

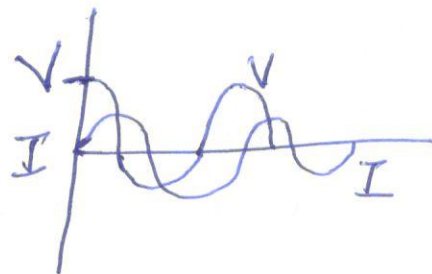
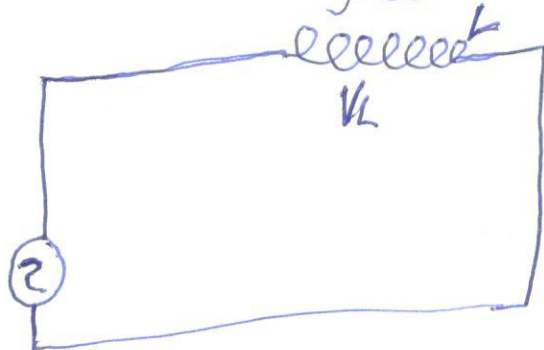
Unit-7

Topic- Alternating current.

Sub-topic - Inductive circuits and alternating current.

Inductor in an A.c circuit

Consider a circuit containing an A.c circuit connected to an inductor only. In practice wires forming the inductor have a resistance but here we assume that their resistance is neglected.



Inductance is a measure of the opposition a circuit element presents to a time varying current. The changing current of the source produces a back emf in the coil (self-induction). Due to the back emf the inductor opposes any change in the current through it. This means the back emf opposes the growth of current when it starts and similarly opposes its decay when it decreases.

$$\mathcal{E} = V_L$$

$$V_L = -L \frac{dI}{dt}$$

$$\text{but } I = I_{\max} \sin \omega t$$

$$V_L = L \frac{d}{dt} I_{\max} \sin \omega t = V_L = L \omega I_{\max} \sin \left(\omega t + \frac{\pi}{2} \right)$$

$$V_L = V_{\max} \sin\left(\omega t + \frac{\pi}{2}\right)$$

Inductive reactance

The opposition of a coil to an alternating current due to the back emf it induces is called inductive reactance.

$$X_L = \frac{V_{\text{rms}}}{I_{\text{rms}}} = \frac{V_{\max}}{I_{\max}} = \frac{\omega L I_{\max}}{I_{\max}} = \omega L$$

$$\underline{X_L = 2\pi f L}$$

Examples 1. A 500H inductor is connected to a 200V, 50Hz, A.C. Source.. calculate

- the inductive reactance?
- the rms current?

~~Given~~ a. $X_L = 2\pi f L = 2 \times 3.14 \times 50\text{Hz} \times 500\text{H} = 157,000\Omega$

b. $I_{\text{rms}} = \frac{V_{\text{rms}}}{X_L} = \frac{200\text{V}}{157,000\Omega} = 0.0012\text{A}$

2. In a pure inductive circuit if $V_{\max} = 200\text{V}$, $\omega = 20\pi\text{rad/s}$ and $L = 50\text{mH}$, calculate the current in the circuit when $t = \frac{1}{2\pi}\text{sec}$?

$$I_{\max} = \frac{V_{\max}}{\omega L} = \frac{200\text{V}}{20\pi\text{rad/s} \times 50\text{mH}} = 200\text{A}$$

$$I = I_{\max} \sin \omega t = 200\text{A} \sin 20\pi\text{rad/s} \times \frac{1}{2\pi}\text{sec}$$

Exercise

1. A coil has an inductive circuit if $V_{\max} = 100V$ and $\omega = 50\pi \text{ rad/s}$ and $L = 80mH$. Calculate the current in the circuit when $t = \frac{1}{10\pi} \text{ sec}$?
2. A coil has an inductive reactance of 200Ω when connected to a $60Hz$ line. what will be its inductive reactance when connected to a $100Hz$ line?
3. An AC supply has a frequency of $100Hz$. An inductor with inductance $5mH$ is connected in series with the power supply. Calculate the inductive reactance?

