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Nam[redacted]
IDNo[redacted]

Wollo University, KIOT
Department Of Pre-Engineering
Mathematics for Natural Science Mid-Examination.

Instruction:

- **Read all the instructions carefully.**
- **Don't detach any paper from the exam paper.**
- **Any attempt of cheating will make your result disqualify.**
- **Switch off your cell phone**

PART I: Choose the best answer from the given alternatives and put your choice in the provided space. (5%)

[redacted] 1) Which one of the following statements is **not true?**

A) $\emptyset \in P(\emptyset)$ C) For any set A, $A \in P(A)$

B) For any set A, $\emptyset \subseteq P(A)$ D) For any set A, $P(A) \subset A$

[redacted] 2) One of the following is an open proposition

A) $x + 9 = 9$

B) There is no real number x such that $x^2 = 4$.

C) For all $x \in \mathbb{R}$, $x^2 + 6x - 12 = 0$.

D) For every real number x, $x^2 + 1 \geq 0$.

[redacted] 3) The negation of a proposition $(P \wedge \neg Q) \Rightarrow Q$ is

A) $\neg P \vee Q$ C) $P \wedge \neg Q$

B) $\neg P \wedge Q$ D) $P \vee \neg Q$

[redacted] 4) If $B \subseteq A$, $A \cap B' = \{1,4,5\}$ and $A \cup B = \{1,2,3,4,5,6\}$, then $B =$ _____

A) $\{1,4,5\}$ C) $\{1,2,3,4,5,6\}$

B) $\{2,3,6\}$ D) $\{1,2,3,4,5\}$

[redacted] 5) Which one of the following statement is correct?

A) The sum of any two rational numbers is rational.

B) The sum of any two irrational numbers is irrational.

C) The product of any two irrational numbers is rational.

D) The product of any two irrational numbers is irrational.

PART II: Write "True" if the statement is correct and "False" if it is incorrect.(5%)

[redacted] 1) The truth value of a proposition **"2 is an odd and even number if Addis Ababa is the Capital city of Ethiopia."** is false.

[redacted] 2) The set of natural numbers is closed under addition and subtraction.

[redacted] 3) Two sets A and B are said to be disjoint if $A \cap B = \{\phi\}$.

[redacted] 4) For some real numbers x, such that $|x| = -1$ is an example of proposition.

[redacted] 5) The argument of product of two complex numbers is the product of their arguments.

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II. Choose the Correct Answer (1.5pt each)

1. Which one of the following proposition is a tautology [redacted]

A. $(pvq) \Longrightarrow q$ B. $pv(q \Longrightarrow p)$ C. $pv(p \Longrightarrow \neg q)$ D. B and C

2. The proposition $p\wedge(\neg p \wedge q)$ is [redacted]

A. A tautology B.A contradiction C. Conditional D. Neither contradiction nor Tautology

3. Which one of the following is not an open proposition [redacted]

A. y is the cube of x B. x is a square of a number C. x is a street in Addis Ababa

D. Asmara is a city in Europe

4. If P(x,y):x<y then which of the following proposition has **False** truth value? [redacted]

A. $(\exists x)(\exists y)p(x,y$ B. $(\forall x)(\exists y)p(x,y)$ C. $(\forall x)(\forall y)p(x,y)$ D. None of the above

5. Let $A\cap B = \{1,2\}, A \cup B = \{1,2,3,4,5\}$, *then* $A\Delta B$ *is equal to* . [redacted]

A. {3,4,5} B. {1,2,3,4,5} C. {1,3,5} D. {1,2}

6. Which of the following proposition is equivalent to $P\vee q$? [redacted]

A. $\neg(p\wedge q)$ B. $\neg(\neg p\vee\neg q)$ C. $p\Rightarrow \neg q$ D. $\neg p\Rightarrow q$

7. Let A= {x: x is a letter in the word FOLLOW}, B={y: y is a letter in the word WOLF}. Which o

of the following is true about A and B ?. [redacted]

A. A and B are disjoint

B. $A\cap B = A \setminus B$

C. $A\cap B = A \cup B$

D. None of the above

PART III: Write the most simplified answer on the space provided. (6*1=6)

