NUCLEAR PHYSICS 7 classes

Further study is required in this course. Evolving field. Gets maximum Nobel Prizes.

1) Basic Nuclear Properties

Size
Constituents - their properties
Angular Momentum
Magnetic Moment
Quadrupole Moment
Parity
Binding Energy

2) Models of Nucleus

① Semi Empirical Model (calculates Binding Energy)
Mass Formula
Mass Parabola

② Shell Model (Calculates rest of properties)

3) Nature of Nuclear Force

Characteristics of strong nuclear force
Yukawa's Meson Theory
Grand state of deuteron & Magnetic Moment

4) β decay

Parity Violation

⑤ γ-decay

Internal Conversion

Mossbauer Effect

⑥ Nuclear Fission & Fusion

Particle Physics

3 classes

① Particle Classification

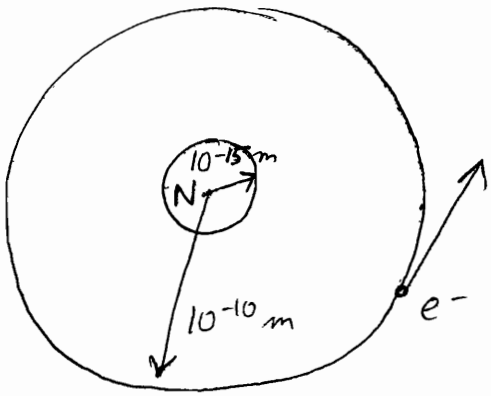
② Conservation laws ③ Quark Structure Hadrons

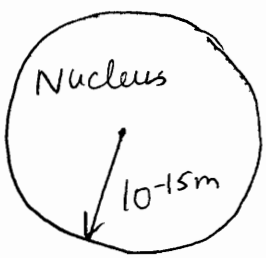
④ Basic idea about Unification

Tayal / Pandey

NUCLEAR PHYSICS (1)

→ Remember α-particle experiment by bombarding them on a Gold Foil. It led to discovery of Nucleus by Rutherford. By H·U·P., we now know e^- cannot reside in nucleus. Neutron was discovered much later, to solve mass discrepancy.





1 fm : 10^{-15} m

[femto meter or fermi meter]

① Constituents of Nucleus

(For proton, expected $\mu = 1\,\mu_N$ ⇒ Non uniform charge distribution)

Proton : $q = +e$

$m_p = 1836\, m_e \simeq 1.66 \times 10^{-27}$ kg

$m_0 c^2 = 938$ MeV

$I = s = \left(\frac{1}{2}\right)$ $\quad I^2 = \frac{3}{4}\hbar^2$

$g \cdot \frac{e}{2m_0} \cdot \frac{1}{2}\hbar$

$= \frac{g\mu_n}{2}$ $\quad$ used for nucleus

$= \frac{5.586}{2}\mu_n = 2.793\,\mu_n$

used for e^-

$(\mu_p)_z = \pm\ 2.793\,\mu_N \approx +2.8\,\mu_N$

Neutron $q = 0$

$m_n = 1840\, m_e \simeq 1.67 \times 10^{-27}\ kg$

$m_0c^2 = 939.5\ MeV \simeq 940\ MeV$

$(\mu_N)_z = \mp 1.91\ \mu_N \approx -1.9\ \mu_N$ (inspite of the fact that it is uncharged ∴ expected $\mu = 0$)

[Note that it is μ_z that we are talking about]

$I = \left(\frac{1}{2}\right)$

$I^2 = \frac{3}{4}\hbar^2$

We represent Nucleus as (Z, A)

A ↑ mass number

Z ↑ atomic number

No. of Protons = Z

No. of Neutrons = $A - Z$

Special Types of Nucleus :

[redacted] : Z same, A difference — same element

[redacted] : Z different, A same — different elements

[redacted] : Neutrons and Protons are interchanged.

$^{15}_{7}N$ Unstable nitrogen

$^{15}_{8}O$ unstable Oxygen

& $|N \sim Z| = 1$

$A \sim 2Z = 1$

$^{3}_{1}H$ (tritium)

$^{3}_{2}He$

Mass of Nucleus = Mass of Z protons $(Z m_p)$

+

Mass of $(A-Z)$ neutrons $((A-Z) m_N)$

−

Mass equivalent of Binding Energy $\left(\frac{E_B}{c^2}\right)$

here E_B is taken as positive value i.e. $|E_B|$

→ In any atom, more than 99% mass is contributed by nucleus.

$$\Rightarrow \frac{E_B}{c^2} = [ZM_p + (A-Z)M_N] - M(Z,A)$$

[representation of mass of nucleus]

$$E_B = \left(-M(Z,A) + [ZM_p + (A-Z)M_N]\right) c^2$$

difference in a.m.u & m_p/m_n due to B.E

due to "Mass defect", we have $\boxed{1\ a.m.u. = 931\ MeV}$ < m_p or m_n

Mass of Constituents − Mass of Nucleus : Mass defect

Binding Energy per nucleon = Average Binding Energy

Nuclei is written in 4 ways :

Pairing [N − Z]	Mass No. A	No. of stable nuclei observed
Even − Even	A even	165
Odd − Odd		4
Even − Odd	A odd	55
Odd − Even		50
N Z		

→ ${}_1^2H$, ${}_3^6Li$, ${}_5^{10}B$, ${}_7^{14}N$ (deuterium) $N = Z$

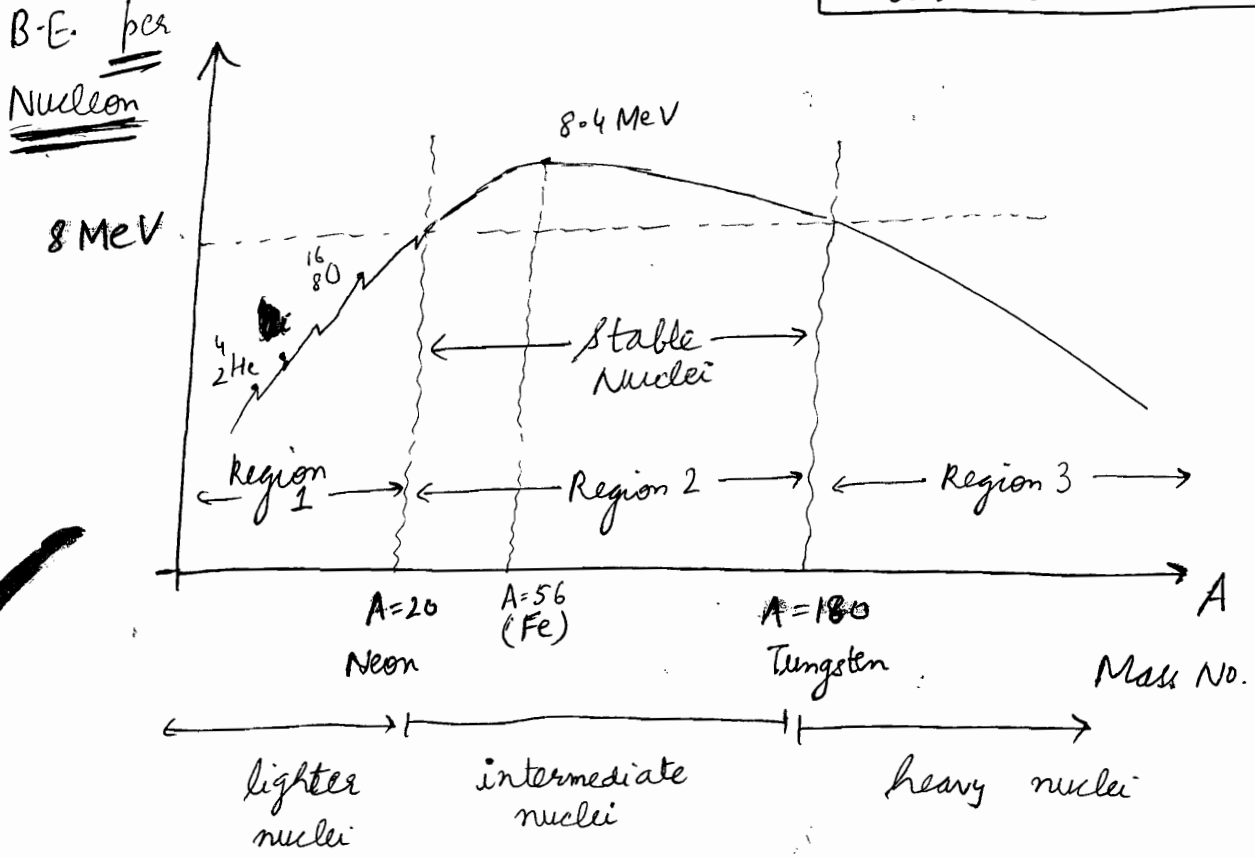
≒ 117 elements ≒ 270 nucleus (stable)

[Many stable isotopes]

- Even - Even are most abundand ⇒ most stable
- Odd - Odd are least abundant ⇒ least stable
- Even - Odd / Odd - Even are normal.

② Binding Energy per Nucleon Curve

> note that in nuclear physics, we are least interested in Z. We care about A.



It is an empirical (experimental) curve.

→ More the binding energy, more the energy needs to be supplied to break it ⇒ more the stability.

→ Nuclei of group 1 fuse together to gain higher binding energy to achieve stability: Nuclear Fusion

→ Nuclei of group 2 break to come to region 2, to increase Binding energy in order to achieve stability.

Nuclear Fission & Spallation

★ More stable the nuclei, more Binding Energy will be released
⇒ reason for energy in fusion & fission

Higher the mass, higher the volume

⇒ High A nucleus will have more Volume

⇒ more radius (if assumed spherical)

$$\left(\frac{4}{3}\pi R^3\right) \propto A$$

$$\Rightarrow \boxed{R \propto A^{\frac{1}{3}}}$$

$$\boxed{R = R_0 A^{\frac{1}{3}}}$$

Take $R_0 = 1.3$ fm

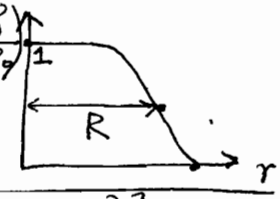
⊙ The size and shape of nuclei are studied by scattering experiments using high speed electrons and neutrons as Bombarding Particles.

Electrons interact only with protons and neutrons interact only with special nuclear forces.

The former provides information on distribution of electric charge and the latter on the distribution of nuclear matter in the nucleus.

Radius Const. 1.2 - 1.3 fm

⊛ ρ is distribution of charge in the nucleus:



$$\text{Density } \rho = \frac{A \times 1.66 \times 10^{-27} \text{ kg}}{\frac{4\pi}{3} R_0^3 \cdot A} \simeq \frac{10^{-27}}{10^{-45}}$$

$$\simeq \underline{\underline{10^{17} \text{ kg/m}^3}}$$

Now we see density is independent of size or shape as in real world. Similarly is the case with nucleus.

Density of Nucleus is independent of mass.

Hence, Nucleus is compared to a liquid drop

eg: (1) o o → O Fusion (coalescence)

(2) O → o o Fission (breaking down)

(3) ρ independent of shape & size

(4) Both are spherical in shape.

Both neutrons and protons have spin $= (1/2)$ and possess

(3) angular momentum.

Hence L-S coupling occurs within nucleus.

$$\vec{I} = \vec{L} + \vec{S} \qquad \text{(L-S coupling)}$$

$$\vec{L} = \sum (\vec{l_p} + \vec{l_n})$$

$$\vec{S} = \sum (\vec{s_p} + \vec{s_n})$$

Nucleon: Neutrons & Protons are together called Nucleons

$$I = I_p + I_s \qquad \text{(I-I coupling)}$$

$$|I| = \sqrt{I(I+1)}\ \hbar \qquad \text{Magnitude Quantization}$$

I: Nuclear Spin Quantum Number

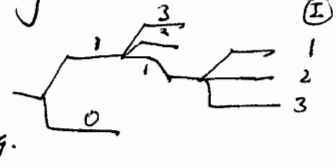
[Total Ang. momentum of Nucleus it may be due to Orbital or Spin or both. Note that I am not interested in individual momentums I am interested only in total I]

$$I_z = M_I \hbar \qquad \text{space Quantization}$$

Here space Quantization will lead to division of energy levels. Order of $\Delta E \simeq 10^{-8}$ eV (NMR)

(*) [Here "inverted coupling" occurs b/coz now μ is positive quantity. $\Rightarrow$ higher I, lower energy]

Nuclear LS coupling is inverted LS coupling i.e. Higher I, lower energy



eg.

④ Magnetic Moment

$$\vec{\mu} \propto \vec{I}$$

$$\vec{\mu} = \underbrace{\frac{e}{2m_p}}_{\text{Gyromagnetic Ratio}} g \vec{I}$$

g : Land 'g' splitting factor

$$I = L + S$$

$$g = 1 + \frac{I(I+1) + S(S+1) - L(L+1)}{2I(I+1)}$$

As always l*** ना negative का हो !!

in L-S coupled state only ....

→ "g" can be obtained from NMR.

$$|\vec{\mu_z}| = \frac{e}{2m_p} g |\vec{I_z}| = \mu_N g M_I$$

$$\boxed{\mu_z = \mu_N g M_I}$$

→ Raabi's method to measure μ of neutron

⊛ To measure μ of proton, we can use simple NMR of water.

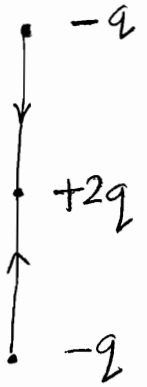
Q Why neutron has magnetic moment ??

It is not explained classically.
It is due to internal structure of neutron. Overall it is electrically neutral; it is surrounded by cloud of π^- mesons.

(5) Quadrupole Moment

[nuclei do not have dipole moment but have a quadrupole moment]

Quadrupole : arrangement of 4 particles



In EM, $q l^2 (3\cos^2\theta - 1)$ ....

or

+	−
−	+

$\rho(r)$: charge per unit volume at distance r

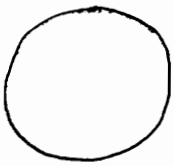
⊛ For quadrupole moment symmetric about z axis

$$Q = \frac{1}{e}\int (3z^2 - r^2)\, \rho(r)\, d\tau$$

It has dimension of Area

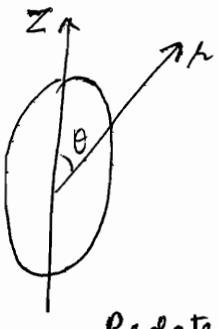
Measure in ⊔ $10^{-28}\ m^2$

Area of $10^{-28}\ m^2$: 1 Barn



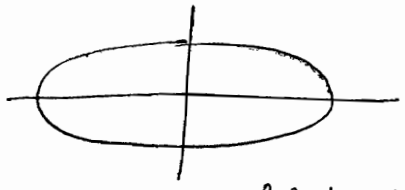
$Q = 0$ if charge dist^n is spherical

Otherwise $Q \neq 0$



$Q > 0$ $\quad Q = \frac{1}{e}\int (3\cos^2\theta - 1)\, r^2\, \rho(r)\, d\tau$

Prolate spheroid [Rugby Ball]



$Q < 0$

⊛ धरती का negative $\vec{E}$.

Oblate spheroid [like Earth]

We can assume, Nucleus fluid to be in shape of Prolate Ellipsoid or Oblate Ellipsoid.

Quadrupole Moment is the departure from spherical shape of nucleus, i.e. spherical charge distribution

It is measured in area in units of Barns.

(6) Parity

Behaviour of particle or system or phenomenon under mirror symmetry. (inversion of coordinates)

If same behaviour ⇒ [redacted] Even Parity

If Opposite behaviour ⇒ [redacted] Odd Parity

If $\psi(x,y,z) = \psi(-x,-y,-z) \Rightarrow$ Even Parity

If $\psi(x,y,z) = -\psi(-x,-y,-z) \Rightarrow$ Odd Parity

$P[\psi(x,y,z)] = +1\ \psi(x,y,z)$ Even Parity

where P opposes the coordinates

if $P[\psi(x,y,z)] = -1\ \psi(x,y,z)$ Odd Parity

± 1 are eigen value of 'P' operator

Note that $P^2[\psi(x,y,z)] = +1\ [\psi(x,y,z)]$

P^2 has only 1 eigen value i.e. +1

उल्टे का उल्टा : सीधा

for further reference, refer to किताब

If even parity ⇒ no change in behaviour in mirror image.

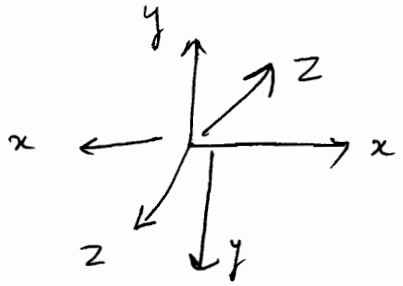
Typical example of even parity wave function : $\cos x$

odd parity wave function : $\sin x$

⊙ Parity is a multiplicative quantity.

Space Parity : $+x, +y, +z \longrightarrow -x, -y, -z$

'Right-Handed System' to 'Left Handed System'



$(X, y) \rightarrow (-x, -y)$: Translational Symmetry

If also $(z) \rightarrow (-z)$: Rotational Symmetry

Every system has 2 parities : Intrinsic Parity
Space Parity

For any particle $P_{Total} = P_{space} * P_{Intrinsic}$

For any reaction, $A + B \rightarrow C + D$

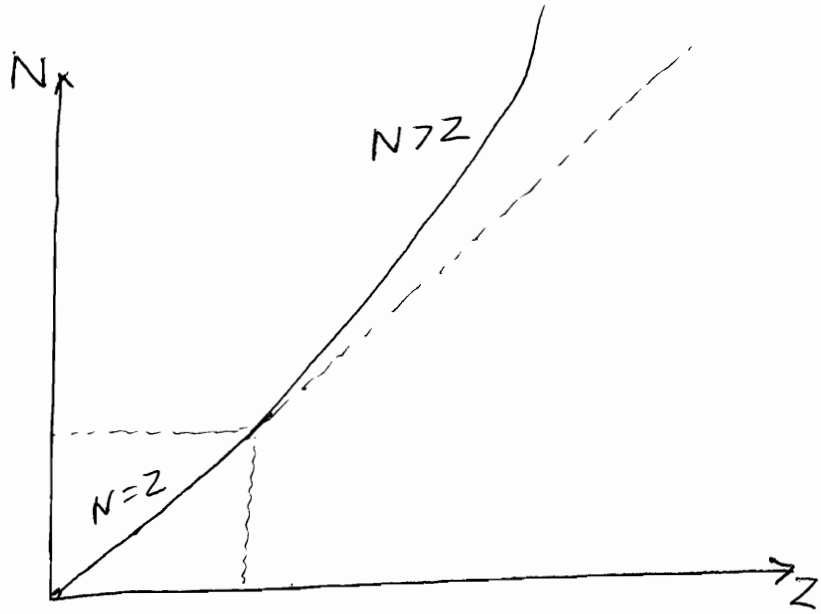
We have to see what type of phenomenon is there. Depending on type of phenomenon, Parity will either be conserved or reversed.

$P_{(A)} P_{(B)} = P_{(C)} P_{(D)}$ Parity Conservation

If Parity is conserved ⇒ reaction recording is independent of coordinate system whether left handed or right handed.

Stability Curve

Most stable nuclei for lighter nuclei :- No. of Protons = No. of Neutrons



All nuclei with $Z > 83$ and $A > 209$ are unstable

For heavier nuclei :- No. of Neutrons > No. of Protons

to achieve stabily :: Coulumb force of repulsion increase due to more Z. Shielding of n is required to achieve stability

~~Magnetic~~ ~~more large~~ ~~the Packing, more stable~~ is the ~~nucleus~~

Packing Fraction

$$\text{Packing Fraction} = \frac{M - A}{A} = \left(\frac{M}{A}\right) - 1$$

→ Negative quantity. Used to measure Binding Energy as a fraction of total Energy Contained in nucleus ($= m \cdot c^2$)

M : mass of nucleus
A : mass no. of nucleus

here A is actually, [mass number * 1 a.m.u]

Interactions

(1) Gravitational Interaction

due to mass

$$\vec{F} = -\frac{GMm}{r^2}\hat{r}$$

No use in Nuclear Physics.

Its acting on all particles, but magnitude is less.

(2) Electromagnetic Interaction

due to charge

Electrostatic Force: $F = -\frac{kq_1q_2}{r^2}\hat{r}$

Again it acts on all particles that are charged.

Rest 2 interactions are there in Nuclear Physics only:
They are dependend neither on charge nor on mass.

(3) Weak Interaction

These are interaction b/w point particles that do not occupy any space. These particles are called LEPTONS.

Neither dependent on charge nor on mass.

Range of interaction $\approx 10^{-17}$ m

Leptons: $\begin{bmatrix} e^-, & \nu_e, \\ \mu^-, & \nu_\mu, \\ \tau^-, & \nu_\tau \end{bmatrix}$

They are responsible for decay reactions.

eg. β decay, muonic decay

Strong Interaction

eg. Force that keeps nucleons together in nucleus.

It is also not due to mass or charge or decay.

In production of certain heavier particles called strange particles, strong interactions play a role.

It is the strongest known force in nature.

Symmetry Statistics of Nucleus

The quantum mechanical description of a system with a no. of particles like the nucleus is given by either Bose-Einstein statistics or Fermi-Dirac statistics.

Bose Einstein All particles with integral spin or zero spin obey Bose Einstein (B-E.) statistics and are called bosons eg proton, deuteron. All nuclei with even A obey B.E. statistics. Wave Function of a system obeying BE statistics is symmetric. This means that the wave function for such a system remains unaltered by interchange of the coordinate for any pair of identical particles.

$$\psi(x_1, x_2, \ldots x_i \ldots x_j \ldots x_n) = \psi(x_1, x_2, \ldots x_j \ldots x_i \ldots x_n)$$

Fermi Dirac All particles with half integral spin obey Fermi Dirac (F.D.) statistics and are called Fermions. eg. e^-, proton, neutron. All nuclei with odd A obey Fermi Dirac statistics. All fermions obey Pauli's exclusion Principle. Wave Function of a system of particles obeying F.D. statistics is anti symmetric. This means that if the coordinates of any pair of identical particles are interchanged in the wave function, the new system will be identical with the original except for a change of sign in the wave function.

$$\psi(x_1, x_2, \ldots x_i, \ldots x_j \ldots x_n) = -\psi(x_1, x_2, \ldots x_j, \ldots x_i \ldots x_n$$

~~Nuclear Physics (2)~~

1/03/2012

THE TABLE : Particle Physics (1)

Type of Interaction	Particles Involved	Range	Lifetime of Interaction	Particle Exchanged	Non-dimensional Coupling Const.	Relative Strength	Characteristics
Strong	HADRONS (Quarks) Colour Field	10^{-15} m	10^{-23} s	Gluons or Pi Mesons (in case of Nuclear forces)	$\left(\frac{gf^2}{4\pi\varepsilon_0\hbar c}\right) \simeq \left(\frac{14}{\blacksquare}\right)$	$\frac{14}{14} = 1$ This is reference	→
EM interaction	ALL CHARGED PARTICLES	∞	$\leq 10^{-14}$ s	Photons	$\frac{e^2}{4\pi\varepsilon_0\hbar c} = \left(\frac{1}{137}\right)$	$\frac{\left(\frac{1}{137}\right)}{14} \approx 10^{-4}$ i.e. 10000 times weaker than Strong Force	We know
Weak	LEPTONS eg. $[e^-, \mu^-, \tau^-, \nu_e, \nu_\mu, \nu_\tau]$	10^{-17} m	$\geq 10^{-8}$ s	Intermediate Vector Bosons $W^\pm, Z^0$	$\left(\frac{m\pi c}{\hbar}\right)^4 \left(\frac{gf}{\hbar c}\right)^2$ Its called COMPTON WAVELENGTH	10^{-14}	→
Gravitational	All Particles	∞	—	GRAVITON	—	10^{-40}	We know

(*) Ironically, weak force is not the weakest force, but rather gravitational force is the weakest force.

Characteristics / Salient features of strong nuclear force:

(covered in nuclear)

(1) Interaction is charge independent
⇒ same b/w n-n, n-p etc.

(2) Attractive in nature

(3) Spin dependent

(4) Short Range. Within nuclear dimension

(5) Satisfies saturation property.

(6) Non-Central in nature i.e. Tensor force

Field of Strong Forces is called "Colour Field".
What "Charge" play role in EM field, "colour" plays same role in Strong Field.

eg. Strong Nuclear Force
Production of strange Particles

Weak Force

eg.
β-decay
Decay of Strange Particle

⊙ Strong Force is some in $n-n$
$n-p$
$p-p$

It is explained in terms of exchange theory.

→ Field is a force per unit quanta, this we know. It is the sphere of influence of that force.

How the force field is created ??

