CHAPTER TWO
GAS VAPOUR MIXTURES
AND
AIR CONDITIONING PROCESSES

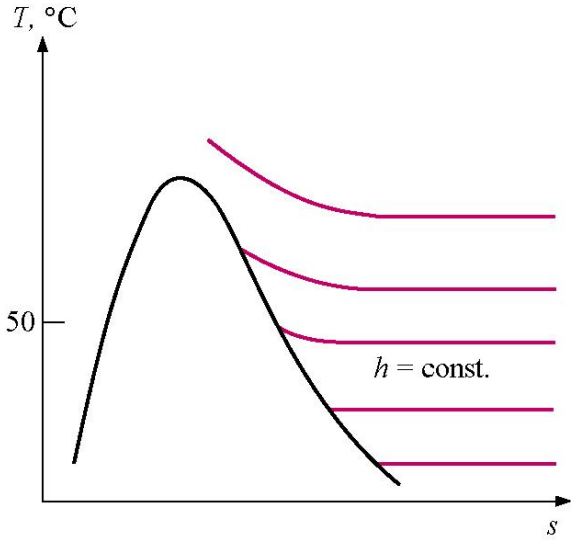
We will be concerned with the mixture of dry air and water vapor. This mixture is often called atmospheric air.

The temperature of the atmospheric air in air-conditioning applications ranges from about –10 to about 50°C. Under these conditions, we treat air as an ideal gas with constant specific heats. Taking C_{pa} = 1.005 kJ/kg·K, the enthalpy of the dry air is given by (assuming the reference state to be 0°C where the reference enthalpy is taken to be 0 kJ/kga)

$$h_a = C_{pa} T \quad \left(\frac{kJ}{kg_a} \right) \qquad T \, in \; ^o C$$

$$= \left[1.005 \frac{kJ}{kg \cdot ^o C} \right] T \quad \frac{kJ}{kg_a}$$

The assumption that the water vapor is an ideal gas is valid when the mixture temperature is below 50°C. This means that the saturation pressure of the water vapor in the air-vapor mixture is below 12.3 kPa. For these conditions, the enthalpy of the water vapor is approximated by $h_v(T) = hg$ at mixture temperature T. The following T-s diagram for water illustrates the ideal-gas behavior at low vapor pressures. See Figure A-9 for the actual T-s diagram.



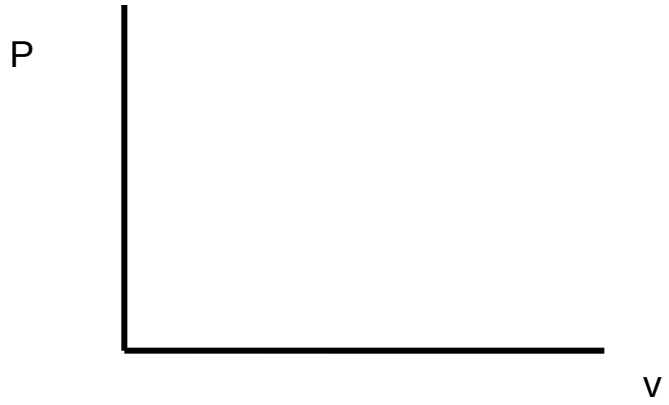
The saturated vapor value of the enthalpy is a function of temperature and can be expressed as

$$h_v = h_g(T) \cong 2501.3 + 1.82T \left(\frac{kJ}{kg_v}\right) \quad T \text{ in } {}^oC$$

Note: For the dry air-water vapor mixture, the partial pressure of the water vapor in the mixture is less that its saturation pressure at the temperature.

$$P_v \leq P_{sat@T_{mix}}$$

Consider increasing the total pressure of an air-water vapor mixture while the temperature of the mixture is held constant. See if you can sketch the process on the *P-v* diagram relative to the saturation lines for the water alone given below. Assume that the water vapor is initially superheated.



When the mixture pressure is increased while keeping the mixture temperature constant, the vapor partial pressure increases up to the vapor saturation pressure at the mixture temperature and condensation begins. Therefore, the partial pressure of the water vapor can never be greater than its saturation pressure corresponding to the temperature of the mixture.

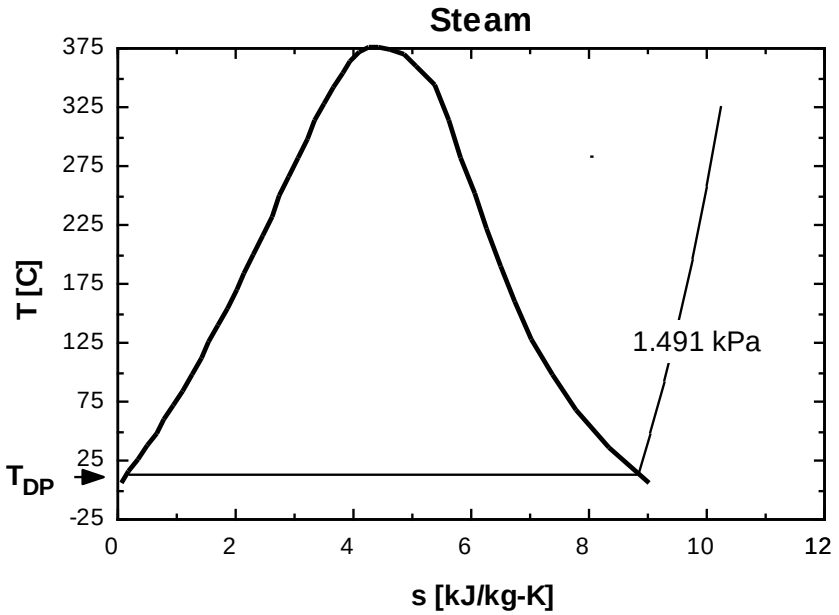
Definitions

Dew Point, *T*dp

The dew point is the temperature at which vapor condenses or solidifies when cooled at constant pressure.

Consider cooling an air-water vapor mixture while the mixture total pressure is held constant. When the mixture is cooled to a temperature equal to the saturation temperature for the water-vapor partial pressure, condensation begins.

When an atmospheric air-vapor mixture is cooled at constant pressure such that the partial pressure of the water vapor is 1.491 kPa, then the dew point temperature of that mixture is 12.95°C.

