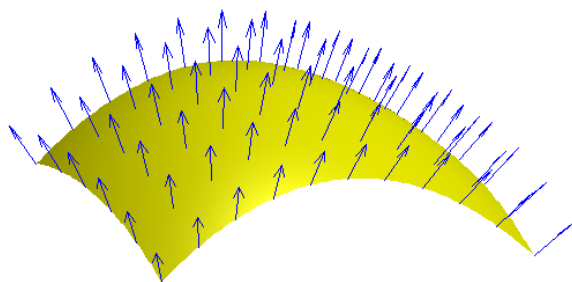


Chapter 6 Magnetism

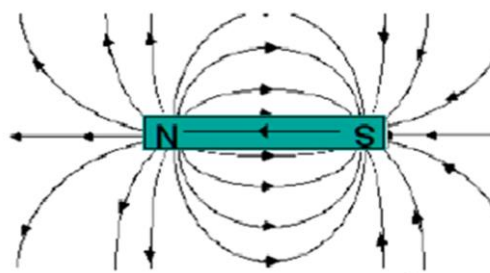
6.1. Concepts of magnetism field

The magnetic field is the area around a magnet in which there is magnetic force. Moving electric charges can make magnetic fields. Magnetic fields can usually be seen by magnetic flux lines. At all times the direction of the magnetic field is shown by the direction of the magnetic flux lines. The strength of a magnet has to do with the spaces between the magnetic flux lines. The closer the flux lines are to each other, the stronger the magnet is. The farther away they are, the weaker. The flux lines can be seen by placing iron filings over a magnet. The iron filings move and arrange into the lines. Magnetic fields give power to other particles that are touching the magnetic field.

Magnetic field lines indicate the direction of a magnetic field in space. Magnetic field lines do not end on magnetic poles, but form closed loops instead. Magnetic Field lines: Outside magnet: N to S, Inside magnet: S to N.



magnetic flux



Magnetic flux lines

The magnetic flux through some surface, in this simplified picture, is proportional to the number of field lines passing through that surface (in some contexts, the flux may be defined to be precisely the number of field lines passing through that surface; although technically misleading, this distinction is not important). The magnetic flux is the *net* number of field lines passing through that surface;

Magnetic Field **B** is sometimes called the *flux density* in (Webers per square meter (Wb/m^2))

- Magnetic flux lines are continuous and closed. Direction is that of the **B** vector at any point.
- Flux lines are NOT in direction of force but direction of force but $\perp$.

$$\Phi = BA \cos \theta$$

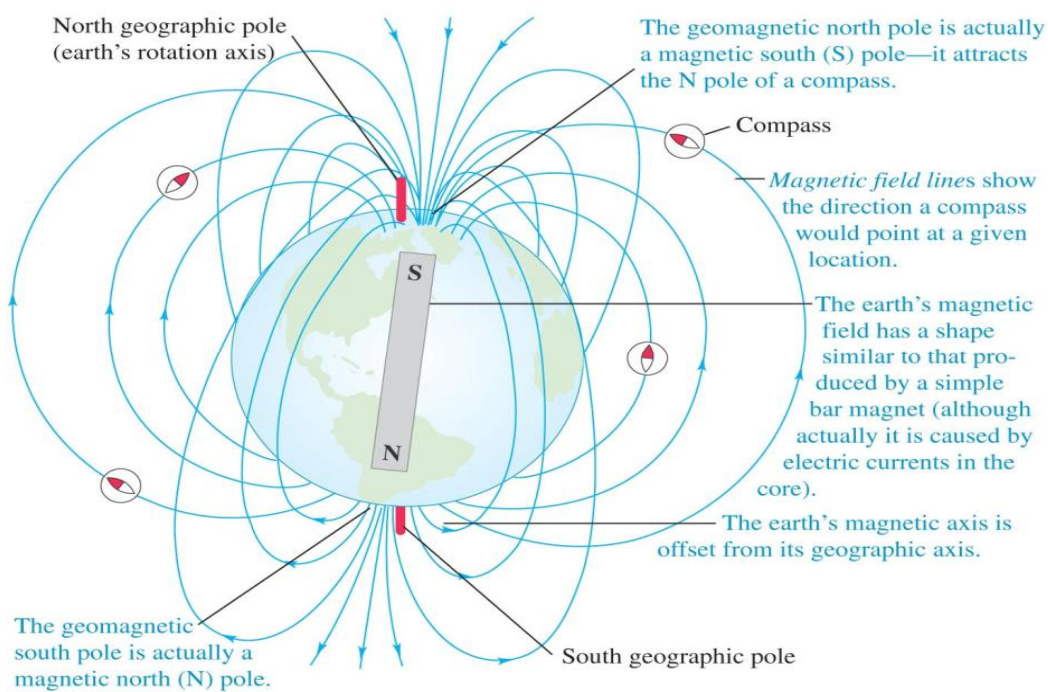
Magnetic property of matters

Magnetism describes how the atoms of materials respond to magnetic field. so we can classify materials as:

- **diamagnetism** the tendency of a material to oppose applied magnetic field
- **Paramagnetic** are materials which have unpaired electrons and tends to align themselves in the same direction as the applied magnetic field, thus reinforcing it.
- **Ferromagnetic** are materials which have unpaired electrons and tends to align them in the same direction as the *applied magnetic field and parallel* to each other. They keep this alignment even when the applied field is removed.

The Earth's magnetic field

Earth's magnetic field, also known as the **geomagnetic field**, is the magnetic field that extends from the Earth's interior out into space, where it interacts with the solar wind, a stream of charged particles emanating from the Sun. The magnetic field is generated by electric currents due to the motion of convection currents of molten iron in the Earth's outer core: these convection currents are caused by heat escaping from the core, a natural process called a geodynamo. The magnitude of the Earth's magnetic field at its surface ranges from 25 to 65 microteslas (0.25 to 0.65 gauss).



N.B Magnetic declination / magnetic variation: the Earth's magnetic axis is not parallel to its geographic axis (axis of rotation) a compass reading deviates from geographic north.

Magnetic inclination: the magnetic field is not horizontal at most of earth's surface, its angle up or down. The magnetic field is vertical at magnetic poles.

6.3. Motion of charged particle in magnetic field

- A positive charged particle moves in a uniform magnetic field $\vec{B}$ with a velocity $\vec{v}$. It is found experimentally that a moving charge experiences a magnetic force $\vec{F}_B$ by a magnetic field $\vec{B}$. This force, $\vec{F}_B$ is given by

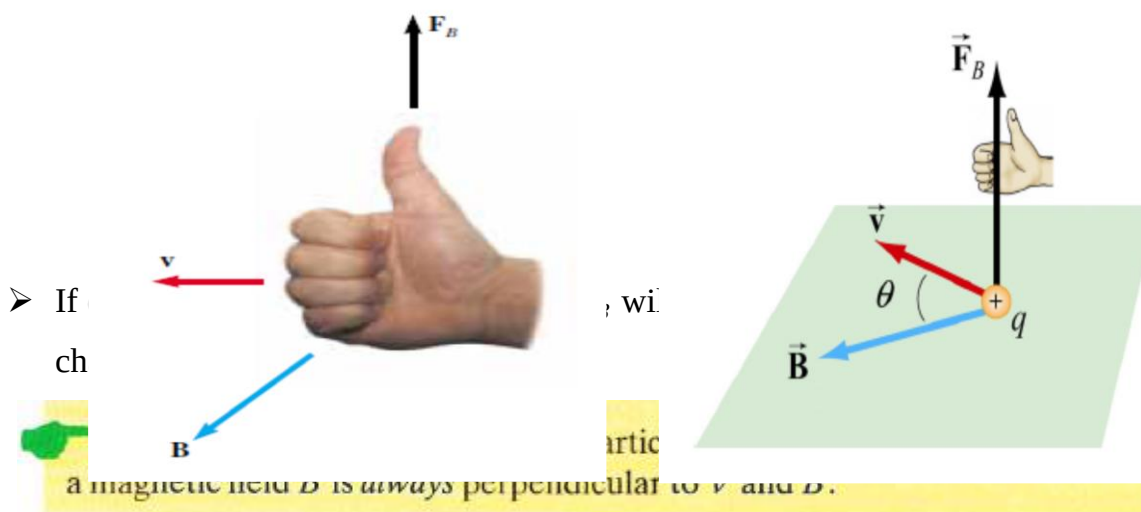
$$\left. \begin{array}{l} F_B \sim q \\ F_B \sim B_{\perp} \\ F_B \sim v \end{array} \right\} F_B = qvB_{\perp} = qvB \sin \theta$$

$$F_B = qvB \sin \theta$$

In vector form,

$$\vec{F}_B = q\vec{v} \times \vec{B}, \quad F_B = \max, \theta = 90^\circ (\vec{B} \perp \vec{v}) \text{ and } F_B = \min = 0, \theta = 0^\circ (180^\circ) (\vec{B} \parallel \vec{v})$$

