that the graphs were modified so that they became the graphs of $y = 2x^2 + 4$ and $y = -2x^2 + 4$. Predict the vertex of each function, and explain your prediction.

9. For each of the following, both points, (x, y), are located on the same parabola. Determine the equation of the axis of symmetry for each parabola.

a) (0, 2) and (6, 2) **c)** (−6, 0) and (2, 0)

b) (1, −3) and (9, −3) **d)** (−5, −1) and (3, −1)

10. A parabola has x-intercepts $x = 3$ and $x = -9$. Determine the equation of the axis of symmetry for the parabola.

11. a) Graph each function.

i) $f(x) = 2x^2 + 3$ **iii)** $f(x) = x^2 - 6x + 9$

ii) $f(x) = -x^2 - 7x + 4$ **iv)** $f(x) = \frac{1}{2}x^2 - 4x + 3$

b) Determine the equation of the axis of symmetry and the coordinates of the vertex for each parabola.

c) State the domain and range of each function.

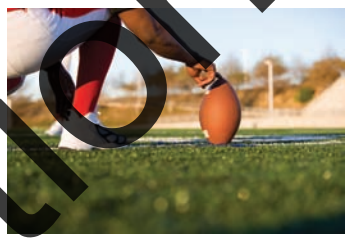
Pre-Publication

12. In southern Alberta, near Fort Macleod, you will find the famous Head-Smashed-In Buffalo Jump. In a form of hunting, Blackfoot once herded buffalo and then stampeded the buffalo over the cliffs. If the height of a buffalo above the base of the cliff, $f(x)$, in metres, can be modelled by the function

$$f(x) = -4.9x^2 + 12$$

where x is the time in seconds after the buffalo jumped, how long was the buffalo in the air, to the nearest hundredth of a second?

13. In the game of football, a team can score by kicking the ball over a bar and between two uprights. For a kick in a particular game, the height of the ball above the ground, y, in metres, can be modelled by the function



$$y = -4.9x^2 + 25x$$

where x is the time in seconds after the ball left the foot of the player.

a) Determine the maximum height that this kick reached, to the nearest tenth of a metre.

b) State any restrictions that the context imposes on the domain and range of the function.

c) How long was the ball in the air?

14. An annual fireworks festival, held near the seawall in downtown Vancouver, choreographs rocket launches to music. The height of one rocket, $h(t)$, in metres over time, t, in seconds, is modelled by the function



$$h(t) = -4.9t^2 + 80t$$

Determine the domain and range of the function that defines the height of this rocket, to the nearest tenth of a metre.

15. Melinda and Genevieve live in houses that are next to each other. Melinda lives in a two-storey house, and Genevieve lives in a bungalow. They like to throw a tennis ball to each other through their open windows. The height of a tennis ball thrown from Melinda to Genevieve, $f(x)$, in feet, over time, x, measured in seconds is modelled by the function

$$f(x) = -5x^2 + 6x + 12$$

What are the domain and range of this function if Genevieve catches the ball 4 ft above the ground? Draw a diagram to support your answer.

Pre-Publication

16. Sid knows that the points $(-1, 41)$ and $(5, 41)$ lie on a parabola defined by the function

$$f(x) = 4x^2 - 16x + 21$$

a) Does $f(x)$ have a maximum value or a minimum value? Explain.

b) Determine, in two different ways, the coordinates of the vertex of the parabola.

Closing

17. **a)** Explain the relationship that must exist between two points on a parabola if the x-coordinates of the points can be used to determine the equation of the axis of symmetry for the parabola.

b) How can the equation of the axis of symmetry be used to determine the coordinates of the vertex of the parabola?

Extending



18. Gamez Inc. makes handheld video game players. Last year, accountants modelled the company's profit using the equation

$$P = -5x^2 + 60x - 135$$

This year, accountants used the equation

$$P = -7x^2 + 70x - 63$$

In both equations, P represents the profit, in hundreds of thousands of dollars, and x represents the number of game players sold, in hundreds of thousands. If the same number of game players were sold in these years, did Gamez Inc.'s profit increase? Justify your answer.

19. A parabola has a range of $\{y \mid y \leq 14.5, y \in \mathrm{R}\}$ and a y-intercept of 10. The axis of symmetry of the parabola includes point $(-3, 5)$. Write the equation that defines the parabola in standard form if $a = \frac{-1}{2}$.

Pre-Publication

7.3 Solving Quadratic Equations by Graphing

GOAL

Solve quadratic equations by graphing the corresponding function.

YOU WILL NEED

- graphing technology
- graph paper

EXPLORE...

- Graph the quadratic function $y = x^2 + 5$. How could you use your graph to solve the equation $21 = x^2 + 5$? What are some other equations you could solve with your graph?

INVESTIGATE the Math

Bonnie launches a model rocket from the ground with an initial velocity of 68 m/s. The following function, $h(t)$, can be used to model the height of the rocket, in metres, over time, t, in seconds:

$$h(t) = -4.9t^2 + 68t$$

Bonnie's friend Sasha is watching from a lookout point at a safe distance. Sasha's eye level is 72 m above the ground.



? How can you determine the times during the flight when the rocket will be at Sasha's eye level?

A. What is the value of $h(t)$ when the rocket is at Sasha's eye level?

B. Substitute the value of $h(t)$ that you calculated in part A into the function

$$h(t) = -4.9t^2 + 68t$$

to create a **quadratic equation**. You can solve this quadratic equation to determine when the rocket is at Sasha's eye level. Rewrite the quadratic equation in standard form.

C. Graph the function that corresponds to your equation. Use the **zeros** of the function to determine the t-intercepts.

D. Graph $h(t) = -4.9t^2 + 68t$. On the same axes, graph the horizontal line that represents Sasha's eye level. Determine the t-coordinates of the points where the two graphs intersect.

E. What do you notice about the t-coordinates of these points?

F. When will the rocket be at Sasha's eye level?

quadratic equation

A polynomial equation of the second degree; the standard form of a quadratic equation is $ax^2 + bx + c = 0$
For example:
$2x^2 + 4x - 3 = 0$

zero

In a function, a value of the variable that makes the value of the function equal to zero.

Pre-Publication

Reflecting

G. How were your two graphs similar? How were they different?

H. Describe the two different strategies you used to solve the problem. What are the advantages of each?

APPLY the Math

EXAMPLE 1 Verifying solutions to a quadratic equation

The flight time for a long-distance water ski jumper depends on the initial velocity of the jump and the angle of the ramp. For one particular jump, the ramp has a vertical height of 5 m above water level. The height of the ski jumper in flight, $h(t)$, in metres, over time, t, in seconds, can be modelled by the following function:

$$h(t) = 5.0 + 24.46t - 4.9t^2$$

How long does this water ski jumper hold his flight pose?



The skier holds his flight pose until he is 4.0 m above the water.

Olana's Solution

$h(t) = 5.0 + 24.46t - 4.9t^2$
$4.0 = 5.0 + 24.46t - 4.9t^2$

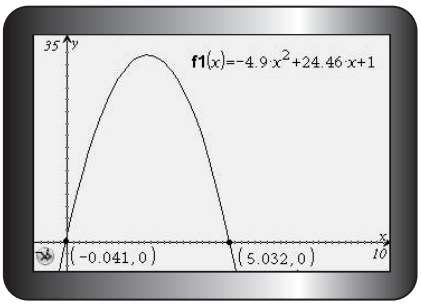
I substituted 4.0 for $h(t)$ to get a quadratic equation I can use to determine the time when the skier's height above the water is 4.0 m.

$0 = 1.0 + 24.46t - 4.9t^2$

I subtracted 4.0 from both sides to put the equation in standard form.

In standard form, $h(t) = 0$. Therefore, the solutions to the equation are the t-intercepts of the graph of this function.

Pre-Publication



I graphed the function on a calculator. I adjusted the window to show the vertex and the *x*-intercepts. I used the calculator to determine the *x*-intercepts.

The t-intercepts are 5.032 and -0.041.

I reread the problem to make sure each solution made sense. Time can't be negative in this situation, so the jumper did not come out of his pose at -0.041 s. Although $(-0.041, 0)$ is a point on the graph, it doesn't make sense in the context of this problem.

Verify:

$4.0 = 5.0 + 24.46t - 4.9t^2$

$t = 5.032$

LS	RS
4.0	$5.0 + 24.46(5.032) - 4.9(5.032)^2$
	$5.0 + 123.082 \ldots - 124.073 \ldots$
	$4.009 \ldots$

LS $\doteq$ RS

I verified the other solution by substituting it into the original equation. The left side was not quite equal to the right side, but I knew that this was because the calculator is set to show values to three decimal places. The solution is not exact, but it is correct.

The ski jumper holds his flight pose for about 5 s.

Your Turn

Curtis rearranged the equation $4.0 = 5.0 + 24.46t - 4.9t^2$ a different way and got the following equation:

$$4.9t^2 - 24.46t - 1.0 = 0$$

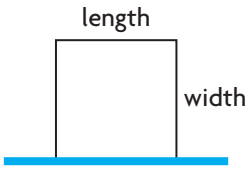
a) Graph the function that is represented by Curtis's equation. How does this graph compare with Olana's graph?

b) Will Curtis get the same solution that Olana did? Explain.

Pre-Publication

EXAMPLE 2 Graphing to determine the number of roots

Lamont runs a boarding kennel for dogs. He wants to construct a rectangular play space for the dogs, using 40 m of fencing and an existing fence as one side of the play space.





a) Write a function that describes the area, A, in square metres, of the play space for any width, w, in metres.

b) Write equations you could use to determine the widths for areas of 250 m^2, 200 m^2, and 150 m^2.

c) Determine the number of possible widths for each equation using a graph.

Lamont's Solution

Let A represent the area of the play space in square metres.
Let l and w represent the dimensions of the play space in metres.

a) $l + 2w = 40$
$l = 40 - 2w$

From the diagram, I could see that the total length of fencing can be expressed as two widths plus one length. I needed a function that just used variables for area and width, so I rewrote my equation to isolate l.

$lw = A$
$(40 - 2w)w = A$
$40w - 2w^2 = A$

I wrote the formula for the area of the play space and substituted $40 - 2w$ for l. Then I simplified the equation.

b) $40w - 2w^2 = 250$
$-2w^2 + 40w - 250 = 0$

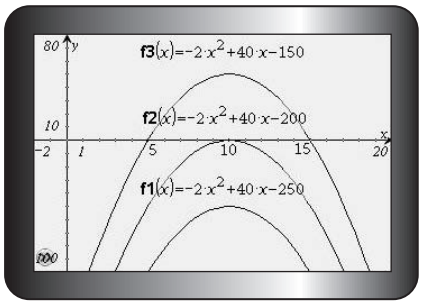
$40w - 2w^2 = 200$
$-2w^2 + 40w - 200 = 0$

$40w - 2w^2 = 150$
$-2w^2 + 40w - 150 = 0$

To determine the equation for each area, I substituted the area for A. Then I rewrote each quadratic equation in standard form.

Pre-Publication

c)



I graphed the corresponding function for each equation.

I can't make a play space with an area of 250 m^2 using 40 m of fencing.

The graph of the first function,

$f_1(w) = -2w^2 + 40w - 250$,

did not cross the w-axis. There are no w-intercepts, so there are no solutions, or **roots**, to the equation.

If I make the play space 10 m wide, the area will be 200 m^2.

The graph of the next function,

$f_2(w) = -2w^2 + 40w - 200$,

intersected the w-axis at its vertex. There is one w-intercept, $w = 10$, so there is one root.

If I make the play space 5 m wide or 15 m wide, the area will be 150 m^2.

The graph of the third function,

$f_3(w) = -2w^2 + 40w - 150$,

has two w-intercepts, $w = 5$ and $w = 15$. This equation has two roots.

roots

The values of the variable that make an equation in standard form equal to zero. These are also called solutions to the equation. These values are also the zeros of the corresponding function and the x-intercepts of its graph.

Pre-Publication

Your Turn

Is it possible for a quadratic equation to have more than two roots? Use a graph to explain.

EXAMPLE 3 Solving a quadratic equation in non-standard form

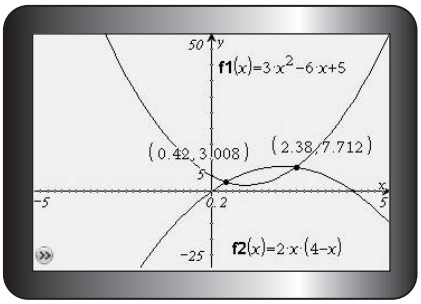
Determine the roots of this quadratic equation. Verify your answers.

$$3x^2 - 6x + 5 = 2x(4 - x)$$

Marwa's Solution

$f(x) = 3x^2 - 6x + 5$
$g(x) = 2x(4 - x)$

I wrote corresponding functions, $f(x)$ and $g(x)$, for each side of the equation to determine the roots.



I graphed each function on a calculator. Then I used the calculator to determine the points of intersection.

The solutions are $x = 0.420$ and $x = 2.380$.

I knew that the solutions of the quadratic equation are the x-coordinates of the points of intersection.

Verify:
$3x^2 - 6x + 5 = 2x(4 - x)$
$x = 0.420$

LS	RS
$3(0.420)^2 - 6(0.420) + 5$	$2(0.420)(4 - 0.420)$
3.009 ...	3.007 ...
LS ≐ RS	

I verified the roots by substituting them into the original equation. Both solutions are valid.

Verify:
$3x^2 - 6x + 5 = 2x(4 - x)$
$x = 2.380$

LS	RS
$3(2.380)^2 - 6(2.380) + 5$	$2(2.380)(4 - 2.380)$
7.713 ...	7.711 ...
LS ≐ RS	

The roots are $x = 0.420$ and $x = 2.380$.

Your Turn

Rewrite $3x^2 - 6x + 5 = 2x(4 - x)$ in standard form. If you graphed the function that corresponds to your equation in standard form, what x-intercepts would you expect to see? Why?

Pre-Publication

In Summary

Key Ideas

- A quadratic equation can be solved by graphing the corresponding quadratic function.
- The standard form of a quadratic equation is

$$ax^2 + bx + c = 0$$

- The roots of a quadratic equation are the x-intercepts of the graph of the corresponding quadratic function. They are also the zeros of the corresponding quadratic function.

Need to Know

- The zeros of a quadratic function correspond to the x-intercepts of the parabola that is defined by the function.
- If a quadratic equation is in standard form
  - you can graph the corresponding quadratic function and determine the zeros of the function to solve the equation
- If the quadratic function is not in standard form
  - you can graph the expression on the left side and the expression on the right side as functions on the same axes
  - the x-coordinates of the points of intersection of the two graphs are the roots of the equation
- For any quadratic equation, there can be zero, one, or two real roots. This is because a parabola can intersect the x-axis in zero, one, or two places.

CHECK Your Understanding

1. Solve each equation by graphing the corresponding function and determining the zeros.

a) $2x^2 - 5x - 3 = 0$ **b)** $9x - 4x^2 = 0$

2. Solve each equation by graphing the expressions on both sides of the equation.

a) $x^2 + 5x = 24$ **b)** $0.5x^2 = -2x + 3$

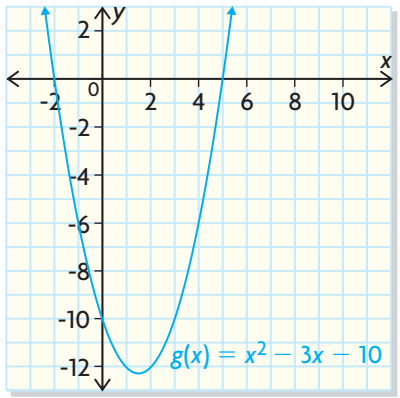
3. Rewrite each equation in standard form. Then solve the equation in standard form by graphing.

a) $6a^2 = 11a + 35$ **b)** $2p^2 + 3p = 1 - 2p$

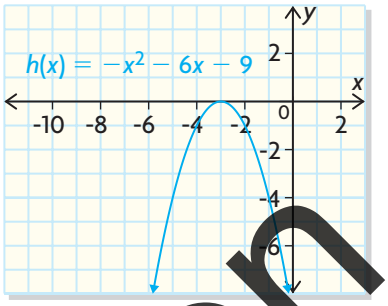
Pre-Publication

4. For each graph, determine the roots of the corresponding quadratic equation.

a)



b)



PRACTISING

5. Solve each equation by graphing the corresponding function and determining the zeros.

a) $3x^2 - 6x - 7 = 0$

b) $0.5z^2 + 3z - 2 = 0$

c) $3b^2 + 8b + 7 = 0$

d) $0.09x^2 + 0.30x + 0.25 = 0$

6. Solve each equation by graphing the expressions on both sides of the equation.

a) $3a^2 = 18a - 21$

b) $5p = 3 - 2p^2$

c) $4x(x + 3) = 3(4x + 3)$

d) $x^2 - 3x - 8 = -2x^2 + 8x + 1$



7. A ball is thrown into the air from a bridge that is 14 m above a river. The function that models the height, $h(t)$, in metres, of the ball over time, t, in seconds, is

$$h(t) = -4.9t^2 + 8t + 14$$

a) When is the ball 16 m above the water?

b) When is the ball 12 m above the water? Explain.

c) Is the ball ever 18 m above the water? Explain how you know.

d) When does the ball hit the water?

8. Solve each quadratic equation by graphing.

a) $5x^2 - 2x = 4x + 3$

b) $-2x^2 + x - 1 = x^2 - 3x - 7$

c) $3x^2 - 12x + 17 = -4(x - 2)^2 + 5$

d) $5x^2 + 4x + 3 = -x^2 - 2x$

9. The stopping distance, d, of a car, in metres, depends on the speed of the car, s, in kilometres per hour. For a certain car on a dry road, the equation for stopping distance is

$$d = 0.0059s^2 + 0.187s$$

The driver of the car slammed on his brakes to avoid an accident, creating skid marks that were 120 m long. He told the police that he was driving at the speed limit of 100 km/h. Do you think he was speeding? Explain.