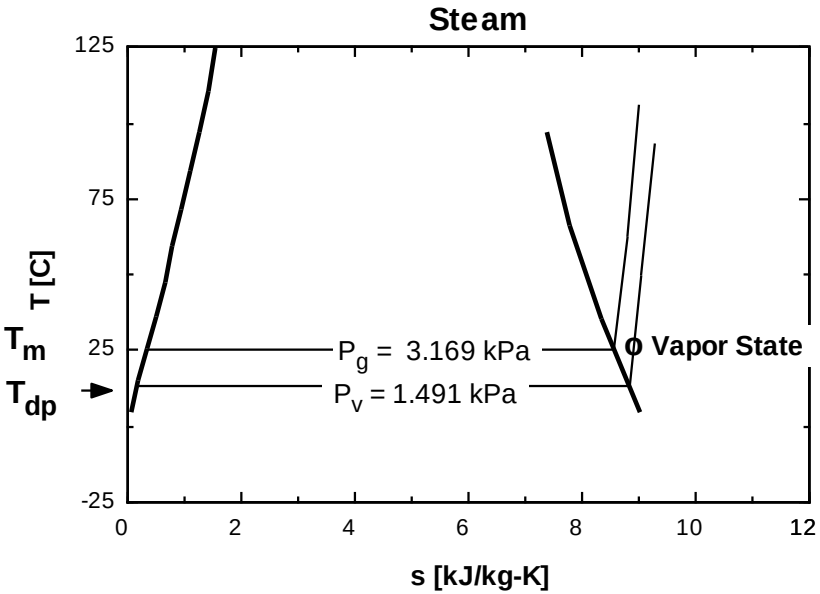


Relative Humidity, ϕ

$$\phi = \frac{Mass\ of\ \ vapor\ in\ air}{Mass\ of\ \ in\ saturated\ \ air} = \frac{m_v}{m_g}$$

$$= \frac{P_v}{P_g}$$

P_v and P_g are shown on the following T-s diagram for the water-vapor alone.



Since $P_g \geq P_v$, $\phi \leq 1$ *or* 100%, $\phi = \frac{P_v}{P_g} = \frac{1.491\ kPa}{3.169\ kPa} = 0.47$

Absolute humidity or specific humidity (sometimes called humidity ratio), ω

$$\omega = \frac{Mass\ of\ \ water\ vapor\ in\ air}{Mass\ of\ \ dry\ air} = \frac{m_v}{m_a}$$

$$= \frac{P_v V M_v / (R_u T)}{P_a V M_a / (R_u T)} = \frac{P_v M_v}{P_a M_a}$$

$$= 0.622 \frac{P_v}{P_a} = 0.622 \frac{P_v}{P - P_v}$$

Using the definition of the specific humidity, the relative humidity may be expressed as

$$\phi = \frac{\omega P}{(0.622 + \omega) P_g} \quad and \quad \omega = \frac{0.622 \phi P_g}{P - \phi P_g}$$

Volume of mixture per mass of dry air, *v*

$$v = \frac{V}{m_a} = \frac{m_m R_m T_m / P_m}{m_a}$$

After several steps, we can show (you should try this)

$$v = \frac{V}{m_a} = v_a = \frac{R_a T_m}{P_a}$$

So the volume of the mixture per unit mass of dry air is the specific volume of the dry air calculated at the mixture temperature and the partial pressure of the dry air.

Mass of mixture

$$m = m_a + m_v = m_a(1 + \frac{m_v}{m_a}) = m_a(1 + \omega)$$

Mass flow rate of dry air, $\dot{m}_a$

Based on the volume flow rate of mixture at a given state, the mass flow rate of dry air is

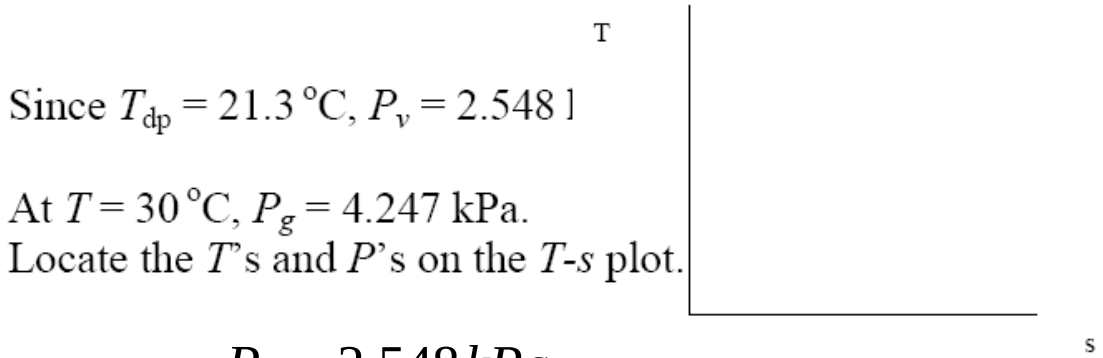
$$\dot{m}_a = \frac{\dot{V}}{v} \qquad \frac{m^3/s}{m^3/kg_a} = \frac{kg_a}{s}$$

Enthalpy of mixture per mass dry air, *h*

$$h = \frac{H_m}{m_a} = \frac{H_a + H_v}{m_a} = \frac{m_a h_a + m_v h_v}{m_a}$$
$$= h_a + \omega h_v$$

Example 14-1

Atmospheric air at 30°C, 100 kPa, has a dew point of 21.3°C. Find the relative humidity, humidity ratio, and h of the mixture per mass of dry air.



$$\phi = \frac{P_v}{P_g} = \frac{2.548\,kPa}{4.247\,kPa} = 0.6 \; or \; 60\%$$

$$\omega = 0.622\,\frac{2.548\,kPa}{(100 - 2.548)\,kPa} = 0.01626\,\frac{kg_v}{kg_a}$$

$$\begin{aligned} h &= h_a + \omega h_v \\ &= C_{p,a}T + \omega(2501.3 + 1.82T) \\ &= 1.005\,\frac{kJ}{kg_a \cdot {}^{o}C}(30^{o}C) + 0.01626\,\frac{kg_v}{kg_a}(2501.3 + 1.82(30^{o}C))\,\frac{kJ}{kg_v} \\ &= 71.71\,\frac{kJ}{kg_a} \end{aligned}$$

Example 14-2

If the atmospheric air in the last example is conditioned to 20°C, 40 percent relative humidity, what mass of water is added or removed per unit mass of dry air?

At 20°C, Pg = 2.339 kPa.

$$P_v = \phi P_g = 0.4(2.339\,kPa) = 0.936\,kPa$$

$$w = 0.622\frac{P_v}{P - P_v} = 0.622\frac{0.936\,kPa}{(100 - 0.936)\,kPa}$$

$$= 0.00588\frac{kg_v}{kg_a}$$

The change in mass of water per mass of dry air is

$$\frac{m_{v,2} - m_{v,1}}{m_a} = \omega_2 - \omega_1$$

$$\frac{m_{v,2} - m_{v,1}}{m_a} = (0.00588 - 0.01626)\frac{kg_v}{kg_a}$$

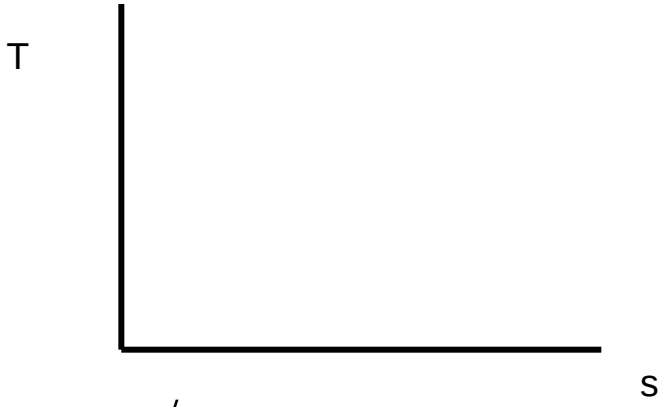
$$= -0.01038\frac{kg_v}{kg_a}$$

Or, as the mixture changes from state 1 to state 2, 0.01038 kg of water vapor is condensed for each kg of dry air.