- Direction of F_B is given by right-hand rule or right-hand screw thread rule;



- The magnitude of magnetic field is given by

$$B = \frac{F_B}{qv \sin \theta}$$

- The SI unit of magnetic field is the Tesla (T),

$$1T = 1 \frac{N}{C \cdot m/s} = 1 \frac{Ns}{Cm}$$

$$1T = 1 \frac{N}{Am}$$

- And another commonly used non-SI unit for $\vec{B}$ is the *gauss* (G),

$$1T = 10^4 G$$

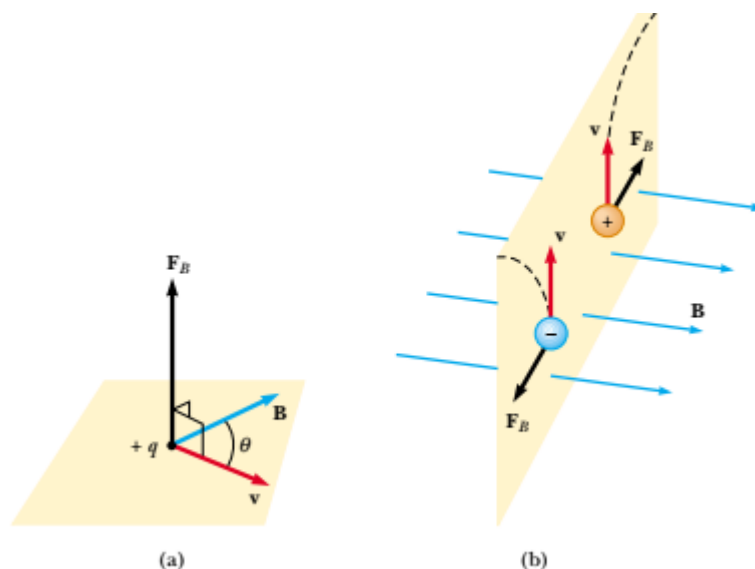
- The work done by magnetic force on a moving charge is given by

$$W = \int \vec{F} \cdot d\vec{s}, \text{ where } d\vec{s} \text{ is in the direction of velocity } \vec{v}.$$

And hence $\vec{F}_B \perp d\vec{s}$, or $\vec{F}_B \cdot d\vec{s} = 0$,

$$W = \int \vec{F}_B \cdot d\vec{s} = 0$$

- Therefore, when a charged particle moves in a magnetic field, the work done by the magnetic force on the particle is zero because the displacement is always perpendicular to the direction of the force. The magnetic field can alter the direction of the particle's velocity vector, but it cannot change its speed.
- The magnitude F_B of the magnetic force exerted on the particle is proportional to the charge q and to the speed v of the particle.
- The magnitude and direction of F_B depend on the velocity of the particle and on the magnitude and direction of the magnetic field $\mathbf{B}$.
 - When a charged particle moves parallel to the magnetic field vector, the magnetic force acting on the particle is zero.
 - When the particle's velocity vector makes any angle $\theta \neq 0$ with the magnetic field, the magnetic force acts in a direction perpendicular to both $\mathbf{v}$ and $\mathbf{B}$; that is, F_B is perpendicular to the plane formed by $\mathbf{v}$ and $\mathbf{B}$ (Fig.a).
 - The magnetic force exerted on a positive charge is in the direction opposite the direction of the magnetic force exerted on a negative charge moving in the same direction (Fig. b).



- *There are several important differences between electric and magnetic forces:*
 - The electric force acts along the direction of the electric field, whereas the magnetic force acts perpendicular to the magnetic field.
 - The electric force acts on a charged particle regardless of whether the particle is moving, whereas the magnetic force acts on a charged particle only when the particle is in motion.
- The electric force does work in displacing a charged particle, whereas the magnetic force associated with a steady magnetic field does no work when a particle is displaced because the force is perpendicular to the displacement.

Example

Uniform magnetic field B , with magnitude 1.2 mT , is directed vertically upward. A proton with kinetic energy 5.3 MeV enters the magnetic field, moving horizontally from south to north. What magnetic deflecting force acts on the proton as it enters the magnetic field? The proton mass is $1.67 \times 10^{-27} \text{ kg}$.

Solution

To find the magnitude of F_B , we first find the proton's speed v . We can find v from the given kinetic energy $K = \frac{1}{2}mv^2$. Since, $1\text{eV} = 1.60 \times 10^{-19}\text{J}$

Solving for v , we obtain

$$v = \sqrt{\frac{2K}{m}} = \sqrt{\frac{(2)(5.3 \text{ MeV})(1.60 \times 10^{-13} \text{ J/MeV})}{1.67 \times 10^{-27} \text{ kg}}}$$

$$= 3.2 \times 10^7 \text{ m/s.}$$

Hence, the magnitude of F_B can be

$$F_B = qvB \sin \theta$$

$$= (1.60 \times 10^{-19} \text{ C})(3.2 \times 10^7 \text{ m/s})$$

$$\times (1.2 \times 10^{-3} \text{ T})(\sin 90^\circ)$$

$$= 6.1 \times 10^{-15} \text{ N.}$$

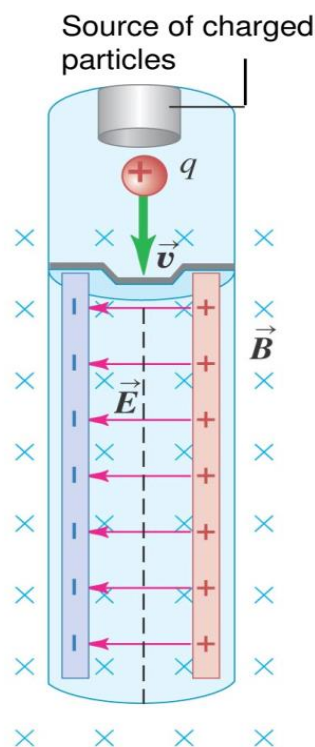
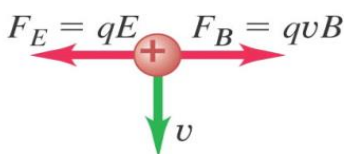
Activity

1. The north-pole end of a bar magnet is held near a positively charged piece of plastic. Is the plastic (a) attracted, (b) repelled, or (c) unaffected by the magnet?
2. A charged particle moves with velocity $\mathbf{v}$ in a magnetic field $\mathbf{B}$. The magnetic force on the particle is a maximum when $\mathbf{v}$ is (a) parallel to $\mathbf{B}$, (b) perpendicular to $\mathbf{B}$, (c) zero.
3. An electron moves in the plane of this paper toward the top of the page. A magnetic field is also in the plane of the page and directed toward the right. The direction of the magnetic force on the electron is (a) toward the top of the page, (b) toward the bottom of the page, (c) toward the left edge of the page, (d) toward the right edge of the page, (e) upward out of the page, (f) downward into the page.