Pre-Publication

10. Solve the following quadratic equation using the two methods described below.

$$4x^2 + 3x - 2 = -2x^2 + 5x + 1$$

a) Graph the expressions on both sides of the equation, and determine the points of intersection.

b) Rewrite the quadratic equation in standard form, graph the corresponding function, and determine the zeros.

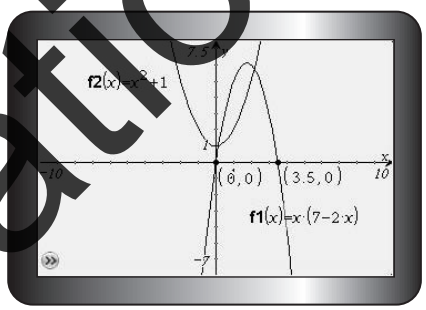
c) Which method do you prefer for this problem? Explain.

11. The length of a rectangular garden is 4 m more than its width. Determine the dimensions of the garden if the area is 117 m^2.

12. Kevin solved the following quadratic equation by graphing the expressions on both sides on the same axes.

$$x(7 - 2x) = x^2 + 1$$

His solutions were $x = 0$ and $x = 3.5$. When he verified his solutions, the left side did not equal the right side.



Verify:

$x(7 - 2x) = x^2 + 1$

$x = 0$

LS	RS
$x(7 - 2x)$	$x^2 + 1$
$(0)(7 - 2(0))$	$(0)^2 + 1$
$(0)(7)$	$0 + 1$
0	1

LS ≠ RS

$x = 3.5$

LS	RS
$x(7 - 2x)$	$x^2 + 1$
$(3.5)(7 - 2(3.5))$	$(3.5)^2 + 1$
$(3.5)(7 - 7)$	$12.25 + 1$
0	13.25

LS ≠ RS

a) Identify Kevin's error.

b) Determine the correct solution.

13. Solve each equation.

a) $0.25x^2 - 1.48x - 1.78 = 0$

b) $4.9x(6 - x) + 36 = 2(x + 9) - x^2$

Closing

14. Explain how you could use a graph to determine the number of roots for an equation in the form $ax^2 + bx = c$.

Extending

15. On the same axes, graph these quadratic functions:

$$y = -2x^2 + 20x - 42$$

$$y = x^2 - 10x + 21$$

Write three different equations whose roots are the points of intersection of these graphs.

Pre-Publication

7.4 Factored Form of a Quadratic Function

YOU WILL NEED

- graph paper and ruler OR graphing technology

GOAL

Relate the factors of a quadratic function to the characteristics of its graph.

EXPLORE...

- John has made a catapult to launch baseballs. John positions the catapult and then launches a ball. The height of the ball, $h(t)$, in metres, over time, t, in seconds, can be modelled by the function $h(t) = -4.9t^2 + 14.7t$ From what height did John launch the ball? How long was the ball in the air?



INVESTIGATE the Math

Ataneq takes tourists on dogsled rides. He needs to build a kennel to separate some of his dogs from the other dogs in his team. He has budgeted for 40 m of fence. He plans to place the kennel against part of his home, to save on materials.



? What dimensions should Ataneq use to maximize the area of the kennel?

A. Using x to represent the width of the kennel, create an expression for the length of the kennel.

B. Write a function, in terms of x, that defines the area of the kennel. Identify the factors in your function.

C. Create a table of values for the function, and then graph it.

D. Does the function contain a maximum or a minimum value? Explain.

E. Determine the x-intercepts of the parabola.

F. Determine the equation of the axis of symmetry of the parabola and the coordinates of the vertex.

G. What are the dimensions that maximize the area of the kennel?

Communication Tip

A quadratic function is in factored form when it is written in the form $y = a(x - r)(x - s)$

Reflecting

H. How are the x-intercepts of the parabola related to the factors of your function?

I. Explain why having a quadratic function in factored form is useful when graphing the parabola.

Pre-Publication

APPLY the Math

EXAMPLE 1 Graphing a quadratic function given in standard form

Sketch the graph of the quadratic function:

$f(x) = 2x^2 + 14x + 12$

State the domain and range of the function.

Arvin's Solution

$f(x) = 2x^2 + 14x + 12$
The coefficient of x^2 is 2,
so the parabola opens upward.

The parabola opens upward when a is positive in the standard form of the function.

$f(x) = 2(x^2 + 7x + 6)$
$f(x) = 2(x + 1)(x + 6)$

I factored the expression on the right side so that I could determine the zeros of the function.

Zeros:
$0 = 2(x + 1)(x + 6)$
$x + 1 = 0$ or $x + 6 = 0$
$x = -1$ $x = -6$
The x-intercepts are $x = -1$ and $x = -6$.

To determine the zeros, I set $f(x)$ equal to zero. I knew that a product is zero only when one or more of its factors are zero, so I set each factor equal to zero and solved each equation.

The values of x at the zeros of the function are also the x-intercepts.

y-intercept:
$f(0) = 2(0 + 1)(0 + 6)$
$f(0) = 2(1)(6)$
$f(0) = 12$
The y-intercept is 12.

I knew that the y-intercept is 12 from the standard form of the quadratic function. However, I decided to verify that my factoring was correct.

I noticed that this value can be obtained by multiplying the values of a, r, and s from the factored form of the function: $f(x) = a(x - r)(x - s)$

Pre-Publication

Axis of symmetry:

$$x = \frac{-6 + (-1)}{2}$$

$$x = -3.5$$

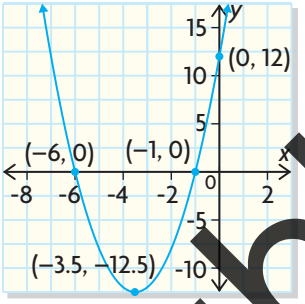
The axis of symmetry passes through the midpoint of the line segment that joins the x-intercepts. I calculated the mean of the two x-intercepts to determine the equation of the axis of symmetry.

$$f(x) = 2(x + 1)(x + 6)$$
$$f(-3.5) = 2(-3.5 + 1)(-3.5 + 6)$$
$$f(-3.5) = 2(-2.5)(2.5)$$
$$f(-3.5) = -12.5$$

The vertex of the parabola is $(-3.5, -12.5)$.

The vertex lies on the axis of symmetry, so its x-coordinate is -3.5. I substituted -3.5 into the equation to determine the y-coordinate of the vertex.

$f(x) = 2x^2 + 14x + 12$



I plotted the x-intercepts, y-intercept, and vertex and then joined these points with a smooth curve.

Domain and range:

$\{(x, y) \mid x \in \mathrm{R}, y \geq -12.5, y \in \mathrm{R}\}$

The only restriction on the variables is that y must be greater than or equal to -12.5, the minimum value of the function.

Your Turn

Sketch the graph of the following function:

$$f(x) = -3x^2 + 6x - 3$$

a) How does the graph of this function differ from the graph in *Example 1*?

b) How are the x-intercepts related to the vertex? Explain.

EXAMPLE 2 Using a partial factoring strategy to sketch the graph of a quadratic function

Sketch the graph of the following quadratic function:

$$f(x) = -x^2 + 6x + 10$$

State the domain and range of the function.

Pre-Publication

Elliot's Solution

$f(x) = -x^2 + 6x + 10$
$f(x) = -x(x - 6) + 10$

I couldn't identify two integers with a product of 10 and a sum of 6, so I couldn't factor the expression. I decided to remove a partial factor of $-x$ from the first two terms. I did this so that I could determine the x-coordinates of the points that have 10 as their y-coordinate.

$-x = 0$ $\qquad$ $x - 6 = 0$
$x = 0$ $\qquad$ $x = 6$
$f(0) = 10$ $\qquad$ $f(6) = 10$
The points (0, 10) and (6, 10) belong to the given quadratic function.

I determined two points in the function by setting each partial factor equal to zero.

When either factor is zero, the product of the factors is zero, so the value of the function is 10.

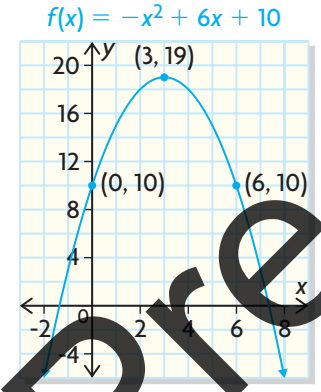
$$x = \frac{0 + 6}{2}$$
$x = 3$

Because (0, 10) and (6, 10) have the same y-coordinate, they are the same horizontal distance from the axis of symmetry. I determined the equation of the axis of symmetry by calculating the mean of the x-coordinates of these two points.

$f(3) = -(3)^2 + 6(3) + 10$
$f(3) = -9 + 18 + 10$
$f(3) = 19$

I determined the y-coordinate of the vertex.

The vertex is (3, 19).



The coefficient of the x^2 term is negative, so the parabola opens downward.

I used the vertex, as well as (0, 10) and (6, 10), to sketch the parabola.

Domain: $\{x \mid x \in \mathrm{R}\}$
Range: $\{y \mid y \leq 19, y \in \mathrm{R}\}$

The only restriction on the variables is that y must be less than or equal to 19, the maximum value of the function.

Pre-Publication

Your Turn

a) i) Apply the partial factoring strategy to locate two points that have the same y-coordinate on the following function:

$$f(x) = -x^2 - 3x + 12$$

ii) Determine the axis of symmetry and the location of the vertex of the function from part i).

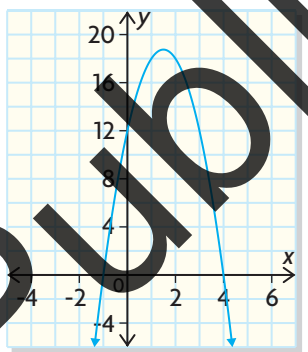
iii) Explain how the process you used in parts i) and ii) is different from factoring a quadratic function.

b) Explain whether you would use partial factoring to graph the function

$$g(x) = -x^2 - 4x + 12$$

EXAMPLE 3 Determining the equation of a quadratic function, given its graph

Determine the function that defines this parabola. Write the function in standard form.



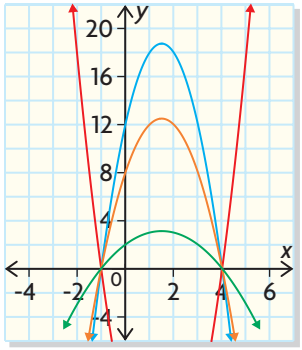
Indira's Solution

The x-intercepts are $x = -1$ and $x = 4$.
The zeros of the function occur when x has values of -1 and 4.
$y = a(x - r)(x - s)$
$y = a[x - (-1)][x - (4)]$
$y = a(x + 1)(x - 4)$

The graph is a parabola, so it is defined by a quadratic function.

I located the x-intercepts and used them to determine the zeros of the function. I wrote the factored form of the quadratic function, substituting -1 and 4 for r and s.

Pre-Publication



I knew that there are infinitely many quadratic functions that have these two zeros, depending on the value of a. I had to determine the value of a for the function that defines the blue graph.

The y-intercept is 12.

$y = a(x + 1)(x - 4)$

$(12) = a[(0) + 1][(0) - 4]$

$12 = a(1)(-4)$

$12 = -4a$

$-3 = a$

From the graph, I determined the coordinates of the y-intercept.

Because these coordinates are integers, I decided to use the y-intercept to solve for a.

In factored form, the quadratic function is

$y = -3(x + 1)(x - 4)$

I substituted the value of a into my equation.

In standard form, the quadratic function is

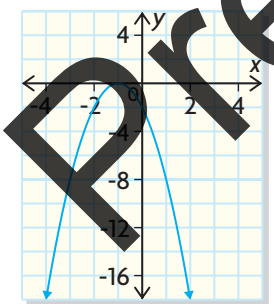
$y = -3(x^2 - 3x - 4)$

$y = -3x^2 + 9x + 12$

My equation seems reasonable, because it defines a graph with a y-intercept of 12 and a parabola that opens downward.

Your Turn

If a parabola has only one x-intercept, how could you determine the quadratic function that defines it, written in factored form? Explain using the given graph.



Pre-Publication

EXAMPLE 4 Solving a problem modelled by a quadratic function in factored form

The members of a Ukrainian church hold a fundraiser every Friday night in the summer. They usually charge $6 for a plate of perogies. They know, from previous Fridays, that 120 plates of perogies can be sold at the $6 price but, for each $1 price increase, 10 fewer plates will be sold. What should the members charge if they want to raise as much money as they can for the church?



Krystina's Solution: Using the properties of the function

Let y represent the total revenue.
$y =$ (Number of plates)(Price)

Let x represent the number of $1 price increases.
$y = (120 - 10x)(6 + x)$

For each price increase, x, I knew that $10x$ fewer plates will be sold.

If I expanded the factors in my function, I would create an x^2 term. This means that the function I have defined is quadratic and its graph is a parabola.

$0 = (120 - 10x)(6 + x)$

$120 - 10x = 0$ or $6 + x = 0$

$-10x = -120$ $x = -6$

$x = 12$

The x-intercepts are $x = -6$ and $x = 12$.

To determine the zeros of the function, I substituted zero for y. A product is zero only when one or both of its factors are zero, so I set each factor equal to zero and solved each equation.

$$x = \frac{12 + (-6)}{2}$$

$x = 3$

$y = (120 - 10x)(6 + x)$

$y = [120 - 10(3)][6 + (3)]$

$y = (90)(9)$

$y = 810$

The coordinates of the vertex are (3, 810).

I determined the equation of the axis of symmetry for the parabola by calculating the mean distance between the x-intercepts.

I determined the y-coordinate of the vertex by substituting into my initial equation.

To generate as much revenue as possible, the members of the church should charge $6 + $3 or $9 for a plate of perogies. This will provide revenue of $810.

The vertex describes the maximum value of the function. Maximum sales of $810 occur when the price is raised by $3.

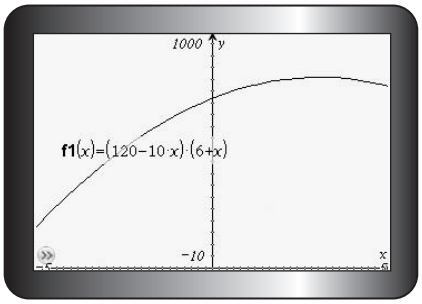
Pre-Publication

Jennifer's Solution: Using graphing technology

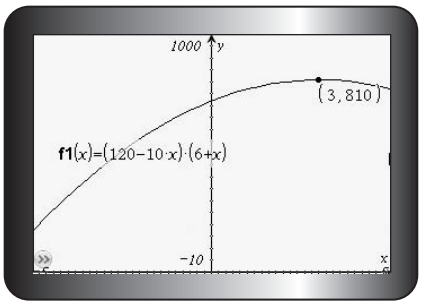
Revenue = (Number of plates)(Price)

$y = (120 - 10x)(6 + x)$

I let y represent Revenue and I let x represent the number of $1 price increases. For each $1 price increase, I knew that 10 fewer plates will be sold.



I graphed the equation on a calculator. Since a reduced price may result in maximum revenue, I set my domain to a minimum value of −5 and a maximum value of 5.



I used the calculator to locate the vertex of the parabola.

The members of the church should charge $3 more than the current price of $6 for a plate of perogies. If they charge $9, they will reach the maximum revenue of $810.

Your Turn

A career and technology class at a high school in Langley, British Columbia, operates a small T-shirt business out of the school. Over the last few years, the shop has had monthly sales of 300 T-shirts at a price of $15 per T-shirt. The students have learned that for every $2 increase in price, they will sell 20 fewer T-shirts each month. What should they charge for their T-shirts to maximize their monthly revenue?

Pre-Publication

In Summary

Key Ideas

- When a quadratic function is written in factored form

$$y = a(x - r)(x - s)$$

each factor can be used to determine a zero of the function by setting each factor equal to zero and solving.

- If a parabola has one or two x-intercepts, the equation of the parabola can be written in factored form using the x-intercept(s) and the coordinates of one other point on the parabola.
- Quadratic functions without any zeros cannot be written in factored form.

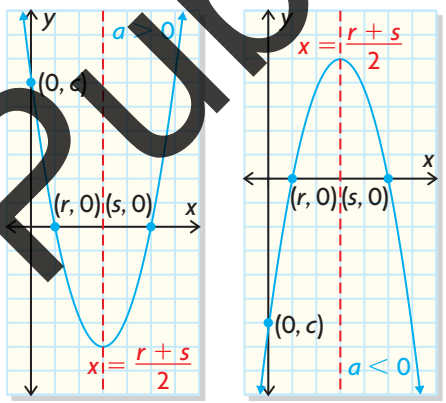
Need to Know

- A quadratic function that is written in the form

$$f(x) = a(x - r)(x - s)$$

has the following characteristics:

  - The x-intercepts of the graph of the function are $x = r$ and $x = s$.
  - The linear equation of the axis of symmetry is $x = \frac{r + s}{2}$.
  - The y-intercept, c, is $c = a \cdot r \cdot s$.



