



ANSWERS OF CHAPTER ONE:-TRIGONOMETRY

SECTION: - 1.1

Part one: using the sum and difference identities, condense each of the following and express as a trig function of a single angle

- $\sin 97 \cos 43 + \cos 97 \sin 43 = \sin(97+43) = \sin 140$
- $\cos 72 \cos 130 + \sin 72 \sin 130 = \cos(72-130) = \cos(-58) = \cos 58$
- $\frac{\tan 140 - \tan 60}{1 + \tan 140 \tan 60} = \tan(140-60) = \tan 80$
- $\sin \frac{\pi}{5} \cos \frac{2\pi}{3} - \cos \frac{\pi}{5} \sin \frac{2\pi}{3} = \sin\left(\frac{\pi}{5} - \frac{2\pi}{3}\right) = \sin\left(-\frac{7\pi}{15}\right) = -\sin \frac{7\pi}{15}$
- $\cos \frac{\pi}{6} \cos \frac{\pi}{7} - \sin \frac{\pi}{6} \sin \frac{\pi}{7} = \cos\left(\frac{\pi}{6} + \frac{\pi}{7}\right) = \cos \frac{13\pi}{42}$
- $\frac{\tan \frac{\pi}{3} + \tan \frac{\pi}{4}}{1 - \tan \frac{\pi}{3} \tan \frac{\pi}{4}} = \tan\left(\frac{\pi}{3} + \frac{\pi}{4}\right) = \tan \frac{7\pi}{12}$

Part two: use the sum and difference identities with unit circle values to find exact answers for the following.

- $\tan(-105^\circ) = -\tan 105^\circ = -\tan(45+60) = -\left[\frac{\tan 45 + \tan 60}{1 - \tan 45 \tan 60}\right] = -\left[\frac{1+\sqrt{3}}{1-1*\sqrt{3}}\right]$
 $= -\left[\frac{1+\sqrt{3}}{1-\sqrt{3}}\right] = \frac{-1-\sqrt{3}}{1-\sqrt{3}}$
- $\sin 345 = \sin(300 + 45) = \sin 300 \cos 45 + \cos 300 \sin 45 = -\frac{\sqrt{3}}{2} * \frac{\sqrt{2}}{2} + \frac{1}{2} * \frac{\sqrt{2}}{2}$
 $= -\frac{\sqrt{6}}{4} + \frac{\sqrt{2}}{4} = \frac{\sqrt{2}-\sqrt{6}}{4}$



Part three: - given $\csc \alpha = \frac{13}{5}$ $\frac{\pi}{2} \leq \alpha \leq \pi$ $\tan \beta = -\frac{3}{4}$ $\frac{3\pi}{2} \leq \beta \leq 2\pi$

$$\sin \alpha = \frac{5}{13} \quad \sin^2 \alpha + \cos^2 \alpha = 1$$

$$\cos^2 \alpha = 1 - \sin^2 \alpha \quad 1 + \tan^2 \beta = \sec^2 \beta \quad \sin^2 \beta + \cos^2 \beta = 1$$

$$\cos^2 \alpha = 1 - (5/13)^2 \quad 1 + \left(-\frac{3}{4}\right)^2 = \sec^2 \beta \quad \sin^2 \beta = 1 - \cos^2 \beta$$

$$\cos^2 \alpha = 1 - \frac{25}{169} \quad 1 + \frac{9}{16} = \sec^2 \beta \quad \sin^2 \beta = 1 - \left(\frac{4}{5}\right)^2$$

$$\cos^2 \alpha = \frac{144}{169} \quad \sec^2 \beta = \frac{25}{16} \quad \sin^2 \beta = 1 - \frac{16}{25}$$

$$\cos \alpha = -\frac{12}{13} \quad \sec \beta = \frac{5}{4} \quad \sin^2 \beta = \frac{9}{25}$$

$$\tan \alpha = -\frac{5}{12} \quad \therefore \cos \beta = \frac{4}{5} \quad \sin \beta = -\frac{3}{5}$$

$$9. \sin(\alpha - \beta) = \sin \alpha \cos \beta + \cos \alpha \sin \beta = \frac{5}{13} * \frac{4}{5} - \left(\frac{12}{13}\right) * \left(-\frac{3}{5}\right) = \frac{20}{65} - \frac{36}{65} = -\frac{16}{65}$$

$$10. \cos(\beta + \alpha) = \cos \beta \cos \alpha - \sin \beta \sin \alpha = \frac{4}{5} * -\frac{12}{13} - \left(-\frac{3}{5}\right) * \left(\frac{5}{13}\right) \\ = -\frac{48}{65} + \frac{15}{65} = -\frac{33}{65}$$

$$11. \tan(\alpha - \beta) = \frac{\tan \alpha - \tan \beta}{1 + \tan \alpha \tan \beta} = \frac{-\frac{5}{12} + \frac{3}{4}}{1 + \left(-\frac{5}{12}\right) * \left(-\frac{3}{4}\right)} = \frac{\frac{1}{3}}{1 + \frac{5}{16}} = \frac{\frac{1}{3}}{\frac{21}{16}} = \frac{16}{63}$$



Part four: - if $\sin\theta = -\frac{3}{5}$ and θ is in third quadrant find the following

$$\sin^2 \theta + \cos^2 \theta = 1$$

$$12. \cos\left(\theta + \frac{\pi}{3}\right) = \cos\theta \cos\frac{\pi}{3} - \sin\theta \sin\frac{\pi}{3}$$

$$\cos^2 \theta = 1 - \sin^2 \theta$$

$$-\frac{4}{5} * -\frac{1}{2} - \left(-\frac{3}{5}\right) * \left(\frac{\sqrt{3}}{2}\right) = -\frac{4}{10} + \frac{3\sqrt{3}}{10} = \frac{3\sqrt{3}-4}{10}$$

$$\cos^2 \theta = 1 - \left(-\frac{3}{5}\right)^2$$

$$13. \tan 2\theta = \tan(\theta + \theta) = \frac{\tan\theta + \tan\theta}{1 - \tan\theta \tan\theta} =$$

$$\cos^2 \theta = 1 - \frac{9}{25}$$

$$\frac{\frac{\frac{3}{4} + \frac{3}{4}}{1 - \left(\frac{3}{4}\right) * \left(\frac{3}{4}\right)}} = \frac{\frac{\frac{6}{4}}{1 - \frac{9}{16}}} = \frac{\frac{\frac{6}{4}}{\frac{7}{16}}} = \frac{\frac{6}{4} * \frac{16}{7}}{7} = \frac{24}{7}$$

$$\cos^2 \theta = \frac{16}{25}$$

$$\cos \theta = -\frac{4}{5}$$

$$\tan \theta = \frac{\sin \theta}{\cos \theta} = \frac{-\frac{3}{5}}{-\frac{4}{5}} = \frac{3}{4}$$

Part five: - verify the following identities

$$14. \sin(\pi - x) = \sin x$$

$$\sin \pi \cos x - \cos \pi \sin x = \sin x$$

$$0 * \cos x - (-1) * \sin x = \sin x$$

$$0 + \sin x = \sin x$$

$$\therefore \sin x = \sin x$$



$$15. \sin\left(\frac{3\pi}{2} + x\right) = -\cos x$$

$$\sin\frac{3\pi}{2} \cos x + \cos\frac{3\pi}{2} \sin x = -\cos x$$

$$-1 \cdot \cos x + 0 \cdot \sin x = -\cos x$$

$$-\cos x + 0 = -\cos x$$

$$\therefore -\cos x = -\cos x$$

$$16. \cos(30-x) + \cos(30+x) = \sqrt{3} \cos x$$

$$\cos 30 \cos x + \sin 30 \sin x + \cos 30 \cos x - \sin 30 \sin x = \sqrt{3} \cos x$$

$$2 \cos 30 \cos x = \sqrt{3} \cos x$$

$$2\left(\frac{\sqrt{3}}{2}\right) \cos x = \sqrt{3} \cos x$$

$$\therefore \sqrt{3} \cos x = \sqrt{3} \cos x$$

$$17. \frac{\sin(\beta-\alpha)}{\sin \alpha \sin \beta} = \cot \alpha - \cot \beta$$

$$\frac{\sin \beta \cos \alpha - \cos \beta \sin \alpha}{\sin \alpha \sin \beta} = \cot \alpha - \cot \beta$$

$$\frac{\sin \beta \cos \alpha}{\sin \alpha \sin \beta} - \frac{\cos \beta \sin \alpha}{\sin \alpha \sin \beta} = \cot \alpha - \cot \beta$$

$$\frac{\cos \alpha}{\sin \alpha} - \frac{\cos \beta}{\sin \beta} = \cot \alpha - \cot \beta$$

$$\therefore \cot \alpha - \cot \beta = \cot \alpha - \cot \beta$$



$$18. \cos(\alpha + \beta) + \cos(\alpha - \beta) = 2\cos\alpha\cos\beta$$

$$\cos\alpha\cos\beta - \sin\alpha\sin\beta + \cos\alpha\cos\beta + \sin\alpha\sin\beta = 2\cos\alpha\cos\beta$$

$$\therefore 2\cos\alpha\cos\beta = 2\cos\alpha\cos\beta$$

$$19. \tan \frac{5\pi}{12} = \tan\left(\frac{\pi}{4} + \frac{\pi}{6}\right) = \frac{\tan\frac{\pi}{4} + \tan\frac{\pi}{6}}{1 - \tan\frac{\pi}{4}\tan\frac{\pi}{6}} = \frac{1 + \frac{1}{\sqrt{3}}}{1 - 1 \cdot \frac{1}{\sqrt{3}}} = \frac{\frac{\sqrt{3}+1}{\sqrt{3}}}{\frac{\sqrt{3}-1}{\sqrt{3}}} = \frac{\sqrt{3}+1}{\sqrt{3}-1}$$

$$\tan\left(-\frac{\pi}{12}\right) = \tan\left(\frac{\pi}{6} - \frac{\pi}{4}\right) = \frac{\tan\frac{\pi}{6} - \tan\frac{\pi}{4}}{1 + \tan\frac{\pi}{6}\tan\frac{\pi}{4}} = \frac{\frac{\sqrt{3}}{3} - 1}{1 + \frac{\sqrt{3}}{3} \cdot 1} = \frac{\frac{\sqrt{3}-3}{3}}{\frac{\sqrt{3}+3}{3}} = \frac{\sqrt{3}-3}{\sqrt{3}+3}$$

20. Given $\sin x = \frac{3}{5}$, $\cos y = -\frac{5}{13}$ where $0 \leq x \leq \frac{\pi}{2}$ and $\frac{\pi}{2} \leq y \leq \pi$ evaluate $\sin(x + y)$

$$\sin^2 x + \cos^2 x = 1$$

$$\sin^2 y + \cos^2 y = 1$$

$$\cos^2 x = 1 - \sin^2 x$$

$$\sin^2 y = 1 - \cos^2 y$$

$$\cos^2 x = 1 - (3/5)^2$$

$$\sin^2 y = 1 - (-5/13)^2$$

$$\cos^2 x = 1 - \frac{9}{25}$$

$$\sin^2 y = 1 - \frac{25}{169}$$

$$\cos^2 x = \frac{16}{25}$$

$$\sin^2 y = \frac{144}{169}$$

$$\cos x = \frac{4}{5}$$

$$\sin y = \frac{12}{13}$$

$$\therefore \sin(x + y) = \sin x \cos y + \cos x \sin y$$

$$\left(\frac{3}{5}\right)\left(-\frac{5}{13}\right) + \left(\frac{4}{5}\right)\left(\frac{12}{13}\right) = -\frac{15}{65} + \frac{48}{65} = \frac{33}{65}$$



21. show that $\cos(x - y) = \cos x \cos y + \sin x \sin y$ is true for $x = \frac{\pi}{3}$ and $y = \frac{\pi}{6}$

$$\cos\left(\frac{\pi}{3} - \frac{\pi}{6}\right) = \cos \frac{\pi}{3} \cos \frac{\pi}{6} + \sin \frac{\pi}{3} \sin \frac{\pi}{6}$$

$$\cos \frac{\pi}{6} = \frac{1}{2} * \frac{\sqrt{3}}{2} + \frac{\sqrt{3}}{2} * \frac{1}{2}$$

$$\frac{\sqrt{3}}{2} = \frac{\sqrt{3}}{4} + \frac{\sqrt{3}}{4}$$

$$\frac{\sqrt{3}}{2} = \frac{2\sqrt{3}}{4}$$

$$\frac{\sqrt{3}}{2} = \frac{\sqrt{3}}{2}$$

22. show that $\sin(x + y) = \sin x \cos y + \cos x \sin y$ is true for $x = \frac{\pi}{2}$ and $y = \frac{3\pi}{4}$

$$\sin\left(\frac{\pi}{2} + \frac{3\pi}{4}\right) = \sin \frac{\pi}{2} \cos \frac{3\pi}{4} + \cos \frac{\pi}{2} \sin \frac{3\pi}{4}$$

$$\sin \frac{5\pi}{4} = 1 * -\frac{\sqrt{2}}{2} + 0 * \frac{\sqrt{2}}{2}$$

$$-\frac{\sqrt{2}}{2} = -\frac{\sqrt{2}}{2} + 0$$

$$-\frac{\sqrt{2}}{2} = -\frac{\sqrt{2}}{2}$$



23. Use an appropriate compound angle formula to determine the exact value of $\sin \frac{\pi}{12}$.

$$\begin{aligned}\sin \frac{\pi}{12} &= \sin\left(\frac{\pi}{3} - \frac{\pi}{4}\right) = \sin \frac{\pi}{3} \cos \frac{\pi}{4} - \cos \frac{\pi}{3} \sin \frac{\pi}{4} \\ &= \frac{\sqrt{3}}{2} * \frac{\sqrt{2}}{2} - \frac{1}{2} * \frac{\sqrt{2}}{2} = \frac{\sqrt{6}}{4} - \frac{\sqrt{2}}{4} \\ &= \frac{\sqrt{6} - \sqrt{2}}{4}\end{aligned}$$

Part six:- prove each of the following identities.

$$24. \cos(x + y) * \cos(x - y) = \cos^2 x + \cos^2 y - 1$$

$$[\cos x \cos y - \sin x \sin y][\cos x \cos y + \sin x \sin y] = \cos^2 x + \cos^2 y - 1$$

$$\cos^2 x \cos^2 y - \sin^2 x \sin^2 y = \cos^2 x + \cos^2 y - 1$$

$$\cos^2 x \cos^2 y - [1 - \cos^2 x][1 - \cos^2 y] = \cos^2 x + \cos^2 y - 1$$

$$\cos^2 x \cos^2 y - [1 - \cos^2 y - \cos^2 x + \cos^2 x \cos^2 y] = \cos^2 x + \cos^2 y - 1$$

$$\cos^2 x \cos^2 y - 1 + \cos^2 y + \cos^2 x - \cos^2 x \cos^2 y = \cos^2 x + \cos^2 y - 1$$

$$\therefore \cos^2 x + \cos^2 y - 1 = \cos^2 x + \cos^2 y - 1$$

$$25. \cos\left(\frac{\pi}{2} - x\right) = \sin x$$

$$\cos \frac{\pi}{2} \cos x + \sin \frac{\pi}{2} \sin x = \sin x$$

$$0 * \cos x + 1 * \sin x = \sin x$$

$$\therefore \sin x = \sin x$$



26. if $\sin x = \frac{4}{5}$ find the value of $\cos(x + 180)$

$$\sin^2 x + \cos^2 x = 1 \quad \therefore \cos(x + 180) = \cos x \cos 180 - \sin x \sin 180$$

$$\cos^2 x = 1 - \sin^2 x \quad = \frac{3}{5} * -1 - \frac{4}{5} * 0$$

$$\cos^2 x = 1 - (4/5)^2 \quad = -\frac{3}{5}$$

$$\cos^2 x = 1 - \frac{16}{25}$$

$$\cos^2 x = \frac{9}{25}$$

$$\cos x = \frac{3}{5}$$

27. If $\sin A = \frac{4}{5}$ $\tan B = \frac{5}{12}$ and angles A and B are in quadrant one what is the value of $\sin(A + B)$

$$\cos^2 A = 1 - \sin^2 A \quad 1 + \tan^2 B = \sec^2 B \quad \sin^2 B + \cos^2 B = 1$$

$$\cos^2 A = 1 - (4/5)^2 \quad 1 + \left(\frac{5}{12}\right)^2 = \sec^2 B \quad \sin^2 B = 1 - \cos^2 B$$

$$\cos^2 A = 1 - \frac{16}{25} \quad 1 + \frac{25}{144} = \sec^2 B \quad \sin^2 B = 1 - \left(\frac{12}{13}\right)^2$$

$$\cos^2 A = \frac{9}{25} \quad \sec^2 B = \frac{169}{144} \quad \sin^2 B = 1 - \frac{144}{169}$$

$$\cos A = \frac{3}{5} \quad \sec B = \frac{13}{12} \quad \sin^2 B = \frac{25}{169}$$

$$\therefore \cos B = \frac{12}{13} \quad \sin B = \frac{5}{13}$$

$$\sin(A + B) = \sin A \cos B + \cos A \sin B = \frac{4}{5} * \frac{12}{13} + \frac{3}{5} * \frac{5}{13} = \frac{48}{65} + \frac{15}{65} = \frac{63}{65}$$



28. find the exact value of each of the following

a. $\sin 75 = \sin(45 + 30) = \sin 45 \cos 30 + \cos 45 \sin 30$

$$= \frac{\sqrt{2}}{2} * \frac{\sqrt{3}}{2} + \frac{\sqrt{2}}{2} * \frac{1}{2} = \frac{\sqrt{6}}{4} + \frac{\sqrt{2}}{4} = \frac{\sqrt{6} + \sqrt{2}}{4}$$

b. $\cos \frac{7\pi}{12} = \cos \left(\frac{\pi}{3} + \frac{\pi}{4} \right) = \cos \frac{\pi}{3} \cos \frac{\pi}{4} - \sin \frac{\pi}{3} \sin \frac{\pi}{4}$

$$= \frac{1}{2} * \frac{\sqrt{2}}{2} - \frac{\sqrt{3}}{2} * \frac{\sqrt{2}}{2} = \frac{\sqrt{2}}{4} - \frac{\sqrt{6}}{4} = \frac{\sqrt{2} - \sqrt{6}}{4}$$

c. $\tan 105 = \tan(60 + 45) = \frac{\tan 60 + \tan 45}{1 - \tan 60 \tan 45} = \frac{\sqrt{3} + 1}{1 - \sqrt{3} * 1} = \frac{1 + \sqrt{3}}{1 - \sqrt{3}}$

d. $\sin \frac{19\pi}{12} = \sin \left(\frac{4\pi}{3} + \frac{\pi}{4} \right) = \sin \frac{4\pi}{3} \cos \frac{\pi}{4} + \cos \frac{4\pi}{3} \sin \frac{\pi}{4}$

$$= -\frac{\sqrt{3}}{2} * \frac{\sqrt{2}}{2} + \left(-\frac{1}{2}\right) * \frac{\sqrt{2}}{2} = \frac{-\sqrt{6}}{4} - \frac{\sqrt{2}}{4} = \frac{-\sqrt{2} - \sqrt{6}}{4}$$

29. verify each of the following use the sum and difference formulas to verify the identities.

a. $\sin \left(x - \frac{3\pi}{2} \right) = \cos x$

$$\sin x \cos \frac{3\pi}{2} - \cos x \sin \frac{3\pi}{2} = \cos x$$

$$\sin x * 0 - \cos x * -1 = \cos x$$

$$0 + \cos x = \cos x$$

$$\cos x = \cos x$$



b. $\cos 3x = 4\cos^3 x - 3\cos x$

$$\cos(2x + x) = 4\cos^3 x - 3\cos x$$

$$\cos 2x \cos x - \sin 2x \sin x = 4\cos^3 x - 3\cos x$$

$$(2\cos^2 x - 1) * \cos x - (2\sin x \cos x) * \sin x = 4\cos^3 x - 3\cos x$$

$$2\cos^3 x - \cos x - 2\sin^2 x \cos x = 4\cos^3 x - 3\cos x$$

$$2\cos^3 x - \cos x - 2(1 - \cos^2 x) * \cos x = 4\cos^3 x - 3\cos x$$

$$2\cos^3 x - \cos x - 2\cos x + 2\cos^3 x = 4\cos^3 x - 3\cos x$$

$$\therefore 4\cos^3 x - 3\cos x = 4\cos^3 x - 3\cos x$$

C. $\sin(x + y) * \sin(x - y) = \sin^2 x - \sin^2 y$

$$[\sin x \cos y + \cos x \sin y] * [\sin x \cos y - \cos x \sin y] = \sin^2 x - \sin^2 y$$

$$\sin^2 x \cos^2 y - \cos^2 x \sin^2 y = \sin^2 x - \sin^2 y$$

$$\sin^2 x [1 - \sin^2 y] - [1 - \sin^2 x] \sin^2 y = \sin^2 x - \sin^2 y$$

$$\sin^2 x - \sin^2 x \sin^2 y - \sin^2 y + \sin^2 x \sin^2 y = \sin^2 x - \sin^2 y$$

$$\therefore \sin^2 x - \sin^2 y = \sin^2 x - \sin^2 y$$



$$d. \tan(x + \pi) - \tan(\pi - x) = 2\tan x$$

$$\frac{\tan x + \tan \pi}{1 - \tan x \tan \pi} - \left[\frac{\tan \pi - \tan x}{1 + \tan \pi \tan x} \right] = 2\tan x$$

$$\frac{\tan x + 0}{1 - \tan x \cdot 0} - \left[\frac{0 - \tan x}{1 + 0 \cdot \tan x} \right] = 2\tan x.$$

$$\tan x + \tan x = 2\tan x$$

$$\therefore 2\tan x = 2\tan x$$

$$e. \sin\left(\frac{\pi}{3} + x\right) = \frac{1}{2} (\sqrt{3} \cos x + \sin x)$$

$$\sin \frac{\pi}{3} \cos x + \cos \frac{\pi}{3} \sin x = \frac{1}{2} (\sqrt{3} \cos x + \sin x)$$

$$\frac{\sqrt{3}}{2} \cos x + \frac{1}{2} \sin x = \frac{1}{2} (\sqrt{3} \cos x + \sin x)$$

$$\therefore \frac{1}{2} (\sqrt{3} \cos x + \sin x) = \frac{1}{2} (\sqrt{3} \cos x + \sin x)$$

$$f. \cos(A + B) + \cos(A - B) = 2\cos A \cos B$$

$$\cos A \cos B - \sin A \sin B + \cos A \cos B + \sin A \sin B = 2\cos A \cos B$$

$$\therefore 2\cos A \cos B = 2\cos A \cos B$$



30. evaluate $\sin(a + b)$, where a and b are obtuse angles ;

$$\sin a = \frac{3}{5} \text{ and } \sin b = \frac{5}{13}.$$

$$\sin^2 a + \cos^2 a = 1$$

$$\sin^2 b + \cos^2 b = 1$$

$$\cos^2 a = 1 - \sin^2 a$$

$$\cos^2 b = 1 - \sin^2 b$$

$$\cos^2 a = 1 - (3/5)^2$$

$$\cos^2 b = 1 - (5/13)^2$$

$$\cos^2 a = 1 - \frac{9}{25}$$

$$\cos^2 b = 1 - \frac{25}{169}$$

$$\cos^2 a = \frac{16}{25}$$

$$\cos^2 b = \frac{144}{169}$$

$$\cos a = -\frac{4}{5}$$

$$\cos b = -\frac{12}{13}$$

$$\therefore \sin(a + b) = \sin a \cos b + \cos a \sin b.$$

$$= \frac{3}{5} * -\frac{12}{13} + -\frac{4}{5} * \frac{5}{13} = -\frac{36}{65} + \frac{-20}{65} = -\frac{56}{65}$$



SECTION: - 1.2

1. Express as a single sine or cosine function

a. $10 \sin x \cos x = 5 * 2 \sin x \cos x = 5 \sin 2x$

b. $5 \sin 2x \cos 2x = \frac{5}{2} * 2 \sin 2x \cos 2x = \frac{5}{2} \sin 4x$

c. $\cos^2 \frac{\theta}{2} - \sin^2 \frac{\theta}{2} = \cos 2 \left(\frac{\theta}{2} \right) = \cos \theta$

d. $1 - 2 \sin^2 \frac{2\theta}{3} = \cos 2 \left(\frac{2\theta}{3} \right) = \cos \frac{4}{3} \theta$

