Kotebe Metropolitan University

Department of Mathematics



Solution Manual for Ethiopian University Entrance Examination (EUEE) Mathematics for Social Science GINBOT 2010 & 2011/May 2018 & 2019

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ETHIOPIAN UNIVERSITY ENTRANCE EXAMINATION (EUEE)

MATHEMATICS FOR SOCIAL SCIENCE

GINBOT 2010/May 2018

SOLUTION MANUAL

1. Which one of the following is true?

A) If the mean, mode and median of a distribution are 7, 5.5 and 6, respectively, then the distribution is negatively skewed.

B) If the mean, median and standard deviation of a distribution are 4, 6 and 2, respectively, then the distribution is positively skewed.

C) If a distribution is negatively skewed, then the mean of the distribution is greater than its second quartile.

D) If the first, second and third quartiles of a distribution are 7, 10 and 13, respectively, then the distribution is symmetrical.

Solution: Recall that the mean is pulled in the direction of skewness, that is, in the direction of the extreme observations. For a right-skewed distribution, the mean is greater than the median; for a symmetric distribution, the mean and the median are equal; and for a left-skewed distribution, the mean is less than the median. The first quartile, Q1, is the number that divides the bottom 25% of the data from the top 75%; the second quartile, Q2, is the median, which is the number that divides the bottom 50% of the data from the top 50%; and the third quartile, Q3, is the number that divides the bottom 75% of the data from the top 25%. Note that the first and third quartiles are the 25th and 75th percentiles, respectively. In a symmetric distribution, Q3-Q2=Q2-Q1.

Answer: D

2. A hospital wants to buy three *X*-ray machines from a supplier. Suppose the price of each machine including 15% VAT is Birr 161,000. If the hospital wants to subtract a 2% withholding tax before VAT, what is the amount the hospital should pay (in Birr) to the supplier after withholding 2% is subtracted?

A) 474,600 C) 473,340

B) 475,340 D) 483,340

Solution: Let x be the price of each X −-ray machine. Then, $x + 15\%x = 1.15x = 161{,}000 \implies x =$ 140,000, which is the price of each X −ray machine before vat. The withholding tax before vat is $140{,}000 - 0.02x140{,}000 = (1 - 0.02)140{,}000 = 137{,}200$. After 15% VAT, the hospital should pay $137{,}200 + 15\%(137{,}200) = 1.15(137{,}200) = 157,780$ for each X −ray machine. The hospital should pay $3 \times 157,780 = 473{,}340$ birr for three X −ray machines.

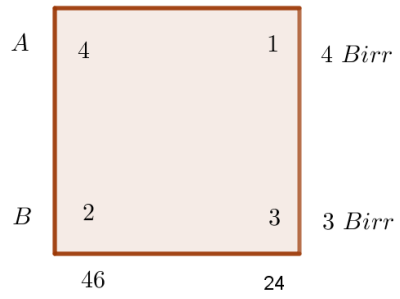
Answer: C

3. A firm deals with two kinds of fruit juices-pineapple and orange juice. These are mixed and two types of mixtures are obtained, which are sold as soft drinks A and B. One tin of A needs 4 liters pineapple juice and 1 liter of orange juice. One tin of B needs 2 liters of pineapple and 3 liters of orange juice. The firm has only 46 liters of pineapple juice and 24 liters of orange juice. Each tin of A and B is sold at a profit of 4 birr and 3 birr, respectively. How many tins of A and B should the firm produce to maximize its profit?

A) 6 tins of A and 9 tins of B C) 9 tins of A and 5 tins of B

B) 9 tins of A and 6 tins of B D) 5 tins of A and 9 tins of B

Solution: . Let x and y be the number of tins of type A and B respectively.



Maximize $Z = 4x + 3y$

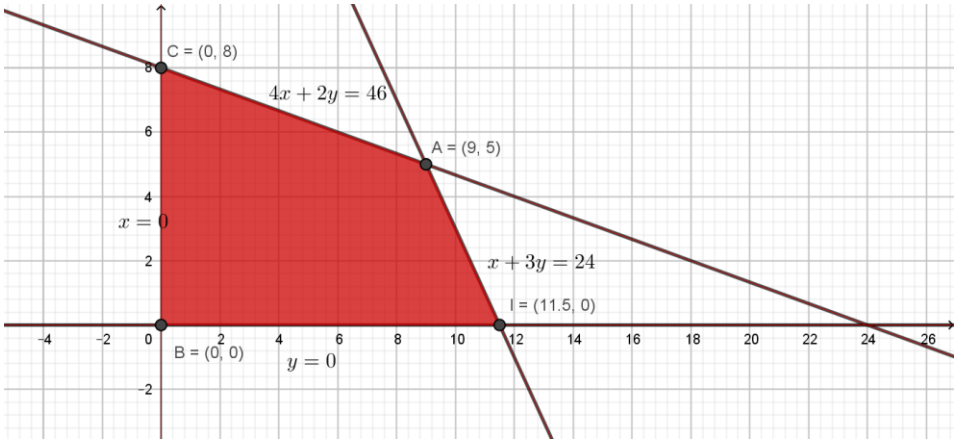
Subject to

$$4x + 2y \leq 46$$

$$x + 3y \leq 24$$

$$x, y \geq 0$$

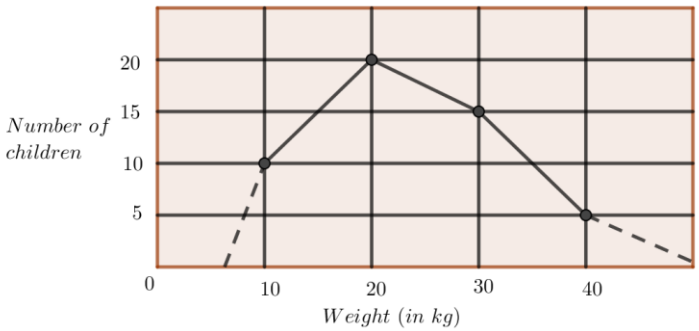
By graphical method, the region of the constrained condition is as shown in the fig below.



Evaluating $z = 4x + 3y$ at the vertices of the region, we see that the maximum value is $4(9) + 3(5) = 51$.

Answer: C

4. The weights (in kg) of children in certain nursery are grouped into four class intervals of equal width and represented by the following frequency polygon.



Which one of the following is true about the data?

A) The mode of the weights is 20 kg.

B) The distribution of the weight is positively skewed.

C) The median weight is 25 kg.

D) The mean weight is 25 kg.

Solution: From the frequency polygon, we see that the weights of the children is grouped as

Weights of children	5-15	15-25	25-35	35-45	Total
Class mark (x_i)	10	20	30	40	
Number of children (f_i)	10	20	15	5	50
Cumulative frequency	10	30	45	50	
$x_i f_i$	100	400	450	200	1150

The mean of the weights is $\bar{x} = \frac{\sum_{i=1}^{4} x_i f_i}{\sum_{i=1}^{4} f_i} = \frac{1150}{50} = 23kg.$

The median of the weights is $= L + \frac{\frac{n}{2} - C_{fb}}{f_m} \times w$ where L is the lower class boundary of the median class, $n = \sum_{i=1}^{4} f_i$, C_{fb} is the cumulative frequency of groups before the median class, f_m is the frequency of the median class and w is class interval.

That is, the median group is 15-25. The median is then $15 + \frac{25-10}{20} \times 10 = 22.5kg$. The formula for calculating the mode of a grouped frequency distribution is given by $Mode = L + \frac{a}{a+b} \times h$

Where L is the lower class boundary of the modal class, a difference between modal frequency and the frequency above it, b difference between modal frequency and the frequency below it, h class interval. The modal class is 15-25.

The mode of the weights is $15 + \frac{20-10}{(20-10)+(20-15)} \times 10 = 21.67kg.$

Since the mean is greater than the median and the median is greater than the mode, the distribution is positively skewed.

Answer: B

5. A man bought a one bed room house for birr 250,000 and paid a down payment of 20%. The remaining is a mortgage to be paid monthly for 15 years with 6% annual interest. What is the monthly payment in Birr?

[Given: $(1.005)^{-15} = 0.93$, $(1.005)^{-180} = 0.41$, $(1.006)^{-15} = 0.91$, $(1.006)^{-180} = 0.34$]

A) 1694

B) 1428

C) 1333

D) 1818

Solution: The formula for calculating the payment amount is given by $\boldsymbol{A = P\left(\frac{i}{1-(1+i)^{-n}}\right)}$ where A is payment amount per period, P initial loan, i interest rate per period and n total number of payments.

As 20% of the price of the house is already paid, $P = 0.8 \times 250{,}000 = 200{,}000$. $i = \frac{0.06}{12} = 0.005$ and $n = 12 \times 15 = 180$. Thus, $A = 200{,}000 \times \frac{0.005}{1-(1+0.005)^{-180}} = 200{,}000 \times \frac{0.005}{1-0.41} = 200{,}000 \times \frac{0.005}{0.59} \approx 1694$.