- If a quadratic function has only one x-intercept, the factored form can be written as follows:

$$f(x) = a(x - r)(x - r)$$

$$f(x) = a(x - r)^2$$

Pre-Publication

CHECK Your Understanding

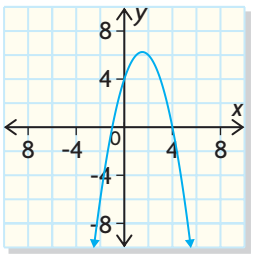
1. Match each quadratic function with its corresponding parabola.

a) $f(x) = (x - 1)(x + 4)$ **d)** $f(x) = (x - 1)(x - 4)$

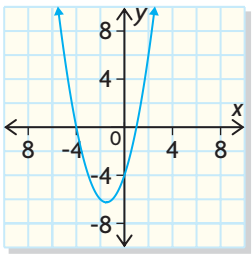
b) $f(x) = (x + 1)(x - 4)$ **e)** $f(x) = (1 - x)(x + 4)$

c) $f(x) = (x + 1)(x + 4)$ **f)** $f(x) = (x + 1)(4 - x)$

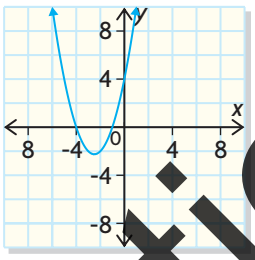
i)



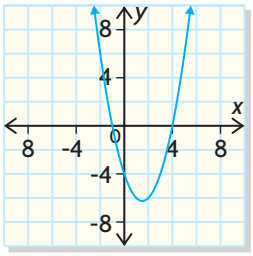
iii)



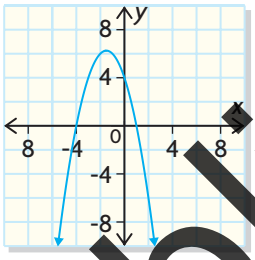
v)



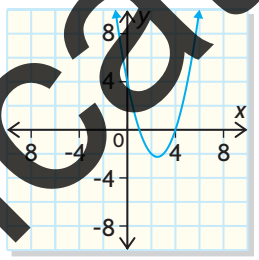
ii)



iv)



vi)



2. For each quadratic function below

i) determine the x-intercepts of the graph

ii) determine the y-intercept of the graph

iii) determine the equation of the axis of symmetry

iv) determine the coordinates of the vertex

v) sketch the graph

a) $f(x) = (x + 4)(x - 2)$ **c)** $h(x) = 2(x + 1)(x - 7)$

b) $g(x) = -2x(x - 3)$

3. A quadratic function has an equation that can be written in the form $f(x) = a(x - r)(x - s)$. The graph of the function has x-intercepts $x = -2$ and $x = 4$ and passes through point $(5, 7)$. Write the equation of the quadratic function.

PRACTISING

4. For each quadratic function, determine the x-intercepts, the y-intercept, the equation of the axis of symmetry, and the coordinates of the vertex of the graph.

a) $f(x) = (x - 1)(x + 1)$ **d)** $f(x) = -2(x - 2)(x + 1)$

b) $f(x) = (x + 2)(x + 2)$ **e)** $f(x) = 3(x - 2)^2$

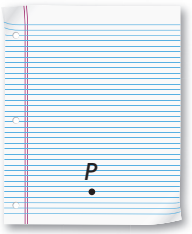
c) $f(x) = (x - 3)(x - 3)$ **f)** $f(x) = 4(x - 1)^2$

Pre-Publication

Math in Action

Paper Parabolas

On an 8.5 in. by 11 in. piece of lined paper, mark a point that is close to the centre, near the bottom edge. Label this point *P*.



- Fold the paper, at any angle, so that any point on the bottom edge of the paper touches point *P*. Crease the paper along the fold line, and then open the paper.
- Fold the paper again, but at a different angle, so that another point on the bottom edge of the paper touches point *P*. Crease the paper.
- Continue this process until you have many different creases in the paper on both sides of *P*.
  - What shape emerges?
  - Compare your shape with the shapes made by other students. How are the shapes the same? How are they different?
  - How does changing the location of point *P* affect the shape that is formed?
  - The creases intersect at several points. How could you determine whether a set of these points is on a parabola?

5. Sketch the graph of each function in question 4, and state the domain and range of the function.

6. Sketch the graph of

$$y = a(x - 3)(x + 1)$$

for $a = 3$. Describe how the graph would be different from your sketch if the value of a were 2, 1, 0, -1, -2, and -3.

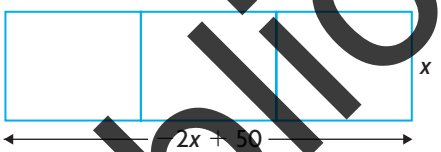
7. Sketch the graph of

$$y = (x - 3)(x + s)$$

for $s = 3$. Describe how the graph would be different from your sketch if the value of s were 2, 1, 0, -1, -2, and -3.

8. Byron is planning to build three attached rectangular enclosures for some of the animals on his farm. He bought 100 m of fencing. He wants to maximize the total area of the enclosures. He determined a function, $A(x)$, that models the total area in square metres, where x is the width of each rectangle:

$$A(x) = -2x^2 + 50x$$



a) Determine the maximum total area.

b) State the domain and range of the variables in the function.

9. Paulette owns a store that sells used video games in Red Deer, Alberta. She charges $10 for each used game. At this price, she sells 70 games a week. Experience has taught her that a $1 increase in the price results in five fewer games being sold per week. At what price should Paulette sell her games to maximize her sales? What will be her maximum revenue?

10. For each quadratic function below

i) use partial factoring to determine two points that are the same distance from the axis of symmetry

ii) determine the coordinates of the vertex

iii) sketch the graph

a) $f(x) = x^2 + 4x - 6$

b) $f(x) = x^2 - 8x + 13$

c) $f(x) = 2x^2 + 10x + 7$

d) $f(x) = -x^2 - 8x - 5$

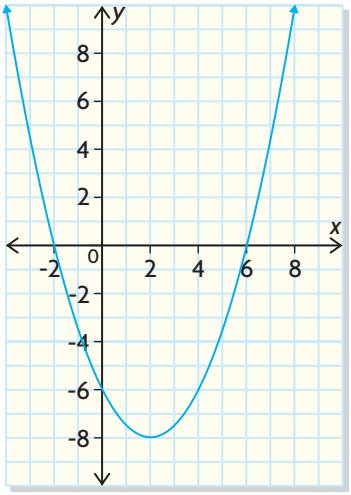
e) $f(x) = -\frac{1}{2}x^2 + 2x - 3$

f) $f(x) = -2x^2 + 10x - 9$

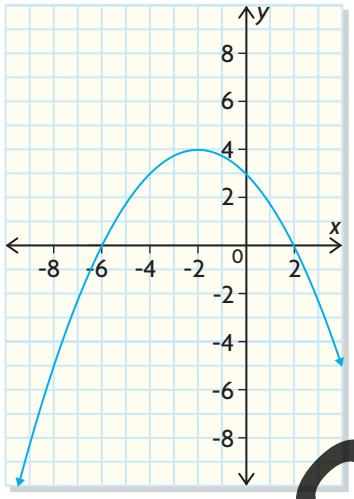
Pre-Publication

11. Determine the equation of the quadratic function that defines each parabola.

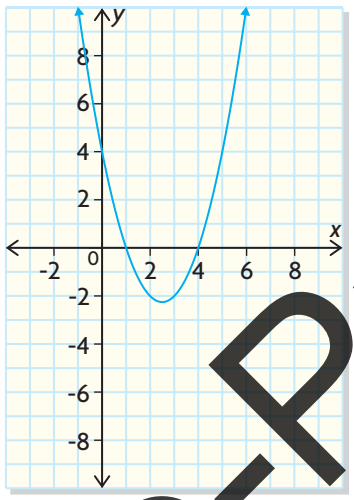
a)



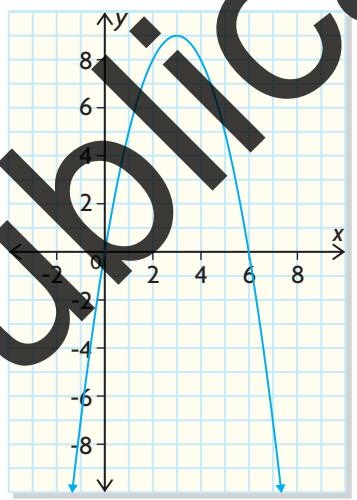
c)



b)



d)



12. **a)** Use two different algebraic strategies to determine the equation of the axis of symmetry and the vertex of the parabola defined by the following function:

$$f(x) = -2x^2 + 16x - 24$$

b) Which strategy do you prefer? Explain.

13. Determine the quadratic function that defines a parabola with x-intercepts $x = -1$ and $x = 3$ and y-intercept $y = -6$. Provide a sketch to support your work.

Pre-Publication



14. How many zeros can a quadratic function have? Provide sketches to support your reasoning.

15. On the north side of Sir Winston Churchill Provincial Park, located near Lac La Biche, Alberta, people gather to witness the migration of American white pelicans. The pelicans dive underwater to catch fish. Someone observed that a pelican's depth underwater over time could be modelled by a parabola. One pelican was underwater for 4 s, and its maximum depth was 1 m.

a) State the domain and range of the variables in this situation.

b) Determine the quadratic function that defines the parabola.

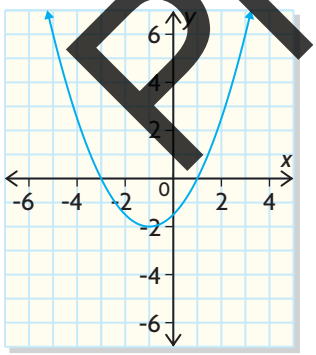
16. Elizabeth wants to enclose the backyard of her house on three sides to form a rectangular play area for her children. She has decided to use one wall of the house and three sections of fence to create the enclosure. Elizabeth has budgeted $800 for the fence. The fencing material she has chosen costs $16/ft. Determine the dimensions that will provide Elizabeth with the largest play area.

17. A water rocket was launched from the ground, with an initial velocity of 32 m/s. The rocket achieved a height of 44 m after 2 s of flight. The rocket was in the air for 6 s.

a) Determine the quadratic function that models the height of the rocket over time.

b) State the domain and range of the variables.





Closing

18. Identify the key characteristics you could use to determine the quadratic function that defines this graph. Explain how you would use these characteristics.

Pre-Publication

Extending



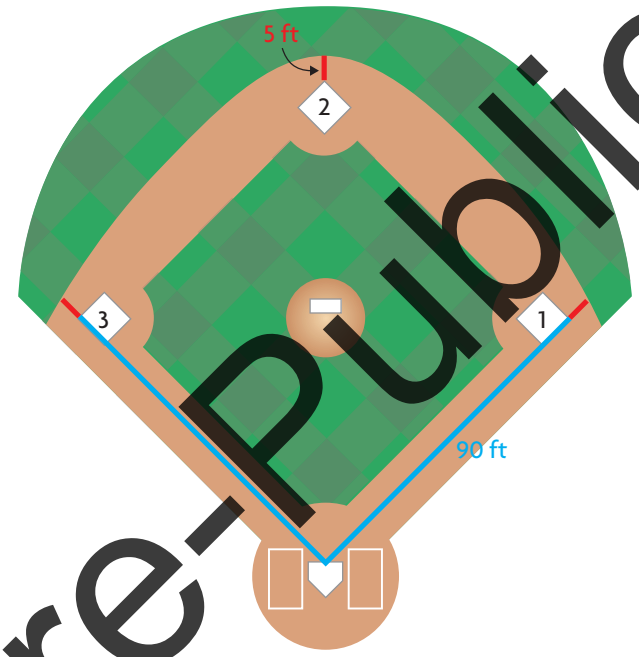
19. The Chicago Bean is a unique sculpture that was inspired by liquid mercury. It is 66 ft long and 33 ft high, with a 12 ft arch underneath. The top curved section that connects the three red dots in the photograph forms a parabola.

a) Determine the quadratic function that connects the three red dots. Assume that the ground (the green line) represents the x-axis of the graph. Write the function in standard form.

b) What are the domain and range of the variables?

c) If the parabola extended to the ground, what would the x-intercepts be, rounded to the nearest tenth?

20. A local baseball team has raised money to put new grass on the field. The curve where the infield ends can be modelled by a parabola. The foreman has marked out the key locations on the field, as shown in the diagram.



a) Determine a quadratic function that models the curve where the infield ends.

b) State the domain and range of the variables. Justify your decision.

c) Graph the quadratic function.

21. The National Basketball Association (NBA) mandates that every court must have the same dimensions. The length of the court must be 6 ft less than twice the width. The area of the court must be 4700 ft^2. Use this information to determine the dimensions of a basketball court used by the NBA.

Pre-Publication

7 Mid-Chapter Review

FREQUENTLY ASKED Questions

Study *Aid*
- See Lesson 7.1.
- Try Mid-Chapter Review Question 1.

Q: What are the characteristics of a quadratic function?

A: The following are the key characteristics of a quadratic function:

- The equation is of degree 2.
- The graph is a parabola.
- The y-coordinate of the vertex of the parabola is a maximum if the parabola opens down and a minimum if the parabola opens up.
- The domain of the function is the set of real numbers. The range is restricted by the y-coordinate of the vertex. However, if the function is being used to model a situation, then the situation may restrict the domain and the range.
- The graph of the function contains a vertical axis of symmetry that passes through the vertex.

Study *Aid*
- See Lesson 7.2, Examples 2 and 3, and Lesson 7.4, Examples 1, 3, and 4.
- Try Mid-Chapter Review Questions 2 to 4, 10, and 12.

Q: How can you use the information that is available from the standard or factored form of a quadratic function to sketch its graph?

A: The standard form is

$$y = ax^2 + bx + c$$

From this form, you can determine that the y-intercept of the graph is $y = c$.

The factored form is

$$y = a(x - r)(x - s)$$

From this form, you can determine

- the zeros (r and s), which provide the x-intercepts $x = r$ and $x = s$
- the y-intercept, determined by multiplying a, r, and s
- the equation of the axis of symmetry, $x = \frac{r + s}{2}$
- the location of the vertex, determined by substituting the x-coordinate of the vertex, $\frac{r + s}{2}$, into the equation

From both forms, you can determine the direction in which the parabola opens: upward when $a > 0$ and downward when $a < 0$.

Pre-Publication

Q: How can I solve a quadratic equation by graphing?

A1: If the quadratic equation is in standard form, enter the corresponding function on a graphing calculator. Determine the x-intercepts of the parabola. These are the solutions to the equation.

A2: If the quadratic equation is not in standard form, you can graph the expressions on the left and right sides separately. The solutions to the equation are the x-coordinates of the points of intersection of the two functions.

Study Aid

- See Lesson 7.3, Examples 1 to 3.
- Try Mid-Chapter Review Questions 5 and 6.

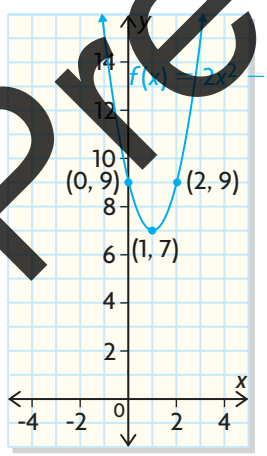
Q: What is partial factoring, and how is it used to sketch a graph?

A: Starting with the standard form of a quadratic function, factor only the terms that contain the variable x. The two partial factors can be used to locate two points that have the same y-coordinate, and so are equidistant from the axis of symmetry. Partial factoring can be used when a function cannot be factored completely. For example:

$$f(x) = 2x^2 - 4x + 9$$

Study Aid

- See Lesson 7.4, Example 2.
- Try Mid-Chapter Review Question 11.

$f(x) = 2x^2 - 4x + 9$ $f(x) = 2x(x - 2) + 9$	Factor the terms that include x.
$2x = 0 \qquad x - 2 = 0$ $x = 0 \qquad\quad x = 2$ (0, 9) and (2, 9) are points on the parabola.	Set each partial factor equal to zero. This allows you to identify two points on the graph with a y-coordinate of 9.
$x = \frac{0 + 2}{2}$ $x = 1$	Determine the equation of the axis of symmetry, which is located midway between (0, 9) and (2, 9).
$f(1) = 2(1)^2 - 4(1) + 9$ $f(1) = 7$ The vertex of the parabola is at (1, 7).	Locate the y-coordinate of the vertex by substituting into the quadratic function.
	From these three known points, you can sketch the graph.

Pre-Publication

PRACTISING

Lesson 7.1

1. Which of the following are quadratic functions?

a) $y = 3x + 4$ **c)** $y = x^2 + 2x - 9$

b) $y = 2x(x - 5)$ **d)** $y = 2x^3 + 4x^2 - 5$

Lesson 7.2

2. a) Determine the y-intercept of the following quadratic function:

$$y = -x^2 + 8x$$

b) Graph the function.

c) From your graph, determine the equation of the axis of symmetry, the location of the vertex, the x-intercepts, and the domain and range of the function.

3. Consider the standard form of a quadratic function:

$$y = ax^2 + bx + c$$

a) Explain how the value of a affects the graph of the parabola.

b) Provide supporting examples, with their graphs.

4. A flare is often used as a signal to attract rescue personnel in an emergency. When a flare is shot into the air, its height, $h(t)$, in metres, over time, t, in seconds can be modelled by

$$h(t) = -5t^2 + 120t$$

a) Identify the x-intercepts of the parabola.

b) When did the flare reach its maximum height, and what was this height?

c) What was the height of the flare after 15 s?

d) State the domain and range of the function.

