THE KENYA NATIONAL EXAMINATIONS COUNCIL
Kenya Certificate of Secondary Education

121/1 — MATHEMATICS — Paper 1

ALT A

Nov. 2019 – 2½ hours

Name ..... Index Number .....

Candidate's Signature ..... Date .....

SCHEME

Instructions to candidates

(a) Write your name and index number in the spaces provided above.
(b) Sign and write the date of examination in the spaces provided above.
(c) This paper consists of **two** sections; **Section I** and **Section II**.
(d) Answer **all** the questions in **Section I** and any **five** questions from **Section II**.
(e) **Show all the steps in your calculations, giving your answers at each stage in the spaces provided below each question.**
(f) Marks may be given for correct working even if the answer is wrong.
(g) **Non-programmable** silent electronic calculators and KNEC Mathematical tables may be used, except where stated otherwise.
(h) **This paper consists of 18 printed pages.**
(i) **Candidates should check the question paper to ascertain that all the pages are printed as indicated and that no questions are missing.**
(j) **Candidates should answer the questions in English.**

For Examiner's Use Only

Section I

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	Total

Section II

17	18	19	20	21	22	23	24	Total

Grand Total

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SECTION I (50 marks)

*Answer **all** the questions in this section in the spaces provided.*

1. Without using mathematical tables or a calculator, evaluate:

$$\frac{5.4}{0.025\times3.6}\times\frac{10000}{10000}$$

(3 marks)

$$\frac{1500}{25}=60$$

2. Express 1728 and 2025 in terms of their prime factors. Hence evaluate:

$$\frac{\sqrt[3]{1728}}{\sqrt{2025}}$$

(4 marks)

$$1728=2\times2\times2\times2\times2\times2\times3\times3\times3$$

$$2025=3\times3\times3\times3\times5\times5$$

$$\frac{\sqrt[3]{2^6\times3^3}}{\sqrt{3^4\times5^2}}=\frac{2^2\times3}{3^2\times5}=\frac{12}{45}=\frac{4}{15}$$

3. Juma left his home at 8.30 a.m. He drove a distance of 140 km and arrived at his aunt's home at 10.15 a.m. Determine the average speed, in km/h, for Juma's journey. (3 marks)

$$S=\frac{D}{T}$$

8.30 am → 10.15 am = 1hr 45min

$$\frac{140}{1.75}=80\text{ km/h}$$

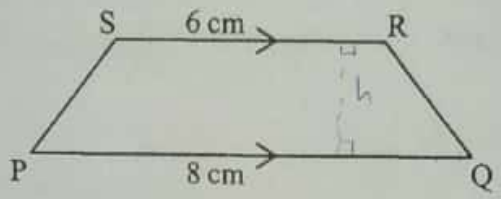
4. Expand and simplify:
$4(q + 6) + 7(q - 3)$.

(2 marks)

$4q + 24 + 7q - 21$

$11q + 3$

5. In the trapezium PQRS shown below, PQ = 8 cm and SR = 6 cm.



If the area of the trapezium is 28 cm², find the perpendicular distance between PQ and SR.

(2 marks)

$A = \frac{1}{2}(a+b)h$

$28 = \frac{1}{2} \times (6+8) \times h$

$\frac{56}{14} = \frac{14h}{14}$

$h = 4$ cm

6. Given that $\sqrt[3]{9^4} = 3^n$, find the value of n.

(3 marks)