Applications of Motion of Charged Particles

Velocity selector

- Particles of a specific speed can be selected from the beam using an arrangement of E and B fields.
- F_m (magnetic) for + charge towards right ($q \mathbf{v} \times \mathbf{B}$).
- F_E (electric) for + charge to left ($q \mathbf{E}$).
- $F_{\text{net}} = 0$ if $F_m = F_E \rightarrow -qE + qvB = 0 \rightarrow \boxed{v = E/B}$
- Only particles with speed E/B can pass through without being deflected by the fields.

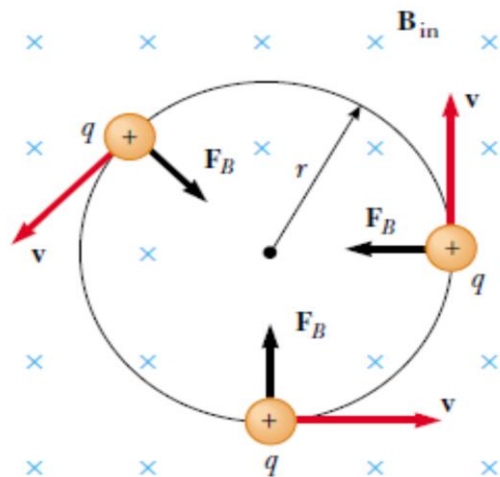


Motion of Charged Particles in Magnetic Field(Circular motion of charged particle in a magnetic field)

- A charged particle is projected into uniform magnetic field $\vec{B}$ yields a circular motion.
- When the velocity of a charged particle is perpendicular to a uniform magnetic field, the particle moves in a circular path in a plane perpendicular to $\vec{B}$. The magnetic force $\vec{F}_B$ acting on the charge is always directed toward the center of the circle and has a constant magnitude

$$F_B = qvB$$

- Hence, the work done by the magnetic force $\vec{F}_B$ on the particle is zero.
- For motion perpendicular to a magnetic field, as shown in figure, the force required to keep a particle moving with speed v in a circle with radius r is the centripetal force:



$$\sum F = ma_c$$

$$\text{Thus, } F_B = qvB = \frac{mv^2}{r}$$

$$r = \frac{mv}{qB}$$

- The angular speed of the particle is

$$\omega = \frac{v}{r} = \frac{qB}{m}$$

- Again, The angular speed of the particle is

$$\omega = 2\pi f = \frac{qB}{m}$$

- The angular speed ω is often referred to as the **cyclotron frequency** because charged particles circulate at this angular frequency in the type of accelerator called a **cyclotron**. A **cyclotron** is a device that can accelerate charged particles to very high speeds.

- Thus, the frequency is given by

$$f = \frac{qB}{2\pi m}$$

- and, the period of the motion is

$$T = \frac{2\pi m}{qB}$$

- The kinetic energy is

$$K = \frac{1}{2}mv^2 = \frac{1}{2}m\left(\frac{qBr}{m}\right)^2$$

$$K = \frac{1}{2}\frac{q^2B^2r^2}{m} = \text{Constant}$$

Example

A proton is moving in a circular orbit of radius 14 cm in a uniform 0.35-T magnetic field perpendicular to the velocity of the proton. Find the linear speed of the proton.

$$\begin{aligned} v &= \frac{qBr}{m_p} = \frac{(1.60 \times 10^{-19} \text{ C})(0.35 \text{ T})(0.14 \text{ m})}{1.67 \times 10^{-27} \text{ kg}} \\ &= 4.7 \times 10^6 \text{ m/s} \end{aligned}$$

The Mass Spectrometer

A **mass spectrometer** separates ions according to their mass-to-charge ratio. In one version of

this device, known as the *Bainbridge mass spectrometer*, a beam of ions first passes through a velocity selector and then enters a second uniform magnetic field $\mathbf{B}_0$ that has the same direction as the magnetic field in the selector (Fig. below). Upon entering the second magnetic field, the ions move in a semicircle of radius r before striking a detector array at P . If the ions are positively charged, the beam deflects upward, as Figure below shows. If the ions are negatively charged, the beam deflects downward

In the last section, we learnt that in a magnetic field, a charged particle experiences a force as a result of the magnetic field,

$$F = Bqv$$

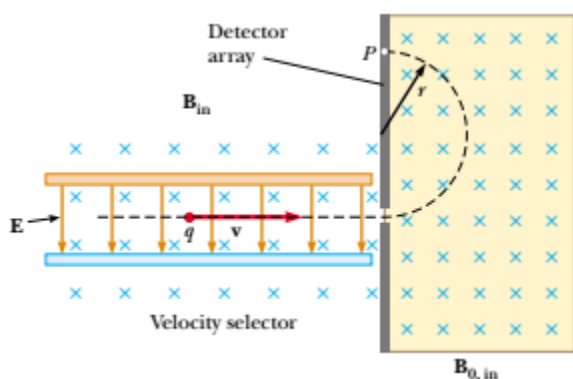
and a centripetal force

$$F = \frac{mv^2}{r}$$

Particles that follow the central dotted path in Figure 6.9 will reach the detector. For these particles

$$F = Bqv = \frac{mv^2}{r} \text{ where } r \text{ is the radius of the circular path.}$$

$$\text{We can rearrange this to } \frac{q}{m} = \frac{v}{Br}$$

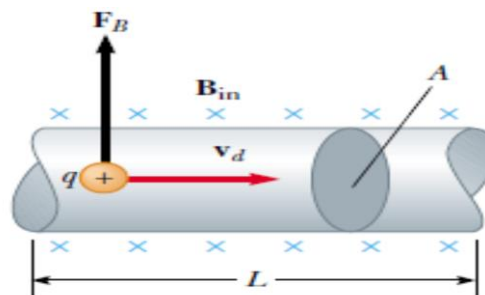


Therefore, we can determine m/q by measuring the radius of curvature and knowing the field magnitudes $\mathbf{B}$, $\mathbf{B}_0$, and B . In practice, one usually measures the masses of various isotopes of a given ion, with the ions all carrying the same charge q . In this way, the mass ratios can be

determined even if q is unknown. A variation of this technique was used by J. J. Thomson (1856–1940) in 1897 to measure the ratio e/me for electrons.

6.2. Magnetic Force on a Current Carrying Conductor

- If a magnetic force is exerted on a single charged particle when the particle moves through a magnetic field.
- Similarly, a current-carrying wire also experiences a force when placed in a magnetic field.
- Consider a wire carrying a current, I in a coil figure below.



The charge, q flowing through in the wire in a given time, t is

$$q = tI$$

During this time, the charge occupies a length, L of wire given by

$$L = v_d t, \text{ where } v_d \text{ is the drift speed of the charge carriers in the}$$

wire.

Thus, we obtain

$$q = tI = \frac{L}{v_d} I$$

The magnitude of the magnetic force is then

$$F_B = qv_d B \sin \theta$$

$$F_B = \left(\frac{L}{v_d} I \right) v_d B \sin \theta$$

$$F_B = ILB \sin \theta$$

where θ is the angle between the direction of the current flow and the direction of the magnetic field.

In vector form,

$$\vec{F}_B = I\vec{L} \times \vec{B}$$

magnitude and direction of Torque on a Current Loop in Uniform Magnetic Field

- Consider a rectangular loop carrying a current I in the presence of a uniform magnetic field $\vec{B}$ directed parallel to the plane of the loop as shown in figure below.