Grade 12 physics lesson note Lesson-9

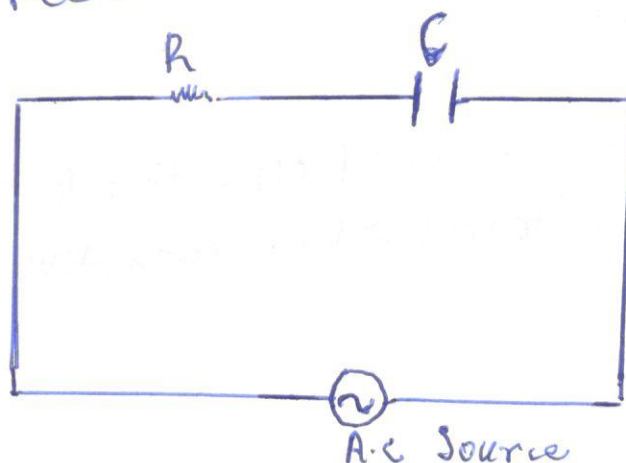
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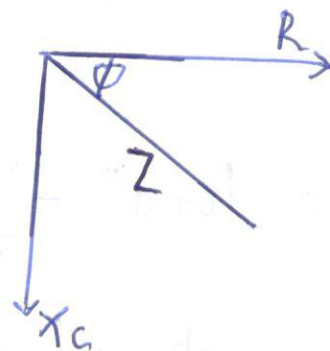
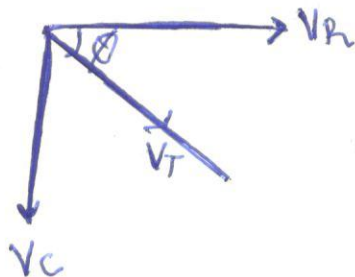
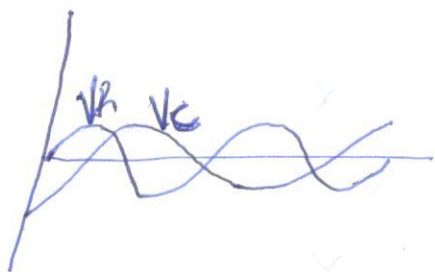
Topic - The RC Series Circuit

- The RC Series Circuit

The figure as shown below which contains a capacitor and a resistor connected in series across A.C Source.



- We know that for a resistor, a current and a Voltage are in phase. This means the resistor Voltage phasor V_R is the same direction as I_R . We also know that the current through a capacitor leads the Voltage across the capacitor by 90° .



So the total Voltage is given as shown below

$$V_T = \sqrt{V_C^2 + V_R^2}$$

The total resistance in the circuit

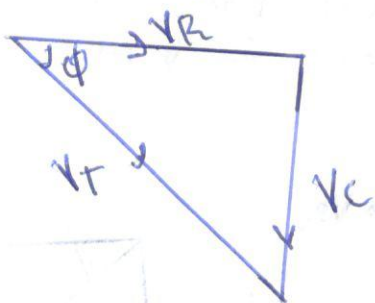
$$R_T = \sqrt{R^2 + X_C^2}$$

$$\begin{aligned} V_T &= I_T \cdot R_T \\ &= I_T \sqrt{R^2 + X_C^2} \end{aligned}$$

Impedance (Z) is the combined opposition to current due to resistance and reactance (either capacitive or inductive or both)

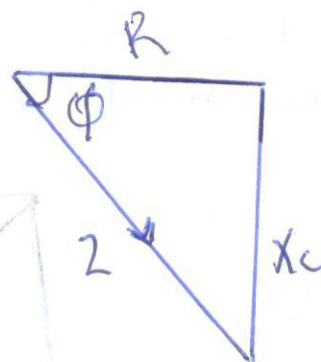
$$Z = \sqrt{R^2 + X_C^2}$$

- The angle that the resultant Voltage phasor (V_T) makes with the current (I_T) or is known as the phase angle ϕ of the circuit.



$$\tan \phi = -\frac{V_C}{V_R}$$

$$\phi = \tan^{-1} \left(-\frac{V_C}{V_R} \right)$$



$$\tan \phi = -\frac{X_C}{R}$$

$$\phi = \tan^{-1} \left(-\frac{X_C}{R} \right)$$

$I_{\max} = I_{\max} \sin \omega t$ but the Voltage lags the current by an angle ϕ

$$V = V_{\max} \sin(\omega t - \phi)$$

$$V_{\max} = I_{\max} \cdot Z = I_{\max} \sqrt{R^2 + X_C^2}$$

Examples

1. A $20 \mu\text{F}$ capacitor is connected to a 50Ω resistor and an AC source whose rms output is 100 V , 50 Hz .

Calculate

- the rms current in the circuit?
- the Voltage drop across the resistor?
- the Voltage drop across the capacitor?
- the phase angle for the circuit?
- the time required to cover the phase angle?

