

Answer key for General physics Mid exam

Part I (1pt each)

- 1) A 2) A 3) B 4) C

Part II

- 5) a) $\Delta P = 0$ (0.5pt) b) $Impulse = \Delta P = 0$ (0.5pt)

- 6) Elastic and Plastic behavior of the materials (1pt).

Elastic materials: are materials that regain their original shape and size when the deforming force is removed. (0.5pt)

Plastic materials: are materials that do not regain their original shape and size when the deforming force is removed. (0.5pt)

- 7) Stress and strain (1pt)

Stress: is the force per unit area that is causing some deformation on an object. (0.5pt)

Strain: is the amount of deformation experienced by the body in the direction of force applied, divided by the initial dimensions of the body. (0.5pt)

- 8) State the Pascal's and Archimedes' principles (1pt each).

Pascal's principles state that change in pressure at any point of an enclosed fluid that is at rest is transmitted, without any loss, to every point of the fluid, including the walls of the container. (1pt)

Archimedes' principle states that when a body is partially or fully dipped into a fluid at rest, the fluid exerts an upward force of buoyancy equal to the weight of the displaced fluid. (1pt)

Part III

9) Solution (3pt: (1pt each)):

a) The scalar product (dot product) of the two vector is:

$$\begin{aligned}\mathbf{A} \cdot \mathbf{B} &= (3\hat{i} - \hat{j} + \hat{k}) \cdot (-5\hat{i} + 2\hat{j} - \hat{k}) = A_x B_x + A_y B_y + A_z B_z \\ &= (3)(-5) + (-1)(2) + (1)(-1) = -15 - 2 - 1 = -18 \\ |\mathbf{A}||\mathbf{B}| &= \sqrt{11}\sqrt{30} = \sqrt{330} = 18.17\end{aligned}$$

b) The cross product (vector product) of the two vectors is:

$$\begin{aligned}\mathbf{A} \times \mathbf{B} &= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & -1 & 1 \\ -5 & 2 & -1 \end{vmatrix} = (A_y B_z - A_z B_y)\hat{i} + (A_z B_x - A_x B_z)\hat{j} + (A_x B_y - A_y B_x)\hat{k} \\ &= ((-1)(-1) - (1)(2))\hat{i} + ((1)(-5) - (3)(-1))\hat{j} + ((3)(2) - (-1)(-5))\hat{k} \\ &= (1 - 2)\hat{i} + (-5 + 3)\hat{j} + (6 - 5)\hat{k} = -\hat{i} - 2\hat{j} + \hat{k} \\ |\mathbf{A} \times \mathbf{B}| &= \sqrt{6} = 2.45\end{aligned}$$

c) The angle between two vectors can be found as:

$$\mathbf{A} \cdot \mathbf{B} = |\mathbf{A}||\mathbf{B}| \cos \theta \quad \text{where } \theta \text{ is angle between } \mathbf{A} \text{ and } \mathbf{B}$$

$$\theta = \cos^{-1} \left(\frac{\mathbf{A} \cdot \mathbf{B}}{|\mathbf{A}||\mathbf{B}|} \right) = \cos^{-1} \left(\frac{-18}{\sqrt{11}\sqrt{30}} \right) = 172.25^\circ$$

$$\text{or } \theta = \sin^{-1} \left(\frac{|\mathbf{A} \times \mathbf{B}|}{|\mathbf{A}||\mathbf{B}|} \right) = \sin^{-1} \left(\frac{2.45}{18.17} \right) = 7.75^\circ \text{ or } \theta = 172.25^\circ$$

10) Solution (2pt: (0.5pt each)):

- a) The position of the object at $t = 2.00\text{s}$ and $t = 3.00\text{s}$ is

$$x(t = 2\text{second}) = 3(2^2) - 2(2) + 3 = 11.0\text{m}$$

$$x(t = 3\text{second}) = 3(3^2) - 2(3) + 3 = 24.0\text{m}$$

Then the average speed between $t = 2.00\text{s}$ and $t = 3.00\text{s}$ is

$$v_{av} = \frac{\Delta x}{\Delta t} = \frac{(24 - 11)\text{m}}{(3 - 2)\text{s}} = 13.0\text{m/s}$$

$$\text{or } \frac{v(3\text{sec}) + v(2\text{sec})}{2} = \frac{16.0 + 10.0}{2} = 13.0\text{m/s}$$

