ANSWERS OF CHAPTER ONE:-TRIGNOMETRY

SECTION: - 1.1

Part one: using the sum and difference identities, condense each of the following and express as a trig function of a single angle

1. $\sin 97 \cos 43 + \cos 97 \sin 43 = \sin(97+43) = \sin 140$
2. $\cos 72 \cos 130 + \sin 72 \sin 130 = \cos(72-130) = \cos(-58) = \cos 58$
3. $\frac{tan140 - \tan 60}{1+\tan 140 tan60} = \tan(140-60) = \tan 80$
4. $\sin\frac{\pi}{5}\cos\frac{2\pi}{3} - \cos\frac{\pi}{5}\sin\frac{2\pi}{3} = \sin(\frac{\pi}{5} - \frac{2\pi}{3}) = \sin(-\frac{7\pi}{15}) = -\sin\frac{7\pi}{15}$
5. $\cos\frac{\pi}{6}\cos\frac{\pi}{7} - \sin\frac{\pi}{6}\sin\frac{\pi}{7} = \cos(\frac{\pi}{6} + \frac{\pi}{7}) = \cos\frac{13\pi}{42}$
6. $\frac{tan\frac{\pi}{3} + \tan\frac{\pi}{4}}{1 - tan\frac{\pi}{3}\tan\frac{\pi}{4}} = \tan(\frac{\pi}{3} + \frac{\pi}{4}) = \tan\frac{7\pi}{12}$

Part two: use the sum and difference identities with unit circle values to find exact answers for the following.

7. $\tan(-105^0) = -\tan 105^0 = -\tan(45+60) = -[\frac{tan45 + \tan 60}{1 - \tan 45 tan60}] = -[\frac{1+\sqrt{3}}{1-1*\sqrt{3}}]$

   $= -[\frac{1+\sqrt{3}}{1-\sqrt{3}}] = \frac{-1-\sqrt{3}}{1-\sqrt{3}}$

8. $\sin 345 = \sin(300 + 45) = \sin 300\cos 45 + \cos 300\sin 45 = -\frac{\sqrt{3}}{2} * \frac{\sqrt{2}}{2} + \frac{1}{2} * \frac{\sqrt{2}}{2}$

   $= -\frac{\sqrt{6}}{4} + \frac{\sqrt{2}}{4} = \frac{\sqrt{2} - \sqrt{6}}{4}$

Part three: - given $\csc \propto = \frac{13}{5}$ $\frac{\pi}{2} \leq \propto \leq \pi$ $\tan\beta = -\frac{3}{4}$ $\frac{3\pi}{2} \leq \beta \leq 2\pi$

$\sin\propto = \frac{5}{13}$ $sin^2 \propto + cos^2 \propto = 1$

$cos^2 \propto = 1 - sin^2 \propto$ $1+tan^2\beta = sec^2\beta$ $sin^2\beta + cos^2\beta = 1$

$cos^2 \propto = 1 - (5/13)^2$ $1 + (-\frac{3}{4})^2 = sec^2\beta$ $sin^2\beta = 1 - cos^2\beta$

$cos^2 \propto = 1 - \frac{25}{169}$ $1 + \frac{9}{16} = sec^2\beta$ $sin^2\beta = 1 - \left(\frac{4}{5}\right)^2$

$cos^2 \propto = \frac{144}{169}$ $sec^2\beta = \frac{25}{16}$ $sin^2\beta = 1 - \frac{16}{25}$

$\cos \propto = -\frac{12}{13}$ $\sec\beta = \frac{5}{4}$ $sin^2\beta = \frac{9}{25}$

$\tan \propto = -\frac{5}{12}$ $\therefore cos\beta = \frac{4}{5}$ $\sin\beta = -\frac{3}{5}$

9. $\sin(\propto -\beta) = \sin\propto cos\beta + \cos\propto \sin\beta = \frac{5}{13} * \frac{4}{5} - (\frac{12}{13}) * (-\frac{3}{5}) = = \frac{20}{65} - \frac{36}{65} = -\frac{16}{65}$
10. $\cos(\beta + \propto) = \cos\beta\ cos\alpha - sin\beta\ sin\alpha = \frac{4}{5} * -\frac{12}{13} - (-\frac{3}{5}) * (\frac{5}{13})$
    $= -\frac{48}{65} + \frac{15}{65} = -\frac{33}{65}$
11. $\tan(\propto -\beta) = \frac{tan\propto - \tan\beta}{1 + tan\alpha \tan\beta} = \frac{-\frac{5}{12} + \frac{3}{4}}{1 + \left(-\frac{5}{12}\right) * (-\frac{3}{4})} = \frac{\frac{1}{3}}{1 + \frac{5}{16}} = \frac{\frac{1}{3}}{\frac{21}{16}} = \frac{16}{63}$

Part four: - if $\sin\theta = -\frac{3}{5}$ and θ is in third quadrant find the following

$sin^2\theta + cos^2\theta = 1$

$cos^2\theta = 1 - sin^2\theta$

$cos^2\theta = 1 - (-3/5)^2$

$cos^2\theta = 1 - \frac{9}{25}$

$cos^2\theta = \frac{16}{25}$

$\cos\theta = -\frac{4}{5}$

$\tan\theta = \frac{sin\theta}{cos\theta} = \frac{-\frac{3}{5}}{\frac{-4}{5}} = \frac{3}{4}$

12. $\cos(\theta + \frac{\pi}{3}) = \cos\theta cos\frac{\pi}{3} - sin\theta sin\frac{\pi}{3}$

$-\frac{4}{5} * -\frac{1}{2} - (-\frac{3}{5}) * (\frac{\sqrt{3}}{2}) = -\frac{4}{10} + \frac{3\sqrt{3}}{10} = \frac{3\sqrt{3}-4}{10}$

13. $\tan 2\theta = \tan(\theta + \theta) = \frac{tan\theta + \tan\theta}{1 - tan\theta tan\theta} =$

$\frac{\frac{3}{4}+\frac{3}{4}}{1-(\frac{3}{4})*(\frac{3}{4})} = \frac{\frac{6}{4}}{1-\frac{9}{16}} = \frac{\frac{6}{4}}{\frac{7}{16}} = \frac{24}{7}$