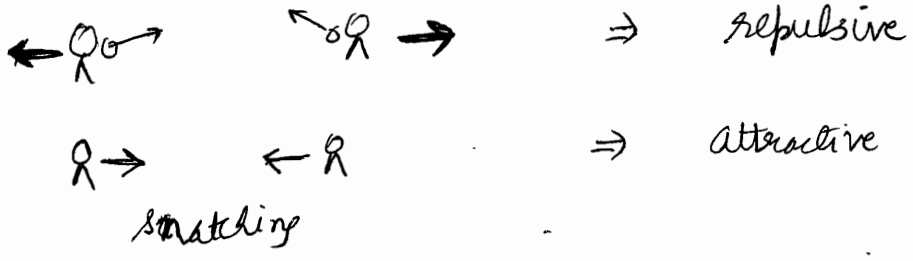
Yukawa's Exchange Theory

Strong Interaction is a broader term whose subset is Nuclear Force. For Nucleus Force, Yukawa gave Exchange Theory or Meson Theory.

It explains nature of nuclear force between $n-n$, $n-p$ and $p-p$.

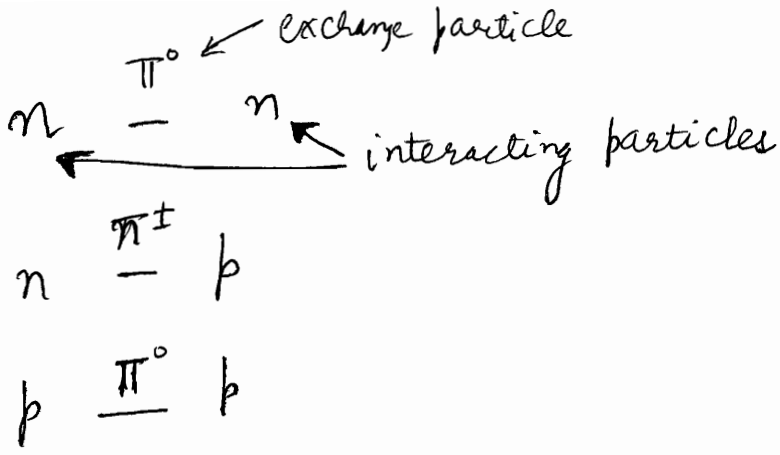
According to the theory, interaction is arising due to exchange of particles

2 players playing some game of exchanging balls.



★ If exchange particle of high mass ⇒ short range of interaction

$m \propto \frac{1}{R}$



Range of Strong Force = 1.7 F to 2.2 F

(Short Range)

I can fix up mass of exchange particle via H.U.P.

$$\Delta E \, \Delta t \geq \frac{\hbar}{2}$$

Use $\Delta E \cdot \Delta t \simeq \hbar$

$$m_\pi c^2 \frac{R}{c} \geq \left(\frac{\hbar}{2}\right)$$

$$\boxed{m_\pi \geq \frac{\hbar}{2Rc}}$$

1.4 fm

Range of m_π : 235 m_e to 270 m_e

if we take Range of R : 1.7 F to 2.2 F

→ Mesons are 0 spin particles, without charge if π°, & charge $\pm q$ if $\pi^\pm$ depending upon equation.

$n \longrightarrow n$ Charge $\pi^\circ = 0$

$n \rightleftharpoons p$ Charge $\pi^\pm \neq 0$

This force satisfies saturation property. One particles interacts only with its immediate neighbour. After interacting with its neighbour, it gets saturated. No further interaction.

i.e. snatching of balls from immediate neighbours only. (जितने मरजी बातूनी हो, कितनों से बात कर लोगे तुम !!)

Specific Case

In nuclear force, nucleons interact via π mesons.

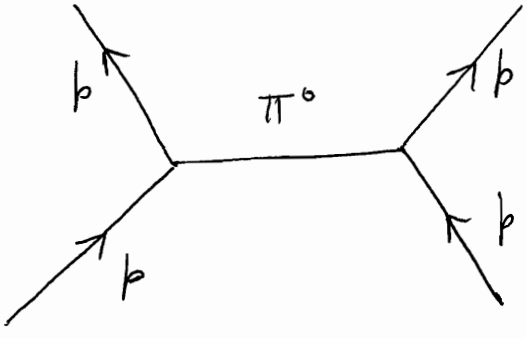
General Case

✓ In strong force, quarks interacts via gluons.

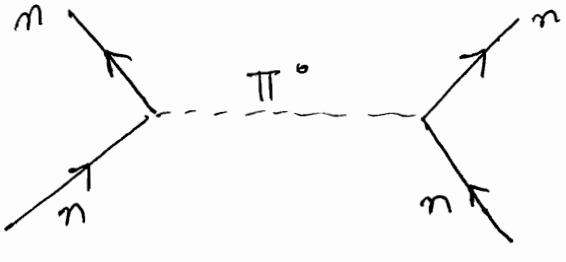
✓ In weak field, leptons interact via intermediate vector bosons.

✓ In gravitational field, particles interact via gravitons

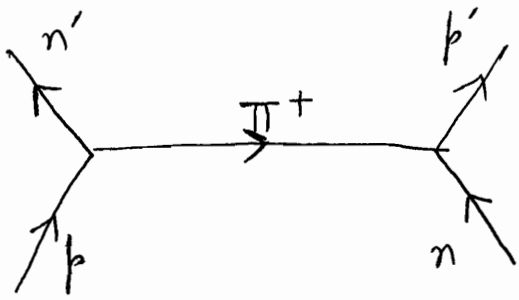
✓ In EM field, charge interact via photons.



$p \xrightarrow{\pi^0} p$

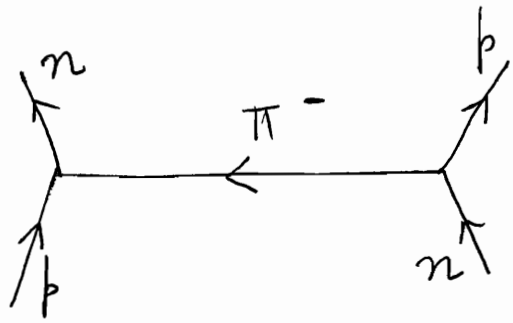


$n \xrightarrow{\pi^0} n$



Proton आया, $\pi^{(+)}$ दिया, खुद n' बन गया !! ⊙ Flow of π^+ is equivalent to

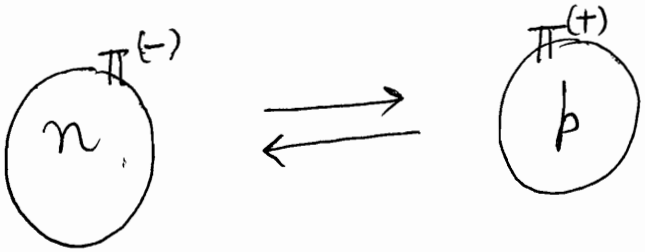
Neutron आया, π^+ accept किया, p' बन गया !! ⊙ Flow of π^- in opposite direction



From the uncertainty principle in the form

$$\Delta E \Delta t \geq (\hbar/2)$$

an event in which an amount of energy ΔE is not conserved is not prohibited so long as the duration of the event does not exceed the Δt This condition lets us estimate mass of pion



Neutrons are surrounded by $\pi^{(-)}$ mesons

Protons are surrounded by $\pi^{(+)}$ mesons

$$n + \pi^{(+)} \longrightarrow p$$

$$p + \pi^{(-)} \longrightarrow n$$

Note that Mesons have 0 spin. (according to Conservation law)

Now since neutrons are totally neutral but due to being surround by negative $\pi^{(-)}$ mesons ⇒ they have negative magnetic moment.

✓ **Elementary Particles** or Fundamental Particle: Particles whose internal structure cannot be written in terms of other particles are fundamental particles.

✓ Every particle has an anti particle.

Particle & Anti Particle have same mass, spin and lifetime. (Note that lifetime is defined for unstable particles)
But they have opposite charge & opposite orientation of spin wrt. magnetic moment.

✓ Whenever particle and anti particle meet, they annihilate and energy equal to $2mc^2$ is released.

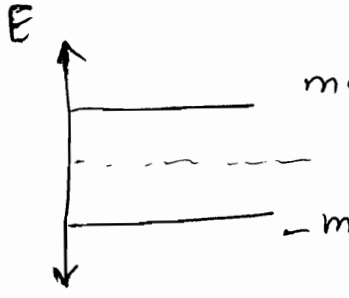
$$e^- + e^+ \longrightarrow \text{Energy released}$$

electron positron

Concept of Anti Particle is a consequence of Dirac's Quantum Mechanic theory (Dirac's equation is a synthesis of Quantum theory & relativity)

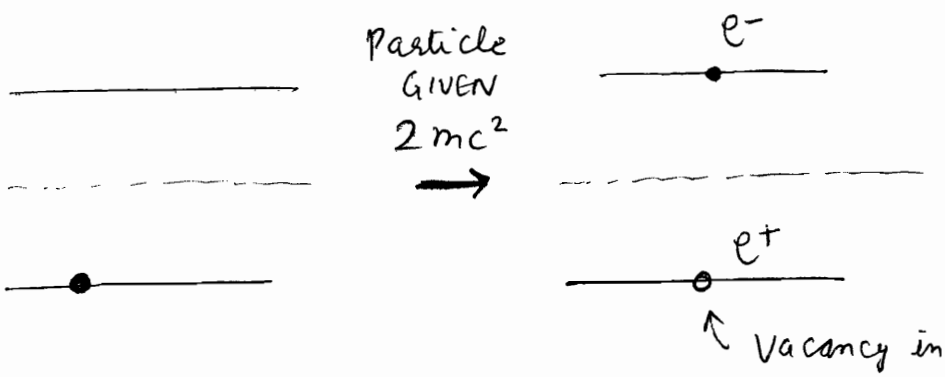
$$E^2 = (pc)^2 + (m_0c^2)^2$$

$$E = \pm\sqrt{(pc)^2 + (m_0c^2)^2} = \pm mc^2$$



[Called dirac sea, Containing negative energy state particles. But all negative energy states are fully occupied. Note that ∞ electrons $\in E$!!]

→ If $(+mc^2)$ state is given $(2mc^2)$ energy, it will reach $(+mc^2)$ state. Hence, anti particle becomes particle.



This vacancy is theoretically equal to same mass, opposite charge

All particles have antiparticles except π^0, η^0 mesons.

π^0 and η^0 have π^0 and η^0 only as antiparticles.

Particle Classification

4 criteria of classification :

(1) On basis of spin : Only 2 types of particles viz.

(i) Fermions — Half Integral Spin

(ii) Bosons — Full Integral Spin

(2) On basis of interaction :

(i) Leptons : weak interaction

(ii) Hadrons : strong interaction

(3) On basis of mass ↓

(4) Gellmann Nishijima Classification

(i) Strange ⊙ Produced by strong force, decay by weak interactions ⊙ have strangeness quantum numbers.

(ii) Non Strange ⊙ No strangeness quantum number.

Classification on basis of mass

⊙ In the standard model, there are 6+6 = 12 types of elementary fermions:
6 Quarks and
6 leptons.

Elementary particles ⊙ are classified according to spin as fermions ($\frac{1}{2}$ integral) and BOSONS (integral)

increasing mass →

CLASS	Particles	A.P.	Spin	Lepton Quantum Number S	I_3
Lepton	Electron Neutrino ν_e	$\bar{\nu}_e$	(1/2)		
	Muon Neutrino ν_μ	$\bar{\nu}_\mu$	(1/2)		
	Tau Neutrino ν_τ	$\bar{\nu}_\tau$	(1/2)		
	electron e^-	e^+	(1/2)		
	muon μ^-	μ^+	(1/2)		
	tau τ^-	τ^+	(1/2)		
Mesons (made of Quarks)	π^+ π^0 π^-	π^- π^0 π^+	0	0	1, 0, −1
	η η^0	η η^0	0	0	0
	K^+ K^0	K^- $\bar{K}^0$		+1	$\frac{1}{2}$, $-\frac{1}{2}$
Baryons	Nucleons P, N	$\bar{P}$, $\bar{N}$	(1/2)	0	$\frac{1}{2}$, $-\frac{1}{2}$
	Hyperons Λ	$\bar{\Lambda}^0$	(1/2)	−1	0
	Σ^+ Σ^0 Σ^-	$\bar{\Sigma}^-$ $\bar{\Sigma}^0$ $\bar{\Sigma}^+$	(1/2)	−1	1, 0, −1
	Ξ^- Ξ^0	$\bar{\Xi}^+$ $\bar{\Xi}^0$	(1/2)	−2	$-\frac{1}{2}$, $\frac{1}{2}$
	Ω^-	$\bar{\Omega}^+$	(3/2)	−3	0

Mass के according misfit

Stable Particles

⊛ Only π and η mesons are exceptions that their antiparticles are among themselves. π^0, η^0: self antiparticle

⊛ Note that for Ω^-, spin = (3/2)

Hadrons

In addition to these, also include resonance particles whose lifetime < 10^{-23} sec

Leptons are $(\frac{1}{2})$ spin particles: fermions

electronic neutrino ν_e / electron e^- : +1 electronic lepton quantum number $L_e = 1$

$\bar{\nu}_e$ / e^+ : −1 electronic lepton quantum number $L_e = -1$

muonic neutrino ν_μ / muon μ^- : 1 muonic lepton quantum number $L_\mu = 1$

$\bar{\nu}_\mu$ / μ^+ : −1 muonic lepton quantum no. $L_\mu = -1$

For rest of particles $L = 0$

tau neutrino ν_τ / tau τ^- : +1 tauonic lepton quantum no. $\longrightarrow$ $L_\tau = 1$

$\bar{\nu}_\tau$ / τ^+ : −1 $\longrightarrow$ $L_\tau = -1$

In all allowed nuclear reactions, lepton number have to be conserved individually i.e. L_e, L_μ, L_τ must be conserved

✓ Mesons are 0 spin particles : bosons
They do not have any quantum numbers.

✓ All Baryons are assigned quantum no. +1 $B = 1$
Anti Baryons assigned : −1 $B = -1$
All other: Baryon Quantum No. = 0 $B = 0$
Baryon No. should be conserved for all Nuclear Reactions.

Conservation Laws

Every allowed nuclear reaction has to satisfy following 8 conservation laws:

(1) L_e
(2) L_μ
(3) L_τ
(4) Baryon Quantum Number 'B'
(5) Conservation of charge
(6) Conservation of Mass-Energy
i.e.
Conservation of relativistic energy

Q Value of Reaction : Amount of energy released or given to a reaction so that it proceeds.

Q value of any reaction is consequence of Conservation of relativistic energy.

$$X + x \longrightarrow Y + y + Q$$

$$X(x, y)Y$$

$$\text{Q value} = \text{k.E.(products)} - \text{k.E.(reactants)}$$

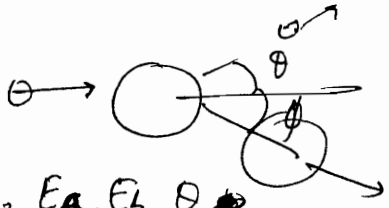
$$= (\text{Rest mass energy of reactants}) - (\text{Rest mass energy of products})$$

$k = E - m_0c^2$ ↑ conserved!!

$$A + a \longrightarrow B + b$$

$$Q = E_b + E_B - E_a - E_A$$

Replace E_B and find Q in terms of m, E_a, E_b, θ



(7) Conservation of linear Momentum

(8) Conservation of Angular Momentum
i.e.
Conservation of Spin

✓ Generally 6 are checked : linear Momentum & Mass are difficult to check

Note that they are conservation laws for weak force, strong force or EM force causing nuclear reaction.

	Strong	EM	Wea
(9) Strangeness Quantum Number "S" S is conserved in Strong & EM reactions S is not conserved in Weak field reactions.	✓	✓	X
(10) Isotopic Spin Component "I_3" Conserved in Strong & EM.	✓	✓	X
(11) Hyper charge $Y = S + B$ Conserved in Strong & EM.	✓	✓	X
(12) Isotopic Spin "I" Conserved in Strong Only	✓	X	X
(13) C charge conjugation	✓	✓	X
(14) P Parity	✓	✓	X
(15) T time reversal	✓	✓	✓
(16) CP	✓	✓	X
(17) CPT	✓	✓	✓

Nuclear Physics (2)

Parity related to ⊕ve & ⊖ve Coordinates .....

Representation of Parity

$I = \frac{1}{2}^{(+)}$ $\quad I = \frac{1}{2}^{(-)}$

Intrinsic ★ Parity of a whole system is the product of parities of individual particles. A system having even no. of odd parity parity and any no. of even parity particles will have even parity. Similarly odd no. of odd parity particles ⇒ odd Parity.

- Photon can have any parity depending upon how created
  - Electric Interaction → ~~even~~ odd
  - Magnetic Interaction → ~~odd~~ even

  for $\Delta l = 1$

★ Space Parity = Parity of wave function that describes the coordinates of its constituent particles

- Parity of any nucleus is due to its nucleons

★ Parity = Intrinsic * Space $= P\psi P_1 P_2 P_3 \dots P_n$

- [redacted]

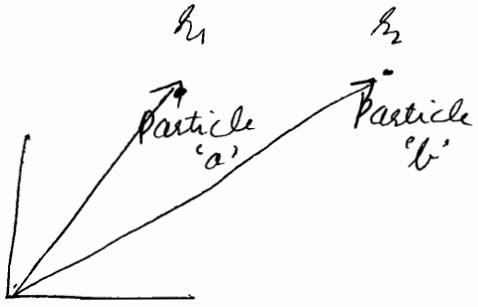
Neutrino, Neutron, Proton : Even Parity
π Meson : Odd Parity

Symmetry related to interchange of particles.....

- Statistics :
  - Fermi Dirac : Antisymmetric Particle
  - Bose Einstein : Symmetric Particle

depending on

$\psi_a(r_1)$ $\psi_b(r_2)$
a at r_1 b at r_2
$\psi_b(r_1)$ $\psi_a(r_2)$
b at r_1 b at r_2



Hence $\psi = \frac{1}{\sqrt{2}} [\psi_a(r_1)\psi_b(r_2) \pm \psi_b(r_1)\psi_a(r_2)]$

Symmetric Wave Function ; Antisymmetric Wave Funct.

$$\psi_S(r_1, r_2) = \frac{1}{\sqrt{2}} \left[\psi_a(r_1)\, \psi_b(r_2) + \psi_b(r_1)\, \psi_a(r_2) \right]$$

Bose-Einstein — Symmetric

∴ Boson are described by symmetric Wave Functions
i.e. $\psi_a \psi_b = \psi_b \psi_a$

$$\psi_A(r_1, r_2) = \frac{1}{\sqrt{2}} \left(\psi_a(r_1)\, \psi_b(r_2) - \psi_b(r_1)\, \psi_a(r_2) \right]$$

Fermi Dirac — Antisymmetric

Antisymmetric Function gives Pauli Exclusion Principle.

∴ Fermions are described by Antisymmetric Wave Functions
i.e. $\psi_a \psi_b = -\psi_b \psi_a$

$$\psi_A(r_1, r_2) = \frac{1}{\sqrt{2}} \left(\psi_a(r_1)\, \psi_b(r_2) - \psi_b(r_1)\, \psi_a(r_2) \right)$$

$$= 0$$

Hence no probability of 2 particles having same position.
⇒ Pauli Exclusion Principle.

⊙ Do not confuse Antisymmetry & Parity. Fermions are, by definition, antisymmetric under wavefunction exchange, which means that if you have 2 fermions, represented by states $\psi_1(x)$ and $\psi_2(y)$, then $\psi_1(x)\psi_2(y) = -\psi_2(x)\psi_1(y)$
⊙ But odd Parity is something different. It applies to SINGLE particle and says that if you reverse all space coordinates then wavefunction switches sign. $\psi(x) = -\psi(-x)$

Models of Nucleus

Semi Empirical Mass Formula

To calculate/predict mass of nucleus w/o doing measurement.

$$M(Z, A) = Z M_p + (A - Z) M_n + \left(\frac{E_b}{c^2} \right)$$

do not forget the c^2

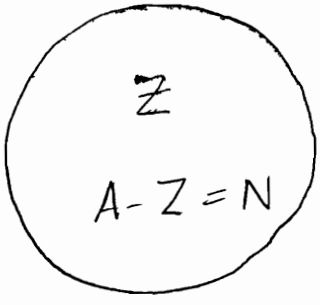
E_b is negative here
$E_b = -|E_b|$

We need to predict Binding Energy to predict nuclear mass. Hence it is also called Semi Empirical Mass Formula.
or
Semi Empirical Binding Energy Formula

Binding Energy is the energy by which nucleons are held together. Or the Energy required to break the nucleus into constituent elements.

Bohr said that nucleus is similar to liquid drop. So he gave Liquid Drop Model in 1936.
Density is independent of shape & size.

Semi Empirical Mass Formula is correction upon this model.



Volume Energy

Assume that nucleon - nucleon Bond Energy is $U \Rightarrow$ energy per nucleon is $\left(\frac{U}{2}\right)$

Consider the nucleons to be packed having 12 adjoining nucleons, the energy per nucleon $= 12 \times \frac{U}{2} = 6U$

As there are A nucleons in the nucleus
Total Binding Energy $= 6UA$
$\Rightarrow E_V = a_1 A$ (say)

E_V is called volume energy

According to this formula, E_b has 5 components

① E_V volume energy
② E_S surface Energy
③ E_C Coulomb
} borrowed from liquid drop model of Bohr

④ $E_{Ass.}$ Asymmetry Energy * Asymmetry in no. of protons and neutrons.

⑤ E_P pairing Energy * {Even-Even} Pairs / {Odd-Odd} Pairs

① Volume Energy is quantification of Strong Nuclear Force

$E_V \propto$ Volume $\quad E_V \propto -\frac{4}{3}\pi R_0^3$ (A) (attractive)

$$E_V = -a_1 A$$ attractive

Nucleons on surface are less stable as they have less neighbours. Hence can be removed more easily.

② Surface Energy $\alpha + 4\pi R_0^2$ $A^{2/3}$

$$E_S = + a_2 A^{2/3}$$ (repulsive)

It will adversely effect the stability of nucleus.

③ Coulumb Energy

On an average, we can assume 1 proton at centre, 1 proton at radius

It is repulsive and hence it will negatively effect the stability of nucleus.

$$E_C = \frac{+e^2}{4\pi\varepsilon_0 R}$$

$$\text{Self Energy} = \frac{3}{5}\cdot\frac{Q^2}{4\pi\varepsilon_0 R} = \frac{+a_3 Z^2}{A^{1/3}}$$ ($R = R_0 A^{\frac{1}{3}}$)

$$\text{No. of pairs} = {}^{Z}C_2 = \frac{Z(Z-1)}{2} \Rightarrow E_C = \frac{+e^2 Z(Z-1)}{8\pi\varepsilon_0 R}$$

$$\Rightarrow E_C \simeq \frac{+a_3 Z^2}{(A)^{1/3}}$$

$$E_C = \frac{a_3 Z(Z-1)}{A^{\frac{1}{3}}}$$

⇒ By liquid drop model,

$$B.E. = -a_1 A + a_2 A^{2/3} + a_3 \frac{Z(Z-1)}{A^{\frac{1}{3}}}$$

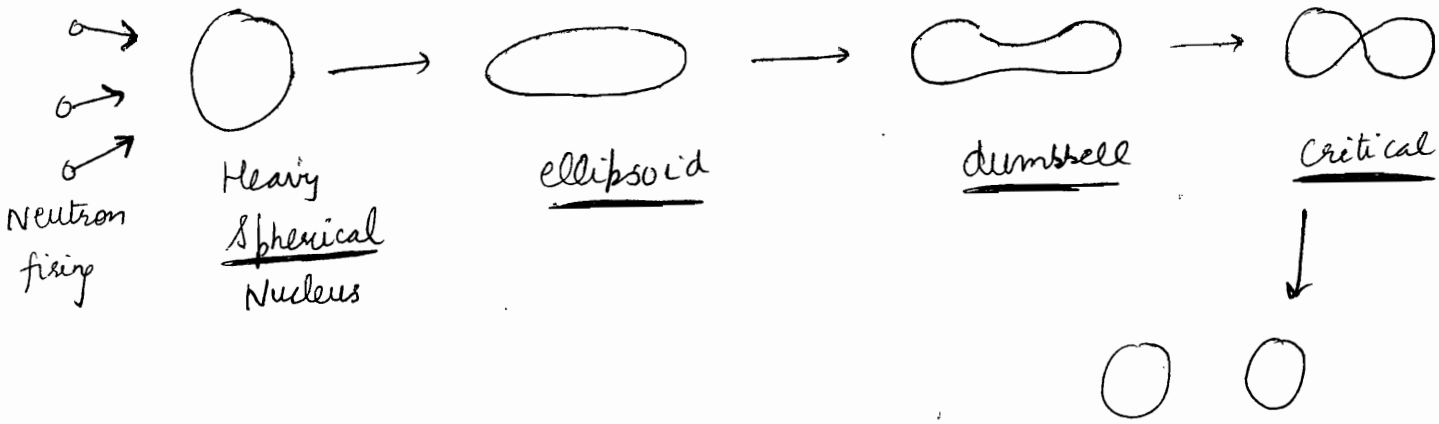
How to determine a_1, a_2, a_3 ??

We use semi-empirical techniques, similar to curve matching done in Einstein's $G_{\mu\nu}$ curve matching semi empirical approach.