Example 14-3

Atmospheric air is at 25°C, 0.1 MPa, 50 percent relative humidity. If the mixture is cooled at constant pressure to 10°C, find the amount of water removed per mass of dry air.

Sketch the water-vapor states relative to the saturation lines on the following *T-s* diagram.



At 25°C, P_{sat} = 3.170 kPa, and with ϕ_1 = 50%

$$P_{v,1} = \phi_1 P_{g,1} = 0.5(3.170\,kPa) = 1.585\,kPa$$

$$T_{dp,1} = T_{sat@P_v} = 13.8^o C$$

$$w_1 = 0.622\frac{P_{v,1}}{P - P_{v,1}} = 0.622\frac{1.5845\,kPa}{(100 - 1.5845)\,kPa}$$

$$= 0.01001\frac{kg_v}{kg_a}$$

Therefore, when the mixture gets cooled to T_2 = 10°C < $T_{dp,1}$, the mixture is saturated, and ϕ_2 = 100%. Then $P_{v,2}$ = $P_{g,2}$ = 1.228 kPa.

$$w_2 = 0.622\frac{P_{v,2}}{P - P_{v,2}} = 0.622\frac{1.228\ kPa}{(100 - 1.228)\,kPa}$$

$$= 0.00773\frac{kg_v}{kg_a}$$

The change in mass of water per mass of dry air is

$$\frac{m_{v,2} - m_{v,1}}{m_a} = \omega_2 - \omega_1$$

$$= (0.00773 - 0.01001)\frac{kg_v}{kg_a}$$

$$= -0.00228\frac{kg_v}{kg_a}$$

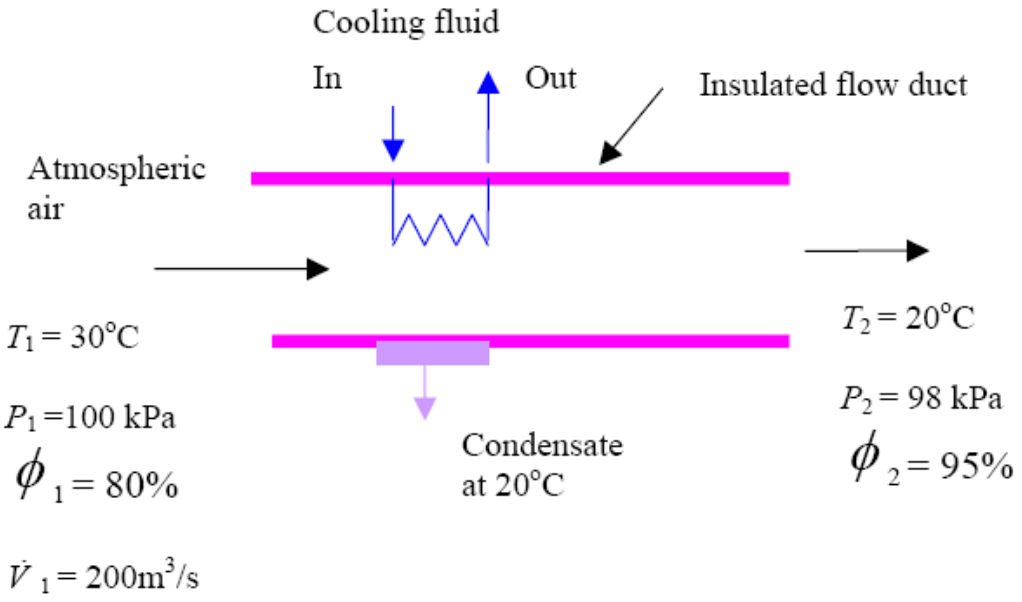
Or as the mixture changes from state 1 to state 2, 0.00228 kg of water vapor is condensed for each kg of dry air.

Steady-Flow Analysis Applied to Gas-Vapor Mixtures

We will review the conservation of mass and conservation of energy principles as they apply to gas-vapor mixtures in the following example.

Example 14-3

Given the inlet and exit conditions to an air conditioner shown below. What is the heat transfer to be removed per kg dry air flowing through the device? If the volume flow rate of the inlet atmospheric air is 200 m^3/min, determine the required rate of heat transfer.



Before we apply the steady-flow conservation of mass and energy, we need to decide if any water is condensed in the process. Is the mixture cooled below the dew point for state 1?

$$P_{v,1} = \phi_1 P_{g,1} = 0.8(4.247\,kPa) = 3.398\,kPa$$

$$T_{dp,1} = T_{sat@P_v} = 26.01^o C$$

So for $T_2 = 20^oC < T_{dp,1}$, some water-vapor will condense. Let's assume that the condensed water leaves the air conditioner at 20°C. Some say the water leaves at the average of 26 and 20°C; however, 20°C is adequate for our use here.

Apply the conservation of energy to the steady-flow control volume

$$\dot{Q}_{net} + \sum_{inlets} \dot{m}_i \left(h + \frac{\vec{V}^2}{2} + gz\right)_i = \dot{W}_{net} + \sum_{exits} \dot{m}_e \left(h + \frac{\vec{V}^2}{2} + gz\right)_e$$

Neglecting the kinetic and potential energies and noting that the work is zero, we get

$$\dot{Q}_{net} + \dot{m}_{a1} h_{a1} + \dot{m}_{v1} h_{v1} = \dot{m}_{a2} h_{a2} + \dot{m}_{v2} h_{v2} + \dot{m}_{l2} h_{l2}$$

Conservation of mass for the steady-flow control volume is

$$\sum_{inlets} \dot{m}_i = \sum_{exits} \dot{m}_e$$

For the dry air:

$$\dot{m}_{a1} = \dot{m}_{a2} = \dot{m}_a$$

For the water vapor:

$$\dot{m}_{v1} = \dot{m}_{v2} + \dot{m}_{l2}$$

The mass of water that is condensed and leaves the control volume is

$$\begin{aligned} \dot{m}_{l2} &= \dot{m}_{v1} - \dot{m}_{v2} \\ &= \dot{m}_a (\omega_1 - \omega_2) \end{aligned}$$