- No magnetic forces act on sides 1 and 3 because these wires are parallel to the field; hence,

$$\vec{F}_1 = \vec{F}_3 = I\vec{b} \times \vec{B} = 0$$

- Magnetic forces do, however, act on sides 2 and 4 because these sides are oriented perpendicular to the field. The magnitude of these forces is,

$$\begin{cases} \vec{F}_2 = I(-a\hat{j}) \times (B\hat{i}) = IaB\hat{k} \\ \vec{F}_4 = I(a\hat{j}) \times (B\hat{i}) = -IaB\hat{k} \end{cases}$$

Thus, the net force on the rectangular loop is

$$\vec{F}_{net} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3 + \vec{F}_4 = 0$$

- Notice that the two forces point in opposite directions, $\vec{F}_2$ pointing out of the page and $\vec{F}_4$ into the page. The forces $\vec{F}_2$ and $\vec{F}_4$ will produce a torque which causes the loop to rotate about the y-axis.

The magnitude of this torque is

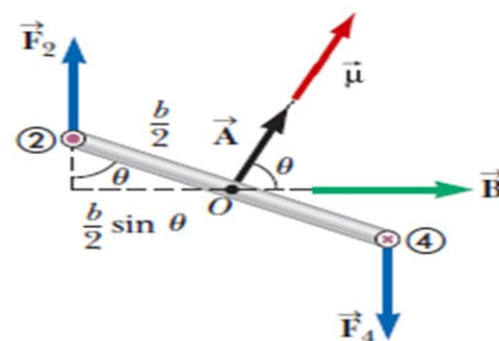
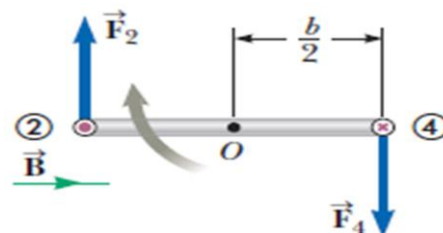
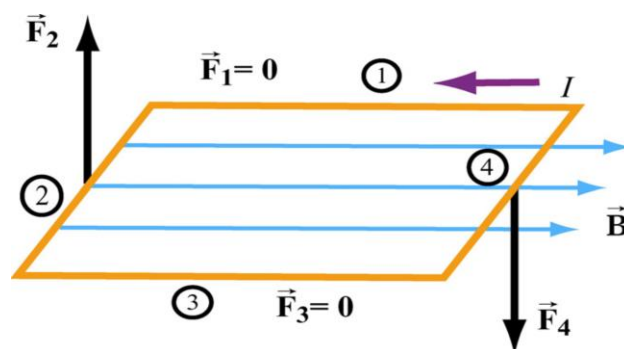
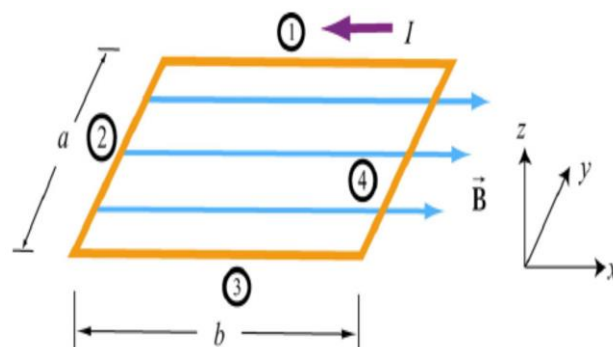
$$\tau = F_2 \frac{b}{2} + F_4 \frac{b}{2} = (IaB) \frac{b}{2} + (IaB) \frac{b}{2} = IabB$$

$$\text{Where, } A = ab$$

The above expression for torque can then be rewritten as

$$\tau = IAB$$

- Consider now the more general situation where the loop (or the area vector $\vec{A}$) makes an angle θ with respect to the magnetic field.



- We see that the moment arm of $\vec{F}_2$ about the point O is equal to $(\frac{b}{2}) \sin \theta$. Likewise, the moment arm of $\vec{F}_4$ about O is also equal to $(\frac{b}{2}) \sin \theta$. Because $F_2 = F_4 = IaB$, the magnitude of the net torque about O is

$$\tau = F_2 \frac{b}{2} \sin \theta + F_4 \frac{b}{2} \sin \theta = IaB \left(\frac{b}{2} \sin \theta \right) + IaB \left(\frac{b}{2} \sin \theta \right)$$

$$\tau = IabB \sin \theta$$

$$\tau = IAB \sin \theta$$

Where, $A = ab$

- In vector form we write,

$$\vec{\tau} = I\vec{A} \times \vec{B}$$

Where $\vec{A}$ is area vector, is perpendicular to the plane of the loop and has a magnitude equal to the area of the loop.

- For a loop consisting of N turns, the magnitude of the torque is

$$\tau = NIAB \sin \theta$$

- The quantity $NI\vec{A}$ is called the magnetic dipole moment $\vec{\mu}$:

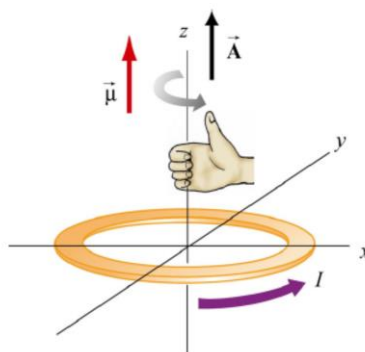
$$\vec{\mu} = NI\vec{A}$$

The SI unit for the magnetic dipole moment is ampere-meter square (Am^2).

- Using the expression for $\vec{\mu}$, the torque exerted on a current-carrying loop can be rewritten as

$$\vec{\tau} = \vec{\mu} \times \vec{B}$$

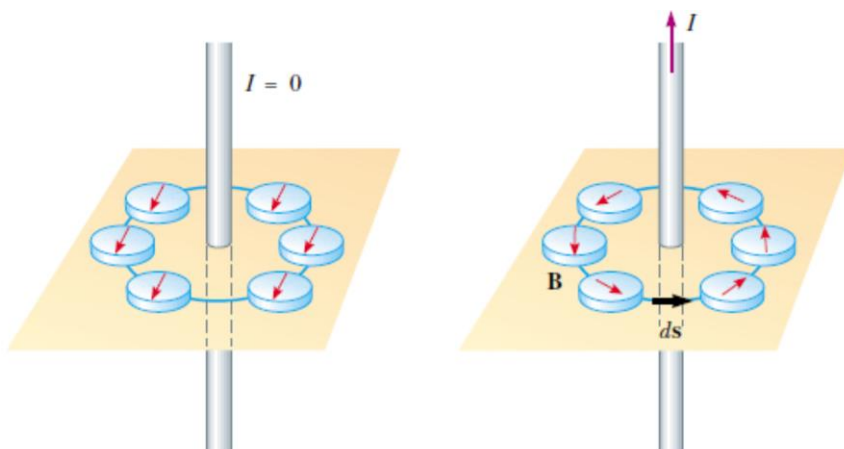
The direction of $\vec{\mu}$ is the same as the area vector $\vec{A}$ (perpen



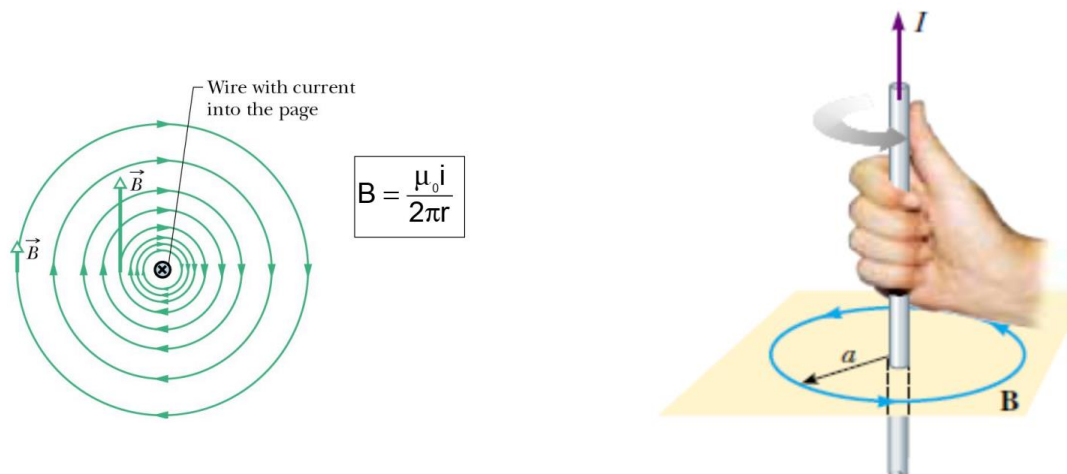
- The above equation is analogous to $\vec{\tau} = \vec{p} \times \vec{E}$ the torque exerted on an electric dipole moment $\vec{p}$ in the presence of an electric field $\vec{E}$.