2. Simplify each expression

a. $\frac{\sin 2a}{\cos a} = \frac{2 \sin a \cos a}{\cos a} = 2 \sin a$

b. $2 \tan a \cos^2 a = 2 \frac{\sin a}{\cos a} * \cos^2 a = 2 \sin a \cos a = \sin 2a$

c. $2 \sin^2 a + \cos 2a = 2 \sin^2 a + 1 - 2 \sin^2 a = 1$

3. Expand using a double angle formula

a. $3 \sin 4x = 3 * 2 \sin 2x \cos 2x = 6 \sin 2x \cos 2x$

b. $1 - \cos 8x = 1 - (\cos^2 4x - \sin^2 4x)$

$$= 1 - \cos^2 4x + \sin^2 4x$$

$$= \sin^2 4x + \sin^2 4x$$

$$= 2 \sin^2 4x$$



$$c. \cos 2x - \frac{\sin 2x}{\sin x} = \cos 2x - \frac{2 \sin x \cos x}{\sin x}$$

$$= \cos 2x - 2 \cos x = 2 \cos^2 x - 1 - 2 \cos x$$

$$= 2 \cos^2 x - 2 \cos x - 1$$

4. Write each expression in terms of a single trigonometric function

a. $2 \sin(0.6) \cos(0.6) = \sin 2(0.6) = \sin(1.2)$

b. $2 \sin(3) \cos(3) = \sin 2(3) = \sin(6)$

c. $2 \sin(2) \cos(2) = \sin 2(2) = \sin(4)$

d. $\cos^2(0.45) - \sin^2(0.45) = \cos 2(0.45) = \cos(0.9)$

e. $2 \cos^2(5) - 1 = \cos 2(5) = \cos(10)$

f. $1 - 2 \sin^2(3) = \cos 2(3) = \cos(6)$

5. Write each of the following expressions in terms of single trigonometric function

a. $2 \sin \frac{\pi}{6} \cos \frac{\pi}{6} = \sin 2\left(\frac{\pi}{6}\right) = \sin\left(\frac{\pi}{3}\right)$

b. $\cos^2 \frac{\pi}{10} - \sin^2 \frac{\pi}{10} = \cos 2\left(\frac{\pi}{10}\right) = \cos\left(\frac{\pi}{5}\right)$

c. $2 \cos^2(0.5) - 1 = \cos 2(0.5) = \cos(1)$



6. If $\sin\theta = \frac{1}{3}$ and θ is in quadrant 1 , evaluate each expression

$$\cos^2 \theta + \sin^2 \theta = 1$$

$$\cos^2 \theta = 1 - \sin^2 \theta$$

$$\cos^2 \theta = 1 - \left(\frac{1}{3}\right)^2$$

$$\cos^2 \theta = 1 - \frac{1}{9}$$

$$\cos^2 \theta = \frac{8}{9}$$

$$\cos \theta = \pm \frac{\sqrt{8}}{3}$$

$$\cos \theta = \frac{\sqrt{8}}{3}$$

$$\tan \theta = \frac{\sin \theta}{\cos \theta} = \frac{\frac{1}{3}}{\frac{\sqrt{8}}{3}} = \frac{1}{\sqrt{8}}$$

$$a. \sin 2\theta = 2 \sin \theta \cos \theta$$

$$= 2 \left(\frac{1}{3}\right) \left(\frac{\sqrt{8}}{3}\right) = \frac{2\sqrt{8}}{9}$$

$$b. \cos 2\theta = \cos^2 \theta - \sin^2 \theta$$

$$= \left(\frac{\sqrt{8}}{3}\right)^2 - \left(\frac{1}{3}\right)^2 = \frac{8}{9} - \frac{1}{9} = \frac{7}{9}$$

$$c. \tan 2\theta = \frac{2 \tan \theta}{1 - \tan^2 \theta} = \frac{2 \left(\frac{1}{\sqrt{8}}\right)}{1 - \left(\frac{1}{\sqrt{8}}\right)^2} = \frac{\frac{2}{\sqrt{8}}}{1 - \frac{1}{8}}$$

$$= \frac{\frac{2}{\sqrt{8}}}{\frac{7}{8}} = \frac{2}{\sqrt{8}} * \frac{8}{7} = \frac{8}{\sqrt{8}} * \frac{2}{7} = \sqrt{8} * \frac{2}{7} = \frac{2\sqrt{8}}{7}$$



7. A value of θ is defined evaluate the expressions $\sin^2 \theta$, $\cos^2 \theta$, $\tan^2 \theta$

a. $\cos \theta = -\frac{1}{2}$ and θ is in Quadrant 2

$$\cos^2 \theta + \sin^2 \theta = 1$$

$$\text{a. } \sin 2\theta = 2\sin \theta \cos \theta$$

$$\sin^2 \theta = 1 - \cos^2 \theta$$

$$= 2 \left(\frac{\sqrt{3}}{2} \right) \left(-\frac{1}{2} \right) = -\frac{\sqrt{3}}{2}$$

$$\sin^2 \theta = 1 - \left(-\frac{1}{2} \right)^2$$

$$\text{b. } \cos 2\theta = \cos^2 \theta - \sin^2 \theta$$

$$\sin^2 \theta = 1 - \frac{1}{4}$$

$$= \left(-\frac{1}{2} \right)^2 - \left(\frac{\sqrt{3}}{2} \right)^2 = \frac{1}{4} - \frac{3}{4} = -\frac{2}{4} = -\frac{1}{2}$$

$$\sin^2 \theta = \frac{3}{4}$$

$$\text{c. } \tan 2\theta = \frac{2\tan \theta}{1 - \tan^2 \theta} = \frac{2(-\sqrt{3})}{1 - (\sqrt{3})^2} = \frac{-2\sqrt{3}}{1 - 3}$$

$$\sin \theta = \pm \frac{\sqrt{3}}{2}$$

$$= \frac{-2\sqrt{3}}{-2} = \sqrt{3}$$

$$\sin \theta = \frac{\sqrt{3}}{2}$$

$$\tan \theta = \frac{\sin \theta}{\cos \theta} = \frac{\frac{\sqrt{3}}{2}}{-\frac{1}{2}} = -\sqrt{3}$$

b. $\sin \theta = -\frac{2}{3}$ and θ is in Quadrant 3

$$\cos^2 \theta + \sin^2 \theta = 1$$

$$\text{a. } \sin 2\theta = 2\sin \theta \cos \theta$$

$$\cos^2 \theta = 1 - \sin^2 \theta$$

$$= 2 \left(-\frac{2}{3} \right) \left(\frac{\sqrt{5}}{3} \right) = \frac{4\sqrt{5}}{9}$$

$$\cos^2 \theta = 1 - \left(-\frac{2}{3} \right)^2$$

$$\text{b. } \cos 2\theta = \cos^2 \theta - \sin^2 \theta$$

$$\cos^2 \theta = 1 - \frac{4}{9}$$

$$= \left(-\frac{\sqrt{5}}{3} \right)^2 - \left(\frac{2}{3} \right)^2 = \frac{5}{9} - \frac{4}{9} = \frac{1}{9}$$

$$\cos^2 \theta = \frac{5}{9}$$

$$\text{c. } \tan 2\theta = \frac{2\tan \theta}{1 - \tan^2 \theta} = \frac{2\left(\frac{2}{\sqrt{5}}\right)}{1 - \left(\frac{2}{\sqrt{5}}\right)^2} = \frac{\frac{4}{\sqrt{5}}}{1 - \frac{4}{5}}$$



$$\cos \theta = \pm \frac{\sqrt{5}}{3}$$

$$\cos \theta = \pm \frac{\sqrt{5}}{3}$$

$$\cos \theta = -\frac{\sqrt{5}}{3}$$

$$\tan \theta = \frac{\sin \theta}{\cos \theta} = \frac{-\frac{2}{3}}{\frac{\sqrt{5}}{3}} = \frac{2}{\sqrt{5}}$$

$$\text{c. } \tan \theta = 0.75 = \frac{3}{4} \text{ and } \pi < \theta < \frac{3\pi}{2}$$

$$1 + \tan^2 \theta = \sec^2 \theta$$

$$1 + (0.75)^2 = \sec^2 \theta$$

$$1 + \left(\frac{3}{4}\right)^2 = \sec^2 \theta$$

$$1 + \frac{9}{16} = \sec^2 \theta$$

$$\frac{25}{16} = \sec^2 \theta$$

$$\sec \theta = -\frac{5}{4}$$

$$\cos \theta = -\frac{4}{5}$$

$$\cos^2 \theta + \sin^2 \theta = 1$$

$$\sin^2 \theta = 1 - \cos^2 \theta$$

$$\sin^2 \theta = 1 - \left(-\frac{4}{5}\right)^2$$

$$\sin^2 \theta = 1 - \frac{16}{25}$$

$$\sin^2 \theta = \frac{9}{25}$$

$$\sin \theta = \pm \frac{3}{5} = -\frac{3}{5}$$

$$\text{c. } \tan 2\theta = \frac{2 \tan \theta}{1 - \tan^2 \theta} = \frac{2 \left(\frac{2}{\sqrt{5}}\right)}{1 - \left(\frac{2}{\sqrt{5}}\right)^2} = \frac{\frac{4}{\sqrt{5}}}{1 - \frac{4}{5}}$$

$$= \frac{4}{\sqrt{5}} * \frac{5}{1} = \frac{5}{\sqrt{5}} * \frac{4}{1} = \sqrt{5} * 4 = 4\sqrt{5}$$

$$\text{a. } \sin 2\theta = 2 \sin \theta \cos \theta$$

$$= 2 \left(-\frac{3}{5}\right) \left(-\frac{4}{5}\right) = \frac{24}{25}$$

$$\text{b. } \cos 2\theta = \cos^2 \theta - \sin^2 \theta$$

$$= \left(-\frac{4}{5}\right)^2 - \left(-\frac{3}{5}\right)^2 = \frac{16}{25} - \frac{9}{25} = \frac{7}{25}$$

$$\text{c. } \tan 2\theta = \frac{2 \tan \theta}{1 - \tan^2 \theta} = \frac{2 \left(\frac{3}{4}\right)}{1 - \left(\frac{3}{4}\right)^2} = \frac{\frac{3}{2}}{1 - \frac{9}{16}}$$

$$= \frac{\frac{3}{2}}{\frac{7}{16}} = \frac{3}{2} * \frac{16}{7} = \frac{24}{7}$$

8. Prove each Identity

$$\text{a. } 1 + \sin 2\theta = (\sin \theta + \cos \theta)^2$$

$$1 + \sin 2\theta = \sin^2 \theta + 2 \sin \theta \cos \theta + \cos^2 \theta$$

$$1 + \sin 2\theta = 1 + \sin 2\theta$$



b. $\sin 2\theta = 2 \cot \theta \sin^2 \theta$

$$\sin 2\theta = 2 * \frac{\cos \theta}{\sin \theta} * \sin^2 \theta$$

$$\sin 2\theta = 2 \cos \theta \sin \theta$$

$$\sin 2\theta = \sin 2\theta$$

c. $\cos 2\theta = \frac{1 - \tan^2 \theta}{1 + \tan^2 \theta}$

$$\cos 2\theta = \frac{1 - \frac{\sin^2 \theta}{\cos^2 \theta}}{1 + \frac{\sin^2 \theta}{\cos^2 \theta}},$$

$$\cos 2\theta = \frac{\frac{\cos^2 \theta - \sin^2 \theta}{\cos^2 \theta}}{\frac{\cos^2 \theta + \sin^2 \theta}{\cos^2 \theta}}$$

$$\cos 2\theta = \frac{\frac{\cos 2\theta}{\cos^2 \theta}}{\frac{1}{\cos^2 \theta}}$$

$$\cos 2\theta = \cos 2\theta$$

d. $\sec^2 \theta = \frac{2}{1 + \cos 2\theta}$

$$\sec^2 \theta = \frac{2}{1 + \cos^2 \theta - \sin^2 \theta}$$

$$\sec^2 \theta = \frac{2}{\cos^2 \theta + \cos^2 \theta}$$

$$\sec^2 \theta = \frac{2}{2 \cos^2 \theta}$$

$$\sec^2 \theta = \frac{1}{\cos^2 \theta}$$

$$\sec^2 \theta = \sec^2 \theta$$



9. Prove each identity

a. $\frac{1-\cos 2\theta}{2} = \sin^2 \theta$

$$\frac{1-(\cos^2 \theta - \sin^2 \theta)}{2} = \sin^2 \theta$$

$$\frac{1-\cos^2 \theta + \sin^2 \theta}{2} = \sin^2 \theta$$

$$\frac{\sin^2 \theta + \sin^2 \theta}{2} = \sin^2 \theta$$

$$\frac{2\sin^2 \theta}{2} = \sin^2 \theta$$

$$\sin^2 \theta = \sin^2 \theta$$

b. $\frac{\cos^2 \theta + \sin^2 \theta}{\sin^2 \theta - \cos^2 \theta} = -\sec 2\theta$

$$\frac{\cos^2 \theta + \sin^2 \theta}{-(\cos^2 \theta - \sin^2 \theta)} = -\sec 2\theta$$

$$-\frac{1}{\cos 2\theta} = -\sec 2\theta$$

$$-\sec 2\theta = -\sec 2\theta$$

c. $\frac{(\sin \theta + \cos \theta)^2}{\sin 2\theta} = \csc \theta + 1$

$$\frac{\sin^2 \theta + 2\sin \theta \cos \theta + \cos^2 \theta}{\sin 2\theta} = \csc 2\theta + 1$$

$$\frac{1 + \sin 2\theta}{\sin 2\theta} = \csc 2\theta + 1$$

$$\frac{1}{\sin 2\theta} + \frac{\sin 2\theta}{\sin 2\theta} = \csc 2\theta + 1$$

$$\csc 2\theta + 1 = \csc 2\theta + 1$$



Section 1.3

Part one: - use half angle identities to find each value

$$1. \tan 22.5 = \tan \frac{45}{2} = \frac{1 - \cos 45}{\sin 45} = \frac{1 - \frac{\sqrt{2}}{2}}{\frac{\sqrt{2}}{2}} = \frac{\frac{2 - \sqrt{2}}{2}}{\frac{\sqrt{2}}{2}} = \frac{2 - \sqrt{2}}{\sqrt{2}} = \frac{2}{\sqrt{2}} - \frac{\sqrt{2}}{\sqrt{2}} = \sqrt{2} - 1$$

$$2. \cos \frac{5\pi}{8} = \cos \frac{5\pi}{4} = -\sqrt{\frac{1 + \cos \frac{5\pi}{4}}{2}} = -\sqrt{\frac{1 - \frac{\sqrt{2}}{2}}{2}} = -\sqrt{\frac{\frac{2 - \sqrt{2}}{2}}{2}} = -\sqrt{\frac{2 - \sqrt{2}}{4}} = -\frac{\sqrt{2 - \sqrt{2}}}{2}$$

$$3. \sin \frac{19\pi}{12} = \sin \frac{19\pi}{6} = -\sqrt{\frac{1 - \cos \frac{19\pi}{6}}{2}} = -\sqrt{\frac{1 + \frac{\sqrt{3}}{2}}{2}} = -\sqrt{\frac{\frac{2 + \sqrt{3}}{2}}{2}} = -\sqrt{\frac{2 + \sqrt{3}}{4}} = -\frac{\sqrt{2 + \sqrt{3}}}{2}$$

$$4. \cos 67.5 = \cos \frac{135}{2} = \sqrt{\frac{1 + \cos 135}{2}} = \sqrt{\frac{1 + (-\frac{\sqrt{2}}{2})}{2}} = \sqrt{\frac{\frac{2 - \sqrt{2}}{2}}{2}} = \sqrt{\frac{2 - \sqrt{2}}{4}} = \frac{\sqrt{2 - \sqrt{2}}}{2}$$

Part two:- Given $\sin A = \frac{4}{5}$, $\frac{\pi}{2} < A < \pi$ $\tan B = -\frac{12}{5}$ $\frac{3\pi}{2} < B < 2\pi$ find each of the following.

$$\sin^2 A + \cos^2 A = 1$$

$$\tan B = -\frac{12}{5}$$

$$\sin^2 B + \cos^2 B = 1$$

$$\cos^2 A = 1 - \sin^2 A$$

$$1 + \tan^2 B = \sec^2 B$$

$$\sin^2 B = 1 - \cos^2 B$$

$$\cos^2 A = 1 - \left(\frac{4}{5}\right)^2$$

$$1 + \left(-\frac{12}{5}\right)^2 = \sec^2 B$$

$$\sin^2 B = 1 - \left(\frac{5}{13}\right)^2$$

$$\cos^2 A = 1 - \frac{16}{25}$$

$$1 + \frac{144}{25} = \sec^2 B$$

$$\sin^2 B = 1 - \frac{25}{169}$$

$$\cos^2 A = \frac{9}{25}$$

$$\sec B = \frac{13}{5}$$

$$\sin^2 B = \frac{144}{169}$$

$$\cos A = -\frac{3}{5}$$

$$\cos B = \frac{5}{13}$$

$$\sin B = -\frac{12}{13}$$



$$a. \sin\left(\frac{1}{2}A\right) = \sqrt{\frac{1-\cos A}{2}} = \sqrt{\frac{1-\left(-\frac{3}{5}\right)}{2}} = \sqrt{\frac{1+\frac{3}{5}}{2}} = \sqrt{\frac{\frac{8}{5}}{2}} = \sqrt{\frac{8}{10}} = \frac{2}{\sqrt{5}}$$

$$b. \cos\left(\frac{1}{2}A\right) = \sqrt{\frac{1+\cos A}{2}} = \sqrt{\frac{1+\left(-\frac{3}{5}\right)}{2}} = \sqrt{\frac{1-\frac{3}{5}}{2}} = \sqrt{\frac{\frac{2}{5}}{2}} = \sqrt{\frac{2}{10}} = \frac{1}{\sqrt{5}}$$

$$c. \sin\left(\frac{1}{2}B\right) = \sqrt{\frac{1-\cos B}{2}} = \sqrt{\frac{1-\frac{5}{13}}{2}} = \sqrt{\frac{\frac{8}{13}}{2}} = \sqrt{\frac{8}{26}} = \frac{2}{\sqrt{13}}$$

$$d. \sec\left(\frac{1}{2}B\right) = \frac{1}{\cos\frac{1}{2}B} = \frac{1}{\sqrt{\frac{1+\cos B}{2}}} = \frac{1}{-\sqrt{\frac{1+\frac{5}{13}}{2}}} = \frac{1}{-\sqrt{\frac{\frac{18}{13}}{2}}} = \frac{1}{-\sqrt{\frac{18}{26}}} = \frac{1}{-\sqrt{\frac{9}{13}}} = \frac{1}{-\frac{3}{\sqrt{13}}} = -\frac{\sqrt{13}}{3}$$

Part three: - If $\sin A = \frac{24}{25}$ and $\angle A$ is a positive acute angle

$$\sin^2 A + \cos^2 A = 1$$

$$\cos^2 A = 1 - \sin^2 A$$

$$\cos^2 A = 1 - \left(\frac{24}{25}\right)^2$$

$$\cos^2 A = 1 - \frac{576}{625}$$

$$\cos^2 A = \frac{49}{625}$$

$$\cos A = \frac{7}{25}$$

$$A) \sin\frac{1}{2}A = \sqrt{\frac{1-\cos A}{2}} = \sqrt{\frac{1-\frac{7}{25}}{2}} = \sqrt{\frac{\frac{18}{25}}{2}} = \sqrt{\frac{18}{50}} = \sqrt{\frac{9}{25}} = \frac{3}{5}$$

$$B) \cos\frac{1}{2}A = \sqrt{\frac{1+\cos A}{2}} = \sqrt{\frac{1+\frac{7}{25}}{2}} = \sqrt{\frac{\frac{32}{25}}{2}} = \sqrt{\frac{32}{50}} = \sqrt{\frac{16}{25}} = \frac{4}{5}$$

$$c) \tan\frac{1}{2}A = \frac{1-\cos A}{\sin A} = \frac{1-\frac{7}{25}}{\frac{24}{25}} = \frac{\frac{18}{25}}{\frac{24}{25}} = \frac{18}{24} = \frac{3}{4}$$



Part four: - if $\cos B = \frac{5}{9}$ and $\pi \leq B \leq \frac{3\pi}{2}$ find

$$\begin{aligned} \sin^2 B + \cos^2 B &= 1 \\ \sin^2 B &= 1 - \cos^2 B \\ \sin^2 B &= 1 - \left(\frac{5}{9}\right)^2 \\ \sin^2 B &= 1 - \frac{25}{81} \\ \sin^2 B &= \frac{56}{81} \\ \sin B &= -\frac{2\sqrt{14}}{9} \end{aligned}$$

$$\begin{aligned} \text{A) } \sin \frac{1}{2} B &= \sqrt{\frac{1 - \cos B}{2}} = \sqrt{\frac{1 - \frac{5}{9}}{2}} = \sqrt{\frac{\frac{4}{9}}{2}} = \sqrt{\frac{4}{18}} = \frac{\sqrt{2}}{3} \\ \text{B) } \cos \frac{1}{2} B &= -\sqrt{\frac{1 + \cos B}{2}} = -\sqrt{\frac{1 + \frac{5}{9}}{2}} = -\sqrt{\frac{14}{18}} = -\frac{\sqrt{7}}{3} \\ \text{c) } \tan \frac{1}{2} B &= \frac{1 - \cos B}{\sin B} = \frac{1 - \frac{5}{9}}{-\frac{2\sqrt{14}}{9}} = \frac{\frac{4}{9}}{-\frac{2\sqrt{14}}{9}} = -\frac{2}{\sqrt{14}} \end{aligned}$$

part five: - find the exact value without a calculator using double angle and half identities

$$\begin{aligned} \text{a. } \sin 22.5^\circ &= \sin \frac{45}{2} = \sqrt{\frac{1 - \cos 45}{2}} = \sqrt{\frac{1 - \frac{\sqrt{2}}{2}}{2}} = \sqrt{\frac{2 - \sqrt{2}}{4}} = \frac{\sqrt{2 - \sqrt{2}}}{2} \\ \text{b. } \tan 75^\circ &= \tan \frac{150}{2} = \frac{1 - \cos 150}{\sin 150} = \frac{1 + \frac{\sqrt{3}}{2}}{\frac{1}{2}} = \frac{2 + \sqrt{3}}{\frac{1}{2}} = 2 + \sqrt{3} \\ \text{c. } \cos 67.5^\circ &= \cos \frac{135}{2} = \sqrt{\frac{1 + \cos 135}{2}} = \sqrt{\frac{1 + (-\frac{\sqrt{2}}{2})}{2}} = \sqrt{\frac{2 - \sqrt{2}}{4}} = \frac{\sqrt{2 - \sqrt{2}}}{2} \\ \text{d. } \tan 15^\circ &= \tan \frac{30}{2} = \frac{1 - \cos 30}{\sin 30} = \frac{1 - \frac{\sqrt{3}}{2}}{\frac{1}{2}} = \frac{2 - \sqrt{3}}{\frac{1}{2}} = 2 - \sqrt{3} \end{aligned}$$