$(9^4)^{\frac{1}{3}}$

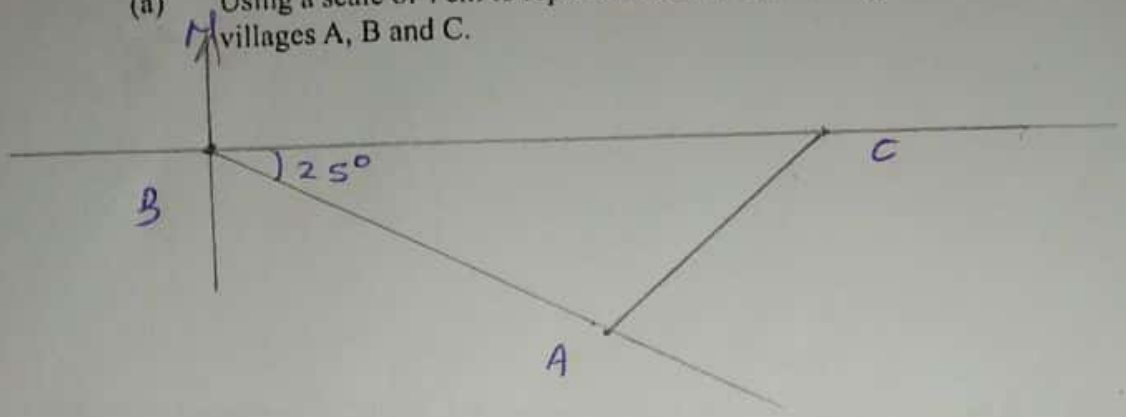
$3^{\frac{8}{3}} = 3^n$

$\frac{8}{3} = n$

$\therefore n = 2\frac{2}{3}$

7. Three villages A, B and C are such that B is 53 km on a bearing of 295° from A and C is 75 km east of B.

(a) Using a scale of 1 cm to represent 10 km, draw a diagram to show the relative positions of villages A, B and C. (2 marks)



(b) Determine the distance, in km, of C from A. (2 marks)

AC = 3.5 cm × 10 = 35 KM

8. A retailer bought a bag of tea leaves. If the retailer were to repack the tea leaves into smaller packets of either 40 g, 250 g or 350 g, determine the least mass, in grams, of the tea leaves in the bag. (3 marks)

2	40	250	350
2	20	125	175
2	10	125	175
5	5	125	175
5	1	25	35
5	1	5	7
7	1	1	7

$2^3 \times 5^3 \times 7$

$= 7000$ g

9. Given that $\sin 2x = \cos(3x - 10°)$, find $\tan x$, correct to 4 significant figures. (3 marks)

$2x + 3x - 10 = 90°$

$\frac{5x}{5} = \frac{100}{5}$

$X = 20$

10. A tourist converted 5820 US dollars into Kenya Shillings at the rate of Ksh 102.10 per dollar. While in Kenya, he spent Ksh 450000 and converted the balance into dollars at the rate of Ksh 103.00 per dollar. Calculate the amount of money, to the nearest dollar, that remained. (3 marks)

$5820 \times 102.10 = Sh.\ 594,222$

$\text{Dollars that remained} = \frac{594222 - 450000}{103.}$

$= 1400 \text{ dollas}$

11. Given that $\boldsymbol{b} = \begin{pmatrix} 2 \\ 4 \end{pmatrix}$, $\boldsymbol{c} = \begin{pmatrix} 3 \\ 2 \end{pmatrix}$ and $\boldsymbol{a} = 3\boldsymbol{c} - 2\boldsymbol{b}$, find the magnitude of $\boldsymbol{a}$, correct to 2 decimal places. (4 marks)

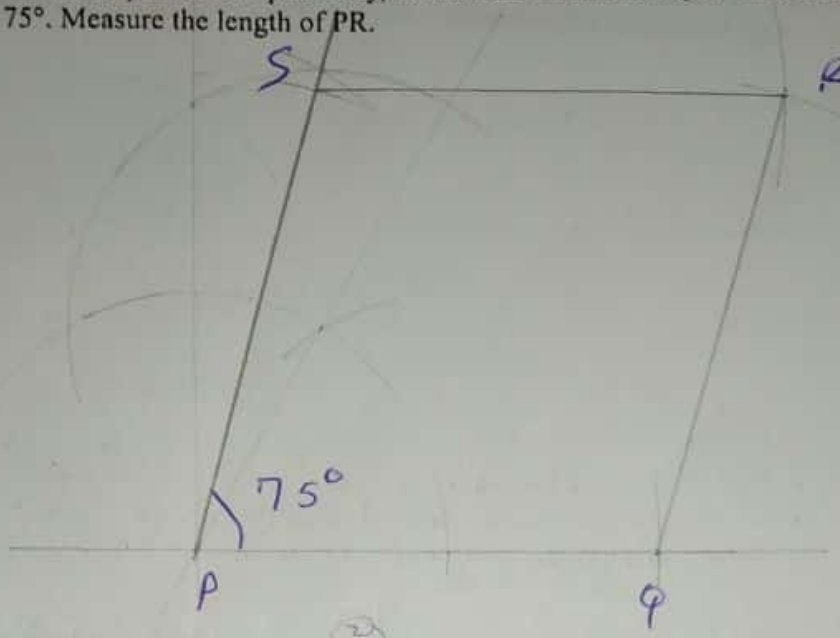
$\underset{\sim}{a} = 3\begin{pmatrix} 3 \\ 2 \end{pmatrix} - 2\begin{pmatrix} 2 \\ 4 \end{pmatrix}$