Answer: A

6. In the previous month, the regular prices of sugar and salt were Birr 12 and 8 per kg, respectively. This month, the price of sugar increased by 20% and the price of salt decreased by 30%. If a family must buy 5 kg of sugar and 3 kg of salt every month, how much more or less does the family need to pay for the two items this month compared to their previous month bill?

A) Birr 4.80 less

B) The same as in the previous month

C) Birr 8.40 more

D) Birr 4.80 more

Solution: Let x be the price of sugar per kg and y the price of salt per kg. In the previous month, $x = 12$ Birr and $y = 8$ Birr. In this month, $x = 12 + 20\%(12) = (1 + 0.2)12 = 14.4$ Birr and $y = 8 - 30\%(8) = (1 - 0.3)(8) = (0.7)(8) = 5.6$ Birr.

The family needed to pay 5(12)+3(8)=84 Birr in the previous month and need to pay $5x14.4 + 3x5.6 = 88.80$ Birr this month. Thus, The family need $88.80 - 84 = 4.80$ more in this month.

Answer: D

7. The grouped frequency distribution of a data is given in the following table.

Class interval	8 – 12	13-17	18-22	23-27	28-32
Frequency	4	8	10	5	3

What is its median(m), and the mean deviation (MD) about the median?

A) m = 20, MD = 4.7

B) $m = 19,\ MD$ = 4.2

C) $m = 19,\ MD$ = 4.7

D) $m = 20, MD$ = 4.2

Solution: The class intervals given are not continuous. Convert it to continuous frequency distribution by subtracting 0.5 from the lower limit and adding 0.5 to the upper limit of each class interval.

Class boundaries	f	cumulative frequency	Mid.value x_i	$\lvert x_i - m\rvert$	$f_i\lvert x_i - m\rvert$
7.5-12.5	4	4	10	8.67	34.68
12.5-17.5	8	12	15	3.67	29.36
17.5-22.5	10	22	20	1.33	13.3
22.5-27.5	5	27	25	6.33	31.65
27.5-32.5	3	30	30	11.33	33.99
Total	**30**				**143**

Since $N/2$ = 15, the class 17.5 – 22.5 is the median class.

$m = L + \left(\frac{\frac{N}{2} - C_{fb}}{f_m}\right) w = 17.5 + \frac{15-8}{10} x5 = 18.67 \approx 19$

The mean deviation about median = $\sum_{i=1}^{n} \frac{f_i|x_i-m|}{N} = \frac{143}{30}$ = 4.7. That implies, m = 19 and MD = 4.7.

Answer: B

8. What is the maximum value of $Z = 3x - 2y$,

$$\text{Subject to} \quad \begin{aligned} -x + y &\leq 0 \\ -x + 2y &\leq 1 \\ 2x - y &\leq 4 \\ x \geq 0, y &\geq 0 \end{aligned} \qquad ?$$

A) 5

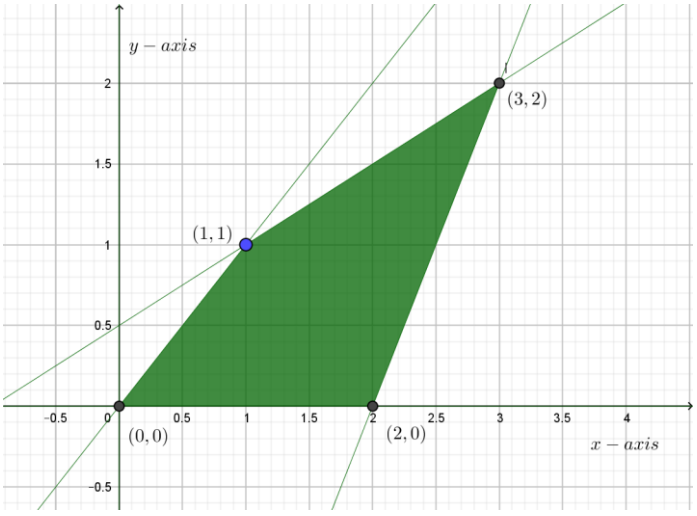
B) 10

C) 8

D) 6

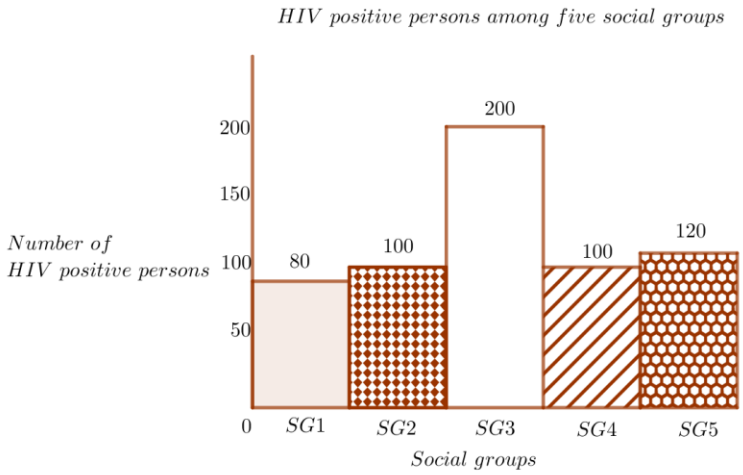
Solution: Using graphical method, the maximum value of $Z = 3x - 2y$ subject to the given constraints attains at the vertices of the shaded polygon, see the fig below.

Since $Z(0,0) = 0$, $Z(1,1) = 1$, $Z(3,2) = 5$ and $Z(2,0) = 6$, the maximum value is 6 which occurs at $(2,0)$.



Answer: D

9. The following histogram presents the number of HIV positive persons among five social groups coded as $SG1$, SG2, $SG3, SG4$ and $SG5$, in certain town.



Moreover, the numbers of persons in each of these social groups $SG1, SG2, SG3, SG4$ and $SG5$ are 500, 2500, 4000, 5000 and 1200 respectively. If so, which of the following can be deduced from the data?

A) $SG1$ consists of the highest percentage of HIV positive persons followed by $SG5$.

B) 3% of those in $SG4$ are HIV positive.

C) $SG3$ consists of the highest percentage of HIV positive persons.

D) 7.4% of the entire population of the five social groups are HIV positive.

Solution: The percentage of HIV positive in SG1 is $\frac{80}{500} \times 100$ =16%, in SG2 = 4%, in SG3 =5%, in SG4 = 2% and in SG5 = 10%. Of the total population, $\frac{600}{12700} \times 100 = 4.72\%$ is HIV positive.

Answer: A

10. A person is planning to make a regular monthly saving over the next five years in an account that pays no interest. If his monthly income is Birr 4000, then what percentage of his income should he save monthly so that his total saving will be Birr 26400 by the end of 5 years?

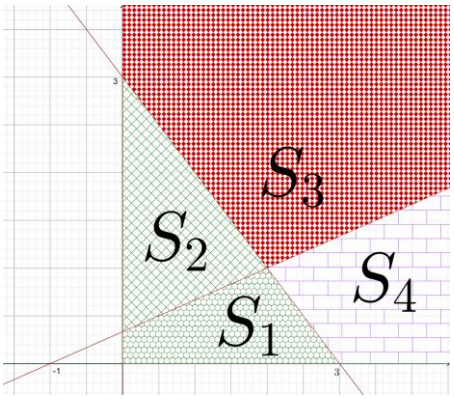
A) 9% B) 10% C) 12% D) 11%

Solution: $I = Pnr$, where P-principal amount, n-number of periods and r-percentage of P.

Hence, $26400 = 4000 \times 12 \times 5 \times r \Rightarrow \boldsymbol{r = 11\%}$.

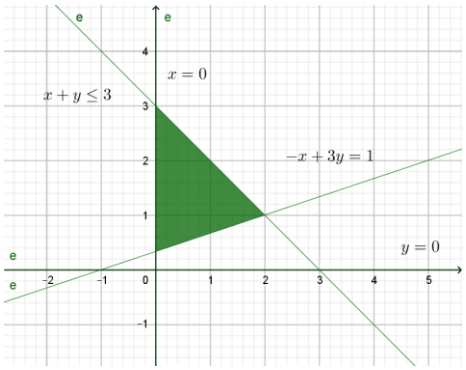
Answer: D

11. Which one of the regions shaded in the figure below is the solution region of the following system of inequalities? $\begin{cases} x + y \leq 3 \\ -x + 3y \geq 1 \\ x \geq 0,\ y \geq 0 \end{cases}$



A) S_1 B) S_3 C) S_2 D) S_4

Solution: Sketching the all the inequalities and taking the intersection, the region of the solution is S_2 as shown in the fig.



Answer: C

12. Birr 1000 is deposited in a saving account that pays 6% annual interest compounded monthly. Which one of the following is the amount (in Birr) that will be in the account by the end of 3 years?