$$\begin{aligned} a. \quad X_C &= \frac{1}{2\pi f C} = \frac{1}{2 \times 3.14 \times 50 \text{ Hz} \times 20 \times 10^{-6} \text{ F}} = \frac{1}{6.28 \times 10^{-3}} = \frac{1000}{6.28} \Omega \\ &= \underline{\underline{159 \Omega}} \end{aligned}$$

$$\begin{aligned} b. \quad Z &= \sqrt{R^2 + X_C^2} = \sqrt{(50 \Omega)^2 + (159 \Omega)^2} = \sqrt{2500 + 25281} \Omega \\ &= \sqrt{27781} \Omega \end{aligned}$$

$$\begin{aligned} I_{\text{rms}} &= \frac{V_{\text{rms}}}{Z} = \frac{100 \text{ V}}{\sqrt{27781}} \text{ A} = \frac{1000}{166.67} \text{ A} = \underline{\underline{6 \text{ A}}} \end{aligned}$$

$$\begin{aligned}
 b. \quad V_R &= I_{rms} \cdot R \\
 &= 6A \times 50\Omega \\
 &= 300V
 \end{aligned}$$

$$\begin{aligned}
 c. \quad V_C &= I_{rms} \cdot X_C \\
 &= 6A \times 159\Omega \\
 &= \underline{954V}
 \end{aligned}$$

$$\begin{aligned}
 d. \quad \phi &= \tan^{-1} \left(\frac{V_C}{V_R} \right) = \tan^{-1} \left(\frac{I_{rms} X_C}{I_{rms} R} \right) = \tan^{-1} \left(\frac{X_C}{R} \right) \\
 &= \tan^{-1} \left(\frac{954V}{300V} \right) = \tan^{-1}(3.18) = \underline{72.54^\circ}
 \end{aligned}$$

$$e. \quad \theta = 72.54^\circ = \phi$$

$$\theta = \omega \cdot t$$

$$t = \frac{\theta}{\omega} = \frac{\theta}{2\pi f} = \frac{72.54^\circ}{360^\circ \cdot 50Hz} = \frac{72.54}{18000} \text{ sec} = \underline{0.004 \text{ sec}}$$

2. A resistance of 10Ω and a capacitor of unknown capacitance connected in series to a $40V$, $20Hz$ line. The voltage drop across the resistor is $30V$. Calculate

- the voltage drop across the capacitor?
- the reactance of the capacitor?
- the capacitance of the capacitor?

$$a. V_c = \sqrt{V^2 - V_R^2}$$

$$V_c = \sqrt{40V^2 - 30V^2}$$

$$= \sqrt{1600V^2 - 900V^2}$$

$$= \sqrt{700V^2}$$

$$= \underline{\underline{26.45V}}$$

$$b. I_{rms} = \frac{V_R}{R_{rms}} = \frac{30V}{10\Omega} = 3A$$

$$X_c = \frac{V_c}{I_{rms}} = \frac{26.45V}{3A} = 8.8\Omega$$

$$c. X_c = \frac{1}{2\pi fC}$$

$$C = \frac{1}{2\pi fX_c} = \frac{1}{2 \times 3.14 \times 20Hz \cdot 8.8\Omega} = \frac{1}{1106.536} = 0.000903F$$

$$= \underline{\underline{903\mu F}}$$

Exercises

1. An alternating supply of 24V and a frequency of 10kHz is used in a circuit which has a resistor of 10 Ω and a capacitor of 100 μ F in series.

Calculate

- total impedance of the circuit?
- Current in the circuit?
- phase angle b/n the supply and the current?

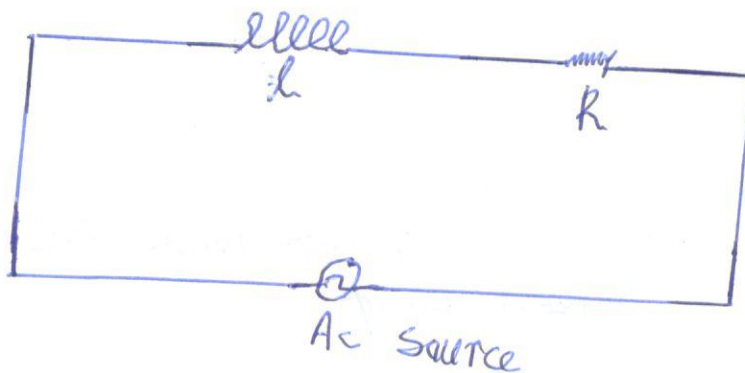
2. Calculate the impedance of an 8 microfarad capacitor and 10 Ω resistor connected in series in alternating current source?