Take any 3 repreesentative nuclei. Calculate a_1, a_2, a_3 for them by using their Binding Energies (that are known beforehand)

But a lot of deviations are there when we use this model on other nucleus.

Fission is successfully explained by Liquid Drop Model.



This model was not discarded by Weizsacker & Bethe. They added more terms to this model.

4) Assymmetric Energy

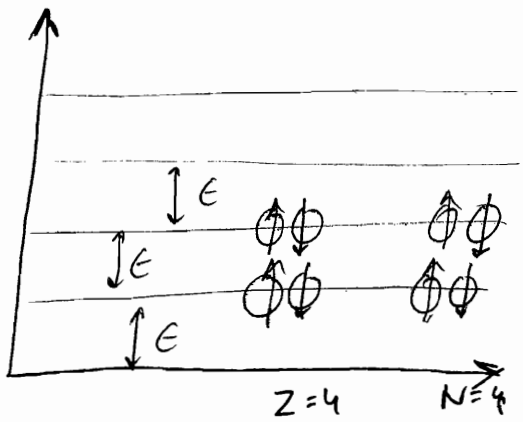
Energy corresponding to excess of neutrons / protons

Stability line upto Ca, N=Z line

Symmetry: $N = Z$

Assymmetry: $N > Z$ or $N < Z$

Assymmetry is due to excess of nucleons i.e. either $N > Z$ or $Z > N$.



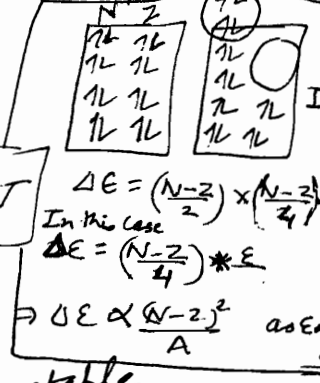
Nuclear Energy levels

$N+Z=8$

$Z=N$: stable

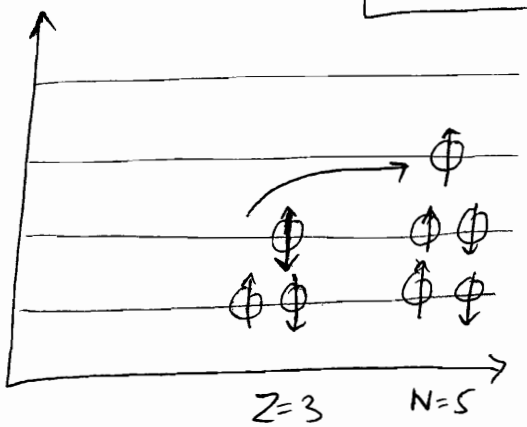
Both are fermions. Both obey Pauli's Exclusion Principle

In fully filled states, nucleus is most stable



do this derivation from BT

Note that we are considering $A = \text{const}$



$N > Z$ or $Z > N$: unstable

$N+Z=8$

$N - Z = 2$ ← excess

Note that proton can be converted into neutrons.

$$E_{assymmetry} = +\left(\frac{N-Z}{2}\right) \times \left(\frac{N-Z}{4}\right)\frac{\Delta E}{A}$$

↑ No. of shift ↑ Energy shift ↑

$$E_{Ass} = + a_4 \frac{(N-Z)^2}{A}$$

Perfect

$\epsilon \propto \frac{1}{A}$

→ note that $(N-Z)^2 \Rightarrow (Z-N)^2 \Rightarrow E_{ass}$ is similar for any excess

$$E_b = -a_1 A + a_2 A^{2/3} + a_3 \frac{Z(Z-1)}{A^{1/3}} + a_4 \frac{(A-2Z)^2}{A}$$

5) Pairing Energy

Statistical Prediction by using discrepancy in E_b and sum of $(E_V + E_S + E_C + E_{Asy.})$

For odd 'A' nucleus : $E_p = 0$

For even 'A', even-even : $E_p = -a_5 A^{-3/4}$

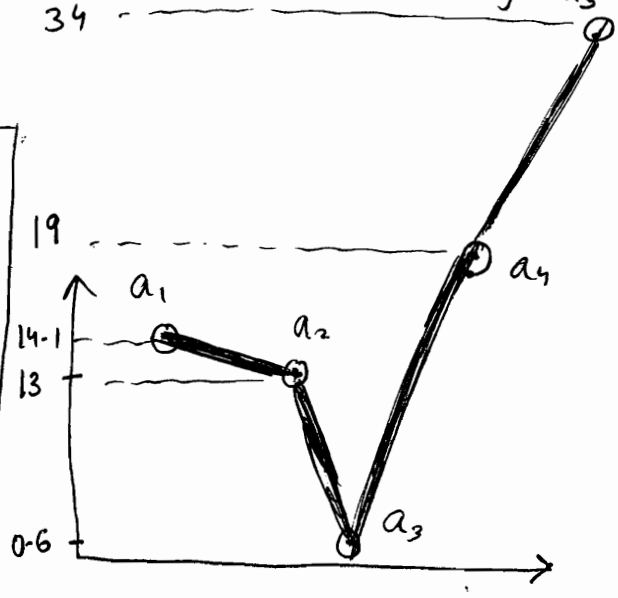
odd-odd : $E_p = +a_5 A^{-3/4}$

$$\boxed{E_p = (\pm, 0)\, a_5 A^{-3/4}}$$

a_1, a_2, a_3, a_4, a_5 are determined semi empirically by method of curve fitting.

a_1	=	14.1	MeV
a_2	=	13	MeV
a_3	=	0.6	MeV
a_4	=	19	MeV
a_5	=	34	MeV

Note this MeV



This model predicts Binding Energy and Mass of most of the nuclei.

Isobaric Series

$A = 64$

$Z = 26$ to $Z = 30$

Most stable :- Binding Energy Maximum or Mass is minimum

ISOBAR

$$\left.\frac{\partial E_B}{\partial Z}\right|_{Z=Z_0} = 0$$

$$\frac{a_3}{A^{1/3}}(2Z_0 - 1) + a_4\left(-4 + \frac{8Z_0}{A}\right) = 0$$

$A = 64$

$$\frac{0.6}{4}(2Z_0 - 1) + 19\left(-4 + \frac{8Z_0}{16}\right) = 0$$

Use "3" so that no confusion between 2 and 2

$$\Rightarrow 0.3Z_0 - \frac{0.6}{4} - 76 + \frac{38Z_0}{16} = 0$$

$$(2.4 + 0.3)Z_0 = 76.15$$

$$Z_0 = \left(\frac{76.15}{2.7}\right) = 28.2$$

✓

take $A = 27$

- ★ Note that Mass Formula calculates energy for whole nucleus and not per nucleon.

Mass Parabola

$$M(Z,A) = ZM_p + (A-Z)M_N + E_b$$

Note that we din't take c^2.... but its immaterial as (i) $m_p \simeq m_n$ (assumed) (ii) we are interested in shapes only

$$M(Z,A) = ZM_p + (A-Z)M_N + a_1 A + a_2 A^{2/3} + \frac{a_3 (Z^2 - Z)}{A^{1/3}} + a_4\left(A - 4Z + \frac{4Z^2}{A}\right) + (\mp, 0)\, a_5 A^{-3/4}$$

★ USE ENERGY PARABOLA, RATHER THAN DIRECT DIFFERENTIATION AS
(1) IT WILL FETCH MORE MARKS
(2) ΔE calculation is much simpler

Now this equation is Parabola in Z with A taken as const.

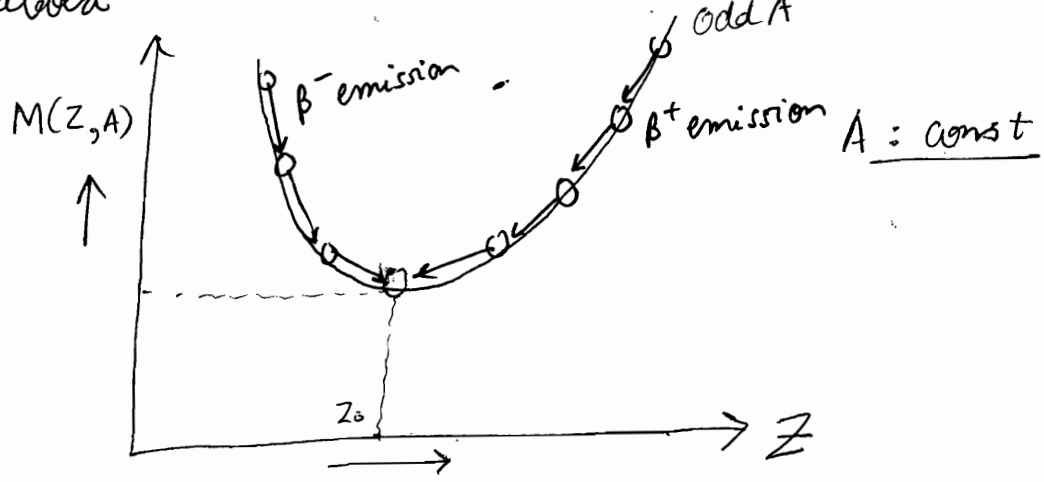
$$M(Z,A) = \left(M_N - a_1 + a_2 A^{-\frac{1}{3}} + a_4\right) A + \left(M_p - M_N - \frac{a_3}{A^{1/3}} - 4a_4\right) Z + \left(\frac{a_3}{A^{\frac{1}{3}}} + \frac{4a_4}{A}\right) Z^2 + (0, \pm) E_p$$

$$\Rightarrow \boxed{M(Z,A) = \alpha A + \beta Z + \gamma Z^2 + (\mp, 0) E_p}$$

For odd A nucleus (odd - even / even - odd)

$$M(Z,A) = \alpha A + \beta Z + \gamma Z^2$$

Mass Parabola



Note that Mass Parabola is valid for any isobaric series.
Whatever is lying on vertex is most stable isobar.

For Mass minimum $\Rightarrow \left(\frac{\partial M}{\partial z}\right)\Big|_{z=z_0} = 0$

$$= \left(\frac{a_3}{A^{1/3}} + 4a_4\right)$$

$$2\left(\frac{a_3}{A^{1/3}} + \frac{4a_4}{A}\right)$$

Minimum Mass @ $z_0 = -\left(\frac{\beta}{2\gamma}\right)$

$\therefore A \doteq 64$

$\Rightarrow z_0 = -\frac{\beta}{2\gamma} = 28.4 \approx 28$

(assume $M_p \neq M_N$)

It also predicts stability against $\beta^{\pm}$ decay. Any isobaric series try to converge at z_0 via $\beta^{\pm}$-emission

eg. 91 series

$^{91}_{36}Kr$ .... for this series : most stable element is 40.

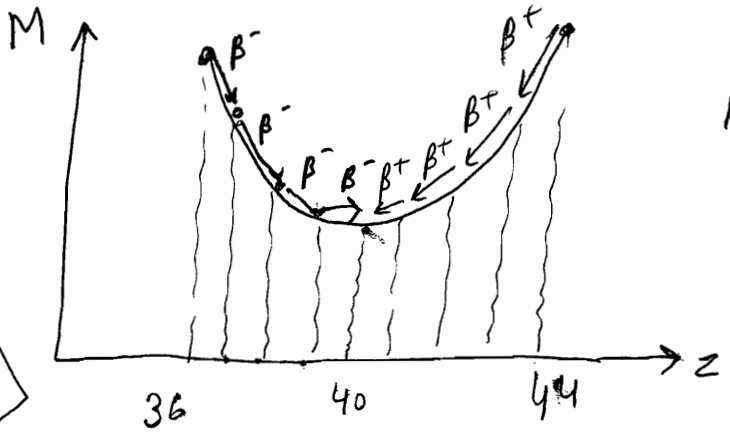
Note that in β decay, mass number does not undergo any change

For $A = 91$,

$z_0 = -\frac{\beta}{2\gamma}$

$= 39.3$

≈ 40



A = 91

This parabola also gives energy given out $[z \to z_0]$

$$\Delta E = M(z_0, A) - M(z, A)$$

$$= \beta(z_0 - z) + \gamma(z_0^2 - z^2)$$

(replace β using z_0 expression)

$$= -2\gamma z_0^2 + \gamma z_0^2 + 2\gamma z_0 z - \gamma z^2$$

$$\Delta E = -\gamma(z - z_0)^2$$

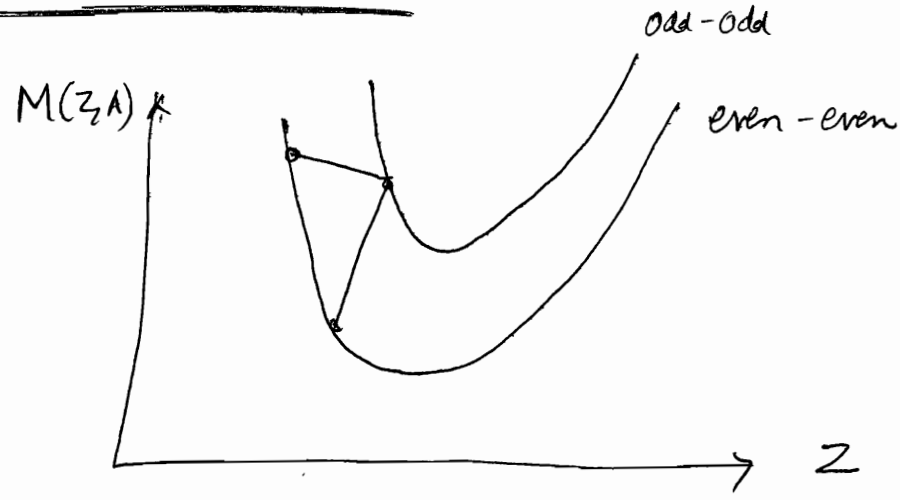
$z_0 = -\left(\frac{\beta}{2\gamma}\right)$

$\gamma = \left[\frac{a_3}{A^{1/3}} + \frac{4a_4}{A}\right]$

⊛ Mass Parabola, therefore, gives :

1. Stability against β decay
2. Most stable isobar
3. ΔE in transition

For even A nucleus, [odd-odd / even-even]



$n \rightarrow p$

$n \rightarrow n-1$ odd → even

$p \rightarrow p+1$ odd → even

(odd-odd) to (even-even)

while total A is conserved..!!

For even nuclei

[redacted]

1. B.E. or mass of nucleus
2. Most stable element
3. Size of nucleus & coulumb repulsion term can be obtained.
4. Nature of Force can be predicted.

Take 2 mirror nuclei

Size of Nucleus

$$M_A(Z,A) = ZM_p + (A-Z)M_n - a_1 A + a_2 A^{2/3} + a_3 \frac{Z(Z-1)}{A^{1/3}} + a_4 \frac{(A-2Z)^2}{A}$$

$\underbrace{\qquad}_{= \frac{(a_4)}{A}}$

For a mirror nucleus,

$N - Z \sim 1$

$A - 2Z \sim 1$ & N and Z are interchanged

⊙ They will then be odd - even ⇒ no factor of a_5

For other nuclei

$$M_B(Z,A) = (A-Z)M_P + ZM_n - a_1 A + a_2 A^{2/3} + \text{[illegible]} + a_3 \frac{(A-2)(A-2-1)}{A^{1/3}} + a_4 \underbrace{\frac{(A-2Z)^2}{A}}_{= \frac{a_4}{A}}$$

Difference of Mass of 2 nucleus

$\Rightarrow$ Difference of Binding energy of 2 nucleus:

$$\Rightarrow \boxed{E_b - E_b' = a_3 \frac{Z(Z-1)}{A^{1/3}} - a_3 \frac{(A-2)(A-2-1)}{A^{1/3}}}$$

[Hence by mirror-nuclei method, a_3 can be predicted.

Hence R_0 is predicted]

$\Rightarrow \underline{R = R_0 A^{1/3}}$: Radius of nucleus is known.

Yukawa Potential

(35)

$$\boxed{\phi(r) = -\left(\frac{g}{4\pi\varepsilon}\right) e^{-(r/r_0)}} \qquad r_0 = \left(\frac{\hbar}{mc}\right)$$

$\circledast$ $E_c = Z C_2 \cdot \frac{e^2}{4\pi\varepsilon_0 r}$

$= \frac{Z(Z-1)}{A^{1/3}} \left(\frac{e^2}{8\pi\varepsilon_0 r_0}\right)$

$\Rightarrow \boxed{a_3 = \frac{e^2}{8\pi\varepsilon_0 r_0}}$ ✓

$r = r_0 A^{1/3}$

$\Rightarrow \boxed{a_3 \propto \left(\frac{1}{r_0}\right)}$

Particle Physics (2)

⊛ **Isotopic Spin** ("I") is fictitious spin.

"I" represents family of elementary particles having all properties same except charge.

Its a vector quantity. 'Spin' in name is misnomer.

✱ { I_3 is the component of isotopic spin along z-axis. It represents member in the family. (i.e. charge state) }

I_3 varies varies $-I$ to $+I$ $(2I+1$ values$)$

$(2I+1)$: no. of observed charged states of an elementary particle.

I is defined only for Hadrons, i.e. particles taking part in Strong & EM field interactions.

For Nucleons $2I+1 = 2$ [no. of states = 0 and 1 (0 ↑ neutron, 1 ↑ proton)]

$\Rightarrow I = \left(\frac{1}{2}\right)$

$\Rightarrow$ for nucleons: $I = \left(\frac{1}{2}\right)$

$I_3 = -\frac{1}{2}$ for neutron
$I_3 = +\frac{1}{2}$ for proton

$\Leftarrow$ Note that we are treating 'n' and 'p' as 2 charge states of nucleon family !!

π Meson

$2I+1 = 3$

$\Rightarrow I = 1$

$[\pi^+, \pi^0]$

$\Rightarrow$ $I = 1$ for all Mesons

$I_3 =$	-1	for π^-
$I_3 =$	0	for π^0
$I_3 =$	$+1$	for π^+

Antiparticles have Opposite Isotopin Spin...

k Meson

$2I+1 = 2$

$I = \left(\frac{1}{2}\right)$

$I_3 =$	$\frac{-1}{2}$	for k^0
$I_3 =$	$+\frac{1}{2}$	for k^+

η Meson

$2I+1 = 1 \Rightarrow I = 0$

$I_3 = 0$ for η^0

Λ^0 Hyperons $\equiv$ Hyperon

$2I+1 =$

$\Rightarrow I = 0$

$2I+1 = 2$

$I = \left(\frac{1}{2}\right)$

$\equiv^0$:	$+\left(\frac{1}{2}\right)$
$\equiv^-$:	$-\left(\frac{1}{2}\right)$

$I_3 = 0$ for Λ^0 hyperons

$I_3 = 0$ for Ω^- hyperon

Σ Hyperons

Σ^+	:	$+1$
Σ^0	:	0
Σ^-	:	-1

→ I_3 is conserved in strong as well as EM interactions.

→ I is conserved in only strong interactions

इनमें वरा I_3 से ही बतलाव है!!

Strangeness Number

$S = 2(Q - I_3) - B$

i.e. $\boxed{Q = I_3 + \left(\frac{S+B}{2}\right)}$ ✓

Gell-Mann Nishijima Formula

This is magnitude of charge. Charge = Qe

$Q = I_3 + \frac{Y}{2}$

I_3: Isotopic Spin Component, Y: hypercharge $= S+B$

For π mesons :

Illustration of Gellmann Nishijima Formula :-

π^+ Meson

$$S = 2(1-1) - B = 2(0) - 0$$

$$= 0$$

$\Rightarrow$ π^+ Meson is non strange particle

π^- Meson

$$S = 2(-1+1) - 0 = 0$$

π^0 Meson

$$S = 2(0-0) - 0 = 0$$

$\Rightarrow$ π mesons are non strange particles

For η^0 Meson

$$S = 2(0-0) - 0 = 0$$

For neutron

$$S = 2\left(0 + \frac{1}{2}\right) - 1 = 0$$

For Proton

$$S = 2\left(1 - \frac{1}{2}\right) - 1 = 0$$

$S=0$: Non strange : $\pi^{\pm}$, π°, n, ϕ, $\bar{n}$, $\bar{p}$, η°

$S=1$: Strange : $\bar{\Sigma}^{-}$, $\bar{\Sigma}^{+}$, $\bar{\Sigma}^{\circ}$, $\bar{\lambda}^{\circ}$, k°, k^{+}

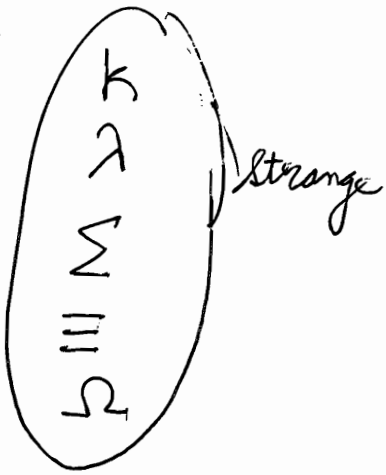
All hyperons are strange $\longrightarrow$ $S=-1$ (λ°, $\Sigma^{\pm}$, Σ°), $S=-2$ (Ξ°

K-Mesons $\longrightarrow$ $S=+1$ $S=-3$ (Ω^{-})

$S=2$: $\bar{\Xi}^{\circ}$ $\Xi^{(+)}$

$S=3$: Ω^{+}

Antiparticles have Opposite Strangeness number.



$S=-1$: λ°, Σ^{+}, Σ^{-}, Σ°,

$S=-2$: $\Xi^{(\circ)}$, $\Xi^{(-)}$ $\longrightarrow$ They are produced by Strong Interaction & decaying is via weak interaction.

$\longrightarrow$ They are produced as pair production

$S=-3$: Ω^{-}

Hypercharge

$Y = S + B$

(-3)	(-2)	Strangeness (-1)		(1)	(2)
Ω^{-} $2(-1-0)-1 = \underline{\underline{-3}}$	$\Xi^{(\circ)}$ $2(0-\frac{1}{2})-1 = \underline{\underline{-2}}$	λ° $2(0-0)-1 = \underline{\underline{-1}}$	Σ° $2(0-0)-1 = \underline{\underline{-1}}$	k° $2(0+\frac{1}{2})-0 = \underline{\underline{+1}}$	None
	$\Xi^{(-)}$ $2(-1+\frac{1}{2})-1 = \underline{\underline{-2}}$	$\Sigma^{(+)}$ $2(1-1)-1 = \underline{\underline{-1}}$		$k^{(+)}$ $2(1-\frac{1}{2})-0 = \underline{\underline{+1}}$	Note that antiparticle has $S = -S_{Particle}$
		$\Sigma^{(-)}$ $2(-1+1)-1 = \underline{\underline{-1}}$			

Tut

Leptons take part in Weak Interaction. & Cons. laws
Baryons take part in strong & EM interactions. They follow 17 Conservation laws.

Q/ leptons in EM interaction ? Of course since they have Charge & mass ⇒ EM & gravitation reaction में होंगे ही !!

(★) Mesons have 0 spin, 0 lepton number, 0 baryon number. Therefore, can be easily fit into any reaction.

(9) (ii) $\mu^- \longrightarrow e^- + \bar{\nu}_e$

linear momentum and mass energy not checked !!