$\begin{pmatrix} 9 \\ 6 \end{pmatrix} - \begin{pmatrix} 4 \\ 8 \end{pmatrix} = \begin{pmatrix} 5 \\ -2 \end{pmatrix}$

$|\underset{\sim}{a}| = \sqrt{5^2 + (-2)^2}$

$|\underset{\sim}{a}| = 5.39$

12. Using a ruler and a pair of compass only, construct a rhombus PQRS such that PQ = 6 cm and ∠SPQ = 75°. Measure the length of PR. (4 marks)



RR = 9.5 cm

13. Solve the inequality $2x - 1 \leq 3x + 4 < 7 - x$. (3 marks)

$2x - 1 \leq 3x + 4$

$\frac{-5}{5} \leq \frac{5x}{5}$

$x \geq -1$

$3x + 4 < 7 - x$

$\frac{4x}{4} < \frac{3}{4} \Rightarrow x < \frac{3}{4}$

$-1 \leq x < \frac{3}{4}$

14. Given that $\mathbf{A} = \begin{pmatrix} 2 & 3 \\ 4 & 4 \end{pmatrix}$, $\mathbf{B} = \begin{pmatrix} x & 1 \\ 2 & 3 \end{pmatrix}$ and that **AB** is a singular matrix, find the value of x. (3 marks)

$A \cdot B = \begin{pmatrix} 2 & 3 \\ 4 & 4 \end{pmatrix} \cdot \begin{pmatrix} x & 1 \\ 2 & 3 \end{pmatrix} = \begin{pmatrix} 2x+6 & 11 \\ 4x+8 & 16 \end{pmatrix}$

$16(2x+6) - 11(4x+8) = 0$

$32x + 96 - 44x - 88 = 0$

$\frac{-12x}{-12} = \frac{-8}{-12}$

$x = \frac{2}{3}$

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15. A trader bought two types of bulbs A and B at Ksh 60 and Ksh 56 respectively. She bought a total of 50 bulbs of both types at a total of Ksh 2872. Determine the number of type A bulbs that she bought. (3 marks)

$A+B = 50$ - (i)

$60A+56B = 2872$ - (ii)

$A = 50-B$

$60(50-B)+56B = 2872$

$3000 - 60B + 56B = 2872$

$\frac{4B}{4} = \frac{128}{4}$

$B = 32$; $A = 18$

16. A bus plies between two towns P and R via town Q daily. On each day it departs from P at 8.15 a.m. and stops for 40 minutes at Q before proceeding to R.

On a certain day, the bus took 5 hours 40 minutes to travel from P to Q and 3 hours 15 minutes to travel from Q to R. Find, in 24 hour clock system, the time the bus arrived at R. (3 marks)

PQ $\Rightarrow$ 8:15 am + 5hrs 40min = 1:55 PM

at Q 1:55 PM + 40min = 2:35 PM

QR $\Rightarrow$ 2:35 PM + 3hrs 15min = 5:50 PM

The bus arrived at R 1750 hours

SECTION II (50 marks)

Answer any five questions from this section in the spaces provided.

17. A rectangular water tank measures 2.4 m long, 2 m wide and 1.5 m high. The tank contained some water up to a height of 0.45 m.

(a) Calculate the amount of water, in litres, needed to fill up the tank. (3 marks)

$V = 2.4 \times 2 \times 1.05 = 5.04 m^3 \times 1000$

$= 5040 l$

(b) An inlet pipe was opened and water let to flow into the tank at a rate of 10 litres per minute. After one hour, a drain pipe was opened and water allowed to flow out of the tank at a rate of 4 litres per minute.

Calculate:

(i) the height of water in the tank after 3 hours; (4 marks)

In 1hr = $60 \times 10 = 600$ litres were allowed to flow into the tank.