Lesson 7.3

5. Solve by graphing and determining the x-intercepts.

$$0.5x^2 + 3x - 3.5 = 0$$

6. Solve by graphing the expressions on both sides of the equation and determining the x-coordinates of the points of intersection.

$$-3x^2 + 4x = x^2 - 7$$

7. If a skydiver jumps from an airplane and free falls for 828 m before he safely deploys his parachute, his free fall could be modelled by the function

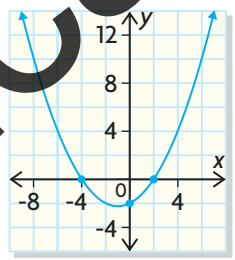
$$h(t) = -4.9t^2 + 10t + 828$$

where $h(t)$ is the height in metres and t is the time in seconds (ignoring air resistance). How long did this skydiver free fall?

Lesson 7.4

8. The points $(-4, 6)$ and $(2, 6)$ lie on a parabola. Determine the equation of the axis of symmetry of the parabola.

9. Determine the equation of this quadratic function.



10. The zeros of a quadratic function are -6 and 12. The graph of the function intersects the y-axis at -36.

a) Determine the equation of the quadratic function.

b) Determine the coordinates of the vertex.

c) State the domain and range of the function.

11. Sketch the graph of the following quadratic function:

$$y = 3x^2 + 6x - 18$$

12. Pedalworks rents bicycles to tourists who want to explore the local trails. Data from previous rentals show that the shop will rent 7 more bicycles per day for every $1.50 decrease in rental price. The shop currently rents 63 bicycles per day, at a rental price of $39. How much should the shop charge to maximize revenue?

Pre-Publication

7.5 Solving Quadratic Equations by Factoring

GOAL

Solve quadratic equations by factoring.

YOU WILL NEED

- calculator

LEARN ABOUT the Math

EXPLORE...

- What values could you substitute for n and x to make this equation true? $(3x + n)(7x - 7) = 0$

The entry to the main exhibit hall in an art gallery is a parabolic arch. The arch can be modelled by the function

$$h(w) = -0.625w^2 + 5w$$

where the height, $h(w)$, and width, w, are measured in feet. Several sculptures are going to be delivered to the exhibit hall in crates. Each crate is a square-based rectangular prism that is 7.5 ft high, including the wheels. The crates must be handled as shown, to avoid damaging the fragile contents.



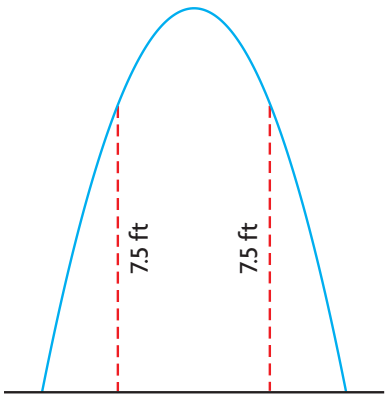
? What is the maximum width of a 7.5 ft high crate that can enter the exhibit hall through the arch?

Pre-Publication

EXAMPLE 1 Solving a quadratic equation by factoring

Determine the distance between the two points on the arch that are 7.5 ft high.

Brooke's Solution



I sketched the situation.

The crate can only fit through the part of the arch that is at least 7.5 ft high. The arch is exactly 7.5 ft high at two points.

The following function describes the arch:
$h(w) = -0.625w^2 + 5w$

The height of the crate is 7.5 ft.

$$7.5 = -0.625w^2 + 5w$$

I wrote an equation, substituting 7.5 for $h(w)$.

$$0.625w^2 - 5w + 7.5 = 0$$

I rewrote the equation in standard form.

I decided to subtract $-0.625w^2 + 5w$ from both sides so the coefficient of w^2 would be positive.

$$\frac{0.625w^2}{0.625} - \frac{5w}{0.625} + \frac{7.5}{0.625} = \frac{0}{0.625}$$

I divided by 0.625 to simplify the equation.

$$w^2 - 8w + 12 = 0$$
$$(w - 2)(w - 6) = 0$$

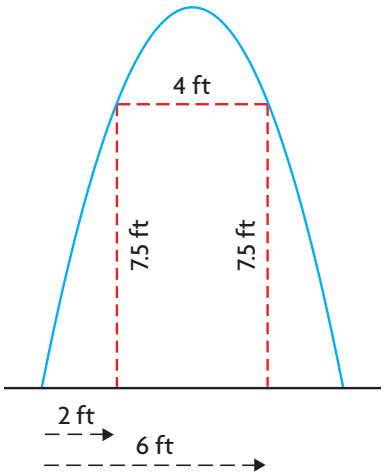
I factored the equation.

$w - 2 = 0$ or $w - 6 = 0$
$w = 2$ $w = 6$

If the product of two factors is 0, then at least one factor must equal 0.

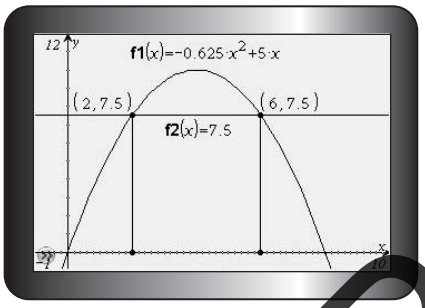
The parabola reaches a height of exactly 7.5 ft at widths of 2 ft and 6 ft.

Pre-Publication



I determined the difference between the widths to determine the maximum width of the crate.

To fit through the archway, the crate cannot be more than 4 ft wide.



I checked my solution by graphing $y = -0.625x^2 + 5x$ and $y = 7.5$.

The x-coordinates of the points of intersection are 2 and 6, so my solution is correct.

Reflecting

A. How did rewriting the equation in standard form and then factoring it help Brooke determine the roots?

B. Was Brooke's decision to divide both sides of the equation by 0.625 (the coefficient of w^2) reasonable? Explain.

C. Describe another way that Brooke could verify the solutions to her equation.

D. Tim says that if you know the roots of an equation, you can use factors to determine the equation. How could Tim use the roots 2 and 6 to determine the equation that Brooke solved?

E. Can you always use factoring to solve a quadratic equation? Explain.

Pre-Publication

APPLY the Math

EXAMPLE 2 | Solving a quadratic equation using a difference of squares

Determine the roots of the following equation:

$$75p^2 - 192 = 0$$

Verify your solution.

Alberto's Solution

$$75p^2 - 192 = 0$$

I noticed that 3 is a factor of both 75 and 192.

$$\frac{75p^2}{3} - \frac{192}{3} = \frac{0}{3}$$

$$25p^2 - 64 = 0$$

I noticed that $25p^2$ and 64 are both perfect squares, so $25p^2 - 64$ is a difference of squares.

$$(5p - 8)(5p + 8) = 0$$

$5p - 8 = 0$ or $5p + 8 = 0$

$5p = 8$ $\qquad$ $5p = -8$

I determined the roots.

$p = \frac{8}{5}$ $\qquad$ $p = -\frac{8}{5}$

The roots are $\frac{8}{5}$ and $-\frac{8}{5}$.

$$75p^2 - 192 = 0$$

I decided to verify my solutions by solving the equation using a different method.

$$75p^2 = 192$$

$$p^2 = \frac{192}{75}$$

I isolated p^2 and then took the square root of each side. I knew that p^2 has two possible square roots, one positive and the other negative.

$$p^2 = \frac{64}{25}$$

My solution matched the solution I obtained by factoring.

$$p = \pm\sqrt{\frac{64}{25}}$$

$$p = \pm\frac{8}{5}$$

Your Turn

How can you tell that any equation with a difference of squares is factorable? What can you predict about the roots?

Pre-Publication

EXAMPLE 3 Solving a quadratic equation with only one root

Solve and verify the following equation:

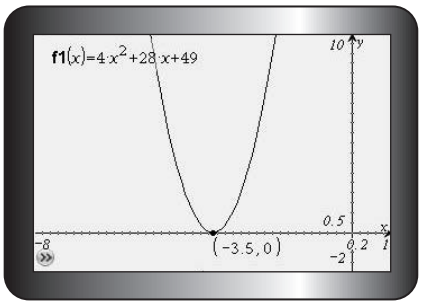
$$4x^2 + 28x + 49 = 0$$

Arya's Solution

$4x^2 + 28x + 49 = 0$
$(2x + 7)(2x + 7) = 0$

$2x + 7 = 0$
$x = -3.5$

I factored the trinomial. I noticed that both factors are the same, so there is only one root.



I decided to verify my solution by graphing the corresponding quadratic function.

I noticed that the vertex of the function is on the x-axis at -3.5, so my solution makes sense.

Your Turn

How can factoring an equation help you determine whether the equation has two roots or one root?

EXAMPLE 4 Using reasoning to write an equation from its roots

Tori says she solved a quadratic equation by graphing. She says the roots were -5 and 7. How can you determine an equation that she might have solved?

Philip's Solution

$x = -5$ or $x = 7$

The x-intercepts of the quadratic function are the roots of the equation.

$x + 5 = 0$ $x - 7 = 0$
One factor is $x + 5$.
The other factor is $x - 7$.

I decided to use the roots to help me write the factors of the equation.

$(x + 5)(x - 7) = 0$
$x^2 + 5x - 7x - 35 = 0$
$x^2 - 2x - 35 = 0$

I wrote the factors as a product. Since each root is equal to 0, their product is also equal to 0.

I simplified to write the equation in standard form.

Pre-Publication

Your Turn

The x-intercepts of the graph of a quadratic function are 3 and -2.5. Write a quadratic equation that has these roots.

EXAMPLE 5 Describing errors in a solution

Matthew solved a quadratic equation as shown. Identify and correct the error in Matthew's solution.

$$4x^2 = 9x$$
$$\frac{4x^2}{x} = \frac{9x}{x}$$
$$4x = 9$$
$$x = 2.25$$

Raj's Solution

Matthew made an error in the second line of his solution. When he divided both sides by x, he eliminated a possible factor, $x = 0$.

$\frac{4x^2}{x}$ and $\frac{9x}{x}$ are not defined when $x = 0$, so Matthew cannot divide by x.

Correctly solving the equation:

$$4x^2 - 9x = 0$$
$$x(4x - 9) = 0$$

To solve the equation, I rewrote it in standard form and then factored the left side.

$$x = 0 \quad \text{or} \quad 4x - 9 = 0$$
$$x = 0 \quad \text{or} \quad 4x = 9$$
$$x = 2.25$$

For my equation to be true, either x or $4x - 9$ must equal 0.

Verify:

$4x^2 - 9x = 0$

$x = 0$		$x = 2.25$	
LS	RS	LS	RS
$4x^2 - 9x$	0	$4x^2 - 9x$	0
$4(0)^2 - 9(0)$		$4(2.25)^2 - 9(2.25)$	
$0 - 0$		$20.25 - 20.25$	
0		0	
LS = RS		LS = RS	

I verified each solution by substituting it into the original equation. For both solutions, the left side is equal to the right side. Therefore, both solutions are correct.

Your Turn

What number will always be a root of an equation that can be written in standard form as $ax^2 + bx = 0$? Explain how you know.

Pre-Publication

In Summary

Key Idea

- Some quadratic equations can be solved by factoring.

Need to Know

- To factor an equation, start by writing the equation in standard form.
- You can set each factor equal to zero and solve the resulting linear equations. Each solution is a solution to the original equation.
- If the two roots of a quadratic equation are equal, then the quadratic equation is said to have one solution.

CHECK Your Understanding

1. Solve by factoring. Verify each solution.

a) $x^2 - 11x + 28 = 0$

b) $x^2 - 7x - 30 = 0$

c) $2y^2 + 11y + 5 = 0$

d) $4t^2 + 7t - 15 = 0$

2. Solve by factoring.

a) $x^2 - 121 = 0$

b) $9r^2 - 100 = 0$

c) $x^2 - 15x = 0$

d) $3y^2 + 48y = 0$

e) $s^2 - 12s + 36 = 0$

f) $16p^2 + 8p + 1 = 0$

g) $-14z^2 + 35z = 0$

h) $5q^2 - 9q = 0$

PRACTISING

3. Solve by factoring. Verify each solution.

a) $x^2 - 9x - 70 = 0$

b) $x^2 + 19x + 48 = 0$

c) $3a^2 + 11a - 4 = 0$

d) $6t^2 - 7t - 20 = 0$

4. Solve each equation.

a) $12 - 5x = 2x^2$

b) $4x^2 = 9 - 9x$

c) $49d^2 + 9 = -42d$

d) $169 = 81g^2$

5. Geeta solved this equation:

$$20x^2 - 21x - 27 = 0$$

Her solutions were $x = 0.75$ and $x = -1.8$.

a) Factor and solve the equation.

b) What error do you think Geeta made?

6. Determine the roots of each equation.

a) $5u^2 - 10u - 315 = 0$

b) $0.25x^2 + 1.5x + 2 = 0$

c) $1.4y^2 + 5.6y - 16.8 = 0$

d) $\frac{1}{2}k^2 + 5k + 12.5 = 0$

Pre-Publication

7. The graph of a quadratic function has x-intercepts -5 and -12. Write a quadratic equation that has these roots.

8. A bus company charges \$2 per ticket but wants to raise the price. The daily revenue that could be generated is modelled by the function

$$R(x) = -40(x - 5)^2 + 25\,000$$

where x is the number of 10¢ price increases and $R(x)$ is the revenue in dollars. What should the price per ticket be if the bus company wants to collect daily revenue of \$21 000?



9. Solve and verify the following equation:

$$5x - 8 = 20x^2 - 32x$$

10. Identify and correct any errors in the following solution:

$$5a^2 - 100 = 0$$
$$5a^2 = 100$$
$$a^2 = 25$$
$$\sqrt{a^2} = \sqrt{25}$$
$$a = 5$$

11. Identify and correct the errors in this solution:

$$4r^2 - 9r = 0$$
$$(2r - 3)(2r + 3) = 0$$
$$2r - 3 = 0 \quad \text{or} \quad 2r + 3 = 0$$
$$2r = 3 \qquad\qquad 2r = -3$$
$$r = 1.5 \quad \text{or} \quad r = -1.5$$

12. **a)** Write a quadratic function with zeros at 0.5 and -0.75.

b) Compare your function with a classmate's function. Did you get the same function?

c) Working with a classmate, determine two other possible functions with the same zeros.

Pre-Publication

13. Sanela sells posters to stores. The profit function for her business is

$$P(n) = -0.25n^2 + 6n - 27$$

where n is the number of posters sold per month, in hundreds, and $P(n)$ is the profit, in thousands of dollars.

a) How many posters must Sanela sell per month to break even?

b) If Sanela wants to earn a profit of \$5000 ($P(n) = 5$), how many posters must she sell?

c) If Sanela wants to earn a profit of \$9000, how many posters must she sell?

d) What are the domain and range of the profit function? Explain your answer.

14. Samuel is hiking along the top of First Canyon on the South Nahanni River in the Northwest Territories. When he knocks a rock over the edge, it falls into the river, 1260 m below. The height of the rock, $h(t)$, at t seconds, can be modelled by the following function:

$$h(t) = -25t^2 - 5t + 1260$$

a) How long will it take the rock to reach the water?

b) What is the domain of the function? Explain your answer.



The Nahanni River in the Northwest Territories is a popular area for wilderness tours and whitewater rafting.

15. a) Create a quadratic equation that can be solved by factoring. Exchange equations with a classmate, and solve each other's equations.

b) Modify the equation you created so that it cannot be factored. Explain how you modified it. Then exchange equations with a classmate again, and solve each other's equations a different way.

Closing

16. a) Explain the steps you would follow to solve a quadratic equation by factoring.

b) When does it make sense to solve a quadratic equation by factoring? When does it make sense to use graphing?

Extending

17. One root of an equation in the form $ax^2 + c = 0$ is 6.

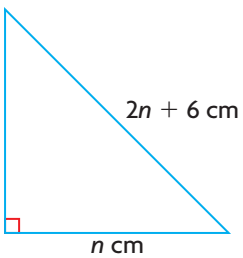
a) What can you predict about the factors if there is no bx term in the equation?

b) Determine the other root.

c) Write the equation in factored form.

d) Write the equation in standard form.

18. The perimeter of this right triangle is 60 cm. Determine the lengths of all three sides.



Pre-Publication

7.6 Vertex Form of a Quadratic Function

YOU WILL NEED

- graph paper and ruler
- graphing technology
- calculator

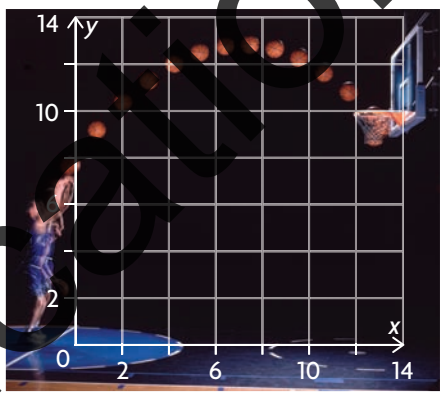
GOAL

Graph a quadratic function in the form $y = a(x - h)^2 + k$, and relate the characteristics of the graph to its equation.

EXPLORE...

- Quadratic functions can be written in different forms. The basic quadratic function is $y = x^2$.
  Use a calculator to graph the following quadratic functions. Explain how the basic function is related to each function. Describe how the changes in the function affect the graph.
  a) $y = (x - 3)^2$
  b) $y = x^2 - 5$
  c) $y = (x + 1)^2 - 2$
  d) $y = (x + 4)^2 + 6$
  e) $y = -2(x + 1)^2 + 3$
  f) $y = 3(x - 2)^2 - 4$

INVESTIGATE the Math



A high-school basketball coach brought in Judy, a trainer from one of the local college teams, to talk to the players about shot analysis. Judy demonstrated, using stroboscopic photographs, how shots can be analyzed and represented by quadratic functions. She used the following function to model a shot:

$$y = -0.1(x - 8)^2 + 13$$

In this function, x represents the horizontal distance, in feet, of the ball from the player and y represents the vertical height, in feet, of the ball above the floor.