Part five: - verify the following identities

14. $\sin(\pi - x) = \sin x$

$\sin\pi cosx - cos\pi \sin x = \sin x$

$0 * \cos x - (-1) * sinx = \sin x$

$0 + \sin x = \sin x$

$\therefore \sin x = \sin x$

15. sin ($\frac{3\pi}{2}$ +x) = -cosx

sin$\frac{3\pi}{2}cosx + cos\frac{3\pi}{2}sinx$ = -cosx

-1*cosx + 0*sinx= -cosx

-cosx+0= -cosx

∴ - cosx = - cosx

16. cos(30-x) + cos(30+x)= $\sqrt{3}$ cosx

Cos30cosx+sin30sinx+cos30cosx-sin30sinx= $\sqrt{3}$ cosx

2cos30cosx= $\sqrt{3}$ cosx

2($\frac{\sqrt{3}}{2}$)* cosx = $\sqrt{3}$ cosx

∴ $\sqrt{3}$ cosx = $\sqrt{3}$ cosx

17. $\frac{\sin(\beta-\alpha)}{sin\alpha sin\beta}$ = cot$\alpha - cot\beta$

$\frac{sin\beta cos\alpha - cos\beta sin\alpha}{sin\alpha sin\beta} = cot\alpha - cot\beta$

$\frac{sin\beta cos\alpha}{sin\alpha sin\beta} - \frac{cos\beta sin\alpha}{sin\alpha sin\beta} = cot\alpha - cot\beta$

$\frac{cos\alpha}{sin\alpha} - \frac{cos\beta}{sin\beta} = cot\alpha - cot\beta$

$\therefore cot\alpha - cot\beta = cot\alpha - cot\beta$

18. $\cos(\alpha+\beta)+\cos(\alpha-\beta)=2cos\alpha cos\beta$

$cos\alpha cos\beta - sin\alpha sin\beta + cos\alpha cos\beta + sin\alpha sin\beta = 2cos\alpha cos\beta$

$\therefore 2cos\alpha cos\beta = 2cos\alpha cos\beta$

19. $tan\frac{5\pi}{12} = \tan(\frac{\pi}{4}+\frac{\pi}{6}) = \frac{tan\frac{\pi}{4}+\tan\frac{\pi}{6}}{1-tan\frac{\pi}{4}\tan\frac{\pi}{6}} = \frac{1+\frac{1}{\sqrt{3}}}{1-1*\frac{1}{\sqrt{3}}} = \frac{\frac{\sqrt{3}+1}{\sqrt{3}}}{\frac{\sqrt{3}-1}{\sqrt{3}}} = \frac{\sqrt{3}+1}{\sqrt{3}-1}$

$\tan(-\frac{\pi}{12}) = \tan((\frac{\pi}{6}-\frac{\pi}{4}) = \frac{tan\frac{\pi}{6}-\tan\frac{\pi}{4}}{1+tan\frac{\pi}{6}\tan\frac{\pi}{4}} = \frac{\frac{\sqrt{3}}{3}-1}{1+\frac{\sqrt{3}}{3}*1} = \frac{\frac{\sqrt{3}-3}{3}}{\frac{\sqrt{3}+3}{3}} = \frac{\sqrt{3}-3}{\sqrt{3}+3}$

20. Given $\sin x=\frac{3}{5}$, $cosy=-\frac{5}{13}$ $where\ 0\le x\le\frac{\pi}{2}$ $and\ \frac{\pi}{2}\le y\le \pi$ $evaluate\ \sin(x+y)$

$sin^2 x + cos^2 x = 1$ $\qquad$ $sin^2 y + cos^2 y = 1$

$cos^2 x = 1 - sin^2 x$ $\qquad$ $sin^2 y = 1 - cos^2 y$

$cos^2 x = 1-(3/5)^2$ $\qquad$ $sin^2 y = 1-(-5/13)^2$

$cos^2 x = 1 - \frac{9}{25}$ $\qquad$ $sin^2 y = 1 - \frac{25}{169}$

$cos^2 x = \frac{16}{25}$ $\qquad$ $sin^2 y = \frac{144}{169}$

$\cos x = \frac{4}{5}$ $\qquad$ $\sin y = \frac{12}{13}$

: - $\sin(x+y) = sinxcosy + cosxsiny$

$(\frac{3}{5})(-\frac{5}{13}) + (\frac{4}{5})(\frac{12}{13}) = -\frac{15}{65}+\frac{48}{65} = \frac{33}{65}$

21. *show that* $\cos(x-y) = cosxcosy + sinxsiny$ *is true for* $x = \frac{\pi}{3}$ *and* $y = \frac{\pi}{6}$

$\cos(\frac{\pi}{3} - \frac{\pi}{6}) = cos\frac{\pi}{3}cos\frac{\pi}{6} + sin\frac{\pi}{3}sin\frac{\pi}{6}$

$cos\frac{\pi}{6} = \frac{1}{2} * \frac{\sqrt{3}}{2} + \frac{\sqrt{3}}{2} * \frac{1}{2}$

$\frac{\sqrt{3}}{2} = \frac{\sqrt{3}}{4} + \frac{\sqrt{3}}{4}$

$\frac{\sqrt{3}}{2} = \frac{2\sqrt{3}}{4}$

$\frac{\sqrt{3}}{2} = \frac{\sqrt{3}}{2}$

22. *show that* $\sin(x+y) = sinxcosy + cosxsiny$ *is true for* $x = \frac{\pi}{2}$ *and* $y = \frac{3\pi}{4}$

$\sin\left(\frac{\pi}{2} + \frac{3\pi}{4}\right) = sin\frac{\pi}{2}cos\frac{3\pi}{4} + cos\frac{\pi}{2}sin\frac{3\pi}{4}$

$\sin\frac{5\pi}{4} = 1 * -\frac{\sqrt{2}}{2} + 0 * \frac{\sqrt{2}}{2}$

$-\frac{\sqrt{2}}{2} = -\frac{\sqrt{2}}{2} + 0$

$-\frac{\sqrt{2}}{2} = -\frac{\sqrt{2}}{2}$

23. Use an appropriate compound angle formula to determine the exact value of $\sin\frac{\pi}{12}$.

$\sin\frac{\pi}{12} = \sin(\frac{\pi}{3}-\frac{\pi}{4}) = sin\frac{\pi}{3}cos\frac{\pi}{4} - cos\frac{\pi}{3}sin\frac{\pi}{4}$

$= \frac{\sqrt{3}}{2} * \frac{\sqrt{2}}{2} - \frac{1}{2} * \frac{\sqrt{2}}{2} = \frac{\sqrt{6}}{4} - \frac{\sqrt{2}}{4}$

$= \frac{\sqrt{6}-\sqrt{2}}{4}$

Part six:-<u>prove each of the following identities.</u>

24. $\cos(x+y) * \cos(x-y) = cos^2 x + cos^2 y - 1$

$[cosxcosy - sinxsiny][cosxcosy + sinxsiny] = cos^2 x + cos^2 y - 1$

$cos^2 xcos^2 y - sin^2 xsin^2 y = cos^2 x + cos^2 y - 1$

$cos^2 xcos^2 y - [1 - cos^2 x][1 - cos^2 y] = cos^2 x + cos^2 y - 1$

$cos^2 xcos^2 y - [1 - cos^2 y - cos^2 x + cos^2 xcos^2 y] = cos^2 x + cos^2 y - 1$