A) $1000 + 1000 \times (1.005)36$ C) $1000 \times (1.05)36$

B) $1000 \times (1.005)36$ D) $1000 + 1000 \times (1.05)3$

Solution: Because $A = P\left(1 + \frac{r}{n}\right)^{nt} = 1000\left(1 + \frac{0.06}{12}\right)^{36} = 1000(1 + 0.005)^{36}$.

Answer: B

13. A jacket discounted by 20% for holiday has a price tag of Birr 576. What is the amount of discount?

(A) 144 Birr (C) 135.8 Birr

(B) 154 Birr (D) 115.2 Birr

Solution: Let the previous price of the jacket be x. Then $x - 20\%(x) = x - 0.2x = 0.8x = 576$. That is $x = \frac{576}{0.8} = 720$. Therefore, the discount = 720 – 576 = 144 Birr.

Answer: A

14. If $f(x) = \frac{|x|}{x}$ and $g(x) = \frac{x+2}{x^3-4x}$, then what is the value of $\lim_{x\to-2}(f(x) + g(x))$?

A) $\frac{-9}{8}$ B) ∞ C) $\frac{9}{8}$ D) $\frac{-7}{8}$

Solution: $\lim_{x\to-2}(f(x) + g(x)) = \lim_{x\to-2}\left(\frac{|x|}{x} + \frac{x+2}{x^3-4x}\right)$

$= \lim_{x\to-2}\left(\frac{|x|}{x}\right) + \lim_{x\to-2}\left(\frac{x+2}{x^3-4x}\right)$

$= \lim_{x\to-2}\left(\frac{-x}{x}\right) + \lim_{x\to-2}\left(\frac{x+2}{x(x+2)(x-2)}\right)$ as $|x| = \begin{cases} x, x \geq 0 \\ -x, x < 0 \end{cases}$

$= \lim_{x\to-2}(-1) + \lim_{x\to-2}\left(\frac{1}{x(x-2)}\right) = -1 + \frac{1}{-2(-2-2)} = -1 + \frac{1}{8} = \frac{-7}{8}$

Answer: D

15. Let $= \left(\frac{1-i}{1+i}\right)^{18}$. Then what is the value of z?

A) -1 B) i C) $-i$ D) $1 - i$

Solution: $z = \left(\frac{1-i}{1+i}\right)^{18} = \left(\frac{(1-i)(1-i)}{(1+i)(1-i)}\right)^{18} = \left(\frac{1-2i+i^2}{1-i^2}\right)^{18} = \left(\frac{1-2i-1}{1+1}\right)^{18} = \left(\frac{-2i}{2}\right)^{18} = (-i)^{18} = (-1)^{18}(i)^{18} = (1)(i^2)^9 = (-1)^9 = -1.$

Answer: A

16. Let A be a 3x3 invertible matrix and B be any 3x3 matrix. If $|A| = a$, and $|B| = b$, then which one of the following is **NOT** true?

A) $|A^TA| = a^2$ B) $|kA| = k^3|A|$, for any $k \in R$

C) $|A^{-1}B| = ab$ D) If $b = 0$, then B is not invertible.

Solution: (A) $|A^TA| = |A^T||A| = a * a = a^2$(Property of determinant)

(B) $|kA| = k^3|A|$ (Property of determinant)

(C) $|A^{-1}B| = |A^{-1}||B|$ and $|A^{-1}A| = |A^{-1}||A| = |I| = 1$ and $|A^{-1}| = \frac{1}{a}$, hence $|A^{-1}||B| = \frac{b}{a}$. (D) If $b = 0$, then B is not invertible.

Answer: C

17. The time needed to type a sample of 8 business letters in an office is 7, 8, 6, 8, 9, 7, 5, 6 minutes. What are the mean($\bar{x}$) and the standard deviation(s) of the data in minute?

A) $\bar{x} = 7, s = \sqrt{1.5}$ C) $\bar{x} = 8, s = \sqrt{2}$

B) $\bar{x} = 7, s = \sqrt{2}$ D) $\bar{x} = 8, s = \sqrt{1.5}$

Solution: Number of sample =8 and Data:7,8,6,8,9,7,5,6

Mean $= \bar{x} = \frac{7+8+6+8+9+7+5+6}{8} = \frac{56}{8} = 7,$ and Standard Deviation(s)$= \sqrt{\frac{\sum_{i=1}^{8}(x_i-\bar{x})^2}{n}} = \sqrt{\frac{(7-7)^2+(8-7)^2+(6-7)^2+(8-7)^2+(9-7)^2+(7-7)^2+(5-7)^2+(6-7)^2}{8}}$

$= \sqrt{\frac{0+1+1+1+4+0+4+1}{8}} = \sqrt{\frac{12}{8}} = \sqrt{1.5}$

Answer: A

18. A private college has 1000 students. 60% of these students are males. 45% of these students pay their payment by credit card including 175 females. What is the probability that the student is a male or a credit card user?

A) 0.675 B) 0.225 C) 0.325 D) 0.775

Solution: As 60% of 1000 are males, the number of males, $m = \frac{60}{100} \times 1000 = 600$. 1000-600 =400 students are females. Let x be the number of students who pay by credit card. Then $\frac{45}{100} \times 1000 = 450$. The number of male students who pay their payment by credit card is 450-175=275.

The probability that the student is a male or a credit card user is $P(m\ or\ x) = P(m) + P(x) - P(m \cap x) = \frac{600}{1000} + \frac{450}{1000} - \frac{275}{1000} = \frac{775}{1000} = 0.775$.

Answer: D

19. Which one of the following is true about the graph of $f(x) = \frac{2x^3+2x^2+3x}{x^2+x}$?

A) The vertical asymptote of the graph is only $x = -1$ and its oblique asymptote is $y = 2x$.

B) The graph has y-intercept at$(0,3)$.

C) The graph has at least one x- intercept.

D) The vertical asymptotes of the graph are at $x = 0$ and $x = -1$, but it has no horizontal asymptote.

Solution: $f(x) = \frac{2x^3+2x^2+3x}{x^2+x} = \frac{x(2x^2+2x+3)}{x(x+1)} = \frac{2x^2+2x+3}{x+1}$, for $x \neq 0$

The vertical asymptote is $x = -1$, and

By long division $2x^2 + 2x + 3 \div x + 1$, y=2x is its oblique asymptote.

The graph has no y- intercept, as 0 is not in the domain of f and $2x^2 + 2x + 3 \neq 0$ for any x as $D = b^2 - 4ac = 2^2 - 4(2)(3) = -20 < 0$(No solution).

Answer: A

20. Let $f(x) = \frac{3x+1}{x-2}$. Then what is the range of $f(x)$?

A) $\mathbb{R}\backslash\{2\}$ B) $\mathbb{R}$ C) $\mathbb{R}\backslash\{3\}$ D) $\mathbb{R}\backslash\left\{\frac{-1}{3}\right\}$

Solution: range of $f = \mathbb{R}\backslash\{3\}$, as $3 = \frac{3x+1}{x-2}$, $3x - 6 = 3x + 1 \Leftrightarrow -6 = 1$(absurd!)

Answer: C

21. What is the value of $\int \frac{1}{x}(lnx + x^2e^{-x})dx$?

A) $\frac{1}{2}x^2lnx - (x+1)e^{-x} + C$ C) $\frac{1}{2}x^2lnx + (2-x)e^{-x} + C$

B) $\frac{1}{2}lnx^2 - (x+1)e^{-x} + C$ D) $\frac{1}{2}lnx^2 + (2-x)e^{-x} + C$

Solution: $\int \frac{1}{x}(lnx + x^2e^{-x})dx = \int \frac{1}{x}lnxdx + \int xe^{-x}dx$.

Using substitution: let $u = lnx, du = \frac{1}{x}dx \Leftrightarrow \int \frac{1}{x}lnxdx = \int udu = \frac{1}{2}u^2 + c_1 = \frac{1}{2}(lnx)^2 + c_1$, and using integration by parts for $\int xe^{-x}dx$, let $u = x$ and $dv = e^{-x}dx \Leftrightarrow v = -e^{-x}$, then

$\int xe^{-x}dx = uv - \int vdu = -x\,e^{-x} - \int -e^{-x}dx = -xe^{-x} - e^{-x} + c_2.$

Thus, $\int \frac{1}{x}(lnx + x^2e^{-x})dx = \frac{1}{2}(lnx)^2 + -xe^{-x} - e^{-x} + C = \frac{1}{2}(lnx)^2 - (x+1)e^{-x} + C$.

Answer: B

22. If $f(x) = \frac{1}{3}x^3 + cx^2 + ax + 5$ has a local minimum value at $x = 1$, then which one of the following is true about the possible values of a and c?