Judy mentioned that once she had a quadratic equation in this form, she did not need the photographs. She could quickly sketch a graph of the path of the ball just by looking at the equation.

? How could Judy predict what the graph of the quadratic function would look like?

A. Graph the following function:

$$y = x^2$$

Change the graph by changing the coefficient of x^2. Try both positive and negative values. How do the parabolas change as you change this coefficient?

B. For each function you graphed in part A, determine the coordinates of the vertex and the equation of the axis of symmetry.

Pre-Publication

C. Graph this function:

$$y = x^2 + 1$$

Change the graph by changing the constant. Try both positive and negative values. How do the parabolas change as you change the constant? How do the coordinates of the vertex and the equation of the axis of symmetry change?

D. Graph this function:

$$y = (x - 1)^2$$

Change the graph by changing the constant. Try both positive and negative values. How do the parabolas change as you change the constant? How do the coordinates of the vertex and the equation of the axis of symmetry change?

E. The equation that Judy used was expressed in vertex form:

$$y = a(x - h)^2 + k$$

Make a conjecture about how the values of a, h, and k determine the characteristics of a parabola.

Communication Tip

A quadratic function is in vertex form when it is written in the form $y = a(x - h)^2 + k$

F. Test your conjecture by predicting the characteristics of the graph of the following function:

$$y = -0.1(x - 8)^2 + 13$$

Use your predictions to sketch a graph of the function.

G. Using a graphing calculator, graph the function from part F:

$$y = -0.1(x - 8)^2 + 13$$

How does your sketch compare with this graph? Are your predictions supported? Explain.

Reflecting

H. Does the value of a in a quadratic function always represent the same characteristic of the parabola, whether the function is written in standard form, factored form, or vertex form? Explain.

I. Neil claims that when you are given the vertex form of a quadratic function, you can determine the domain and range without having to graph the function. Do you agree or disagree? Explain.

J. Which form of the quadratic function—standard, factored, or vertex—would you prefer to start with, if you wanted to sketch the graph of the function? Explain.

Pre-Publication

APPLY the Math

EXAMPLE 1 Sketching the graph of a quadratic function given in vertex form

Sketch the graph of the following function:

$$f(x) = 2(x - 3)^2 - 4$$

State the domain and range of the function.

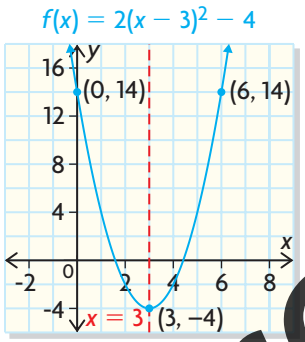
Samuel's Solution

$f(x) = 2(x - 3)^2 - 4$
Since $a > 0$, the parabola opens upward.
The vertex is at $(3, -4)$.
The equation of the axis of symmetry is $x = 3$.

The function was given in vertex form. I listed the characteristics of the parabola that I could determine from the equation.

$f(0) = 2(0 - 3)^2 - 4$
$f(0) = 2(-3)^2 - 4$
$f(0) = 2(9) - 4$
$f(0) = 18 - 4$
$f(0) = 14$
Point $(0, 14)$ is on the parabola.

To determine another point on the parabola, I substituted 0 for x.



I plotted the vertex and the point I had determined, (0, 14). Then I drew the axis of symmetry. I used symmetry to determine the point that is the same horizontal distance from (0, 14) to the axis of symmetry. This point is (6, 14). I connected all three points with a smooth curve.

Domain and range:
$\{(x, y) \mid x \in \mathbb{R}, y \geq -4, y \in \mathbb{R}\}$

Your Turn

Sketch the graph of the following function:

$$f(x) = -\frac{1}{2}(x + 6)^2 + 1$$

State the domain and range of the function. Justify your decision.

Pre-Publication

EXAMPLE 2 Determining the equation of a parabola using its graph

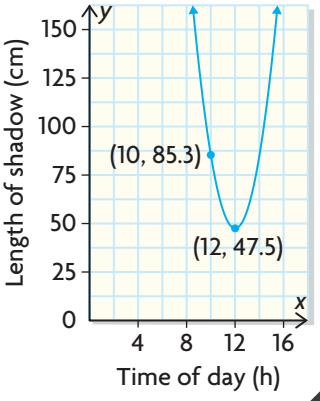
Liam measured the length of the shadow that was cast by a metre stick at 10 a.m. and at noon near his home in Saskatoon. Other students in his class also measured the shadow at different times during the day. They had read that, when graphed as shadow length versus time, the data should form a parabola with a minimum at noon, because the shadow is shortest at noon. Liam decided to try to predict the equation of the parabola, without the other students' data.



Determine the equation that represents the relationship between the time of day and the length of the shadow cast by a metre stick.

Liam's Solution

I have the points (10, 85.3) and (12, 47.5).



I measured the length of the shadow in centimetres. My measurements were 85.3 cm at 10 a.m. and 47.5 cm at noon.

I drew a sketch of a parabola using (12, 47.5) as the vertex, since the length of the shadow at noon should be the minimum value of the function.

$f(x) = a(x - h)^2 + k$
$f(x) = a(x - 12)^2 + 47.5$

I decided to use the vertex form of the quadratic function, since I already knew the values of h and k in this form.

Solving for a:
$85.3 = a(10 - 12)^2 + 47.5$
$85.3 = a(-2)^2 + 47.5$
$85.3 = 4a + 47.5$
$37.8 = 4a$
$9.45 = a$

I knew that (10, 85.3) is a point on the parabola. I substituted the coordinates of this point into the equation and then solved for a.

The function that represents the parabola is
$f(x) = 9.45(x - 12)^2 + 47.5$

The domain and range of this function depend on the hours of daylight, which depends on the time of year.

Pre-Publication

Your Turn

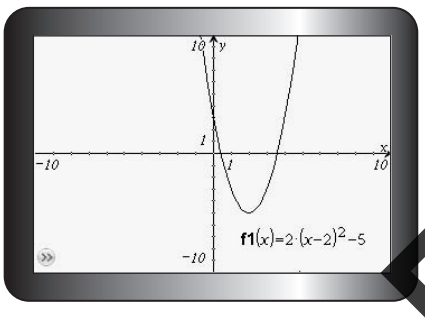
Donald, a classmate of Liam's, lives across the city. Donald measured the length of the shadow cast by a metre stick as 47.0 cm at noon and 198.2 cm at 4:00 p.m. Determine a quadratic function using Donald's data, and explain how his function is related to Liam's function.

EXAMPLE 3 Reasoning about the number of zeros that a quadratic function will have

Randy claims that he can predict whether a quadratic function will have zero, one, or two zeros if the function is expressed in vertex form. How can you show that he is correct?

Eugene's Solution

$f(x) = 2(x - 2)^2 - 5$
Conjecture: two zeros

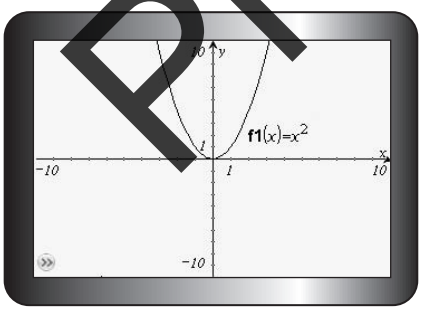


The graph supports my conjecture.

The vertex of the parabola that is defined by the function is at (2, −5), so the vertex is below the *x*-axis. The parabola must open upward because *a* is positive. Therefore, I should observe two *x*-intercepts when I graph the function.

To test my conjecture, I graphed the function on a calculator. I can see two *x*-intercepts on my graph, so the function has two zeros.

$f(x) = x^2$
$f(x) = (x - 0)^2 + 0$
Conjecture: one zero



The graph supports my conjecture.

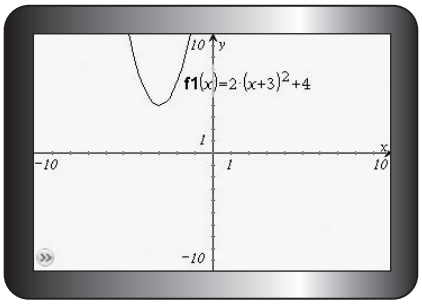
I decided to use the basic quadratic function, since this provided me with a convenient location for the vertex, (0, 0).

Since the vertex is on the *x*-axis and the parabola opens up, this means that I should observe only one *x*-intercept when I graph the function.

To test my conjecture, I graphed the function on a calculator. Based on my graph, I concluded that the function has only one zero.

Pre-Publication

$f(x) = 2(x + 3)^2 + 4$
Conjecture: no zeros



The graph supports my conjecture.

The vertex of the parabola that is defined by this function is at $(-3, 4)$, and the parabola opens upward. The vertex lies above the x-axis, so I should observe no x-intercepts when I graph the function.

To test my conjecture, I graphed the function on a calculator. I concluded that the function has no zeros.

Your Turn

a) Define three different quadratic functions, in vertex form, that open downward. One function should have two zeros, another should have one zero, and the third should have no zeros.

b) Explain how you were able to connect the number of zeros to each function.

EXAMPLE 4 Solving a problem that can be modelled by a quadratic function

A soccer ball is kicked from the ground. After 2 s, the ball reaches its maximum height of 20 m. It lands on the ground at 4 s.

a) Determine the quadratic function that models the height of the kick.

b) Determine any restrictions that must be placed on the domain and range of the function.

c) What was the height of the ball at 1 s? When was the ball at the same height on the way down?



Pre-Publication

Tia's Solution

a) Let x represent the elapsed time in seconds, and let y represent the height in metres.

$y = a(x - h)^2 + k$

The maximum height is 20 m at the elapsed time of 2 s.

Vertex:

$(x, y) = (2, 20)$

Since I knew the maximum height and when it occurred, I also knew the coordinates of the vertex. I decided to use the vertex form to determine the equation.

$y = a(x - 2)^2 + 20$

I substituted the known values.

Solving for a:

$f(x) = a(4 - 2)^2 + 20$

$0 = a(2)^2 + 20$

$0 = 4a + 20$

$-20 = 4a$

$-5 = a$

To determine the value of a, I substituted the coordinates of the point that corresponds to the ball hitting the ground, (4, 0).

The following quadratic function models the height of the kick:

$f(x) = -5(x - 2)^2 + 20$

b) Time at beginning of kick:

$x = 0$

Time when ball hits ground:

$x = 4$

Domain: $\{x \mid 0 \leq x \leq 4, x \in R\}$

At the beginning of the kick, the time is 0 *s*. When the ball lands, the time is 4 *s*. I can only use *x*-values in this interval. Time in seconds is continuous, so the set is real numbers.

Vertex: (2, 20)

Height of ball at beginning of kick: 0 m

Height of ball at vertex: 20 m

Range: $\{y \mid 0 \leq y \leq 20, y \in R\}$

The ball starts on the ground, at a height of 0 m, and rises to its greatest height, 20 m. The ball is not below the ground at any point. Height in metres is continuous, so the set is real numbers.

Pre-Publication

c) $f(x) = -5(x - 2)^2 + 20$
$f(1) = -5(1 - 2)^2 + 20$
$f(1) = -5(-1)^2 + 20$
$f(1) = -5 + 20$
$f(1) = 15$

The ball was at a height of 15 m after 1 s.
This occurred as the ball was rising.

I used the vertex form of the quadratic function to determine the height of the ball at 1 s.

Equation of the axis of symmetry:
$x = 2$

Symmetry provides the point (3, 15).
The ball was also 15 m above the ground at 3 s.
This occurred as the ball was on its way down.

I knew that point (1, 15) is 1 unit to the left of the axis of symmetry of the parabola. The other point on the parabola, with height 15 m, should be 1 unit to the right of the axis of symmetry. This means that the x-coordinate of the point must be 3.

Your Turn

The goalkeeper kicked the soccer ball from the ground. It reached its maximum height of 24.2 m after 2.2 s. The ball was in the air for 4.4 s.

a) Define the quadratic function that models the height of the ball above the ground.

b) How is the equation for this function similar to the equation that Tia determined? Explain.

c) After 4 s, how high was the ball above the ground?

Pre-Publication

In Summary

Key Idea

- The vertex form of the equation of a quadratic function is written as follows:

$$y = a(x - h)^2 + k$$

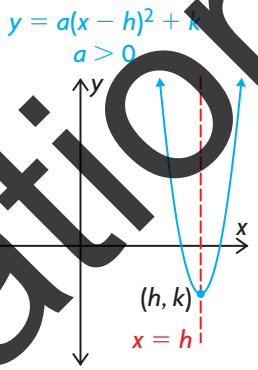
The graph of the function can be sketched more easily using this form.

Need to Know

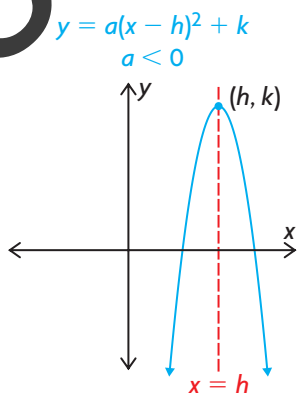
- A quadratic function that is written in vertex form,

$$y = a(x - h)^2 + k$$

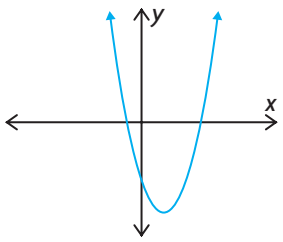
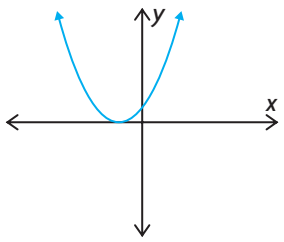
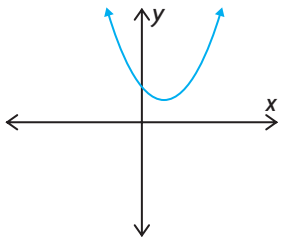
has the following characteristics:
  - The vertex of the parabola has the coordinates (h, k).
  - The equation of the axis of symmetry of the parabola is $x = h$.



  - The parabola opens upward when $a > 0$, and the function has a minimum value of k when $x = h$.
  - The parabola opens downward when $a < 0$, and the function has a maximum value of k when $x = h$.



- A parabola may have zero, one, or two x-intercepts, depending on the location of the vertex and the direction in which the parabola opens. By examining the vertex form of the quadratic function, it is possible to determine the number of zeros, and therefore the number of x-intercepts.

Two x-intercepts	One x-intercept	No x-intercepts
		

Pre-Publication

CHECK Your Understanding

1. For each quadratic function below, identify the following:

i) the direction in which the parabola opens

ii) the coordinates of the vertex

iii) the equation of the axis of symmetry

a) $f(x) = (x - 3)^2 + 7$

b) $m(x) = -2(x + 7)^2 - 3$

c) $g(x) = 7(x - 2)^2 - 9$

d) $n(x) = \frac{1}{2}(x + 1)^2 + 10$

e) $r(x) = -2x^2 + 5$

2. Predict which of the following functions have a maximum value and which have a minimum value. Also predict the number of x-intercepts that each function has. Test your predictions by sketching the graph of each function.

a) $f(x) = -x^2 + 3$

b) $q(x) = -(x + 2)^2 - 5$

c) $m(x) = (x + 4)^2 + 2$

d) $n(x) = (x - 3)^2 - 6$

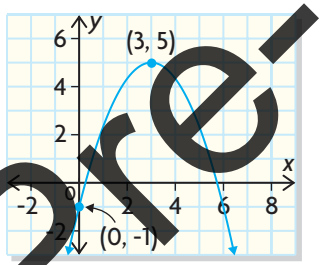
e) $r(x) = 2(x - 4)^2 + 2$

3. Determine the value of a, if point $(-1, 4)$ is on the quadratic function:

$$f(x) = a(x + 2)^2 + 7$$

PRACTISING

4. Which equation represents the graph? Justify your decision.



A. $y = -\frac{2}{3}x^2 + 5$

B. $y = -(x - 3)^2 + 5$

C. $y = -\frac{2}{3}(x - 3)^2 + 5$

D. $y = \frac{2}{3}(x - 3)^2 + 5$

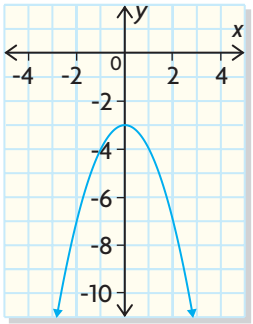
Pre-Publication

5. Match each equation with its corresponding graph. Explain your reasoning.

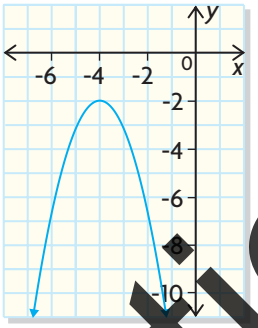
a) $y = (x - 3)^2$ **c)** $y = -x^2 - 3$

b) $y = -(x + 4)^2 - 2$ **d)** $y = (x - 4)^2 + 2$

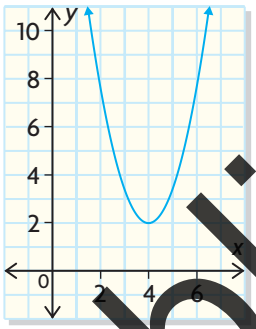
i)



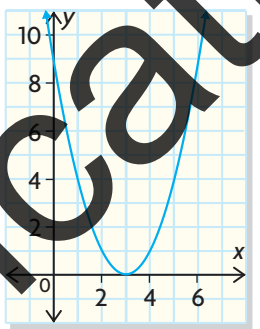
iii)



ii)



iv)

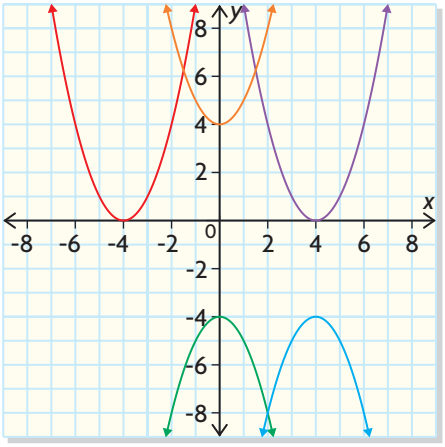


6. Explain how you would determine whether a parabola contains a minimum value or maximum value when the quadratic function that defines it is in vertex form:

$$y = a(x - h)^2 + k$$

Support your explanation with examples of functions and graphs.