Q	-1	-1	0	$\Delta Q = 0$
S	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	Not Conserved
L_e	0	1	-1	$\Delta L_e = 0$
L_μ	1	0	0	Not Conserved
L_τ	0	0	0	$\Delta L_\tau = 0$
B	0	0	0	$\Delta B = 0$

* Hence not allowed due to non conservation of S and L_μ

(ii) $n \Rightarrow p + e^-$

Q	0	1	−1
✓ S	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$
✓ Le	0	0	1

$\Delta Q = 0$

Not Conserved

$\Delta L_e \neq 0$ not conserved

Neutron cannot decay into proton & electron

(iii) $k^\circ \rightarrow \pi^\circ + \pi^\circ$

- ⊙ leptons are not involved
- ⊙ no Baryons involved

$\Delta S = 0$

S : 0	0	0
Q : 0	0	0

- ⊙ All Mesons have 0 Spin

Allowed

Strangeness need not be conserved for weak interaction

eg. decay process

Strangeness +1 0 0

Δ Strangeness $\neq 0$

$\Downarrow$

$\Leftrightarrow$ decay reaction goes by weak interaction

(iv) $e^+ + e^- \longrightarrow \gamma + \gamma$

L_e	+1	+1	0	$\Delta L_e = 0$
S	$(\frac{1}{2})$	$(\frac{1}{2})$	1	$\Delta S = 0$

(II)

(i) $n \rightarrow p + e^- + \bar{\nu}_e$

L_e	0	0	1	+1
B	1	1	0	0
S	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$

$\Delta L_e \neq 0$

$\Delta B = 0$

$\Delta S = 0$

(*) $S = \frac{3}{2}$ $S = \frac{1}{2}$ are same

✓ $\boxed{n \rightarrow p + e^- + \bar{\nu}_e}$ ⇐ allowed reaction

$p \rightarrow \pi^0 + e^+ + \nu_e$

Q	1	0	1	0
ΔL_e	0	0	+1	1
B	1	0	0	0

$\Delta Q \neq 0$

$\Delta L_e \neq 0$

$\Delta B \neq 0$

$p \rightarrow \pi^0 + e^+ + \nu_e$

$\Delta L_e = 0$		0	0	0
B	1	$(1/2)$	$(1/2)$	0
				$(1/2)$

$\Delta B \neq 0$

$n \rightarrow p + e^- + \gamma$

Photon is a boson

S	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	1
Le	0	0	1	0

✓ $\Delta S \neq 0$

✓ $\Delta Le \neq 0$

✓ $n \rightarrow p + e^- + \bar{\nu}_e$ β decay (β^-)

✓ $p \rightarrow n + e^+ + \nu_e$ β decay (β^+)

✓ $p + e^- \longrightarrow n + \nu_e$ β decay (electron capture)

$\pi^- + p \longrightarrow k^+ + k^-$ ✗

Pair Production Reaction

Q	−1	+1	+1	−1	$\Delta Q = 0$
(S)	0	$\frac{1}{2}$	0	0	Not Conserved
(B)	0	1	0	0	Not Conserved

$\pi^- + p \rightarrow \Sigma^+ + k^-$ ✓

Q	−1	+1	+1	−1	$\Delta Q = 0$
B	0	+1	+1	0	$\Delta B = 0$
S	0	$\frac{1}{2}$	$\frac{1}{2}$	0	$\Delta S = 0$

Strangeness

$$\pi^- + p \longrightarrow \Sigma^+ + K^-$$

$$0 \quad 0 \quad -1 \quad +1$$

$\Delta S = 0$

⇓

either EM or Strong

I_3 $\quad -1 \quad +\frac{1}{2} \quad 1 \quad -\frac{1}{2}$: Not Conserved

⇒

Check @ Home

---

(18) $\Sigma^0 \longrightarrow \lambda^0 + \gamma$ (lifetime $\leq 10^{-14}$ s)

⇓

EM

(Q) $\quad 0 \quad 0 \quad 0$ Conserved ✓

(S) $\quad \frac{1}{2} \quad \frac{1}{2} \quad 1$ Conserved ✓

(B) $\quad 1 \quad 1 \quad 0$ Conserved ✓

$\Delta S = 0$

⇓

Reaction proceeds via EM or Strong

I_3 $\quad 0 \quad 0 \quad 0$ Conserved

Now lifetime $\leq 10^{-14}$ s and γ photon involved

⇒ EM interaction

Strange Particle { production : conserved ; decay : not conserved } Strangeness Quantum No.

Strange Particle Production & Decay

Q-13 Tut Sheet

अरे यार यहाँ, यहाँ भी है !!

Strong Interaction Reactions

(1) $\pi^- + p \rightarrow \lambda^0 + k^0$

$\pi^+ + n \rightarrow \lambda^0 + k^+$

(2) $k^- + p \rightarrow \Omega^- + k^+ + k^0$ (antiparticle)

⇒ Strong/EM reaction

⇓

check I, Str. after checking & Conservation rules.

(3) $p + p \rightarrow p + \lambda^0 + k^+$

⊙ Prod^n by strong force ⊛
⇒ Strangeness conserved
⇒ λ^0 के साथ k^0/k^+ तो आयगा

Decay Reactions

(1) $\Omega^- \rightarrow \Sigma^0 + \pi^-$

I_3: 0 0 ; 0 −1

I 0 ; 1 1 : integral

(2) $\Sigma^+ \rightarrow p + \pi^0$

$\rightarrow n + \pi^+$

I_3: $+\frac{1}{2}$; $(\frac{1}{2})$ (1)

(3) $\Xi^- \rightarrow \lambda^0 + \pi^-$

(4) $\Xi^0 \rightarrow \lambda^0 + \pi^0$

(5) $\lambda^0 \rightarrow p + \pi^-$ ⊛

$k^0 \rightarrow \pi^+ + \pi^-$ ⊛

⇒ weak interaction

All 8 Conserved, then 'I$_3$' and 'Strangeness' not Conserved

Also Parity not Conserved

Note that Parity is multiplicative .... rest are additive

k^+/k^0 का mass π से ~~दुगना~~

∴ production or decay from π particles only

k^0 from / to π^-

k^+ from / to π^+

बाकी λ^0 की necessity (due to strangeness) के लिए nucleon

* 8 Conservation laws ⇒ conserved ⇒ allowed

Now check, I_3, Strange, : ⇒ EM / Strong (if conserved)
Parity or Weak (if not conserved)

Quark Structure of Hadrons

✓ These are very heavy particles that take part in Strong Interactions.

✓ lifetime < 10^{-26} s : Product of Strong Interactions ← Resonant Particles.

They are not written in table

✓ Internal Structure of Hadrons in terms of 6 quarks.

None of the quarks have been discovered.
Its only a model.

→ Due to phenomenon called "Colour Confinement", quarks are never observed or found in isolation, they can only be found within baryons or mesons.

Therefore, much of what is known about quarks, has been drawn from observation of hadron themselves. Quarks, by default, clump together to form groups, or hadrons. The 2 types of hadrons are Mesons (1 quark, 1 antiquark) and Baryons (3 quarks). The constituent quarks in a group cannot be separated from their parent hadron & that is why quarks can never be studied at more direct level than at hadron level

Nuclear Physics (3)

3/03/12

Magic Numbers $Z=20$: Ca, $Z=28$: Fe, $Z=50$: Tin, $Z=82$: Pb

2, 8, 20, 28, 50, 82, 126

Number of neutrons OR protons, whose nuclei are most stable.

eg. $^{4}_{2}He$: doubly magic nuclei P and N; both are of magic no. 2

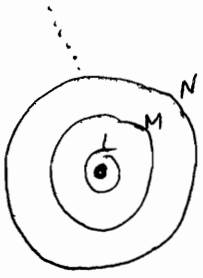
Shell Model is an attempt to explain stability of such nuclei.

$^{16}_{8}O$

20 : Calcium

28 : Iron

82 : lead



K : $n=1$ — 2e max

L : $n=2$ — 8e⁻ max

M : $n=3$ — 18e⁻ max

$2n^2$

We know, Closed shell provide stability to atom

Similarly, Nucleus with magic number of proton / neutron are most stable.

Q) What is the [redacted]??

→ 2 types of motion :

Space motion (r, θ, ϕ)

Spin (s)

Similar to H atom problem, we can solve for other atoms. We will get quantum numbers for all these parameters

$$\underset{\downarrow}{(r,} \ \underset{\downarrow}{\theta,} \ \underset{\downarrow}{\phi)} \quad \underset{\downarrow}{(s)}$$
$$n \quad l \quad m_l \quad m_s$$

n : unrestricted

l : $0, 1, 2, \ldots (n-1)$

m_l : $-l$ to l

m_s : $\pm\left(\frac{1}{2}\right)$

⇒ Total no. of states of e^- in a given n

$$= 2\sum_{l=0}^{n-1} 2l+1$$

$$= 2\left[2\,\frac{(n-1)(n)}{2} + (n) \right]$$

$$= 2\left[n^2 - n + n \right]$$

$$= 2n^2$$

→ Similarly, we can derive magic number reasoning in shell model.

Shell Model

(1) We have to consider motion of 1 nucleon due to Potential of (A-1) nucleons. We can assume it Yukawa's Potential or any other potential. [spherically symmetrical]

(2) Every nucleon will possess n, l and m_l. We are interested in l and m_l.

$$L = \sqrt{l(l+1)}\ \hbar \qquad L_z = m_l \hbar$$

Similar to e^-, even for nucleons,

l =	0	1	2	... 3	4	5	6
	↓	↓	↓	↓	↓	↓	↓
	S	P	D	... F.	G	H	I

(3) Each nucleon will have Spin & Angular Momentum.

$$|\vec{S}| = \sqrt{S(S+1)}\ \hbar \qquad S_z = m_s \hbar$$

(4) Total Angular Momentum (Nuclear Spin) will be obtained by L-S coupling or I-I coupling.

$$\vec{I} = \vec{L} + \vec{S} \qquad \vec{I} = \vec{I}_1 + \vec{I}_2$$

$$\vec{L} = \Sigma \vec{l}_i \qquad \vec{I}_i = (\vec{l}_i + \vec{s}_i)$$

$$\vec{S} = \Sigma_i$$

(5) In 1 level of I, $(2I+1)$ states

(6) Nucleons form closed sub-shells within the nucleus in a similar manner as the e^- do in the case of atoms. Hence, its named shell model.

SPUDS if PUG

DISH of PIG

Spuds of pug dish of pig

deletion of all vowels except the last gives us the order,

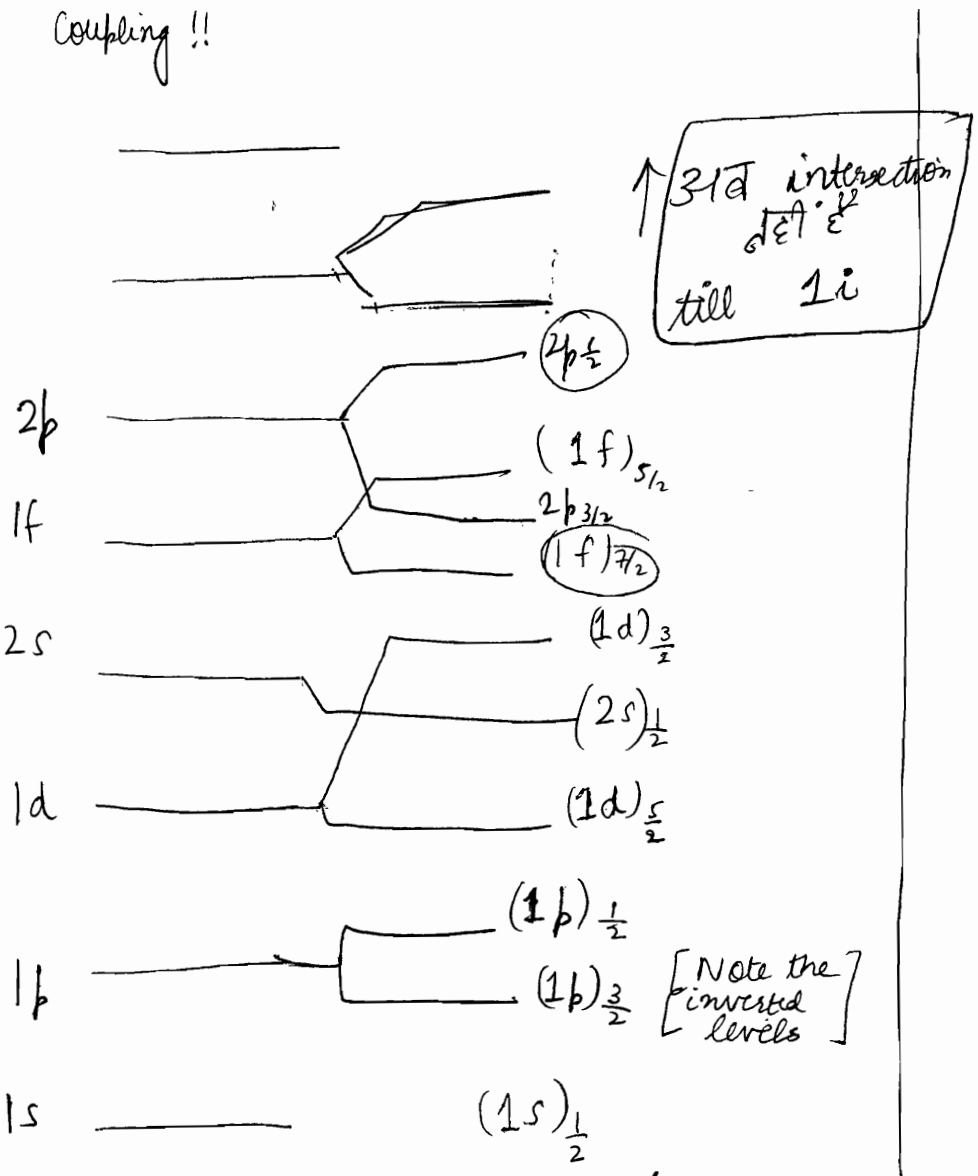
1s 1p 1d 2s 1f 2p 1g 2d 3s 1h 2f

3p 1i 2g

Representation $(nl)_I$ $\boxed{I = l+s}$

Note the inverted LS Coupling !!

Representation



No. of nucleons = (2I+1)

2
6
4
8
4
2
6
2
4
2 1/2 + 1 = 2

Magic No.

50
Energy gap is according division
20
20
8
8
2
2

write like this

No coupling → Upon LS coupling / Spin Orbit Interaction

$A = 13$

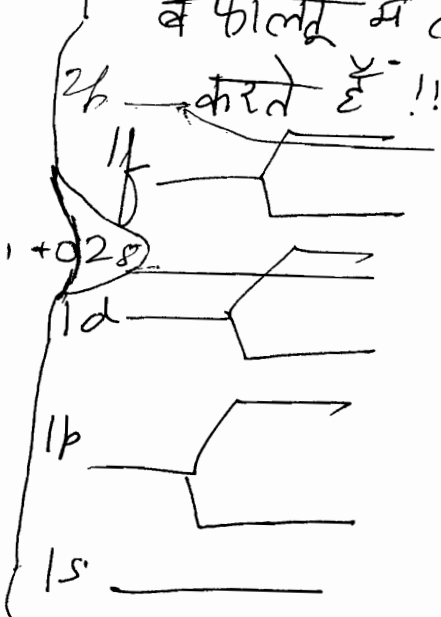
$$(1s_{\frac{1}{2}})^{2+2} \quad (1p_{\frac{3}{2}})^{4+4} \quad (1p_{\frac{1}{2}})^{1}$$

eg. $^{13}_{6}C$

$$\underbrace{(1s_{\frac{1}{2}})^{2+2} \quad (1p_{\frac{3}{2}})^{4+4}}_{\text{fully filled nucleons contribute nothing to energy}} \quad (1p_{\frac{1}{2}})^{1}$$

हमें के दल 2p तक decay करना है !!

2s and 2p के फालतू में दो करते हैं !!



$$\text{Parity} = (-1)^{l}$$ multiplication

For even nucleons, $I = 0$, Parity is even Case 1

So we will write $I = \left(\frac{1}{2}\right)^{-}$ for $^{13}_{6}C$ Odd A i.e. odd-even or even-odd

Spin I, Parity

It predicts ground states & excited states of all nuclei correctly. It predicts spin & parity of all nuclei correctly.

Parity

$$(-1)^{Zl_1}\,(-1)^{Nl_2} = (-1)^{(Zl_1 + Nl_2)}$$ for all types

For odd-odd nucleons :

For even-even : even parity Case 2 even A

even-even

$\vec{I} = \vec{I}_p + \vec{I}_N$

Cases: even A

odd-odd

Ispin

$I = I_p \pm I_N$ depending on odd/even ness of

$|I_1 + I_2 + l_1 + l_2|$

If $|I_1 + I_2 + l_1 + l_2|$ is even $\Rightarrow I = I_p - I_N$

If $|I_1 + I_2 + l_1 + l_2|$ is odd $\Rightarrow I = I_p + I_N$

Parity multiplicative

Nordheim's Strong Rule

~~2~~ more successes

Total 6 success

(3) It predicts magnetic moment of few nuclei correctly.

(4) It predicts Quadrupole Moment of few nuclei correctly.

$$\vec{\mu} = \frac{e}{2m_p} g \vec{I}$$

$$Q = \left(\frac{3}{5}\right)\left(\frac{2I-1}{2I+2}\right) R_0^2 A^{2/3} \text{ Barns}$$

$R^2 = R_0^2 A^{\frac{2}{3}}$

For nuclei far off from magic number, their $\vec{\mu}$ and Q are not predicted.

(5) It successfully explains Nuclear Isomerism.

Nuclear Isomers are long lived excited state of nuclei.

(A,Z) are same for isomers but states are different.

X X^* : isomer of X. Its lifetime is higher.

long lived excited states arise when Probability of Transition is low, i.e. when it violates selection rules of transition

eg $1^1f_{\frac{5}{2}} \rightarrow 2^1p_{(\frac{3}{2})}$ $\Delta l = \pm 1$ is violated

(6) It explains extra-stability enjoyed by magic numbers. Hence elements ~~Extra~~ near magic no. have many isobars & isotopes.

(1) 4 stable nuclei do not fall into magic numbers.

^2_1H ^6_3Li $^{10}_5B$ $^{14}_7N$ (odd-odd)

(2) Few orbitals are overlapping eg. 2s and 1d

This is not in accordance with shell model.

(3) It does not predict spin of $^{23}_{11}Na$ nucleus correctly.

$(1s)^2$ $(1p)^6$ $(1d_{\frac{5}{2}})^4$ even : 0 : Neutrons

$(1s_{\frac{1}{2}})^2$ $(1p_{\frac{3}{2}})^4$ $(1p_{\frac{1}{2}})^2$ $(1d_{\frac{5}{2}})^3$ odd $\Rightarrow (\frac{5}{2})^+$: Protons

~~Parity~~ is Predicted : $(\frac{5}{2})$

Observed : $I = \frac{3}{2}$

(4) It does not explain Magnetic Moment & Quadrupole Moment of numbers away from magic numbers.

(4#) $^{13}_{7}N$

Only due to Z

$(1s_{\frac{1}{2}})^2 \quad (1p_{\frac{3}{2}})^4 \quad (1p_{\frac{1}{2}})^1$

Parity: Odd

$I = \left(\frac{1}{2}\right)^-$

$^{17}_{8}O$

Only due to N

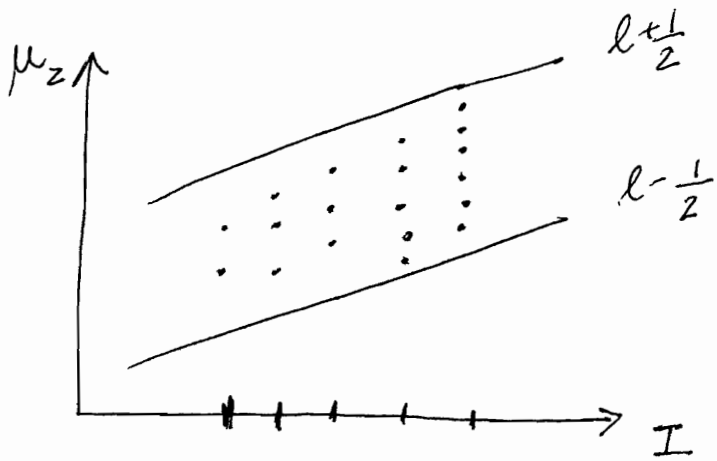
$N = 9$

$(1s_{\frac{1}{2}})^2 \quad (1p_{\frac{3}{2}})^4 \quad (1p_{\frac{1}{2}})^2 (1d_{\frac{5}{2}})^1$

$I = \left(\frac{5}{2}\right)^+$

(★) μ_z not predicted due to

1. Meson distortion in internal structure of nucleons
2. Spherically symmetrical Potential assumed for deriving shell model.



Another Points about shell model

- The neutron levels are equally spaced whereas the spacing between the proton levels increases gradually as we move from the ground level to the higher level. This is due to Coulumb force or repulsion of protons.

In adding nucleons to the nucleus, the proton and the neutron levels should be equally filled up for maximum stability. Hence for small values of mass number A, number of neutrons is very nearly equal to number of protons. As the no. of nucleons becomes large, the most stable nuclides will again be formed when lowest energy levels available to protons and neutrons are filled up. As the spacing between proton levels goes on increasing, no. of neutrons will be greater than no. of protons for stable nuclei. Hence another success for shell model.

Evidences for shell model

(1) Nucleons have tendency to form pairs. It is easier to remove a single unpaired nucleon from the nucleus than the one which is paired.

(2) Nuclei containing even no. of protons and neutrons are most abundant; nuclei with both p and n odd are least.

(3) High stability for magic numbers.

(4) Nuclides whose Z correspond to magic number have highest no. of isotopes. eg. $_{50}Sn$ has 10 isotopes.

(5) Stable end-product of natural radio active series $^{208}_{82}Pb$
$Z = 82$, $N = 126$: both magic numbers.

(6) 1st excitation energy of magic numbers of nucleon in a nuclei is very high. When the no. of protons or neutrons exceed magic number by one, Binding energy of extra nucleon is much less than avg. value.

(7) Island of isomerism for $19 < Z < 37$ due to transition from $2p$ to $1f$ subshells which is against selection rule.

Elementary Particles

Lecture 2
Appendix

½ integral spin → Fermions
integral spin → Bosons

Fermions

[all are dirac fermions i.e. have their distinct antiparticle]

[They are basic building block of all matter]

interact via strong force → Quarks
interact via weak force → Leptons

Quarks

[fundamental constituent of hadrons and interact via strong force]

up-type (⊖ve) | down-type (⊖ve)
up | down
charm | strange
top | bottom

Leptons

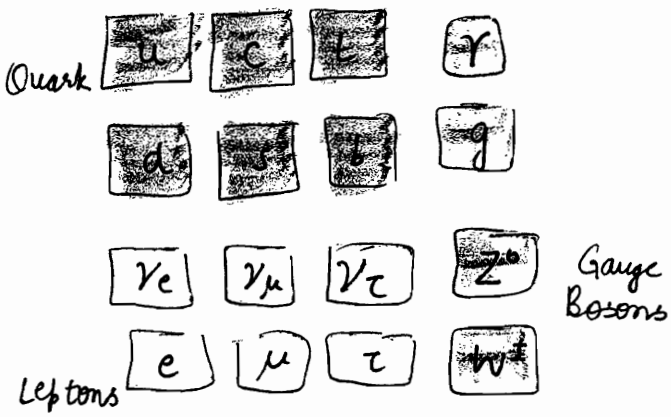
e⁻ like (⊖ve) | neutrinos (neutral)
e^- | ν_e
μ^- | ν_μ
τ^- | ν_τ

Bosons

[Fundamental forces of nature are mediated by gauge bosons.]
[Mass is hypothesized to be created by Higgs Boson]
[Graviton is added to the list, though not predicted]

Graviton | Higgs Boson | Gauge Boson

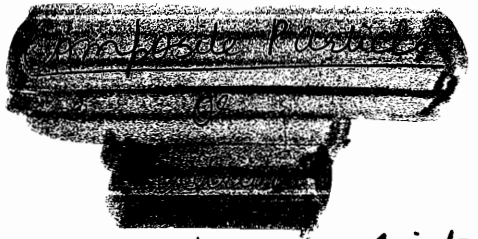
Gauge Boson → intermediate vector bosons: Photon; $W^\pm$ & Z^0 boson; Gluons



Standard Model
(elementary particles)

⊛ either be in upper CUT वरना नीचे तो BHASAD hai

⊙ all exchange particles are bosons



[defined as strongly interacting composite particles ..... made up of quarks]

fermions → Baryons (3 quark) ← odd no. of quarks
$S = \frac{1}{2}, \frac{3}{2}$

bosons → Mesons (1 quark, 1 anti quark) ← even no. of quarks
$S = 0$
[They mediate residual strong force between nucleons]

Baryons → Nucleons
- p
- N

Baryons → 1 or more strange quark → Hyperons
- Λ^0
- Σ ($\Sigma^+, \Sigma^0, \Sigma^-$)
- Ξ (Ξ^0 & Ξ^-)
- Ω (Ω^-)

Mesons
- Pion: π^+, π^0, π^-
- Eta Meson: η
- kaon: k^+, k^0

[redacted]

Smallest neutral particles into which matter can be divided by chemical reactions

[redacted]

Smallest particles into which a non-elemental substance can be divided while maintaining PHYSICAL properties of substance.

Particle [redacted] Physics (3)

05/03/2012

Conservation laws (contd...)

→ Process involving photon proceeds by EM interaction. Also Π_e is not conserved.

[redacted]

Replace particle by antiparticle. If reaction proceeds ⇒ charge conjugation is conserved.

[redacted]

$$A + B \longrightarrow C + D$$

if $P(A) * P(B) = P(C) * P(D)$ ⇒ Parity conserved

Also, $P = (-1)^l$

[redacted]

Replace t by $-t$.

looking at movie backwards & we are not able to differentiate b/w $f(t)$ and $f(-t)$

⇒ Time Reversal conservation

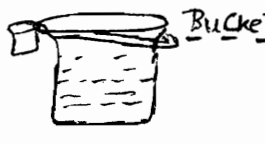
→ CPT is supposed to be conserved in all nuclear reactions.

Gellmann Nishijima Relationship (modified)

$$Q = \left[I_z + \frac{1}{2}(S + B + C + t + b) \right]$$

$q = Qe^-$

Charm, top, bottom



Quark Model of Hadrons

In 1964, Gellmann and Neemann proposed that hadrons are massive particle who can further be written as combination of 'aces', later on called quarks.