Source of Magnetic Field

- The magnetic effect of electric current was discovered by the Danish physicist Robert Oersted 1819.
- As of the Oersted experiment, the compass needle situated around a conductor deflects when the current passes through the conductor.
- When no current is present in the conductor (wire), all compass needles point in the same direction (toward the Earth's North Pole).
- When the wire carries a strong current, the compass needles deflect in a direction tangent to the circle, which is the direction of the magnetic field created by the current.

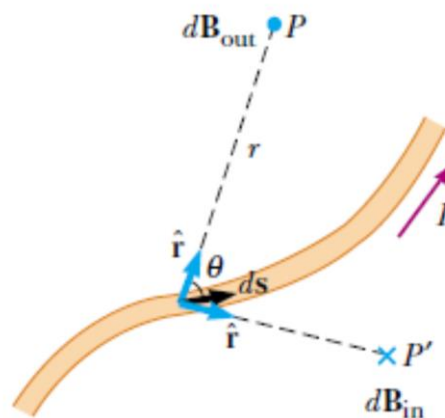


- The right-hand rule for determining the direction of the magnetic field surrounding a long straight wire carrying a current. Note that the magnetic field lines form circles around the wire.



7.2. Biot-Savart's law

- In this section, we will see how to use the law of Biot and Savart to calculate the magnetic field produced at some point in space by a small current element.
- Now, let us considering a wire of arbitrary shape carrying a current I .
- We divide the wire into small elements $d\vec{s}$ and then the current-length element to be $I ds$.
- Hence, based on the Biot and Savart experimental observations for the magnetic field $d\vec{B}$ at a point P associated with a length element $d\vec{s}$ of a wire carrying a steady current I :



- ✓ $d\vec{B}$ is perpendicular to $d\vec{s}$ and $\hat{r}$,
 - ✓ The magnitude of dB is inversely proportional to r^2 ,
 - ✓ The magnitude of dB is proportional to the current I ,
 - ✓ The magnitude of dB is proportional to $\sin \theta$ and ds , where θ is the angle between the $d\vec{s}$ and $\hat{r}$.
- Therefore, The magnetic field $d\vec{B}$ at a point P at distance r due to the current I through a length element $d\vec{s}$ is given by the **Biot–Savart law**:

$$d\vec{B} = \frac{\mu_0 I d\vec{s} \times \hat{r}}{4\pi r^2}$$

Where μ_0 is a constant called **the permeability of free space**:

$$\mu_0 = 4\pi \times 10^{-7} \text{ T} \cdot \text{m/A}$$

- The magnitude of the magnetic field dB is given by

$$dB = \frac{\mu_0 I ds \sin \theta}{4\pi r^2}$$

- The *total* magnetic field $\vec{B}$ created at some point by a current of finite size can be

$$\vec{B} = \frac{\mu_0 I}{4\pi} \int \frac{d\vec{s} \times \hat{r}}{r^2}$$

where the integral is taken over the entire current distribution.

Examples

1. A thin, straight wire carrying a constant current I and placed along the x -axis as shown in figure below. Determine the magnitude and direction of the magnetic field at point P due to this current.

Solution

- ⇒ We start by considering a length element ds located a distance r from P .
- ⇒ Then, the magnetic field at point P due to the current in each element ds of the wire is

$$d\vec{B} = \frac{\mu_0 I d\vec{s} \times \hat{r}}{4\pi r^2}$$

- ⇒ The total magnetic field is

$$\vec{B} = \frac{\mu_0 I}{4\pi} \int \frac{d\vec{s} \times \hat{r}}{r^2}, \text{ Since } d\vec{s} = d\vec{x}$$

$$B = \frac{\mu_0 I}{4\pi} \int \frac{dx \sin \theta}{r^2}$$

Hence, $\tan \theta = \frac{a}{-x} \Rightarrow x = \frac{a}{\tan \theta} = -a \cot \theta \Rightarrow dx = a \csc^2 \theta d\theta$, and

$$\sin \theta = \frac{a}{r} \Rightarrow r = \frac{a}{\sin \theta} = a \csc \theta \Rightarrow r^2 = a^2 \csc^2 \theta$$

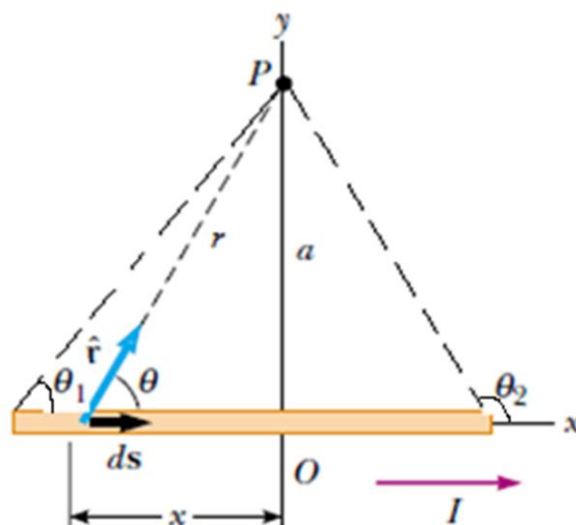
Thus,

$$B = \frac{\mu_0 I}{4\pi} \int \frac{a \csc^2 \theta \sin \theta d\theta}{a^2 \csc^2 \theta} = \frac{\mu_0 I}{4\pi a} \int \sin \theta d\theta$$

- ⇒ Finally, we now obtain the magnitude of the magnetic field at point P , where the subtending angles range from θ_1 to θ_2 as defined in figure above:

$$B = \frac{\mu_0 I}{4\pi a} \int_{\theta_1}^{\theta_2} \sin \theta d\theta = \frac{\mu_0 I}{4\pi a} [-\cos \theta]_{\theta_1}^{\theta_2} = \frac{\mu_0 I}{4\pi a} (\cos \theta_1 - \cos \theta_2)$$

- ⇒ Therefore, the net magnetic field $\vec{B}$ at point P is out of the page ($\hat{k}$),



$$\vec{B} = \frac{\mu_0 I}{4\pi a} (\cos \theta_1 - \cos \theta_2) \hat{k}$$

⇒ For the special case of an infinitely long straight wire, *i.e.*, for length elements ranging between positions $x = -\infty$ and $x = \infty$. We see that

$$\theta_1 = 0 \text{ and } \theta_2 = \pi$$

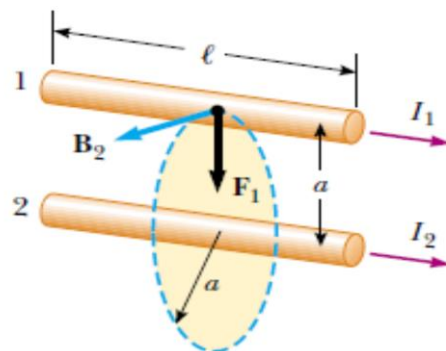
$$\text{Thus, } (\cos \theta_1 - \cos \theta_2) = (\cos 0 - \cos \pi) = 1 - (-1) = 2$$

⇒ The above equation becomes

$$\vec{B} = \frac{\mu_0 I}{2\pi a} \hat{k}$$

7.3. The Magnetic Force between Two Parallel Conductors

- We have already seen that a current-carrying wire produces a magnetic field.
- Consider two long, straight, parallel wires separated by a distance a and carrying currents I_1 and I_2 in the same direction, as shown in figure.