7. State the equation of each function, if all the parabolas are congruent and if $a = 1$ or $a = -1$.



Pre-Publication

8. Marleen and Candice are both 6 ft tall, and they play on the same college volleyball team. In a game, Candice set up Marleen with an outside high ball for an attack hit. Using a video of the game, their coach determined that the height of the ball above the court, in feet, on its path from Candice to Marleen could be defined by the function



$$h(x) = -0.03(x - 9)^2 + 8$$

where x is the horizontal distance, measured in feet, from one edge of the court.

a) Determine the axis of symmetry of the parabola.

b) Marleen hit the ball at its highest point. How high above the court was the ball when she hit it?

c) How high was the ball when Candice set it, if she was 2 ft from the edge of the court?

d) State the range for the ball's path between Candice and Marleen. Justify your answer.

9. **a)** Write quadratic functions that define three different parabolas, all with their vertex at $(3, -1)$.

b) Predict how the graphs of the parabolas will be different from each other.

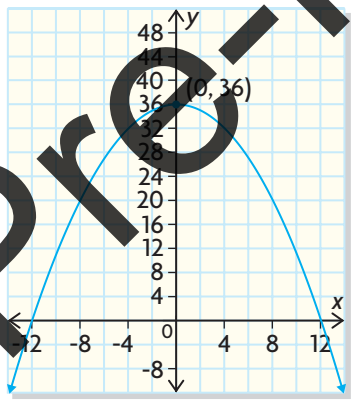
c) Graph each parabola on the same coordinate plane. How accurate were your predictions?

10. Without using a table of values or a graphing calculator, describe how you would graph the following function:

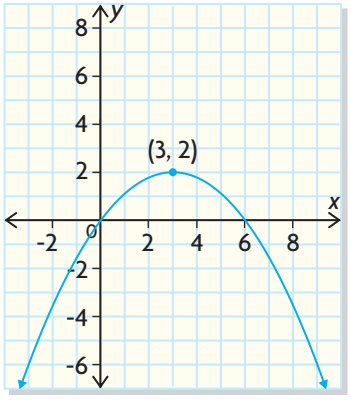
$$f(x) = 2(x - 1)^2 - 9$$

11. For each graph, determine the equation of the quadratic function in vertex form.

a)



b)



Pre-Publication

12. The vertex of a parabola is at $(4, -12)$.

a) Write a function to define all the parabolas with this vertex.

b) A parabola with this vertex passes through point (13, 15). Determine the function for the parabola.

c) State the domain and range of the function you determined in part b).

d) Graph the quadratic function you determined in part b).

13. The height of the water, $h(t)$, in metres, that is sprayed from a sprinkler at a local golf course, can be modelled by the function

$$h(t) = -4.9(t - 1.5)^2 + 11.3$$

where time, t, is measured in seconds.

a) Graph the function, and estimate the zeros of the function.

b) What do the zeros represent in this situation?

14. A parabolic arch has x-intercepts $x = -6$ and $x = -1$. The parabola has a maximum height of 15 m.

a) Determine the quadratic function that models the parabola.

b) State the domain and range of the function.



15. Serge and a friend are throwing a paper airplane to each other. They stand 5 m apart from each other and catch the airplane at a height of 1 m above the ground. Serge throws the airplane on a parabolic flight path that achieves a minimum height of 0.5 m halfway to his friend.

a) Determine a quadratic function that models the flight path for the height of the airplane.

b) Determine the height of the plane when it is a horizontal distance of 1 m from Serge's friend.

c) State the domain and range of the function.

Closing

16. Liz claims that she can sketch an accurate graph more easily if a quadratic function is given in vertex form, rather than in standard or factored form. Do you agree or disagree? Explain.



An atlatl is used to launch a dart. It has been used as a hunting tool by peoples all over the world for thousands of years.

Extending

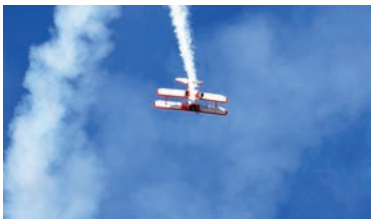
17. Peter is studying the flight path of an atlatl dart for a physics project. In a trial toss on the sports field, Peter threw his dart 80 yd and hit a platform that was 2 yd above the ground. The maximum vertical height of the atlatl was 10 yd. The dart was 2 yd above the ground when released.

a) Sketch a graph that models the flight path of the dart thrown by Peter.

b) How far from Peter, horizontally, was the atlatl dart when it reached a vertical height of 8 yd? Explain.

Pre-Publication

18. When an airplane is accelerated downward by combining its engine power with gravity, the airplane is said to be in a power dive. At the Abbotsford International Air Show, one of the stunt planes began such a manoeuvre. Selected data from the plane's flight log is shown below.



t	0	4	8	16
$h(t)$	520	200	40	200

a) Define a function, $h(t)$, that models the height of the plane above the ground, in metres, over time, t, in seconds, after the manoeuvre began.

b) How low to the ground did the plane get on this manoeuvre?

c) How long did it take for the plane to return to its initial altitude?

19. A bridge is going to be constructed over a river, as shown below. The supporting arch of the bridge will form a parabola. At the point where the bridge is going to be constructed, the river is 20 m wide from bank to bank. The arch will be anchored on the ground, 4 m from the edge of the riverbank on each side. The maximum height of the arch can be between 18 m and 22 m above the surface of the water. Create two different quadratic functions that model the supporting arch. Include a labelled graph for each arch.



Pre-Publication

7.7 Solving Quadratic Equations Using the Quadratic Formula

YOU WILL NEED

- graphing technology

GOAL

Use the quadratic formula to determine the roots of a quadratic equation.

EXPLORE...

- Kyle was given the following function:

  $y = 2x^2 + 12x - 14$

  He wrote it in vertex form:

  $y = 2(x + 3)^2 - 32$

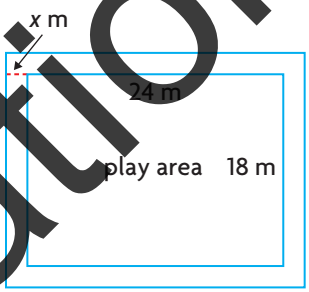
  How can you use the vertex form to solve this equation?

  $2x^2 + 12x - 14 = 0$



LEARN ABOUT the Math

Ian has been hired to lay a path of uniform width around a rectangular play area, using crushed rock. He has enough crushed rock to cover 145 m^2.



? If Ian uses all the crushed rock, how wide will the path be?

EXAMPLE 1 Using the quadratic formula to solve a quadratic equation

Determine the width of the path that will result in an area of 145 m^2.

Alima's Solution

Area of border = Total area − Play area
The play area is a constant, (length)(width)
or (24 m)(18 m) or 432 m^2.

The total area of the playground, P, can be represented as

P = (length)(width)
$P = (2x + 24)(2x + 18)$

I wrote a function that describes how the area of the path, A square metres, changes as the width of the path, x metres, changes.

The area of the path, $A(x)$, can be represented as

$A(x) = (2x + 24)(2x + 18) - 432$
$A(x) = 4x^2 + 84x + 432 - 432$
$A(x) = 4x^2 + 84x$

Pre-Publication

$145 = 4x^2 + 84x$

I substituted the area of 145 m^2 for $A(x)$.

$4x^2 + 84x - 145 = 0$
$a = 4$, $b = 84$, and $c = -145$

I rewrote the equation in standard form:

$ax^2 + bx + c = 0$

Then I determined the values of the coefficients a, b, and c.

Quadratic formula:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-84 \pm \sqrt{84^2 - 4(4)(-145)}}{2(4)}$$

$$x = \frac{-84 \pm \sqrt{9376}}{8}$$

$$x = \frac{-84 + \sqrt{9376}}{8} \quad \text{or}$$

$$x = \frac{-84 - \sqrt{9376}}{8}$$

$x = 1.603 \ldots$ or $x = -22.603 \ldots$

The **quadratic formula** can be used to solve any quadratic equation. I wrote the quadratic formula and then substituted the values of a, b, and c from my equation into the formula.

I simplified the right side.

I separated the quadratic expression into two solutions.

quadratic formula

A formula for determining the roots of a quadratic equation in the form $ax^2 + bx + c = 0$, where $a \neq 0$; the quadratic formula is written using the coefficients of the variables and the constant in the quadratic equation that is being solved:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

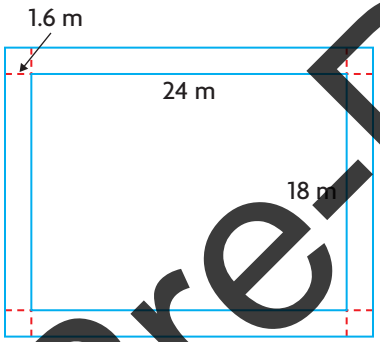
This formula is derived from $ax^2 + bx + c = 0$ by isolating x.

The solution -22.603 is inadmissible.

I knew that the width of the path couldn't be negative, so $-22.603 \ldots$ is an **inadmissible solution**.

inadmissible solution

A root of a quadratic equation that does not lead to a solution that satisfies the original problem.



I sketched the path and verified my solution by determining the area of the path. To do this, I added the areas of all the rectangles that make up the path.

$18(1.603 \ldots) = 28.866 \ldots$ m^2
$24(1.603 \ldots) = 38.489 \ldots$ m^2
$(1.603 \ldots)(1.603 \ldots) = 2.571 \ldots$ m^2

Area of path $= 2(28.866 \ldots) + 2(38.489 \ldots) + 4(2.571 \ldots)$
Area of path $= 144.999 \ldots$ m^2

The total area is very close to 145 m^2.
The path should be about 1.6 m wide.

Pre-Publication

Reflecting

A. Why did Alima need to write her equation in standard form?

B. Which part of the quadratic formula shows that there are two possible solutions?

C. Why did Alima decide not to use the negative solution?

D. In this chapter, you have learned three methods for solving quadratic equations: graphing, factoring, and using the quadratic formula. What are some advantages and disadvantages of each method?

APPLY the Math

EXAMPLE 2 Connecting the quadratic formula to factoring

Solve the following equation:

$$6x^2 - 3 = 7x$$

Adrianne's Solution

$$6x^2 - 3 = 7x$$
$$6x^2 - 7x - 3 = 0$$
$$a = 6,\ b = -7, \text{ and } c = -3$$

First, I rewrote the equation in standard form to determine the values of a, b, and c.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$x = \frac{-(-7) \pm \sqrt{(-7)^2 - 4(6)(-3)}}{2(6)}$$
$$x = \frac{7 \pm \sqrt{121}}{12}$$
$$x = \frac{7 \pm 11}{12}$$

I wrote the quadratic formula and substituted the values of a, b, and c.

I simplified the right side. I realized that 121 is a perfect square.

$$x = \frac{18}{12} \quad \text{or} \quad x = \frac{-4}{12}$$
$$x = \frac{3}{2} \quad \text{or} \quad x = \frac{-1}{3}$$

I determined the two solutions.

Verify:

$$6x^2 - 7x - 3 = 0$$
$$(3x + 1)(2x - 3) = 0$$
$$3x + 1 = 0 \quad \text{or} \quad 2x - 3 = 0$$
$$3x = -1 \qquad 2x = 3$$
$$x = \frac{-1}{3} \qquad x = \frac{3}{2}$$

If the radicand in the quadratic formula is a perfect square, then the original equation can be factored. I decided to verify my solution by factoring the original equation.

The solutions match those I got using the quadratic formula.

Pre-Publication

Your Turn

Sandy was given the following equation:

$$12x^2 - 47x + 45 = 0$$

She used the quadratic formula to solve it.

Could Sandy use factoring to verify her solutions? Explain how you know.

$$x = \frac{-(-47) \pm \sqrt{(-47)^2 - 4(12)(45)}}{2(12)}$$

$$x = \frac{47 \pm \sqrt{49}}{24}$$

$$x = 2\frac{1}{4} \quad \text{or} \quad x = \frac{5}{3}$$

EXAMPLE 3 Determining the exact solution to a quadratic equation

Solve this quadratic equation:

$$2x^2 + 8x - 5 = 0$$

State your answer as an exact value.

Quyen's Solution

$2x^2 + 8x - 5 = 0$

$a = 2$, $b = 8$, and $c = -5$

The equation was in standard form. I determined the values of a, b, and c.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(8) \pm \sqrt{8^2 - 4(2)(-5)}}{2(2)}$$

I wrote the quadratic formula and substituted the values of a, b, and c.

$$x = \frac{-8 \pm \sqrt{104}}{4}$$

$$x = \frac{-8 \pm \sqrt{4}\sqrt{26}}{4}$$

$$x = \frac{-8 \pm 2\sqrt{26}}{4}$$

$$x = \frac{-4 \pm \sqrt{26}}{2}$$

I simplified the expression.

I noticed that one factor of 104 is 4, which is a perfect square. I simplified the radical.

I simplified the fraction.

$$x = \frac{-4 + \sqrt{26}}{2} \quad \text{or} \quad x = \frac{-4 - \sqrt{26}}{2}$$

Another way to write my solution is to show two separate values.

Your Turn

Solve the following quadratic equation:

$$5x^2 - 10x + 3 = 0$$

State your answer as an exact value.

Pre-Publication

EXAMPLE 4 Solving a pricing problem

A store rents an average of 750 video games each month at the current rate of $4.50. The owners of the store want to raise the rental rate to increase the revenue to $7000 per month. However, for every $1 increase, they know that they will rent 30 fewer games each month. The following function relates the price increase, p, to the revenue, r:

$$(4.5 + p)(750 - 30p) = r$$

Can the owners increase the rental rate enough to generate revenue of $7000 per month?



Christa's Solution

$$(4.5 + p)(750 - 30p) = r$$
$$3375 + 615p - 30p^2 = r$$

I simplified the function.

$$3375 + 615p - 30p^2 = 7000$$

I substituted the revenue of $7000 for r and wrote the equation in standard form.

$$-30p^2 + 615p + 3375 = 7000$$
$$-30p^2 + 615p - 3625 = 0$$

I divided each term by -5 to simplify the equation.

$$\frac{-30p^2}{-5} + \frac{615p}{-5} - \frac{3625}{-5} = \frac{0}{-5}$$
$$6p^2 - 123p + 725 = 0$$

I didn't try to factor the equation since the numbers were large. I decided to use the quadratic formula.

$$p = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

I substituted the values of a, b, and c into the quadratic formula.

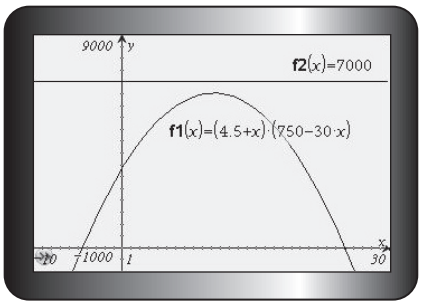
$$p = \frac{-(-123) \pm \sqrt{(-123)^2 - 4(6)(725)}}{2(6)}$$
$$p = \frac{123 \pm \sqrt{-2271}}{12}$$

I simplified the right side.

$\sqrt{-2271}$ is not a real number, so there are no real solutions to this equation. It is not possible for the store to generate revenue of $7000 per month by increasing the rental rate.

I noticed that the radicand is negative.

Pre-Publication



To verify my answer, I graphed

$y = (4.5 + x)(750 - 30x)$ and $y = 7000$

There is no point of intersection.

Your Turn

Is it possible for the store to generate revenue of \$6500 per month by increasing the rental rate? Explain.

In Summary

Key Idea

- The roots of a quadratic equation in the form $ax^2 + bx + c = 0$, where $a \neq 0$, can be determined by using the quadratic formula:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Need to Know

- The quadratic formula can be used to solve any quadratic equation, even if the equation is not factorable.
- If the radicand in the quadratic formula simplifies to a perfect square, then the equation can be solved by factoring.
- If the radicand in the quadratic formula simplifies to a negative number, then there is no real solution for the quadratic equation.

CHECK Your Understanding

1. Solve each equation using the quadratic formula. Verify by graphing.

a) $x^2 + 7x - 5 = 0$ **c)** $2a^2 - 5a + 1 = 0$

b) $8x^2 + 35x + 12 = 0$ **d)** $-20p^2 + 7p + 3 = 0$

2. Solve each equation using the quadratic formula.

a) $x^2 + 5x - 6 = 0$ **c)** $25x^2 - 121 = 0$

b) $4x + 9x^2 = 0$ **d)** $12x^2 - 17x - 40 = 0$

3. Solve each equation in question 2 by factoring. Which method did you prefer for each equation? Explain.

Pre-Publication

PRACTISING

4. Solve each quadratic equation.

a) $3x^2 + 5x = 9$ **c)** $6x - 3 = 2x^2$

b) $1.4x - 3.9x^2 = -2.7$ **d)** $x^2 + 1 = x$

5. The roots for the quadratic equation

$$1.44a^2 + 2.88a - 21.6 = 0$$

are $a = 3$ and $a = -5$. Verify these roots.