Divide the conservation of energy equation by $\dot{m}_a$, then

$$\frac{\dot{Q}_{net}}{\dot{m}_a} + h_{a1} + \omega_1 h_{v1} = h_{a2} + \omega_2 h_{v2} + (\omega_1 - \omega_2) h_{l2}$$

$$\frac{\dot{Q}_{net}}{\dot{m}_a} = h_{a2} - h_{a1} + \omega_2 h_{v2} - \omega_1 h_{v1} + (\omega_1 - \omega_2) h_{l2}$$

$$\frac{\dot{Q}_{net}}{\dot{m}_a} = C_{pa}(T_2 - T_1) + \omega_2 h_{v2} - \omega_1 h_{v1} + (\omega_1 - \omega_2) h_{l2}$$

Now to find the ω's and *h*'s.

$$\omega_1 = \frac{0.622 P_{v1}}{P_1 - P_{v1}} = \frac{0.622(3.398)}{100 - 3.398}$$

$$= 0.02188 \frac{kg_v}{kg_a}$$

$$P_{v2} = \phi_2 P_{g2}$$

$$= (0.95)(2.339\, kPa) = 2.222\, kPa$$

$$\omega_2 = \frac{0.622 P_{v2}}{P_2 - P_{v2}} = \frac{0.622(2.222)}{98 - 2.222}$$

$$= 0.01443 \frac{kg_v}{kg_a}$$

Using the steam tables, the *h*'s for the water are

$$h_{v1} = 2555.6 \frac{kJ}{kg_v}$$

$$h_{v2} = 2537.4 \frac{kJ}{kg_v}$$

$$h_{l2} = 83.91 \frac{kJ}{kg_v}$$

The required heat transfer per unit mass of dry air becomes

$$\frac{\dot{Q}_{net}}{\dot{m}_a} = C_{pa}(T_2 - T_1) + \omega_2 h_{v2} - \omega_1 h_{v1} + (\omega_1 - \omega_2) h_{l2}$$

$$= 1.005 \frac{kJ}{kg_a \cdot^o C}(20 - 30)^o C + 0.01443 \frac{kg_v}{kg_a}(2537.4 \frac{kJ}{kg_v})$$

$$- 0.02188 \frac{kg_v}{kg_a}(2555.6 \frac{kJ}{kg_v}) + (0.02188 - 0.01443) \frac{kg_v}{kg_a}(83.91 \frac{kJ}{kg_v})$$

$$= -8.627 \frac{kJ}{kg_a}$$

The heat transfer from the atmospheric air is

$$q_{out} = \frac{-\dot{Q}_{net}}{\dot{m}_a} = +8.627\,\frac{kJ}{kg_a}$$

The mass flow rate of dry air is given by

$$\dot{m}_a = \frac{\dot{V}_1}{v_1}$$

$$v_1 = \frac{R_a T_1}{P_{a1}} = \frac{0.287\,\frac{kJ}{kg_a \cdot K}(30+273)K}{(100-3.398)kPa}\,\frac{m^3 kPa}{kJ}$$

$$= 0.90\,\frac{m^3}{kg_a}$$

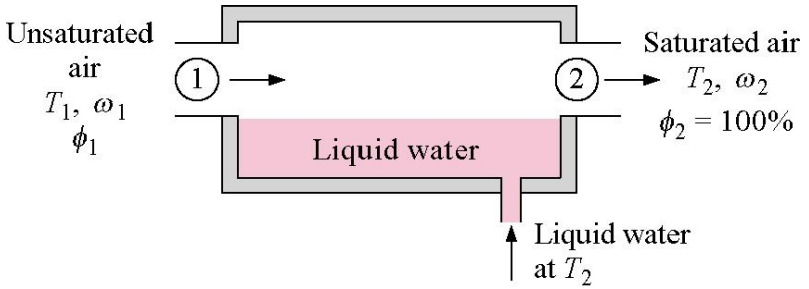
$$\dot{m}_a = \frac{200\,\frac{m^3}{\min}}{0.90\,\frac{m^3}{kg_a}} = 222.2\,\frac{kg_a}{\min}$$

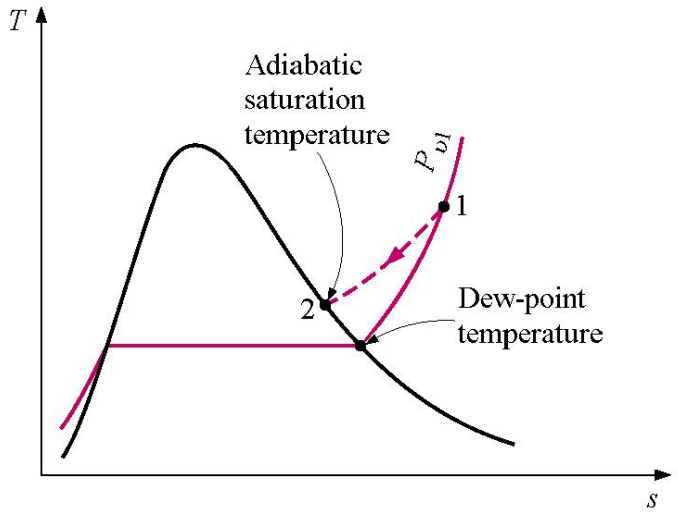
$$\dot{Q}_{out} = \dot{m}_a q_{out} = 222.2 \frac{kg_a}{\min}(8.627 \frac{kJ}{kg_a}) \frac{1\min}{60s} \frac{1kWs}{kJ}$$

$$= 31.95 kW = 9.08 Tons$$

The Adiabatic Saturation Process

Air having a relative humidity less than 100 percent flows over water contained in a well-insulated duct. Since the air has $\phi < 100$ percent, some of the water will evaporate and the temperature of the air-vapor mixture will decrease.





If the mixture leaving the duct is saturated and if the process is adiabatic, the temperature of the mixture on leaving the device is known as the **adiabatic saturation temperature**.

For this to be a steady-flow process, makeup water at the adiabatic saturation temperature is added at the same rate at which water is evaporated.

We assume that the total pressure is constant during the process.