- The two wires exert magnetic forces on each other because the magnetic field of one wire exerts a force on the moving charges in the second wire.
- I_1 , sets up $\vec{B}_1$ at site of I_2 and I_2 , sets up $\vec{B}_2$ at site of I_1 .
- The direction of B_1 is perpendicular to wire 2 and the direction of B_2 is perpendicular to wire 1.
- The magnetic force on a current-carrying wire is given by

$$\vec{F} = I\vec{l} \times \vec{B}$$

- The magnitude of the magnetic force on a length, l , of wire 1 is then

$$F_{12} = I_1 l B_2$$

- The magnitude of the magnetic field a perpendicular distance a from wire 2 is

$$B_2 = \frac{\mu_0 I_2}{2\pi a}$$

- Thus,

$$F_{12} = \frac{\mu_0 l I_1 I_2}{2\pi a}$$

- We can rewrite this magnitude in terms of the force per unit length:

$$\frac{F_B}{l} = \frac{\mu_0 I_1 I_2}{2\pi a}$$

- The magnetic force on a length, l , of wire 2 has the same magnitude and opposite direction:

$$\vec{F}_{12} = -\vec{F}_{21} \Rightarrow |\vec{F}_{12}| = |\vec{F}_{21}|$$

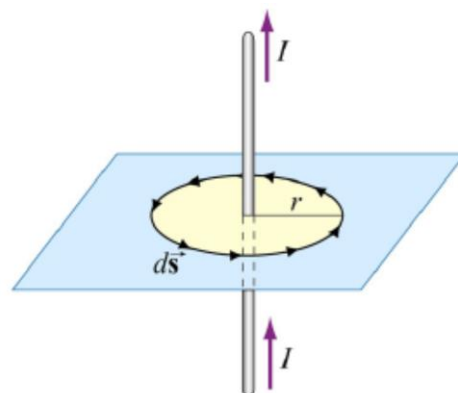
- Hence, parallel conductors carrying currents in the same direction attract each other, and parallel conductors carrying currents in opposite directions repel each other.

7.4. Ampere's Law and Its Application

- Ampere's law states that the line integral of $\vec{B} \cdot d\vec{s}$ around any closed path equals $\mu_0 I$, where I is the total steady current through any surface bounded by the closed path:

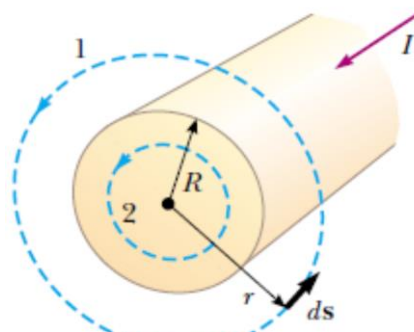
$$\oint \vec{B} \cdot d\vec{s} = \mu_0 I$$

- Ampere's law describes the creation of magnetic fields by all continuous current configurations.
- Its use is similar to that of Gauss's law in calculating electric fields for highly symmetric charge distributions.
- Ampere's law is applicable to the following current configurations.
1. A long current-carrying wire carrying a steady current I .
 2. An infinite current sheet with a current density J .
 3. A solenoid.
 4. A Toroid.



Example 1: The Magnetic Field Created By a Long Current-Carrying Wire

A long, straight wire of radius R carries a steady current I that is uniformly distributed through the cross section of the wire as shown in figure. Calculate the magnetic field a distance r from the center of the wire in the regions $r \geq R$ and $r < R$.



Solution

⇒ The magnetic field at any point can be calculated from Ampere's law using a circular path of radius r , concentric with the wire.

⇒ For $r \geq R$, let us choose for our path of integration circle 1. Ampere's law gives

$$\oint \vec{B} \cdot d\vec{s} = \mu_0 I$$

⇒ The total current passing through the plane of the circle 1 is I , thus the magnitude of magnetic field is

$$\oint \vec{B} \cdot d\vec{s} = B \oint ds = B (2\pi r) = \mu_0 I$$

$$B = \frac{\mu_0 I}{2\pi r} \quad (\text{for } r \geq R)$$

⇒ For $r < R$, let us choose for our path of integration circle 2. Here the current I' passing through the plane of circle 2 is less than the total current I . Thus, the current density J

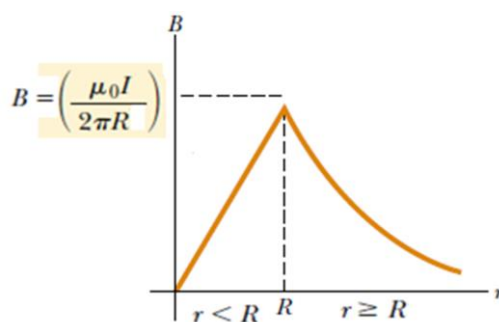
$$J = \frac{I}{A} = \frac{I'}{A'} \Rightarrow \frac{I}{\pi R^2} = \frac{I'}{\pi r^2} \Rightarrow I' = \frac{r^2}{R^2} I$$

Now, we apply Ampere's law to circle 2:

$$\oint \vec{B} \cdot d\vec{s} = B \oint ds = B (2\pi r) = \mu_0 I'$$

$$B(2\pi r) = \mu_0 I' = \mu_0 \left(\frac{r^2}{R^2} I \right)$$

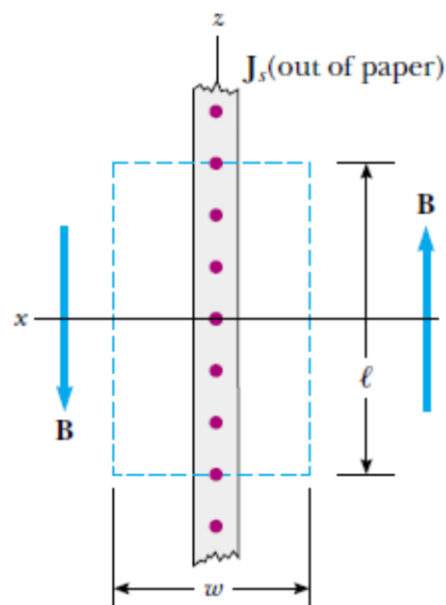
$$B = \left(\frac{\mu_0 I}{2\pi R^2} \right) r \quad (\text{for } r < R)$$

**Example 2:** Magnetic Field Created by an Infinite Current Sheet

A thin, infinitely large sheet lying in the yz plane carries a current of linear current density J_s . The current is in the y direction, and J_s represents the current per unit length measured along the z axis. Find the magnetic field near the sheet.

Solution

⇒ To evaluate the line integral in Ampere's law, we



construct a rectangular path through the sheet, as in shown in figure.