6. Solve each equation. State the solutions as exact values.

a) $3x^2 - 6x - 1 = 0$ **c)** $8x^2 + 8x - 1 = 0$

b) $x^2 + 8x + 3 = 0$ **d)** $9x^2 - 12x - 1 = 0$

7. A student council is holding a raffle to raise money for a charity fund drive. The profit function for the raffle is

$$p(c) = -25c^2 + 500c - 350$$

where $p(c)$ is the profit and c is the price of each ticket, both in dollars.

a) What ticket price will result in the student council breaking even on the raffle?

b) What ticket price will raise the most money for the school's donation to charity?

8. Akpatok Island in Nunavut is surrounded by steep cliffs along the coast. The cliffs range in height from about 125 m to about 250 m.

a) Suppose that someone accidentally dislodged a stone from a 125 m cliff. The height of the stone, $h(t)$, in metres, after t seconds can be represented by the following function:

$$h(t) = -4.9t^2 + 4t + 125$$

How long would it take the stone dislodged from this height to reach the water below?

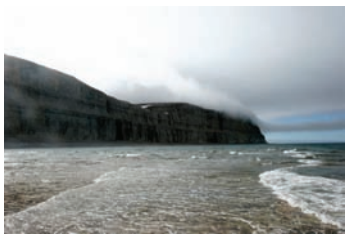
b) Predict how much longer it would take for the stone to reach the water if it fell from a height of 250 m. Discuss this with a partner.

c) The height of a stone, $h(t)$, in metres, falling from a 250 m cliff over time, t, in seconds, can be modelled by this function:

$$h(t) = -4.9t^2 + 4t + 250$$

Determine how long it would take the stone to reach the water.

d) How close was your prediction to your solution?



Akpatok Island gets its name from the word *Akpat*, the Innu name for the birds that live on the cliffs.

9. Keisha and Savannah used different methods to solve this equation:

$$116.64z^2 + 174.96z + 65.61 = 0$$

a) Could one of these students have used factoring? Explain.

b) Solve the equation using the method of your choice.

c) Which method did you use? Why?

Pre-Publication

10. The Moon's gravity affects the way that objects travel when they are thrown on the Moon. Suppose that you threw a ball upward from the top of a lunar module, 5.5 m high. The height of the ball, $h(t)$, in metres, over time, t, in seconds could be modelled by this function:

$$h(t) = -0.81t^2 + 5t + 6.5$$

a) How long would it take for the ball to hit the surface of the Moon?

b) If you threw the same ball from a model of the lunar module on Earth, the height of the ball could be modelled by this function:

$$h(t) = -4.9t^2 + 5t + 6.5$$

Compare the time that the ball would be in flight on Earth with the time that the ball would be in flight on the Moon.



Six lunar modules landed on the Moon from 1969 to 1972.

11. A landscaper is designing a rectangular garden, which will be 5.00 m wide by 6.25 m long. She has enough crushed rock to cover an area of 6.0 m^2 and wants to make a uniform border around the garden. How wide should the border be, if she wants to use all the crushed rock?



Closing

12. Discuss the quadratic formula with a partner. Make a list of everything you have both learned, from your work in this lesson, about using the quadratic formula to solve quadratic equations.

Extending

13. The two roots of any quadratic equation are

$$x = \frac{-b + \sqrt{b^2 - 4ac}}{2a} \quad \text{and} \quad x = \frac{-b - \sqrt{b^2 - 4ac}}{2a}$$

a) Determine the sum of the roots of any quadratic equation.

b) Determine the product of the roots of any quadratic equation.

c) Solve the following quadratic equation:

$$10x^2 - 13x + 4 = 0$$

Determine the sum and the product of its roots.

d) Determine the sum and the product of the roots of the quadratic equation in part c), using your formulas from parts a) and b). Do your answers match your answers from part c)?

e) Determine the sum and the product of the solutions to questions 1 d), 2 a), 5, and 7.

f) How could you use your formulas from parts a) and b) to check your solutions to any quadratic equation?

Pre-Publication

History | Connection

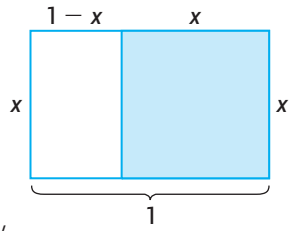
The Golden Ratio

The golden ratio has been discovered and rediscovered by many civilizations. Its uses in architecture include the Great Pyramids in Egypt and the Parthenon in Greece. The golden ratio is the ratio of length to width in a rectangle with special properties, called the golden rectangle. This rectangle appears often in art, architecture, and photography.



The Manitoba Legislative Building

If you section off a square inside a golden rectangle so that the side length of the square equals the width of the golden rectangle, you will create a smaller rectangle with the same length : width ratio. Mathematicians sometimes use the Greek letter *phi*, φ, to represent this ratio.



To determine the golden ratio, you need to know that the ratio of length to width in the original rectangle, $\frac{1}{x}$, is equal to the ratio of length to width in the smaller rectangle, $\frac{x}{1-x}$.

A. Solve the following equation for x to determine the width of a rectangle with length 1. Then determine $\frac{1}{x}$ to get the golden ratio, φ.

$$\frac{1}{x} = \frac{x}{1-x}$$

B. Work with a partner or group to find golden rectangles in the photograph of the Manitoba Legislative Building.

C. Find more golden rectangles in architecture, art, and nature. Present your findings to the class.

Pre-Publication

7.8 Solving Problems Using Quadratic Models

GOAL

Analyze and solve problems that involve quadratic functions and equations.

YOU WILL NEED

- graphing technology

EXPLORE...

- A right triangle has sides of length x, $2x + 4$, and $3x - 4$. Write a quadratic equation to determine the value of x. Is there more than one solution?

LEARN ABOUT the Math

The engineers who designed the Coal River Bridge on the Alaska Highway in British Columbia used a supporting arch with twin metal arcs.

The function that describes the arch is

$$h(x) = -0.005\,061x^2 + 0.499\,015x$$

where $h(x)$ is the height, in metres, of the arch above the ice at any distance, x, in metres, from one end of the bridge.

? How can you use the width of the arch to determine the height of the bridge?

EXAMPLE 1 Solving a problem by factoring a quadratic equation

Determine the distance between the bases of the arch. Then determine the maximum height of the arch, to the nearest tenth of a metre.

Morgan's Solution

The coordinates of the maximum are (x, y), where x is halfway between the two bases of the arch and y is the height of the arch.

I reasoned that the bridge is symmetrical and resting on the vertex of the arch.

$$h(x) = -0.005\,061x^2 + 0.499\,015x$$
$$0 = -0.005\,061x^2 + 0.499\,015x$$

I wrote an equation to determine the x-coordinates of the bases of the arch. The height at each base is 0 m, so the value of $h(x)$ at these points is 0.

Pre-Publication

$0 = -5.061x^2 + 499.015x$
$0 = x(-5.061x + 499.015)$

I multiplied both sides by 1000 and factored the equation.

$x = 0$ or $-5.061x + 499.015 = 0$
$-5.061x = -499.015$
$x = 98.600\ldots$

I solved the equation.

One base is at 0 m, and the other is at 98.600... m. The width of the arch is 98.600... m.

The width of the arch is the distance between the two bases.

Equation of axis of symmetry:

$$x = \frac{0 + 98.600\ldots}{2}$$

$x = 49.300\ldots$

The x-coordinate of the vertex is 49.300...

The function is quadratic, so the arch is a parabola with an axis of symmetry that passes through the vertex.

$h(x) = -0.005\,061x^2 + 0.499\,015x$
For $x = 49.300\ldots$,
$y = -0.005\,061\,(49.300\ldots)^2 + 0.499\,015\,(49.300\ldots)$
$y = -12.300\ldots + 24.601\ldots$
$y = 12.300\ldots$ m

The height of the arch is the y-coordinate of the vertex of the parabola.

The distance between the bases of the bridge is 98.6 m.
The height of the arch above the ice is 12.3 m.

Reflecting

A. How did determining the x-coordinates of the bases of the arch help Morgan determine the height of the arch?

B. What reasoning might have led Morgan to multiply both sides of the equation by 1000?

C. How did Morgan know that the equation $0 = -5.061x^2 + 499.015x$ could be factored?

D. How else could Morgan have solved her quadratic equation?

Pre-Publication

APPLY the Math

EXAMPLE 2 Solving a number problem by graphing

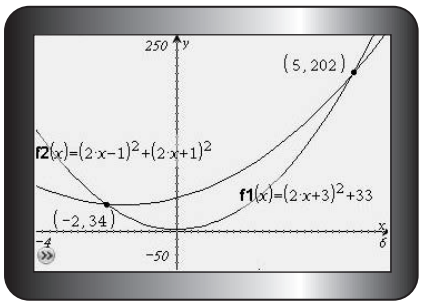
Determine three consecutive odd integers, if the square of the largest integer is 33 less than the sum of the squares of the two smaller integers.

Hailey's Solution

Let the three integers be $2x - 1$, $2x + 1$, and $2x + 3$.

$(2x + 3)^2 + 33 = (2x - 1)^2 + (2x + 1)^2$

Odd numbers are not divisible by 2. According to the problem, if I add 33 to the square of the largest number, my result will equal the sum of the squares of the two smaller numbers.



I graphed both sides of my equation and determined the points of intersection.

The points of intersection are $(-2, 34)$ and $(5, 202)$. The two possible values of x are -2 and 5.

If $x = -2$,

$2x - 1$	$2x + 1$	$2x + 3$
$2(-2) - 1$	$2(-2) + 1$	$2(-2) + 3$
-5	-3	-1

The integers are -5, -3, and -1.

If $x = 5$,

$2x - 1$	$2x + 1$	$2x + 3$
$2(5) - 1$	$2(5) + 1$	$2(5) + 3$
9	11	13

The integers are 9, 11, and 13.

I determined three consecutive odd integers for each value of x.

The consecutive odd integers could be -5, -3, and -1, or they could be 9, 11, and 13.

My answers seem reasonable. $13^2 = 169$ and this is 33 less than $9^2 + 11^2$, which is 202. $(-1)^2 = 1$ and this is 33 less than $(-5)^2 + (-3)^3$, which is 34.

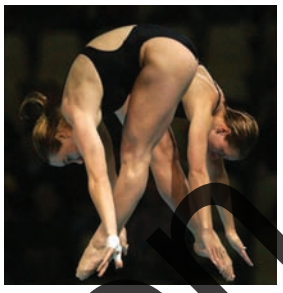
Your Turn

Why was Hailey's method better for solving the problem than simply guessing and testing numbers?

Pre-Publication

EXAMPLE 3 Solving a problem by creating a quadratic model

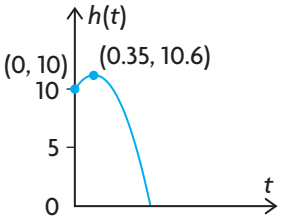
Synchronized divers perform matching dives from opposite sides of a platform that is 10 m high. If two divers reached their maximum height of 0.6 m above the platform after 0.35 s, how long did it take them to reach the water?



Canadians Émilie Heymans and Blythe Hartley won bronze medals at the 2004 Olympic Games.

Oliver's Solution

Let t represent the time in seconds.
Let $h(t)$ represent the height in metres over time.



I sketched a graph to show how the divers' height changed as time passed. I knew that the vertex of the parabola was (0.35, 10.6) because the maximum height of 10.6 m (0.6 m above the 10 m platform) was attained after 0.35 s.

$h(t) = a(t - 0.35)^2 + 10.6$

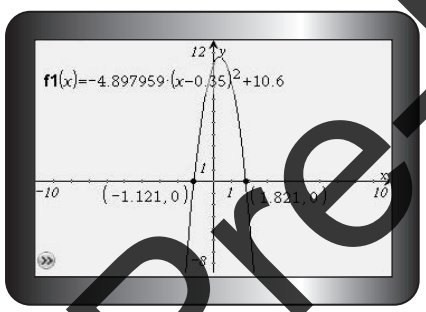
I wrote a quadratic function in vertex form.

$10 = a(0 - 0.35)^2 + 10.6$
$10 = a(-0.35)^2 + 10.6$
$10 = 0.1225a + 10.6$
$-0.6 = 0.1225a$
$-4.897... = a$

The platform is 10 m high. Therefore, when $t = 0$, $h(t) = 10$. I substituted these values into my equation and solved for a.

$f(x) = -4.897...(x - 0.35)^2 + 10.6$

I wrote a function to represent the dive. I knew that the height would be 0 when the divers hit the water.



I graphed my function and determined the x-intercepts.

The zeros of my function are -1.121 and 1.821.
The solution -1.121 s is inadmissible.

Time cannot be negative in this situation.

The divers reached the water after about 1.821 s.

Your Turn

How does Oliver's first graph show that there is only one solution to the problem?

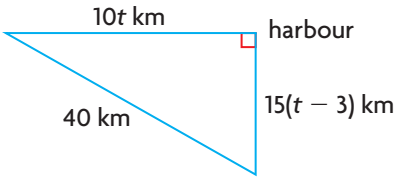
Pre-Publication

EXAMPLE 4 Visualizing a quadratic relationship

At noon, a sailboat leaves a harbour on Vancouver Island and travels due west at 10 km/h. Three hours later, another sailboat leaves the same harbour and travels due south at 15 km/h. At what time, to the nearest minute, will the sailboats be 40 km apart?

Nikki's Solution

Let t be the number of hours it will take for the sailboats to be 40 km apart.



I drew a diagram to show the paths of the two sailboats. They are sailing at right angles to each other.

The first boat travels t hours at 10 km/h.

The second boat leaves 3 h later, so it travels for $t - 3$ h at 15 km/h.

$$(10t)^2 + [15(t - 3)]^2 = 40^2$$

I used the Pythagorean theorem to write an equation that relates the distances travelled to the 40 km distance between them.

$$(10t)^2 + (15t - 45)^2 = 40^2$$
$$100t^2 + 225t^2 - 1350t + 2025 = 1600$$
$$325t^2 - 1350t + 425 = 0$$
$$13t^2 - 54t + 17 = 0$$

I simplified my quadratic equation and wrote it in standard form. Then I divided both sides by 25.

$$t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$t = \frac{-(-54) \pm \sqrt{(-54)^2 - 4(13)(17)}}{2(13)}$$
$$t = \frac{54 \pm \sqrt{2032}}{26}$$
$$t = 0.343\ldots \quad \text{or} \quad t = 3.810\ldots$$

I used the quadratic formula to solve for t, the number of hours that it will take for the boats to be 40 km apart.

The solution 0.343... h is inadmissible.
(0.81... h)(60 min/1 h) = 48.6 min
The boats will be 40 km apart at 3:49 p.m.

The boats could not be 40 km apart after 0.343... h, because the second boat has not yet left the harbour and the first boat is less than 10 km out.

Your Turn

a) Tomas solved the same problem. However, he used t to represent the time for the second boat's journey. How would the labels on Tomas's diagram be different from the labels on Nikki's diagram?

b) Use Tomas's method to solve the problem.

Pre-Publication

In Summary

Key Ideas

- A function, a graph, or a table of values can represent a relation. Use the form that is most helpful for the context of the problem.
- Depending on the information that is given in a problem, you can use a quadratic function in vertex form or in standard form to model the situation.

Need to Know

- A problem may have only one admissible solution, even though the quadratic equation that is used to represent the problem has two real solutions. When you solve a quadratic equation, verify that your solutions make sense in the context of the problem.

CHECK Your Understanding

1. The engineers who built the Coal River Bridge on the Alaska Highway in British Columbia used scaffolding during construction. At one point, scaffolding that was 9 m tall was placed under the arch. The arch is modelled by the function

$$h(x) = -0.005\,061x^2 + 0.499\,015x$$

a) Describe a strategy you could use to determine the minimum distance of this scaffolding from each base of the arch.

b) Use your strategy from part a) to solve the problem.

c) Compare your strategy and solutions with a classmate's strategy and solutions. What other strategies could you have used?

PRACTISING

2. A company manufactures aluminum cans. One customer places an order for cans that must be 18 cm high, with a volume of 1150 cm^3.

a) Use the formula $V = \pi r^2 h$ to determine the radius that the company should use to manufacture these cans.

b) Graph the function that corresponds to $0 = \pi r^2 h - V$ to determine the radius.

c) Which method do you prefer? Explain why.

3. The sum of two numbers is 11. Their product is -152. What are the numbers?

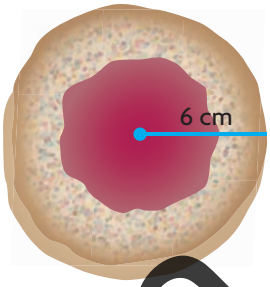
Communication Tip

The formula $V = \pi r^2 h$ can also be written as a quadratic function:

$f(r) = \pi r^2 h - V$

Pre-Publication

4. A doughnut store sells doughnuts with jam centres. The baker wants the area of the jam to be about equal to the area of the cake part of the doughnut, as seen from the top. The outer radius of a whole doughnut is 6 cm. Determine the radius of the jam centre.



5. Duncan dives with a junior swim club. In a dive off a 7.5 m platform, he reaches a maximum height of 7.94 m after 0.30 s. How long does it take him to reach the water?

6. A jet skier leaves a dock at 8 a.m. and travels due west at 36 km/h. A second jet skier leaves the same dock 10 min later and travels due south at 44 km/h. At what time of day, to the nearest minute, will the two jet skis be 20 km apart?