$cos^2 xcos^2 y - 1 + cos^2 y + cos^2 x - cos^2 xcos^2 y = cos^2 x + cos^2 y - 1$

$\therefore cos^2 x + cos^2 y - 1 = cos^2 x + cos^2 y - 1$

25. $\cos\left(\frac{\pi}{2} - x\right) = sinx$

$cos\frac{\pi}{2}cosx + sin\frac{\pi}{2}sinx = sinx$

0*cosx+1*sinx=sinx

$\therefore sinx = sinx$

26. if $sinx = \frac{4}{5}$ *find the value of* $\cos(x + 180)$

$sin^2 x + cos^2 x = 1$

$cos^2 x = 1 - sin^2 x$

$cos^2 x = 1 - (4/5)^2$

$cos^2 x = 1 - \frac{16}{25}$

$cos^2 x = \frac{9}{25}$

$\cos x = \frac{3}{5}$

$\therefore \cos(x + 180) = cosxcos180 - sinxsin180$

$= \frac{3}{5} * -1 - \frac{4}{5} * 0$

$= -\frac{3}{5}$

27. If $\sin A = \frac{4}{5}$ $tanB = \frac{5}{12}$ *and angles A and B are in quadrant* one *what is the value of* $\sin(A + B)$

$cos^2 A = 1 - sin^2 A$

$cos^2 A = 1 - (4/5)^2$

$cos^2 A = 1 - \frac{16}{25}$

$cos^2 A = \frac{9}{25}$

$\cos A = \frac{3}{5}$

$1 + tan^2 B = sec^2 B$

$1 + (\frac{5}{12})^2 = sec^2 B$

$1 + \frac{25}{144} = sec^2 B$

$sec^2 B = \frac{169}{144}$

$\sec B = \frac{13}{12}$

$\therefore cosB = \frac{12}{13}$

$sin^2 B + cos^2 B = 1$

$sin^2 B = 1 - cos^2 B$

$sin^2 B = 1 - \left(\frac{12}{13}\right)^2$

$sin^2 B = 1 - \frac{144}{169}$

$sin^2 B = \frac{25}{169}$

$\sin B = \frac{5}{13}$

$\sin(A + B) = sinAcosB + cosAsinB = \frac{4}{5} * \frac{12}{13} + \frac{3}{5} * \frac{5}{13} = \frac{48}{65} + \frac{15}{65} = \frac{63}{65}$

28. *find the exact value of each of the following*

a. $sin75 = \sin(45 + 30) = sin45cos30 + cos45sin30$

$= \frac{\sqrt{2}}{2} * \frac{\sqrt{3}}{2} + \frac{\sqrt{2}}{2} * \frac{1}{2} = \frac{\sqrt{6}}{4} + \frac{\sqrt{2}}{4} = \frac{\sqrt{6}+\sqrt{2}}{4}$

b. $cos\frac{7\pi}{12} = \cos\left(\frac{\pi}{3} + \frac{\pi}{4}\right) = cos\frac{\pi}{3}cos\frac{\pi}{4} - sin\frac{\pi}{3}sin\frac{\pi}{4}$

$= \frac{1}{2} * \frac{\sqrt{2}}{2} - \frac{\sqrt{3}}{2} * \frac{\sqrt{2}}{2} = \frac{\sqrt{2}}{4} - \frac{\sqrt{6}}{4} = \frac{\sqrt{2}-\sqrt{6}}{4}$

c. $tan105 = \tan(60 + 45) = \frac{tan60 + \tan 45}{1 - tan60\tan 45} = \frac{\sqrt{3}+1}{1-\sqrt{3}*1} = \frac{1+\sqrt{3}}{1-\sqrt{3}}$

d. $sin\frac{19\pi}{12} = \sin\left(\frac{4\pi}{3} + \frac{\pi}{4}\right) = sin\frac{4\pi}{3}cos\frac{\pi}{4} + cos\frac{4\pi}{3}sin\frac{\pi}{4}$

$= -\frac{\sqrt{3}}{2} * \frac{\sqrt{2}}{2} + \left(-\frac{1}{2}\right) * \frac{\sqrt{2}}{2} = \frac{-\sqrt{6}}{4} - \frac{\sqrt{2}}{4} = \frac{-\sqrt{2}-\sqrt{6}}{4}$

29. *verify each of the following use the sum and difference formulas to verify the identities.*

a. $\sin\left(x - \frac{3\pi}{2}\right) = cosx$

$sinxcos\frac{3\pi}{2} - cosxsin\frac{3\pi}{2} = cosx$

$sinx * 0 - cosx * -1 = cosx$

$0 + cosx = cosx$

$cosx = cosx$

b. $cos3x = 4cos^3x - 3cosx$

$\cos(2x + x) = 4cos^3x - 3cosx$

$cos2xcosx - sin2xsinx = 4cos^3x - 3cosx$

$(2cos^2x - 1) * cosx - (2sinxcosx) * sinx = 4cos^3x - 3cosx$

$2cos^3x - cosx - 2sin^2xcosx = 4cos^3x - 3cosx$

$2cos^3x - cosx - 2(1 - cos^2x) * cosx = 4cos^3x - 3cosx$

$2cos^3x - cosx - 2cosx + 2cos^3x = 4cos^3x - 3cosx$

$\therefore 4cos^3x - 3cosx = 4cos^3x - 3cosx$

C. $\sin(x + y) * \sin(x - y) = sin^2x - sin^2y$

$[sinxcosy + cosxsiny] * [sinxcosy - cosxsiny] = sin^2x - sin^2y$

$sin^2xcos^2y - cos^2xsin^2y = sin^2x - sin^2y$

$sin^2x[1 - sin^2y] - [1 - sin^2x]sin^2y = sin^2x - sin^2y$

$sin^2x - sin^2xsin^2y - sin^2y + sin^2xsin^2y = sin^2x - sin^2y$

$\therefore sin^2x - sin^2y = sin^2x - sin^2y$

d. $\tan(x + \pi) - \tan(\pi - x) = 2tanx$

$\frac{tanx + \tan\pi}{1 - tanx\tan\pi} - \left[\frac{tan\pi - tanx}{1 + tan\pi\tan x}\right] = 2tanx$

$\frac{tanx+0}{1-tanx*0} - \left[\frac{0-tanx}{1+0*tanx}\right] = 2tanx.$

$tanx + tanx = 2tanx$

$\therefore 2tanx = 2tanx$

e. $\sin\left(\frac{\pi}{3} + x\right) = \frac{1}{2}(\sqrt{3}\,cosx + sinx)$

$sin\frac{\pi}{3}cosx + cos\frac{\pi}{3}sinx = \frac{1}{2}(\sqrt{3}\,cosx + sinx)$

$\frac{\sqrt{3}}{2}cosx + \frac{1}{2}sinx = \frac{1}{2}(\sqrt{3}\,cosx + sinx)$

$\therefore \frac{1}{2}(\sqrt{3}\,cosx + sinx) = \frac{1}{2}(\sqrt{3}\,cosx + sinx)$

f. $\cos(A + B) + \cos(A - B) = 2cosAcosB$

$cosAcosB - sinAsinB + cosAcosB + sinAsinB = 2cosAcosB$

$\therefore 2cosAcosB = 2cosAcosB$

30. *evaluate* $\sin(a+b)$, *where a and b are obtuse angles* ;

$sina = \frac{3}{5}$ *and* $sinb = \frac{5}{13}$.