Apply the conservation of energy to the steady-flow control volume

$$\dot{Q}_{net} + \sum_{inlets} \dot{m}_i \left(h + \frac{\vec{V}^2}{2} + gz\right)_i = \dot{W}_{net} + \sum_{exits} \dot{m}_e \left(h + \frac{\vec{V}^2}{2} + gz\right)_e$$

Neglecting the kinetic and potential energies and noting that the heat transfer and work are zero, we get

$$\dot{m}_{a1} h_{a1} + \dot{m}_{v1} h_{v1} + \dot{m}_{l2} h_{l2} = \dot{m}_{a2} h_{a2} + \dot{m}_{v2} h_{v2}$$

Conservation of mass for the steady-flow control volume is

$$\sum_{inlets} \dot{m}_i = \sum_{exits} \dot{m}_e$$

For the dry air:

$$\dot{m}_{a1} = \dot{m}_{a2} = \dot{m}_a$$

For the water vapor:

$$\dot{m}_{v1} + \dot{m}_{l2} = \dot{m}_{v2}$$

The mass flow rate water that must be supplied to maintain steady-flow is,

$$\begin{aligned} \dot{m}_{l2} &= \dot{m}_{v2} - \dot{m}_{v1} \\ &= \dot{m}_a(\omega_2 - \omega_1) \end{aligned}$$

Divide the conservation of energy equation by $\dot{m}_a$, then

$$h_{a1} + \omega_1 h_{v1} + (\omega_2 - \omega_1) h_{l2} = h_{a2} + \omega_2 h_{v2}$$

What are the knowns and unknowns in this equation?

Solving for ω_1

$$\begin{aligned} \omega_1 &= \frac{h_{a2} - h_{a1} + \omega_2(h_{v2} - h_{l2})}{(h_{v1} - h_{l2})} \\ &= \frac{C_{pa}(T_2 - T_1) + \omega_2 h_{fg2}}{(h_{g1} - h_{f2})} \end{aligned}$$

Since ω_1 is also defined by

$$\omega_1 = 0.622 \frac{P_{v1}}{P_1 - P_{v1}}$$

We can solve for P_{v1}.

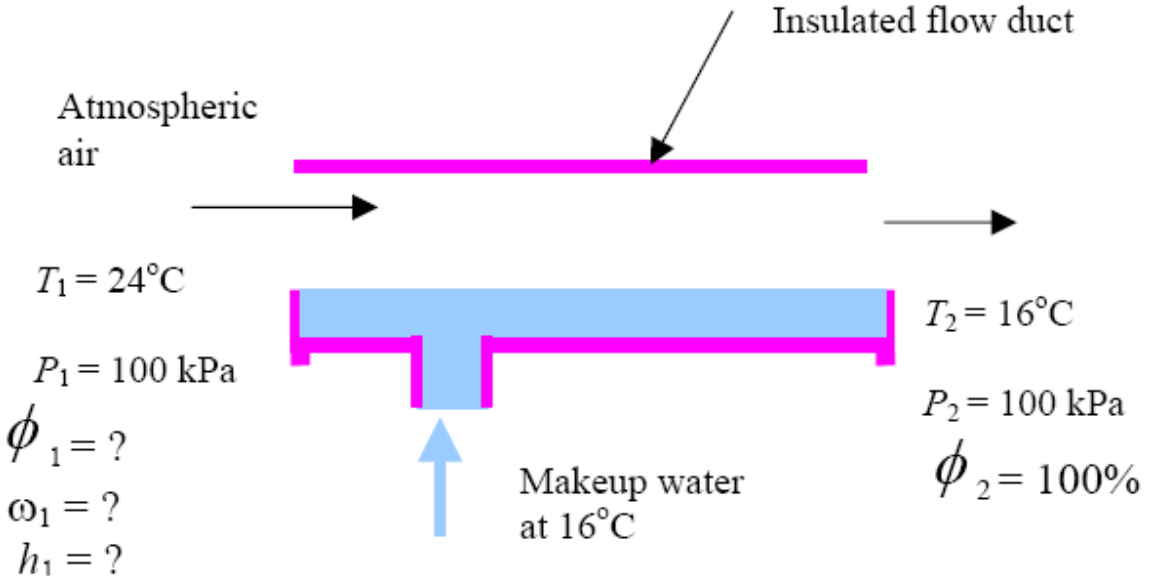
$$P_{v1} = \frac{\omega_1 P_1}{0.622 + \omega_1}$$

Then the relative humidity at state 1 is

$$\phi_1 = \frac{P_{v1}}{P_{g1}}$$

Example 14-4

For the adiabatic saturation process shown below, determine the relative humidity, humidity ratio (specific humidity), and enthalpy of the atmospheric air per mass of dry air at state 1.



$$\left.\begin{matrix}\phi_2 = 100\% \\ T_2 = 16^o C\end{matrix}\right\} \begin{cases} P_{v2} = \phi_2 P_{g2} = 1.0(1.818kPa) \\ \quad = 1.818kPa \\ \omega_2 = \dfrac{0.622P_{v2}}{P_2 - P_{v2}} = \dfrac{0.622(1.818)}{100 - 1.818} \\ \quad = 0.0115 \dfrac{kg_v}{kg_a} \end{cases}$$

Using the steam tables:

$$h_{f2} = 67.2 \frac{kJ}{kg_v}$$

$$h_{v1} = 2544.7 \frac{kJ}{kg_v}$$

$$h_{fg2} = 2463.0 \frac{kJ}{kg_v}$$

From the above analysis

$$\omega_1 = \frac{C_{pa}(T_2 - T_1) + \omega_2 h_{fg2}}{(h_{g1} - h_{f2})}$$

$$= \frac{1.005 \frac{kJ}{kg_a{}^{o}C}\left(16 - 24\right)^{o} C + 0.0115 \frac{kg_v}{kg_a}\left(2463.0 \frac{kJ}{kg_v}\right)}{(2544.7 - 67.2)\frac{kJ}{kg_v}}$$

$$= 0.00822 \frac{kg_v}{kg_a}$$

We can solve for P_{v1}.

$$P_{v1} = \frac{\omega_1 P_1}{0.622 + \omega_1}$$
$$= \frac{0.00822(100kPa)}{0.622 + 0.00822}$$
$$= 1.3\,kPa$$

Then the relative humidity at state 1 is

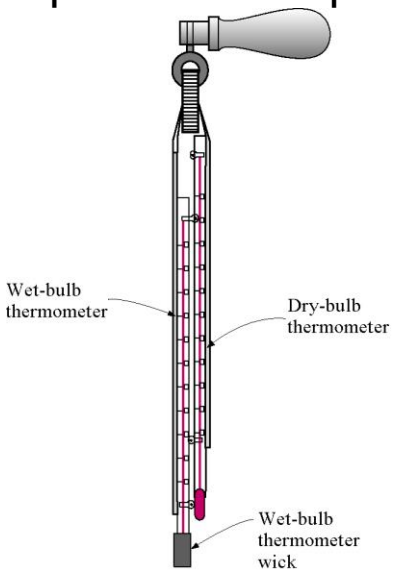
$$\phi_1 = \frac{P_{v1}}{P_{g1}} = \frac{P_{v1}}{P_{sat@24^oC}}$$
$$= \frac{1.3\,kPa}{3.004\,kPa} = 0.433 \; or \; 43.3\%$$