7. Alexis sells chocolate mousse tortes for $25. At this price, she can sell 200 tortes every week. She wants to increase her earnings, but, from her research, she knows that she will sell 5 fewer tortes per week for each price increase of $1.

a) What function, $E(x)$, can be used to model Alexis's earnings, if x represents the price increase in dollars?

b) What higher price would let Alexis earn the same amount of money she earns now?

c) What should Alexis charge for her tortes if she wants to earn the maximum amount of money?

8. Two consecutive integers are squared. The sum of these squares is 365. What are the integers?

Pre-Publication

9. Brianne is a photographer in southern Alberta. She is assembling a display of photographs of endangered local wildlife. She wants each photograph in her display to be square, and she wants the matte surrounding each photograph to be 6 cm wide. She also wants the area of the matte to be equal to the area of the photograph itself. What should the dimensions of each photograph be, to the nearest tenth of a centimetre?



Closing

10. Quadratic equations that describe problem situations are sometimes complicated. What are some methods you can use to simplify these equations and make them easier to solve?

Extending

11. Aldrin and Jan are standing at the edge of a huge field. At 2:00 p.m., Aldrin begins to walk along a straight path at a speed of 3 km/h. Two hours later, Jan takes a straight path at a 60° angle to Aldrin's path, walking at 5 km/h. At what time will the two friends be 13 km apart?

12. Frances is an artist. She wants the area of the matte around her new painting to be twice the area of the painting itself. The matte that she wants to use is available in only one width. The outside dimensions of the same matte around another painting are 80 cm by 60 cm. What is the width of the matte?

Pre-Publication

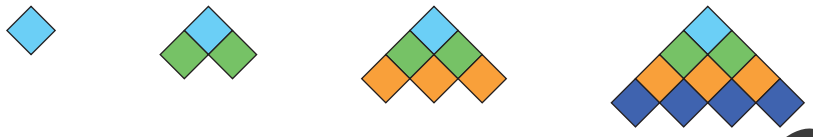
Applying Problem-Solving Strategies

Determining Quadratic Patterns

Many geometric patterns have connections to algebra. Examining a pattern can help you develop a formula that describes the general rule for the pattern.

The Puzzle

This pattern grows as a new row of tiles is added to each figure.



How many tiles would you need to construct a figure with 12 rows?

The Strategy

A. Copy this table.

Number of Rows	0	1	2	3	4			
Number of Tiles in Bottom Row	0	1	2	3				
Total Number of Tiles	0	1	3	6				

B. Complete your table for the next three figures in the pattern above.

C. Explain how you would determine the total number of tiles in figures with 8, 9, and 10 rows.

D. Write a quadratic equation that gives the total number of tiles in a figure with any number of rows.

E. Test your equation by using it to determine the total number of tiles in a figure with 12 rows. Check your answer by extending your table.

F. When you developed your equation for the pattern, did you use inductive or deductive reasoning? Explain.

Pre-Publication

7 Chapter Self-Test

1. Sketch each of the following quadratic functions. Explain why you chose the method you used.

a) $f(x) = x^2 - 8x + 12$

b) $f(x) = -2(x + 1)(x - 5)$

c) $f(x) = 0.5(x + 2)^2 - 7$

d) $f(x) = -2x^2 - 8x$

2. Determine the y-intercepts, x-intercepts, equation of the axis of symmetry, and vertex of the parabola that is defined by each quadratic function.

a) $y = -1(x + 3)(x - 5)$

b) $y = (2x - 3)(x + 4)$

3. Workers who were improving a section of highway near Rogers Pass, British Columbia, used dynamite to remove a rock obstruction. When the rock shattered, the height of one piece of rock, $h(t)$, in feet, could be modelled by the function

$$h(t) = -16t^2 + 160t$$

where t represents the time, in seconds, after the blast.

a) How long was the piece of rock in the air?

b) How high was the piece of rock after 2 s?

c) What was the maximum height of the piece of rock?

4. A parabola has a y-intercept of -4 and a vertex at $(3, -7)$. Determine the equation of the parabola in standard form.

5. Dimples the Clown has been charging $260 to perform at a children's party. He thinks that each raise of $80 in the charge for a party will result in one fewer booking per month. Dimples performs at 20 children's parties each month at his current price. How much should he charge to maximize his monthly revenue?



The Yukon Bridge spans the Tutshi River in northern British Columbia. To build the Yukon Bridge, engineers had to rig a temporary skyline. This skyline was used to transfer drilling equipment and to anchor rods to the far side of the river. A helicopter was used to install the support towers on the far side of the river.

6. Solve by factoring. Verify each solution.

a) $x^2 + 11x + 24 = 0$

b) $8a^2 + 31a - 4 = 0$

c) $5c = c^2 - 6$

d) $25x^2 + 10x + 5 = 5x^2 - 3x + 3$

7. Solve by using the quadratic formula.

a) $x^2 + 5x - 8 = 0$

b) $4x^2 - 12x - 3 = 0$

c) $0.25x^2 - 0.3x + 0.09 = 0$

d) $5x^2 + 6x + 7 = 0$

8. The Yukon Bridge is a suspension bridge with a parabolic shape. Its height, $h(w)$, in metres, can be represented by the equation

$$h(w) = 0.005\,066w^2 - 0.284\,698w$$

where the height is 0 m at the endpoints and w is the length of a straight line from one endpoint to the other.

a) Determine the length of line w.

b) What is the maximum drop in height from line w to the bridge?

***WHAT** DO You Think Now?* Revisit **What Do You Think?** on page 357. How have your answers and explanations changed?

Pre-Publication

7 Chapter Review

Q: How can I solve a quadratic equation algebraically?

A: Write the equation in standard form:

$$ax^2 + bx + c = 0$$

Then determine the roots of the equation by factoring or by using the quadratic formula.

Study Aid

- See Lessons 7.5, Examples 1, 4, and 5, and 7.7, Examples 1, 3, and 4.
- Try Chapter Review Questions 8 and 13.

Factoring

If the expression $ax^2 + bx + c$ is factorable, then the equation $ax^2 + bx + c = 0$ is true when either of the factors is equal to 0.

For example:

$$2x^2 + 2x = 5x + 20$$
$$2x^2 - 3x - 20 = 0$$
$$(2x + 5)(x - 4) = 0$$

$2x + 5 = 0$ or $x - 4 = 0$

$2x = -5$ $\quad x = 4$

$x = -\frac{5}{2}$

The roots are

$-\frac{5}{2}$ and 4

Using the quadratic formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

For example:

$3x^2 - 4x - 5 = 0$

$a = 3$, $b = -4$, and $c = -5$

Substitute these values into the quadratic formula.

$$x = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(3)(-5)}}{2(3)}$$

$$x = \frac{4 \pm \sqrt{76}}{6}$$

The radicand is positive, so the equation has a solution.

$x = \frac{4 + \sqrt{76}}{6}$ or $x = \frac{4 - \sqrt{76}}{6}$

The roots are

$\frac{2 + \sqrt{19}}{3}$ and $\frac{2 - \sqrt{19}}{3}$

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Study Aid

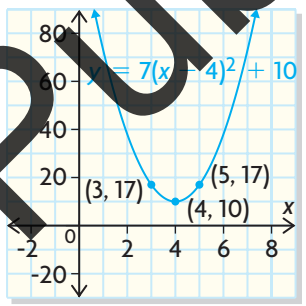
- See Lesson 7.6, Examples 1 and 2.
- Try Chapter Review Question 9.

Q: How can you graph a quadratic function in vertex form, $y = a(x - h)^2 + k$?

A: Use the information provided by the form of the quadratic equation.

For example: Sketch the graph of the following quadratic function:

$$y = 7(x - 4)^2 + 10$$

The vertex is at (4, 10).	Determine the coordinates of the vertex, (h, k).
$y = 7[(5) - 4]^2 + 10$ $y = 7(1)^2 + 10$ $y = 17$ One other point on the graph is (5, 17).	Locate one other point on the function by substituting a value for x into the equation. In this example, substitute 5 for x because the calculation is easy to check.
Another point on the graph is (3, 17), because 3 is the same distance from 4 as 5 is. (Another way of looking at this is that $(x - 4)^2 = 1$ when x is 5 and when x is 3.)	Apply symmetry to the first located point. In this example, the vertical line of symmetry is $x = 4$.
	Connect the three points with a smooth curve.

Study Aid

- See Lesson 7.8, Examples 1 to 4.
- Try Chapter Review Questions 14 to 17.

Q. When using a quadratic model, how do you decide whether you should determine the vertex of the parabola or solve a quadratic equation?

A. If you want to determine a maximum or minimum value, then you should locate the vertex of the function. If you are given a specific value for the dependent variable (any number, including 0), then you should solve the corresponding quadratic equation by graphing, factoring, or using the quadratic formula.

Pre-Publication

PRACTISING

Lesson 7.1

1. Graph the following quadratic functions without using technology.

a) $f(x) = x^2 - 6x + 8$

b) $g(x) = -2(x + 1)(x - 3)$

c) $h(x) = 0.5(x + 4)^2 - 2$

Lesson 7.2

2. The points $(-2, -41)$ and $(6, -41)$ are on the following quadratic function:

$$f(x) = -3x^2 + 12x - 5$$

Determine the vertex of the function.

3. In the photograph, the fisherman is holding his fishing rod 0.5 m above the water. The fishing rod reaches its maximum height 1.5 m above and 1 m to the left of his hand.



a) Determine the quadratic function that describes the arc of the fishing rod. Assume that the y-axis passes through the fisherman's hand and the x-axis is at water level.

b) State the domain and range for the function that models the fishing rod.

Lesson 7.3

4. Solve by graphing.

a) $6x^2 - 13x + 6 = 0$

b) $-5x^2 - 8x + 3 = 0$

c) $4n^2 + 1 = n + 3$

d) $c^2 - 38c + 340 = 3c^2 - 96c + 740$

Lesson 7.4

5. a) Rewrite the following quadratic function in factored form:

$$f(x) = 2x^2 - 12x + 10$$

b) Identify the zeros of the function, and determine the equation of the axis of symmetry of the parabola it defines.

c) State the domain and range of the function.

d) Graph the function.

6. Determine the x-intercepts of the graph of this quadratic function:

$$f(x) = 2x^2 - 5x - 12$$

7. Determine the vertex of the parabola that is defined by each quadratic function. Explain your process.

a) $f(x) = 3x^2 - 6x + 5$

b) $g(x) = -1(x + 2)(x + 3)$

Lesson 7.5

8. Solve by factoring. Verify each solution.

a) $s^2 - 7s - 60 = 0$

b) $2a^2 + 10a + 12 = 0$

c) $16d^2 - 169 = 0$

d) $3x^2 - 2x = 81 - 2x - x^2$

Lesson 7.6

9. a) State the direction of opening of the parabola that is defined by the following quadratic function:

$$y = 2(x - 3)^2 - 7$$

b) Provide the equation of the axis of symmetry and the coordinates of the vertex of the parabola.

c) State the domain and range of the function.

d) Sketch the parabola.

Pre-Publication

10. Determine the quadratic function with zeros of -4 and -2, if the point $(-1, -9)$ is also on the graph of this function.

11. Determine the quadratic function that defines the parabola that has a vertex at $(3, -5)$ and passes through $(-1, -9)$.

12. The High Level Bridge in Edmonton is the source of the Great Divide Waterfall, which is open to the public on holiday weekends in the summer. The water falls a vertical distance of 45 m from the bridge and reaches the North Saskatchewan River 10 m horizontally from the base of the bridge. Determine a quadratic function that models the path of the water.



Lesson 7.7

13. Solve by using the quadratic formula.

a) $117x^2 - 307x + 176 = 0$

b) $f^2 + 2f - 2 = 0$

c) $7h^2 + 6h = 5$

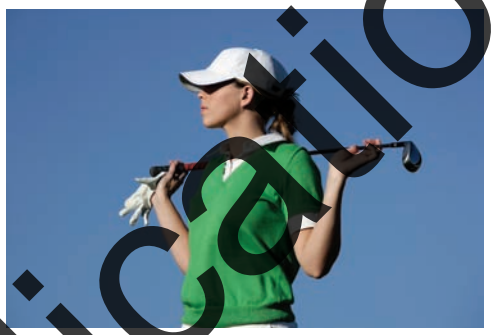
d) $6x^2 + 8x + 4 = 0$

Lesson 7.8

14. Determine three consecutive positive odd integers, if the sum of the squares of the first two integers is 15 less than the square of the third integer.

15. A right triangle has a perimeter of 120 cm. One side of the triangle is 24 cm long. Determine the length of the other side and the length of the hypotenuse.

16. On the 13th hole of a golf course, Saraya hits her tee shot to the right of the fairway. Saraya estimates that she now has 130 yd to reach the front of the green. However, she needs to clear some pine trees that are 40 yd from the green. The trees are about 10 yd high. Determine two different quadratic equations that model the flight of a golf ball over the trees and onto the green. Write one of your functions in factored form and the other in standard form.



17. A fishing boat leaves a dock at noon and travels due west at 40 km/h. A second boat leaves the same dock 20 min later and travels due south at 51 km/h. At what time, to the nearest minute, will the two boats be 116 km apart?



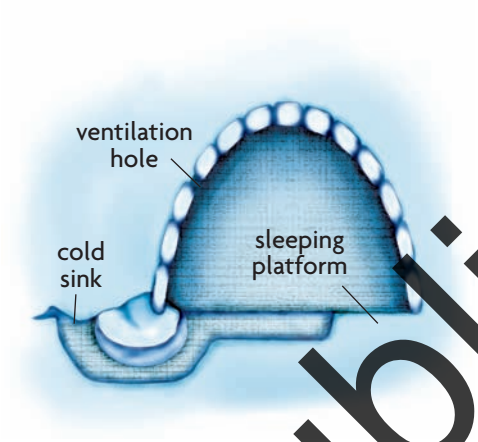
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7 Chapter Task

Parabolas in Inuit Culture

Traditional Inuit homes include arched domes, called igloos, that are made of snow blocks. The internal design of an igloo makes use of the principle that hot air rises and cold air sinks. In a typical igloo, hot air from the qulliq (seal oil lamp) and from human bodies rises and is trapped in the dome. Cold air sinks and is pooled at the entrance, which is lower than the living area. There are ventilation holes for the release of carbon dioxide.





Pre-Publication

? How can quadratic functions be used to model the cross-section of an igloo?

A. From the Internet or another source, obtain a picture of an igloo that appears to have a parabolic cross-section that you can use to estimate the size of an igloo.

B. Draw and label cross-sectional models of the interior of an igloo. Assume that each snow block is 1 ft thick. Also assume that the entrance to the igloo is 2 ft below the snow line.

C. Model the cross-sections of the arches on the exterior and interior of the igloo using quadratic functions, assuming that both arches are parabolic. State the domain and range of each function.

D. Based on your quadratic models, how close to the entrance of the igloo can a person who is 5 ft 6 in. tall stand, without ducking or bending?

Task *Checklist*

- ✔ Is your model neatly drawn and labelled?
- ✔ Have you stated and justified all the assumptions that support your quadratic models?
- ✔ Have you used appropriate mathematical language?

7 Project Connection

The Final Product and Presentation

Your final presentation should be more than just a factual written report of the information you have found. To make the most of your hard work, select a format for your final presentation that will suit your strengths, as well as your topic.

Presentation Styles

To make your presentation interesting, use a format that suits your own style. Here are some ideas:

- a report on an experiment or an investigation
- a summary of a newspaper article or a case study
- a short story, musical performance, or play
- a web page
- a slide show, multimedia presentation, or video
- a debate
- an advertising campaign or pamphlet
- a demonstration or the teaching of a lesson

Here are some decisions that other students have made about the format for their presentation:

Project 1: Weather Predictions

Muhamud has researched the mathematics of weather predictions. He has decided to make his presentation a demonstration of how a weather report is prepared, including the mathematics used, followed by an actual television weather report. He plans to submit a written report on his research and conclusions, as well.

Project 2: Gender Differences

Ming has studied the differences between the responses of females and males on cognitive aptitude tests. To illustrate her findings, she will have the class complete one of the assessment tasks during her presentation and then compare the results with standardized norms. In her report, Ming plans to include testing she has done on randomly selected students at her school.

Executive Summary

Sometimes, it is effective to give your audience an executive summary of your presentation. This is a one-page summary of your presentation, which includes your research question and the conclusions you have made. Ask your teacher about making copies of your summary for the class.

Pre-Publication

Project Example **Creating Your Presentation**

Sarah chose the changes in population of the Western provinces and the territories over the last century as her topic. Below, she describes how she determined which format to use for her presentation.

Sarah's Presentation

Because most of my supporting information is graphical, I am going to use a multimedia slide show. I will include some tables and graphs to show that the population of British Columbia and Alberta grew faster than the population of the rest of the Western and Northern provinces and territories. I will give all of the audience members an executive summary of my research, which will include my research question, my data (with the necessary supporting visuals), and my conclusions. I will give my teacher the full report.

Evaluating Your Own Presentation

Before giving your presentation, you can use these questions to decide if your presentation will be effective:

- Did I define my topic well? What is the best way to define my topic?
- Is my presentation focused? Will my classmates find it focused?
- Did I organize my information effectively? Is it obvious that I am following a plan in my presentation?
- Am I satisfied with my presentation? What might make it more effective?
- What unanswered questions might my audience have?

Your Turn

A. Does your topic suit some presentation formats better than others? Explain why.

B. From which presentation format, do you think your audience will gain the greatest understanding? Why?

C. Choose a format for your presentation, and create your presentation.

D. Use the questions provided in Evaluating Your Own Presentation to assess your presentation. Make any changes that you think are needed, as a result of your evaluation.

Pre-Publication