$sin^2 a + cos^2 a = 1$

$cos^2 a = 1 - sin^2 a$

$cos^2 a = 1 - (3/5)^2$

$cos^2 a = 1 - \frac{9}{25}$

$cos^2 a = \frac{16}{25}$

$\cos a = -\frac{4}{5}$

$sin^2 b + cos^2 b = 1$

$cos^2 b = 1 - sin^2 b$

$cos^2 b = 1 - (5/13)^2$

$cos^2 b = 1 - \frac{25}{169}$

$cos^2 b = \frac{144}{169}$

$\cos b = -\frac{12}{13}$

$\therefore \sin(a+b) = sinacosb + cosasinb.$

$= \frac{3}{5} * -\frac{12}{13} + \frac{-4}{5} * \frac{5}{13} = -\frac{36}{65} + \frac{-20}{65} = -\frac{56}{65}$

SECTION: - 1.2

1. **Express as a single sine or cosine function**

a. 10 sinxcosx= 5*2sinxcosx = 5sin2x

b. 5sin2xcos2x=$\frac{5}{2} * 2$ sin2xcos2x = $\frac{5}{2}$ sin4x

c. $Cos^2 \frac{\theta}{2} - sin^2 \frac{\theta}{2}$ = cos 2 ($\frac{\theta}{2}$) = cos θ

d. 1-2$sin^2 \frac{2\theta}{3}$= cos2 ($\frac{2\theta}{3}$)= cos $\frac{4}{3}\theta$

2. **Simplify each expression**

a. $\frac{sin2a}{cosa} = \frac{2sinacosa}{cosa}$ = 2sina

b. 2tanacos^{2}a = 2 $\frac{sina}{cosa}$* cos^{2}a = 2sinacosa = sin2a

c. 2sin^{2}a+cos2a = 2sin^{2}a+ 1-2sin^2 a = 1

3. **Expand using a double angle formula**

a. 3sin4x = 3*2sin2x cos2x = 6sin2xcos2x

b. 1-cos8x = 1-(cos^2 4x-sin^2 4x)

=1- cos^{2}4x+sin^{2}4x

= sin^{2}4x+sin^{2}4x

= 2sin^{2}4x

c. $\text{Cos2x} - \frac{\sin 2x}{\sin x} = \cos 2x - \frac{2\sin x \cos x}{\sin x}$

$=\cos 2x - 2\cos x = 2\cos^2 x - 1 - 2\cos x$

$=2\cos^2 x - 2\cos x - 1$

4. Write each expression in terms of a single trigonometric function

a. $2\sin(0.6)\cos(0.6) = \sin 2(0.6) = \sin(1.2)$

b. $2\sin(3)\cos(3) = \sin 2(3) = \sin(6)$

c. $2\sin(2)\cos(2) = \sin 2(2) = \sin(4)$

d. $\text{Cos}^2(0.45) - \sin^2(0.45) = \cos 2(0.45) = \cos(0.9)$

e. $2\cos^2(5) - 1 = \cos 2(5) = \cos(10)$

f. $1 - 2\sin^2(3) = \cos 2(3) = \cos(6)$

5. Write each of the following expressions in terms of single trigonometric function

a. $2\sin\frac{\pi}{6}\cos\frac{\pi}{6} = \sin 2(\frac{\pi}{6}) = \sin(\frac{\pi}{3})$

b. $\text{Cos}^2\frac{\pi}{10} - \sin^2\frac{\pi}{10} = \cos 2(\frac{\pi}{10}) = \cos(\frac{\pi}{5})$

c. $2\cos^2(0.5) - 1 = \cos 2(0.5) = \cos(1)$

6. If $\sin\theta = \frac{1}{3}$ and θ is in quadrant 1 , evaluate each expression

$\cos^2\theta + \sin^2\theta = 1$

$\cos^2\theta = 1 - \sin^2\theta$

$\cos^2\theta = 1 - (\frac{1}{3})^2$

$\cos^2\theta = 1 - \frac{1}{9}$

$\cos^2\theta = \frac{8}{9}$

$\cos\theta = \pm\frac{\sqrt{8}}{3}$

$\cos\theta = \frac{\sqrt{8}}{3}$

$\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{\frac{1}{3}}{\frac{\sqrt{8}}{3}} = \frac{1}{\sqrt{8}}$

a. $\sin2\theta = 2\sin\theta\cos\theta$

$= 2\,(\frac{1}{3})\,(\frac{\sqrt{8}}{3}) = \frac{2\sqrt{8}}{9}$

b. $\cos2\theta = \cos^2\theta - \sin^2\theta$

$= (\frac{\sqrt{8}}{3})^2 - (\frac{1}{3})^2 = \frac{8}{9} - \frac{1}{9} = \frac{7}{9}$

c. $\tan2\theta = \frac{2\tan\theta}{1-\tan^2\theta} = \frac{2(\frac{1}{\sqrt{8}})}{1-(\frac{1}{\sqrt{8}})2} = \frac{\frac{2}{\sqrt{8}}}{1-\frac{1}{8}}$

$= \frac{\frac{2}{\sqrt{8}}}{\frac{7}{8}} = \frac{2}{\sqrt{8}} * \frac{8}{7} = \frac{8}{\sqrt{8}} * \frac{2}{7} = \sqrt{8} * \frac{2}{7} = \frac{2\sqrt{8}}{7}$