Part six: - verify the identities

$$\begin{aligned} 1. (\sin x + \cos x)^2 &= 1 + \sin 2x \\ \sin^2 x + \cos^2 x + 2\sin x \cos x &= 1 + \sin 2x \\ 1 + \sin 2x &= 1 + \sin 2x \end{aligned}$$



$$2. \sin 2x = (\tan x) (1 + \cos 2x)$$

$$\sin 2x = \frac{\sin x}{\cos x} (1 + \cos^2 x - \sin^2 x)$$

$$\sin 2x = \frac{\sin x}{\cos x} (2\cos^2 x)$$

$$\sin 2x = \sin x * 2 \cos x$$

$$\sin 2x = 2 \sin x \cos x$$

$$\sin 2x = \sin 2x$$

$$3. \sin^2 x = \frac{1}{2} (1 - \cos 2x)$$

$$\sin^2 x = \frac{1}{2} - \frac{1}{2} \cos 2x$$

$$\sin^2 x = \frac{1}{2} - \frac{1}{2} (1 - 2\sin^2 x)$$

$$\sin^2 x = \frac{1}{2} - \frac{1}{2} + \sin^2 x$$

$$\sin^2 x = \sin^2 x$$

$$4. \cos^2 x = \frac{1}{2} (\cos 2x + 1)$$

$$\cos^2 x = \frac{1}{2} [\cos^2 x - \sin^2 x + 1]$$

$$\cos^2 x = \frac{1}{2} [\cos^2 x - (1 - \cos^2 x) + 1]$$

$$\cos^2 x = \frac{1}{2} [2\cos^2 x]$$

$$\cos^2 x = \cos^2 x$$

$$5. 1 - \cos 2x = \tan x \sin 2x$$

$$1 - (1 - 2\sin^2 x) = \frac{\sin x}{\cos x} * 2 \sin x \cos x$$

$$1 - 1 + 2\sin^2 x = 2\sin^2 x$$

$$2\sin^2 x = 2\sin^2 x$$

$$6. 1 + \sin 2t = (\sin t + \cos t)^2$$

$$1 + \sin 2t = \sin^2 t + \cos^2 t + 2 \sin t \cos t$$

$$1 + \sin 2t = 1 + 2 \sin t \cos t$$

$$1 + \sin 2t = 1 + \sin 2t$$



Section 1.4

Part one: - Express each sum or difference as a product.

$$\begin{aligned} 1. \sin 6x + \sin 2x &= 2\sin \frac{6x+2x}{2} \cos \frac{6x-2x}{2} \\ &= 2\sin 4x \cos 2x \end{aligned}$$

$$\begin{aligned} 2. \sin 8x + \sin 2x &= 2\sin \frac{8x+2x}{2} \cos \frac{8x-2x}{2} \\ &= 2\sin 5x \cos 3x \end{aligned}$$

$$\begin{aligned} 3. \sin 7x - \sin 3x &= 2\cos \frac{7x+3x}{2} \sin \frac{7x-3x}{2} \\ &= 2\cos 5x \sin 2x \end{aligned}$$

$$\begin{aligned} 4. \sin 11x - \sin 5x &= 2\cos \frac{11x+5x}{2} \sin \frac{11x-5x}{2} \\ &= 2\cos 8x \sin 3x \end{aligned}$$

$$\begin{aligned} 5. \cos 4x + \cos 2x &= 2\cos \frac{4x+2x}{2} \cos \frac{4x-2x}{2} \\ &= 2\cos 3x \cos x \end{aligned}$$

$$\begin{aligned} 6. \cos 9x - \cos 7x &= -2\sin \frac{9x+7x}{2} \sin \frac{9x-7x}{2} \\ &= -2\sin 8x \sin x \end{aligned}$$

$$\begin{aligned} 7. \sin x + \sin 2x &= 2\sin \frac{x+2x}{2} \cos \frac{x-2x}{2} \\ &= 2\sin \frac{3x}{2} \cos \frac{-x}{2} = 2\sin \frac{3}{2}x \cos \frac{x}{2} \end{aligned}$$

$$\begin{aligned} 8. \sin x - \sin 2x &= 2\cos \frac{x+2x}{2} \sin \frac{x-2x}{2} \\ &= 2\cos \frac{3}{2}x \sin -\frac{x}{2} \\ &= -2\cos \frac{3}{2}x \sin \frac{x}{2} \end{aligned}$$

$$\begin{aligned} 9. \cos \frac{3x}{2} + \cos \frac{x}{2} &= 2\cos \frac{\frac{3x}{2} + \frac{x}{2}}{2} \cos \frac{\frac{3x}{2} - \frac{x}{2}}{2} \\ &= 2\cos \frac{2x}{2} \cos \frac{x}{2} = 2\cos x \cos \frac{x}{2} \end{aligned}$$



$$10. \quad \sin 75^\circ + \sin 15^\circ = 2 \sin \frac{75+15}{2} \cos \frac{75-15}{2}$$

$$= 2 \sin 45 \cos 30$$

$$= 2 \left(\frac{\sqrt{2}}{2} \right) \left(\frac{\sqrt{3}}{2} \right) = \frac{\sqrt{6}}{2}$$

$$11. \quad \cos 75^\circ - \sin 15^\circ = -2 \sin \frac{75+15}{2} \sin \frac{75-15}{2}$$

$$= -2 \sin 45 \sin 30$$

$$= -2 \left(\frac{\sqrt{2}}{2} \right) \left(\frac{1}{2} \right)$$

$$= -\frac{\sqrt{2}}{2}$$

$$12. \quad \sin \frac{\pi}{12} - \sin \frac{5\pi}{12} = 2 \cos \frac{\frac{\pi}{12} + \frac{5\pi}{12}}{2} \sin \frac{\frac{\pi}{12} - \frac{5\pi}{12}}{2}$$

$$= -2 \cos \frac{\pi}{4} \sin \frac{\pi}{6}$$

$$= 2 \left(\frac{\sqrt{2}}{2} \right) \left(\frac{1}{2} \right) = -\frac{\sqrt{2}}{2}$$

Part two: - verify each identity.

$$1. \quad \frac{\sin 3x - \sin x}{\cos 3x - \cos x} = -\cot 2x$$

2.

$$\frac{2 \cos \frac{3x+x}{2} \sin \frac{3x-x}{2}}{-2 \sin \frac{3x+x}{2} \sin \frac{3x-x}{2}} = -\cot 2x$$

$$= \frac{2 \cos 2x \sin x}{-2 \sin 2x \sin x} = -\cot 2x$$

$$-\cot 2x = -\cot 2x$$



$$3. \frac{\sin x + \sin 3x}{\cos x + \cos 3x} = \tan 2x$$

$$\frac{2\sin \frac{x+3x}{2} \cos \frac{x-3x}{2}}{2\cos \frac{x+3x}{2} \cos \frac{x-3x}{2}} = \tan 2x$$

$$\frac{2\sin 2x \cos x}{2 \cos 2x \cos x} = \tan 2x$$

$$\tan 2x = \tan 2x$$

$$4. \frac{\sin 2x + \sin 4x}{\cos 2x + \cos 4x} = \tan 3x$$

$$\frac{2\sin \frac{2x+4x}{2} \cos \frac{2x-4x}{2}}{2\cos \frac{2x+4x}{2} \cos \frac{2x-4x}{2}} = \tan 3x$$

$$\frac{2\sin 3x \cos x}{2\cos 3x \cos x} = \tan 3x$$

$$\tan 3x = \tan 3x$$

$$4. \frac{\cos 4x - \cos 2x}{\sin 2x - \sin 4x} = \tan 3x$$

$$\frac{-2\sin \frac{4x+2x}{2} \sin \frac{4x-2x}{2}}{2\cos \frac{4x+2x}{2} \sin \frac{2x-4x}{2}} = \tan 3x$$

$$\frac{-2\sin 3x \sin x}{-2\cos 3x \sin x} = \tan 3x$$



$$\frac{\sin 3x}{\cos 3x} = \tan 3x$$

$$\tan 3x = \tan 3x$$

$$5. \frac{\sin x - \sin y}{\sin x + \sin y} = \tan \frac{x-y}{2} \cot \frac{x+y}{2}$$

$$\frac{2 \cos \frac{x+y}{2} \sin \frac{x-y}{2}}{2 \sin \frac{x+y}{2} \cos \frac{x-y}{2}} = \tan \frac{x-y}{2} \cot \frac{x+y}{2}$$

$$\tan \frac{x-y}{2} \cot \frac{x+y}{2} = \tan \frac{x-y}{2} \cot \frac{x+y}{2}$$

$$6. \frac{\sin x + \sin y}{\sin x - \sin y} = \tan \frac{x+y}{2} \cot \frac{x-y}{2}$$

$$\frac{2 \sin \frac{x+y}{2} \cos \frac{x-y}{2}}{2 \cos \frac{x+y}{2} \sin \frac{x-y}{2}} = \tan \frac{x+y}{2} \cot \frac{x-y}{2}$$

$$\tan \frac{x+y}{2} \cot \frac{x-y}{2} = \tan \frac{x+y}{2} \cot \frac{x-y}{2}$$

$$7. \frac{\sin x + \sin y}{\cos x + \cos y} = \tan \frac{x+y}{2}$$

$$\frac{2 \sin \frac{x+y}{2} \cos \frac{x-y}{2}}{2 \cos \frac{x+y}{2} \cos \frac{x-y}{2}} = \tan \frac{x+y}{2}$$

$$\tan \frac{x+y}{2} = \tan \frac{x+y}{2}$$

$$8. \frac{\sin x - \sin y}{\cos x - \cos y} = -\cot \frac{x+y}{2}$$

$$= \frac{2 \cos \frac{x+y}{2} \sin \frac{x-y}{2}}{-2 \sin \frac{x+y}{2} \sin \frac{x-y}{2}} = -\cot \frac{x+y}{2}$$

$$-\cot \frac{x+y}{2} = -\cot \frac{x+y}{2}$$



Section 1.5

Part one: - find the exact value of $\cos 165 - \cos 75$ using an appropriate sum – product identity

$$\begin{aligned}\cos 165 - \cos 75 &= -2 \sin \frac{165+75}{2} \sin \frac{165-75}{2} = -2 \sin 120 \sin 45 \\ &= -2 \left(\frac{\sqrt{3}}{2} \right) \left(\frac{\sqrt{2}}{2} \right) = -\frac{\sqrt{6}}{2}\end{aligned}$$

Part two: - write each product as sum or difference involving sine and cosine

1. $\sin 3m \cos 3m = \frac{1}{2} [\sin(3m + m) + \sin(3m - m)] = \frac{1}{2} [\sin 4m + \sin 2m]$
2. $\cos 7A \cos 5A = \frac{1}{2} [\cos(7A + 5A) + \cos(7A - 5A)] = \frac{1}{2} [\cos 12A + \cos 2A]$
3. $\sin B \sin 3B = \frac{1}{2} [\cos(B - 3B) - \cos(B + 3B)] = \frac{1}{2} [\cos 2B - \cos 4B]$
4. $\cos 2C \sin 3C = \frac{1}{2} [\sin(2C + 3C) - \sin(2C - 3C)] = \frac{1}{2} [\sin 5C + \sin C]$

Part three:- write each difference or sum as product involving sine and cosine

5. $\sin 3t + \sin t = 2 \sin \frac{3t+t}{2} \cos \frac{3t-t}{2} = 2 \sin 2t \cos t$
6. $\cos 7U + \cos 5U = 2 \cos \frac{7U+5U}{2} \cos \frac{7U-5U}{2} = 2 \cos 6U \cos U$
7. $\cos 5W - \cos 9W = -2 \sin \frac{5W+9W}{2} \sin \frac{5W-9W}{2} = 2 \sin 7W \sin 2W$
8. $\sin u - \sin 5u = 2 \cos \frac{u+5u}{2} \sin \frac{u-5u}{2} = -2 \cos 3u \sin 2u$

Part four: - evaluate exactly using an appropriate identity

9. $\cos 285 + \cos 195 = 2 \cos \frac{285+195}{2} \cos \frac{285-195}{2} = 2 \cos 240 \cos 45 = 2 \left(-\frac{1}{2} \right) \left(\frac{\sqrt{2}}{2} \right) = -\frac{\sqrt{2}}{2}$
10. $\sin 195 + \sin 105 = 2 \cos \frac{195+105}{2} \sin \frac{195-105}{2} = 2 \cos 150 \sin 45 = 2 \left(-\frac{\sqrt{3}}{2} \right) \left(\frac{\sqrt{2}}{2} \right) = -\frac{\sqrt{6}}{2}$
11. $\cos 15 - \cos 105 = -2 \sin \frac{15+105}{2} \sin \frac{15-105}{2} = 2 \sin 60 \sin 45 = 2 \left(\frac{\sqrt{3}}{2} \right) \left(\frac{\sqrt{2}}{2} \right) = \frac{\sqrt{6}}{2}$
12. $\sin 75 * 2 \sin 165 = 2 * \frac{1}{2} [\cos(75 - 165) - \cos(75 + 165)] = \cos 90 - \cos 240$
 $= 0 + \frac{1}{2} = \frac{1}{2}$



Section 1.6

Solving trigonometric equations

1. $\sin \frac{1}{2} x = -1$

$$\sin \frac{1}{2} x = \sin 270$$

$$\frac{1}{2} x = 270$$

$$x = 540 \text{ or } 3\pi$$

2. No result

3. $\sin 2x \cos x = \sin x$

$$2 \sin x \cos x \cos x = \sin x$$

$$2 \sin x \cos^2 x = \sin x$$

$$2 \sin x \cos^2 x - \sin x = 0$$

$$\sin x (2 \cos^2 x - 1) = 0$$

$$\sin x = 0$$

$$x = 0, \pi$$

$$x = 0, 180$$

$$(2 \cos^2 x - 1) = 0$$

$$\cos x = \pm \frac{1}{\sqrt{2}}$$

$$\cos x = \pm \frac{1}{2}$$

$$2 \cos^2 x = 1 \quad \cos^2 x = \frac{1}{2}$$

$$x = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$$

$$x = 45, 135, 225, 315$$

4. $\cos 2x = -2 \cos^2 x$

$$2 \cos^2 x - 1 = -2 \cos^2 x$$

$$2 \cos^2 x - 1 + 2 \cos^2 x = 0$$

$$4 \cos^2 x = 1$$

$$\cos^2 x = \frac{1}{4}$$

$$x = \frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}$$

$$x = 60, 120, 240, 300$$



5. $\tan 2x - \tan x = 0$

$$\frac{2\tan x}{1-\tan^2 x} - \tan x = 0$$

$$2\tan x - \tan x + \tan^3 x = 0$$

$$\tan^3 x + \tan x = 0$$

$$\tan x(\tan^2 x + 1) = 0$$

$$\tan x = 0$$

$$\tan^2 x + 1 = 0$$

$$x = 0, \pi \text{ or } 0, 180$$

$$\tan^2 x = -1$$

$$\tan x = \pm\sqrt{-1} \quad x = \text{No real solution}$$

6. $\sin \frac{x}{2} + \cos x = 1$

$$\sin \frac{x}{2} = 1 - \cos x$$

$$\left(\sqrt{\frac{1-\cos x}{2}} \right)^2 = (1 - \cos x)^2$$

$$\frac{1-\cos x}{2} = 1 - 2\cos x + \cos^2 x$$

$$1 - \cos x = 2 - 4\cos x + 2\cos^2 x$$

$$2\cos^2 x - 4\cos x + \cos x + 2 - 1 = 0$$

$$2\cos^2 x - 3\cos x + 1 = 0$$

$$2\cos^2 x - 2\cos x - \cos x + 1 = 0$$

$$2\cos x(\cos x - 1) - 1(\cos x - 1) = 0$$

$$(2\cos x - 1)(\cos x - 1) = 0$$

$$(\cos x - 1) = 0$$

$$2\cos x - 1 = 0$$

$$\cos x = 1$$

$$2\cos x = 1$$

$$x = 0, 2\pi \text{ or } 0, 360$$

$$\cos x = -\frac{1}{2} \quad x = \frac{\pi}{3}, \frac{5\pi}{3} \text{ or } 60, 300$$



7. No result

$$8. \tan \frac{x}{2} = \sin x$$

$$\frac{1 - \cos x}{\sin x} = \sin x$$

$$1 - \cos x = \sin^2 x$$

$$1 - \cos x - \sin^2 x = 0$$

$$1 - \cos x - (1 - \cos^2 x) = 0$$

$$1 - \cos x - 1 + \cos^2 x = 0$$

$$\cos^2 x - \cos x = 0$$

$$\cos x (\cos x - 1) = 0$$

$$\cos x = 0$$

$$\cos x - 1 = 0$$

$$x = \frac{\pi}{2}, \frac{3\pi}{2} \text{ or } 90, 270$$

$$\cos x = 1 \quad x = 0, 2\pi \text{ or } 0, 360$$

$$9. \frac{\cos 2x}{\cos^2 x} = 1$$

$$\frac{2\cos^2 x - 1}{\cos^2 x} = 1$$

$$2\cos^2 x - 1 = \cos^2 x$$

$$2\cos^2 x - \cos^2 x - 1 = 0$$

$$\cos^2 x - 1 = 0$$

$$\cos^2 x = 1$$

$$\sqrt{\cos^2 x} = \sqrt{1}$$

$$\cos x = \pm 1 \quad x = 0, \pi, 2\pi \text{ or } 0, 180, 360$$

finding general solution , find the general solution of the equations

$$10. \cos 2x = -1$$

$$\cos 2x = \cos 180$$

$$2x = 180$$



$$x = 90 + 2n\pi \quad \text{or} \quad x = \frac{\pi}{2} + 2n\pi$$

$$11. \sin 2x + \sin x = 0$$

$$2\sin x \cos x + \sin x = 0$$

$$\sin x(2\cos x + 1) = 0$$

$$\sin x = 0$$

$$2\cos x + 1 = 0$$

$$x = 0 + 2n\pi$$

$$2\cos x = -1$$

$$x = \pi + 2n\pi$$

$$\cos x = -\frac{1}{2} \quad x = \frac{2\pi}{3} + 2n\pi \quad x = \frac{4\pi}{3} + 2n\pi$$

$$12. \cos 2x - \cos x = 0$$

$$2\cos^2 x - 1 - \cos x = 0$$

$$2\cos^2 x - \cos x - 1 = 0$$

$$2\cos^2 x - 2\cos x + \cos x - 1 = 0$$

$$2\cos x(\cos x - 1) + 1(\cos x - 1) = 0$$

$$(2\cos x + 1)(\cos x - 1) = 0$$

$$2\cos x + 1 = 0$$

$$\cos x - 1 = 0$$

$$2\cos x = -1$$

$$\cos x = 1$$

$$\cos x = -\frac{1}{2}$$

$$x = 0 + 2n\pi$$

$$x = \frac{2\pi}{3} + 2n\pi$$

$$x = 2\pi + 2n\pi$$

$$x = \frac{4\pi}{3} + 2n\pi$$



$$13. \cos \frac{x}{2} - \sin x = 0$$

$$\left(\sqrt{\frac{1+\cos x}{2}} \right)^2 = (\sin x)^2$$

$$\frac{1+\cos x}{2} = \sin^2 x$$

$$1 + \cos x = 2\sin^2 x$$

$$1 + \cos x - 2\sin^2 x = 0$$

$$1 - \cos x - 2(1 - \cos^2 x) = 0$$

$$1 + \cos x - 2 + 2\cos^2 x = 0$$

$$2\cos^2 x + \cos x - 1 = 0$$

$$2\cos^2 x + 2\cos x - \cos x - 1 = 0$$

$$2\cos x(\cos x + 1) - 1(\cos x + 1) = 0$$

$$(2\cos x - 1)(\cos x + 1) = 0$$

$$2\cos x - 1 = 0$$

$$\cos x + 1 = 0$$

$$2\cos x = 1$$

$$\cos x = -1$$

$$\cos x = \frac{1}{2}$$

$$x = 0 + 2n\pi$$

$$x = \frac{2\pi}{3} + 2n\pi$$

$$x = 2\pi + 2n\pi$$

$$x = \frac{4\pi}{3} + 2n\pi$$

14. No result



$$15. \cos 2x = 3\sin x + 2$$

$$1 - 2\sin^2 x = 3\sin x + 2$$

$$2\sin^2 x + 3\sin x + 1 = 0$$

$$2\sin^2 x + 2\sin x + \sin x + 1 = 0$$

$$2\sin x(\sin x + 1) + 1(\sin x + 1) = 0$$

$$(2\sin x + 1)(\sin x + 1) = 0$$

$$2\sin x + 1 = 0$$

$$\sin x + 1 = 0$$

$$2\sin x = -1$$

$$\sin x = -1$$

$$\sin x = -\frac{1}{2}$$

$$x = \frac{3\pi}{2} + 2n\pi$$

$$X = \frac{7\pi}{6} + 2n\pi$$

$$X = \frac{11\pi}{6} + 2n\pi$$



Section 1.7

$$1)p = 180$$

$$L = a$$

$$A = ?$$

$$P = a + a + a$$

$$S = \frac{a + a + a}{2} = \frac{60 + 60 + 60}{2} = \frac{180}{2} = 90$$

$$P = 3a$$

$$180 = 3a$$

$$A = \sqrt{s(s-a)(s-a)(s-a)}$$

$$a = 60$$

$$A = \sqrt{90(90-60)(90-60)(90-60)}$$

$$A = \sqrt{90 \times 30 \times 30 \times 30} = 1558.8 \text{ cm}^2$$

$$2) \quad a = 122\text{m} \quad b = 22\text{m} \quad c = 120\text{m}$$

$$S = \frac{122 + 22 + 120}{2} = \frac{264}{2} = 132\text{m}$$

$$\text{Area of side walls} = \sqrt{132(132-122)(132-22)(132-120)}$$

$$A = \sqrt{132 \times 10 \times 110 \times 12} = 1320\text{m}^2$$

$$\text{if } 1\text{m}^2 \rightarrow 5000 \text{ per year}$$

$$1320\text{m}^2 \rightarrow X$$

$$X = 6600000 \text{ per year}$$

if the company pays 6600000 for 12 months, how much will it pay for 3 months

$$6600000 \rightarrow 12 \text{ months}$$

$$X \rightarrow 3 \text{ months}$$

$$X = 1650000$$



3) sides of painted in colour are $a = 15m$ $b = 11m$ $c = 6m$

Then Area painted in colour is $A = \sqrt{s(s-a)(s-b)(s-c)}$

$$S = \frac{a+b+c}{2} = \frac{15+11+6}{2} = 16m$$

$$A = \sqrt{16(16-15)(16-11)(16-6)} = \sqrt{16 \times 1 \times 5 \times 10} = \sqrt{800} = 28.3m^2$$

