ADDIS ABABA UNIVERSITY
DEPARTMENT OF MATHEMATICS
Mathematics for Natural Sciences (Math 1011)
Worksheet I - July 2021

Part I: Propositional Logic.

(1) Which of the following sentences are propositions? Determine the truth value whenever possible?
   (a) $5 < 9$ and there are infinitely many prime numbers.
   (b) Nairobi is the capital of Kenya.
   (c) Thank you for your consideration.
   (d) Answer this question.
   (e) $x^2 = y^2$ provided that $x = y$.
   (f) How many courses are you taking this semester?
   (g) $x + 5 = 11$.
   (h) The first human kind lived in Ethiopia.

(2) What is the negation of each of the following propositions?
   (a) Today is Friday.
   (b) There is no pollution in Europe.
   (c) $6 + 4 = 10$.
   (d) The summer in Dire Dawa is hot and sunny.

(3) Let p and q be propositions:
   p : It is below freezing.
   q : It is snowing.
   Write the following propositions using p, q and logical connectives.
   (a) It is below freezing and snowing.
   (b) It is below freezing but not snowing.
   (c) It is not below freezing and it is not snowing.
   (d) It is either snowing or below freezing (or both).
   (e) If it is below freezing, it is also snowing.
   (f) It is either below freezing or it is snowing, but it is not snowing if it is below freezing.
   (g) That it is below freezing is necessary and sufficient for it to be snowing.

(4) Given the following propositions
   p : You have the flu.
   r : You pass the course.
   q : You miss the final examination.
   Express each of the following propositions as an ordinary English sentence.
   a) $\neg q \Leftrightarrow r$ b) $\neg(p \wedge q)$ c) $(p \Rightarrow \neg r) \vee (q \Rightarrow \neg r)$.

(5) Determine the truth value of p if
   (a) $(q \Rightarrow \neg p) \vee r$ is False
   (b) $\neg q$ and $p \Rightarrow q$ are True
   (c) $(p \vee \neg q) \Rightarrow (\neg r \vee p)$ is False.

(6) If $\neg[\neg r \Rightarrow \neg(p \wedge q)]$ is true, then find the truth value of $[(p \Leftrightarrow r) \vee q] \Leftrightarrow (\neg p \wedge r)$.

(7) Prove that

(a) $(p \wedge q) \Rightarrow r \equiv p \Rightarrow (q \Rightarrow r)$
(b) $(q \vee r) \Rightarrow p \equiv (q \Rightarrow p) \wedge (r \Rightarrow p)$.

(8) For the following propositions, indicate whether it is a tautology, a contradiction, or neither. Use a truth table to decide.
(a) $[p \wedge (p \Rightarrow q)] \Rightarrow q$
(b) $(p \Rightarrow q) \wedge \neg(\neg q \Rightarrow \neg p)$.
(c) $(\neg q \Rightarrow \neg p) \Rightarrow ((\neg q \Rightarrow p) \Rightarrow q)$.

(9) Determine whether the following statements are valid or invalid.
(a) If I do not wake up, then I cannot go to work.
If I cannot go to work, then I will not get paid.
Therefore, if I do not wake up, then I will not get paid.
(b) If I study, then I will not fail Math 1011.
If I do not play cards to often, then I will study.
I failed Math 1011.
Therefore, I played cards too often.
(c) Today is either Thursday or Friday.
On Thursdays I have to go to a lecture.
Today is not Friday.
Therefore,I have to go to a lecture today.

(10) Let $P(x) : x$ is an integer greater than 5. $Q(x) : x$ is a natural number.
Determine the truth value of the following propositions.
(a) $P(3) \wedge Q(-2)$
(b) $P(2) \Rightarrow Q(2)$
(c) $[P(3) \Rightarrow Q(4)] \vee Q(9)$.

(11) Let $P(x) : x$ is a prime number $Q(x) : x$ is an even number.
$R(x) : x$ is an odd number. $S(x) : x$ is an integer.
Write a sentence, which corresponds to each of the following:
(a) $R(x) \vee Q(x) \Rightarrow P(x)$
(b) $P(x) \Leftrightarrow Q(x) \vee R(x)$.

(12) If $U = \mathbb{R}$, then find the truth value of $(\forall x)(\exists y)(\sqrt{x^2} = 16 \Rightarrow y + x = 10)$.

(13) Let $P(x) : x$ is a composite number. $Q(x) : x$ is a prime number.
Find the truth values of (i) $(\exists x)[P(x) \Leftrightarrow Q(x)]$ (ii) $(\forall x)[P(x) \Leftrightarrow Q(x)]$.

(14) Find the truth values of the following where $U = \mathbb{R}$
(a) $(\exists x)(\forall y)(x^2 < y^2)$
(b) $(\forall x)[x \neq 0 \Rightarrow (\exists y)(xy = 4)]$.

Part II: Set Theory.

(1) Write the following sets in complete listing or partial listing method.
(a) $A = \{x | x \text{ is an integer and } 3 < x \leq 10\} \cap \{x | x \text{ is even integer}\}$.
(b) $B = \{x \in \mathbb{N} | (x-1)(x-3) \neq 0 \Leftrightarrow x + 1 = x\}$.
(c) $C = \{x \in \mathbb{N} | x \geq 3 \Leftrightarrow x < 0\}$.
(d) $D = \{x | x \in \mathbb{N} \wedge (x-1)(x-3) \neq 0 \Leftrightarrow x + 1 = x\}$.

(2) If $n(A \backslash B) = 18$, $n(A \cup B) = 70$ and $n(A \cap B) = 25$, then find $n(B)$.

(3) For all subsets A, B and C of some Universal set U, prove or disprove the following statements.

(a) $(A \subseteq B \wedge A \subseteq C) \Rightarrow A \subseteq B \cap C$
(b) $A \subseteq B \Leftrightarrow A \cap B = A$
(c) $A^c \backslash B^c = B \backslash A$
(d) $A \subseteq B \Rightarrow A \cup (B \backslash A) = B$.

(4) Let $U = \{x \in \mathbb{Z} | -12 \leq x \leq 6\}$.
$A = \{x \in \mathbb{Z} | x = 2n \wedge x = 3m, \text{for} -4 \leq n \leq 8 \text{ and} -2 \leq m \leq 3\}$.
$B = \{0, 1, 3\}$ $C = \{x \in \mathbb{Z} | x = \frac{n}{3}, n \in A\}$.
Then find
(a) $A \backslash [B \cup C]^c$
(b) $(A \backslash B) \backslash (A \cup C)^c$
(c) $(A \cap B) \backslash C$.

(5) (a) For every natural number n, define $A_n = \{x \in \mathbb{Z} | \frac{-24}{n} \leq 2x + 1 \leq \frac{24}{n}\}$.
Then find
(i) $\cup_{n=1}^{6} A_n$
(ii) $[\,(\cup_{n=1}^{4} A_n) \backslash (\cap_{n=1}^{3} A_n)\,]^c$
(iii) $\cap A_n$, $n \in \mathbb{N}$.

(b) For every natural number n, define
$A_n = \{x \in \mathbb{N} | x = n \Rightarrow x < n\}$.
Then find
(i) $\cup_{n=1}^{10} A_n$
(ii) $\cup A_n$, $n \in \mathbb{N}$.