7. A value of θ is defined evaluate the expressions sin2 θ , cos2 θ, tan2 θ

a. Cos θ = $-\frac{1}{2}$ and θ is in Quadrant 2

$\cos^2\theta + \sin^2\theta = 1$

$\sin^2\theta = 1 - \cos^2\theta$

$\sin^2\theta = 1 - (-\frac{1}{2})^2$

$\sin^2\theta = 1 - \frac{1}{4}$

$\sin^2\theta = \frac{3}{4}$

$\sin\theta = \pm\frac{\sqrt{3}}{2}$

$\sin\theta = \frac{\sqrt{3}}{2}$

$\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{\frac{\sqrt{3}}{2}}{-\frac{1}{2}} = -\sqrt{3}$

a. $\sin2\theta = 2\sin\theta\cos\theta$

$= 2\left(\frac{\sqrt{3}}{2}\right)\left(-\frac{1}{2}\right) = -\frac{\sqrt{3}}{2}$

b. $\cos2\theta = \cos^2\theta - \sin^2\theta$

$= (-\frac{1}{2})^2 - (\frac{\sqrt{3}}{2})^2 = \frac{1}{4} - \frac{3}{4} = -\frac{2}{4} = -\frac{1}{2}$

c. $\tan2\theta = \frac{2\tan\theta}{1-\tan^2\theta} = \frac{2(-\sqrt{3})}{1-(\sqrt{3})^2} = \frac{-2\sqrt{3}}{1-3}$

$= \frac{-2\sqrt{3}}{-2} = \sqrt{3}$

b. Sin θ = $-\frac{2}{3}$ and θ is in Quadrant 3

$\cos^2\theta + \sin^2\theta = 1$

$\cos^2\theta = 1 - \sin^2\theta$

$\cos^2\theta = 1 - (-\frac{2}{3})^2$

$\cos^2\theta = 1 - \frac{4}{9}$

$\cos^2\theta = \frac{5}{9}$

a. $\sin2\theta = 2\sin\theta\cos\theta$

$= 2\left(-\frac{2}{3}\right)\left(\frac{\sqrt{5}}{3}\right) = \frac{4\sqrt{5}}{9}$

b. $\cos2\theta = \cos^2\theta - \sin^2\theta$

$= (-\frac{\sqrt{5}}{3})^2 - (\frac{2}{3})^2 = \frac{5}{9} - \frac{4}{9} = \frac{1}{9}$

c. $\tan2\theta = \frac{2\tan\theta}{1-\tan^2\theta} = \frac{2(\frac{2}{\sqrt{5}})}{1-(\frac{2}{\sqrt{5}})2} = \frac{\frac{4}{\sqrt{5}}}{1-\frac{4}{5}}$

$\text{Cos}\theta = \pm\frac{\sqrt{5}}{3}$

$\text{Cos}\theta = \pm\frac{\sqrt{5}}{3}$

$\text{Cos}\theta = -\frac{\sqrt{5}}{3}$

$\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{-\frac{2}{3}}{\frac{\sqrt{5}}{3}} = \frac{2}{\sqrt{5}}$

c. $\tan 2\theta = \frac{2\tan\theta}{1-\tan^2\theta} = \frac{2(\frac{2}{\sqrt{5}})}{1-(\frac{2}{\sqrt{5}})2} = \frac{\frac{4}{\sqrt{5}}}{1-\frac{4}{5}}$

$= \frac{4}{\sqrt{5}} * \frac{5}{1} = \frac{5}{\sqrt{5}} * \frac{4}{1} = \sqrt{5} * 4 = 4\sqrt{5}$

c. Tan θ= 0.75 = $\frac{3}{4}$ and $\pi < \theta \frac{3\pi}{2}$

$1+\tan^2\theta = \sec^2\theta$

$1+(0.75)^2 = \sec^2\theta$

$1+(\frac{3}{4})^2 = \sec^2\theta$

$1+\frac{9}{16} = \sec^2\theta$

$\frac{25}{16} = \sec^2\theta$

$\text{Sec}\,\theta = -\frac{5}{4}$

$\text{Cos}\,\theta = -\frac{4}{5}$

$\cos^2\theta + \sin^2\theta = 1$

$\sin^2\theta = 1-\cos^2\theta$

$\sin^2\theta = 1-(\frac{-4}{5})^2$

$\sin^2\theta = 1-\frac{16}{25}$

$\sin^2\theta = \frac{9}{25}$

$\sin\theta = \pm\frac{3}{5} = -\frac{3}{5}$

a. $\sin 2\theta = 2\sin\theta\cos\theta$

$= 2(-\frac{3}{5})(-\frac{4}{5}) = \frac{24}{25}$

b. $\cos 2\theta = \cos^2\theta - \sin^2\theta$

$= (-\frac{4}{5})^2 - (-\frac{3}{5})^2 = \frac{16}{25} - \frac{9}{25} = \frac{7}{25}$

c. $\tan 2\theta = \frac{2\tan\theta}{1-\tan^2\theta} = \frac{2(\frac{3}{4})}{1-(\frac{3}{4})^2} = \frac{\frac{3}{2}}{1-\frac{9}{16}}$

$= \frac{\frac{3}{2}}{\frac{7}{16}} = \frac{3}{2} * \frac{16}{7} = \frac{24}{7}$

8. Prove each Identity

a. $1+\sin 2\theta = (\sin\theta + \cos\theta)^2$

$1+\sin 2\theta = \sin^2\theta + 2\sin\theta\cos\theta + \cos^2\theta$

$1+\sin 2\theta = 1+\sin 2\theta$

b. $\sin 2\theta = 2\cot\theta\sin^2\theta$

$\sin 2\theta = 2 * \frac{\cos\theta}{\sin\theta} * \sin^2\theta$

$\sin 2\theta = 2\cos\theta\sin\theta$

$\sin 2\theta = \sin 2\theta$

c. $\cos 2\theta = \frac{1-\tan^2\theta}{1+\tan^2\theta}$

$\cos 2\theta = \frac{1-\frac{\sin^2\theta}{\cos^2\theta}}{1+\frac{\sin^2\theta}{\cos^2\theta}}$,

$\cos 2\theta = \frac{\frac{\cos^2\theta-\sin^2\theta}{\cos^2\theta}}{\frac{\cos^2\theta+\sin^2\theta}{\cos^2\theta}}$

$\cos 2\theta = \frac{\frac{\cos 2\theta}{\cos^2\theta}}{\frac{1}{\cos^2\theta}}$