4) $a = 18cm$ $b = 10cm$ $p = 42cm$

$$P = a + b + c \quad s = \frac{a+b+c}{2} = \frac{18+10+14}{2} = 21$$

$$42 = 18 + 10 + c \quad A = \sqrt{21(21-18)(21-10)(21-14)}$$

$$C = 42 - 28 \quad A = \sqrt{21 \times 3 \times 11 \times 7}$$

$$C = 14cm \quad A = 69.6cm^2$$

5) sides of triangle in ratio 12: 17: 25 $p = 540cm$ $A = ?$

$$P = a + b + c \quad a = 12x = 12(10) = 120$$

$$540 = 12x + 17x + 25x \quad b = 17x = 17(10) = 170$$

$$540 = 54x \quad c = 25x = 25(10) = 250$$

$$X = 10cm$$

$$s = \frac{a+b+c}{2} = \frac{120+170+250}{2} = 270cm$$

$$A = \sqrt{270(270-120)(270-170)(270-250)}$$

$$A = \sqrt{270 \times 150 \times 100 \times 20} = 9000cm^2$$



$$6. p = 30cm \quad a = 12cm \quad b = 12cm \quad A = ?$$

$$P = a + b + c \quad A = \sqrt{15(15 - 12)(15 - 12)(15 - 6)} = \sqrt{15 * 3 * 3 * 9}$$

$$30 = 12 + 12 + c \quad A = 34.9cm^2$$

$$C = 30 - 24 = 6cm$$

$$S = \frac{a + b + c}{2} = \frac{12 + 12 + 6}{2} = 15cm$$

$$7. a) x = 6in \quad y = 4in \quad z = 7in$$

$$S = \frac{x + y + z}{2} = \frac{6 + 4 + 7}{2} = 8.5in$$

$$A = \sqrt{s(s - x)(s - y)(s - z)}$$

$$A = \sqrt{8.5(8.5 - 6)(8.5 - 4)(8.5 - 7)} = \sqrt{8.5 * 2.5 * 4.5 * 1.5} = 11.9in^2 \approx 12in^2$$

$$b) d = 16ft \quad e = 14ft \quad f = 6ft$$

$$S = \frac{d + e + f}{2} = \frac{16 + 14 + 6}{2} = 18ft$$

$$A = \sqrt{s(s - d)(s - e)(s - f)}$$

$$A = \sqrt{18(18 - 16)(18 - 14)(18 - 6)} = \sqrt{18 * 2 * 4 * 12}$$

$$A = 41.6ft^2$$

$$c) d = 14km \quad e = 16km \quad f = 11km$$

$$S = \frac{d + e + f}{2} = \frac{14 + 16 + 11}{2} = 20.5km$$

$$A = \sqrt{s(s - d)(s - e)(s - f)}$$

$$A = \sqrt{20.5(20.5 - 14)(20.5 - 16)(20.5 - 11)} = \sqrt{20.5 * 6.5 * 4.5 * 9.5} = 75.5km^2$$



d) $a = 12mi$ $b = 16mi$ $c = 9mi$

$$S = \frac{a + b + c}{2} = \frac{12 + 16 + 9}{2} = 18.5mi$$

$$A = \sqrt{s(s-a)(s-b)(s-c)}$$

$$A = \sqrt{18.5(18.5 - 12)(18.5 - 16)(18.5 - 9)} = \sqrt{18.5 * 6.5 * 2.5 * 9.5}$$

$$A = 53.4mi^2$$

e) $d = 12ft$ $e = 11.9ft$ $f = 14ft$

$$S = \frac{d + e + f}{2} = \frac{12 + 11.9 + 14}{2} = 18.95ft$$

$$A = \sqrt{s(s-d)(s-e)(s-f)}$$

$$A = \sqrt{18.95(18.95 - 12)(18.95 - 11.9)(18.95 - 14)} = \sqrt{18.95 * 6.95 * 7.05 * 4.95}$$

$$A = 67.8ft^2$$

f) $d = 8ft$ $e = 12ft$ $f = 13ft$

$$S = \frac{d + e + f}{2} = \frac{8 + 12 + 13}{2} = 16.5ft$$

$$A = \sqrt{s(s-d)(s-e)(s-f)}$$

$$A = \sqrt{16.5(16.5 - 8)(16.5 - 12)(16.5 - 13)} = \sqrt{16.5 * 8.5 * 4.5 * 3.5}$$

$$A = 47ft^2$$

g) in $\triangle CAB$ $c = 6in$ $a = 12.8in$ $b = 13in$

$$s = \frac{a+b+c}{2} = \frac{12.8+13+6}{2} = 15.9in$$

$$A = \sqrt{15.9(15.9 - 12.8)(15.9 - 13)(15.9 - 6)} = \sqrt{15.9 * 3.1 * 2.9 * 9.9} = 37.6in^2$$



h) in $\triangle DEF$ $d = 10.3ft$ $e = 9ft$ $f = 15ft$

$$s = \frac{d + e + f}{2} = \frac{10.3 + 9 + 15}{2} = 17.15ft$$

$$A = \sqrt{17.15(17.15 - 10.3)(17.15 - 9)(17.15 - 15)} = \sqrt{17.15 \times 6.85 \times 8.15 \times 2.15} = 45.4ft^2$$

l) in $\triangle FDE$ $f = 13mi$ $d = 4mi$ $e = 13mi$

$$s = \frac{d + e + f}{2} = \frac{4 + 13 + 13}{2} = 15mi$$

$$A = \sqrt{15(15 - 13)(15 - 4)(15 - 13)} = \sqrt{15 \times 2 \times 11 \times 2} = 25.7mi^2$$

j) in $\triangle XYZ$ $y = 15km$ $x = 5km$ $z = 12km$

$$s = \frac{x + y + z}{2} = \frac{5 + 15 + 12}{2} = 16km$$

$$A = \sqrt{16(16 - 15)(16 - 5)(16 - 12)} = \sqrt{16 \times 1 \times 11 \times 4} = 26.5km^2$$



Answers of chapter two:- complex numbers

Section 2.1

Page 35 exercises 1 – 24 perform the operation and write the result in standard form

1. $(5 + i) + (6 - 2i) = 11 - i$
2. $(13 - 2i) + (-5 + 6i) = 8 + 4i$
3. $(8 - i) - (4 - i) = 4 + 0i$
4. $(3 + 2i) - (6 + 13i) = -3 - 11i$
5. $(-2 + \sqrt{-8}) + (5 - \sqrt{-50}) = (-2 + 2\sqrt{2}i) + (5 - 5\sqrt{2}i) = 3 - 3\sqrt{2}i$
6. $(8 + \sqrt{-18}) - (4 + 3\sqrt{2}i) = (8 + 3\sqrt{2}i) - (4 + 3\sqrt{2}i) = 4 + 0i$
7. $13 - (14 - 7i) = 13 - 14 + 7i = -1 + 7i$
8. $22 + (-5 + 8i) + 10i = 22 - 5 + 8i + 10i = 17 + 18i$
9. $-\left(\frac{3}{2} + \frac{5}{2}i\right) + \left(\frac{5}{3} + \frac{11}{3}i\right) = \left(-\frac{3}{2} - \frac{5}{2}i\right) + \left(\frac{5}{3} + \frac{11}{3}i\right) = \frac{1}{6} + \frac{7}{6}i$
10. $(1.6 + 3.2i) + (-5.8 + 4.3i) = -4.2 + 7.5i$
11. $\sqrt{-6} \cdot \sqrt{-2} = \sqrt{6}i \cdot \sqrt{2}i = \sqrt{12}i^2 = -\sqrt{12}$
12. $\sqrt{-5} \cdot \sqrt{-10} = \sqrt{5}i \cdot \sqrt{10}i = \sqrt{50}i^2 = -\sqrt{50}$
13. $(\sqrt{-10})^2 = (\sqrt{10}i)^2 = 10i^2 = -10$
14. $(\sqrt{-75})^2 = (\sqrt{75}i)^2 = 75i^2 = -75$
15. $(1 + i)(3 - 2i) = 3 - 2i + 3i - 2i^2 = 3 + i + 2 = 5 + i$
16. $(6 - 2i)(2 - 3i) = 12 - 18i - 4i + 6i^2 = 12 - 22i - 6 = 6 - 22i$
17. $6i(5 - 2i) = 30i - 12i^2 = 30i + 12 = 12 + 30i$
18. $-8i(9 + 4i) = -72i - 32i^2 = -72i + 32 = 32 - 72i$
19. $(\sqrt{14} + \sqrt{10}i)(\sqrt{14} - \sqrt{10}i) = 14 - 10i^2 = 14 + 10 = 24$
20. $(3 + \sqrt{-5})(7 - \sqrt{-10}) = (3 + \sqrt{5}i)(7 - \sqrt{10}i) = 21 - 3\sqrt{10}i + 7\sqrt{5}i - \sqrt{50}i^2$
 $= (21 + \sqrt{50}) + (7\sqrt{5} - 3\sqrt{10})i$
21. $(4 + 5i)^2 = 16 + 40i + 25i^2 = 16 + 40i - 25 = -9 + 40i$
22. $(2 - 3i)^2 = 4 - 12i + 9i^2 = 4 - 12i - 9 = -5 - 12i$
23. $(2 + 3i)^2 + (2 - 3i)^2 = 4 + 12i + 9i^2 + 4 - 12i + 9i^2 = 8 + 18i^2 = 8 - 18 = -10$
24. $(1 - 2i)^2 - (1 + 2i)^2 = 1 - 4i + 4i^2 - 1 - 4i - 4i^2 = -8i$



Section 2.2

Write the conjugate of the complex number multiply the number and its conjugate .

1. $5 + 3i = ?$

the conjugate of $5 + 3i$ is $5 - 3i$

$$(5 + 3i)(5 - 3i) = 25 - 9i^2 = 25 + 9 = 34$$

2. $-2 - \sqrt{5}i = ?$

The conjugate of $-2 - \sqrt{5}i$ is $-2 + \sqrt{5}i$

$$(-2 - \sqrt{5}i)(-2 + \sqrt{5}i) = 4 - 5i^2 = 4 + 5 = 9$$

3. $20i = ?$

The conjugate of $20i$ is $-20i$

$$(20i)(-20i) = -400i^2 = 400$$

4. $\sqrt{8} = ?$

the conjugate of $\sqrt{8}$ is $\sqrt{8}$

$$\sqrt{8} * \sqrt{8} = 8$$

5. $9 - 12i = ?$

The conjugate of $9 - 12i$ is $9 + 12i$

$$(9 - 12i)(9 + 12i) = 81 - 144i^2 = 81 + 144 = 225$$

6. $-4 + \sqrt{2}i = ?$

The conjugate of $-4 + \sqrt{2}i$ is $-4 - \sqrt{2}i$

$$(-4 + \sqrt{2}i)(-4 - \sqrt{2}i) = 16 - 2i^2 = 16 + 2 = 18$$

7. $\sqrt{-15} = \sqrt{15}i = ?$

The conjugate of $\sqrt{15}i$ is $-\sqrt{15}i$

$$\sqrt{15}i * -\sqrt{15}i = -15i^2 = 15$$



8. $1 + \sqrt{8} = ?$

The conjugate of $1 + \sqrt{8}$ is $1 + \sqrt{8}$

$$(1 + \sqrt{8}) * (1 + \sqrt{8}) = 1 + \sqrt{8} + \sqrt{8} + 8 = 9 + 2\sqrt{8} = 9 + 4\sqrt{2}$$

Part two: in exercise 1-14 perform the operation and write the result in standard form

$$1. \frac{6}{i} = \frac{6}{i} * \frac{-i}{-i} = \frac{-6i}{-i^2} = \frac{-6i}{1} = -6i$$

$$2. \frac{-10}{2i} = \frac{-10}{2i} * \frac{-2i}{-2i} = \frac{20i}{-4i^2} = \frac{20i}{4} = 5i$$

$$3. \frac{4}{4-5i} = \frac{4}{4-5i} * \frac{4+5i}{4+5i} = \frac{16+20i}{16-25i^2} = \frac{16+20i}{16+25} = \frac{16+20i}{41} = \frac{16}{41} + \frac{20}{41}i$$

$$4. \frac{3}{1-i} = \frac{3}{1-i} * \frac{1+i}{1+i} = \frac{3+3i}{1-i^2} = \frac{3+3i}{1+1} = \frac{3+3i}{2} = \frac{3}{2} + \frac{3}{2}i$$

$$5. \frac{2+i}{2-i} = \frac{2+i}{2-i} * \frac{2+i}{2+i} = \frac{4+2i+2i+i^2}{4-i^2} = \frac{4+4i-1}{4+1} = \frac{3+4i}{5} = \frac{3}{5} + \frac{4}{5}i$$

$$6. \frac{8-7i}{1-2i} = \frac{8-7i}{1-2i} * \frac{1+2i}{1+2i} = \frac{8+16i-7i-14i^2}{1-4i^2} = \frac{8+9i+14}{1+4} = \frac{22+9i}{5} = \frac{22}{5} + \frac{9}{5}i$$

$$7. \frac{6-7i}{i} = \frac{6-7i}{i} * \frac{-i}{-i} = \frac{-6i+7i^2}{-i^2} = \frac{-6i-7}{1} = -7 - 6i$$

$$8. \frac{8+20i}{2i} = \frac{8+20i}{2i} * \frac{-2i}{-2i} = \frac{-16i-40i^2}{-4i^2} = \frac{-16i+40}{4} = \frac{40}{4} - \frac{16}{4}i = 10 - 4i$$

$$9. \frac{1}{(4-5i)^2} = \frac{1}{16-40i+25i^2} = \frac{1}{16-40i-25} = \frac{1}{-9-40i}$$

$$\frac{1}{-9-40i} = \frac{1}{-9-40i} * \frac{-9+40i}{-9+40i} = \frac{-9+40i}{81-1600i^2} = \frac{-9+40i}{81+1600}$$

$$= \frac{-9}{1681} + \frac{40}{1681}i$$

$$10. \frac{(2-3i)(5i)}{2+3i} = \frac{10i-15i^2}{2+3i} = \frac{15+10i}{2+3i}$$

$$\frac{15+10i}{2+3i} * \frac{2-3i}{2-3i} = \frac{30-45i+20i-30i^2}{4-9i^2} = \frac{30-25i+30}{4+9} = \frac{60-25i}{13}$$

$$= \frac{60}{13} - \frac{25}{13}i$$



$$\begin{aligned}
 11. \frac{2}{1+i} - \frac{3}{1-i} &= \frac{2(1-i)-3(1+i)}{(1+i)(1-i)} = \frac{2-2i-3-3i}{1-i^2} = \frac{-1-5i}{1+1} = \frac{-1-5i}{2} = \frac{-1}{2} - \frac{5}{2}i \\
 12. \frac{2i}{2+i} + \frac{5}{2-i} &= \frac{2i(2-i)+5(2+i)}{(2+i)(2-i)} = \frac{4i-2i^2+10+5i}{4-i^2} = \frac{4i+2+10+5i}{4+1} = \frac{12+9i}{5} = \frac{12}{5} + \frac{9}{5}i \\
 13. \frac{i}{3-2i} + \frac{2i}{3+8i} &= \frac{i(3+8i)+2i(3-2i)}{(3-2i)(3+8i)} = \frac{3i+8i^2+6i-4i^2}{9+24i-6i-16i^2} = \frac{9i+4i^2}{9+18i+16} = \frac{-4+9i}{25+18i} \\
 &= \frac{-4+9i}{25+18i} * \frac{25-18i}{25-18i} = \frac{-100+72i+225i-162i^2}{625-324i^2} \\
 &= \frac{-100+297i+162}{625+324} = \frac{62+297i}{949} = \frac{62}{949} + \frac{297}{949}i \\
 14. \frac{1+i}{i} - \frac{3}{4-i} &= \frac{(1+i)(4-i)-3(i)}{i(4-i)} = \frac{4-i+4i-i^2-3i}{4i-i^2} = \frac{4+3i+1-3i}{4i+1} = \frac{5}{1+4i} \\
 &= \frac{5}{1+4i} * \frac{1-4i}{1-4i} = \frac{5-20i}{1-16i^2} = \frac{5-20i}{1+16} = \frac{5-20i}{17} = \frac{5}{17} - \frac{20}{17}i
 \end{aligned}$$



Section 2.3

Part one:-In exercise 2.3 use the quadratic formula to solve the quadratic equation

1. $x^2 - 2x + 2 = 0$

$a = 1$

$b = -2$

$c = 2$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(1)(2)}}{2(1)} = \frac{2 \pm \sqrt{4 - 8}}{2} = \frac{2 \pm \sqrt{-4}}{2}$$

$$x = \frac{2 \pm 2i}{2}$$

$$x = \frac{2}{2} \pm \frac{2i}{2} = 1 \pm i$$

2. $x^2 + 6x + 10 = 0$

$a = 1$

$b = 6$

$c = 10$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-(6) \pm \sqrt{(6)^2 - 4(1)(10)}}{2(1)} = \frac{-6 \pm \sqrt{36 - 40}}{2} = \frac{-6 \pm \sqrt{-4}}{2}$$

$$x = \frac{-6 \pm 2i}{2}$$

$$x = \frac{-6}{2} \pm \frac{2i}{2} = -3 \pm i$$

3. $4x^2 + 16x + 17 = 0$

$a = 4$

$b = 16$

$c = 17$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-(16) \pm \sqrt{(16)^2 - 4(4)(17)}}{2(4)} = \frac{-16 \pm \sqrt{256 - 272}}{8}$$

$$x = \frac{-16 \pm \sqrt{-16}}{8} = \frac{-16 \pm 4i}{8}$$

$$x = \frac{-16}{8} \pm \frac{4i}{8} = -2 \pm \frac{1}{2}i$$

4. $9x^2 - 6x + 37 = 0$

$a = 9$

$b = -6$

$c = 37$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-(-6) \pm \sqrt{(-6)^2 - 4(9)(37)}}{2(9)} = \frac{6 \pm \sqrt{36 - 1332}}{18}$$

$$x = \frac{6 \pm \sqrt{-1296}}{18} = \frac{6 \pm 36i}{18}$$

$$x = \frac{6}{18} \pm \frac{36}{18}i = \frac{1}{3} \pm 2i$$

5. $4x^2 + 16x + 15 = 0$

$a = 4$

$b = 16$

$c = 15$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-(16) \pm \sqrt{(16)^2 - 4(4)(15)}}{2(4)} = \frac{-16 \pm \sqrt{256 - 240}}{8}$$



$$x = \frac{-16 \pm \sqrt{16}}{8} = \frac{-16 \pm 4}{8}$$

$$x = \frac{-16}{8} \pm \frac{4}{8} = -2 \pm \frac{1}{2}$$

$$x_1 = -2 + \frac{1}{2} = \frac{-3}{2}$$

$$x_2 = -2 - \frac{1}{2} = \frac{-5}{2}$$

6. $9x^2 - 6x - 35 = 0$

$$a = 9$$

$$b = -6$$

$$c = -35$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-(-6) \pm \sqrt{(-6)^2 - 4(9)(-35)}}{2(9)} = \frac{6 \pm \sqrt{36 + 1260}}{18}$$

$$x = \frac{6 \pm \sqrt{1296}}{18} = \frac{6 \pm 36}{18}$$

$$x = \frac{6}{18} \pm \frac{36}{18} = \frac{1}{3} \pm 2$$

$$x_1 = \frac{1}{3} + 2 = \frac{7}{3}$$

$$x_2 = \frac{1}{3} - 2 = \frac{-5}{3}$$

7. $16t^2 - 4t + 3 = 0$

$$a = 16$$

$$b = -4$$

$$c = 3$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(16)(3)}}{2(16)} = \frac{4 \pm \sqrt{16 - 192}}{32}$$

$$x = \frac{4 \pm \sqrt{-176}}{32} = \frac{4 \pm \sqrt{-16 * 11}}{32} = \frac{4 \pm 4i\sqrt{11}}{32}$$

$$x = \frac{4}{32} \pm \frac{4i\sqrt{11}}{32} = \frac{1}{8} \pm \frac{\sqrt{11}}{8}i$$

8. $5s^2 + 6s + 3 = 0$

$$a = 5$$

$$b = 6$$

$$c = 3$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-(6) \pm \sqrt{(6)^2 - 4(5)(3)}}{2(5)} = \frac{-6 \pm \sqrt{36 - 60}}{10} = \frac{-6 \pm \sqrt{-24}}{10}$$

$$= \frac{-6 \pm \sqrt{-4 * 6}}{10} = \frac{-6 \pm 2i\sqrt{6}}{10}$$

$$x = \frac{-6}{10} \pm \frac{2i\sqrt{6}}{10} = \frac{-3}{5} \pm \frac{\sqrt{6}}{5}i$$

Part two:- in exercise 1-8 simplify the complex number and write it in standard form

1. $-6i^3 + i^2 = -6(-i) + (-1) = 6i - 1 = -1 + 6i$

2. $4i^2 - 2i^3 = 4(-1) - 2(-i) = -4 + 2i$

3. $-5i^5 = -5(i) = 0 - 5i$

4. $(-i)^3 = -i^3 = -(-i) = 0 + i$



$$5. (\sqrt{-75})^3 = (\sqrt{-25 * 3})^3 = (5i\sqrt{3})^3 = 125i^3(3\sqrt{3}) = 0 - 375\sqrt{3}i$$

$$6. (\sqrt{-2})^6 = (i\sqrt{2})^6 = (i)^6(\sqrt{2})^6 = (-1)(8) = -8 = -8 = -8 + 0i$$

$$7. \frac{1}{i^3} = \frac{1}{i^3} * \frac{-i^3}{-i^3} = \frac{-i^3}{-i^6} = \frac{i^3}{i^6} = \frac{-i}{-1} = i = 0 + i$$

$$8. \frac{1}{(2i)^3} = \frac{1}{8i^3} = \frac{1}{8(-i)} = -\frac{1}{8i} = -\frac{1}{8i} * \frac{-8i}{-8i} = \frac{-8i}{-64i^2} = \frac{-8i}{-64(-1)} = \frac{-8i}{64} = -\frac{1}{8}i$$



Section 2.4

Exercise 2.4 in exercise 1-6 plot the complex number and find its absolute value

1. $-5i = ?$ $a = 0$ $b = -5$
the absolute value of $-5i$ is $= \sqrt{a^2 + b^2} = \sqrt{0 + 25} = \sqrt{25} = 5$
2. $-5 = ?$ $a = -5$ $b = 0$
the absolute value of -5 is $= \sqrt{a^2 + b^2} = \sqrt{25 + 0} = \sqrt{25} = 5$
3. $-4 + 4i = ?$ $a = -4$ $b = 4$
the absolute value of $-4 + 4i$ is $= \sqrt{a^2 + b^2} = \sqrt{16 + 16} = \sqrt{32} = 4\sqrt{2}$
4. $5 - 12i = ?$ $a = 5$ $b = -12$
the absolute value of $5 - 12i$ is $= \sqrt{a^2 + b^2} = \sqrt{25 + 144} = \sqrt{169} = 13$
5. $6 - 7i = ?$ $a = 6$ $b = -7$
the absolute value of $6 - 7i$ is $= \sqrt{a^2 + b^2} = \sqrt{36 + 49} = \sqrt{85}$
6. $-8 + 3i = ?$ $a = -8$ $b = 3$
the absolute value of $-8 + 3i$ is $= \sqrt{a^2 + b^2} = \sqrt{64 + 9} = \sqrt{73}$

Part two: - in exercise 1-8 represent the complex number graphically and find the polar form of the number

1. $3 - 3i = ?$ $a = 3$ $b = -3$
 $r = \sqrt{a^2 + b^2} = \sqrt{9 + 9} = \sqrt{18} = 3\sqrt{2}$
 $\theta = \tan^{-1}\left(\frac{-3}{3}\right) = \tan^{-1}(-1) = -45 = 315$
 $\therefore 3\sqrt{2}(\cos 315 + i \sin 315)$
2. $2 + 2i = ?$ $a = 2$ $b = 2$
 $r = \sqrt{a^2 + b^2} = \sqrt{4 + 4} = \sqrt{8} = 2\sqrt{2}$
 $\theta = \tan^{-1}\left(\frac{2}{2}\right) = \tan^{-1}(1) = 45$
 $\therefore 2\sqrt{2}(\cos 45 + i \sin 45)$
3. $\sqrt{3} + i = ?$ $a = \sqrt{3}$ $b = 1$
 $r = \sqrt{a^2 + b^2} = \sqrt{3 + 1} = \sqrt{4} = 2$
 $\theta = \tan^{-1}\left(\frac{1}{\sqrt{3}}\right) = 30$
 $\therefore 2(\cos 30 + i \sin 30)$
4. $-1 + \sqrt{3}i = ?$ $a = -1$ $b = \sqrt{3}$
 $r = \sqrt{a^2 + b^2} = \sqrt{1 + 3} = \sqrt{4} = 2$
 $\theta = \tan^{-1}\left(\frac{\sqrt{3}}{-1}\right) + \pi = \frac{5\pi}{3} + \pi = \frac{8\pi}{3}$ or $\frac{2\pi}{3}$
 $\therefore 2\left(\cos \frac{2\pi}{3} + i \sin \frac{2\pi}{3}\right)$
5. $-2(1 + \sqrt{3}i) = -2 - 2\sqrt{3}i = ?$ $a = -2$ $b = -2\sqrt{3}$
 $r = \sqrt{a^2 + b^2} = \sqrt{4 + 12} = \sqrt{16} = 4$



$$\theta = \tan^{-1} \left(\frac{-2\sqrt{3}}{-2} \right) + \pi = \frac{\pi}{3} + \pi = \frac{4\pi}{3}$$

$$\therefore 4 \left(\cos \frac{4\pi}{3} + i \sin \frac{4\pi}{3} \right)$$

6. $\frac{5}{2}(\sqrt{3} - i) = \frac{5\sqrt{3}}{2} - \frac{5}{2}i = ? \quad a = \frac{5\sqrt{3}}{2} \quad b = -\frac{5}{2}$

$$r = \sqrt{a^2 + b^2} = \sqrt{\frac{75}{4} + \frac{25}{4}} = \sqrt{\frac{100}{4}} = \sqrt{25} = 5$$

$$\theta = \tan^{-1} \left(\frac{\frac{-5}{2}}{\frac{5\sqrt{3}}{2}} \right) = \tan^{-1} \left(\frac{-1}{\sqrt{3}} \right) = \frac{11\pi}{6}$$

$$\therefore 5 \left(\cos \frac{11\pi}{6} + i \sin \frac{11\pi}{6} \right)$$

7. $6i = ? \quad a = 0 \quad b = 6$

$$r = \sqrt{a^2 + b^2} = \sqrt{0 + 36} = \sqrt{36} = 6$$

$$\theta = \tan^{-1} \left(\frac{6}{0} \right) = 90 \text{ or } \frac{\pi}{2}$$

$$\therefore 6 \left(\cos \frac{\pi}{2} + i \sin \frac{\pi}{2} \right)$$

8. $4 = ? \quad a = 4 \quad b = 0$

$$r = \sqrt{a^2 + b^2} = \sqrt{16 + 0} = \sqrt{16} = 4$$

$$\theta = \tan^{-1} \left(\frac{0}{4} \right) = 0 \text{ or } 180$$

$$\therefore 4(\cos \pi + i \sin \pi)$$

Part three: - in exercise 1-6 represent the complex number graphically and find the standard form of the result