- ⇒ The rectangle has dimensions l and w , with the sides of length l parallel to the sheet surface.
- ⇒ The net current in the plane of the rectangle is $J_s l$.
- ⇒ The two sides of length w do not contribute to the line integral because the component of B along the direction of these paths is zero.
- ⇒ However, a perpendicular field would pass through the current, which is inconsistent with the Biot–Savart law.
- ⇒ Assuming a field that is constant in magnitude and parallel to the plane of the sheet.
- ⇒ Now, we apply Ampere’s law

$$\oint \vec{B} \cdot d\vec{s} = \mu_0 I = \mu_0 J_s \ell$$

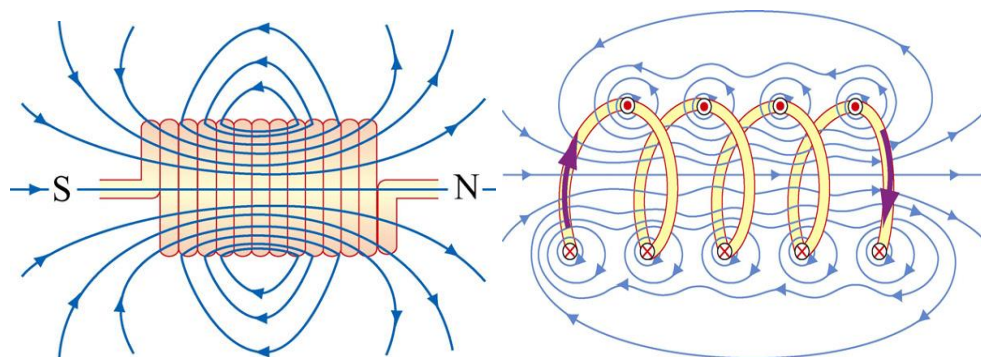
$$2B\ell = \mu_0 J_s \ell$$

$$B = \mu_0 \frac{J_s}{2}$$

- ✓ This result shows that the magnetic field is independent of distance from the current sheet.

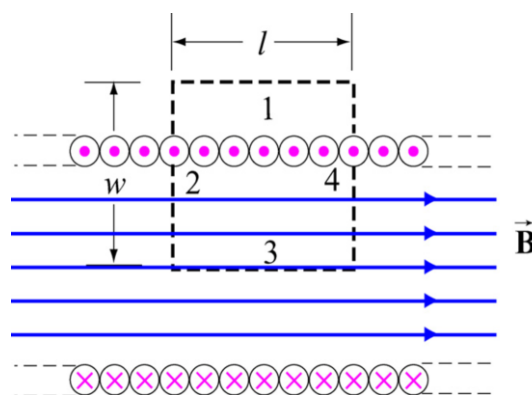
Example 3: The Magnetic Field of a Solenoid

A solenoid is a long coil of wire tightly wound in the helical form. Figure below shows the magnetic field lines of a solenoid carrying a steady current I . We see that if the turns are closely spaced, the resulting magnetic field inside the solenoid becomes fairly uniform, provided that the length of the solenoid is much greater than its diameter. For an “ideal” solenoid, which is infinitely long with turns tightly packed, the magnetic field inside the solenoid is uniform and parallel to the axis, and vanishes outside the solenoid. Calculate the magnetic field strength inside an ideal solenoid.



Solution

- ⇒ Take cross-sectional view of an ideal solenoid, where the interior magnetic field is uniform and the exterior field is close to zero.



- ⇒ To compute $\vec{B}$, we consider a rectangular path of length l and width w and traverse the path in a counterclockwise manner. The line integral of $\vec{B}$ along this loop is

$$\begin{aligned}\oint \vec{B} \cdot d\vec{s} &= \int_1 \vec{B} \cdot d\vec{s} + \int_2 \vec{B} \cdot d\vec{s} + \int_3 \vec{B} \cdot d\vec{s} + \int_4 \vec{B} \cdot d\vec{s} \\ &= 0 + 0 + Bl + 0\end{aligned}$$

- ⇒ In the above, the contributions along sides 2 and 4 are zero because $\vec{B}$ is perpendicular to $d\vec{s}$. In addition $\vec{B} = 0$, along side 1 because the magnetic field is non-zero only inside the solenoid.
- ⇒ On the other hand, the total current enclosed by the Amperian loop is $I_{enc} = NI$, where N is the total number of turns. Applying Ampere's law yields

$$\begin{aligned}\oint \vec{B} \cdot d\vec{s} &= \mu_0 I_{enc} \\ \oint \vec{B} \cdot d\vec{s} &= Bl = \mu_0 NI\end{aligned}$$

$$B = \frac{\mu_0 NI}{l} = \mu_0 nI$$

Where $n = \frac{N}{l}$ represents the number of turns per unit length.

Example 4: The Magnetic Field Created by a Toroid

If a solenoid is bent so that the two ends meet, with the wire forming a series of loops, each with the same current flowing through it. This device is called a toroid. Consider a toroid which consists of N turns, as shown in figure. Find the magnetic field everywhere.

Solution

- ⇒ Let us choose a concentric circle of radius r , where $b < r < c$ as an Amperian loop.
- ⇒ Furthermore, the wire passes through the loop N times, so that the total current through the loop is $I_{enc} = NI$.
- ⇒ Applying Ampere's law to the circle, we obtain

$$\oint \vec{B} \cdot d\vec{s} = \mu_0 I_{enc}$$

$$\oint \vec{B} \cdot d\vec{s} = B \oint ds = B(2\pi r) = \mu_0 NI$$

$$B = \frac{\mu_0 NI}{2\pi r}$$

where I is the current in the toroid windings and N is the total number of turns.

- ⇒ Imagine inside a toroid, where $r < b$ of the Amperian loop encloses zero current, so

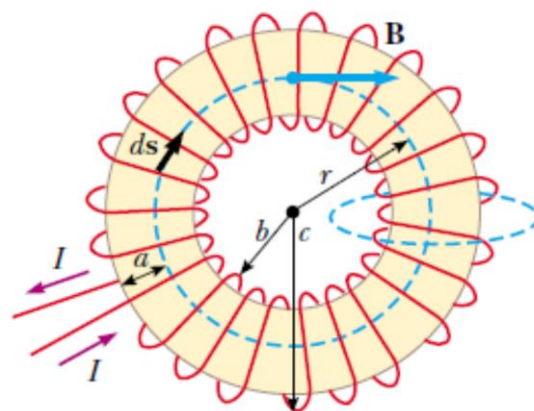
$$\oint \vec{B} \cdot d\vec{s} = \mu_0 I_{enc} = 0$$

$$B = 0$$

- ⇒ Again, outside a toroid, where $r > c$ of the Amperian loop encloses zero net current,

$$\oint \vec{B} \cdot d\vec{s} = \mu_0 (NI - NI) = 0$$

$$B = 0$$

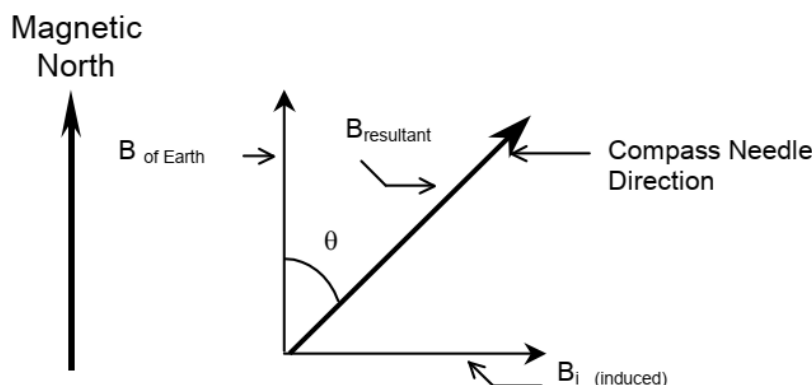


Tangent Galvanometer: A tangent galvanometer works based on Tangent law. It consists of a number of turns of copper wire wound on a hoop. At the center of the hoop a compass is mounted. When a direct current flows through the wires, a magnetic field is induced in the space

surrounding the loops of wire. This magnetic flux is designated by Φ . The strength of the magnetic field induced by the current at the center of the loops of wire is given by Amperes law:

$$B = \frac{\mu_0 NI}{2R}$$

(2) where μ_0 is the permeability of free space and has the value of $4\pi \times 10^{-7}$ Newton/Amp², N is the number of turns of wire, ' i ' is the current through the wire, and R is the radius of the loop. When the wire loops of the tangent galvanometer are aligned with the magnetic field direction of the Earth, and a current is sent through the wire loops, then the compass needle will align with the vector sum of the field of the Earth and the induced field as shown in Figure 1.