$\cos 2\theta = \cos 2\theta$

d. $\sec^2\theta = \frac{2}{1+\cos 2\theta}$

$\sec^2\theta = \frac{2}{1+\cos^2\theta-\sin^2\theta}$

$\sec^2\theta = \frac{2}{\cos^2\theta+\cos^2\theta}$

$\sec^2\theta = \frac{2}{2\cos^2\theta}$

$\sec^2\theta = \frac{1}{\cos^2\theta}$

$\sec^2\theta = \sec^2\theta$

9. Prove each identity

a. $\frac{1-\cos 2\theta}{2} = \sin^2\theta$

$\frac{1-(\cos^2\theta-\sin^2\theta)}{2} = \sin^2\theta$

$\frac{1-\cos^2\theta+\sin^2\theta}{2} = \sin^2\theta$

$\frac{\sin^2\theta+\sin^2\theta}{2} = \sin^2\theta$

$\frac{2\sin^2\theta}{2} = \sin^2\theta$

$\sin^2\theta = \sin^2\theta$

b. $\frac{\cos^2\theta+\sin^2\theta}{\sin^2\theta-\cos^2\theta} = -\sec 2\theta$

$\frac{\cos^2\theta+\sin^2\theta}{-(\cos^2\theta-\sin^2\theta)} = -\sec 2\theta$

$-\frac{1}{\cos 2\theta} = -\sec 2\theta$

$-\sec 2\theta = -\sec 2\theta$

C. $\frac{(\sin\theta+\cos\theta)^2}{\sin 2\theta} = \csc\theta+1$

$\frac{\sin^2\theta+2\sin\theta\cos\theta+\cos^2\theta}{\sin 2\theta} = \csc 2\theta+1$

$\frac{1+\sin 2\theta}{\sin 2\theta} = \csc 2\theta+1$

$\frac{1}{\sin 2\theta} + \frac{\sin 2\theta}{\sin 2\theta} = \csc 2\theta+1$

$\csc 2\theta+1 = \csc 2\theta+1$

Section 1.3

Part one: - use half angle identities to find each value

1. Tan22.5 = $\tan\frac{45}{2} = \frac{1-cos45}{sin45} = \frac{1-\frac{\sqrt{2}}{2}}{\frac{\sqrt{2}}{2}} = \frac{\frac{2-\sqrt{2}}{2}}{\frac{\sqrt{2}}{2}} = \frac{2-\sqrt{2}}{\sqrt{2}} = \frac{2}{\sqrt{2}} - \frac{\sqrt{2}}{\sqrt{2}} = \sqrt{2}$ -1
2. Cos $\frac{5\pi}{8}$ = cos $\frac{\frac{5\pi}{4}}{2}$ = - $\sqrt{\frac{1+cos\frac{5\pi}{4}}{2}}$ = - $\sqrt{\frac{1-\frac{\sqrt{2}}{2}}{2}}$ = - $\sqrt{\frac{\frac{2-\sqrt{2}}{2}}{2}}$ = - $\sqrt{\frac{2-\sqrt{2}}{4}}$ = - $\frac{\sqrt{2-\sqrt{2}}}{2}$
3. Sin $\frac{19\pi}{12}$ = sin $\frac{\frac{19\pi}{6}}{2}$ = - $\sqrt{\frac{1-cos\frac{19\pi}{6}}{2}}$ = - $\sqrt{\frac{1+\frac{\sqrt{3}}{2}}{2}}$ = - $\sqrt{\frac{\frac{2+\sqrt{3}}{2}}{2}}$ = - $\sqrt{\frac{2+\sqrt{3}}{4}}$ = - $\frac{\sqrt{2+\sqrt{3}}}{2}$
4. Cos67.5=cos$\frac{135}{2}$= $\sqrt{\frac{1+cos135}{2}}$ = $\sqrt{\frac{1+(-\frac{\sqrt{2}}{2})}{2}}$ = $\sqrt{\frac{\frac{2-\sqrt{2}}{2}}{2}}$ = $\sqrt{\frac{2-\sqrt{2}}{4}}$ = $\frac{\sqrt{2-\sqrt{2}}}{2}$

Part two:-Given sinA= $\frac{4}{5}$, $\frac{\pi}{2} < A < \pi$ tanB= $\frac{-12}{5}$ $\frac{3\pi}{2} < B < 2\pi$ find each of the following.

$Sin^2A+cos^2A = 1$	$\tan B = -\frac{12}{5}$	$sin^2B+cos^2B = 1$
$Cos^2A = 1-sin^2A$	$1+\tan^2B = sec^2B$	$sin^2B = 1-cos^2 B$
$Cos^2A = 1-(\frac{4}{5})^2$	$1+(-\frac{12}{5})^2 = sec^2B$	$sin^2B = 1-(\frac{5}{13})^2$
$Cos^2A = 1-\frac{16}{25}$	$1+\frac{144}{25} = sec^2B$	$sin^2B = 1-\frac{25}{169}$
$Cos^2A = \frac{9}{25}$	$Sec\ B = \frac{13}{5}$	$sin^2B = \frac{144}{169}$
$Cos\ A = -\frac{3}{5}$	$Cos\ B = \frac{5}{13}$	$sin\ B = -\frac{12}{13}$

a. $\text{Sin}(\frac{1}{2}A) = \sqrt{\frac{1-cosA}{2}} = \sqrt{\frac{1-(-\frac{3}{5})}{2}} = \sqrt{\frac{1+\frac{3}{5}}{2}} = \sqrt{\frac{\frac{8}{5}}{2}} = \sqrt{\frac{8}{10}} = \frac{2}{\sqrt{5}}$

b. $\text{Cos}(\frac{1}{2}A) = \sqrt{\frac{1+cosA}{2}} = \sqrt{\frac{1+(-\frac{3}{5})}{2}} = \sqrt{\frac{1-\frac{3}{5}}{2}} = \sqrt{\frac{\frac{2}{5}}{2}} = \sqrt{\frac{2}{10}} = \frac{1}{\sqrt{5}}$

c. $\text{Sin}(\frac{1}{2}B) = \sqrt{\frac{1-cosB}{2}} = \sqrt{\frac{1-\frac{5}{13}}{2}} = \sqrt{\frac{\frac{8}{13}}{26}} = \sqrt{\frac{8}{26}} = \frac{2}{\sqrt{13}}$

d. $\text{Sec}(\frac{1}{2}B) = \frac{1}{cos\frac{1}{2}B} = \frac{1}{\sqrt{\frac{1+cosB}{2}}} = \frac{1}{-\sqrt{\frac{1+\frac{5}{13}}{2}}} = \frac{1}{\sqrt{\frac{\frac{18}{13}}{2}}} = \frac{1}{-\sqrt{\frac{18}{26}}} = \frac{1}{-\sqrt{\frac{9}{13}}} = \frac{1}{-\frac{3}{\sqrt{13}}} = -\frac{\sqrt{13}}{3}$

Part three: - If sin A= $\frac{24}{25}$ and < A is a positive acute angle

$\text{Sin}^2A + \cos^2A = 1$

$\text{Cos}^2A = 1-\text{sin}^2A$

$\text{Cos}^2A = 1-(\frac{24}{25})^2$

$\text{Cos}^2A = 1-\frac{576}{625}$

$\text{Cos}^2A = \frac{49}{625}$

$\text{Cos } A = \frac{7}{25}$

A) $\text{sin}\frac{1}{2}A = \sqrt{\frac{1-cosA}{2}} = \sqrt{\frac{1-\frac{7}{25}}{2}} = \sqrt{\frac{18}{50}} = \sqrt{\frac{9}{25}} = \frac{3}{5}$