1. $2(\cos 150 + i \sin 150) = ? \quad r = 2 \quad \theta = 150$

$$a = r \cos \theta = 2 \cos 150 = 2 \left(-\frac{\sqrt{3}}{2} \right) = -\sqrt{3}$$

$$b = r \sin \theta = 2 \sin 150 = 2 \left(\frac{1}{2} \right) = 1$$

$\therefore$ the standard form of complex number is $= a + bi = -\sqrt{3} + i$

2. $5(\cos 135 + i \sin 135) = ? \quad r = 5 \quad \theta = 135$

$$a = r \cos \theta = 5 \cos 135 = 5 \left(-\frac{\sqrt{2}}{2} \right) = -\frac{5\sqrt{2}}{2}$$

$$b = r \sin \theta = 5 \sin 135 = 5 \left(\frac{\sqrt{2}}{2} \right) = \frac{5\sqrt{2}}{2}$$

$\therefore$ the standard form of complex number is $= a + bi = -\frac{5\sqrt{2}}{2} + \frac{5\sqrt{2}}{2}i$

3. $\frac{3}{2}(\cos 300 + i \sin 300) = ? \quad r = \frac{3}{2} \quad \theta = 300$



$$a = r \cos \theta = \frac{3}{2} \cos 300 = \frac{3}{2} \left(\frac{1}{2} \right) = \frac{3}{4}$$

$$b = r \sin \theta = \frac{3}{2} \sin 300 = \frac{3}{2} \left(-\frac{\sqrt{3}}{2} \right) = -\frac{3\sqrt{3}}{4}$$

$$\therefore \text{the standard form of complex number is } = a + bi = \frac{3}{4} - \frac{3\sqrt{3}}{4}i$$

$$4. \frac{3}{4}(\cos 315 + i \sin 315) = ? \quad r = \frac{3}{4} \quad \theta = 315$$

$$a = r \cos \theta = \frac{3}{4} \cos 315 = \frac{3}{4} \left(\frac{\sqrt{2}}{2} \right) = \frac{3\sqrt{2}}{8}$$

$$b = r \sin \theta = \frac{3}{4} \sin 315 = \frac{3}{4} \left(-\frac{\sqrt{2}}{2} \right) = -\frac{3\sqrt{2}}{8}$$

$$\therefore \text{the standard form of complex number is } = a + bi = \frac{3\sqrt{2}}{8} - \frac{3\sqrt{2}}{8}i$$

$$5. 3.75 \left(\cos \frac{3\pi}{4} + i \sin \frac{3\pi}{4} \right) = ? \quad r = 3.75 \quad \theta = \frac{3\pi}{4}$$

$$a = r \cos \theta = 3.75 \cos \frac{3\pi}{4} = \frac{15}{4} \left(-\frac{\sqrt{2}}{2} \right) = -\frac{15\sqrt{2}}{8}$$

$$b = r \sin \theta = \frac{15}{4} \sin \frac{3\pi}{4} = \frac{15}{4} \left(\frac{\sqrt{2}}{2} \right) = \frac{15\sqrt{2}}{8}$$

$$\therefore \text{the standard form of complex number is } = a + bi = -\frac{15\sqrt{2}}{8} + \frac{15\sqrt{2}}{8}i$$

$$6. 8 \left(\cos \frac{\pi}{12} + i \sin \frac{\pi}{12} \right) = ? \quad r = 8 \quad \theta = \frac{\pi}{12}$$

$$a = r \cos \theta = 8 \cos \frac{\pi}{12} = 8 \left(\frac{\sqrt{6} + \sqrt{2}}{4} \right) = 2(\sqrt{6} + \sqrt{2})$$

$$b = r \sin \theta = 8 \sin \frac{\pi}{12} = 8 \left(\frac{\sqrt{6} - \sqrt{2}}{4} \right) = 2(\sqrt{6} - \sqrt{2})$$

$$\therefore \text{the standard form of complex number is } = a + bi = 2(\sqrt{6} + \sqrt{2}) + 2(\sqrt{6} - \sqrt{2})i$$



Section 2.5

In exercise 1-4 perform the operation and leave the result in standard form

1. $\left| 3 \left(\cos \frac{\pi}{3} + i \sin \frac{\pi}{3} \right) \right| \left| 4 \left(\cos \frac{\pi}{6} + i \sin \frac{\pi}{6} \right) \right| = 12 \left[\cos \left(\frac{\pi}{3} + \frac{\pi}{6} \right) + i \sin \left(\frac{\pi}{3} + \frac{\pi}{6} \right) \right] = 12 \left(\cos \frac{\pi}{2} + i \sin \frac{\pi}{2} \right)$
2. $\left| \frac{3}{2} \left(\cos \frac{\pi}{2} + i \sin \frac{\pi}{2} \right) \right| \left| 6 \left(\cos \frac{\pi}{4} + i \sin \frac{\pi}{4} \right) \right| = 9 \left[\cos \left(\frac{\pi}{2} + \frac{\pi}{4} \right) + i \sin \left(\frac{\pi}{2} + \frac{\pi}{4} \right) \right] = 9 \left(\cos \frac{3\pi}{4} + i \sin \frac{3\pi}{4} \right)$
3. $\left| \frac{5}{3} (\cos 140 + i \sin 140) \right| \left| \frac{2}{3} (\cos 60 + i \sin 60) \right| = \frac{10}{9} [\cos(140 + 60) + i \sin(140 + 60)] = \frac{10}{9} (\cos 200 + i \sin 200)$
4. $\frac{\cos(\frac{5\pi}{3}) + i \sin(\frac{5\pi}{3})}{\cos \pi + i \sin \pi} = \cos \left(\frac{5\pi}{3} - \pi \right) + i \sin \left(\frac{5\pi}{3} - \pi \right) = \cos \frac{2\pi}{3} + i \sin \frac{2\pi}{3}$

Part two: - in exercise 1-8 use demoivre's theorem to find indicated power of the complex number express the result in standard form

1. $(1 + i)^5 = ?$ $a = 1$ $b = 1$
 $r = \sqrt{a^2 + b^2} = \sqrt{1 + 1} = \sqrt{2}$
 $\theta = \tan^{-1} \left(\frac{1}{1} \right) = 45$
 $[\sqrt{2}(\cos 45 + i \sin 45)]^5 = (\sqrt{2})^5 [\cos(45 * 5) + i \sin(45 * 5)]$
 $= 4\sqrt{2}(\cos 225 + i \sin 225)$
The standard form of $4\sqrt{2}(\cos 225 + i \sin 225) = -4 - 4i$
2. $(2 + 2i)^6 = ?$ $a = 2$ $b = 2$
 $r = \sqrt{a^2 + b^2} = \sqrt{4 + 4} = \sqrt{8} = 2\sqrt{2}$
 $\theta = \tan^{-1} \left(\frac{2}{2} \right) = \tan^{-1}(1) = 45$
 $[2\sqrt{2}(\cos 45 + i \sin 45)]^6 = (2\sqrt{2})^6 [\cos(45 * 6) + i \sin(45 * 6)]$
 $= 512(\cos 270 + i \sin 270)$
The standard form of $512(\cos 270 + i \sin 270) = 0 - 512i$
3. $(-1 + i)^{10} = ?$ $a = -1$ $b = 1$
 $r = \sqrt{a^2 + b^2} = \sqrt{1 + 1} = \sqrt{2}$



$$\theta = \tan^{-1}\left(\frac{1}{-1}\right) + \pi = \frac{7\pi}{4} + \pi = \frac{11\pi}{4} \text{ or } \frac{3\pi}{4}$$

$$\left[\sqrt{2}\left(\cos\frac{3\pi}{4} + i\sin\frac{3\pi}{4}\right)\right]^{10} = (\sqrt{2})^{10} \left[\cos\left(\frac{3\pi}{4} * 10\right) + i\sin\left(\frac{3\pi}{4} * 10\right)\right]$$

$$= 32\left(\cos\frac{15\pi}{2} + i\sin\frac{15\pi}{2}\right)$$

The standard form of $32\left(\cos\frac{15\pi}{2} + i\sin\frac{15\pi}{2}\right) = 0 - 32i$

4. $(1 - i)^{12} = ?$ $a = 1$ $b = -1$

$$r = \sqrt{a^2 + b^2} = \sqrt{1 + 1} = \sqrt{2}$$

$$\theta = \tan^{-1}\left(\frac{-1}{1}\right) = 315$$

$$\left[\sqrt{2}(\cos 315 + i\sin 315)\right]^{12} = (\sqrt{2})^{12} [\cos(315 * 12) + i\sin(315 * 12)]$$

$$= 64(\cos 3780 + i\sin 3780)$$

The standard form of $64(\cos 3780 + i\sin 3780) = -64 + 0i$

5. $2(\sqrt{3} + i)^7 = ?$ $a = \sqrt{3}$ $b = 1$

$$r = \sqrt{a^2 + b^2} = \sqrt{3 + 1} = \sqrt{4} = 2$$

$$\theta = \tan^{-1}\left(\frac{1}{\sqrt{3}}\right) = 30$$

$$2[2(\cos 30 + i\sin 30)]^7 = 2\{(2)^7[\cos(30 * 7) + i\sin(30 * 7)]\}$$

$$= 256(\cos 210 + i\sin 210)$$

The standard form of $256(\cos 210 + i\sin 210) = -128\sqrt{3} - 128i$

6. $4(1 - \sqrt{3}i)^3 = ?$ $a = 1$ $b = -\sqrt{3}$

$$r = \sqrt{a^2 + b^2} = \sqrt{1 + 3} = \sqrt{4} = 2$$

$$\theta = \tan^{-1}\left(\frac{-\sqrt{3}}{1}\right) = 300$$

$$4[2(\cos 300 + i\sin 300)]^3 = 4\{(2)^3[\cos(300 * 3) + i\sin(300 * 3)]\}$$

$$= 32(\cos 900 + i\sin 900)$$

The standard form of $32(\cos 900 + i\sin 900) = -32 + 0i$



$$7. \left(\cos \frac{5\pi}{4} + i \sin \frac{5\pi}{4} \right)^{10} = (1)^{10} \left[\cos \left(\frac{5\pi}{4} * 10 \right) + i \sin \left(\frac{5\pi}{4} * 10 \right) \right]$$

$$= \cos \frac{25\pi}{2} + i \sin \frac{25\pi}{2}$$

The standard form of $\left(\cos \frac{25\pi}{2} + i \sin \frac{25\pi}{2} \right) = 0 + i$

$$8. \left[2 \left(\cos \frac{\pi}{2} + i \sin \frac{\pi}{2} \right) \right]^8 = (2)^8 \left[\cos \left(\frac{\pi}{2} * 8 \right) + i \sin \left(\frac{\pi}{2} * 8 \right) \right]$$

$$= 256(\cos 4\pi + i \sin 4\pi)$$

The standard form of $256(\cos 4\pi + i \sin 4\pi) = 256 + 0i$

Part three: -

in exercise 1_6 a) use theorem A.5 on page A64 to find indicated roots of the complex number b)re

1. Square roots of $5(\cos 120 + i \sin 120) = [5(\cos 120 + i \sin 120)]^{\frac{1}{2}}$
 $k = 0, 1$

When $k = 0$ $(5)^{\frac{1}{2}} \left(\cos \frac{120 + 360(0)}{2} + i \sin \frac{120 + 360(0)}{2} \right)$

$$= \sqrt{5}(\cos 60 + i \sin 60)$$

The standard form of $\sqrt{5}(\cos 60 + i \sin 60) = \frac{\sqrt{5}}{2} + \frac{\sqrt{15}}{2}i$

When $k = 1$ $\sqrt{5} \left(\cos \frac{120 + 360(1)}{2} + i \sin \frac{120 + 360(1)}{2} \right)$

$$= \sqrt{5}(\cos 240 + i \sin 240)$$

The standard form of $\sqrt{5}(\cos 240 + i \sin 240) = -\frac{\sqrt{5}}{2} - \frac{\sqrt{15}}{2}i$

Square roots of $16(\cos 60 + i \sin 60) = [16(\cos 60 + i \sin 60)]^{\frac{1}{2}}$
 $k = 0, 1$

When $k = 0$ $(16)^{\frac{1}{2}} \left(\cos \frac{60 + 360(0)}{2} + i \sin \frac{60 + 360(0)}{2} \right) = 4(\cos 30 + i \sin 30)$

The standard form of $4(\cos 30 + i \sin 30) = 2\sqrt{3} + 2i$

When $k = 1$ $4 \left(\cos \frac{60 + 360(1)}{2} + i \sin \frac{60 + 360(1)}{2} \right) = 5(\cos 210 + i \sin 210)$

The standard form of $4(\cos 210 + i \sin 210) = -2\sqrt{3} - 2i$



2. fourth roots of $16 \left(\cos \frac{4\pi}{3} + i \sin \frac{4\pi}{3} \right) = \left[16 \left(\cos \frac{4\pi}{3} + i \sin \frac{4\pi}{3} \right) \right]^{\frac{1}{4}} =$
 $k = 0, 1, 2, 3$

When $k = 0$ $(16)^{\frac{1}{4}} \left(\cos \frac{\frac{4\pi}{3} + 2\pi(0)}{4} + i \sin \frac{\frac{4\pi}{3} + 2\pi(0)}{4} \right) = 2 \left(\cos \frac{\pi}{3} + i \sin \frac{\pi}{3} \right)$

The standard form of $2 \left(\cos \frac{\pi}{3} + i \sin \frac{\pi}{3} \right) = 1 + \sqrt{3}i$

When $k = 1$ $(16)^{\frac{1}{4}} \left(\cos \frac{\frac{4\pi}{3} + 2\pi(1)}{4} + i \sin \frac{\frac{4\pi}{3} + 2\pi(1)}{4} \right)$
 $= 2 \left(\cos \frac{5\pi}{6} + i \sin \frac{5\pi}{6} \right)$

The standard form of $2 \left(\cos \frac{5\pi}{6} + i \sin \frac{5\pi}{6} \right) = -\sqrt{3} + i$

When $k = 2$ $(16)^{\frac{1}{4}} \left(\cos \frac{\frac{4\pi}{3} + 2\pi(2)}{4} + i \sin \frac{\frac{4\pi}{3} + 2\pi(2)}{4} \right)$
 $= 2 \left(\cos \frac{4\pi}{3} + i \sin \frac{4\pi}{3} \right)$

The standard form of $2 \left(\cos \frac{4\pi}{3} + i \sin \frac{4\pi}{3} \right) = -1 - \sqrt{3}i$

When $k = 3$ $(16)^{\frac{1}{4}} \left(\cos \frac{\frac{4\pi}{3} + 2\pi(3)}{4} + i \sin \frac{\frac{4\pi}{3} + 2\pi(3)}{4} \right)$
 $= 2 \left(\cos \frac{11\pi}{6} + i \sin \frac{11\pi}{6} \right)$

The standard form of $2 \left(\cos \frac{11\pi}{6} + i \sin \frac{11\pi}{6} \right) = \sqrt{3} - i$

3. fifth roots of $32 \left(\cos \frac{5\pi}{6} + i \sin \frac{5\pi}{6} \right) = \left[32 \left(\cos \frac{5\pi}{6} + i \sin \frac{5\pi}{6} \right) \right]^{\frac{1}{5}} =$
 $k = 0, 1, 2, 3$

When $k = 0$ $(32)^{\frac{1}{5}} \left(\cos \frac{\frac{5\pi}{6} + 2\pi(0)}{5} + i \sin \frac{\frac{5\pi}{6} + 2\pi(0)}{5} \right) = 2 \left(\cos \frac{\pi}{6} + i \sin \frac{\pi}{6} \right)$

The standard form of $2 \left(\cos \frac{\pi}{6} + i \sin \frac{\pi}{6} \right) = \sqrt{3} + i$



$$\text{When } k = 1 \quad (32)^{\frac{1}{5}} \left(\cos \frac{5\pi}{6} + 2\pi(1) + i \sin \frac{5\pi}{6} + 2\pi(1) \right)$$

$$= 2 \left(\cos \frac{17\pi}{30} + i \sin \frac{17\pi}{30} \right)$$

The standard form of $2 \left(\cos \frac{17\pi}{30} + i \sin \frac{17\pi}{30} \right) = -0.42 + 1.96i$

$$\text{When } k = 2 \quad (32)^{\frac{1}{5}} \left(\cos \frac{5\pi}{6} + 2\pi(2) + i \sin \frac{5\pi}{6} + 2\pi(2) \right)$$

$$= 2 \left(\cos \frac{29\pi}{30} + i \sin \frac{29\pi}{30} \right)$$

The standard form of $2 \left(\cos \frac{29\pi}{30} + i \sin \frac{29\pi}{30} \right) = -1.99 + 0.2i$

$$\text{When } k = 3 \quad (32)^{\frac{1}{5}} \left(\cos \frac{5\pi}{6} + 2\pi(3) + i \sin \frac{5\pi}{6} + 2\pi(3) \right)$$

$$= 2 \left(\cos \frac{41\pi}{30} + i \sin \frac{41\pi}{30} \right)$$

The standard form of $2 \left(\cos \frac{41\pi}{30} + i \sin \frac{41\pi}{30} \right) = -1 - 1.8i$

$$\text{when } k = 4 \quad (32)^{\frac{1}{5}} \left(\cos \frac{5\pi}{6} + 2\pi(4) + i \sin \frac{5\pi}{6} + 2\pi(4) \right)$$

$$= 2 \left(\cos \frac{53\pi}{30} + i \sin \frac{53\pi}{30} \right)$$

The standard form of $2 \left(\cos \frac{53\pi}{30} + i \sin \frac{53\pi}{30} \right) = 1.49 - 1.34i$

4. cube roots of $-\frac{125}{2}(1 + \sqrt{3}i) = \left(-\frac{125}{2} - \frac{125\sqrt{3}}{2}i\right)^{\frac{1}{3}} = [216 (\cos 180 + i \sin 180)]^{\frac{1}{3}}$
 $k = 0, 1, 2$

$$\text{When } k = 0 \quad (216)^{\frac{1}{3}} \left(\cos \frac{180 + 360(0)}{3} + i \sin \frac{180 + 360(0)}{3} \right)$$

$$= 6(\cos 60 + i \sin 60)$$

The standard form of $6(\cos 60 + i \sin 60) = 3 + 5.2i$

$$\text{When } k = 1 \quad (216)^{\frac{1}{3}} \left(\cos \frac{180 + 360(1)}{3} + i \sin \frac{180 + 360(1)}{3} \right)$$

$$= 6(\cos 180 + i \sin 180)$$

The standard form of $6(\cos 180 + i \sin 180) = -6 + 0i$



$$\text{When } k = 2 \quad (216)^{\frac{1}{3}} \left(\cos \frac{180 + 360(2)}{3} + i \sin \frac{180 + 360(2)}{3} \right)$$

$$= 6(\cos 180 + i \sin 180)$$

$$\text{The standard form of} \quad 6(\cos 180 + i \sin 180) = -6 + 0i$$

$$5. \text{ cube roots of } -4\sqrt{2}(1 - i) = (-4\sqrt{2} + 4\sqrt{2}i)^{\frac{1}{3}} = [8(\cos 135 + i \sin 135)]^{\frac{1}{3}} =$$

$$k = 0, 1, 2$$

$$\text{When } k = 0 \quad (8)^{\frac{1}{3}} \left(\cos \frac{135 + 360(0)}{3} + i \sin \frac{135 + 360(0)}{3} \right) = 2(\cos 45 + i \sin 45)$$

$$\text{The standard form of} \quad 2(\cos 45 + i \sin 45) = \sqrt{2} + \sqrt{2}i$$

$$\text{When } k = 1 \quad (8)^{\frac{1}{3}} \left(\cos \frac{135 + 360(1)}{3} + i \sin \frac{135 + 360(1)}{3} \right)$$

$$= 2(\cos 165 + i \sin 165)$$

$$\text{The standard form of} \quad 2(\cos 165 + i \sin 165) = -1.9 + 0.5i$$

$$\text{When } k = 2 \quad (8)^{\frac{1}{3}} \left(\cos \frac{135 + 360(2)}{3} + i \sin \frac{135 + 360(2)}{3} \right)$$

$$= 6(\cos 285 + i \sin 285)$$

$$\text{The standard form of} \quad 6(\cos 285 + i \sin 285) = 1.6 - 5.8i$$

part four: -in exercises 18 use theorem A5 on page A64 to find all the solutions of the equation an

$$1. \quad x^4 - i = 0 \quad x^4 = i$$

$$x = (i)^{\frac{1}{4}} = (\cos 90 + i \sin 90)^{\frac{1}{4}}$$

$$\text{When } k = 0 \quad (1)^{\frac{1}{4}} \left(\cos \frac{90 + 360(0)}{4} + i \sin \frac{90 + 360(0)}{4} \right) = (\cos 22.5 + i \sin 22.5)$$

$$= 0.92 + 0.38i$$

$$\text{When } k = 1 \quad (1)^{\frac{1}{4}} \left(\cos \frac{90 + 360(1)}{4} + i \sin \frac{90 + 360(1)}{4} \right) = (\cos 112.5 + i \sin 112.5)$$

$$= -0.38 + 0.92i$$

$$\text{When } k = 2 \quad (1)^{\frac{1}{4}} \left(\cos \frac{90 + 360(2)}{4} + i \sin \frac{90 + 360(2)}{4} \right) = (\cos 202.5 + i \sin 202.5)$$

$$= -0.92 - 0.38i$$



$$\text{When } k = 3 \quad (1)^{\frac{1}{4}} \left(\cos \frac{90 + 360(3)}{4} + i \sin \frac{90 + 360(3)}{4} \right) = (\cos 292.5 + i \sin 292.5)$$

$$= 0.38 - 0.92i$$

$$2. \quad x^5 + 243 = 0$$

$$x^5 = -243$$

$$x = (-243)^{\frac{1}{5}} = [243(\cos 180 + i \sin 180)]^{\frac{1}{5}}$$

$$\text{When } k = 0 \quad (243)^{\frac{1}{5}} \left(\cos \frac{180 + 360(0)}{5} + i \sin \frac{180 + 360(0)}{5} \right) = 3(\cos 36 + i \sin 36)$$

$$= 2.43 + 1.76i$$

$$\text{When } k = 1 \quad (243)^{\frac{1}{5}} \left(\cos \frac{180 + 360(1)}{5} + i \sin \frac{180 + 360(1)}{5} \right) = 3(\cos 108 + i \sin 108)$$

$$= -0.92 + 2.85i$$

$$\text{When } k = 2 \quad (243)^{\frac{1}{5}} \left(\cos \frac{180 + 360(2)}{5} + i \sin \frac{180 + 360(2)}{5} \right) = 3(\cos 180 + i \sin 180)$$

$$= -3 + 0i$$

$$\text{When } k = 3 \quad (243)^{\frac{1}{5}} \left(\cos \frac{180 + 360(3)}{5} + i \sin \frac{180 + 360(3)}{5} \right)$$

$$= 3(\cos 252 + i \sin 252) = -0.92 - 2.85i$$

$$\text{When } k = 4 \quad (243)^{\frac{1}{5}} \left(\cos \frac{180 + 360(4)}{5} + i \sin \frac{180 + 360(4)}{5} \right) = 3(\cos 324 + i \sin 324)$$