1. The truth value of a quantified proposition $(\exists x \in \mathbb{R})(\forall y \in \mathbb{R})(x + y + 3 = 3)$ is [redacted]
2. If the truth value of $p \Rightarrow q$, $q \Rightarrow r$ and p are "true", then the truth value of r is [redacted]
3. Let $A \cup B = \{a,b,c,d,e,x,y,z\}$ and $A \cap B = \{b,e,\square\}$ be sets. If $B - A = \{x,y\}$, then $A =$ [redacted]
4. If p and q are propositions, then the converse and contrary of $q \Rightarrow \neg q$ are [redacted] and [redacted] respectively.
5. If $A = \{x : -19 \leq x \leq 8 \wedge x \in \mathbb{N}\}$, then the description of set A in roster method is [redacted]
6. The fraction form of a rational number 6.013 is [redacted]
7. How many elements does A contain if it has 255 proper subsets? [redacted]
8. If the GCF of two numbers is 5, their LCM is 1400 and one of the numbers is 175, then the second number is [redacted]
9. The polar form of a complex number $(2 - 2i)^{10}$ is [redacted]

PART IV: **Attempt all questions by showing all the necessary steps clearly and neatly.**

1) For the argument **"If I work hard, then I can get a lot of money. If I get a lot of money, then I can buy a car. I cannot buy a car. Therefore, I did not work hard."** (3+2+2 pts)
   a) Identify the premises.
   b) Write the argument form.
   c) Check the validity.

2) Using Principle of Mathematical Induction, show that the expression $n^2 + 2n$ is divisible by 3 for every natural number n. (5 pts.)

3) Suppose that $z = \frac{1+i}{1-i} - (1+2i)(2+2i) + \frac{3-i}{1+i}$. Then (2.5+2.5 pts.)
   a) Write z in standard form.
   b) Find $|\bar{z}|$

$p \Rightarrow r$ $\quad p \equiv T$

$p \Rightarrow r \equiv T$

$T \Rightarrow r \equiv f$

$A \cup B = \{a, b, c, d, e, x, y, z\}$ $\quad A = \{a, b, c,$

$A \cap B = \{b, e, \square\}$

$B - A = \{x, y\}$

$B = \{b, e, x, y\}$

$z^2 =$ (8)

$p \Rightarrow q$ $\quad p \Rightarrow p$ converse

8. Which one the following pairs of sets are equal.

A. $M = \{a, b, c, d\}$and $K = \{p, q, r, s\}$

B. $M = \{x: x\ is\ a\ factor\ of\ 10\}$ and $K = \{1, 2, 5, 4, 10\}$

C. $M = \{1, 4, 9\}$ and $K = \{x: x = n^2, n \in N\ and\ n < 3\}$

D. $M = \{x: x\ is\ an\ even\ natural\ numbers\ less\ than\ 10\}$ and $K = \{x: x = 2n, n \in N\ and\ n < 5\}$

9. For two non-empty sets A and B if $A \cap B' = \emptyset$ then which one of the following is true?

A. $n(A) = n(B)$

B. B is a proper subset set of A

C. A is a proper subset of B

D. None of the above

10. Let A= $\{5, \{6\}, \{7\}\}$, which of the following is not **True** if P(A) denotes the power set of A?

A. $\emptyset \epsilon P(A)$ B. $\emptyset \subseteq P(A)$ C. $\{5, \{6\}\} \subseteq P(A)$ D. $\{5, \{6\}\} \in P(A)$

III. Answer the Following Questions Clearly and Neatly

1. Let p=Abebe is rich

q= Abebe is happy

Write each of the following in symbolic form **(1.5 pts each)**

a. Abebe is poor but happy

b. Abebe is neither rich nor happy

2. If x is less than zero ,then x is not positive **(1.5 pts each)**

Write the converse and the contra positive of the above proposition

Converse:- If x is not positive, then it is less than zero

Contrapositive:- If x is positive, then it is greater or equal zero

3. Using formal proof, show that $\neg p \Longrightarrow \neg q, q \vdash p$ is valid. **(2 pts each)**

1. q, T ... Premises
2. $\neg q = f$ ... by ne 1 premises
3. $\neg p = \neg \neg q = T$ ... premises
4. p is valid

∴ p is T

p = T

So: $\neg p = \neg \neg q$. is True

∴ p is valid

4. Find the number of subsets of the A={$x: x$ is given inter and $-1 \leq x \leq 3$} **(2 pts each)**

integer

A = {-1, 0, 1, 2, 3}

HAWASSA UNIVERSITY

DEPARTMENT OF MATHEMATICS

December 25, 2019

Name: _____ ID.No. _____ Sec._____

Instructor's name:_____ TIME allowed 2 : 30 hrs.