B) $\cos\frac{1}{2}A = \sqrt{\frac{1+cosA}{2}} = \sqrt{\frac{1+\frac{7}{25}}{2}} = \sqrt{\frac{32}{50}} = \sqrt{\frac{16}{25}} = \frac{4}{5}$

c) $\tan\frac{1}{2}A = \frac{1-cosA}{sinA} = \frac{1-\frac{7}{25}}{\frac{24}{25}} = \frac{\frac{18}{25}}{\frac{24}{25}} = \frac{18}{24} = \frac{3}{4}$

Part four: - if cos B = $\frac{5}{9}$ and $\pi \leq B \leq \frac{3\pi}{2}$ find

$Sin^2B + cos^2B = 1$

$sin^2B = 1-cos^2B$

$sin^2B = 1-(\frac{5}{9})^2$

$sin^2B = 1-\frac{25}{81}$

$sin^2B = \frac{56}{81}$

$Sin\ B = -\frac{2\sqrt{14}}{9}$

A) $sin\frac{1}{2}B = \sqrt{\frac{1-cosB}{2}} = \sqrt{\frac{1-\frac{5}{9}}{2}} = \sqrt{\frac{\frac{4}{9}}{2}} = \sqrt{\frac{4}{18}} = \frac{\sqrt{2}}{3}$

B) $cos\frac{1}{2}B = -\sqrt{\frac{1+cosB}{2}} = \sqrt{\frac{1+\frac{5}{9}}{2}} = -\sqrt{\frac{14}{18}} = -\frac{\sqrt{7}}{3}$

c) $tan\frac{1}{2}B = \frac{1-cosB}{sinB} = \frac{1-\frac{5}{9}}{-\frac{2\sqrt{14}}{9}} = \frac{\frac{4}{9}}{-\frac{2\sqrt{14}}{9}} = -\frac{2}{\sqrt{14}}$

part five: - find the exact value without a calculator using double angle and half identities

a. $Sin22.5^0 = sin\frac{45}{2} = \sqrt{\frac{1-cos45}{2}} = \sqrt{\frac{1-\frac{\sqrt{2}}{2}}{2}} = \sqrt{\frac{2-\sqrt{2}}{4}} = \frac{\sqrt{2-\sqrt{2}}}{2}$

b. $Tan75^0 = tan\frac{150}{2} = \frac{1-cos150}{sin150} = \frac{1+\frac{\sqrt{3}}{2}}{\frac{1}{2}} = \frac{\frac{2+\sqrt{3}}{2}}{\frac{1}{2}} = 2+\sqrt{3}$

c. $Cos67.5^0 = cos\frac{135}{2} = \sqrt{\frac{1+cos135}{2}} = \sqrt{\frac{1+(-\frac{\sqrt{2}}{2})}{2}} = \sqrt{\frac{2-\sqrt{2}}{4}} = \frac{\sqrt{2-\sqrt{2}}}{2}$

d. $Tan15^0 = tan\frac{30}{2} = \frac{1-cos30}{sin30} = \frac{1-\frac{\sqrt{3}}{2}}{\frac{1}{2}} = \frac{\frac{2-\sqrt{3}}{2}}{\frac{1}{2}} = 2-\sqrt{3}$

Part six: - verify the identities

1. $(Sinx+cosx)^2 = 1+sin2x$

   $Sin^2x + cos^2x + 2sinxcosx = 1+sin2x$

   $1+sin2x = 1+sin2x$

2. $\text{Sin2x} = (\text{tanx})\ (1+\text{cos2x})$

   $\text{Sin2x} = \frac{\text{sinx}}{\text{cosx}}\ (1+\text{cos}^2\text{x}-\text{sin}^2\text{x})$

   $\text{Sin2x} = \frac{\text{sinx}}{\text{cosx}}\ (2\text{cos}^2\text{x})$

   $\text{Sin2x} = \text{sinx}*2\ \text{cosx}$

   $\text{Sin2x} = 2\text{sinxcosx}$

   $\text{Sin2x} = \text{sin2x}$

3. $\text{Sin}^2\text{x} = \frac{1}{2}\ (1-\text{cos2x})$

   $\text{Sin}^2\text{x} = \frac{1}{2} - \frac{1}{2}\ \text{cos2x}$

   $\text{Sin}^2\text{x} = \frac{1}{2} - \frac{1}{2}\ (1-2\text{sin}^2\text{x})$

   $\text{Sin}^2\text{x} = \frac{1}{2} - \frac{1}{2} + \text{sin}^2\text{x}$

   $\text{Sin}^2\text{x} = \text{sin}^2\text{x}$

4. $\text{Cos}^2\text{x} = \frac{1}{2}\ (\text{cos2x}+1)$

   $\text{Cos}^2\text{x} = \frac{1}{2}\ [\text{cos}^2\text{x} - \text{sin}^2\text{x} + 1]$

   $\text{Cos}^2\ \text{x} = \frac{1}{2}\ [\text{cos}^2\ \text{x}-(1-\text{cos}^2\text{x}) + 1]$

   $\text{Cos}^2\text{x} = \frac{1}{2}\ [2\text{cos}^2\text{x}]$

   $\text{Cos}^2\text{x} = \text{cos}^2\text{x}$

5. $1-\ \text{cos2x}\ = \text{tanxsin2x}$

   $1-(1-2\text{sin}^2\text{x}) = \frac{\text{sinx}}{\text{cosx}}*2\text{sinx cosx}$

   $1-1+2\text{sin}^2\text{x} = 2\text{sin}^2\text{x}$

   $2\text{sin}^2\text{x} = 2\text{sin}^2\text{x}$

6. $1+\text{Sin2t}\ = (\text{sint} + \text{cost})^2$

   $1+\text{sin2t} = \text{sin}^2\text{t} + \text{cos}^2\text{t} + 2\text{sintcost}$

   $1+\text{sin2t} = 1 + 2\text{sintcost}$

   $1 + \text{sin2t} = 1 + \text{sin2t}$

Section 1.4

Part one: - Express each sum or difference as a product.