$$= 2.43 - 1.76i$$

$$3. \quad x^3 + 64i = 0$$

$$x^3 = -64i$$

$$x = (-64i)^{\frac{1}{3}} = [64(\cos 270 + i \sin 270)]^{\frac{1}{3}}$$

$$\text{When } k = 0 \quad (64)^{\frac{1}{3}} \left(\cos \frac{270 + 360(0)}{3} + i \sin \frac{270 + 360(0)}{3} \right) = 4(\cos 90 + i \sin 90)$$

$$= 0 + 4i$$



$$\text{When } k = 1 \quad (64)^{\frac{1}{3}} \left(\cos \frac{270 + 360(1)}{3} + i \sin \frac{270 + 360(1)}{3} \right) = 4(\cos 210 + i \sin 210)$$

$$= -3.46 - 2i$$

$$\text{When } k = 2 \quad (64)^{\frac{1}{3}} \left(\cos \frac{270 + 360(2)}{3} + i \sin \frac{270 + 360(2)}{3} \right) = 4(\cos 330 + i \sin 330)$$

$$= 3.46 - 2i$$

$$4. x^3 - (1 - i) = 0$$

$$x^3 = 1 - i$$

$$x = (1 - i)^{\frac{1}{3}} = [\sqrt{2}(\cos 315 + i \sin 315)]^{\frac{1}{3}}$$

$$\text{When } k = 0 \quad (\sqrt{2})^{\frac{1}{3}} \left(\cos \frac{315 + 360(0)}{3} + i \sin \frac{315 + 360(0)}{3} \right) = \sqrt[3]{2}(\cos 105 + i \sin 105)$$

$$= -0.3 + 1.2i$$

$$\text{When } k = 1 \quad (\sqrt{2})^{\frac{1}{3}} \left(\cos \frac{315 + 360(1)}{3} + i \sin \frac{315 + 360(1)}{3} \right) = \sqrt[3]{2}(\cos 225 + i \sin 225)$$

$$= -0.9 - 0.9i$$

$$\text{When } k = 2 \quad (\sqrt{2})^{\frac{1}{3}} \left(\cos \frac{315 + 360(2)}{3} + i \sin \frac{315 + 360(2)}{3} \right) = \sqrt[3]{2}(\cos 345 + i \sin 345)$$

$$= 1.2 - 0.3i$$

$$5. x^3 + 1 = 0$$

$$x^3 = -1$$

$$x = (-1)^{\frac{1}{3}} = (\cos 180 + i \sin 180)^{\frac{1}{3}}$$

$$\text{When } k = 0 \quad (1)^{\frac{1}{3}} \left(\cos \frac{180 + 360(0)}{3} + i \sin \frac{180 + 360(0)}{3} \right) = (\cos 60 + i \sin 60) = \frac{1}{2} + \frac{\sqrt{3}}{2}i$$

$$\text{When } k = 1 \quad (1)^{\frac{1}{3}} \left(\cos \frac{180 + 360(1)}{3} + i \sin \frac{180 + 360(1)}{3} \right) = (\cos 180 + i \sin 180) = -1 + 0i$$

$$\text{When } k = 2 \quad (1)^{\frac{1}{3}} \left(\cos \frac{180 + 360(2)}{3} + i \sin \frac{180 + 360(2)}{3} \right) = (\cos 300 + i \sin 300) = \frac{1}{2} - \frac{\sqrt{3}}{2}i$$

$$6. x^4 - 81 = 0$$

$$x^4 = 81$$

$$x = (81)^{\frac{1}{4}} = [81(\cos 0 + i \sin 0)]^{\frac{1}{4}}$$



$$\begin{aligned} \text{When } k = 0 & \quad (81)^{\frac{1}{4}} \left(\cos \frac{0+360(0)}{4} + i \sin \frac{0+360(0)}{4} \right) = 3(\cos 0 + i \sin 0) = 3 + 0i \\ \text{When } k = 1 & \quad (81)^{\frac{1}{4}} \left(\cos \frac{0+360(1)}{4} + i \sin \frac{0+360(1)}{4} \right) = 3(\cos 90 + i \sin 90) = 0 + 3i \\ \text{When } k = 2 & \quad (81)^{\frac{1}{4}} \left(\cos \frac{0+360(2)}{4} + i \sin \frac{0+360(2)}{4} \right) = 3(\cos 180 + i \sin 180) = -3 + 0i \\ \text{When } k = 3 & \quad (81)^{\frac{1}{4}} \left(\cos \frac{0+360(3)}{4} + i \sin \frac{0+360(3)}{4} \right) = 3(\cos 270 + i \sin 270) \\ & = 0 - 3i \end{aligned}$$

$$7. x^6 - 64i = 0$$

$$x^6 = 64i$$

$$x = (64i)^{\frac{1}{6}} = [64(\cos 90 + i \sin 90)]^{\frac{1}{6}}$$

$$\begin{aligned} \text{When } k = 0 & \quad (64)^{\frac{1}{6}} \left(\cos \frac{90+360(0)}{6} + i \sin \frac{90+360(0)}{6} \right) = 2(\cos 15 + i \sin 15) = 1.9 + 0.5i \\ \text{When } k = 1 & \quad (64)^{\frac{1}{6}} \left(\cos \frac{90+360(1)}{6} + i \sin \frac{90+360(1)}{6} \right) = 2(\cos 75 + i \sin 75) = 0.5 + 1.9i \\ \text{When } k = 2 & \quad (64)^{\frac{1}{6}} \left(\cos \frac{90+360(2)}{6} + i \sin \frac{90+360(2)}{6} \right) = 2(\cos 135 + i \sin 135) = -\sqrt{2} + 0i \\ \text{When } k = 3 & \quad (64)^{\frac{1}{6}} \left(\cos \frac{90+360(3)}{6} + i \sin \frac{90+360(3)}{6} \right) = 2(\cos 195 + i \sin 195) \\ & = -2.9 - 0.8i \end{aligned}$$

$$\text{When } k = 4 \quad (64)^{\frac{1}{6}} \left(\cos \frac{90+360(4)}{6} + i \sin \frac{90+360(4)}{6} \right) = 2(\cos 255 + i \sin 255) = -0.5 - 1.9i$$

$$\text{When } k = 5 \quad (64)^{\frac{1}{6}} \left(\cos \frac{90+360(5)}{6} + i \sin \frac{90+360(5)}{6} \right) = 2(\cos 315 + i \sin 315) = \sqrt{2} - \sqrt{2}i$$

$$8. x^4 + (1 + i) = 0$$

$$x^4 = -1 - i$$

$$x = (-1 - i)^{\frac{1}{4}} = [\sqrt{2}(\cos 225 + i \sin 225)]^{\frac{1}{4}}$$

$$\begin{aligned} \text{When } k = 0 & \quad (\sqrt{2})^{\frac{1}{4}} \left(\cos \frac{225+360(0)}{4} + i \sin \frac{225+360(0)}{4} \right) \\ & = \sqrt[4]{2}(\cos 56.3 + i \sin 56.3) = 0.7 + 0.9i \end{aligned}$$



Chapter three: - Probability

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Exercise 3.1

1. a) permutation b) combination c) permutation d) combination
2. a) RBG, RGB b) BRG, BGR c) GRB, GBR d) 6 possible arrangements
3. a) 5 people b) 4 people c) 3 people d) 1 people e) $5! = 120$
4. a) $5! = 120$ b) $C(5, 3) = 5$

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Exercise 3.2

1. $s = \{1, 2, 3, 4, 5, 6\}$
 a) $p(1) = \frac{1}{6}$ $p(2) = \frac{1}{6}$ $p(3) = \frac{1}{6}$ $p(4) = \frac{1}{6}$ $p(5) = \frac{1}{6}$ $p(6) = \frac{1}{6}$
 so these outcomes are equally likely outcomes b/c they have same probability
2. $s = \{HHH, HHT, HTH, HTT, THH, THT, TTH, TTT\}$
 $p(3 \text{ heads}) = \frac{1}{8}$ $p(3 \text{ tails}) = \frac{1}{8}$
 these two events are equally likely outcomes
 $p(2 \text{ heads and } 1 \text{ tails}) = \frac{3}{8}$ $p(2 \text{ tails and } 1 \text{ heads}) = \frac{3}{8}$
 also these two events are equally likely outcomes

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Exercise 3.3

1. a) two tails = $\{TTH, THT, HTT\}$
 b) At least two tails = $\{TTH, THT, HTT, TTT\}$
 c) At most two tails = $\{THH, HTH, THT, HHT, TTH, HTT\}$

Exercise 3.4

- a) two heads are obtained :- $\{HHT, THH, HTH\}$
 complementary :- $\{HHH, THT, TTH, HTT, TTT\}$
- b) at least two heads are obtained :- $\{HHT, THH, HTH, HHH\}$
 complementary :- $\{THT, TTH, HTT, TTT\}$
- c) at most two heads are obtained :- $\{HTT, TTH, THT, HHT, THH, HTH\}$
 complementary :- $\{HHH, TTT\}$

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Exercise 3.5

- I) 1. dependent 2. Independent 3. Independent
 4. independent 5. Dependent
- II) 6. $P(A) = P(1^{\text{st}} \text{ will show } 3) = \frac{1}{6}$
 $P(B) = P(2^{\text{nd}} \text{ will show } 3) = \frac{1}{6}$



$$P(A \text{ and } B) = \frac{1}{6} * \frac{1}{6} = \frac{1}{36}$$

$$7. p(1^{\text{st}} \text{ throw will show } 6) = \frac{1}{6}$$

$$P(2^{\text{nd}} \text{ throw will show } 6) = \frac{1}{6}$$

$$P(3^{\text{rd}} \text{ throw will show } 6) = \frac{1}{6}$$

$$P(\text{all three times will show } 6) = \frac{1}{6} * \frac{1}{6} * \frac{1}{6} = \frac{1}{216}$$

8. red= 5

white= 4

blue= 7

a) no replacement occurs :-

$$p(1^{\text{st}} \text{ red}) = \frac{5}{16}$$

$$p(2^{\text{nd}} \text{ white}) = \frac{4}{15}$$

$$p(3^{\text{rd}} \text{ blue}) = \frac{7}{14}$$

$$p(\text{red}) * p(\text{white following red}) * p(\text{blue following red and white}) = \frac{5}{16} * \frac{4}{15} * \frac{7}{14} = \frac{1}{24}$$

b) replacement occurs each time:-

$$p(1^{\text{st}} \text{ red}) = \frac{5}{16}$$

$$p(2^{\text{nd}} \text{ white}) = \frac{4}{16}$$

$$p(3^{\text{rd}} \text{ blue}) = \frac{7}{16}$$

$$p(\text{red}) * p(\text{white}) * p(\text{blue}) = \frac{5}{16} * \frac{4}{16} * \frac{7}{16} = \frac{35}{1024}$$

$$10. a) p(A) = p(\text{green neither shows at } 2) = \frac{5}{6}$$

$$P(B) = p(\text{red neither shows at } 2) = \frac{5}{6}$$

$$P(A \text{ and } B) = \frac{5}{6} * \frac{5}{6} = \frac{25}{36}$$

b) same numbers in a die are :- { (1,1)(2,2)(3,3)(4,4)(5,5)(6,6) }

$$p(\text{both shows same number}) = \frac{6}{36} = \frac{1}{6}$$

c) different numbers in a die are :- { (1,2)(1,3)(1,4)(1,5)(1,6)(2,1)(2,3)(2,4)(2,5)

(2,6)(3,1)(3,2)(3,4)(3,5)(3,6)(4,1)(4,2)(4,3)(4,5)(4,6)(5,1)(5,2)(5,3)(5,4)(5,6)(6,1)(6,2)(6,3)(6,4)(6,5)

$$P(\text{both shows different numbers}) = \frac{30}{36} = \frac{5}{6}$$

$$d) P(A) = p(\text{red shows } 2) = \frac{1}{6}$$

$$P(B) = P(\text{green shows any other number}) = \frac{5}{6}$$



$$P(A \text{ and } B) = \frac{1}{6} * \frac{5}{6} = \frac{5}{36}$$

12. drawer 1 = 6 pairs of block socks
and 6 coloured handkerchiefs

drawer 2= 11 white handkerchiefs

$$P(A) = P(\text{a pair of block socks}) = \frac{2}{6} = \frac{1}{3}$$

$$P(B) = p(\text{white handkerchiefs}) = \frac{11}{17}$$

$$P(A \text{ and } B) = \frac{1}{3} * \frac{11}{17} = \frac{11}{51}$$

13. S= 100 clocks

4 damaged

6 buys

P (6 clocks that declare bought can 4 of them is damaged in a shipment)=

$$\frac{C(6,4)}{C(100,4)} = \frac{1}{261415}$$

$$P (6 \text{ clocks that declare bought cant be damaged in a shipment }) = 1 - \frac{1}{261415} = 0.99$$

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more exercises

Exercise a

1. Bag A = 4 white and 6 red

$$I . P(\text{white}) = \frac{6}{15} = \frac{2}{5}$$

bag B = 2 white and 3 red

$$II . P(\text{not white}) = \frac{9}{15} = \frac{3}{5}$$

Exercise b

1. Bag A = 3 red and 7 white

bag B= 2 red and 3 white

$$P(\text{white}) = \frac{10}{15} = \frac{2}{3}$$

$$p(\text{red}) = \frac{5}{15} = \frac{1}{3}$$

$$P(\text{white or red}) = \frac{2}{3} + \frac{1}{3} = 1$$

$$2. p(\text{at least two diamonds}) = \frac{C(13,2)}{C(52,2)} + \frac{C(13,3)}{C(52,3)} + \frac{C(13,4)}{C(52,4)} + \frac{C(13,5)}{C(52,5)} + \frac{C(13,6)}{C(52,6)} + \frac{C(13,7)}{C(52,7)} + \frac{C(13,8)}{C(52,8)} + \frac{C(13,9)}{C(52,9)} + \frac{C(13,10)}{C(52,10)} + \frac{C(13,11)}{C(52,11)} + \frac{C(13,12)}{C(52,12)} + \frac{C(13,13)}{C(52,13)} = 0.075$$

3.

Age(yrs)	46	47	48
Number	10	18	2

$$i) p(47 \text{ yrs old}) = \frac{18}{30} = \frac{3}{5}$$

$$ii) p(\text{not 47 yrs old}) = \frac{12}{30} = \frac{2}{5}$$

$$iii) p(\text{over 47 yrs old}) = \frac{2}{30} = \frac{1}{15}$$



$$4. p(A) = p(\text{servant}) = \frac{7}{26}$$

$$p(B) = p(\text{boy}) = \frac{3}{26}$$

$$p(A \text{ or } B) = \frac{7}{26} + \frac{3}{26} = \frac{10}{26}$$

$$5.a) p(\text{red card}) = \frac{26}{52} = \frac{1}{2}$$

$$b) p(\text{black queen}) = \frac{2}{52} = \frac{1}{26}$$

$$c) p(\text{a king}) = \frac{4}{52} = \frac{1}{13}$$

$$d) p(\text{king of spades}) = \frac{1}{52}$$

6.

grade	A	A-	B+	B	B-	C+	C	C-	D+	D	D-	E
No of students	2	6	7	8	8	14	15	9	7	5	3	1

$$i) P(B+ \text{ and above}) = \frac{15}{85} = \frac{3}{17}$$

$$ii) p(\text{will make it to the university}) = \frac{45}{85} = \frac{9}{17}$$

$$iii) p(\text{poor students}) = \frac{9}{85}$$

7. crate of soda = 16 bottles of Fanta and 8 bottles of sprites

$$P(\text{Fanta}) = \frac{16}{24} = \frac{2}{3}$$

$$p(\text{sprites}) = \frac{8}{24} = \frac{1}{3}$$

8. $s = 10$

i) number pairs their sum is 6 = { (0,6)(1,5)(4,2) }

$$P(\text{sum is 6}) = \frac{3}{10}$$

ii) number pairs their sum is 9 = {(0,9)(1,8)(2,7)(3,6)(4,5)}

$$P(\text{sum is 9}) = \frac{5}{10} = \frac{1}{2}$$

iii) number pairs their sum is 12 = {(3,9)(4,8)(5,7)}

$$P(\text{sum is 12}) = \frac{3}{10}$$

9. a box contains 3 red , 4 blue , 3 green

$s = 10$

$$i) p(\text{blue bead}) = \frac{4}{10} = \frac{2}{5}$$

$$ii) p(\text{red bead}) = \frac{3}{10}$$

$$iii) p(\text{blue or red bead}) = \frac{4}{10} + \frac{3}{10} = \frac{7}{10}$$

10. i) $s = 36$

ii) number pairs both are odd = {(1,1)(1,3)(1,4)(3,1)(3,3)(3,5)(5,1)(5,3)(5,5)}



$$P(\text{both numbers are odd}) = \frac{9}{36} = \frac{1}{4}$$

iii) number pairs both are even = {(2,2)(2,4)(2,6)(4,2)(4,4)(4,6)(6,2)(6,4)(6,6)}

$$P(\text{both numbers are even}) = \frac{9}{36} = \frac{1}{4}$$

iv) number pairs their sum is odd =

{(1,2)(2,1)(1,4)(4,1)(1,6)(6,1)(2,3)(3,2)(3,4)(4,3)(2,5)(5,2)(4,5)(5,4)(5,6)(6,5)(3,6)(6,3)}

$$P(\text{that their sum is odd}) = \frac{18}{36} = \frac{1}{2}$$

v) number pairs their sum is greater is 7 =

{(2,6)(3,5)(4,4)(5,3)(6,2)(3,6)(4,5)(5,4)(6,3)(4,6)(6,4)(5,5)(6,5)(5,6)(6,6)}

$$P(\text{sum is greater than 7}) = \frac{15}{36}$$

$$\text{vi) } p(A) = p(\text{1st throw appears 3}) = \frac{1}{6}$$

$$P(B) = p(\text{1st throw appears 4}) = \frac{1}{6}$$

$$p(A \text{ or } B) = \frac{1}{6} + \frac{1}{6} = \frac{2}{6} = \frac{1}{3}$$

$$p(A) = p(\text{2nd throw appears 3}) = \frac{1}{6}$$

$$P(B) = p(\text{2nd throw appears 4}) = \frac{1}{6}$$

$$p(A \text{ or } B) = \frac{1}{6} + \frac{1}{6} = \frac{2}{6} = \frac{1}{3}$$

so $p(A \text{ or } B)$ on 1st throw and $p(A \text{ or } B)$ on 2nd throw is $= \frac{1}{3} * \frac{1}{3} = \frac{1}{9}$



Answers of chapter four

Section 4.1

1.

Class limit	10-19	20-29	30-39	40-49	50-59
Frequency	4	3	2	5	6

a) The upper class limit of the first class interval (10-19) is 19

b)

Class boundary	9.5-19.5	19.5-29.5	29.5-39.5	39.5-49.5	49.5-59.5
----------------	----------	-----------	-----------	-----------	-----------

$\therefore$ the lower class boundary of the last class interval

(49.5 – 59.5) is 49.5

c) The class size:- [(upper class limit – lower class limit) + 1] = [(19-10)+1]=10
So the class size is 10

d)

Class marks	14.5	24.5	34.5	44.5	54.5
-------------	------	------	------	------	------

$\therefore$ the midpoints (class marks) are : –14.5, 24.5, 34.5, 44.5, and 54.5

2.

Class limit	1.10-1.49	1.50-1.89	1.90-2.29
-------------	-----------	-----------	-----------

a)

Class boundaries	1.095-1.495	1.495-1.895	1.895-2.295
------------------	-------------	-------------	-------------

The class boundaries of the three class intervals are 1.095-1.495, 1.495-1.895, 1.895-2.295.

b) The unit is 0.01

c) The class size is 0.4



Section 4.2

1. a) 31,36,32,39,40,32,37

First arrange the data in (descending or in ascending)

31,32,32,36,37,39,40

- The mode is 32
- The Median is 36
- The mean = $\frac{\sum x}{n} = \frac{31+32+32+36+37+39+40}{7} = 35.28$

b) 14, 20,17,17,15,19,18,14

First arrange the data in (descending or in ascending)

14, 14,15,17,17,18,19,20

- The mode is 14 and 17
- The median is $\frac{17+17}{2} = 17$
- The mean = $\frac{\sum x}{n} = \frac{14+14+15+17+17+18+19+20}{8} = 16.75$

c)

i.

Number of family members	2	3	4	5	6	7
Frequency	3	6	9	8	7	2

- The mode is 4

ii.

Number of family members	2	3	4	5	6	7
Frequency	3	6	9	8	7	2
Cumulative frequency	3	9	18	26	33	35

- The median is 18th place which is 4



Number of family members	Frequency	FX
2	3	6
3	6	18
4	9	36
5	8	40
6	7	42
7	2	14
	$\sum F = 35$	$\sum FX = 156$

- The mean is $\frac{\sum FX}{\sum F} = \frac{156}{35} = 4.46$

2.

Class	30-39	40-49	50-59	60-69	70-79
frequency	3	13	14	6	4

I) Modal class(class with highest frequency)= 50-59

II) Median mark $= L_m + \left(\frac{\frac{N}{2} - FB}{F_m} \right) I$

L_m (lower class boundaries of median class) =49.5

$N (\sum F) = 40$

FB ($\sum F$ before median class) = 16

F_m (frequency of median class) =14

I (class interval) = 10

- Median mark $= 49.5 + \left(\frac{\frac{40}{2} - 16}{14} \right) 10 = 52.36$

III)

Class limit	Frequency(f)	Class mark(x)	FX
30-39	3	34.5	103.5
40-49	13	44.5	578.5
50-59	14	54.5	763
60-69	6	64.5	387
70-79	4	74.5	298
	$\sum F = 40$		$\sum FX = 2130$

- The mean score is $= \frac{\sum FX}{\sum F} = \frac{2130}{40} = 53.25$

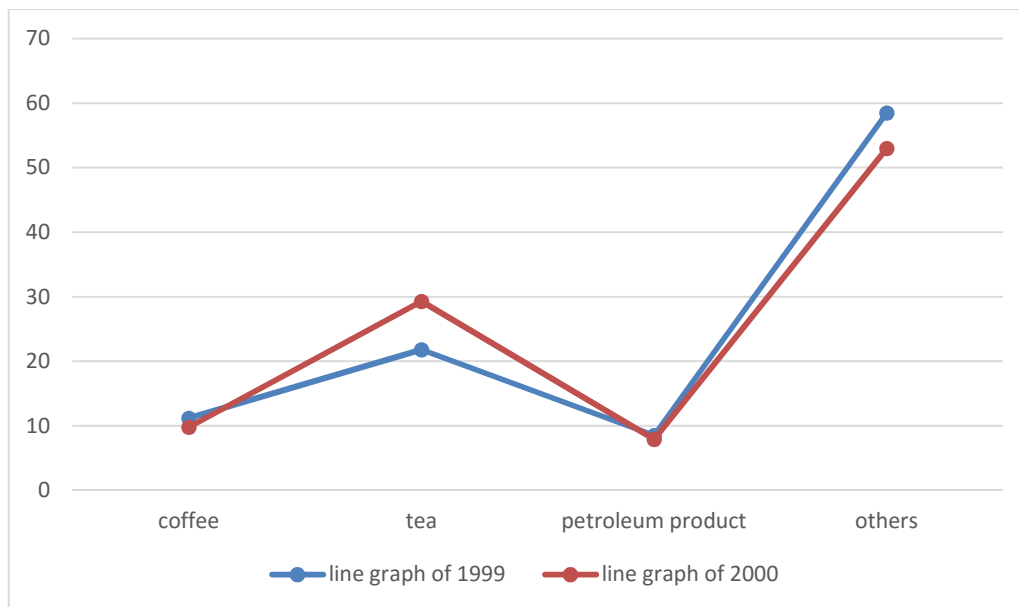


Section 4.3

1.