Part I. Choose the best answer to each of the following questions and write your answer on the space provided in the answer sheet [1.5 points each]

1. Let U= the set of natural numbers.
   $P(x) = x$ is prime. $q(x) = x$ is even. $r(x) = x$ is odd.
   Then which of the following statement is false?
   A) $(\exists x)[p(x) \vee r(x)]$ B) $(\exists x)[p(x) \wedge q(x)]$
   C) $(\exists x)[p(x) \Rightarrow q(x)]$ D) $(\forall x)[(p(x) \wedge r(x)) \Rightarrow q(x)]$

2. Which of the following is not necessarily true for any sets A and B of the universal set U?
   A) If $A \subseteq A'$, then $A = \emptyset$ B) $A \cap B = A \setminus (A \setminus B)$
   C) $B \setminus A = B' \cap A$ D) If $A \setminus B = B \setminus A$ then $A = B$

3. Which of the following pair of statements are not equivalent to each other?
   A) $p \Rightarrow (q \Rightarrow r)$, $(p \wedge q) \Rightarrow r$ B) $p \Rightarrow (q \vee r)$, $(r \wedge p) \Rightarrow q$
   C) $p \Rightarrow q$, $q \vee \neg p$ D) $\neg(\neg p \wedge q)$, $p \vee \neg q$

4. The polar form of $z = \dfrac{1+i\sqrt{3}}{1+i}$ is
   A) $\sqrt{2}\left(\cos(\frac{\pi}{3}) + i\sin(\frac{\pi}{3})\right)$ B) $\sqrt{2}\left(\cos(\frac{\pi}{12}) + i\sin(\frac{\pi}{12})\right)$
   C) $\frac{\sqrt{2}}{2}\left(\cos(105^0) + i\sin(105^0)\right)$ D) $2\left(\cos(\frac{\pi}{6}) + i\sin(\frac{\pi}{6})\right)$

5. If $2 \in A$ but $-2 \notin A$ and $-2 \in B$ but $2 \notin B$, then which of the following is necessarily true?
   A) $\{-2, 2\} \subseteq A' \cup B'$ B) $A \setminus B = \{2\}$ C) $A \subseteq B$ D) $A \cap B = \emptyset$

6. Which of the following is not true on the set $M = \{1, \{a\}, \emptyset, \{\emptyset\}\}$
   A) $\{\emptyset\} \subseteq M$ B) $\{\emptyset\} \in M$ C) $\{\emptyset, \{\emptyset\}\} \in M$ D) $\emptyset \subseteq M$

Part II. Give the most simplified short answers to each of the following questions on the space provided in the answer sheet [1.5 points each]

1. When $\{x \in \mathbb{R} : x(x^2-2) \neq 0\}$ expressed as a union of intervals is _____

2. Let $U = \{n \in \mathbb{N} : n \leq 10\}$, $A = \{1,2,3,4,9\}$,
   $B = \{2,3,4,6,8\}$, $C = \{2,4,6,7,8,9\}$
   Then a) $(A-B) \cup C' =$ _____, b) $(A-C)' \cap B' =$ _____

3. Let $w = 2-i$ and $z = -1+i$, then $z^2 w \overline{z}^2$ is equal to _____

4. If $(5-2i)(a+bi) = 11i+16$, Then $a =$ _____ and $b =$ _____

5. Let $p(x)$: "x is negative" , $q(x)$: "$x < 5$". Then give the negation of $(\forall x)[q(x) \Rightarrow p(x)]$
   _____

6. The fraction form of $2.2\overline{34}$ is _____

7. If $A = \left\{(-1)^n \left(\frac{n-1}{n}\right)\right\}_{n=1}^{\infty}$, then
   a) *lub* of A = _____, b) *glb* of A = _____

8. If $\neg r \wedge (p \Leftrightarrow q)$ has a truth value of True, then the truth value of $p \Leftrightarrow (q \Rightarrow r)$ is _____

9. If $z = (\sqrt{3}+i)^6$, then $Re(z) =$ _____

10. Determine the truth value of the following statements if $x, y \in \mathbb{R}$
    a) $(\forall y)(\exists x)(x^2+y^2=1)$ _____ b) $(\exists x)(\forall y)(x < y^2 - y)$ _____

Part III. Work on the following problems by showing all the pertinent information.

1. For the statements p, q and r, use the truth table to show that the statements $p \Rightarrow (q \vee r)$ and $\neg r \Rightarrow (p \Rightarrow q)$ are logically equivalent. [4 points]

2. Using principle of Mathematical induction Prove that
   $1^3 + 2^3 + 3^3 + \cdots + n^3 = \frac{n^2(n+1)^2}{4}$ for $n \in \mathbb{N}$ and $n \geq 1$ [4 points]

3. Suppose **p**: The rain comes. **q** : Crops are ruined. **r**: The people will starve.
   Give the symbolic argument form and determine the validity of
   "If the rain does not come, the crops are ruined and the people will starve. The crops are not ruined or the people will not starve. Therefore, the rain comes" [4 points]
   using formal proof. [4 points]

4. Find the 4^{th} roots of z if $z = -1 - \sqrt{3}i$