Internal structure of all hadrons (strongly interacting particles) can be expressed in terms of 3 quarks (u, d, s) with following properties:

Quark	Charge	Spin	Baryon No	I_3	S	C	t	b
[redacted]	$+2/3\,e$	$(\frac{1}{2})$	$(\frac{1}{3})$	$+(\frac{1}{2})$	0	0	0	0
[redacted]	$-1/3\,e$	$(\frac{1}{2})$	$(\frac{1}{3})$	$-(\frac{1}{2})$	0	0	0	0
[redacted]	$-1/3\,e$	$(\frac{1}{2})$	$(\frac{1}{3})$	0	s	0	0	0
Charm (c)	$+\frac{2}{3}e$	$(\frac{1}{2})$	$(\frac{1}{3})$	0	0	c	0	0

Quarks are I st generation

II nd generation

For Anti quarks, charge, B are opposite in sign.

III rd generation

top (t)	$+\frac{2}{3}e$	$(\frac{1}{2})$	$(\frac{1}{3})^-$	0	0	0	t	0
bottom (b)	$-\frac{1}{3}e$	$(\frac{1}{2})$	$(\frac{1}{3})$	0	0	0	0	b

The values of s, c, t, b are calculated from Modified Gellman Nissigema Formula.

For Resonance Particle, "ee Upsilon Meson" was discovered. To describe it, t & b was introduced.

$$\boxed{Q = I_3 + \frac{1}{2}(S + B + C + t + b)}$$

For Strange Quarke

$$-\frac{1}{3} = 0 + \frac{1}{2}\left(-1 + \frac{1}{3} + C + 0 + 0\right)$$

$$\Rightarrow -\frac{1}{3} = -\frac{1}{2} + \frac{1}{6} + C$$

$$\Rightarrow -\frac{1}{3} = \frac{-1}{3} + C \quad \Rightarrow \boxed{C = 0}$$

For up Quark

$$\frac{2}{3} = I_2 + \frac{1}{2}\left(0 + \frac{1}{3} + 0 + 0 + 0\right)$$

$$I_2 = \frac{2}{3} - \frac{1}{6} = \frac{3}{6} = \left(\frac{1}{2}\right)$$

$$\boxed{I_2 = \frac{1}{2}}$$

For Strange Quark

$$-\frac{1}{3} = 0 + \frac{1}{2}\left(S + \frac{1}{3}\right)$$

$$\Rightarrow -\frac{2}{3} = S + \frac{1}{3} \quad \Rightarrow \boxed{S = -1}$$

For charm quark

$$\frac{2}{3} = 0 + \frac{1}{2}\left(0 + \frac{1}{3} + C\right) \Rightarrow \boxed{C = 1}$$

Internal Structure of Mesons

For them, Spin = 0 ✓

These are the most important property of Mesons

Also, B = 0 ✓

3 quarks have never make 0 spin.

✓ [Internal structure of Meson is written in terms of 1 quark & 1 antiquark]

⭐ Note that almost all the normal particles are expressed in u, d, s or their antiparticles $\bar{u}, \bar{d}, \bar{s}$

π Mesons

π^+ : { Spin 0, B 0, Charge +1, Strangeness 0 } ⇒ No strange quark.

$I_3 = +1$

unique defⁿ of π^+ meson

$u\bar{d}$

Only possible

Q B S ✓ definition of particle

I_3: Opposite sign in case of antiparticle

π^- : $\bar{u}d$ { S = 0, B = 0, Q = −1, S = 0 }

π^0 ⇒ both { S = 0, B = 0, Q = 0, S = 0 } $u\bar{u}$ or $d\bar{d}$

η^0

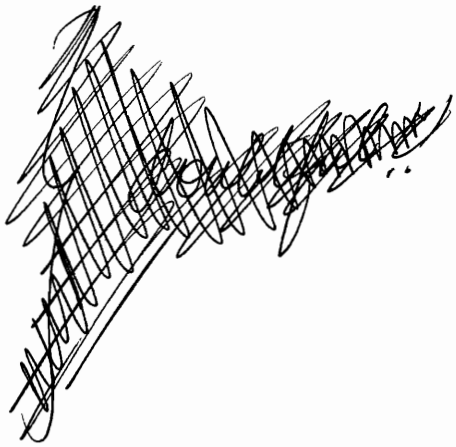
$$\psi = \frac{1}{\sqrt{2}} [u\bar{u} \pm d\bar{d}]$$

Eta Meson

Pi Mesons have odd parity

It happens when anti symmetric wave function.

$$\Rightarrow \quad \pi^0 : \frac{1}{\sqrt{2}} (u\bar{u} - d\bar{d})$$



Hence

$$\eta^0 : \frac{1}{\sqrt{6}} (u\bar{u} + d\bar{d} - 2s\bar{s})$$

k Meson

$$k^+ : \begin{cases} S = 0 \\ B = 0 \\ s = +1 \\ Q = +1 \end{cases} \qquad k^0 : \begin{cases} S = 0 \\ B = 0 \\ s = +1 \\ Q = 0 \end{cases}$$

$$\boxed{k^+ : u\bar{s}}$$

$$\boxed{k^- : \bar{u}s}$$

$$\boxed{k^0 : d\bar{s}}$$

$$\eta^0 : \left[\frac{u\bar{u} + d\bar{d} - 2s\bar{s}}{\sqrt{6}} \right]$$

$$\pi^0 : \frac{u\bar{u} - d\bar{d}}{\sqrt{2}}$$

Internal structure of Baryons

Nucleons

Neutron n $\begin{cases} Q = 0 \\ B = +1 \\ S = 0 \\ Spin = \frac{1}{2} \end{cases}$ udd

Anti neutron $= \bar{u}\bar{d}\bar{d}$

Problem with λ^0 and Σ^0 $\begin{cases} Q = 0 \\ B = +1 \\ S = -1 \\ Spin = \frac{1}{2} \end{cases}$ $\psi_1 = uds$ $\psi_2 =$

→ We will look for intrinsic parity

For Odd $\Rightarrow \frac{1}{\sqrt{2}}(\psi_1 - \psi_2)$

Even $\Rightarrow \frac{1}{2}(\psi_1 + \psi_2)$

Proton p uud

Antiproton $= \bar{u}\bar{u}\bar{d}$

Omega (Ω^-) : $\begin{cases} Q = -1 \\ B = +1 \\ S = -3 \\ Spin = 3/2 \end{cases}$ sss ↑↑↑

Problem as Pauli Exclusion Principle is violated....

Remember this example !!!

Xi $\equiv^-$ $\begin{cases} Q = -1 \\ B = +1 \\ S = -2 \\ Spin = \frac{1}{2} \end{cases}$ dss ↑↓

$\Xi^+ \quad \bar{d}\bar{s}\bar{s}$

$\Xi^0 \quad uss$

To resolve this discrepancy, we have introduced concept of coloured quarks.

Equal combination of red, blue & green. ⇒ all observed particles are colourless.

Every Quark comes in 3 colours: red, blue & green.

⇒ (sss) are all different colours.

$s_R s_B s_G : \Omega$

All colours have anti colours too.

Strong Interactions arises due to colour. Colour plays same role in strong interaction what charge plays in EM interactions. (same colour → repulsion)

Strong Interaction b/w coloured quarks occur via exchange of gluons. Its called Quantum Chromodynamics.

Red - Red → good interaction
Blue - Blue → good interaction
Green - Green → good interaction

12 gluons have been proposed.

Reaction which obey conservation laws but are not observed. To explain that in 1971, Mainani introduced Charm Quark

To explain (short lived resonance particles called Upsilon Mesons) in 1979, top & bottom quarks were introduced.

$^{23}_{11}Na$ — n: udd; p: uud

Z = 11 — 22 u and 11 d

N = 12 — 12 u and 24 d

---

36 u & 35 d ✓

Idea about Unification

4 fundamental interactions.

We have unified 2 interactions

Combined Electro weak Interaction..... interaction b/w leptons & charged particles.

Weinberg & Salaam Model.....

Force arises due to exchange of particles.

Snatched : attraction

thrown : repulsion

Electroweak Interaction arises due to exchange of 4 bosons: $W^{\pm}$, Z^{0}, photon

via phenomenon called Spontaneous Symmetry Breaking (SSB)

3 of the bosons acquire huge mass and 4th becomes massless in this phenomenon. Massless part is photon, travels with $v = c$, and its responsible for ∞ range of EM interaction and other particles are responsible for Weak interactions. ⇒ Range become extremely small as mass is $\left(\frac{80000\ MeV}{c^2}\right)$ $(W^{\pm})$

$\left(\frac{90,000\ MeV}{c^2}\right)$ Z^0 ← intermediate vector bosons

These exchange particles are quanta of Electroweak Interactions.

⊛ "Strong force is spin dependent" is also known from the fact that ortho and para Hydrogen (H_2), (2 spin states combination of 2 protons) have different cross section for neutron scattering experiment.

Ortho & Para H_2 :



2 proton spins aligned parallel / antiparallel.

ortho para

at room temperature : 3 : 1

Nuclear Physics (4)

- Deuteron
- Fusion Cycles

Quadrupole Moment

$$Q = \left(\frac{2I-1}{2I+2}\right) R^2$$

where $R^2 = \frac{3}{5} R_0^2 A^{2/3}$

→ For a shell model, those nuclei with closed shells should be spherically symmetric and $Q=0$

Examples of Shell Model Application

⊙ 10 isotopes of $_{50}Sn$ ⇒ abundance at $Z=50$

⇒ stability around magic numbers.

2, 8, 20, 28, 50, 82, 126, ...

⊙ $_{82}Pb$: 7 nuclei at $Z=82$.

⊙ ^4_2He : very stable

✔ $19 < Z < 37$: island of isomerism.

large no. of nuclear isomers. It amounts to transition : $f_{\frac{1}{2}}$ to $p_{\frac{1}{2}}$

Nature of Strong Nuclear Force

⊙ We have done all characteristics, Yukawa's Meson Theory along with feynmann diagrams, exchange theory.

⊙ We have to do deuteron problem:

- Ground state of deutron
- Magnetic Moment

Deuteron

★ Deutron Problem in 2 steps
Ⓐ Theoretical Modelling
Ⓑ Quantum solution

Deuterium $^{2}_{1}H$ 1 p and 1 n and 1 e

Remove e^-. We get only nucleus. Nucleus of Deuterium is called Deuteron.

There is no other simple & nucleus than deuteron to study nuclear strong force.

There are 7 important characteristics of deuteron.

(1) It is loosely bound [Low Binding Energy] structure of 1N and 1P typically observed in ground states.

(2) Precisely known value of B.E. = 2.225 MeV

(3) Size 'b' fm precisely known b = 4.2 fm

(4) Nuclear Spin I = 1

(5) Magnetic Moment $(\mu)_z = 0.8574\ \mu_N$

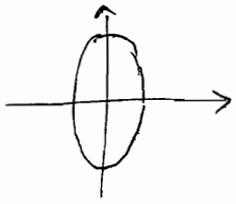
(6) Quadrupole Moment $Q = +0.00282 \times 10^{-28}\ m^2$

(Prolate spheroid)

(7) Parity of deuteron is even

For comparison, B.E. of α particle is 28.3 MeV

empirically known. Can be seen from nordheim strong rule



✓ Size of deuteron is bigger while range of force is small ⇒ loosely bound structure.

Ground state of deuteron is of triplet state

$S = 1;\ n = 2S+1 = 3$

$^{3}S_{1}$

→ If $Q \neq 0 \Rightarrow$ cannot be spherically symmetric

Hence some distortion has occured with spherically symmetric $^3S_1$ state.

$\boxed{\vec{I} = \vec{L} + \vec{S}}$ starting from here

$\vec{L} = \sum_i \vec{l_i} = \vec{l_1} + \vec{l_2}$

$\vec{S} = \sum_i \vec{s_i} = \vec{s_1} + \vec{s_2}$

✓ We know $I = 1$ (1)

$|s_1 - s_2| < S < |s_1 + s_2| \Rightarrow \boxed{0 < S < 1}$

✓ Not observed in excited state ..... most probably $l = 0$ (2)

$\vec{L} = \vec{I} - \vec{S}$

$\Rightarrow |I - S| < l < |I + S|$

If $S = 1$ ↑↑

$\Rightarrow 0 < l < 2$ $\quad l = 0, 1, 2$

(1) and (2) ⇓ (3)

If $S = 0$, ↑↓ $L = I = 1 \Rightarrow$ excited state always

Hence $S \neq 0$ (3)

From above analysis, we can see that the force keeping the 2 nucleons together is spin dependent.

✓ Hence along with $I = 1$, only S acceptable is $S = 1$ (3)

1st conclusion

Quantum Mechanics Principle implies If given potential function is symmetric, this ψ is either of odd/even parity but not together.

ψ_1: $l = 0, 2$ (ground) even parity

ψ_2: $l = 1$ (excited) Odd parity

Even and odd values of l cannot exist in the same wavefunction ψ in order to conserve parity

$\Rightarrow \psi$ valid $\Rightarrow$ $l = 0, 2$. 2nd conclusion

$$\psi = C_1 \psi(1s) + C_2 \psi(1d)$$

(low Prob in excited state)

due to low value of C_2, we have little departure from sphericity.

* बस यही logic है !!

ψ is combination of odd-odd or even-even eigenfunctions. and For ground state we know that $l=0$ is part of $\psi \Rightarrow \psi = \psi(l=0) + \psi(l=2)$

$$(\mu_p)_2 = +2.793\ \mu_N$$
$$(\mu_n)_2 = -1.91\ \mu_N$$

$\mu_{deuteron}$ in grand state

$$(\mu_d)_2 = +0.\underline{88}3\ \mu_N$$

Observed value has discrepancy:

$$(\mu_d)_{2\,observed} = 0.\underline{85}74\ \mu_N$$

✓ if $l=0 \Rightarrow$ Contribution of only spins in moment $\neq$ observed value $\Rightarrow l=2$ will also be there

✓ Hence departure from $Q=0$ & $(\mu_d)_2$ discrepancy gives ✓ excited state of $C_2 \psi(1d)$. (proof of) 3rd conclusion

$$\psi = C_1 \psi(1s) + C_2 \psi(1d)$$

$$(C_1)^2 + |C_2|^2 = 1$$

$$\boxed{\begin{aligned} |C_1|^2 &\simeq 0.96 \\ |C_2|^2 &\simeq 0.04 \end{aligned}}$$

4th conclusion

It reconciles μ_2 and Q.

✓ Force between neutron and proton is spin dependent. 5th conclusion

✓ Also, it transits from 1s to 1d state ⇒ angular momentum is not a const. of motion 6th conclusion

⇒ not a central force. Also $Q \neq 0$ ⇒ Not symmetrical force.

Hence Nuclear Force is not a central force, i.e. it is a Tensor Force.

Neutron & Proton create a strong potential field, V_0.

Quantum Mechanical / Wave Mechanical Model

Ground State of Deuteron

Ground state: $l=0$ ⇒ spherically symmetric ⇒ $\psi = f(r)$ only

$$H\psi = E\psi$$

$$H = \frac{p^2}{2m} + V$$

$$= -\frac{\hbar^2}{2m}\nabla^2 + V$$

$$\nabla^2\psi + \frac{2m}{\hbar^2}(E-V)\psi = 0$$

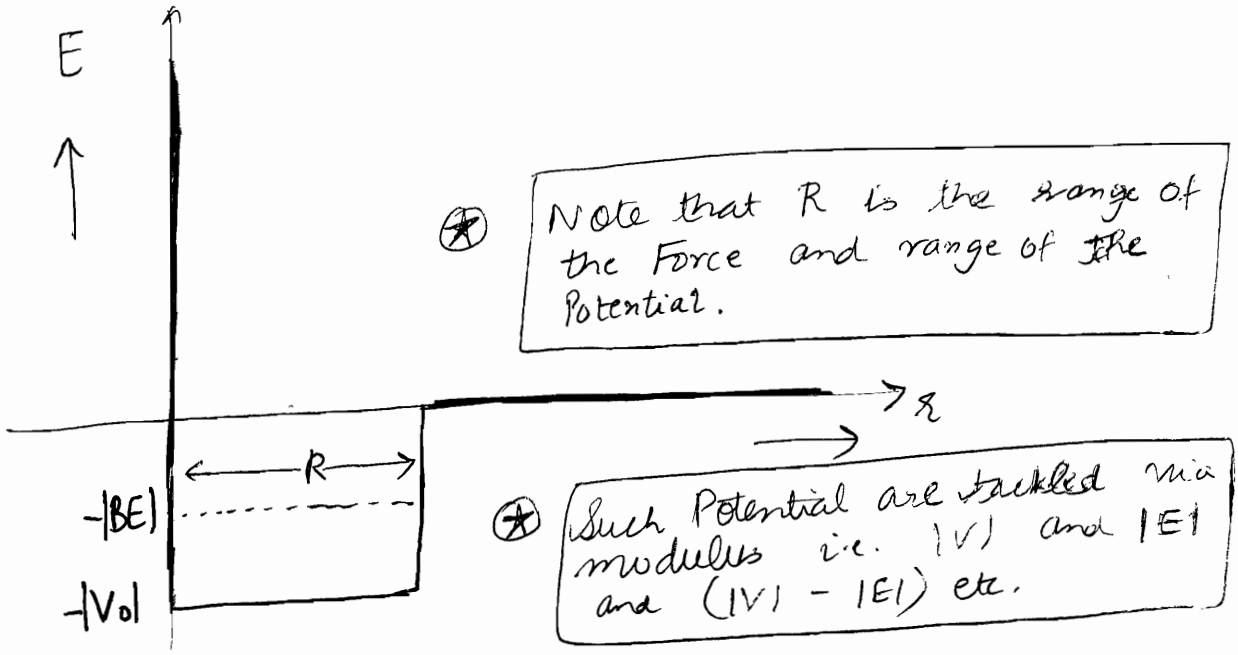
$$\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial\psi}{\partial r}\right) + \frac{2m}{\hbar^2}(E-V)\psi = 0 \quad \left(\text{Spherically Symmetric}\right)$$

Put $\psi(r) = \frac{u(r)}{r}$

$$\Rightarrow \boxed{\frac{d^2u}{dr^2} + \frac{2m}{\hbar^2}(E-V)\,u = 0}$$

$$\psi(r) = \frac{u(r)}{r}$$

$$\frac{d^2u}{dr^2} + \frac{2m}{\hbar^2}(E-V)u = 0$$



* Note that R is the range of the Force and range of the Potential.

* Such Potential are tackled via modulus i.e. |V| and |E| and (|V| - |E|) etc.

No one knows what is Potential well of strong Force is like. We can assume it to be Harmonic Potential or Yukawa Potential. We assume to be simplest well.

From deuteron model, we know Fattractive $\simeq$ 35 MeV.

- R is range of force. It is not equal to radius of deuteron. Force may not operate within whole radius.

B.E. < 0

$$V_0 < E < 0$$

At 1st approximation, we can take $R = l_b$ (radius of nucleus)

For $r < R$

$$\frac{d^2u_1}{dr^2} + \underbrace{\frac{2m}{\hbar^2}(-V_0 + E)}_{k_1^2} u_1 = 0$$

Note that Potential uses Cartesian Coordinates
⇒ Even/Odd Parity but $\psi(r)$ not Symmetric

$$\frac{d^2u_1}{dr^2} + k_1^2 u_1 = 0$$

$$u_1(r) = A \sin(k_1 r) + B' \cos(k_1 r)$$

At $r = 0$, ~~ψ is finite~~ $u(r) = r\psi(r) = 0$

$$\Rightarrow \boxed{u_1(r) = A \sin(k_1 r)} \quad \text{and } B' = 0$$

$$k_1 = \sqrt{\frac{2m}{\hbar^2}(|V_0| - |E|)}$$

For $r > R$

$$\frac{d^2u_2}{dr^2} + \underbrace{\frac{2m}{\hbar^2}(-E)}_{k_2^2} u_1 = 0$$

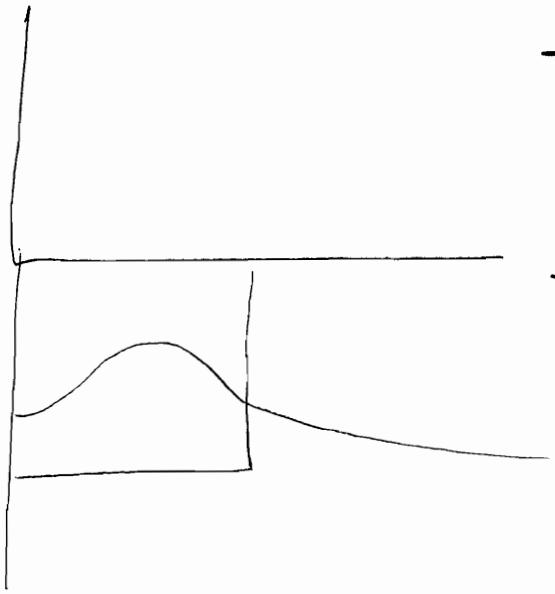
$$\frac{d^2u_2}{dr^2} - k_2^2 u_2 = 0$$

$$\Rightarrow u_2(r) = C e^{k_2 r} + D e^{-k_2 r}$$

At $r = \infty$, ψ is finite

$$\Rightarrow \boxed{u_2(r) = D e^{-k_2 r}} \quad \text{and } C = 0$$

$$k_2 = \sqrt{\frac{2m}{\hbar^2}|E|}$$



→ अरे भाई, जहाँ तक Potential है, वहाँ तक range of force है !! (atleast existence of 1 solution से R_{min} निकालना)

→ जहाँ तक Probability of occurence sufficient है (penetration depth), वहाँ तक radius है !!

(atleast radius $= \frac{1}{k_2}$)

at $r = R$, $u_1(r) = u_2(r)$, $\left.\frac{du_1}{dr}\right|_{r=R} = \left.\frac{du_2}{dr}\right|_{r=R}$

$\Rightarrow A \sin k_1 R = D e^{-k_2 R}$

$A k_1 \cos k_1 R = D(-k_2) e^{-k_2 R}$

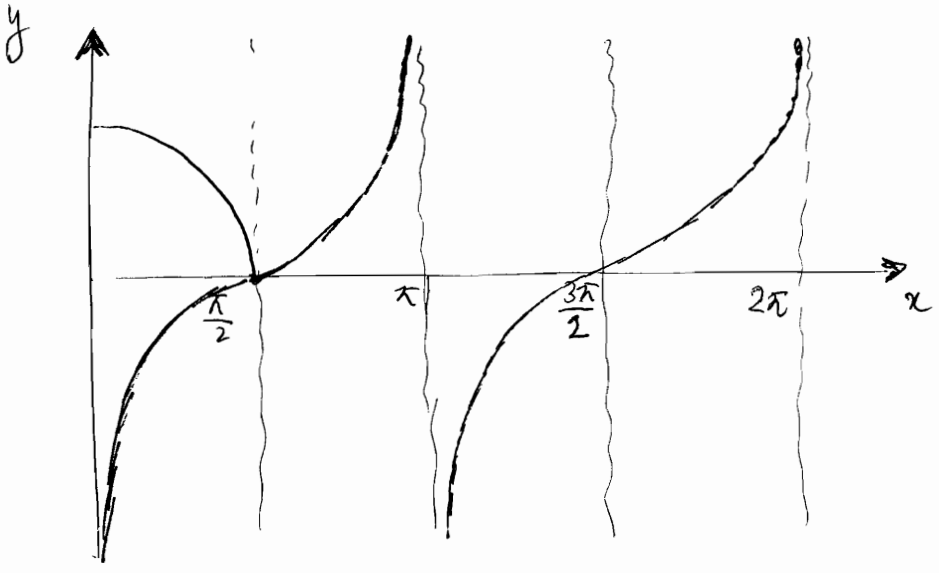
$\Rightarrow k_1 R \cot(k_1 R) = -k_2 R$

$k_1 R = x$

$k_2 R = y$

$\Rightarrow \boxed{x \cot x = -y}$ $\Rightarrow \boxed{y = x \tan\left(\frac{\pi}{2} + x\right)}$

Also $\boxed{x^2 + y^2 = \frac{2m}{\hbar^2} |V_0| R^2}$



I am interested in +y values only.