After 3hrs ⇒ $600 \times 3 = 1800 l$

water out after 2hrs ⇒ $4 \times 60 \times 2 = 480 l$

water in the tank after 3hrs = $(1800 - 480) + 2160$

$3480 \text{ litres} = 3.48 m^3$

$h = \frac{3.48}{2.4 \times 2} = 0.725 m$

(ii) the total time taken to fill up the tank. (3 marks)

If 3hrs = 480l

? = 5040

$\frac{5040 \times 3}{480} = 31.5 \text{ hrs}$

18. (a) A line, L_1 passes through the points (3,3) and (5,7). Find the equation of L_1 in the form $y = mx + c$, where m and c are constants. (3 marks)

$m = \frac{7-3}{5-3} = \frac{4}{2} = 2$

$\frac{2}{1} = \frac{y-3}{x-3}$

$y - 3 = 2x - 6$

$y = 2x - 3$

(b) Another line L_2 is perpendicular to L_1 and passes through (−2,3). Find:

(i) the equation of L_2; (3 marks)

m of $L_2 = -\frac{1}{2}$

$-\frac{1}{2} = \frac{y-3}{x+2}$

$2y - 6 = -x - 2$

$y = -\frac{1}{2}x + 4$

(ii) the x-intercept of L_2. (1 mark)

at x-intercept $y = 0$

$-\frac{1}{2}x + 4 = 0$

$x = 8$

x-intercept is 8

(c) Determine the point of intersection of L_1 and L_2. (3 marks)