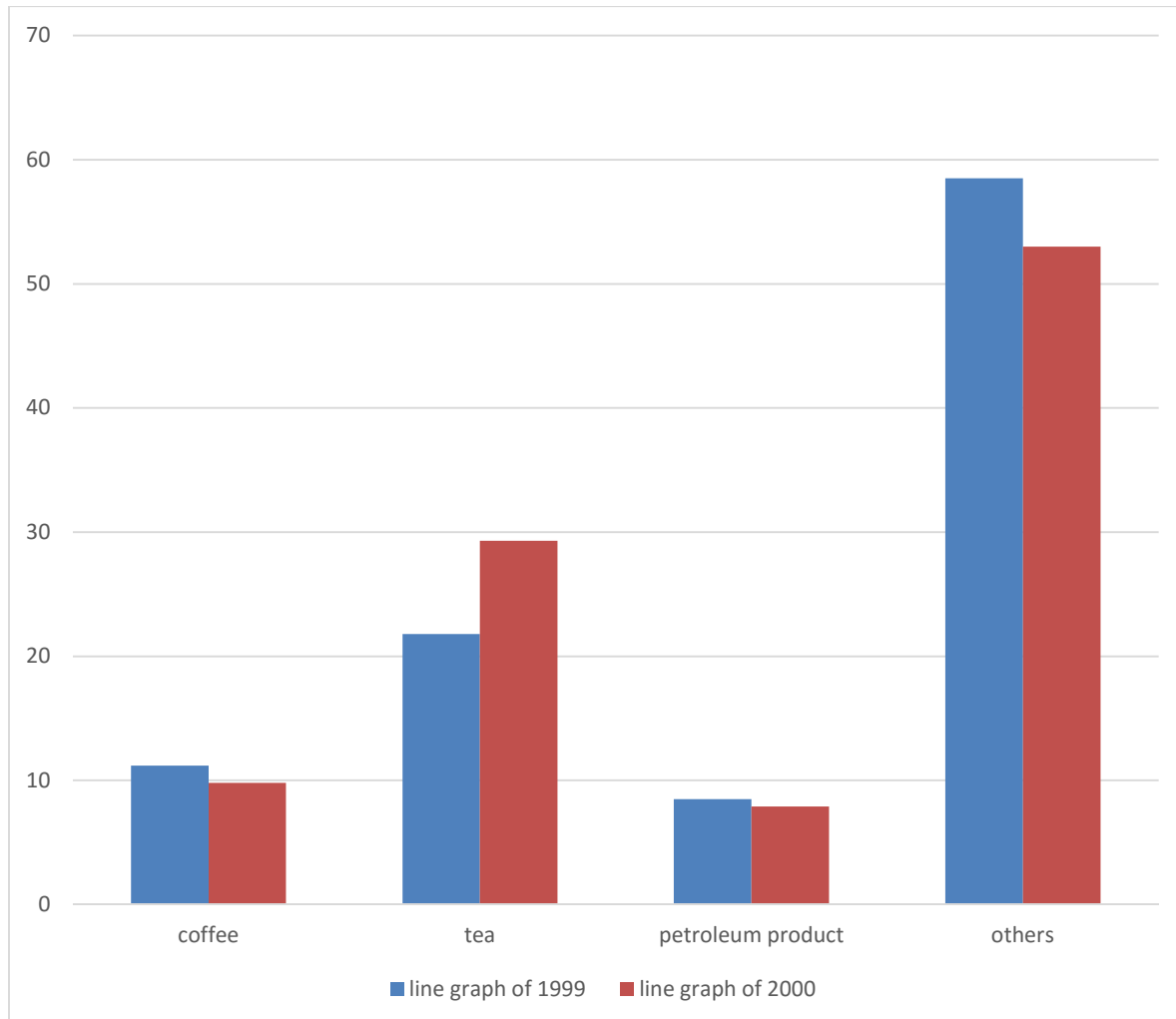
Year	Coffee	Tea	Petroleum Product	Others
1999	11.2	218	8.5	58.5
2000	9.8	29.3	7.9	53

A. Line Graph





B. Bar Graph

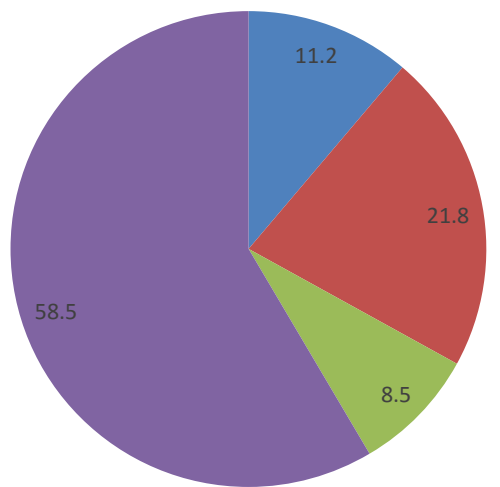




C.1. Pie Chart

- $11.2\% = \frac{11.2}{100} \times 360 = 40.32$
- $21.8\% = \frac{21.8}{100} \times 360 = 78.48$
- $8.5\% = \frac{8.5}{100} \times 360 = 30.6$
- $58.5\% = \frac{58.5}{100} \times 360 = 210.6$

Pie Chart of 1999

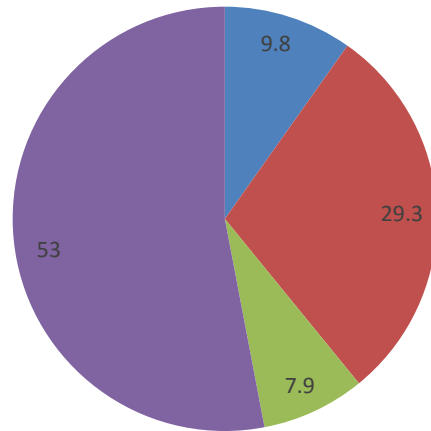


■ coffee ■ tea ■ petroleum product ■ others

C.2. Pie Chart

- $9.8\% = \frac{9.8}{100} \times 360 = 35.28$
- $29.3\% = \frac{29.3}{100} \times 360 = 105.48$
- $7.9\% = \frac{7.9}{100} \times 360 = 28.44$
- $53\% = \frac{53}{100} \times 360 = 190.8$

Pie Chart of 2000

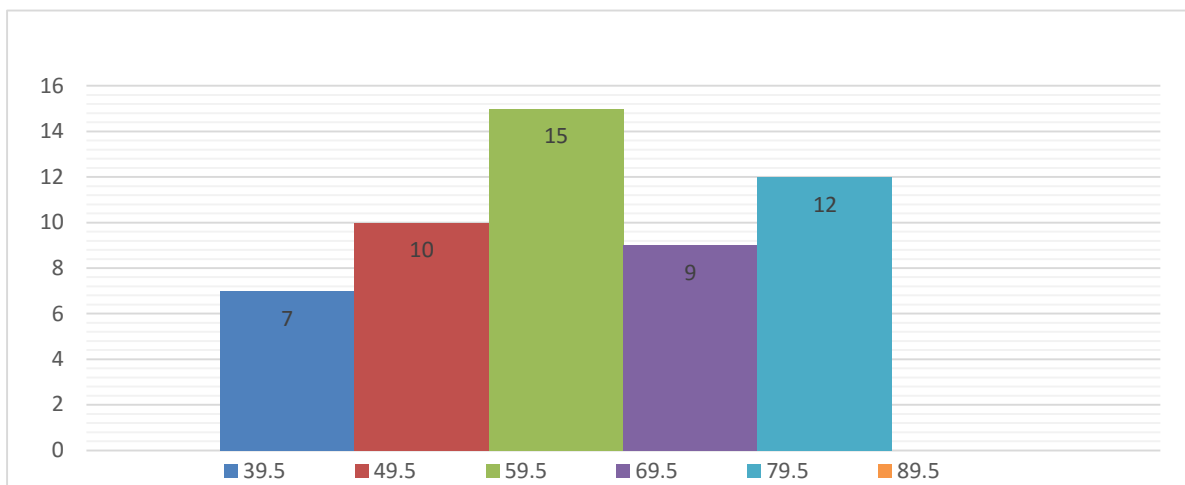


■ coffee ■ tea ■ petroleum product ■ others



Section 4.4

Class	40-49	50-59	60-69	70-79	80-89
Frequency	7	10	15	9	12





Section 4.5

1.

D)

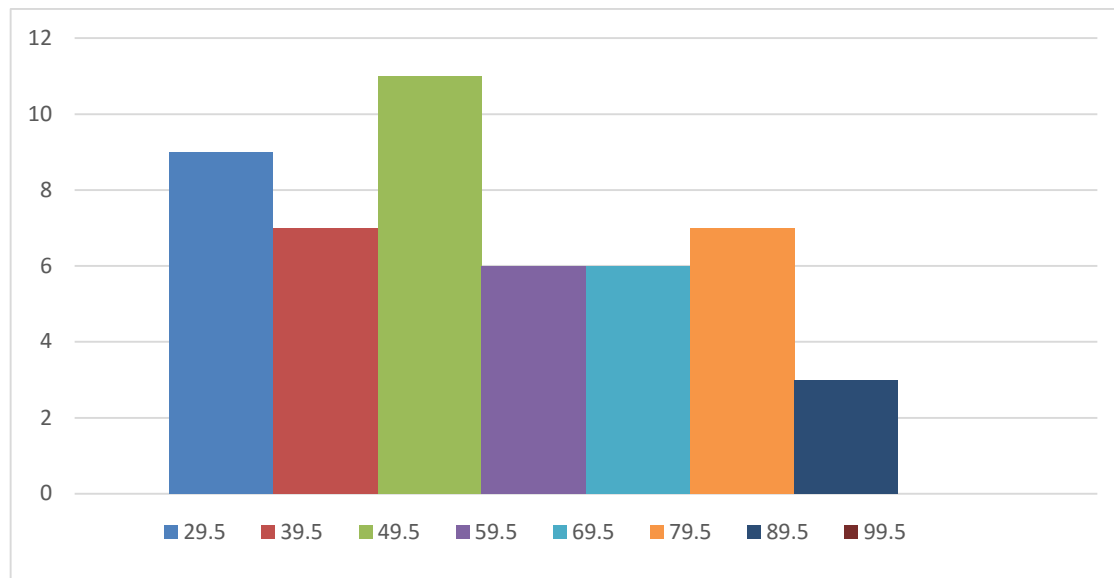
Class	Frequency
30-39	9
40-49	7
50-59	11
60-69	6
70-79	6
80-89	7
90-99	3

II)

Class	Frequency(F)	Class mark(x)	FX
30-39	9	34.5	310.5
40-49	7	44.5	311.5
50-59	11	54.5	599.5
60-69	6	64.5	387
70-79	6	74.5	447
80-89	7	84.5	591.5
90-99	3	94.5	283.5
	$\sum F = 49$		$\sum FX = 2930.5$



III)



2. a) 14,10,11,15,19,10,12

First arrange the data in ascending or in descending

10,10,11,12,14,15,19

The median is 12

b) 55, 50, 51,60,55,54

First arrange the data in ascending or in descending

50, 51, 54,55,55,60

The median is $\frac{54+55}{2} = 54.5$

3.

X	5	6	7	8	9	10
frequency	7	12	4	5	8	4



i)

X	Frequency(F)	FX
5	7	35
6	12	72
7	4	28
8	5	40
9	8	72
10	4	40
	$\sum F = 40$	$\sum FX = 287$

• $\text{mean} = \frac{\sum FX}{\sum F} = \frac{287}{40} = 7.175$

i) mode = 6

ii)

X	Frequency(F)	Cumulative frequency
5	7	7
6	12	19
7	4	23
8	5	28
9	8	36
10	4	40

• $\text{Median} = \frac{20th+21th}{2} = \frac{7+7}{2} = 7$

4.

Class	30-34	35-39	40-44
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- Upper class limit of first class (30-34) is 34
- Lower class boundary of second class (35-39) is 34.5
- The class width is 5



Section 4.6

class	Frequency(F)	Class mark(X)	d(x-A)	Fd
115-119	23	117	-10	-230
120-124	17	122	-5	-85
125-129	20	127	0	0
130-134	16	132	5	80
	$\sum F = 76$			$\sum Fd = -235$

$$\text{Mean} = A + \frac{\sum Fd}{\sum F} = 127 + \frac{-235}{76} = 123.9$$

Section 4.7

Mass(g)	470-479	480-489	490-499	500-509	510-519	520-529	530-539
Number of loaves	5	10	16	2	14	9	6

I) By assumed mean (take A=504.5)

Class	Frequency(F)	Class mark(X)	d(x-A)	Fd
470-479	5	474.5	-30	-150
480-489	10	484.5	-20	-200
490-499	16	494.5	-10	-160
500-509	2	504.5	0	0
510-519	14	514.5	10	140
520-529	9	524.5	20	180
530-539	6	534.5	30	180
	$\sum F = 62$			$\sum Fd = -10$

$$\text{Mean} = A + \frac{\sum Fd}{\sum F} = 504.5 + \frac{-10}{62} = 504.3$$



II) By coding method (take A=504.5)

Class	Frequency(F)	Class mark(X)	$T = \frac{(x-A)}{c}$	Ft
470-479	5	474.5	-3	-15
480-489	10	484.5	-2	-20
490-499	16	494.5	-1	-16
500-509	2	504.5	0	0
510-519	14	514.5	1	14
520-529	9	524.5	2	18
530-539	6	534.5	3	18
	$\sum F = 62$			$\sum Ft = -1$

$$mean = A + \frac{c \sum Ft}{\sum F}$$

$$mean = 504.5 + \frac{10(-1)}{62} = 504.3$$



Section 4.8

Marks		30-39	40-49	50-59	60-69	70-79	80-89	90-99
Number of students	4A	3	4	9	3	8	5	3
	4B	2	6	12	10	5	4	1

i) Mean and standard deviation of 4A

Class	Frequency(F)	Class mark(x)	FX	$(x - \bar{x})^2$	$F(x - \bar{x})^2$
30-39	3	34.5	103.5	918.09	2754.27
40-49	4	44.5	178	412.09	1648.36
50-59	9	54.5	490.5	106.09	954.81
60-69	3	64.5	193.5	0.09	0.27
70-79	8	74.5	596	94.09	752.72
80-89	5	84.5	422.5	388.09	1940.45
90-99	3	94.5	283.5	882.09	2646.27
	$\sum F = 35$		$\sum FX = 2267.5$		$\sum F(x - \bar{x})^2 = 10697.15$

- Mean = $\frac{\sum Fx}{\sum F} = \frac{2267.5}{35} = 64.8$
- Variance = $\frac{\sum F(x - \bar{x})^2}{\sum F} = \frac{10697.15}{35} = 305.6$
- Standard deviation = $\sqrt{\text{variance}} = \sqrt{305.6} = 17.5$



ii) Mean and standard deviation of 4B

Class	Frequency(F)	Class mark(x)	FX	$(x - \bar{x})^2$	$F(x - \bar{x})^2$
30-39	2	34.5	69	702.25	1404.5
40-49	6	44.5	267	272.25	1633.5
50-59	12	54.5	654	42.25	507
60-69	10	64.5	645	12.25	122.5
70-79	5	74.5	372.5	182.25	911.25
80-89	4	84.5	338	552.25	2209
90-99	1	94.5	94.5	1122.25	1122.25
	$\sum F = 40$		$\sum FX = 2440$		$\sum F(x - \bar{x})^2 = 7910$

- Mean = $\frac{\sum Fx}{\sum F} = \frac{2440}{40} = 61$

- Variance = $\frac{\sum F(x - \bar{x})^2}{\sum F} = \frac{7910}{40} = 197.75$

- Standard deviation = $\sqrt{\text{variance}} = \sqrt{197.75} = 14.1$

1. 6,9,10,13,15,17

- Mean:- $\frac{\sum x}{n} = \frac{6+9+10+13+15+17}{6} = 11.7$

- Variance = $\frac{\sum (x - \bar{x})^2}{n}$

$$= \frac{(6 - 11.7)^2 + (9 - 11.7)^2 + (10 - 11.7)^2 + (13 - 11.7)^2 + (15 - 11.7)^2 + (17 - 11.7)^2}{6}$$

$$= \frac{32.49 + 7.29 + 2.89 + 1.69 + 10.89 + 28.09}{6} = 13.89$$

- Standard deviation = $\sqrt{\text{variance}} = \sqrt{13.89} = 3.7$

a) 26,29,30,33,35,37

- Mean:- $\frac{\sum x}{n} = \frac{26+29+30+33+35+37}{6} = 31.7$



$$\begin{aligned} \bullet \text{ Variance} &= \frac{\sum(x-\bar{x})^2}{n} \\ &= \frac{(26 - 31.7)^2 + (29 - 31.7)^2 + (30 - 31.7)^2 + (33 - 31.7)^2 + (35 - 31.7)^2 + (37 - 31.7)^2}{6} \\ &= \frac{32.49 + 7.29 + 2.89 + 1.69 + 10.89 + 28.09}{6} = 13.89 \end{aligned}$$

- Standard deviation = $\sqrt{\text{variance}} = \sqrt{13.89} = 3.7$
b) 30, 45, 50, 65, 75, 85

$$\begin{aligned} \bullet \text{ Mean:- } \frac{\sum x}{n} &= \frac{30 + 45 + 50 + 65 + 75 + 85}{6} = 58.3 \\ \bullet \text{ Variance} &= \frac{\sum(x-\bar{x})^2}{n} \\ &= \frac{(30 - 58.3)^2 + (45 - 58.3)^2 + (50 - 58.3)^2 + (65 - 58.3)^2 + (75 - 58.3)^2 + (85 - 58.3)^2}{6} \\ &= \frac{800.89 + 176.89 + 68.89 + 44.89 + 278.89 + 712.89}{6} = 347.2 \end{aligned}$$

- Standard deviation = $\sqrt{\text{variance}} = \sqrt{347.2} = 18.6$

Section 4.9

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1. 105,109,113,101,120 (A=100)

$$\begin{aligned} \bullet \text{ Mean} &= \frac{\sum(x-A)}{n} \\ &= \frac{(105-100) + (109-100) + (113-100) + (101-100) + (120-100)}{5} \\ &= \frac{5+9+13+1+20}{5} = 9.6 \end{aligned}$$

2. Assumed mean (A=439.5)

Class	Frequency(F)	Class mark(X)	d(x-A)	Fd
415-424	13	419.5	-20	-260
425-434	24	429.5	-10	-240
435-444	12	439.5	0	0
445-454	5	449.5	10	50
455-464	17	459.5	20	340
	$\sum F = 71$			$\sum Fd = -110$

$$\bullet \text{ Mean} = A + \frac{\sum Fd}{\sum F} = 439.5 + \frac{-110}{71} = 437.9$$



4. Mean and standard deviation

Class	Frequency(F)	Class mark(x)	FX	$(x - \bar{x})^2$	$F(x - \bar{x})^2$
65-69	6	67	402	151.29	907.74
70-74	10	72	720	53.29	532.9
75-79	12	77	924	5.29	63.48
80-84	9	82	738	7.29	65.61
85-89	11	87	957	59.29	652.19
90-94	5	92	460	161.29	806.45
	$\sum F = 53$		$\sum FX = 4201$		$\sum F(x - \bar{x})^2 = 3028.37$

- Mean = $\frac{\sum Fx}{\sum F} = \frac{4201}{53} = 79.3$
- Variance = $\frac{\sum F(x - \bar{x})^2}{\sum F} = \frac{3028.37}{53} = 57.14$
- Standard deviation = $\sqrt{\text{variance}} = \sqrt{57.14} = 7.56$



$$\begin{aligned}
 5. \text{ Mean} &= \frac{\sum x}{n} & \text{Variance} &= \frac{\sum (x - \bar{x})^2}{n} \\
 &= \frac{x+13+10+y+9}{5} = 12 & \frac{(x-12)^2 + (13-12)^2 + (10-12)^2 + (y-12)^2 + (9-12)^2}{5} &= 6 \\
 &= \frac{x+y+32}{5} = 12 & \frac{x^2 - 24x + 144 + 1 + 4 + y^2 - 24y + 144 + 9}{5} &= 6 \\
 &= x + y + 32 = 60 & \frac{x^2 + y^2 - 24x - 24y + 302}{5} &= 6 \\
 &= x + y = 60 - 32 & x^2 + y^2 - 24x - 24y + 302 &= 30 \\
 &= x + y = 28 & x^2 + y^2 - 24x - 24y &= -272 \\
 &= y = 28 - x & x^2 + (28 - x)^2 - 24x - 24(28 - x) &= -272 \\
 & & x^2 + 784 - 56x + x^2 - 24x - 672 + 24x &= -272 \\
 & & 2x^2 - 56x + 384 &= 0 \\
 & & X_1 = 12 & X_2 = 16
 \end{aligned}$$

If $x = 16$ then $y = 28 - 16 = 12$

$\therefore$ so we get $x = 16$ because of the condition if $x > y$ and then $y = 12$

6. A) 9, 11, 13, 16, 17

- Mean:- $\frac{\sum x}{n} = \frac{9+11+13+16+17}{5} = 13.2$

- Variance:- $\frac{\sum (x - \bar{x})^2}{n}$

$$= \frac{(9 - 13.2)^2 + (11 - 13.2)^2 + (13 - 13.2)^2 + (16 - 13.2)^2 + (17 - 13.2)^2}{5}$$

$$= \frac{17.64 + 4.84 + 0.04 + 7.84 + 14.4}{5} = 8.96$$



- Standard deviation = $\sqrt{\text{variance}} = \sqrt{8.96} = 2.99$

b) 21, 23, 25, 28, 29

- Mean: $\frac{\sum x}{n} = \frac{21+23+25+28+29}{5} = 25.2$

- Variance = $\frac{\sum (x - \bar{x})^2}{n}$

$$= \frac{(21 - 25.2)^2 + (23 - 25.2)^2 + (25 - 25.2)^2 + (28 - 25.2)^2 + (29 - 25.2)^2}{5}$$

$$= \frac{17.64 + 4.84 + 0.04 + 7.84 + 14.4}{5} = 8.96$$

- Standard deviation = $\sqrt{\text{variance}} = \sqrt{8.96} = 2.99$

c) 108, 132, 156, 192, 204

- Mean: $\frac{\sum x}{n} = \frac{108+132+156+192+204}{5} = 158.4$

- Variance = $\frac{\sum (x - \bar{x})^2}{n}$

$$= \frac{(108 - 158.4)^2 + (132 - 158.4)^2 + (156 - 158.4)^2 + (192 - 158.4)^2 + (204 - 158.4)^2}{5}$$

$$= \frac{2540.16 + 696.96 + 5.76 + 1128.96 + 2079.36}{5} = 1290.24$$

- Standard deviation = $\sqrt{\text{variance}} = \sqrt{1290.24} = 35.92$



Answers of Chapter five: - LOGIC

1. Construct a truth table of each of the following

a) $[p \Rightarrow (q \wedge \sim q)] \Leftrightarrow \sim p$

P	q	$\sim P$	$\sim q$	$q \wedge \sim q$	$p \Rightarrow (q \wedge \sim q)$	$p \Rightarrow (q \wedge \sim q) \Leftrightarrow \sim p$
T	T	F	F	F	F	T
F	F	F	T	F	F	T
F	T	T	F	F	F	T
F	F	T	T	F	T	T

b) $(q \Rightarrow \sim p) \Leftrightarrow (p \wedge q)$

P	q	$\sim P$	$p \wedge q$	$q \Rightarrow \sim p$	$(q \Rightarrow \sim p) \Leftrightarrow (p \wedge q)$
T	T	F	T	F	F
T	F	F	F	T	F
F	T	T	F	T	F
F	F	T	F	T	F

c) $(p \wedge q) \Leftrightarrow (q \wedge p)$

P	Q	$p \wedge q$	$q \wedge p$	$(p \wedge q) \Leftrightarrow (q \wedge p)$
T	T	T	T	T
T	F	F	F	T
F	T	F	F	T
F	F	F	F	T

d) $(p \vee q) \Leftrightarrow (q \vee p)$

P	Q	$p \vee q$	$q \vee p$	$(p \vee q) \Leftrightarrow (q \vee p)$
T	T	T	T	T
T	F	T	T	T
F	T	T	T	T
F	F	F	F	T



e) $[p \wedge (q \wedge r)] \Leftrightarrow [(p \wedge q) \wedge r]$

P	q	r	$q \wedge r$	$p \wedge (q \wedge r)$	$p \wedge q$	$(p \wedge q) \wedge r$	$[p \wedge (q \wedge r)] \Leftrightarrow [(p \wedge q) \wedge r]$
T	T	T	T	T	T	T	T
T	T	F	F	F	T	F	T
T	F	T	F	F	F	F	T
T	F	F	F	F	F	F	T
F	T	T	T	F	F	F	T
F	T	F	F	F	F	F	T
F	F	T	F	F	F	F	T
F	F	F	F	F	F	F	T

f) $[p \vee (q \vee r)] \Leftrightarrow [(p \vee q) \vee r]$

P	q	r	$q \vee r$	$p \vee (q \vee r)$	$p \vee q$	$(p \vee q) \vee r$	$[p \vee (q \vee r)] \Leftrightarrow [(p \vee q) \vee r]$
T	T	T	T	T	T	T	T
T	T	F	T	T	T	T	T
T	F	T	T	T	T	T	T
T	F	F	F	T	T	T	T
F	T	T	T	T	T	T	T
F	T	F	T	T	T	T	T
F	F	T	T	T	F	T	T
F	F	F	F	F	F	F	T

g) $[p \wedge (q \vee r)] \Leftrightarrow [(p \wedge q) \vee (p \wedge r)]$

P	Q	r	$q \vee r$	$p \wedge (q \vee r)$	$p \wedge q$	$p \wedge r$	$(p \wedge q) \vee (p \wedge r)$	$[p \wedge (q \vee r)] \Leftrightarrow [(p \wedge q) \vee (p \wedge r)]$
T	T	T	T	T	T	T	T	T
T	T	F	T	T	T	F	T	T
T	F	T	T	T	F	T	T	T
T	F	F	F	F	F	F	F	T
F	T	T	T	F	F	F	F	T
F	T	F	T	F	F	F	F	T
F	F	T	T	F	F	F	F	T
F	F	F	F	F	F	F	F	T



h) $[p \vee (q \wedge r)] \Leftrightarrow [(p \vee q) \wedge (p \vee r)]$

P	q	r	$q \wedge r$	$p \vee (q \wedge r)$	$p \vee q$	$p \vee r$	$(p \vee q) \wedge (p \vee r)$	$[p \vee (q \wedge r)] \Leftrightarrow [(p \vee q) \wedge (p \vee r)]$
T	T	T	T	T	T	T	T	T
T	T	F	F	T	T	T	T	T
T	F	T	F	T	T	T	T	T
T	F	F	F	T	T	F	F	F
F	T	T	T	T	T	T	T	T
F	T	F	F	F	T	T	T	F
F	F	T	F	F	F	T	F	T
F	F	F	F	F	F	F	F	T

2. De Morgan's law prove that $\sim (p \vee q)$ is equivalent to $(\sim p) \wedge (\sim q)$

P	Q	$p \vee q$	$\sim (p \vee q)$	$\sim p$	$\sim q$	$\sim p \vee \sim q$
T	T	T	F	F	F	F
T	F	T	F	F	T	F
F	T	T	F	T	F	F
F	F	F	T	T	T	T

So $\sim (p \vee q)$ is equivalent to $(\sim p) \wedge (\sim q)$ because they have same truth table

1. Establish the logical equivalence of these compound statements

A.