The enthalpy of the mixture at state 1 is

$$h_1 = h_{a1} + \omega_1 h_{v1}$$
$$= C_{pa} T_1 + \omega_1 h_{v1}$$
$$= 1.005 \frac{kJ}{kg_a \cdot^o C}(24^oC) + 0.00822 \frac{kg_v}{kg_a} 2544.7 \frac{kJ}{kg_v}$$
$$= 45.04 \frac{kJ}{kg_a}$$

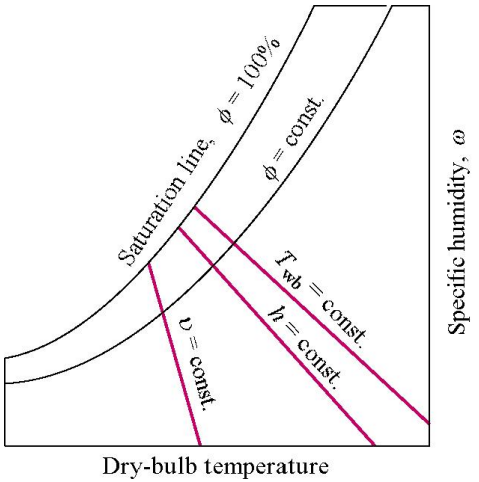
Wet-Bulb and Dry-Bulb Temperatures

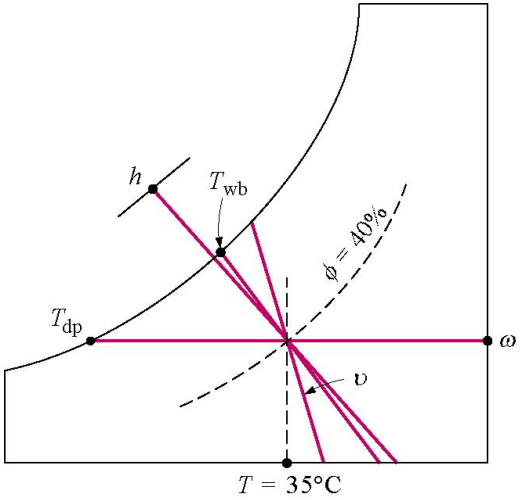
In normal practice, the state of atmospheric air is specified by determining the wet-bulb and dry-bulb temperatures. These temperatures are measured by using a device called a psychrometer. The psychrometer is composed of two thermometers mounted on a sling. One thermometer is fitted with a wet gauze and reads the wet-bulb temperature. The other thermometer reads the dry-bulb, or ordinary, temperature. As the psychrometer is slung through the air, water vaporizes from the wet gauze, resulting in a lower temperature to be registered by the thermometer. The dryer the atmospheric air, the lower the wet-bulb temperature will be. When the relative humidity of the air is near 100 percent, there will be little difference between the wet-bulb and dry-bulb temperatures. The wet-bulb temperature is approximately equal to the adiabatic saturation temperature. The wet-bulb and dry-bulb temperatures and the atmospheric pressure uniquely determine the state of the atmospheric air.



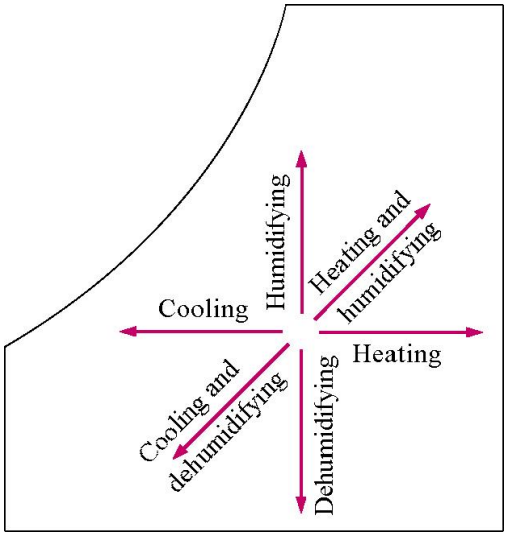
The Psychrometric Chart

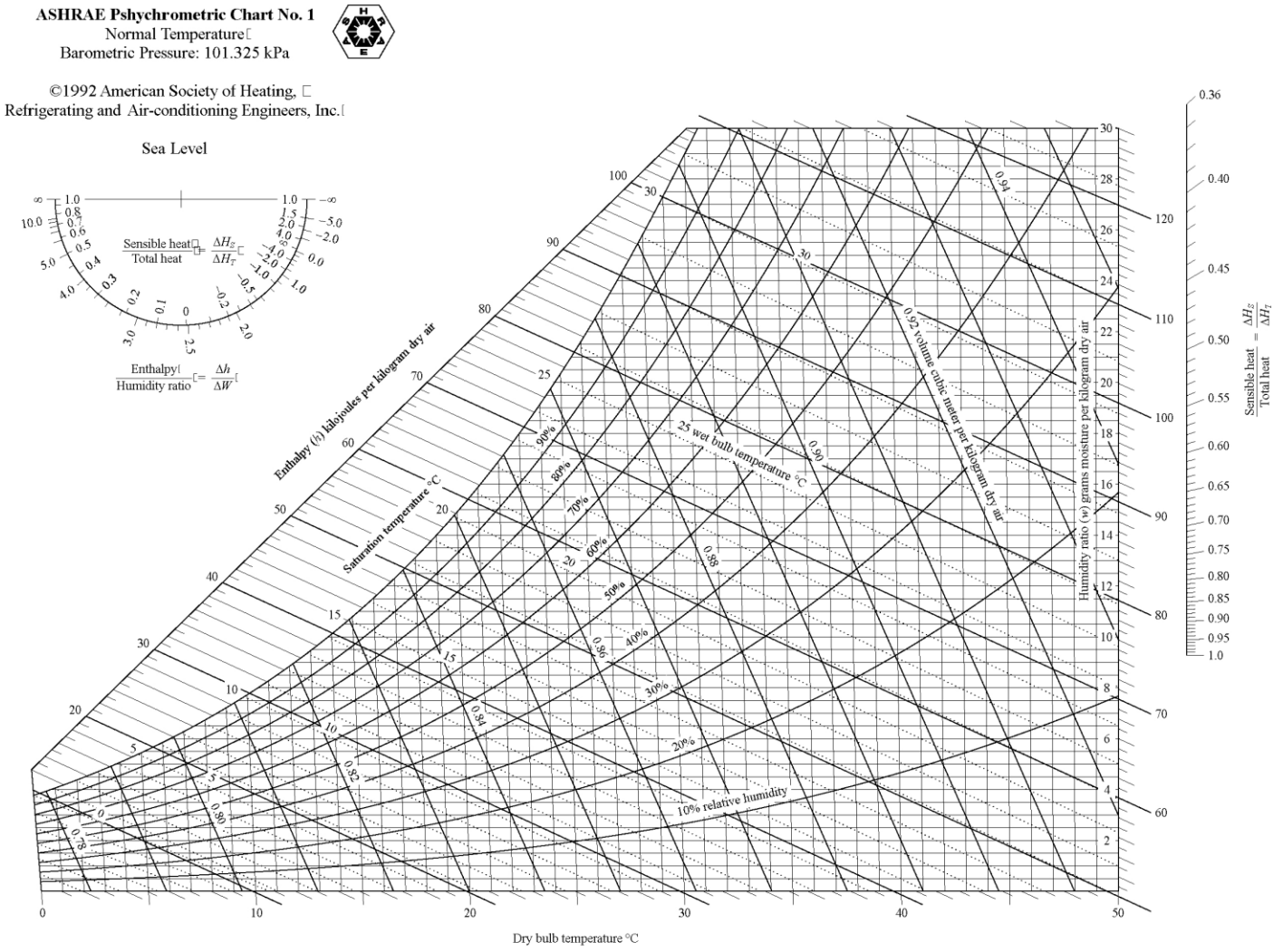
For a given, fixed, total air-vapor pressure, the properties of the mixture are given in graphical form on a psychrometric chart.