A) $a = 3, c = -2$ C) $a = -2c - 1, c < -1$

B) $a = -2c - 1$, c any real number D) $a = -2c - 1, c > -1$

Solution: The function $f(x) = \frac{1}{3}x^3 + cx^2 + ax + 5$ has local minimum at $x = 1$.

$\Leftrightarrow f'(x) = x^2 + 2cx + a \Leftrightarrow f'(1) = (1)^2 + 2c(1) + a = 0$

$\Leftrightarrow a + 2c = -1 \Leftrightarrow a = -2c - 1$, $f''(x) = 2x + 2c \Leftrightarrow f''(1) = 2(1) + 2c >$ 0, (by second derivative test)

$\Leftrightarrow c > -1$. Thus, $a = -2c - 1$ and $c > -1$.

Answer: D

23. Let $\{a_n\}$ be a sequence with $a_1 = a, a_2 = f(a_1) = f(a), a_3 = f(a_2) = f\big(f(a)\big), \ldots . a_{n+1}$ $= f(a_n)$ where f is a continuous function. If $\lim_{n\to\infty} a_n = 5$, what is the value of $f(5)$?

A) 5 B) 5^n C) 5^{n+1} D) 1

Solution: $\lim_{n\to\infty} a_n = 5$ and $a_{n+1} = f(a_n) \Leftrightarrow \lim_{n\to\infty} a_n = \lim_{n\to\infty} a_{n+1} = \lim_{n\to\infty} f(a_n) = 5 = f(\lim_{n\to\infty} a_n)$ since f is continuous.

$= f(5) = 5.$

Answer: A

24. If the sum of the first three consecutive terms of an arithmetic progression is $\{A_n\}$ with $A_n > 0$ for all n, is 9 and the sum of their squares is 35, then what is the sum S_n of the first n terms?

A) $n^2 + 1$ B) $n^2 - 1$ C) n^2 D) $2n^2 + 1$

Solution: Let A_1, A_2, A_3 be three consecutive terms of an arithmetic progression $\{A_n\}$ with $A_n > 0$, for all n, let d be the common difference of the sequence:

$A_1 + A_2 + A_3 = 9 \Leftrightarrow A_1 + A_1 + d + A_1 + 2d = 9$ and ${A_1}^2 + {A_2}^2 + {A_3}^2 = 35$

$\Leftrightarrow 3A_1 + 3d = 9 \Leftrightarrow A_1 = 3 - d$ and $(3-d)^2 + (3-d+d)^2 + (3-d+2d)^2 = 35$

$\Leftrightarrow 9 - 6d + d^2 + 9 + 9 + 6d + d^2 = 35$

$\Leftrightarrow 27 + 2d^2 = 35 \Leftrightarrow d^2 = 4 \Leftrightarrow d = \pm 2 \Leftrightarrow d = 2$ as $A_n > 0$ for all n.

Thus $A_1 = 1, A_2 = 3, A_3 = 5, A_n = A_1 + (n-1)d = 1 + (n-1)2 = 2n - 1$

$S_n = \frac{n}{2}(A_1 + A_n) = \frac{n}{2}(1 + 2n - 1) = \frac{n}{2}(2n) = n^2$

Answer: C

25. Let A be a 3x3 matrix and $|A| = -2$. Then what is the value of $|adj(A)|$?

A) -8 B) C) -1/2 D) 4

Solution: since A is square matrix and $|A| = -2$, then it has an inverse.

$\Leftrightarrow A^{-1}A = I \Leftrightarrow |A^{-1}A| = |A^{-1}||A| = 1 \Leftrightarrow |A^{-1}| = \frac{-1}{2}$,

but $A^{-1} = \frac{1}{|A|}adj(A) \Leftrightarrow adj(A) = |A|A^{-1}$

$\Leftrightarrow |adjA| = |-2A^{-1}| = (-2)^3|A^{-1}| = (-8)\left(\frac{-1}{2}\right) = 4$

Answer: D

26. Let $A = \{1,2,3,4,5,6,7\}$, $B = \{7,8,9\}$ and $C = \{8,9,10\}$. If one of the numbers is deleted randomly from each of these sets, what is the probability that all the three deleted numbers are even or all are multiply of 3?

A) $\frac{4}{5}$ B) $\frac{2}{21}$ C) $\frac{1}{9}$ D) $\frac{8}{63}$

Solution: $A = \{1,2,3,4,5,6,7\}$, $B = \{7,8,9\}$ and $C = \{8,9,10\}$

$P_A(Even) = \frac{3}{7}$, $P_B(Even) = \frac{1}{3}$, and $P_C(Even) = \frac{2}{3}$,

$P_A(Multiple\ of\ 3) = \frac{2}{7}$, $P_B(Multiple\ of\ 3) = \frac{1}{3}$, and $P_C(Multiple\ of\ 3) = \frac{1}{3}$

Hence, $P(Even\ or\ Multiple\ of\ 3) = P(Even) + P(Multiple\ of\ 3)$

$$= \frac{3}{7} * \frac{1}{3} * \frac{2}{3} + \frac{2}{7} * \frac{1}{3} * \frac{1}{3} = \frac{2}{21} + \frac{2}{63} = \frac{8}{63}$$

Answer: D

27. If a function f is differentiable at a, then what is the value of $\lim_{x\to a} f(x)$?

(A) $f(a)$ (B) $f'(a)$ (C) $f'(a) - f(a)$ (D) 0

Solution: $f(x) - f(a) = \frac{f(x)-f(a)}{x-a}(x-a)$, taking the limit on both sides we have,

$$\lim_{x\to a}\big(f(x) - f(a)\big) = \lim_{x\to a}\left(\frac{f(x)-f(a)}{x-a}\right)(x-a) = f'(a) * 0 = 0 \Leftrightarrow \lim_{x\to a} f(x) = f(a)$$

Answer: A

28. Which one of the following is convergent sequence?

A) $\left\{\frac{1+2^n}{3^n}\right\}$ B) $\left\{\frac{1}{n} + \sin(n)\right\}$ C) $\left\{\frac{1-3^n}{2^n}\right\}$ D) $\left\{\frac{(-1)^n}{2}\right\}$

Solution: $\lim_{n\to\infty}\left\{\frac{1+2^n}{3^n}\right\} = \lim_{n\to\infty}\left[\frac{1}{3^n} + \left(\frac{2}{3}\right)^n\right] = 0 + 0 = 0$ and the rest choice have no limit.

Answer: A

29. What is the value of $\sum_{n=2}^{20}\left(\frac{1}{n-1} - \frac{1}{n}\right)$?

A) $\frac{17}{20}$ B) $\frac{23}{20}$ C) $\frac{21}{20}$ D) $\frac{19}{20}$

Solution: $\sum_{n=2}^{20}\left(\frac{1}{n-1} - \frac{1}{n}\right) = \left(\frac{1}{1} - \frac{1}{2}\right) + \left(\frac{1}{2} - \frac{1}{3}\right) + \left(\frac{1}{3} - \frac{1}{4}\right) + \left(\frac{1}{4} - \frac{1}{5}\right) + \cdots + \left(\frac{1}{19} - \frac{1}{20}\right) =$

$1 - \frac{1}{20} = \frac{19}{20}$, as the remaining terms cancelled.

Answer: D

30. Let A and B be two events. Suppose that the probability that neither event occurs is $\frac{3}{8}$. What is the probability that at-least one of the event occur?

A) $\frac{1}{8}$ B) $\frac{5}{8}$ C) $\frac{1}{4}$ D) $\frac{3}{4}$

Solution: $P(AUB)' = \frac{3}{8} = 1 - P(AUB) \Leftrightarrow P(AUB) = 1 - \frac{3}{8} = \frac{5}{8}$

Answer: B

31. If the truth value of $(p \wedge \neg p) \Leftrightarrow [(q \vee \neg q \Rightarrow r)]$ is True, then which one of the following must be True?

$A)\ q$ B) p C) $\neg q$ D) $\neg r$

Solution: $(p \wedge \neg p) \Leftrightarrow [(q \vee \neg q \Rightarrow r)]$ has truth value True means both have the same truth values. And $p \wedge \neg p$ =False, $q \vee \neg q$=True, which implies $q \vee \neg q \Rightarrow r$ = False, then r = False, $\neg r$=True

Answer: D

32. What is the maximum value of the function $f(x) = x^4 - 2x^2$ on $[-2,1]$?

A) 8 B) 12 C) 24 D) 40

Solution: $f(x) = x^4 - 2x^2$, $f'^{(x)} = 4x^3 - 4x = 0 \Leftrightarrow 4x(x^2 - 1) = 0$,

$x = -1, x = 0, x = 1$ are critical numbers. Then evaluate the function at the critical and end points of the interval, $f(-1) = -1, f(0) = 0, f(1) = -1, f(-2) = 8$ and

$f(1) = -1$. Hence the maximum is $f(-2) = 8$.

Answer: A

33. What are the greatest lower bound and least upper bound of the sequence $\left\{(-1)^n \left(1 + \frac{1}{n}\right)\right\}$, respectively?