P	Q	$\sim q$	$(P \Rightarrow q)$	$\sim (P \Rightarrow q)$	$(P \wedge \sim q)$
T	T	F	T	F	F
T	F	T	F	T	T
F	T	F	T	F	F
F	F	T	T	F	F



B.

P	Q	$P \Rightarrow q$	$q \Rightarrow P$
T	T	T	T
T	F	F	T
F	T	T	F
F	F	T	T

4. show $P \vee (Q \wedge R)$ and $(p \vee Q) \wedge (p \vee R)$ are logically equivalent

P	Q	R	$Q \wedge R$	$P \vee (Q \wedge R)$	$(p \vee Q)$	$(p \vee R)$	$(p \vee Q) \wedge (p \vee R)$
T	T	T	T	T	T	T	T
T	T	F	F	T	T	T	T
T	F	T	F	T	T	T	T
T	F	F	F	T	T	T	T
F	T	T	T	T	T	T	T
F	F	T	F	F	F	T	F
F	F	F	F	F	F	F	F

So $P \vee (Q \wedge R)$ and $(p \vee Q) \wedge (p \vee R)$ are logically equivalent because they have same truth table.



Answers of chapter six: - calculus

Estimating a limit numerically In Exercises 5-8 complete the table and use the result to estimate the limit numerically. Determine whether the limit can be reached.

5. $\lim_{x \rightarrow 2} (5x + 4)$

$$\lim_{x \rightarrow 2} (5(2) + 4)$$

$$= 10 + 4$$

$$= 14$$

X	1.9	1.99	1.999	2	2.001	2.01	2.1
F(x)	13.5	13.95	13.995	14	14.005	14.05	14.5

6. $\lim_{x \rightarrow 1} (2x^2 + x - 4)$

$$\lim_{x \rightarrow 1} (2(1)^2 + 1 - 4)$$

$$= (2 + 1 - 4)$$

$$= 3 - 4$$

$$= -1$$

X	0.9	0.99	0.999	1	1.001	1.01	1.1
F(x)	-1.48	-1.04	-1.004	-1	-0.99	-0.94	-0.48

7. $\lim_{x \rightarrow -1} \frac{x+1}{x^2-x-2}$

$$\lim_{x \rightarrow -1} \frac{-1+1}{(-1)^2 - (-1) - 2}$$

$$= \frac{0}{1+1-2}$$

$$= \frac{0}{0}$$

$$\lim_{x \rightarrow -1} \frac{x+1}{(x+1)(x-2)}$$

$$\lim_{x \rightarrow -1} \frac{1}{(x-2)}$$

X	-1.1	-1.01	-1.001	-1	-0.999	-0.99	-0.9
F(x)	$-\frac{10}{13}$	$-\frac{100}{301}$	$-\frac{1000}{3001}$	$-\frac{1}{3}$	$-\frac{1000}{2999}$	$-\frac{100}{299}$	$-\frac{10}{29}$



$$\lim_{x \rightarrow -1} \frac{1}{(-1 - 2)}$$

$$= -\frac{1}{3}$$

$$8. \lim_{x \rightarrow 0} \frac{\sin 2x}{x}$$

$$\lim_{x \rightarrow 0} \frac{2 \sin 2x}{2x} = 2$$

x	-0.1	-0.01	-0.001	0	0.001	0.01	0.1
F(x)	1.9	1.99	1.999	2	1.9999	1.99	1.986

$$9. \lim_{x \rightarrow 0} \frac{\tan x}{2x}$$

$$\lim_{x \rightarrow 0} \frac{\frac{\sin x}{\cos x}}{2x} = \lim_{x \rightarrow 0} \frac{\sin x}{2x \cos x} = \lim_{x \rightarrow 0} \frac{\sin x}{x} * \frac{1}{2 \cos x}$$

$$\lim_{x \rightarrow 0} \frac{1}{2 \cos x} = \frac{1}{2(1)} = \frac{1}{2}$$

-0.1	-0.01	-0.001	0	0.001	0.01	0.1
0.501	0.500	0.5000	0.5	0.50000	0.5000	0.501



Evaluating Limits In Exercises 11 and 14, use the given information to evaluate each limit.

$$11. \lim_{x \rightarrow c} f(x) = 4$$

$$\lim_{x \rightarrow c} g(x) = 8$$

$$(A) \lim_{x \rightarrow c} [-2g(x)]$$

$$(B) \lim_{x \rightarrow c} [f(x) + g(x)]$$

$$\lim_{x \rightarrow c} [-2(8)]$$

$$\lim_{x \rightarrow c} [4 + 8]$$

$$= -16$$

$$= 12$$

$$(C) \lim_{x \rightarrow c} \frac{f(x)}{g(x)}$$

$$(D) \lim_{x \rightarrow c} \sqrt{f(x)}$$

$$\lim_{x \rightarrow c} \frac{4}{8}$$

$$\lim_{x \rightarrow c} \sqrt{4}$$

$$= \frac{1}{2}$$

$$= 2$$

$$12. \lim_{x \rightarrow c} f(x) = 3$$

$$\lim_{x \rightarrow c} g(x) = -1$$

$$(A) \lim_{x \rightarrow c} [f(x) + g(x)]^2$$

$$(B) \lim_{x \rightarrow c} [6f(x)g(x)]$$

$$\lim_{x \rightarrow c} [3 + (-1)]^2$$

$$\lim_{x \rightarrow c} [6(3)(-1)]$$

$$= [2]^2$$

$$= [(18)(-1)]$$

$$= 4$$

$$= -18$$

$$(C) \lim_{x \rightarrow c} \frac{5g(x)}{4f(x)}$$

$$(D) \lim_{x \rightarrow c} \frac{1}{\sqrt{f(x)}}$$

$$\lim_{x \rightarrow c} \frac{5(-1)}{4(3)}$$

$$\lim_{x \rightarrow c} \frac{1}{\sqrt{3}}$$

$$= -\frac{5}{12}$$

$$= \frac{1}{\sqrt{3}}$$

$$13. f(x) = x^3$$

$$g(x) = \frac{\sqrt{x^2+5}}{2x^2}$$

$$(A) \lim_{x \rightarrow 2} f(x)$$

$$(B) \lim_{x \rightarrow 2} g(x)$$



$$\lim_{x \rightarrow 2} x^3$$

$$\lim_{x \rightarrow 2} 2^3$$

$$= 8$$

$$\lim_{x \rightarrow 2} \frac{\sqrt{x^2+5}}{2x^2}$$

$$= \frac{\sqrt{(2)^2+5}}{2(2)^2}$$

$$= \frac{\sqrt{4+5}}{2(4)}$$

$$= \frac{3}{8}$$

(C) $\lim_{x \rightarrow 2} [g(x) \cdot f(x)]$

$$\lim_{x \rightarrow 2} \left[\frac{\sqrt{x^2+5}}{2x^2} \cdot x^3 \right]$$

$$\lim_{x \rightarrow 2} \left[\frac{x^3 \sqrt{x^2+5}}{2x^2} \right]$$

$$= \left[\frac{(2)^3 \sqrt{(2)^2+5}}{2(2)^2} \right]$$

$$= \frac{8\sqrt{9}}{8}$$

$$= \frac{8(3)}{8}$$

$$= \frac{24}{8}$$

$$= 3$$

(D) $\lim_{x \rightarrow 2} [g(x) - f(x)]$

$$\lim_{x \rightarrow 2} \left[\frac{\sqrt{x^2+5}}{2x^2} - x^3 \right]$$

$$\lim_{x \rightarrow 2} \left[\frac{\sqrt{(2)^2+5}}{2(2)^2} - (2)^3 \right]$$

$$= \left[\frac{\sqrt{4+5}}{2(4)} - 8 \right]$$

$$= \left[\frac{3}{8} - 8 \right]$$

$$= -\frac{61}{8}$$

14. $f(x) = \frac{x}{3-x}$

$g(x) = \sin \pi x$

(A) $\lim_{x \rightarrow 2} f(x)$

$$\lim_{x \rightarrow 2} \frac{x}{3-x}$$

$$\lim_{x \rightarrow 2} \frac{2}{3-2}$$

$$= \frac{2}{1}$$

$$= 2$$

(B) $\lim_{x \rightarrow 2} g(x)$

$$\lim_{x \rightarrow 2} \sin \pi x$$

$$\lim_{x \rightarrow 2} \sin 2\pi$$

$$= \sin 2\pi$$

$$= 0$$



(C) $\lim_{x \rightarrow 2} [f(x) \cdot g(x)]$

$$\lim_{x \rightarrow 2} \left[\frac{x}{3-x} \cdot \sin \pi x \right]$$

$$\lim_{x \rightarrow 2} \left[\frac{x \sin \pi x}{3-x} \right]$$

$$= \left[\frac{2 \sin 2\pi}{3-2} \right]$$

$$= \left[\frac{2 \sin 2\pi}{1} \right]$$

$$= 2(0)$$

$$= 0$$

(D) $\lim_{x \rightarrow 2} [g(x) - f(x)]$

$$\lim_{x \rightarrow 2} \left[\sin \pi x - \frac{x}{3-x} \right]$$

$$= \sin 2\pi - \frac{2}{3-2}$$

$$= 0 - \frac{2}{1}$$

$$= 0 - 2$$

$$= -2$$

Evaluating a limit by direct substitution.

16. $\lim_{x \rightarrow 5} (10 - x^2)$

$$\lim_{x \rightarrow 5} (10 - (5)^2)$$

$$= (10 - 25)$$

$$= -15$$

17. $\lim_{x \rightarrow -3} (2x^2 + 4x + 1)$

$$\lim_{x \rightarrow -3} (2(-3)^2 + 4(-3) + 1)$$

$$= 2(9) - 18 + 1$$

$$= 7$$

18. $\lim_{x \rightarrow -3} \frac{3x}{x^2+1}$

$$\lim_{x \rightarrow -3} \frac{3(-3)}{(-3)^2+1}$$

$$= \frac{-9}{9+1}$$

$$= \frac{-9}{10}$$

19. $\lim_{x \rightarrow -2} \frac{5x+3}{2x-9}$

$$\lim_{x \rightarrow -2} \frac{5(-2)+3}{2(-2)-9}$$

$$= \frac{-10+3}{-4-9}$$

$$= \frac{7}{13}$$



20. $\lim_{x \rightarrow -1} \sqrt{x+2}$

$$\lim_{x \rightarrow -1} \sqrt{-1+2}$$

$$= \sqrt{1}$$

$$= 1$$

22. $\lim_{x \rightarrow \pi} \sin 2x$

$$\lim_{x \rightarrow \pi} \sin 2\pi$$

$$= \sin 2\pi$$

$$= 0$$

24. $\lim_{x \rightarrow -2} \left[\frac{1}{2}x^3 - 5x \right]$

$$\lim_{x \rightarrow -2} \left[\frac{1}{2}(-2)^3 - 5(-2) \right]$$

$$= \frac{1}{2}(-8) + 10$$

$$= -4 + 10$$

$$= 6$$

26. $\lim_{x \rightarrow 4} \frac{x-1}{x^2+2x+3}$

$$\lim_{x \rightarrow 4} \frac{4-1}{(4)^2+2(4)+3}$$

$$= \frac{3}{16+8+3}$$

$$= \frac{3}{27}$$

$$= \frac{1}{9}$$

21. $\lim_{x \rightarrow 3} e^x$

$$\lim_{x \rightarrow 3} e^3$$

$$= e^3$$

$$= 20.08$$

23. $\lim_{x \rightarrow \frac{1}{2}} \arcsin x$

$$= \arcsin\left(\frac{1}{2}\right)$$

$$= \sin^{-1}\left(\frac{1}{2}\right)$$

$$= 30$$

25. $\lim_{x \rightarrow -2} [x^3 - 6x + 5]$

$$\lim_{x \rightarrow -2} [(-2)^3 - 6(-2) + 5]$$

$$= [-8 + 12 + 5]$$

$$= [-8 + 17]$$

$$= 9$$

27. $\lim_{x \rightarrow 3} \frac{x^2+1}{x}$

$$\lim_{x \rightarrow 3} \frac{(3)^2+1}{3}$$

$$= \frac{9+1}{3}$$

$$= \frac{10}{3}$$



28. $\lim_{x \rightarrow 3} \sqrt[3]{x^2 - 1}$

$$\lim_{x \rightarrow 3} \sqrt[3]{(3)^2 - 1}$$

$$= \sqrt[3]{9 - 1}$$

$$= \sqrt[3]{8}$$

$$= 2$$

30. $\lim_{x \rightarrow \pi} \tan x$

$$\lim_{x \rightarrow \pi} \tan \pi$$

$$= \tan \pi$$

$$= 0$$

29. $\lim_{x \rightarrow e} \ln x$

$$\lim_{x \rightarrow e} \ln e$$

$$= \ln e$$

$$= 1$$

31. $\lim_{x \rightarrow 1} \arccos \frac{x}{2}$

$$= \arccos \frac{1}{2}$$

$$= \cos^{-1}\left(\frac{1}{2}\right)$$

$$= 60$$



Section 6.2

A. Find the derivative of each function.

1. $y = x^8$

$$y' = 8x^{8-1}$$

$$y' = 8x^7$$

3. $y = x^{-\frac{2}{5}}$

$$y' = -\frac{2}{5}x^{-\frac{2}{5}-1}$$

$$y' = -\frac{2}{5}x^{-\frac{7}{5}}$$

5. $y(t) = 6t^{-9}$

$$y'(t) = 6(-9)t^{-9-1}$$

$$y'(t) = -54t^{-10}$$

7. $g(x) = x^{100} + 50x + 1$

$$g'(x) = 100x^{100-1} + 50$$

$$g'(x) = 100x^{99} + 50$$

9. $g(x) = x^2 + \frac{1}{x^2}$

$$g(x) = x^2 + x^{-2}$$

$$g'(x) = 2x^{2-1} + (-2)x^{-2-1}$$

$$g'(x) = 2x - 2x^{-3}$$

$$g'(x) = 2x - \frac{2}{x^3}$$

2. $y = \sqrt[3]{x}$

$$y = (x)^{\frac{1}{3}}$$

$$y' = \frac{1}{3}(x)^{\frac{1}{3}-1}$$

$$y' = \frac{1}{3}(x)^{-\frac{2}{3}}$$

4. $v(r) = \frac{4}{3}\pi r^3$

$$v'(r) = \frac{4}{3}(3)\pi r^{3-1}$$

$$v'(r) = 4\pi r^2$$

6. $f(x) = x^3 - 10x + 100$

$$f'(x) = 3x^{3-1} - 10$$

$$f'(x) = 3x^2 - 10$$

8. $f(x) = (2x)^3$

$$f'(x) = 3(2x)^{3-1}$$

$$f'(x) = 3(2x)^2$$

$$f'(x) = 6x^2$$

10. $s(t) = t^8 + 6t^7 - 18t^2 + 2t$

$$s'(t) = 8t^{8-1} + 6(7)t^{7-1} - 18(2)t^{2-1} + 2$$

$$s'(t) = 8t^7 + 42t^6 - 36t + 2$$



$$11. y = \frac{x^2+4x+3}{x}$$

$$y' = \frac{x(2x+4)-x^2+4x+3(1)}{x^2}$$

$$y' = \frac{2x^2+4x-x^2-4x-3}{x^2}$$

$$y' = \frac{x^2-3}{x^2}$$

$$y' = 1 - \frac{3}{x^2}$$

$$12. f(x) = x - 3x^{\frac{1}{3}}$$

$$f'(x) = 1 - 3\left(\frac{1}{3}\right)x^{\frac{1}{3}-1}$$

$$f'(x) = 1 - x^{-\frac{2}{3}}$$

$$13. y = 5x^{-4} - \frac{7}{8}x^{-2} + 3x^2 - 6$$

$$y' = 5(-4)x^{-4-1} - \frac{7}{8}(-2)x^{-2-1} + 3(2)x^{2-1}$$

$$y' = -20x^{-5} + \frac{7}{4}x^{-3} + 6x$$

$$y' = -20\frac{1}{x^5} + \frac{7}{4}\frac{1}{x^3} + 6x$$

$$14. y = \frac{x^{12}-2x^9+5x^{-7}}{4}$$

$$y' = \frac{4(12x^{12-1}-2(9)x^{9-1}+5(-7)x^{-7-1})-(x^{12}-2x^9+5x^{-7})(0)}{(4)^2}$$

$$y' = \frac{4(12x^{11}-18x^8-35x^{-8})-0}{(4)^2}$$

$$y' = \frac{(48x^{11}-72x^8-140x^{-8})}{16}$$

$$y' = \frac{48x^{11}}{16} - \frac{72x^8}{16} - \frac{140x^{-8}}{16}$$

$$y' = 3x^{11} - \frac{9}{2}x^8 - \frac{35}{4x^8}$$



B. Differentiate the following:

$$1. f(x) = x^3 - 7$$

$$f'(x) = 3x^{3-2}$$

$$f'(x) = 3x^2$$

$$2. f(x) = 5\sqrt{x} + x$$

$$f(x) = 5(x)^{\frac{1}{2}} + x$$

$$f'(x) = 5(\frac{1}{2})(x)^{\frac{1}{2}-1} + 1$$

$$f'(x) = \frac{5}{2}(x)^{-\frac{1}{2}} + 1$$

$$3. h(x) = \frac{1}{x} - x^4$$

$$h(x) = x^{-1} - x^4$$

$$h'(x) = -x^{-1-1} - 4x^{4-1}$$

$$h'(x) = -x^{-2} - 4x^3$$

$$4. r(y) = 11y^3 - 2y + 7$$

$$r'(y) = 11(3)y^{3-1} - 2$$

$$r'(y) = 33y^2 - 2$$

$$5. s(t) = t^3 - 3t^2 + 2t - \sqrt{6}$$

$$s'(t) = 3t^{3-1} - 3(2)t^{2-1} + 2$$

$$s'(t) = 3t^2 - 6t + 2$$

$$6. f(x) = \frac{x-1}{x}$$

$$f'(x) = \frac{x(1)-(x-1)(1)}{x^2}$$

$$f'(x) = \frac{x-x+1}{x^2}$$

$$f'(x) = \frac{1}{x^2}$$

$$7. f(x) = (x + 6x^2)(3\sqrt{x} - 2)$$

$$f(x) = (x + 6x^2)(3x^{\frac{1}{2}} - 2)$$

$$f'(x) = (1 + 6(2)x^{2-1})(3x^{\frac{1}{2}} - 2) + (x + 6x^2)(3(\frac{1}{2})x^{\frac{1}{2}-1})$$

$$f'(x) = (1 + 12x)(3x^{\frac{1}{2}} - 2) + (x + 6x^2)(\frac{3}{2}x^{-\frac{1}{2}})$$

$$f'(x) = 3x^{\frac{1}{2}} + 36x^{\frac{3}{2}} - 2 - 24x + \frac{3}{2}x^{\frac{1}{2}} + 9x^{\frac{3}{2}}$$

$$f'(x) = \frac{9}{2}x^{\frac{1}{2}} + 45x^{\frac{3}{2}} - 24x - 2$$



$$8. f(x) = (x^5 - 3)^2$$

$$f'(x) = 2(x^5 - 3)(5x^4)$$

$$f'(x) = (x^5 - 3)(10x^4)$$

$$f'(x) = 10x^9 - 30x^4$$

$$9. y = -2x^3 + x^2 - 4x + 7 \quad \text{at } x = 2$$

first we find the value of y by substitution x - value to the function

$$y = -2(2)^3 + (2^2) - 4(2) + 7$$

$$y = -16 + 4 - 8 + 7$$

$$y = -13$$

so we have a point $(x, y) = (2, -13)$

and then we find slope(m) by finding the derivative of the function and then substitute x - value to derivative of the function

$$y' = -6x^2 + 2x - 4$$

$$\text{slope}(m) = y'(x) = -6(2)^2 + 2(2) - 4$$

$$m = -24 + 4 - 4 = -24$$

and then we find equation of tangent line = $y - y_1 = m(x - x_1)$

$$y - (-13) = -24(x - 2)$$

$$y + 13 = -24x + 48$$

$$y = -24x + 48 - 13$$

$$y = -24x + 35$$

$$10. y = 2\sqrt{x} - \frac{1}{x^2} + \frac{1}{x} + 4 \quad \text{at } x = 1$$

first we find the value of y by substitution x - value to the function

$$y = 2\sqrt{1} - \frac{1}{(1)^2} + \frac{1}{1} + 4$$

$$y = 2 - 1 + 1 + 4$$

$$y = 6$$

so we have a point $(x, y) = (1, 6)$

and then we find slope(m) by finding the derivative of the function and then substitute x - value to derivative of the function

$$y' = 2 * \frac{1}{2\sqrt{x}} + \frac{2}{x^3} - \frac{1}{x^2} = \frac{1}{\sqrt{x}} + \frac{2}{x^3} - \frac{1}{x^2}$$

$$\text{slope}(m) = y'(x) = \frac{1}{\sqrt{1}} + \frac{2}{(1)^3} - \frac{1}{(1)^2}$$

$$m = 1 + 2 - 1 = 2$$



and then we find equatin of tangent line = $y - y_1 = m(x - x_1)$

$$y - 6 = 2(x - 1)$$

$$y - 6 = 2x - 2$$

$$y = 2x - 2 + 6$$

$$y = 2x + 4$$