$2x - 3 = -\frac{1}{2}x + 4$

$2.5x = 7$

$x = 2.8 \quad \therefore y = 2.6$

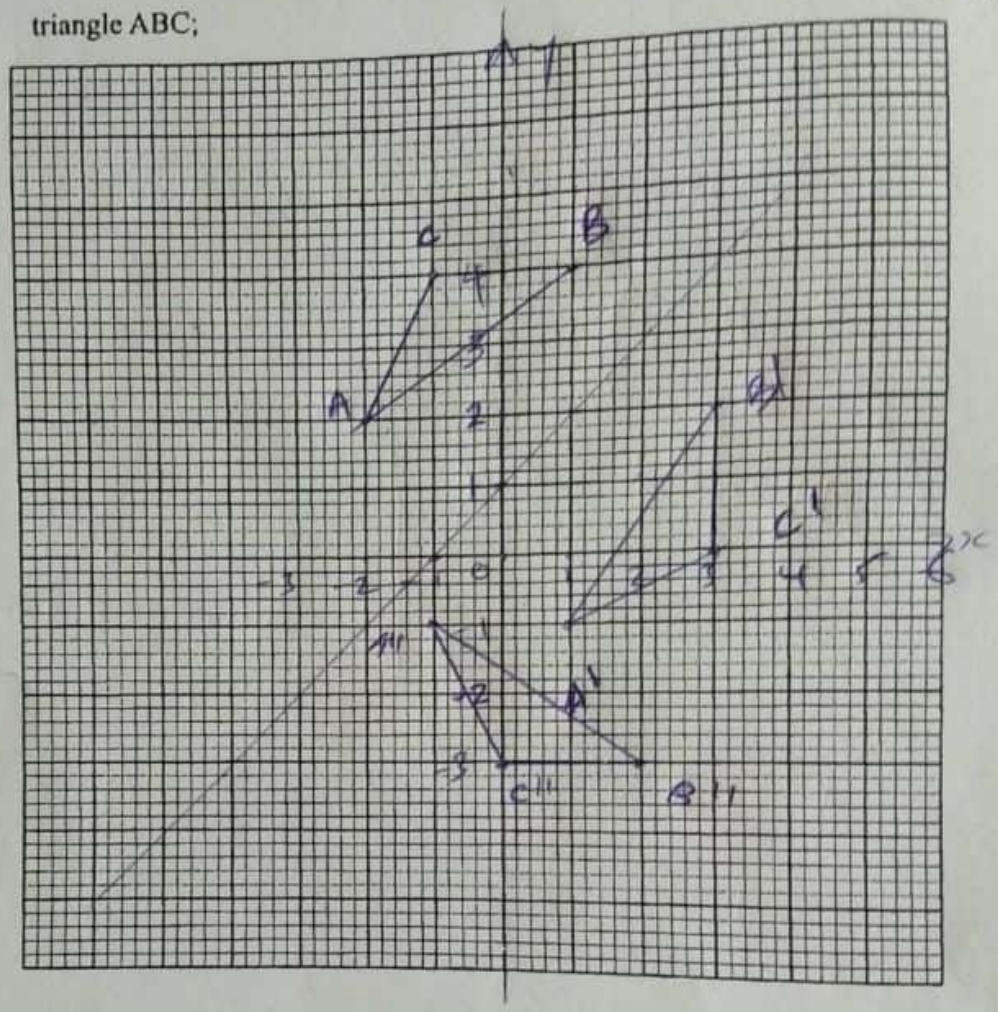
$(2.8, 2.6)$

19. A triangle ABC with vertices A $(-2,2)$, B $(1,4)$ and C $(-1,4)$ is mapped onto triangle A′B′C′ by a reflection in the line $y = x + 1$.

(a) On the grid provided draw:

(1 mark)

(i) triangle ABC;



(ii) the line $y = x + 1$;

(2 marks)

(iii) triangle A′B′C′.

(2 marks)

(b) Triangle A″B″C″ is the image of triangle A′B′C′ under a negative quarter turn about (0,0).

On the same grid, draw triangle A″B″C″.

(3 marks)

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(c) State the type of congruence between triangles:

(i) ABC and A′B′C′; (1 mark)

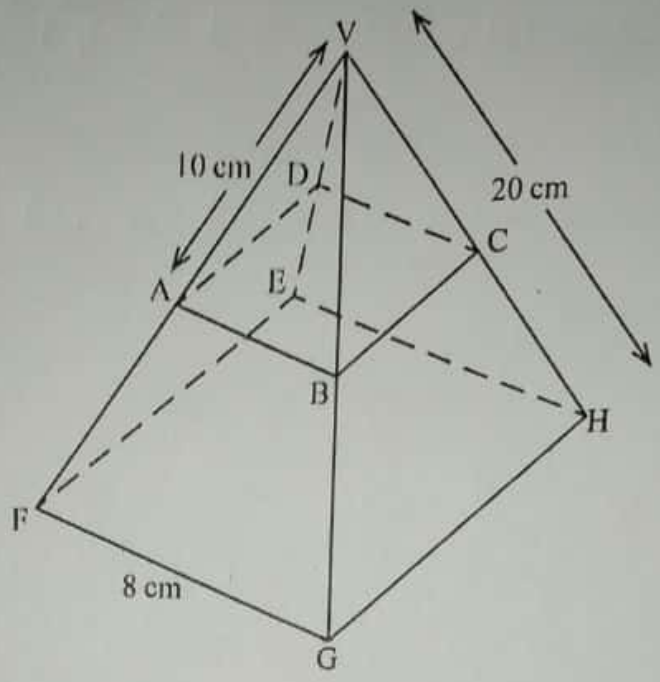
Opposite Congruence

(ii) A′B′C′ and A″B″C″. (1 mark)

Direct Congruence

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20. The figure below is a right pyramid VEFGH with a square base of 8 cm and a slant edge of 20 cm. Points A, B, C and D lie on the slant edges of the pyramid such that VA = VB = VC = VD = 10 cm and plane ABCD is parallel to the base EFGH.



(a) Find the length of AB. (2 marks)

$\frac{8}{20} = \frac{AB}{10}$

$AB = 4$ cm

(b) Calculate, correct to 2 decimal places:

(i) the length of AC; (2 marks)

$AC = \sqrt{4^2+4^2} = 5.66$ cm

(ii) the perpendicular height of the pyramid VABCD. (2 marks)

$h = \sqrt{10^2 - 2.83^2} = 9.59$ cm

(c) The pyramid VABCD was cut off. Find the volume of the frustum ABCDEFGH correct to 2 decimal places. (4 marks)

$$V = \frac{1}{3}\left\{(8\times8\times9.59\times2) - (4\times4\times9.59)\right\}$$

$$= \frac{1}{3}(1227.52 - 153.44)$$

$$= 358.03\ cm^3$$

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21. The heights of 40 athletes in a county athletics competition were as shown in the table below:

Height, *cm*	Frequency	X	fx	C.f
150 – 159	2	154.5	309	2
160 – 169	8	164.5	1316	10
170 – 179	10	174.5	1745	20
180 – 189	x 12	184.5	2214	32
190 – 199	6	194.5	1167	38
200 – 209	2	204.5	409	40
			7160	

(a) Find the value of x. (1 mark)

$x = 40 - 28 = 12$

(b) State the modal class. (1 mark)

180 – 189

(c) Calculate:

(i) the mean height of the athletes; (4 marks)

$\text{Mean} = \frac{7160}{40} = 179$

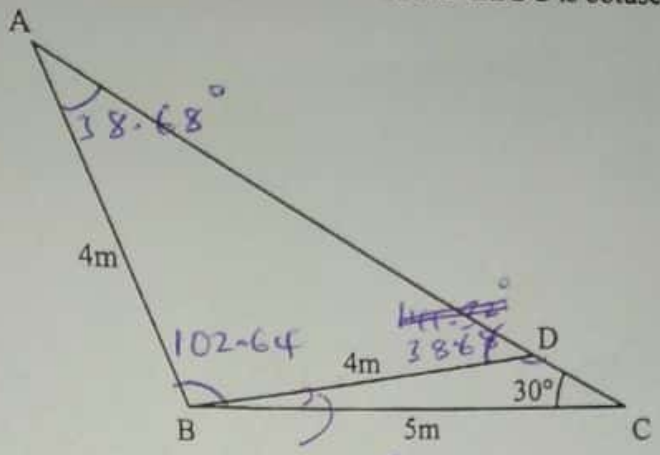
(ii) the median height, correct to 1 decimal place, of the athletes. (4 marks)

$\text{Median} = 169.5 + \left(\frac{20-10}{16}\right) \times 10$

179.5

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22. The figure below represents a triangular flower garden ABC in which AB = 4 m, BC = 5 m and ∠BCA = 30°. Point D lies on AC such that BD = 4 m and ∠BDC is obtuse.



Find, correct to 2 decimal places:

(a) ∠ BDC; (3 marks)

$$\frac{4}{\sin 30} = \frac{5}{\sin D}$$

$$BDC = \sin^{-1}\left(\frac{5\sin 30^\circ}{4}\right) = 38.68^\circ$$

$$\therefore \angle BDC = 180 - 38.68 = 141.32^\circ$$

(b) the length of AD; (3 marks)

$$\frac{AD}{\sin 102.64} = \frac{4}{\sin 38.68}$$

$$AD = \frac{4\sin 102.64}{\sin 38.68} = 6.25\ m$$

(c) the length of DC; (2 marks)

$$DC = \frac{4 \times \sin 8.68}{\sin 30} = 1.21\ m$$

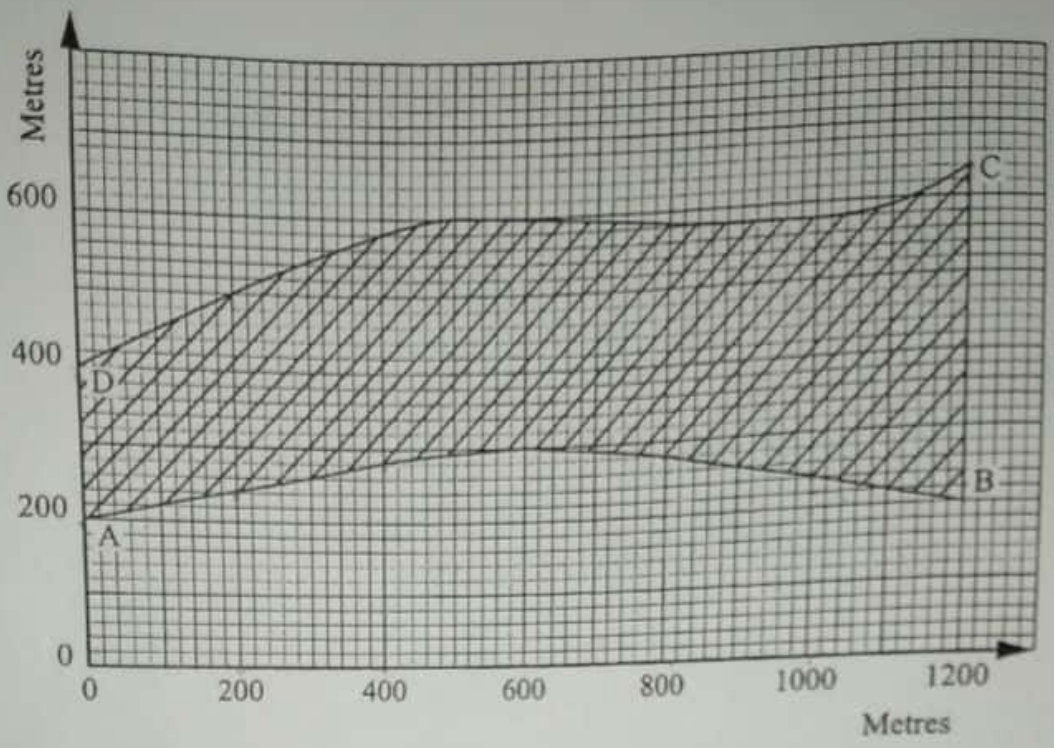
(d) the area of the flower garden ABC. (2 marks)