$A)\ -2$ and 2 B) $\frac{-3}{2}$ and 2 C) -2 and $\frac{3}{2}$ D) -2 and $-\frac{3}{2}$

Solution: $\left\{(-1)^n \left(1 + \frac{1}{n}\right)\right\} = \{-2, \frac{3}{2}, -\frac{4}{3}, \frac{5}{4}, -\frac{6}{5}, \ldots.\}$, the lower bounds are $\forall x \leq -2$ and upper bounds are $\forall x \geq \frac{3}{2}$. Hence, $glb = -2$ and $lub = 3/2$

Answer: C

34. Which one of the following is an equation of the circle whose endpoints of a diameter are $(0, -2)$ and $(2,2)$?

A) $x^2 + y^2 = 4$ C) $(x-1)^2 + y^2 = 4$

B) $x^2+y^2-2y-4=0$ D) $x^2+y^2-2x-4=0$

Solution: Center of the circle $=\left(\frac{0+2}{2},\frac{-2+2}{2}\right)=(1,0)$ and radius $=\sqrt{(2-1)^2+(2-0)^2}=\sqrt{5}$, the equation of the circle: $(x-1)^2+y^2=5 \Leftrightarrow x^2+y^2-2x+1-5=0$ and $x^2+y^2-2x-4=0$

Answer: D

35. What is the equation of the line that passes through $(1,1)$ and is parallel to the line $3y-x=1$?

A) $x-3y+2=0$ C) $3y-x+2=0$

B) $x+3y=4$ D) $3x-y=2$

Solution: The line through $(1,1)$ and parallel to $3y-x=1$, the slope of the required line is $y=\frac{1}{3}$, as the two lines are parallel, the equation of the line is

$y-1=\frac{1}{3}(x-1) \Longrightarrow x-3y+2=0$

Answer: A

36. Which one of the following is true about $signum$, absolute value and greatest integer functions?

(A) $sgn(x)=\pm|x|$, for all $x\in R$ (C) $|x|=xsgn(x)$, for all $x\in R$

(B) $sgn(x)\leq|x|$, for all $x\leq 0$ (D) $sgn(x)\leq|x|$, for all $x\geq 0$

Solution: $sgn(x)=\begin{cases}1\,, x>0\\0\,, x=0\\-1\,, x<0\end{cases}$ $\Leftrightarrow |x|=xsgn(x)$, for all $x\in R$

Answer: C

37. What is the area of the triangle (in Sq. units) formed by the lines joining the vertex of the parabola $x^2=-36y$ to the end points of the latus rectum?

A) 126 B) 261 C) 216 D) 162

Solution: $x^2=-36y \Leftrightarrow -4p=-36, p=9$ Focus$(h,k\pm p)=(0,-9)$ and equation of Latus rectum is $y=-9$ and end point of latus rectum become: -

$x^2 = -36(-9) = 324, x = -18, x = 18$, $(-18, -9)$ and $(18, -9)$, then the length of latus rectum is 36. Hence area of the triangle $A = \frac{1}{2}(9)(36) = 162$

Answer: D

38. What is the partial fraction decomposition of $\frac{x^2+x+1}{(x+2)(x^2+1)}$?

A) $\frac{3}{5(x+2)} + \frac{2x+1}{5(x^2+1)}$ (C) $\frac{2}{5(x+2)} + \frac{3x+1}{5(x^2+1)}$

B) $\frac{5}{3(x+2)} + \frac{2x+1}{3(x^2+1)}$ (D) $\frac{2}{3(x+2)} + \frac{2x+1}{3(x^2+1)}$

Solution: $\frac{x^2+x+1}{(x+2)(x^2+1)} = \frac{A}{x+2} + \frac{Bx+C}{(x^2+1)} \Leftrightarrow \frac{x^2+x+1}{(x+2)(x^2+1)} = \frac{A(x^2+1)+(x+2)(Bx+C)}{(x+2)(x^2+1)}$

$$\frac{x^2+x+1}{(x+2)(x^2+1)} = \frac{Ax^2+A+Bx^2+2Bx+Cx+2C}{(x+2)(x^2+1)}$$

$$= \frac{(A+B)x^2+(2B+C)+A+2C}{(x+2)(x^2+1)}$$

$$x^2+x+1 = (A+B)x^2+(2B+C)+A+2C$$

$A + B = 1 \Rightarrow A = 1 - B, 2B + C = 1$ and $A + 2C = 1$, then

$\begin{cases} 2B + C = 1 \\ -B + 2C = 0 \Rightarrow B = 2C \end{cases}$, then $2(2C) + C = 1 \Rightarrow C = \frac{1}{5}$, $B = \frac{2}{5}$, and $A = 1 - \frac{2}{5} = \frac{3}{5}$

Hence, $\frac{A}{x+2} + \frac{Bx+C}{(x^2+1)} = \frac{3}{5(x+2)} + \frac{2x+1}{5(x^2+1)}$.

Answer: A

39. What is the area of the region enclosed by the graph of $y^2 = x + 1$ and $y^2 = -x + 1$?

A) $\frac{3}{8} sq.units$ B) $\frac{4}{3} sq.units$ C) $\frac{8}{3} sq.units$ D) $\frac{3}{4} sq.units$

Solution: The intersection of $y^2 = x + 1$ and $y^2 = -x + 1$

$y^2 - 1 = x = -y^2 + 1 \Leftrightarrow 2y^2 = 2 \Leftrightarrow y = \pm 1$, then the area

$$A = \int_{-1}^{1} [(1 - y^2) - (y^2 - 1)]\, dy = \int_{-1}^{1} (-2y^2 + 2) dy = \frac{-2}{3} y^3 + 2y$$

$= \frac{-2}{3}(1) + 2(1) - ((\frac{2}{3}(-1) + 2(-1)) = \frac{-2}{3} + \frac{-2}{3} + 2 + 2 = \frac{-4}{3} + 4 = \frac{8}{3}$

Hence, $A = \frac{8}{3}\, sq.units$

Answer: C

40. The age distribution of students in a certain class is given by

Age	10-14	15-19	20-24	25-29
No of students	2	10	6	7

What is the modal value of the distribution?

A) 17.83 B) 17.38 C) 18.37 D) 18.73

Solution: The modal value of continuous data is given by

$x_{mod} = l + w\left(\frac{f_0 - f_1}{2f_0 - f_1 - f_2}\right)$, where $l =$ lower class boundary of modal class,

f_0 =frequency of modal class, f_1 =frequency before frequency of modal class and

w =width of the interval.

The class mark of the data

Class	**Class mark (x_i)**	**Frequency**
10-14	12	2
15-19	17	10 (Modal class)
20-24	22	6
25-29	27	7

Hence, $w = 5, f_0 = 10, f_1 = 2,$ and $f_2 = 6, L = 14.5$

$x_{mod} = L + w\left(\frac{f_0 - f_1}{2f_0 - f_1 - f_2}\right)$

$= 14.5 + 5\left(\frac{10-2}{20-8}\right) = 14.5 + \frac{40}{12}$

$= 17.83$

Answer: A

41. If $f(x) = k\,lnx + e^{sinx}$ and $f''(\pi) = -1$, then what is the value of k?

A) $\mathbf{2\pi^2}$ B) π^2 C) π D) 2π

Solution: $f'(x) = \frac{k}{x} + cosx\, e^{sinx}$

$$f''(x) = -\frac{k}{x^2} + cos^2 x e^{sinx} - sinx e^{\sin x}$$

$$f''(\pi) = -\frac{k}{\pi^2} + 1 = -1$$

$$\Rightarrow -\frac{k}{\pi^2} = -2$$

$$\Rightarrow \boldsymbol{k = 2\pi^2}$$

Answer: A

42. Let $f(x) = x - x^2$ and $g(x) = \frac{1}{x}$. Then what is $g\left(f\left(\frac{1}{x}\right)\right)$ equal to

$A)\ x - x^2$ $B)\ \frac{x^2}{x-1}$ $C)\ \frac{1}{x^2 - x}$ $D)\ \frac{x-1}{x^2}$

Solution: $f(x) = x - x^2$ and $g(x) = \frac{1}{x}$. Then what is $g\left(f\left(\frac{1}{x}\right)\right)$?

$$g\left(f\left(\frac{1}{x}\right)\right) = g\left(\frac{1}{x} - \frac{1}{x^2}\right) = \frac{1}{\frac{1}{x} - \frac{1}{x^2}} = \frac{x^2}{x-1}, \quad x \neq 1$$

Answer: B

43. A cylindrical tank whose inner diameter is $2\ m$ contains $4\pi m^3$ oil. If the oil is discharged from the tank at the rate of $\frac{2\pi}{3}\ m^3/min$, then how long ($in\ min$) does it take for the tank to be empty?