The air-conditioning processes:




ASHRAE Pshychrometric Chart No. 1
Normal Temperature
Barometric Pressure: 101.325 kPa
©1992 American Society of Heating,
Refrigerating and Air-conditioning Engineers, Inc.
Sea Level
Sensible heat / Total heat = ΔHs / ΔHt
Enthalpy / Humidity ratio = Δh / ΔW
Enthalpy (h) kilojoules per kilogram dry air
Saturation temperature °C
Humidity ratio (w) grams moisture per kilogram dry air
Sensible heat / Total heat = ΔHs / ΔHt
25 wet bulb temperature °C
0.92 volume cubic meter per kilogram dry air
90%
80%
70%
60%
50%
40%
30%
20%
10% relative humidity
Dry bulb temperature °C
Prepared by Center for Applied Thermodynamic Studies, University of Idaho

Example 14-5

Determine the relative humidity, humidity ratio (specific humidity), enthalpy of the atmospheric air per mass of dry air, and the specific volume of the mixture per mass of dry air at a state where the dry-bulb temperature is 24°C, the wet-bulb temperature is 16°C, and atmospheric pressure is 100 kPa.

From the psychrometric chart read

$$\phi = 44\%$$

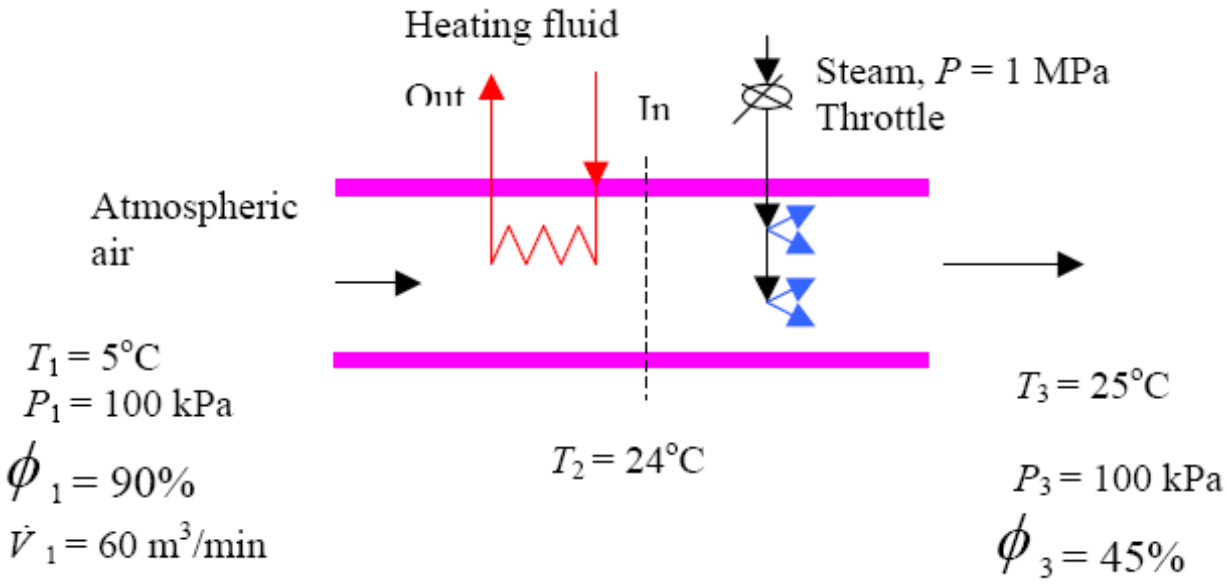
$$\omega = 8.0 \frac{g_v}{kg_a} = 0.008 \frac{kg_v}{kg_a}$$