At least for 1 bound state, $\sqrt{k^2+\gamma^2} \geq \left(\frac{\pi}{2}\right)$

$$\frac{2m}{\hbar^2}|V_0|R^2 \geq \frac{\pi^2}{4}$$

$$\Rightarrow |V_0|R^2 \geq \left(\frac{\hbar^2\pi^2}{8m}\right)$$

$$\boxed{(V_0 R^2)_{min} = \left(\frac{\hbar^2\pi^2}{8m}\right)}$$ ✓

Assumptions :-

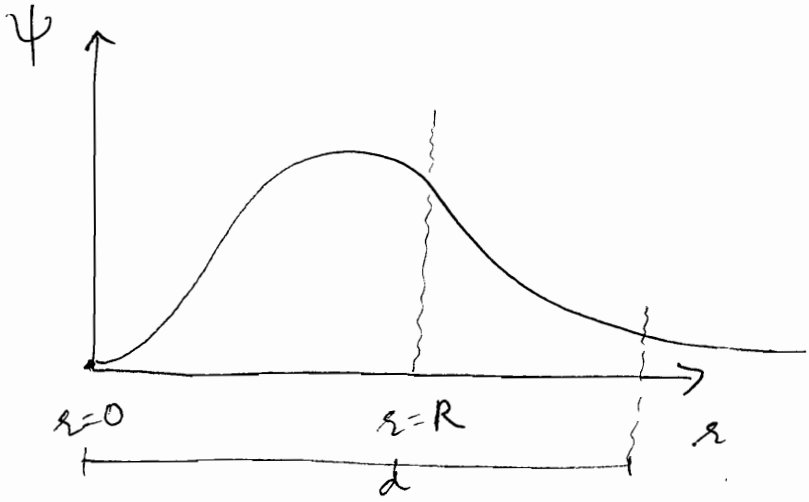
① 1st assumption, $R = b$

ie range of force is acting

(We assumed Probability of finding the particle drops after $r = R$)

② $m = \left(\frac{m_p m_n}{m_p + m_n}\right) \simeq \left(\frac{m_n}{2}\right)$

2 body to 1 body Problem



We can assume Penetration depth till value of ψ drops to $\left(\frac{1}{e}\right)$ of its maximum value in II nd region

$$De^{-k_2 d} = De^{-1} \Rightarrow \boxed{d = \frac{1}{k_2}}$$

Radius of deuteron $= \left(\frac{1}{k_2}\right)$ = Penetration depth of ψ

$$R_d = \left(\frac{1}{k_2}\right) = \frac{\hbar}{\sqrt{2m|B|}}$$

radius of nucleus — till Penetration depth

i.e. till significant Prob. of finding partic

$$R_{min} = \frac{\pi\hbar}{\sqrt{8m|V_0|}}$$

minimum range of force

$$\frac{R_d}{R} = \frac{\sqrt{8m|V_0|}}{\pi\sqrt{2m|B|}} = \frac{2}{\pi}\sqrt{\frac{|V_0|}{|B|}}$$

$V_0 \simeq 30$ MeV (estimate)

$$\frac{R_d}{R} = \frac{2}{\pi}\sqrt{\frac{V_0}{B}}$$

$$\frac{R_d}{R} = \frac{2}{\pi}\sqrt{\frac{30}{2.225}} = 2.338$$

$$\Rightarrow \frac{R_d}{R} = 2.34$$

$$\Rightarrow R = \left(\frac{R_d}{2}\right)$$ ✓ we know R_d from experiments

Hence, range of strong force is half of radius

$\Rightarrow$ loosely bound structure.

RADIUS: $R_d \simeq 4.2$ fm $\Rightarrow$ RANGE OF FORCE: $R \simeq 2.1$ fm

★ another explanation of loosely bound structure :-
B.E of deutron is small Its value is 2.225 MeV while the energy reqd. to eject a nucleon from nucleus of medium mass number is about 8 MeV.

✓ Between 2 proton, Coulumb energy ≃ 0.1 MeV

Explanation of fusion on basis of Binding Energy, we know

$kT \simeq 0.1$ MeV

$T \simeq 10^9$ k

NUCLEAR FUSION

→ This temperature is not available on Earth.

Actually its a Potential Barrier. Quantum Tunneling may hold key to fusion w/o reaching such a high Temp.

But these are ambient temperatures in the cores of stars. 'Thermo Nuclear Fusion' is therefore other name for nuclear fusion reactions.

* KE to overcome Potential of $\frac{e^2}{4\pi\varepsilon_0 r}$
≈ 0.5 MeV
$= \frac{3}{2} kT$
$\Rightarrow T \approx 10^9$ k
but Tunneling allows fusion @ $T = 10^7$ k i.e. SUN temp.

These Nuclear Fusion reaction release large amount of energy (remember the B.E. curve)

2 cycles of nuclear fusion in course of stars.

Proton - Proton Cycle (early phase of stars)

Mass < 1.5 (Mass)$_{sun}$

$$2\times \quad {}_1^1H + {}_1^1H \longrightarrow {}_1^2H + {}_1^0e^+ + \nu_e + Q_1$$

$$2\times \quad {}_1^2H + {}_1^1H \longrightarrow {}_2^3He + Q_2$$

$$1\times \quad {}_2^3He + {}_2^3He \longrightarrow {}_2^4He + 2\,{}_1^1H + Q_3$$

$$4\,({}_1^1H) \longrightarrow ({}_2^4He) + 2({}_1^0e^+) + 2(\nu_e) + 24.7\ \text{MeV}$$

Carbon nitrogen (oxygen) cycle — (mature star)

CNO cycle

Mass > 1.5 Mass(sun)

Occurs in later stages in life of a star, or in stars with huge mass

White Dwarf : huge mass has been compacted into small volume

⇒ density ↑↑

& T ↑↑

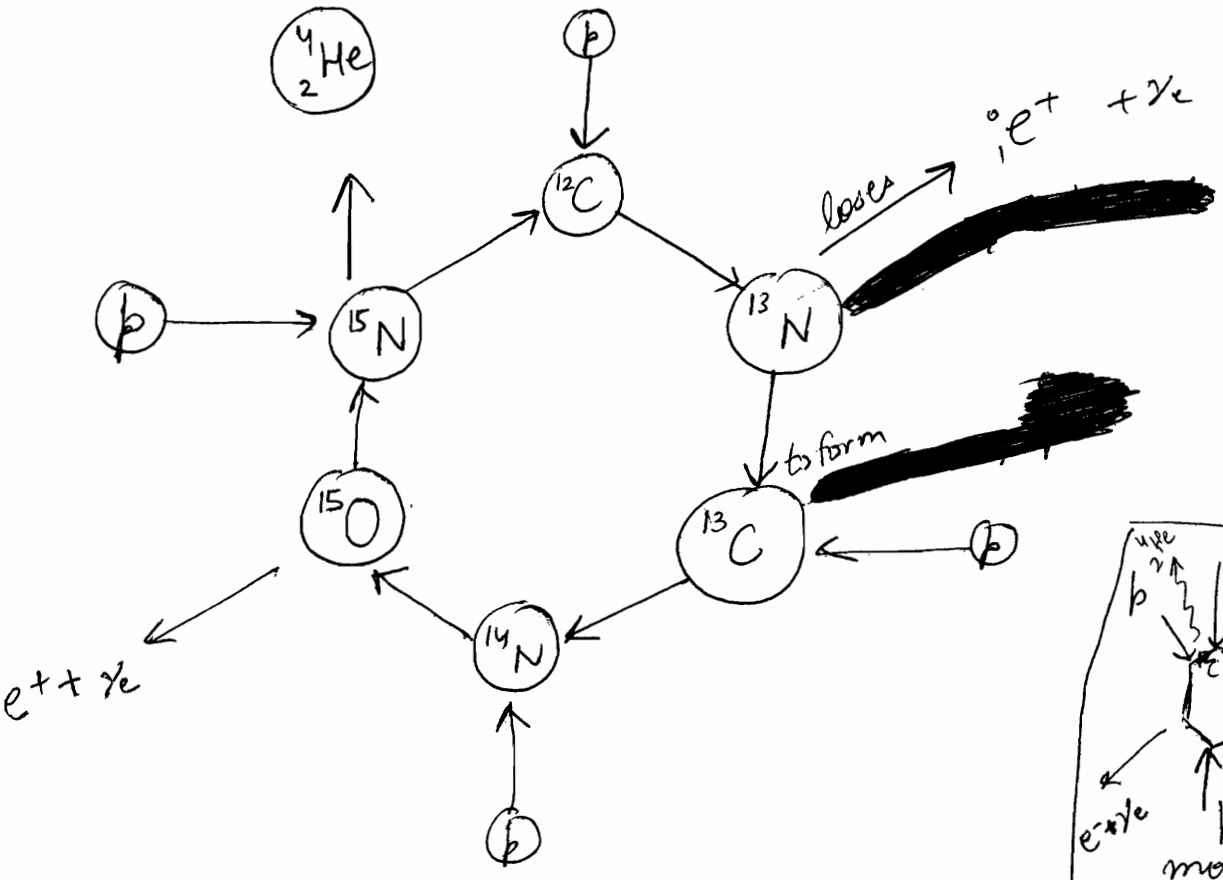
Occurs in white dwarfs, hotter than sun

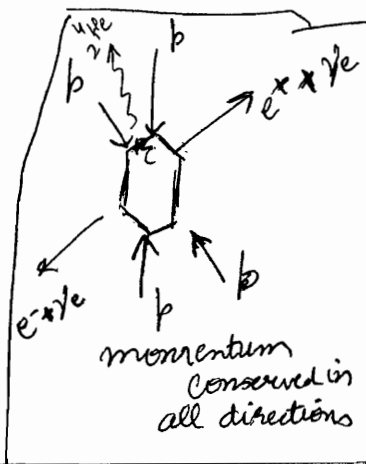
Source of their energy is Carbon - Nitrogen Cycle.

They (C and N) act as catalysts.

Final equation is same

$$4\left({}_{1}^{1}H\right) \xrightarrow{C/N/O} {}_{2}^{4}He + 2e^{+} + 2\gamma_e + 24.7\ MeV$$





Nuclear Physics (5)

06/03/12

- Parity Violation
- β-decay
- Neutrinos

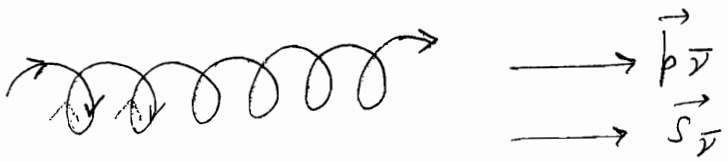
Parity Violation in β-decay

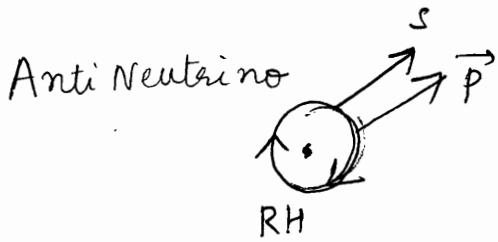
✔ Behaviour of ψ under inversion of coordinates is described by Parity.

Concept of Helicity

✔ In odd parity, Left handed coordinate system changes to Right Handed Coordinate System. Under inversion of coordinates

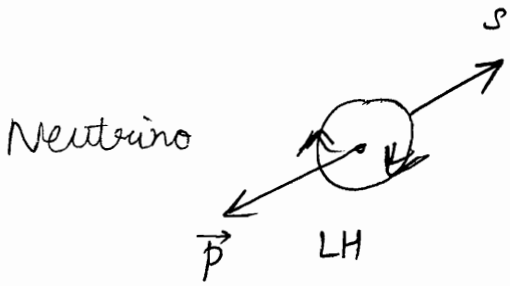
✔ Neutrino & anti-neutrino have difference of LH and RH coordinate system. This is called Helicity.





Helicity = +1





Helicity = −1

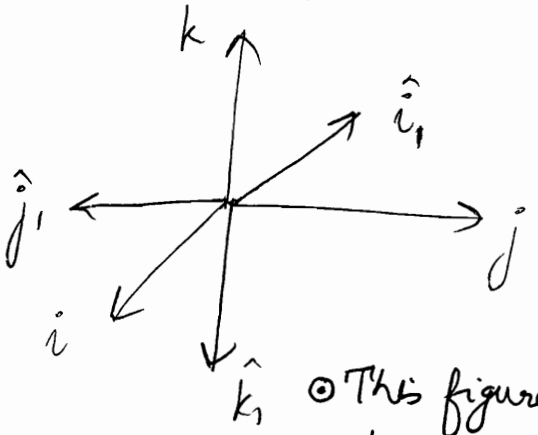
→ LH and RH dependent upon circular motion and $\vec{p}$

→ $\vec{s}$ is dependent only upon circular motion via right hand screw rule.

→ No role of charge

Helicity is the component of $\vec{s}$ along $\vec{p}$

$$\boxed{H = \hat{s} \cdot \hat{p}} = \frac{\vec{s} \cdot \vec{p}}{|\vec{s}||\vec{p}|}$$



⊙ The 2 coordinate system differ in ~~parity~~ handedness :-

$(x,y,z) \longrightarrow (-x,-y,-z)$

RH Parity LH Parity

⊙ This figure shows that inverting the sign of unit vectors is equivalent to inverting the handedness

$\vec{i}_1 = -\vec{i}$

$\vec{j}_1 = -\vec{j}$

$\vec{k}_1 = -\vec{k}$

✓ If particle appears to be same, then even parity

✓ If particle appears opposite, then odd parity.

It is similar to seeing particle in a mirror $(-x,-y,-z)$.

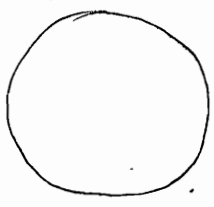
[If same equation of motion in mirror ⇒ even parity. Otherwise we say parity is violated]

Initially, we thought nature is symmetric ⇒ parity should be conserved in all nuclear reactions

But when we observed β- decay. It had different behaviour from then known Strong, EM or Gravity interactions. It was called Weak Interaction. Neutrino is a particle that takes part in all weak interactions.

$> 10^{-8}$ second : maximum time duration of interaction

⇒ low probability of this interaction.



Strongly Interacting Particles



Weakly Interacting Particle

Hadrons are assigned intrinsic parity

$p_p = +1$

$p_n = +1$

→ Proposed by Lee & Yang that parity is not conserved in weak interactions.

Wu and coworkers performed experiment on $^{60}_{27}Co$ nucleus which proved Parity violation in β-decay.

$$^{60}_{27}Co \longrightarrow {}^{60}_{28}Ni + e^- + \bar{\nu}e$$

(even parity) (even parity)

→ Q conserved → Le conserved

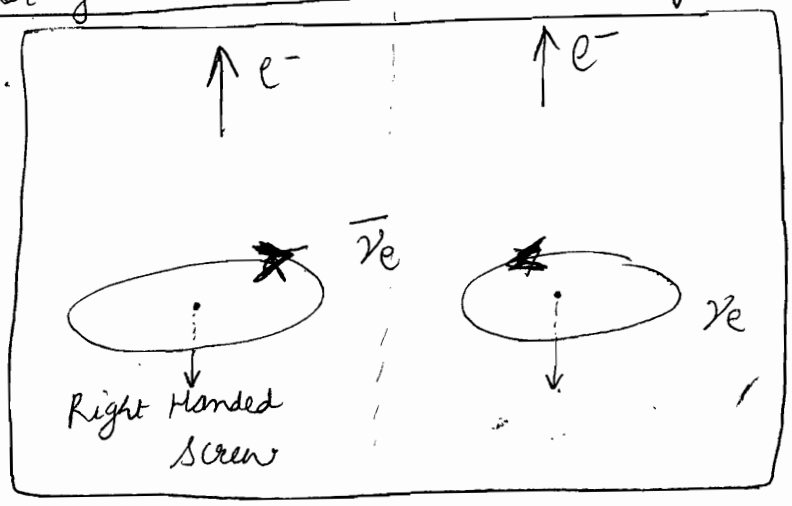
→ A conserved

This reaction occurs only via Weak Interaction force.

$$n \longrightarrow p + e^- + \bar{\nu}e$$

Remember neutron and proton have even parity....

To have Parity Conservation, e^- and $\bar{\nu}e$ must have same parity. By appling $\vec{B}$, I can have motion of e^- in particular direction. Now I see ✗ in mirror. Behaviour of ν in mirror gets reversed due to helicity. Hence parity is not conserved.



✓ Perfect diagram for explanation

Modern Physics : It starts from theory of relativity and Planck Quantum Theory. 4 experiments are landmark in Modern Physics.

1) Frank - Hertz Experiment (1912-13)

Energy level of H atom are discrete.

2) Davisson - Germer Experiment

De Broglie concept of material waves.

3) Stern - Gerlach Experiment

Spin Concept

4) Lee - Yang & Hu Experiment

β - decay.

} in syllabus every year 2 are asked out of 3.

β Decay

β decay or α decay or γ decay are ways of nuclei to achieve stability. These 3 are performed inside the nucleus.

$$ {}^{A}_{Z}X \rightarrow {}^{A'}_{Z'}Y + \beta \leftarrow \text{electron / positron} $$

or

$$ \gamma \leftarrow \text{photon} $$

In β decay, A is conserved

There are 3 types of β decay reactions :

(1) ${}^{A}_{Z}X \rightarrow {}^{A}_{Z+1}Y + e^-$ } β particle emission

i.e. $n \rightarrow p + e^-$

(2) ${}^{A}_{Z}X \rightarrow {}^{A}_{Z-1}Y + e^+$ } Positron emission

i.e. $p \rightarrow n + e^+$

(3) ${}^{A}_{Z}X + e^- \rightarrow {}^{A}_{Z-1}$ } k-shell e⁻ capture

$p + e^- \rightarrow n$

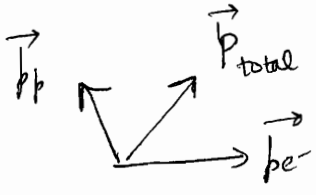
Physics of Neutrinos

Pauli observed that such spontaneous reactions are not possible due to :

(i) $\vec{p}_p + \vec{p}_{e^-} = 0$ for 1st reaction as $\vec{p}_n = 0$ (initially)

$\Rightarrow \vec{p}_p = -\vec{p}_{e^-}$ ⟵ ⟶

It was observed that they are not at 180°



i.e. $\vec{p}_{total} \neq 0$

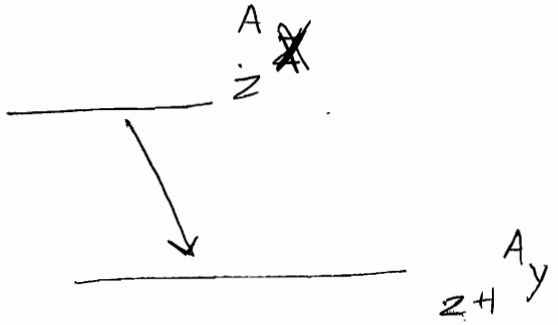
Hence conservation of linear momentum is violated

(ii) Spin Conservation is violated (Conservation of Angular Momentum)

$$\frac{1}{2} \rightarrow \frac{1}{2} + \frac{1}{2}$$

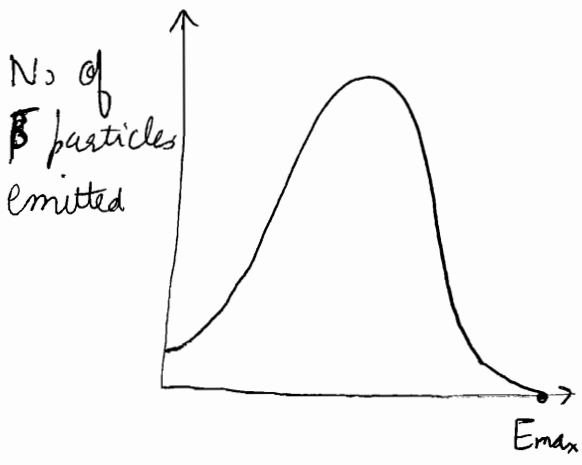
(iii) $${}^{A}_{Z}X \rightarrow {}^{A}_{Z+1}Y + e^{-}$$

(Parent) (Daughter)



Applying total relativistic energy conservation,

The difference of energy of parent & daughter must be energy of electron ⇒ All e^- have same energy. But we do not observe singe value of Energy. We observe Spectra.



→ Emax is characteristics of nucleus, called end point energy. $(E_\beta)_{max}$

Continuous Spectrum of β decay

We can observe some features from it:

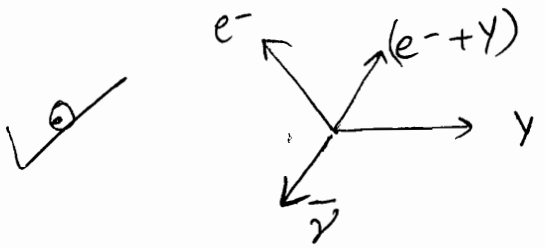
① Energy is continuously distributed b/w $E=0$ and end point energy.

② β-particles are maximum with mid-Energy level.

$\Rightarrow$ relativistic energy conservation is violated.

Hence; its not a 2-body problem. It must be a 3 body problem.

Hence we require neutral particle of $(\frac{1}{2})$ spin along with other 2 products



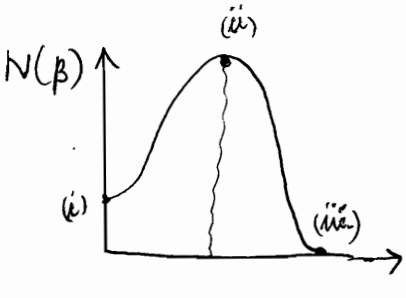
$\vec{p}_{total} = 0$

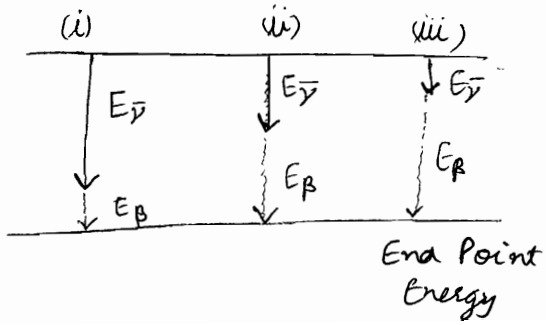
$\frac{1}{2} \rightarrow \frac{1}{2} + \frac{1}{2} + \frac{1}{2}$ $\Rightarrow$ S conserved

(half integral) (half integral)

Relativistic Energy difference 'ΔE' is shared between e⁻ and $\bar{\nu}$

$$E_\nu^2 = (p_\nu c)^2 + (mc^2)^2 \simeq (p_\nu c)^2$$





~~Pauli named the 3rd particle as Neutrino.~~

From lepton number conservation we get the particle as Anti Neutrino

Pauli proposed that there should be a 3rd neutral particle of $(\frac{1}{2})$ spin.

Fermi used it to make quantum mechanical theory of these particles, whom, Fermi called Neutrinos.

He developed a golden rule regarding "Time dependent Perturbation Theory", called Fermi Theory of β decay.

↑ [changes in Hamiltonian]

$$ {}^{A}_{Z}X \rightarrow {}^{A}_{Z+1}Y + \beta^- + \bar{\nu}_e $$

$$ {}^{A}_{Z}X \rightarrow {}^{A}_{Z-1}Y + \beta^+ + \nu_e $$

$$ p + e^- \longrightarrow n + \nu_e $$

According to fermi, β-decay is explained by a 3rd Particle which exists in 2 states : Neutrino & Anti Neutrino.

He applied Quantum Mechanics to obtain Probability of transition.

$$ \begin{cases} i \rightarrow f + e^- + \bar{\nu}_e \\ i \rightarrow f + e^+ + \nu \\ i + e^- \longrightarrow f + \nu_e \end{cases} $$

⊙ $P(i \to f) = \frac{2\pi}{\hbar} |\psi_{i \to f}|^2 \rho_{if}$

where ρ_{if} = density of state $= \left(\frac{dN}{dE}\right)$ Fermi's Golden Rule

& ψ_i : ψ Parent

& ψ_f : ψ daughter $\psi_e - \psi_{\bar{\nu}_e}$

✓ ft parameter is calculated.

Since ft is huge, we talk in terms of log (ft)

✓ We also have Fermi Selection Rules

✓ Nuclear Parity should not Change. $P_{parent} = P_{daughter}$ (overall parity violation may be there but $P_{parent} = P_{daughter}$)

✓ Orbital Angular Momenta [redacted] should not change

✓ $\Delta i = 0$ (for whole reaction) i: nuclear spin

✓ Allowed Reactions are having low value of ft parameter

High ft $\Rightarrow$ $t\uparrow$ $\Rightarrow$ [redacted] low probability

(lifetime)

✓ $\Delta l = 1$: 1st order forbidden transition

✓ $\Delta l = k$: k^{th} order forbidden transition

Gamow Teller changed $\Delta i = 0, \pm 1$ thus modifying fermi transition rules.