A) $\frac{4}{3}$ $B)\ 4$ C) 12 $D)\ 6$

Solution: The volume of cylinder is $v = \pi r^2 h$ and given $v = 4\pi m^3$.

Solving for h we *have* $h = 4m$ for $r = 1m$

$$\frac{dv}{dt} = \frac{d}{dt}(\pi r^2 h), \qquad \frac{dv}{dt} = \frac{2\pi}{3}\ m^3/min$$

$$\frac{dv}{dt} = 2\pi r\ h\frac{dr}{dt}$$

$$= 8\pi m^3\ \frac{dr}{dt}$$

$$\frac{2\pi}{3}\frac{m^3}{min} = 8\pi m^3\frac{dr}{dt}$$

$$\frac{dr}{dt} = \frac{2\pi}{3}\frac{m^3}{min}.\frac{1}{8\pi m^3}$$

$$= \frac{1}{12\ min}$$

$$\frac{dt}{dr} = 12\ min$$

$$dt = 12\ dr\ \ min$$

$$\int dt = 12\ \text{min}\int_0^1 dr$$

$$\text{t} = 12\ \text{min}$$

Answer: C

44. What are the values of $\boldsymbol{a}$ and $\boldsymbol{b}$ so that the function

$$f(x) = \begin{cases} x+1, & 1 < x \\ ax+b, & 1 \le x < 2 \\ 3x, & x \ge 2 \end{cases}$$ is continuous everywhere?

A) $a = 4, b = -2$ B) $a = -4, b = -2$ C) $a = 4, b = 2$ D) $a = -4, b = 2$

Solution: $f(x)$ is continuous if $\lim_{x\to 2^+} f(x) = \lim_{x\to 2^-} f(x)$ *and* $\lim_{x\to 1^+} f(x) = \lim_{x\to 1^-} f(x)$

$6 = 2a + b \cdots\cdots (1)$ *and* $2 = a + b \cdots\cdots (2)$

Solving equation (1) and equation (2), we obtain

$$\begin{cases} 2a + b = 6 \\ a + b = 2 \end{cases} \cdots\cdots a = 4 \text{ and } b = -2$$

Answer: A

45. Let $U = N$ (the set of natural numbers) be the universe. Which one of the following proposition is true?

A) $(\exists x)(x + x = x)$ B) $(\forall x)(\exists y)(y < x)$

C) $(\forall \boldsymbol{x})(\exists \boldsymbol{y})(\boldsymbol{x} \div \boldsymbol{y} = \boldsymbol{y} \div \boldsymbol{x})$ D) $(\forall x)(\exists y)(x - y = x)$

Solution: A) Since there is no a natural number x such that $x + x = 2x = x$, it is false. B) Unless $(\forall x)(\exists y)(y \leq x)$, it is false. C) $(\forall \boldsymbol{x})(\exists \boldsymbol{y})(\boldsymbol{x} \div \boldsymbol{y} = \boldsymbol{y} \div \boldsymbol{x})$ is true as it is possible to take x=y. D) This is false.

Answer: C

46. What is the maximum possible area of a rectangle in square units with diagonal of length 16 units.

A) 48 B) 128 C) 64 D) 256

Solution: Using Pythagoras theorem we have $x^2 + y^2 = r^2$

$$x^2 + y^2 = 16^2, \quad y = (16^2 - x^2)^{\frac{1}{2}}$$

Area $(A) = xy$

$$A = xy = x\,(16^2 - x^2)^{\frac{1}{2}}$$

$$A(x) = x\sqrt{16^2 - x^2}$$

To find the dimension of the rectangle, we have find the critical point for $A(x)$

$$A'(x) = \sqrt{16^2 - x^2} - \frac{2x}{2\sqrt{16^2 - x^2}}$$

$$= \frac{16^2 - 2x^2}{\sqrt{16^2 - x^2}}$$

$$A'(x) = 0$$

$$16^2 - 2x^2 = 0$$

$$\text{x} = \pm\sqrt{\frac{16^2}{2}} = \pm\frac{16}{\sqrt{2}} = 8\sqrt{2},\ \text{y} = (16^2 - (8\sqrt{2})^2)^{\frac{1}{2}} = 8\sqrt{2}$$

$$\text{Area(A)} = \text{xy} = (8\sqrt{2})(8\sqrt{2}) = 128$$

Answer: B

47. Let $f(x) = \ln(x\sqrt{x})$. Then what is $f'(x)$ equal xto?

A) $\frac{2x}{3}$ B) $\frac{\sqrt{x}}{2}$ C) $\frac{2}{x\sqrt{x}}$ D) $\frac{3}{2x}$

Solution:

$$f'(x) = \frac{1}{x\sqrt{x}}.(x\sqrt{x})'$$

$$= \frac{1}{x\sqrt{x}}.(1.\sqrt{x} + x.(\frac{1}{2\sqrt{x}}) = \frac{3}{2x}$$

Answer: D

48. What it is the sum of the series $\sum_{n=1}^{\infty} 3^n 4^{-n}$?

A) ∞ B) $\frac{3}{16}$ C) 3 D) 4

Solution: $\sum_{n=1}^{\infty} 3^n 4^{-n} = \sum_{n=1}^{\infty} \left(\frac{3}{4}\right)^n$ is a geometric series with $r = \frac{3}{4}$. Since $\left|\frac{3}{4}\right| = \frac{3}{4} < 1$, the series is convergent and it converges to $\frac{G_1}{1-r}$ where $G_1 = \frac{3}{4}$ and $r = \frac{3}{4}$.

Thus, $\sum_{n=1}^{\infty} 3^n 4^{-n} = \frac{\frac{3}{4}}{1-\frac{3}{4}} = \frac{\frac{3}{4}}{\frac{1}{4}} = 3.$

Answer: C

49. If $z = (1+\sqrt{3}i)(1+i)$, then which one of the following is the polar representation of z?

A) $z = 4(\cos(105°) + isin\,(105°))$ B) $z = 2\sqrt{2}(\cos(105°) + isin\,(105°))$

C) $z = 2\sqrt{2}(\cos(15°) + isin\,(15°))$ D) $z = 4(\cos(75°) + isin\,(75°))$

Solution: The polar representation of $z = x + yi$ is $z^n = r^n(\cos(n\theta) + isin\,(n\theta))$ where $r = \sqrt{x^2+y^2}$ and $\theta = \tan^{-1}\left(\frac{y}{x}\right)$.

The given number $z = \left(1+\sqrt{3}i\right)(1+i) = 1 + i + \sqrt{3}i + \sqrt{3}i^2 = \left(1-\sqrt{3}\right) + i(1+\sqrt{3})$ which is of the form $z = x + iy$ where $x = 1-\sqrt{3}$ and $y = 1+\sqrt{3}$.

Then $r = \sqrt{\left(1-\sqrt{3}\right)^2 + \left(1+\sqrt{3}\right)^2} = 2\sqrt{2}$ and $\theta = \tan^{-1}\left(\frac{1+\sqrt{3}}{1-\sqrt{3}}\right)$. Note that $\tan(\theta) = \frac{1+\sqrt{3}}{1-\sqrt{3}} = \frac{\tan(45°)+\tan(60°)}{1-(\tan(45°))(\tan(60°))} = \tan(45° + 60°)$. Thus, $\theta = 105°$. The polar representation of $z = 2\sqrt{2}\left(\cos(105°) + isin\,(105°)\right)$.

Answer: B

50. The variance of 20 observations is 5. If each observation is multiplied by 2, then what is the variance of the resulting observations?

A) 5 B) 10 C) 20 D) 40

Solution: Let $x_1, x_2, \ldots, x_{20}$ be records of the observations. Then $\bar{x} = \sum_{i=1}^{20} \frac{x_i}{20}$ and $\sigma_x^2 = \sum_{i=1}^{20} \frac{(x_i-\bar{x})^2}{20} = 5.$

Let $y_1(=2x_1), y_2(=2x_2), \ldots, y_{20}(=2x_{20})$ be the new observations. Then $\bar{y} = \sum_{i=1}^{20} \frac{y_i}{20} = \sum_{i=1}^{20} \frac{2x_i}{20} = 2\sum_{i=1}^{20} \frac{x_i}{20} = 2\bar{x}$ and $\sigma_y^2 = \sum_{i=1}^{20} \frac{(y_i-\bar{y})^2}{20} = \sum_{i=1}^{20} \frac{(2x_i-2\bar{x})^2}{20} = 4\sum_{i=1}^{20} \frac{(x_i-\bar{x})^2}{20} = 4(5) = 20.$

Answer: C

51. A firm has two types of overtime pay rates (payment per hour) for its employees. Normal overtime pay rate and holiday overtime pay rate (for Sundays and holidays). The holiday
overtime pay rate is 1.5 times the normal overtime pay rate; and the normal overtime pay rate is 1.5 times the regular pay rate. The regular pay rate for an employee is given by his weekly basic wage divided by 40. If an employee whose weekly basic wage is Birr 2400
has additionally, 6 hours of normal overtime and 3 hours of holiday overtime in a week, how much (in Birr) is his payment for the week?