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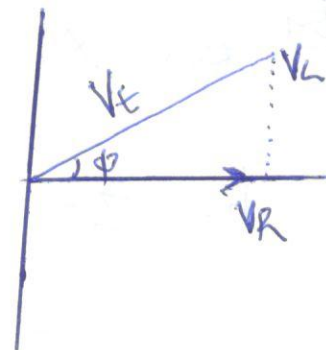
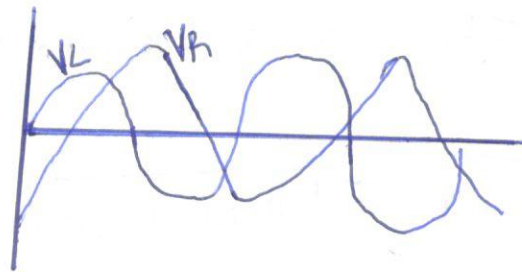
Topic - The RL Series circuit.

- The RL Series circuit

- The figure as shown below which contains inductor and a resistor connected in series across A c Source.



Here again since the inductor and the resistors are in series the same current flows through each. But we know that for the resistors the current and the voltage are in phase while for an inductor the ~~current~~ voltage leads the current by 90° . Hence the voltage across the inductor V_L , therefore leads the voltage across the resistor V_R by 90° as shown below.



$$V_T = V_R^2 + V_L^2$$

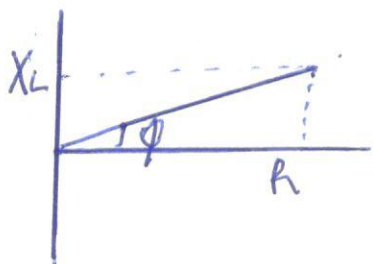
$$V_R = IR$$

$$V_L = I \times X_L$$

$$\tan \phi = \frac{V_L}{V_R}$$

$$\phi = \tan^{-1} \left(\frac{V_L}{V_R} \right)$$

$$V_T = I^2 (R^2 + X_L^2)$$



$$R_T = \sqrt{R^2 + X_L^2}$$

$$\text{Impedance (Z)} = \sqrt{R^2 + X_L^2}$$

$$\tan \phi = \frac{X_L}{R}$$

$$\phi = \tan^{-1}\left(\frac{X_L}{R}\right)$$

$$I = I_{\max} \sin \omega t$$

$$V = V_{\max} \sin(\omega t + \phi) \text{ since the Voltage leads the current by } \phi$$

Examples 1. A 100Ω resistor and a 200mH inductor are connected in series with a 40V , 20Hz source.

Calculate a. the inductive reactance?

b. the circuit impedance?

c. the rms current in the circuit?

d. the phase angle between the current and the voltage?

Solution

a. $X = 2\pi fL = 2 \times 3.14 \times 20\text{Hz} \times 200 \times 10^{-3}\text{H} = 255\Omega$

b. $Z = \sqrt{R^2 + X_L^2} = \sqrt{(100\Omega)^2 + (255\Omega)^2} = \sqrt{10625} \Omega$
 $= \underline{\underline{103\Omega}}$

$$I_{rms} = \frac{V_{rms}}{Z} = \frac{40V}{103\Omega} = 0.38A$$

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$$\begin{aligned} d. \quad \phi &= \tan^{-1} \left(\frac{X_L}{R} \right) \\ &= \tan^{-1} \left(\frac{25\Omega}{100\Omega} \right) \\ &= \tan^{-1}(0.25) \\ &= 21.2^\circ \end{aligned}$$

Exercises

1. An AC source of a frequency of 40Hz, supplies a current of 2A to a series circuit containing a 100Ω resistor and a 400mH inductor. Calculate
 - a. the impedance?
 - b. the Voltage across the resistor?
 - c. the Voltage across the inductor?
 - d. the total Voltage?
 - e. the phase angle?