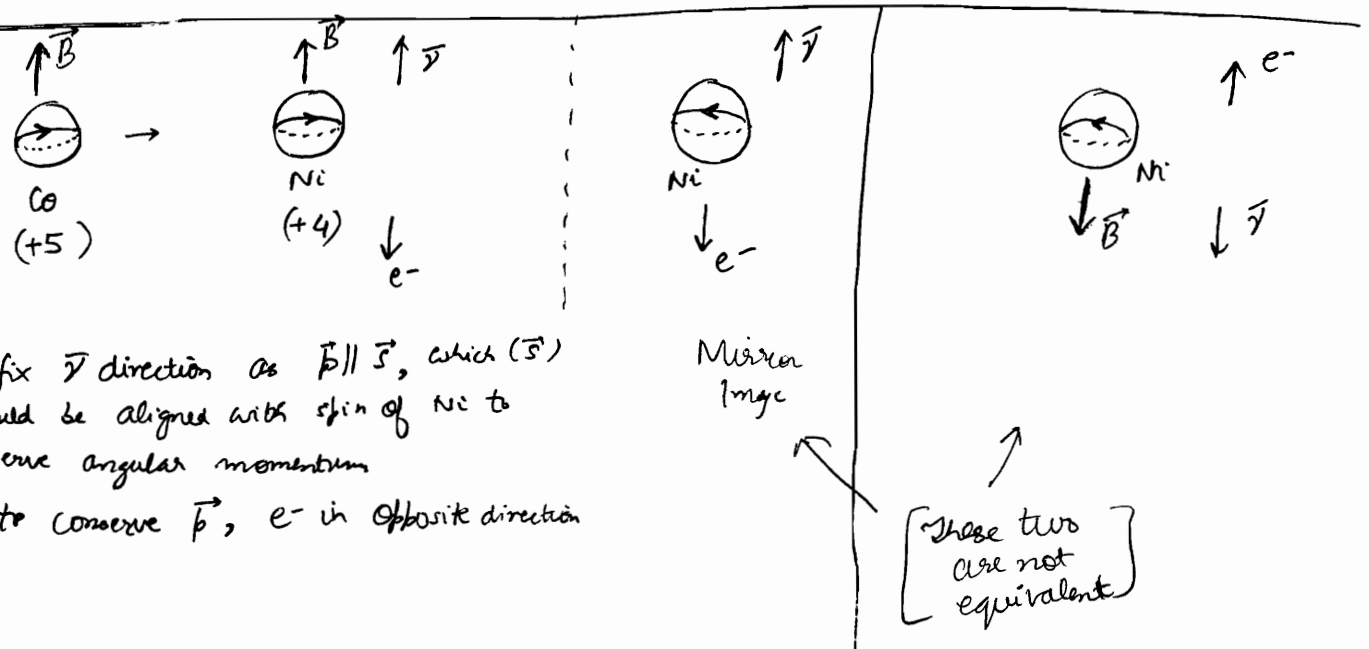
Some key Points

(1) Self sustained fusion reaction occurs in H-bomb. The explosive substance is a mixture of (^2_1D) and (^3_1T) The high temperature needed for reaction is produced by exploding a fission bomb. $^3_1H + ^2_1H \longrightarrow ^4_2He + ^1_0n +$

(2) Controlled fusion reaction have been achieved using deuterium plasma. At temp of around 10^8 K, fusion rxn begins to release energy. At this temp., energy escapes as heat through container walls. Hence, strong magnetic fields are used for thermal insulation of the plasma in a Toroidal Magnetic Chamber TOKAMAC.

(3) For a fusion, energy released = 24.7 MeV
If energy due to annihilation of two positrons is also added, then total energy released $\approx 24.7 + 2 \approx$ 26.7 MeV per cycle.

(4) 3 properties of Neutrino :
(i) electrically neutral
(ii) mass almost zero ⟶ It follows from the fact that that $(E_\beta)_{max}$ observed is equal to mass-energy difference of parent and [product + electron]
If neutrino had finite mass comparable to m_{e^-}, then it must be substracted to get $(E_\beta)_{max}$
(iii) Spin = $(1/2)$ ⇒ F.D. statistics



1st fix $\vec{\nu}$ direction as $\vec{p} \parallel \vec{s}$, which ($\vec{s}$) should be aligned with spin of Ni to conserve angular momentum

Now to conserve $\vec{p}$, e^- in opposite direction

Mirror Image

[These two are not equivalent]

Nuclear Physics (6)

· Gamma decay 06/03/12

6A

$$\odot \quad ft = \frac{2\pi^3 \hbar^7 \ln 2}{g^2 c^4 m^5 |M_{if}|^2} = \frac{0.693\, T_0}{|M_{if}|^2}$$

T_0 ← time period or lifetime

parameter for β decay

$g = 1.41 \times 10^{-62}$ joule m^3

If $\log_{10}(ft) = 2.7$ to $3.7 \Rightarrow$ superallowed transitions

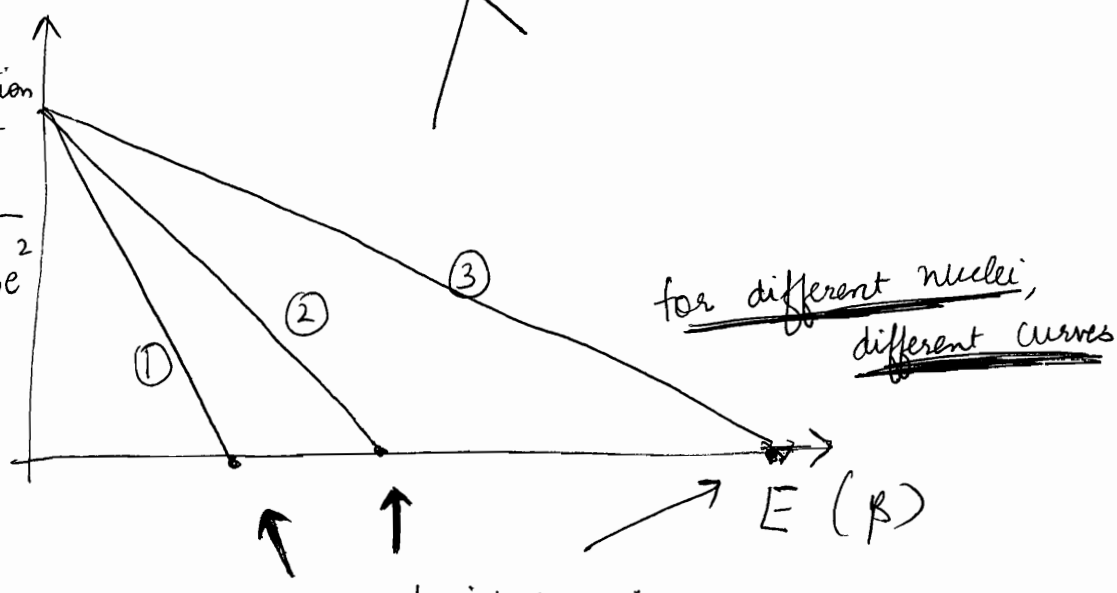
if $\log_{10}(ft) = 4$ to $5.8 \Rightarrow$ allowed transition

if $\log_{10}(ft) = 6$ to $10 \Rightarrow$ 1st forbidden

k^{th} forbidden

Kurie's Plot is obtained.

$$= \sqrt{\frac{P(p_e)\, dp_e}{F(Z, E_e)\, p_e^2}}$$



for different nuclei, different curves

(E const. ⇒ p const. (If 1 γ photon) But p of γ rays will depend on the initial direction of e⁻ and e⁺ ⇒ p is variable ⇒ 2 γ photons minimum

From end point energies, limits on mass of neutrino is obtained. (Beiser)

Another example of β decay:

$$^{6}_{2}He \rightarrow {}^{6}_{3}Li + e^- + \bar{\nu}_e \quad \text{(check)}$$

$0^+ \rightarrow 1^+$

$0^{(+)}$ (3^+)

(allowed according to gamow taylor selection rule)

We know

$$n \rightarrow p + e^- + \bar{\nu}_e$$

β decay : β particle emission

Writing opposite $p + e^- + \bar{\nu}_e \rightarrow n$

⇓ we can write

$$p + \bar{\nu}_e \rightarrow n + e^+$$

inverse β decay (given in Beiser)

Proton can never decay into neutron & positron w/o $\bar{\nu}$ as neutron is heavier.

For showing this reaction, Reines and Cowan got Nobel Prize. This reaction is utilized for indirect detection of anti neutrino. If this reaction occurs ⇒ $\bar{\nu}_e$ is present.

$\bar{\nu}_e$ can be obtained from flux of $\bar{\nu}_e$ emitted in β decay reaction. Here, in the vicinity, I keep some protons. e^+ is produced. It annihilates with e^- to emit specific γ EM radiation. We can detect that γ photons

$$e^- + e^+ \rightarrow \gamma + \gamma$$

★ γ: 0.511 MeV each

→ In the Reines & Cowan Experiment, ★ refer P- 495 Arthur Beiser

Aqueous solution of Cadmium Chloride

Cadmium has large absorption cross-section.

Neutron are captured by Cadmium Nucleus and energy absorbed is recorded by spectroscope.

Also annihilation of e^+ and e^- gives out energy which is recorded.

GB

Gamma Decay
Internal Conversion
Pair Production
} 3 mutually competing processes.

Gamma rays are highest energy EM radiation.

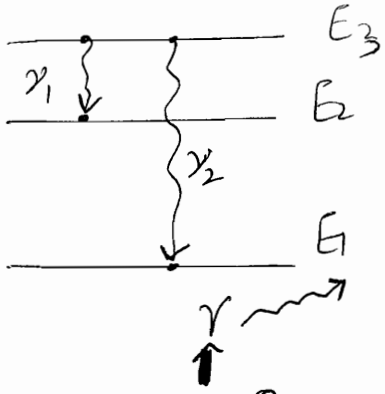
✓ Highest energy X-ray overlaps with lowest energy γ rays.

$$E_\gamma > 0.1 \text{ MeV}$$

✓ Accelaration of charged particles produces EM rays, we know. X-rays are produced by electronic level transitions in tungsten targets

★ note that these levels are different from ★ states. That were of RF region and used in NMR. ★ Zeeman type

✓ Gamma rays are arising due to nuclear energy level transitions. Due to some process, say α or β decay, nucleus is left in excited state. When it comes out to ground state, it emits γ photons.



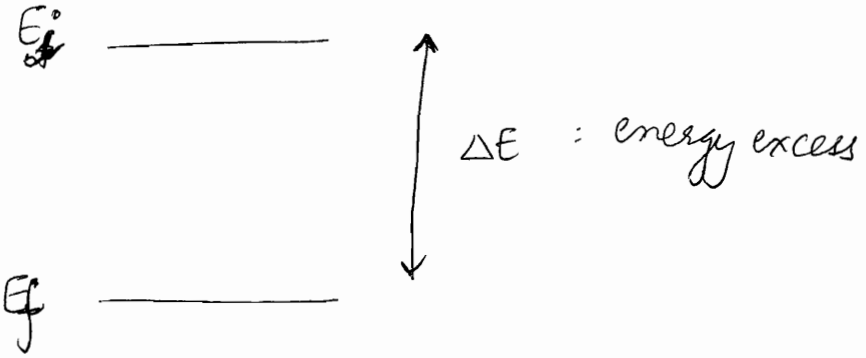
$$\gamma_1 = \frac{E_3 - E_2}{h}$$

$$\gamma_2 = \frac{E_3 - E_1}{h}$$

nuclear photon is absorbed by atomic electron

Either Gamma Decay or Internal Conversion or Pair Production occurs. They are 3 mutually competing processes. ↓ Occuring when nucleus is in excited state.

$\gamma \rightarrow e^- + e^+$



✓ From E_i to E_f, nucleus can come down via,

(i) emitting γ photon (gamma decay)

(ii) ejecting a conversion e^- (internal conversion)

(iii) production of x and $\bar{x}$ (pair production)

eg. $(e^-)+(e^+)$ if $\Delta E > 1.02$ MeV

Internal Conversion (sort of internally occuring Photo electric effect)

If $\Delta E >$ Binding Energy of e^- from some shell

→ k shell has maximum B.E.

↑ (nearby shell)

[The internal conversion is not a two-step process in which γ-photon 1st emitted & then it knocks out an orbital e^-. Its a single step process in which excited nucleus interacts directly with orbital e^-, transferring its excitation energy to it]

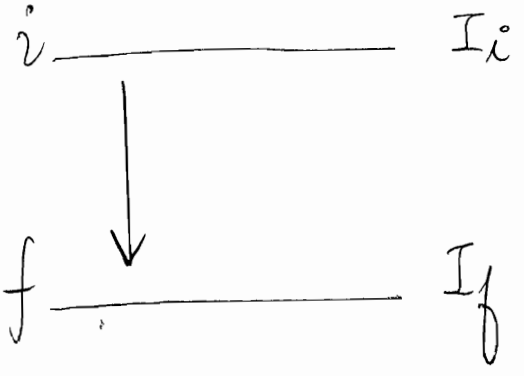
→ K.E. of k-shell e^- = $(\Delta E - B.E._{\text{k-shell } e^-})$

If ΔE is less, we will check for L shell and so on.

Also if k-shell e^- is removed, vacancy is filled by L shell e^-, which is filled by M shell e^- and so on.... Hence we get line spectra. Similar to e^- capture....

Pair Production

ΔE → energy required for pair production.



PARITY CHANGE IN GAMMA TRANSITIONS

⭐ $I_i \rightsquigarrow I_f$ = Angular Momentum of Photon carried

$$\vec{L}_\gamma = \vec{I}_i \rightsquigarrow \vec{I}_f$$

$$|(I_i - I_f)| \leq L_\gamma \leq |(I_i + I_f)|$$

in β^- decay, Gamow Teller conditions for ΔI. But in γ decay, no such conditions

ground state of different nuclei

same but excited nuclei

Every nucleus has nucleons. Its like a liquid drop.

If 1 proton displaced, dipole created.

If 2 p displaced, quadrupole created

If n p displaced, multipole created.

*this is how excitation takes place

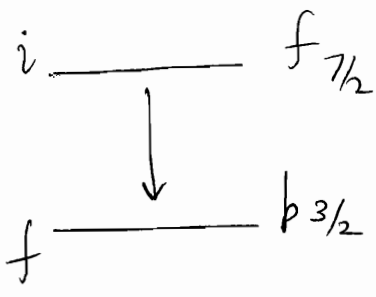
Within nucleus, there is charge flow, these charges are in format of multipole i.e. 2^l.

if $l=0$: monopole

if $l=1$: dipole

if $l=l$: 2^l multipole.

They can be electric or magnetic multipole.



$I_i = 1/2$

$I_f = \frac{3}{2}$

$L = I_i - I_f$ to $I_i + I_f$

$= 2, 3, 4, 5$

↑ quadrupole transition

↑ multipole transition

Parity change $= P_i P_f = (-1)^L$ for electrical multipole

$= (-1)^{L+1}$ for magnetic multipole

for $l=2$: quadrupole transition

If electric : $(-1)^2 : 1 : \Rightarrow$ parity should not change

If magnetic : $(-1)^3 : -1 \Rightarrow$ parity changes.

✓ Now I measure parity and tell whether electric or magnetic multipole transition.

For photon to be ejected, L should be minimum 1 as its spin is 1.

DIFFERENT CONDITIONS FOR DIFFERENT GAMMA RAY CONVERSION

If $L < 1 \Rightarrow$ internal conversion as e^- can have spin < 1.

But If ΔE > Energy required for pair production $\Rightarrow$ Pair Prodn

$$N = N_\gamma + N_k + N_{p.p.}$$

- N_γ: gamma decay emission
- N_k: internal Conversion. e^- emit from i^{th} shell
- $N_{p.p.}$: Pair Production

$$1 = \left(\frac{N_\gamma}{N}\right) + \left(\frac{N_i}{N}\right)$$

if $\Delta E <$ E required for Pair Production

$$\frac{N_i}{N} : \frac{\text{internal Conversion coefficient}}{} = \alpha_i = \left(\frac{N_K + N_L + N_M + N_N \ldots}{N}\right)$$

$$= \frac{N_K}{N} + \frac{N_L}{N} + \frac{N_M}{N} + \ldots$$

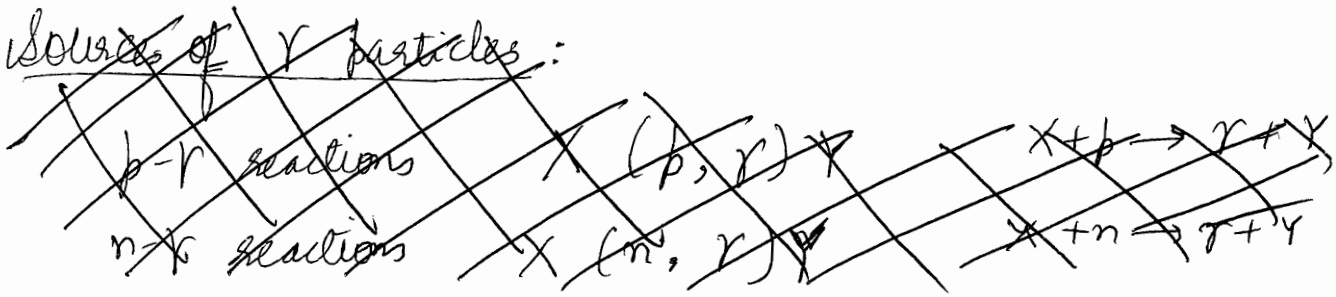
$$= \alpha_K + \alpha_L + \alpha_M + \ldots$$

Partial Conversion Coefficients

$$(\alpha)_k \propto Z^3 \left(\frac{1}{\Delta E}\right)^{L+\left(\frac{3}{2}\right)}$$ (Various studies done upon o

How do we measure energy of γ particles?

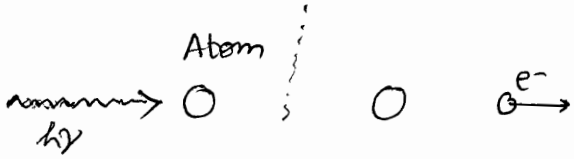
Sources of γ particles:



★ 0.1 MeV to more than 100 MeV : γ Particles

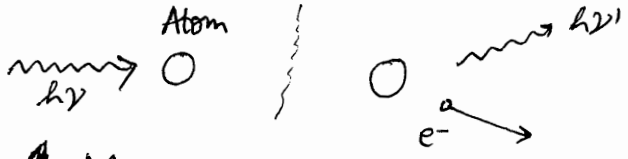
3 mechanism called secondary electron methods to measure E_γ :

(i) Photo electric emission



for lowest energy γ photons i.e. $\simeq$ 0.1 MeV
i.e. 100 keV

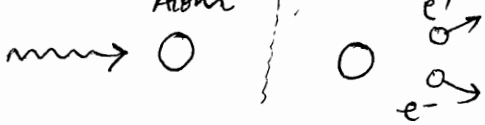
(ii) Compton Effect



0.1 MeV < E_γ < 1 MeV

(less than pair production)

(iii) Pair Production

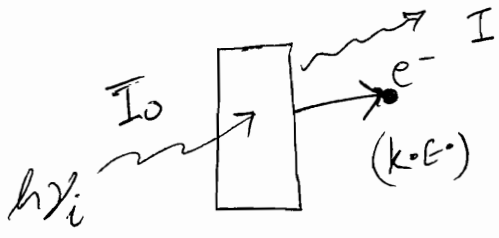


E_γ > 1 MeV

→ Interaction of γ rays with matter is also via these 3 ways.

Photo electric

When suitable energy radiation (E_r > Work Function) is incident, photo electron is ejected.

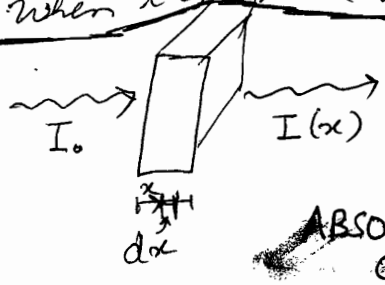


We measure K.E. of electron

$\Rightarrow E_r = h\gamma_i =$ Work Function (ϕ) + K.E. $= (\phi + k)$

Hence measured.

Intensity of photons will decrease when they faced an absorbing obstruction. $I < I_0$



ABSORPTION OF GAMMA RAYS

$$[dI] \propto [-I\,dx]$$

$$dI = -\mu I\,dx$$

$$\frac{dI}{I} = -\mu\,dx$$

$$\Rightarrow I = Ae^{-\mu x}$$

$$\Rightarrow \boxed{I = I_0 e^{-\mu x}}$$

μ: absorption coefficient

$$\boxed{\mu = \frac{1}{x}\ln\left(\frac{I_0}{I}\right)}$$

Compton

$h\gamma_i + [redacted] = h\gamma'_f + (K \cdot E)_{e^-}$ ⇝



↑ measured ↑ measured $= E_e - m_0c^2$

↓ from

$$\lambda' - \lambda = \frac{h}{m_0 c} (1 - \cos\phi)$$

- Measure final wavelength and (λ') ϕ We will get (λ)
- $m_0 c$ ⇝ Compton wavelength λ_0
- $\Delta\lambda = \lambda_0 [1 - \cos\phi]$
- ϕ : deflection of γ gamma ray

Pair Production

If $\Delta E > 3$ MeV

⇒ conversion into [e−] and [e+]

or

other pair

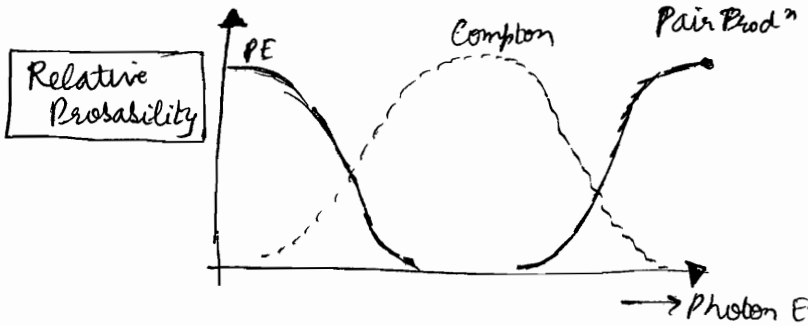
$h\gamma_i = E_x + E_{\bar{x}} = 2E_x$

↑ measured

Measure energies of x and $\bar{x}$

(★) In Gamma Decay, A, Z, Energy, $\vec{P}$, Spin will remain Conserved.

(★) Remember that Spin of photon = $\hbar$



[Remember lecture on Raman Spectra
UV : P.E effect
γ : Compton]

Nuclear Physics (7) : Mossbauer · Fission 07/03/2012

(7A)

Q What kind of γ-transitions occur?

(1) $g_{(\frac{9}{2})^+} \longrightarrow p_{(\frac{1}{2})^-}$ $\quad L = 4, 5$

Parity changing

$\Rightarrow M_4$ & E_5

(2) $f_{(\frac{5}{2})^-} \longrightarrow p_{(\frac{3}{2})^-}$

$L = 1, 2, 3, 4$

$M_1 \; E_2 \; M_3 \; E_4$

Parity not Changing

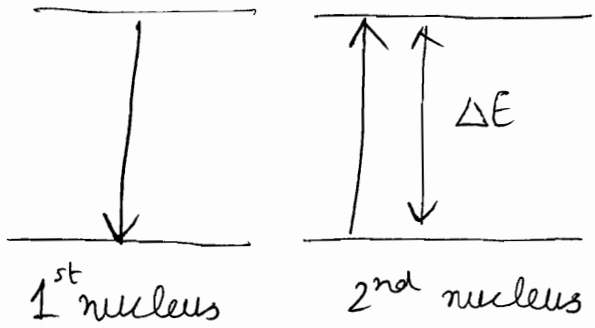
Mossbauer Effect

→ It is recoil-less emission & absorption of γ photons.

ie- Resonant absorption. 'No Recoilness' achieved by trapping nucleus in heavy crystal

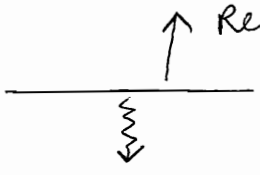
→ It provides a source of monochromatic γ photon

ie- it resolves very fine lines



→ It happen when nucleus levels are low lying, eg 10^4 eV (start of gamma rays)

When γ is thrown out, usually the nucleus recoils.



$\Delta E - E_{recoil} = E_{\gamma\text{-photon}}$

$\gamma = \left(\frac{\Delta E - E_{recoil}}{\hbar}\right)$ Usual case

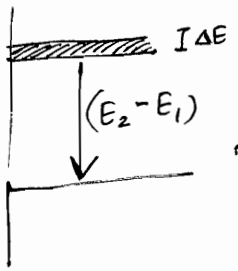
But for resonant absorption γ should be $\left(\frac{\Delta E}{\hbar}\right)$ so that absorption occurs. Hence, I should have recoil less nucleus. It happens when ΔE is low $\simeq 10^{-4}$ eV

When ΔE is low, Δt (lifetime) is high. Mossbauer case

What is Δt?

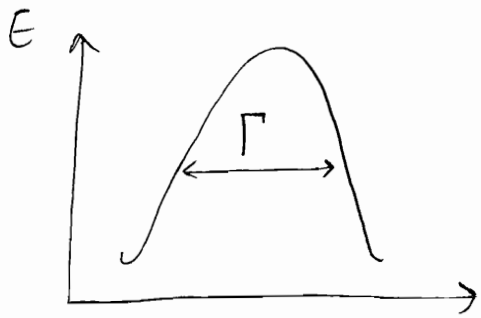
$$\Delta E \, \Delta t \simeq \hbar$$

* Order of $\hbar \simeq 10^{-15}$ eV sec



$$\Gamma \Delta t \simeq 10^{-15} \text{ eV sec}$$

$$T \simeq \Delta t \simeq \frac{10^{-15} \text{ eV sec}}{10^{-4} \text{ eV}} \simeq 10^{-11} \text{ sec}$$



Not every nucleus gives Mossbauer Effect i.e. every nucleus does not give gamma photons of particular frequency.

$$X \longrightarrow Y + \gamma$$

Also note γ's frequency has to be adjusted for doppler effect

Doppler Broadening $\Rightarrow \frac{\Delta\nu}{\nu} = \left(\frac{v}{c}\right) \Rightarrow \left(\frac{\Delta E}{E}\right) = \left(\frac{v}{c}\right)$

due to recoil of nucleus i.e. motion of source.

$$p_y + p_r = 0 \Rightarrow p_y = -p_r = -\left(\frac{E_r}{c}\right)$$

Note the $v_{recoil} \ll c \Rightarrow$ Non Relativistic measures can be applied $\Rightarrow \frac{p_y^2}{2M_{recoil}} = \boxed{\frac{E_r^2}{2M_{recoil}c^2} = E_{recoil}}$

⊙ If whole of crystal recoils instead of single nucleus

$\Rightarrow M_{recoil} \to \infty \Rightarrow E_{recoil} \to 0$ w/o reduction of E_γ. Hence in such crystals, Mössbauer Effect is seen.