A) 3210 B) 3345 C) 3300 D) 3525

Solution: Let x, y, z be normal, holiday, and regular overtime pay rates respectively.

$$y = 1.5x = 1.5(1.5z) = 2.25z$$

Since $z = \frac{2400}{40} = 60$, $x = 1.5(60) = 90$ and $y = 2.25(60) = 135$.

The employee has a weekly payment of $2400 + 6(90) + 3(135) = 3345$.

Answer: B

52. What is the value of $\int_1^2 \frac{x+4}{x(x+2)}dx$?

A)2 B) ln 3 C) ln 2 D) ln 4 − ln 3

Solution: The partial fraction of $\frac{x+4}{x(x+2)}$ takes the form $\frac{a}{x} + \frac{b}{x+2}$. Then from the equation $\frac{x+4}{x(x+2)} = \frac{a}{x} + \frac{b}{x+2}$, we have $a(x+2) + bx = (a+b)x + 2a = x + 4$. Thus, $a + b = 1$ and $2a = 4$. then $a = 2$ and the value of $b = -1$

Then $\int_1^2 \frac{x+4}{x(x+2)}dx = \int_1^2 \frac{2}{x}dx + \int_1^2 \frac{-1}{(x+2)}dx = \int_1^2 \frac{x+4}{x(x+2)}dx = (2\ln x - \ln(x+2))|_{x=1}^{x=2}$

$= (2\ln 2 - \ln 4) - (2\ln 1 - \ln 3) = 2\ln 2 - \ln 2^2 + \ln 3 = 2\ln 2 - 2\ln 2 + \ln 3 = \ln 3$

Answer: B

53. What is the slope of the tangent line to the graph of $f(x) = 3e^x + sin\, x + 2$ at (0,5)?

A) 2 B) 3 C) 5 D) 4

Solution: The slope of the line tangent to the graph of f at any point (x, y) is $m = \frac{df(x)}{dx}$ $= 3e^x + \cos x$. At $(0,5)$, $m = 3e^0 + \cos 0 = 4$.

Answer: D

54. The population of a certain city is increasing at a rate of 3% per year. If the population was 100000 in 2010 E.C., then what will be the population in 2020E.C?

A) 134000 C) 1060000

B) 130000 D) 1378000

Solution: Let $P(t)$ and $P(t_0)$ be the populations of the city in t and t_0 years. Then the population dynamics is modeled by $\frac{1}{P}\left(\frac{dP}{dt}\right) = 0.03$. The solution of this is $P(t) = P(t_0)e^{0.03t}$.

Let $t_0 = 2010 = 0$. Then, $t = 2020 = 10$. Then, $P(t_0 = 0) = 100{,}000$ and the population in 2020 becomes $P(10) = 100{,}000e^{0.03*10} = 100{,}000(e^{0.03})^{10} = 100{,}000(1.03)^{10} = 100{,}000(1.34) = 134{,}000$

Answer: A

55. If $z = -3 + 4i$,and $w = 1 + 2i$,then what is the value of $\frac{2z}{w} + \bar{w}$?

A) $2 + 3i$ C) $3 + 2i$

B) $3 + 5i$ D) $3 - 2i$

Solution: $\frac{2z}{w} + \bar{w} = \frac{2(-3+4i)}{1+2i} + 1 - 2i = \frac{-6+8i}{1+2i} + \frac{(1-2i)(1+2i)}{1+2i} = \frac{-6+8i}{1+2i} + \frac{1+2i-2i+4}{1+2i}$

$= \frac{-6+8i}{1+2i} + \frac{5}{1+2i} = \frac{-1+8i}{1+2i} = \left(\frac{-1+8i}{1+2i}\right)\left(\frac{1-2i}{1-2i}\right) = \frac{-1+2i+8i+16}{5} = \frac{15+10i}{5} = 3 + 2i = 3 + 2i$

Answer: C

56. A salesman sold item x_1 , x_2 ,and x_3 with different rates of commissions as shown in the table below

Months	Sales of unit			Total commission (in birr)
	x_1	x_2	x_3	
February	90	100	20	800
March	130	50	40	900
April	60	100	30	850

What are the rate of commission on items x_1 , x_2 ,and x_3 respectively

a. 4 ,2 and 11
c. 4 ,11 and 2
b. 2 ,4 and 11
d. 11 ,2 and 4

Solution: From the table, we have the following system

$90x_1 + 100x_2 + 20x_3 = 800$ ……. (1)

$130x_1 + 50x_2 + 40x_3 = 900$ ……. (2)

$60x_1 + 100x_2 + 30x_3 = 850$ ……. (3)

From this system of linear equations, we do have the coefficient of matrix

$A = \begin{bmatrix} 90 & 100 & 20 \\ 130 & 50 & 40 \\ 60 & 100 & 30 \end{bmatrix}$, and its determinant is

$$\|A\| = \begin{Vmatrix} 90 & 100 & 20 \\ 130 & 50 & 40 \\ 60 & 100 & 30 \end{Vmatrix} = 90 \begin{Vmatrix} 50 & 40 \\ 100 & 30 \end{Vmatrix} - 100 \begin{Vmatrix} 130 & 40 \\ 60 & 30 \end{Vmatrix} + 20 \begin{Vmatrix} 130 & 50 \\ 60 & 100 \end{Vmatrix}$$

$$= -225000 - 150000 + 200000 = -175000 \neq 0$$

Let A_i be the sub matrix obtained by replacing the ith column of the coefficient matrix with the matrix of constants.

$$\|A_1\| = \begin{Vmatrix} 800 & 100 & 20 \\ 900 & 50 & 40 \\ 850 & 100 & 30 \end{Vmatrix} = 800 \begin{Vmatrix} 50 & 40 \\ 100 & 30 \end{Vmatrix} - 100 \begin{Vmatrix} 900 & 40 \\ 850 & 30 \end{Vmatrix} + 20 \begin{Vmatrix} 900 & 50 \\ 850 & 100 \end{Vmatrix}$$

$$= -2000000 + 700000 + 950000 = -350000,$$

$$\|A_2\| = \begin{Vmatrix} 90 & 800 & 20 \\ 130 & 900 & 40 \\ 60 & 850 & 30 \end{Vmatrix} = 90 \begin{Vmatrix} 900 & 40 \\ 850 & 30 \end{Vmatrix} - 800 \begin{Vmatrix} 130 & 40 \\ 60 & 30 \end{Vmatrix} + 20 \begin{Vmatrix} 130 & 900 \\ 60 & 850 \end{Vmatrix}$$

$= -630000 - 1200000 + 1130000 = -700000$, and

$\|A_3\| = \begin{Vmatrix} 90 & 100 & 800 \\ 130 & 50 & 900 \\ 60 & 100 & 850 \end{Vmatrix} =$

$90 \begin{Vmatrix} 50 & 900 \\ 100 & 850 \end{Vmatrix} - 100 \begin{Vmatrix} 130 & 900 \\ 60 & 850 \end{Vmatrix} + 800 \begin{Vmatrix} 130 & 50 \\ 60 & 100 \end{Vmatrix}$

$= -4275000 - 5650000 + 8000000 = -1925000$

Using crammers rule, we have $x_1 = \frac{\|A_1\|}{\|A\|} = \frac{-350000}{-175000} = 2$,$x_2 = \frac{\|A_2\|}{\|A\|} = \frac{-700000}{-175000} = 4$ and

$x_1 = \frac{\|A_3\|}{\|A\|} = \frac{-1925000}{-175000} = 11$

Answer: B

57. If A is the square matrix of order 3 and $det(A) = 5$, then what is the value of $det(A.adj(A))$?

A) 3 C) 125

B) 5 D) 25

Solution: Recall that $A^{-1} = \frac{adj(A)}{\det(A)}$ and also $det(A^{-1}) = \frac{1}{det(A)}$.

Then $det(A.adj(A)) = \det(A.(\det(A))\,A^{-1}) = \det\left(\det(A).\left((A)(A^{-1})\right)\right)$

$= \det(\det(A).I) = (\det(A))^3 \det(I) = (5)^3 \times 1 = 125.$

Answer: C

58. If $f'(x) = e^{x-1} + 3x^2 - \frac{1}{x}$,and $f(1) = 5$, what is f(x) ?

A) $f(x) = e^{x-1} + 3x^2 + \frac{1}{x^2} + 2$

B) $f(x) = e^{x-1} - x^3 + \ln x + 5$

C) $f(x) = e^{x-1} + 3x^2 - \frac{1}{x} + 3$

D) $f(x) = e^{x-1} + x^3 - \ln x + 3$

Solution: Since$f'(x) = e^{x-1} + 3x^2 - \frac{1}{x}$, $f(x) = \int f'(x)dx$

$$= \int \left(e^{x-1} + 3x^2 - \frac{1}{x}\right) dx$$

$$= e^{x-1} + x^3 - \ln x + c$$

As $f(1) = e^{1-1} + 1^3 - \ln 1 + c = 5$, $2 + c = 5$. That is, $c = 3$

Thus, $f(x) = e^{x-1} + x^3 - \ln x + 3$

Answer: D

59. Suppose the following are the premises of an argument.

He is healthy and he is not angry

He is angry or his plan fails

His plan does not fail if he does not travel abroad

Given that the premises are true, which one of the following can be a conclusion that makes the argument valid?