$$A = \frac{1}{2} \times 4 \times 5 \times \sin 111.32$$

$$9.32\ m^2$$

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23. The shaded region on the graph below shows a piece of land ABCD earmarked for building a sub-county hospital.



(a) Write down the ordinates of curves AB and DC for $x = 0, 200, 400, 600, 800, 1000$ and 1200. (2 marks)

x	0	200	400	600	800	1000	1200
AB	200	240	280	300	280	240	200
DC	400	500	580	600	580	580	640

(b) Use trapezium rule, with 6 strips to estimate the area of the piece of land ABCD, in hectares. (4 marks)

$A = \frac{1}{2} \times 200 \{(400 + 640) + 2(500 + 580 + 600 + 580 + 580)\} -$

$\frac{1}{2} \times 200 \{(200 + 200) + 2(240 + 280 + 300 + 280 + 240)\}$

$= 672{,}000 - 308{,}000 = \frac{364\,000\ m^2}{10\,000}$

36.4 hectares

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(c) Use mid-ordinate rule with 3 strips to estimate the area of the piece of land, in hectares. (4 marks)

$A = 400(500 + 600 + 580) - 400(240 + 300 + 240)$

$672000 - 312000$

$360000 m^2$

$\frac{360000}{10000} = 36$ hectares

24. The equation of a curve is $y = x^3 + x^2 - x - 1$

(a) Determine:

(i) the stationary points of the curve; (3 marks)

$\frac{dy}{dx} = 3x^2 + 2x - 1 = 0$

$3x^2 + 3x - x - 1 = 0$

$3x(x+1) - 1(x+1) = 0$

$(3x-1)(x+1) = 0$

$x = -1$ or $\frac{1}{3}$

at $x = -1$; $y = (-1)^3 + (-1)^2 + 1 - 1 = 0$

at $x = \frac{1}{3}$: $y = (\frac{1}{3})^3 + (\frac{1}{3})^2 + \frac{1}{3} - 1 = -1\frac{5}{27}$

$(-1, 0)$ and $(\frac{1}{3}, -1\frac{5}{27})$

(ii) the nature of the stationary points in (a) (i) above. (2 marks)

at $x = -1$

x	-2	-1	0
$\frac{dy}{dx}$	+ve	0	-ve

$(-1, 0)$ Maximum point

at $x = \frac{1}{3}$

x	0	$\frac{1}{3}$	1
$\frac{dy}{dx}$	-ve	0	+ve

$(\frac{1}{3}, -1\frac{5}{27})$ Minimum point

(b) Determine:

(i) the equation of the tangent to the curve at $x = 1$; (3 marks)

$y = 3(1)^2 + 2(1) - 1 = 4$

$\frac{4}{1} = \frac{y}{x-1}$

$y = 4x - 4$

(ii) the equation of the normal to the curve at $x = 1$. (2 marks)

$y = -\frac{1}{4}$

$-\frac{1}{4} = \frac{y}{x-1}$

$4y = -x + 1$

$y = -\frac{1}{4}x + \frac{1}{4}$

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