Exercise

1. A uniform magnetic field of 30 mT exists in the + X direction. A particle of charge + e and mass 1.67×10^{-27} kg is projected through the field in the + Y direction with a speed of 4.8×10^6 m/s.
 - (a) Find the force on the charged particle in magnitude and direction
 - (b) Find the force if the particle were negatively charged.
 - (c) Describe the nature of path followed by the particle in both the cases.

Solution :

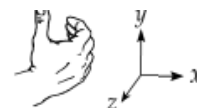
- (a) $F = e v B \sin\theta = (1.6 \times 10^{-19}) (4.8 \times 10^6) (30 \times 10^{-3}) \sin 90^\circ = 230.4 \times 10^{-16}$ N. The direction of the force is in the (-z) direction.
- (b) If the particle were negatively charged, the magnitude of the force will be the same but the direction will be along (+z) direction.
- (c) As $V \perp B$, the path described is a circle, where radius is given by

$$R = mv/qB = (1.67 \times 10^{-27}) (4.8 \times 10^6) / (1.6 \times 10^{-19}) (30 \times 10^{-3}) = 1.67 \text{ m.}$$

2. An electron moving along the positive x axis perpendicular to a magnetic field experiences a magnetic deflection in the negative y direction. What is the direction of the magnetic field?

$$\mathbf{F}_B = q\mathbf{v} \times \mathbf{B}; |\mathbf{F}_B|(-\hat{j}) = -e|\mathbf{v}|\hat{i} \times \mathbf{B}$$

Therefore, $B = |\mathbf{B}|(-\hat{k})$ which indicates the negative z direction.



3. A proton travels with a speed of $3.00 \times 10^6 \text{ m/s}$ at an angle of 37.0° with the direction of a magnetic field of 0.300 T in the y direction. What are (a) the magnitude of the magnetic force on the proton and (b) its acceleration?

$$(a) \quad F_B = qvB \sin \theta = (1.60 \times 10^{-19} \text{ C})(3.00 \times 10^6 \text{ m/s})(3.00 \times 10^{-1} \text{ T}) \sin 37.0^\circ$$

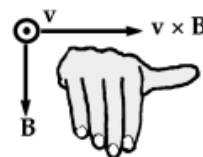
$$F_B = \boxed{8.67 \times 10^{-14} \text{ N}}$$

$$(b) \quad a = \frac{F}{m} = \frac{8.67 \times 10^{-14} \text{ N}}{1.67 \times 10^{-27} \text{ kg}} = \boxed{5.19 \times 10^{13} \text{ m/s}^2}$$

4. A proton moves perpendicular to a uniform magnetic field $\mathbf{B}$ at $1.00 \times 10^7 \text{ m/s}$ and experiences an acceleration of $2.00 \times 10^{13} \text{ m/s}^2$ in the x direction when its velocity is in the z direction. Determine the magnitude and direction of the field.

$$F = ma = (1.67 \times 10^{-27} \text{ kg})(2.00 \times 10^{13} \text{ m/s}^2) = 3.34 \times 10^{-14} \text{ N} = qvB \sin 90^\circ$$

$$B = \frac{F}{qv} = \frac{3.34 \times 10^{-14} \text{ N}}{(1.60 \times 10^{-19} \text{ C})(1.00 \times 10^7 \text{ m/s})} = \boxed{2.09 \times 10^{-2} \text{ T}}$$

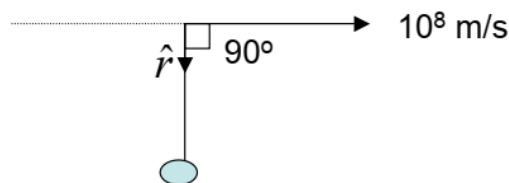


The right-hand rule shows that $\mathbf{B}$ must be in the $-y$ direction to yield a force in the $+x$ direction when $\mathbf{v}$ is in the z direction.

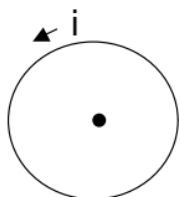
FIG. P29.5

5. A point charge $q = 1 \text{ mC}$ ($1 \times 10^{-3} \text{ C}$) moves in the x direction with $v = 10^8 \text{ m/s}$. It misses a mosquito by 1 mm . What is the $\mathbf{B}$ field experienced by the mosquito?

$$B = \frac{\mu_0}{4\pi} q \frac{v}{r^2} = 10^4 \text{ T}$$



Loop of wire of radius $R = 5 \text{ cm}$ and current $i = 10 \text{ A}$. What is B at the center? Magnitude and direction



$$B = \frac{\mu_0 i}{2R}$$

$$B = 4\pi \times 10^{-7} \frac{N}{A^2} \frac{10 A}{2(.05 m)}$$

$$B = 1.2 \times 10^{-6} \cdot 10^2 T$$

$$B = 1.2 \times 10^{-4} T = 1.2 \text{ Gauss}$$

Direction is out of the page.

Exercises

1. A proton moves with a velocity of $\mathbf{v} = (2\mathbf{i} - 4\mathbf{j} + \mathbf{k}) \text{ m/s}$ in a region in which the magnetic field is $\mathbf{B} = (\mathbf{i} + 2\mathbf{j} - 3\mathbf{k}) \text{ T}$. What is the magnitude of the magnetic force this charge experiences?
2. A proton moving at $4.00 \times 10^6 \text{ m/s}$ through a magnetic field of 1.70 T experiences a magnetic force of magnitude $8.20 \times 10^{-13} \text{ N}$. What is the angle between the proton's velocity and the field?
3. At the equator, near the surface of the Earth, the magnetic field is approximately 50.0 T northward, and the electric field is about 100 N/C downward in fair weather. Find the gravitational, electric, and magnetic forces on an electron in this environment, assuming the electron has an instantaneous velocity of $6.00 \times 10^6 \text{ m/s}$ directed to the east.
4. An electron has a velocity of $1.20 \times 10^4 \text{ m/s}$ (in the positive x direction), and an acceleration of $2.00 \times 10^{12} \frac{m}{s^2}$ (in the positive z direction) in a uniform electric and magnetic field. If the electric field has a magnitude of 20.0 N/C (in the positive z direction), what can you determine about the magnetic field in the region? What can you not determine?
5. A wire having a mass per unit length of 0.500 g/cm carries a 2.00 A current horizontally to the south. What are the direction and magnitude of the minimum magnetic field needed to lift this wire vertically upward?
6. A wire carries a steady current of 2.40 A . A straight section of the wire is 0.750 m long and lies along the x axis within a uniform magnetic field, $\mathbf{B} = 1.60\mathbf{k} \text{ T}$. If the current is in the $+\mathbf{x}$ direction, what is the magnetic force on the section of wire?
7. A wire 2.80 m in length carries a current of 5.00 A in a region where a uniform magnetic field has a magnitude of 0.390 T . Calculate the magnitude of the magnetic force on

the wire assuming the angle between the magnetic field and the current is (a) 60.0° , (b) 90.0° , (c) 120° .

8. A current of 17.0 mA is maintained in a single circular loop of 2.00 m circumference. A magnetic field of 0.800 T is directed parallel to the plane of the loop. (a) Calculate the magnetic moment of the loop. (b) What is the magnitude of the torque exerted by the magnetic field on the loop?
9. A small bar magnet is suspended in a uniform 0.250 T magnetic field. The maximum torque experienced by the bar magnet is $4.60 \times 10^{-3}\text{ N} \cdot \text{m}$. Calculate the magnetic moment of the bar magnet.