A) His plane fails and he is angry

B) His plane does not fail

C) He travels abroad

D) His plan fails and he does not travel abroad

Solution: Suppose that P: he is healthy, Q: he is not angry, R: .his plan fails, S: he travels abroad. According to the given premises, we have

- $P \cap Q$ is true. That is, both P and Q are true.
- $\neg Q \cup R$ is true. Since $\neg Q$ is false, R is true.
- $\neg S \Longrightarrow \neg R$ is true. Since $\neg R$ is false, $\neg S$ is false or S is true.

The conclusion "He travels abroad" makes the argument valid.

Answer: C

60. A man running a race course noted that the sum of the distances from the two flag posts from him is always 10 meters and the distance between the flag posts is 8 meters. What is the equation of the path traced by man? (Take flag posts to be on the x-axis with origin at their midpoint)

A) $\frac{x^2}{9} + \frac{y^2}{25} = 1$

B) $\frac{x^2}{25} + \frac{y^2}{9} = 1$

C) $\frac{x^2}{100} + \frac{y^2}{64} = 1$

D) $\frac{x^2}{64} + \frac{y^2}{100} = 1$

Solution: If a point moves in a plane in such a way that the sum of its distances from two fixed points is constant, then the path is an ellipse. The constant value is equal to the length of the major axis of the ellipse, 10 meters. That is $2a = 10$ or $a = 5$. The Distance between the foci is $2c = 8$ which is $c = 4$, and the distance b of the minor axis satisfies $b^2 = a^2 - c^2 = 25 - 16 = 9$ .Then, $b = 3$. Then the equation of the ellipse is $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ becomes $\frac{x^2}{5^2} + \frac{y^2}{3^2} = 1$. That is, $\frac{x^2}{25} + \frac{y^2}{9} = 1$.

Answer: B

61. What is the value of $\int_1^9 \frac{e^{\sqrt{x}}}{\sqrt{x}} dx$?

A) $e^3 - e$

B) $\frac{e^3}{3} - e$

C) $e\left(e^2 - \frac{1}{3}\right)$

D) $2(e^3 - e)$

Solution: Let $= \sqrt{x}$. Then $2du = \frac{dx}{\sqrt{x}}$. Further, if $x = 1, u = 1$ and $x = 9, u = 3$. Then $\int_1^9 \frac{e^{\sqrt{x}}}{\sqrt{x}} dx = 2\int_1^3 e^u \, du = e^u|_{u=1}^{u=3} = e^3 - e.$

Answer: A

62. What is the value of $\int x\sqrt{1-x^2}\, dx$?

A) $\frac{-1}{3}(1-x^2)^{\frac{3}{2}} + c$

B) $-2x\sqrt{1-x^2} + c$

C) $(1-x^2)^{\frac{3}{2}} + c$

D) $\frac{1}{2}\sqrt{1-x^2} + c$

Solution: Let $= 1 - x^2$. Then $du = -2xdx \Longrightarrow -\frac{du}{2} = xdx$, then

$\int x\sqrt{1-x^2}\, dx = -\frac{1}{2}\int \sqrt{u}\, du = -\frac{1}{2}\frac{u^{\frac{1}{2}+1}}{\frac{1}{2}+1} + c$, where c is any constant

$= -\frac{1}{2}\frac{u^{\frac{3}{2}}}{\frac{3}{2}} + c = -\frac{1}{3}\frac{u^{\frac{3}{2}}}{3} + c = -\frac{1}{3}(1-x^2)^{\frac{3}{2}} + c$

Answer: A

63. Which one of the following is not true about the function $f(x) = 3x^4 - 4x^3$.

A) $(0, 0)$ is the point of inflection of f

B) 0 and 1 are critical numbers of f

C) f is concave upward on $\left(0, \frac{2}{3}\right)$ and concave downward on $(-\infty, 0)$ and on $\left(\frac{2}{3}, \infty\right)$

D) f is decreasing on $(-\infty, 1)$ and increasing on $(1, \infty)$

Solution: $f(x) = 3x^4 - 4x^3 \Rightarrow f'(x) = 12x^3 - 12x^2 = 12x^2(x-1)$

The critical points satisfy the equation $12x^2(x-1) = 0$. Then, $= 0$, $x = 1$ are the critical points.

factors	$(-\infty, 0)$	$x = 0$	$(0, 1)$	$x = 1$	$(1, \infty)$
$12x^2$	+	0	+	+	+
$x - 1$	-	-	-	0	+
$12x^2(x-1)$	-	0	-	0	+

From the table, f is decreasing on $(-\infty, 0) \cup (0, 1)$ and increasing on $(1, \infty)$

Since $f'(x) = 12x^3 - 12x^2$, $f''(x) = 36x^2 - 24x = 6x(6x-4)$.

Setting $f''(x) = 6x(6x-4) = 0$, we have $x = 0$, and $x = \frac{2}{3}$.

factor	$(-\infty, 0)$	$x = 0$	$(0, \frac{2}{3})$	$x = \frac{2}{3}$	$(\frac{2}{3}, \infty)$
$6x$	-	0	+	+	+
$(6x-4)$	-	-	-	0	+
$6x(6x-4)$	+	0	-	0	+

Then the function is Concave upward on $(-\infty, 0) \cup \left(\frac{2}{3}, \infty\right)$, downward on $(0, \frac{2}{3})$, and $(0, 0)$ is the inflection point of f.

Therefor the answer is the function f is concave upward on $(0, \frac{2}{3})$ and concave downward on $(-\infty, 0) \cup \left(\frac{2}{3}, \infty\right)$.

Answer: C

64. What is the value of $\lim_{x \to 0} \frac{1}{x^2} sin^2\left(\frac{x}{2}\right)$?

A) $\frac{1}{2}$

B) $\frac{1}{4}$

C) 2

D) 4

Solution: Since $\lim_{x\to 0} sin^2\left(\frac{x}{2}\right) = 0 = \lim_{x\to 0} x^2$, the limit is an indeterminate form of $\frac{0}{0}$. Applying L'hopitals rule, we getlim$_{x\to 0} \frac{1}{x^2} sin^2\left(\frac{x}{2}\right) = \lim_{x\to 0} \frac{(sin^2\left(\frac{x}{2}\right))'}{(x^2)'} = \lim_{x\to 0} \frac{2\sin(\frac{x}{2})\cos(\frac{x}{2})\frac{1}{2}}{2x} == \lim_{x\to 0} \frac{\sin(\frac{x}{2})\cos(\frac{x}{2})}{2x}$ which is also an indeterminate form of $\frac{0}{0}$.

$$\lim_{x\to 0} \frac{1}{x^2} sin^2\left(\frac{x}{2}\right) = \lim_{x\to 0} \frac{\sin(\frac{x}{2})\cos(\frac{x}{2})}{2x} = \lim_{x\to 0} \frac{\cos(\frac{x}{2})\cos(\frac{x}{2})\frac{1}{2} - \sin(\frac{x}{2})\sin(\frac{x}{2})\frac{1}{2}}{2} =$$

$$\frac{\cos(0)\cos(0)\frac{1}{2} - \sin(0)\sin(0)\frac{1}{2}}{2} = \frac{1}{4}.$$

Answer: B

65. Which one of the following is equal to $\frac{d}{dx}\log_2\sqrt{6x}$?

A) $\frac{3x}{2\ln(2)}$ B) $\frac{3}{2x\ln(2)}$ C) $\frac{1}{6x\ln(2)}$ D) $\frac{1}{2x\ln(2)}$

Solution: Recalling that $\frac{d}{du}(\log_a u) = \frac{1}{u\ ln\ (a)}$ and noting that $f(x) = \log_2\sqrt{6x} = g(h(x))$ where $g(u) = \log_2 u$, $u = h(x) = \sqrt{6x}$, the derivative can be obtained by using chain rule. $\frac{d}{dx}f(x) = \frac{d}{du}g(u).\frac{d}{dx}h(x) = \frac{1}{u\ ln\ (2)} \times \sqrt{6}\frac{1}{2\sqrt{x}}$ where $u = \sqrt{6x}$.

$$= \frac{1}{\sqrt{6x} \times \ln(2)} \times \sqrt{6}\frac{1}{2\sqrt{x}} = \frac{1}{2x\ ln\ (2)}$$

Answer: D