^{191}[Iridium] → 129 keV γ-photon's source

^{57}Fe

$E_{recoil} \to 0$ ✳ 0

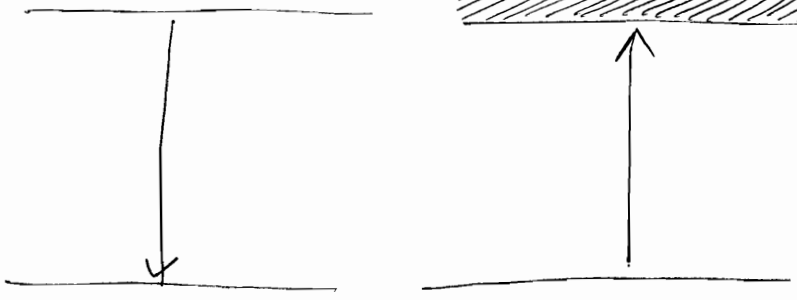
But $p = \sqrt{2mE}$

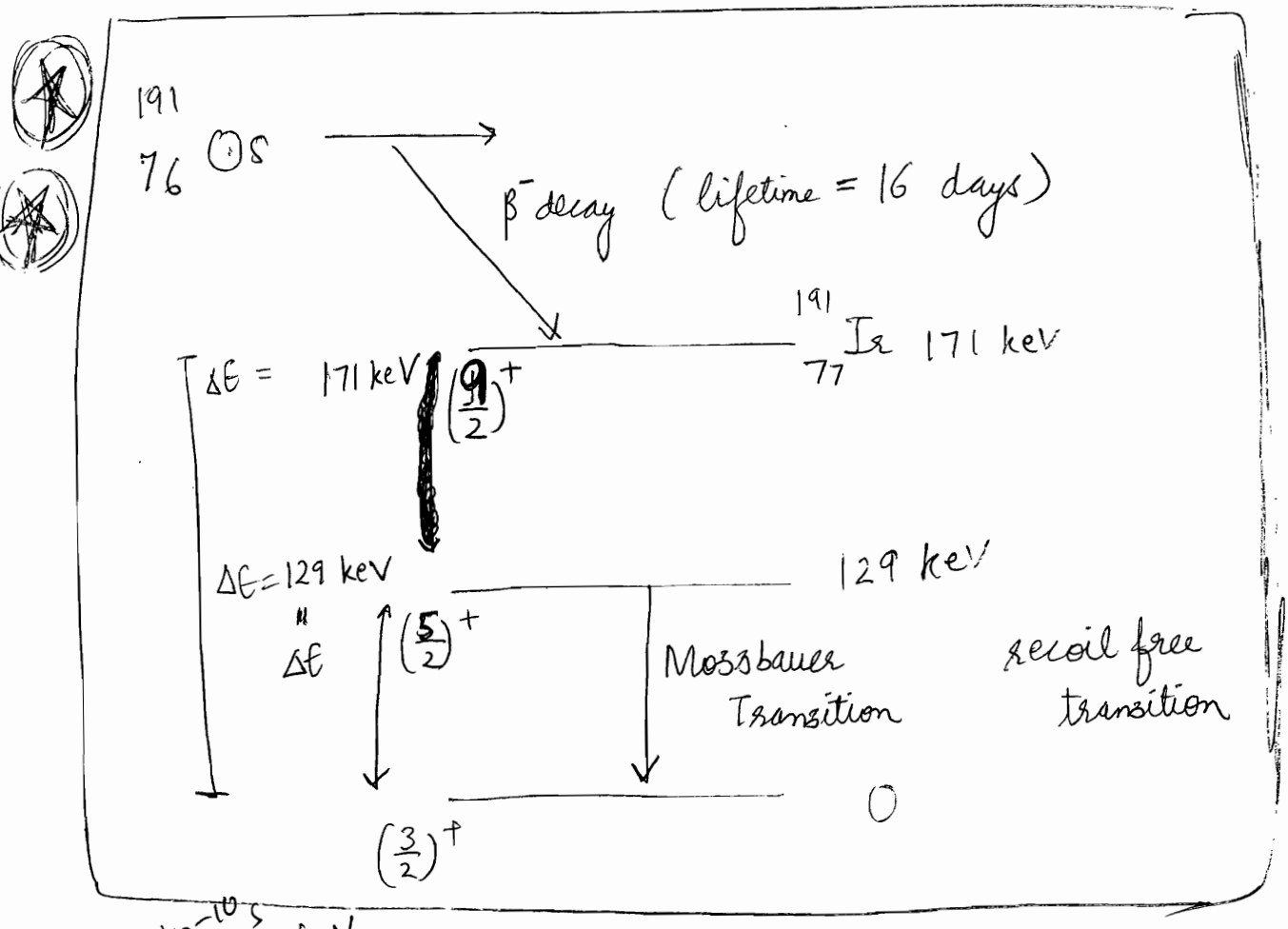
⊙ Note that for Mössbauer

(1) recoilless by embedding in crystal

(2) $\Delta E = \Gamma$ is small for the upper energy level

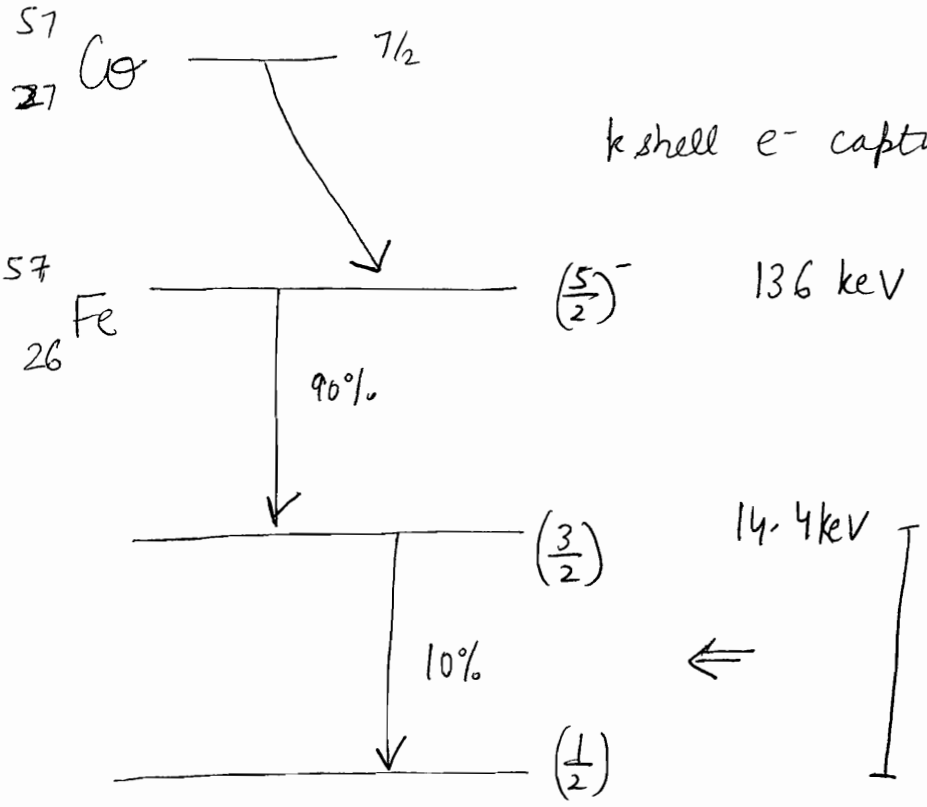
$I \quad \Delta E = \Gamma$





$\Delta t \sim 10^{-10}$ s

$\Delta E \sim 10^{-6}$ eV



⊙ Mossbauer is not given by any nucleus but a few like Ir, Fe, Ru, Ni

f *** RIN

Mossbauer Effect

$E_R = 14.4\ keV$

$T = 1.4 \times 10^{-7}\ sec.$

$\Delta E = \frac{\hbar}{\Delta T} \Rightarrow$ width $= \left(\frac{10^{-15}\ eV\ sec}{1.4 \times 10^{-7}\ sec}\right)$

$= 4.6 \times 10^{-9}\ eV$

It's a very sharp line

Mossbauer Effect:

1. Source of monochromatic γ photons.
2. Experimental Confirmation of General Theory of Relativity

$$\frac{\Delta E}{E} = \left(\frac{v}{c}\right)$$

ΔE can be made due to gravitational energy

We calculate ΔE = $\left(\frac{v}{c}\right) E$

ΔE matches with theoretical prediction.

(3) In chemical ~~shift~~ analysis of nuclei, $(2I+1)$ sublevels of I if Nucleus possesses dipole moment, in presence of ext. field.

Its called hyperfine splitting.

It may may be

(a) Dipole splitting

(b) Quadrupole splitting

(7B)

Nuclear Fission

To achieve stability. Their B.E per nucleon is high for heavier nuclei ; hence nuclear fission occurs to stabilize. All such nuclei, prone to nuclear fission are

Fissile Nucleus

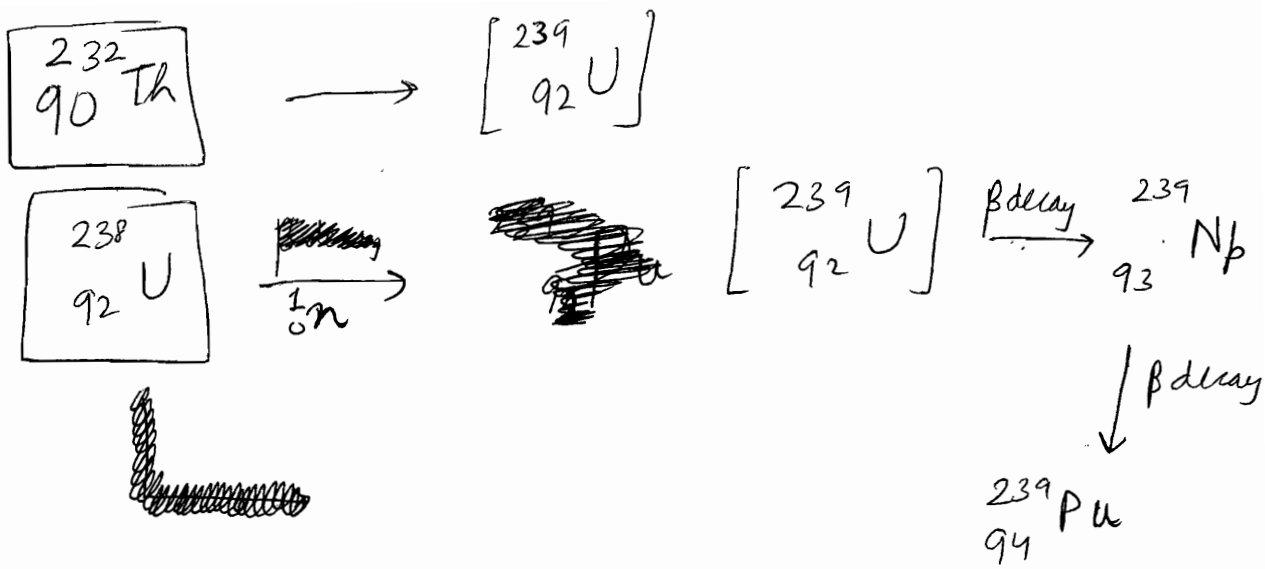
eg. $^{238}_{92}U$

99.3% } by weight

0.7%

- $^{235}_{92}U$
- $^{239}_{92}U$
- $^{239}_{94}Pu$

✓ Fertile or fissionable material means that these are nuclei that can converted to fissile material



- **Fuel : fissile material**
- **Raw Material : fertile material**

$$^{235}_{92}U + {}^{1}_{0}n \longrightarrow {}^{236}_{92}U$$

compound nucleus

$$^{236}_{92}U \longrightarrow {}^{98}_{40}Zr + {}^{136}_{52}Te$$

$$^{98}_{40}Zr \longrightarrow {}^{98}_{41}Cb \longrightarrow {}^{98}_{42}Mo$$

$$^{136}_{52}Te \longrightarrow {}^{136}_{54}Xe$$

$$\Rightarrow \boxed{^{235}_{92}U + {}^{1}_{0}n \longrightarrow {}^{98}_{42}Mo + {}^{136}_{54}Xe + 2\,{}^{1}_{0}n}$$

+210 MeV

Neutrons can be

1. Cold Neutron : lowest energy
2. Thermal Neutron : @ room temp.
3. Fast Neutron — upto MeV
4. Very Fast Neutron — upto 10 MeV
5. Ultra Fast Neutron — > 10 MeV

In the previous reaction, ^{235}U, any neutron, even @ room temperature can be used to break it. Hence spontaneous reaction.

$Q \simeq 210$ MeV for previous reaction

Every reaction requires certain activation energy. Some bombardment is required for nuclear fission reactions. Its provided by neutrons. So it depends upon neutron whether it can provide certain amount of activation energy!!

→ Note that mass no. is conserved in nuclear reaction. difference in ΔE comes due to difference in Binding Energies as $M_{nucleus} \neq \Sigma A$

Q. $U^{235} \rightarrow 200$ MeV

Find out annual requirement of U^{238} for having power output = 1W.

Ans/- Remember ratio: 99.3 : 0.7 ratio.

1 Watt : 1 J/sec = $\frac{1}{(1.6\times10^{-19})}$ eV/sec

$= 6.25 \times 10^{18}$ eV/sec

$= 6.25 \times 10^{12}$ MeV/sec

^{235}U required per sec.

$= \frac{6.25\times10^{12}}{200} = \underline{\underline{3.125\times10^{10} \text{ nuclei}}}$

Mass per sec = 6.023×10^{23} : 235 gram

$\Rightarrow 3.125\times10^{10} \Rightarrow \frac{235}{6.023\times10^{23}} \times 3.125\times10^{10}$ gram

Mass of ^{238}U per sec. $\Rightarrow \frac{99.3}{0.7} \times \frac{235}{6.023\times10^{23}} \times 3.125\times10^{10}$ g.

How reactor is formed?

Component of reactors:

① Fuel i.e. fissile material

② Moderator i.e. 2.5 neutrons emitted on avg. per nuclear reaction....
we have to control chain reaction using moderators eg. Graphite
Water
Heavy water

Remember: 1st class if $m_1 = m_2 \Rightarrow$ velocities are interchanged

But this is not the only criteria. 2nd criteria is absorption cross section. For water, potential to stop neutron is maximum but its absorption cross section is small. Hence heavy water is used.

③ Coolant : liquid is circulated. It may be water or liquid gases.

PHWR
Boiling Water Reactor

liquid → steam → useful work

④ Control Rods.

Note that moderator was slowing down neutrons. Cadmium & Boron Rods are used to control the reaction

⑤ Shield

Thermal Shield or Concrete Shields

Fuel + Moderator : mixed and formed a solution

⇒ homogenous plant

if separate ⇒ heterogenous plant

Application of Semi Empirical Mass Formula in Fission

3 imp points to take care of.

$^{A}_{Z}X \longrightarrow$ $^{A/2}_{Z/2}$ + $^{A/2}_{Z/2}$

R R

① take $R_0 = 1.5$ fm

Coulomb Repulsion in Critical case

$$E_C = \frac{1}{4\pi\varepsilon_0} \frac{\left(\frac{Ze}{2}\right)^2}{2R_0\left(\frac{A}{2}\right)^{\frac{1}{3}}}$$

$$= (0.15)\frac{Z^2}{A^{1/3}} \text{ MeV}$$

$$ {}^{A}_{Z}X \longrightarrow 2\,{}^{\frac{A}{2}}_{\frac{Z}{2}}Y + Q $$

Semi Empirical Mass Formula

$$M(Z,A) = ZM_P + (A-Z)M_N - \left[a_1 A - a_2 A^{2/3} - a_3 \frac{Z^2}{A^{1/3}} - a_4 \frac{(A-2Z)^2}{A} (\pm a_5, 0) \frac{1}{A^{3/4}}\right]$$

$$Q = \left[M(Z,A) - 2M\left(\frac{Z}{2}, \frac{A}{2}\right)\right] c^2$$

All fissile material are of odd $A \Rightarrow$ as not there

$$Q = + a_2\left(A^{2/3} - 2A^{2/3}\left(\frac{1}{2}\right)^{\frac{2}{3}}\right) + a_3\left(\frac{Z^2}{A^{1/3}} - \frac{2\left(\frac{Z}{2}\right)^2}{\left(\frac{A}{2}\right)^{\frac{1}{3}}}\right)$$

(3) we can take $Z(Z-1) \approx Z^2$

(2) $Q = |B.E._F| - |B.E._i|$

$$+ a_4\left(\frac{(A-2Z)^2}{A} - 2\frac{\left(\frac{A}{2}-Z\right)^2}{\left(\frac{A}{2}\right)}\right)$$

⇐ this term cancels out

$$Q = \left(-3.42\, A^{\frac{2}{3}} + 0.22\, \frac{Z^2}{A^{\frac{1}{3}}}\right) \text{MeV}$$

For spontaneous fission $Q \geq 0$

$$\Rightarrow 0.22\, \frac{Z^2}{A^{1/3}} \geq 3.42\, A^{2/3}$$

$$\Rightarrow \left(\frac{Z^2}{A}\right) \geq 15$$

Fission Parameter for spontaneous fission

To better the result,

if $Q < E_{coulumb} \Rightarrow$ no reaction

$Q = P.E. + k.E$ of daughters

$\Rightarrow$ For reaction, $Q \geq E_{coulumb}$

$$\Rightarrow -3.42 A^{\frac{2}{3}} + 0.22 \frac{Z^2}{A^{\frac{1}{3}}} \geq 0.1 \frac{Z^2}{A^{1/3}}$$

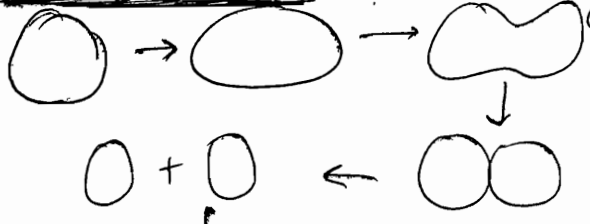
$$\Rightarrow 3.42 A^{2/3} \leq 0.0 \frac{Z^2}{A^{1/3}}$$

$$\Rightarrow \boxed{\frac{Z^2}{A} \geq 49}$$

$$\boxed{\frac{Z^2}{A} \geq 43}$$

Bohr and Wheeler's Approximation using liquid drop model

Bohr & Wheeler theory



and using legendre's polynomial

$R = R_n P_n(\cos\theta)$

Via Quantum Mechanical Tunneling, Fission can take place even with $(Q - E_A) < E_{coulumb}$

E_A : Activation energy

→ Experimental determination of μ nucleus

Ex " " " μ_{proton}, $\mu_{neutron}$ via magnetic Resonance.

[neutron : unstable
proton : stable] via Rabi

→ Neutron is covered by negative charge cloud. Unsymmetric charge distribution in neutron.

→ Absorption Cross Section } 4 factors determining probability of Nuclear Fission

→ Bohr Wheeler Condition

→ Internal Conversion from book

$\left[\text{Prob}_{(\alpha)}\right] \alpha [Z^3]$

→ Mossbauer : Width of nuclear energy levels is extremely narrow, then only resonant absorption.

→ Unification Theory : in Energy Scale

⊛ Remember $\Delta E \Delta t \simeq \hbar$ for pi meson. Enough energy must be supplied to a nucleon so that its emission conserves energy. Thus at least $mc^2 \simeq 140$ MeV is required. This amount of energy is produced in LHC, and therefore such particles could be studied.

~~Solid State Physics~~

Standard Model

Present model of elementary particles and their interaction. In the model, the strong, weak and electromagnetic forces appear as different manifestations of one basic phenomenon, with leptons and quarks finding natural places within the scheme. By placing leptons & quarks in the same framework, it becomes possible to explain why the electrons (leptons) and the protons (composition of quarks) have electric charge of exactly the same magnitude.

Particles

① Spin ½ fermions : 3 generation of leptons
3 generation of quarks
All their antiparticles

② Spin 1 Gauge Boson (12 in number) 1 massless electroweak boson γ
3 massive electroweak boson $W^{\pm}, Z^{0}$
8 Coloured gluons

③ Spin-0 Higgs Boson

Interaction EM coupling of γ
Weak interaction of $W^{\pm}, Z^{0}$
Strong Interaction gluons

Higgs Boson

Higgs mechanism is responsible for giving mass to gauge bosons. Higgs mechanism is the mechanism by which the existence of a spin 0 particle can give a gauge boson mass w/o breaking the gauge symmetry. Higgs showed that a field, called the Higgs field, must exist everywhere in space. By interacting with Higgs field, particles acquire their characteristic mass. HIGGS BOSON mediates the action of Higgs field. Mass of Higgs Boson could be $1\ TeV/c^2$

Grand Unification Theory

According to GUT, the particles should belong to a group representation. The simplest possible GUT is labelled SU(5).

Intro Success of electro-weak Unification has prompted physicists to pursue vigorously the attempts at unifying the strong nuclear and electroweak interactions. Such grand Unification will do away with the distinction between quarks and leptons at sufficiently high energies. All the three interactions are then expected to be manifestations of different facets of single universal gauge force. Such a possibility arises due to the same value of the spin for gauge boson in the 3 cases. Unification Energy is quite large. After Universe began expanding at a time when its thermal energy was comparable to unification energy, Unification effects were large. Now the effects are extremely low bcoz thermal energy of universe is low. Their observed vastly different strengths at low energy result from differing normalizations due to successive spontaneous symmetry breaking.

Consequence A startling consequence of the theory entail the disappearance of distinction between quarkes & leptons, which gives rise to possibility of transformation of protons into leptons and pions. The protons would then not be stable particles

2nd important consequence of GUT is the existence of very massive magnetic monopoles.

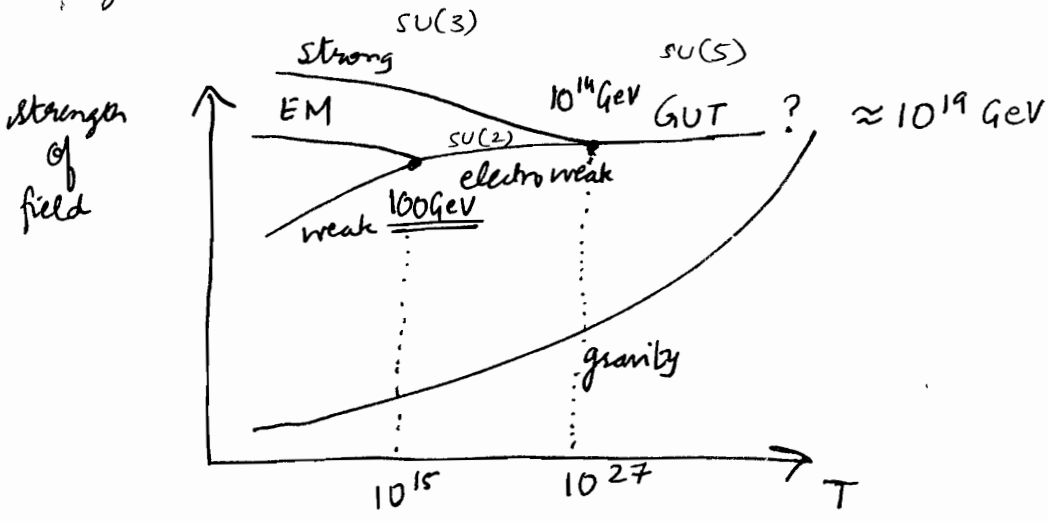
3rd it will provide solid support for big bang theory of creation of universe. Theories of smallest constituents of observable matter are thus seen to have profound implications on cosmology.

Theory

SU(5) representation has a place for all the fermion of one generation and will treat the quarks and leptons on equal footing. The interaction is with gauge bosons associated with an SU(5) local gauge invariance : that means 24 gauge bosons. 12 gauge bosons have already been collected which are γ, $W^{\pm}$, Z^0, 8 gluons. Group theory says that six others are each coloured. one of red, green or blue with electric charge

$$-\frac{1}{3}|e|\ (Y_R, Y_G, Y_B) \quad \text{or} \quad -\frac{4}{3}|e|\ (X_R, X_G, X_B)$$

Remaining 6 are antiparticles of these six.
They are called "lepto quark or diquark bosons".
Thus lepto quark are particles that carry information between quarks & leptons given a generation and allowing quarks and leptons to interact. They are colour triplet bosons that carry both lepton and Baryon number.



use : in π meson : $\Delta t = \left(\frac{1.4 \text{ fm}}{c}\right)$

in fission parameter : $R = 1.5 A^{1/3}$ fm