$$h = 46 \frac{kJ}{kg_a}$$

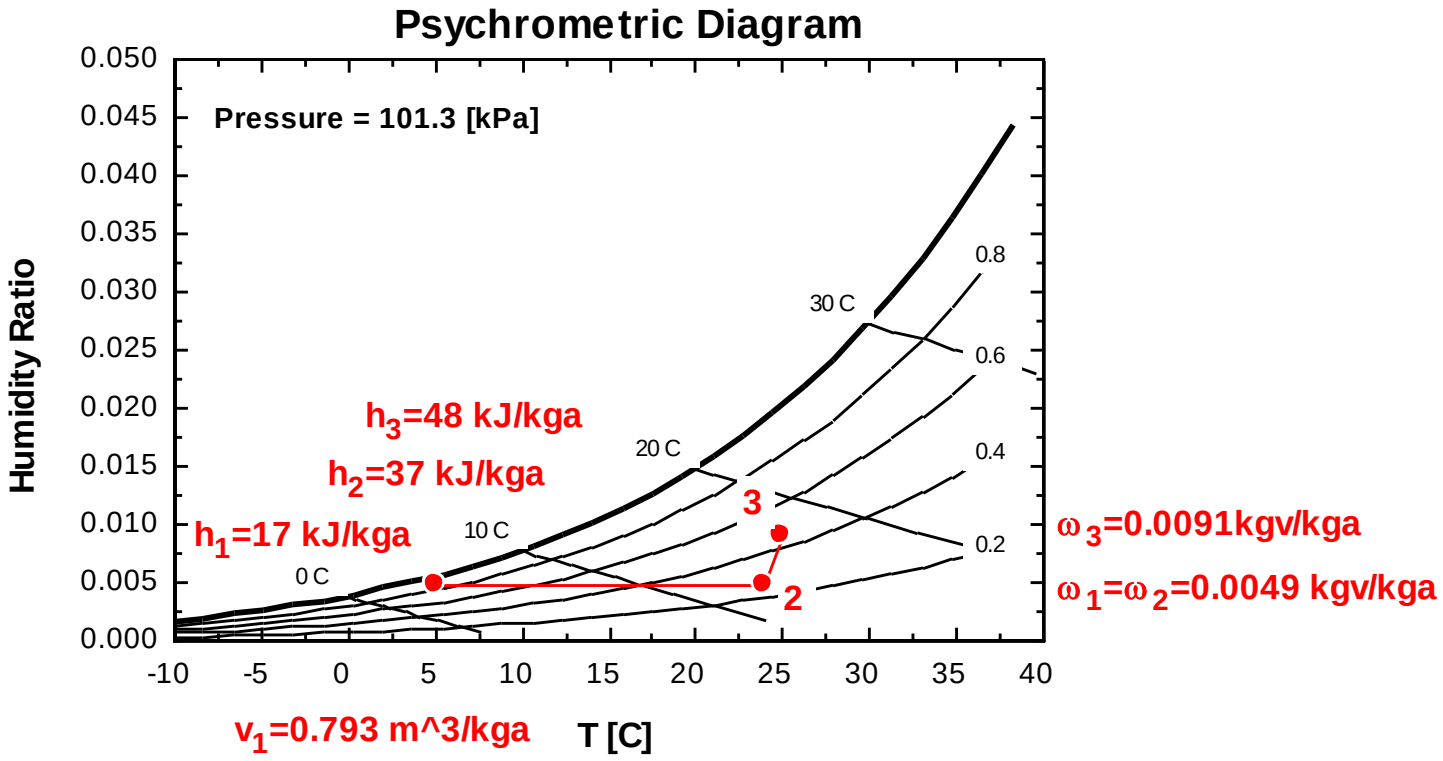
$$v = 0.853 \frac{m^3}{kg_a}$$

Example 14-6

For the air-conditioning system shown below in which atmospheric air is first heated and then humidified with a steam spray, determine the required heat transfer rate in the heating section and the required steam temperature in the humidification section when the steam pressure is 1 MPa.



The psychrometric diagram is



Apply conservation of mass and conservation of energy for steady-flow to process 1-2.

Conservation of mass for the steady-flow control volume is

$$\sum_{inlets} \dot{m}_i = \sum_{exits} \dot{m}_e$$

For the dry air

$$\dot{m}_{a1} = \dot{m}_{a2} = \dot{m}_a$$

For the water vapor (note: no water is added or condensed during simple heating)

$$\dot{m}_{v1} = \dot{m}_{v2}$$

Thus,

$$\omega_2 = \omega_1$$

Neglecting the kinetic and potential energies and noting that the work is zero, and letting the enthalpy of the mixture per unit mass of air *h* be defined as

$$h = h_a + \omega h_v$$

we obtain

$$\dot{E}_{in} = \dot{E}_{out}$$

$$\dot{Q}_{in} + \dot{m}_a h_1 = \dot{m}_a h_2$$

$$\dot{Q}_{in} = \dot{m}_a (h_2 - h_1)$$

Now to find the $\dot{m}_a$ and h's using the psychrometric chart.

At T_1 = 5oC, ϕ_1 = 90%, and T_2 = 24^{o}C:

$$\left.\begin{array}{l} \phi_1 = 90\% \\ T_1 = 5^o C \end{array}\right\} \left\{\begin{array}{l} h_1 = 17\,\dfrac{kJ}{kg_a} \\ \omega_1 = 0.0049\,\dfrac{kg_v}{kg_a} \\ v_1 = 0.793\,\dfrac{m^3}{kg_a} \end{array}\right.$$

$$\left.\begin{array}{l} \omega_2 = \omega_1 = 0.0049\,\dfrac{kg_v}{kg_a} \\ T_2 = 24^o C \end{array}\right\} \left\{ h_2 = 37\,\frac{kJ}{kg_a} \right.$$

The mass flow rate of dry air is given by

$$\dot{m}_a = \frac{\dot{V}_1}{v_1}$$

$$\dot{m}_a = \frac{60\,\frac{m^3}{\min}}{0.793\,\frac{m^3}{kg_a}} = 75.66\,\frac{kg_a}{\min}\,\frac{1\min}{60s} = 1.261\,\frac{kg_a}{s}$$

The required heat transfer rate for the heating section is

$$\dot{Q}_{in} = 1.261\,\frac{kg_a}{s}(37-17)\,\frac{kJ}{kg_a}\,\frac{1kWs}{kJ}$$

$$= 25.22\,kW$$

This is the required heat transfer to the atmospheric air. List some ways in which this amount of heat can be supplied.

At the exit, state 3, T_3 = 25°C and ϕ_3 = 45%. The psychrometric chart gives

$$\left.\begin{array}{l}\phi_3 = 45\% \\ T_3 = 25^o\,C\end{array}\right\} \left\{\begin{array}{l} h_3 = 48\,\frac{kJ}{kg_a} \\ \omega_3 = 0.0089\,\frac{kg_v}{kg_a} \\ v_3 = 0.858\,\frac{m^3}{kg_a}\end{array}\right.$$

Apply conservation of mass and conservation of energy to process 2-3.

Conservation of mass for the steady-flow control volume is

$$\sum_{inlets} \dot{m}_i = \sum_{exits} \dot{m}_e$$

For the dry air

$$\dot{m}_{a2} = \dot{m}_{a3} = \dot{m}_a$$

For the water vapor (note: water is added during the humidification process)

$$\dot{m}_{v2} + \dot{m}_s = \dot{m}_{v3}$$

$$\dot{m}_s = \dot{m}_{v3} - \dot{m}_{v2}$$

$$\dot{m}_s = \dot{m}_a (\omega_3 - \omega_2)$$

$$= 1.261 \frac{kg_a}{s} (0.0089 - 0.0049) \frac{kg_v}{kg_a}$$

$$= 0.00504 \frac{kg_v}{s}$$

Neglecting the kinetic and potential energies and noting that the heat transfer and work are zero, the conservation of energy yields

$$\dot{E}_{in} = \dot{E}_{out}$$

$$\dot{m}_a h_2 + \dot{m}_s h_s = \dot{m}_a h_3$$

$$\dot{m}_s h_s = \dot{m}_a (h_3 - h_2)$$

Solving for the enthalpy of the steam,

$$\dot{m}_a (\omega_3 - \omega_2) h_s = \dot{m}_a (h_3 - h_2)$$

$$h_s = \frac{h_3 - h_2}{\omega_3 - \omega_2}$$

$$h_s = \frac{(48 - 37)\dfrac{kJ}{kg_a}}{(0.0089 - 0.0049)\dfrac{kg_v}{kg_a}}$$

$$= 2750 \frac{kJ}{kg_v}$$

At P_s = 1 MPa and h_s = 2750 kJ/kg$_v$, T_s = 179.88^{o}C and the quality x_s = 0.985.

See the text for applications involving cooling with dehumidification, evaporative cooling, adiabatic mixing of airstreams, and wet cooling towers.