- b) At any time the instantaneous velocity of the object is:

$$v(t) = \frac{dx}{dt} = \frac{d}{dt}(3.00t^2 - 2.00t + 3.00) = 6t - 2$$

The instantaneous speed at $t = 2.00\text{ s}$ is

$$v = 6(2) - 2 = 10.0\text{ m/s}$$

The instantaneous speed at $t = 3.00\text{ s}$ is

$$v = 6(3) - 2 = 16.0\text{ m/s}$$

- c) The average acceleration between $t = 2.00\text{ s}$ and $t = 3.00\text{ s}$

$$a_{av} = \frac{\Delta v}{\Delta t} = \frac{(16.0 - 10.0)\text{m/s}}{(3 - 2)\text{s}} = 6.0\text{ m/s}^2$$

- d) At all times instantaneous acceleration is given by:

$$a = \frac{dv}{dt} = \frac{d}{dt}(6t - 2) = 6.00\text{ m/s}^2 \text{ (This includes both } t = 2\text{ s and } t = 3\text{ s)}$$

11) Solution (2pt):

Applying Newton's second law to both objects and assuming the motion of the block is to the right, we obtain

Motion of the ball

$$\sum F_y = T - m_2 g = m_2 a_y = m_2 a \quad (1)$$

$$\sum F_x = m_2 a_x = 0$$

Motion of block

$$\sum F_x = F \cos \theta - T - f_k = m_1 a_x = m_1 a \quad (2)$$

$$\sum F_y = n - m_1 g - F \sin \theta = m_2 a_y = 0$$

$$n = m_1 g - F \sin \theta$$

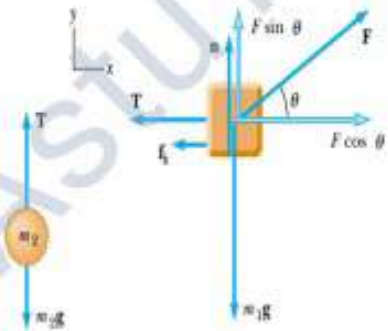
We can rewrite equation (2) as:

$$F \cos \theta - T - \mu_k n = F \cos \theta - T - \mu_k (m_1 g - F \sin \theta) = m_1 a \quad (3)$$

Adding equation (1) and (3) gives:

$$F \cos \theta - m_2 g - \mu_k (m_1 g - F \sin \theta) = (m_1 + m_2) a$$

$$\Rightarrow a = \frac{F(\cos \theta + \mu_k \sin \theta) - (\mu_k m_1 + m_2)g}{m_1 + m_2}$$



12) Solution (3pt):

For each of the paths from O to C, the work done is given by:

$$W = \int \vec{F} \cdot d\vec{S} \quad \text{Here } d\vec{S} = (dx\hat{i} + dy\hat{j})$$

$$\text{Hence } W = \int (2y\hat{i} + x^2\hat{j}) \cdot (dx\hat{i} + dy\hat{j})$$

$$W = \int_0^5 2y dx + \int_0^5 x^2 dy \quad (1)$$

a) The path OAC can be break up into two paths **(0.5pt)**,

From $O \rightarrow A$ and then from $A \rightarrow C$.

First considering path O to A, in this path $y = 0$, hence from equation (1) it is clear that $W_{OA} = 0$.

Now consider path A to C, in this path x is constant (i.e. $dx = 0$) and is equal to $5m$.

From equation (i)

$$W_{AC} = x^2 \int_0^5 dy = (5)^2 [y]_0^5 = 125J$$

Hence total work done $W_{OAC} = 0 + 125J = 125J$

b) The path OBC can be break up into two paths **(0.5pt)**,

From $O \rightarrow B$ and then from $B \rightarrow C$

First considering path O to B, in this path $x = 0$, hence from equation (i) it is clear that $W_{OB} = 0$.

Now consider path B to C, in this path y is constant (i.e. $dy = 0$) and is equal to $5m$.

From equation (1)

$$W_{BC} = 2y \int_0^5 dx = 2(5)[x]_0^5 = 10 * 5 = 50J$$

Hence total work done $W_{OBC} = 0 + 50J = 50J$

c) Along the path $O \rightarrow C$, both x and y are changing **(1pt)**. In this case $x = y \Rightarrow$

$$dx = dy$$

Hence from equation (1)

$$W_{OC} = \int_0^5 2y dx + \int_0^5 x^2 dy = \int_0^5 (2x + x^2) dx = \left[x^2 + \frac{x^3}{3} \right]_0^5 = 25 + \frac{125}{3} = \frac{200}{3}J$$

d) The work done from O to C in three different paths is not same; hence the force $\mathbf{F}$ is a non-conservative force **(1pt)**.