1. $\sin 6x + \sin 2x = 2\sin\frac{6x+2x}{2}\cos\frac{6x-2x}{2}$
   $= 2\sin 4x\cos 2x$
2. $\sin 8x + \sin 2x = 2\sin\frac{8x+2x}{2}\cos\frac{8x-2x}{2}$
   $= 2\sin 5x\cos 3x$
3. $\sin 7x - \sin 3x = 2\cos\frac{7x+3x}{2}\sin\frac{7x-3x}{2}$
   $= 2\cos 5x\sin 2x$
4. $\sin 11x - \sin 5x = 2\cos\frac{11x+5x}{2}\sin\frac{11x-5x}{2}$
   $= 2\cos 8x\sin 3x$
5. $\cos 4x + \cos 2x = 2\cos\frac{4x+2x}{2}\cos\frac{4x-2x}{2}$
   $= 2\cos 3x\cos x$
6. $\cos 9x - \cos 7x = -2\sin\frac{9x+7x}{2}\sin\frac{9x-7x}{2}$
   $= -2\sin 8x\sin x$
7. $\sin x + \sin 2x = 2\sin\frac{x+2x}{2}\cos\frac{x-2x}{2}$
   $= 2\sin\frac{3x}{2}\cos\frac{-x}{2} = 2\sin\frac{3}{2}x\cos\frac{x}{2}$
8. $\sin x - \sin 2x = 2\cos\frac{x+2x}{2}\sin\frac{x-2x}{2}$
   $= 2\cos\frac{3}{2}x\sin -\frac{x}{2}$
   $= -2\cos\frac{3}{2}x\sin\frac{x}{2}$
9. $\cos\frac{3x}{2} + \cos\frac{x}{2} = 2\cos\frac{\frac{3x}{2}+\frac{x}{2}}{2}\cos\frac{\frac{3x}{2}-\frac{x}{2}}{2}$
   $= 2\cos\frac{2x}{2}\cos\frac{x}{2} = 2\cos x\cos\frac{x}{2}$

10. $\text{Sin}75^0 + \sin 15^0 = 2\sin\frac{75+15}{2}\cos\frac{75-15}{2}$

$= 2\sin 45 \cos 30$

$= 2\left(\frac{\sqrt{2}}{2}\right)\left(\frac{\sqrt{3}}{2}\right) = \frac{\sqrt{6}}{2}$

11. $\text{Cos}75^0 - \sin 15^0 = -2\sin\frac{75+15}{2}\sin\frac{75-15}{2}$

$= -2\sin 45 \sin 30$

$= -2\left(\frac{\sqrt{2}}{2}\right)\left(\frac{1}{2}\right)$

$= -\frac{\sqrt{2}}{2}$

12. $\text{Sin}\frac{\pi}{12} - \sin\frac{5\pi}{12} = 2\cos\frac{\frac{\pi}{12}+\frac{5\pi}{12}}{2}\sin\frac{\frac{\pi}{12}-\frac{5\pi}{12}}{2}$

$= -2\cos\frac{\pi}{4}\sin\frac{\pi}{6}$

$= 2\left(\frac{\sqrt{2}}{2}\right)\left(\frac{1}{2}\right) = -\frac{\sqrt{2}}{2}$

Part two: - verify each identity.

1. $\frac{\sin 3x - \sin x}{\cos 3x - \cos x} = \text{-cot}2x$
2. $\frac{2\cos\frac{3x+x}{2}\sin\frac{3x-x}{2}}{-2\sin\frac{3x+x}{2}\sin\frac{3x-x}{2}} = \text{- cot}2x$

$= \frac{2\cos 2x \sin x}{-2\sin 2x \sin x} = \text{- cot}2x$

$-\cot 2x = -\cot 2x$

3. $\frac{\sin x+\sin 3x}{\cos x+\cos 3x}=\tan 2x$

$$\frac{2\sin\frac{x+3x}{2}\cos\frac{x-3x}{2}}{2\cos\frac{x+3x}{2}\cos\frac{x-3x}{2}}=\tan 2x$$

$$\frac{2\sin 2x\cos x}{2\cos 2x\cos x}=\tan 2x$$

$$\tan 2x=\tan 2x$$

4. $\frac{\sin 2x+\sin 4x}{\cos 2x+\cos 4x}=\tan 3x$

$$\frac{2\sin\frac{2x+4x}{2}\cos\frac{2x-4x}{2}}{2\cos\frac{2x+4x}{2}\cos\frac{2x-4x}{2}}=\tan 3x$$

$$\frac{2\sin 3x\cos x}{2\cos 3x\cos x}=\tan 3x$$

$$\tan 3x=\tan 3x$$

4 . $\frac{\cos 4x-\cos 2x}{\sin 2x-\sin 4x}=\tan 3x$

$$\frac{-2\sin\frac{4x+2x}{2}\sin\frac{4x-2x}{2}}{2\cos\frac{4x+2x}{2}\sin\frac{2x-4x}{2}}=\tan 3x$$

$$\frac{-2\sin 3x\sin x}{-2\cos 3x\sin x}=\tan 3x$$

$\frac{sin3x}{cos3x} = tan3x$

$tan3x = tan3x$

5. $\frac{\sin x - \sin y}{\sin x + \sin y} = tan\frac{x-y}{2}\, cot\frac{x+y}{2}$

$\frac{2cos\frac{x+y}{2}\, sin\frac{x-y}{2}}{2sin\frac{x+y}{2}cos\frac{x-y}{2}} = tan\frac{x-y}{2}\ cot\frac{x+y}{2}$

$tan\frac{x-y}{2}\ cot\frac{x+y}{2} = tan\frac{x-y}{2}\ cot\frac{x+y}{2}$

6. $\frac{sinx+siny}{sinx-siny} = tan\frac{x+y}{2}\ cot\frac{x-y}{2}$

$\frac{2sin\frac{x+y}{2}\, cos\frac{x-y}{2}}{2cos\frac{x+y}{2}sin\frac{x-y}{2}} = tan\frac{x+y}{2}\ cot\frac{x-y}{2}$

$tan\frac{x+y}{2}\ cot\frac{x-y}{2} = tan\frac{x+y}{2}\ cot\frac{x-y}{2}$

7. $\frac{sinx+siny}{cosx+cosy} = tan\frac{x+y}{2}$

$\frac{2sin\frac{x+y}{2}\, cos\frac{x-2}{2}}{2cos\frac{x+y}{2}cos\frac{x-y}{2}} = tan\frac{x+y}{2}$

$tan\frac{x+y}{2} = tan\frac{x+y}{2}$

8. $\frac{sinx-siny}{cosx-cosy} = -\, cot\frac{x+y}{2}$

$= \frac{2cos\frac{x+y}{2}\, sin\frac{x-y}{2}}{-2sin\frac{x+y}{2}sin\frac{x-y}{2}} = -\, cot\frac{x+y}{2}$

$-\, Cot\frac{x+y}{2} = -\, cot\frac{x+y}{2}$

Section 1.5

Part one: - find the exact value of $\cos 165 - \cos 75$ *using an appropriate sum – product identity*

$\cos 165 - \cos 75 = -2\sin\frac{165+75}{2}\ sin\ \frac{165-75}{2} = -2\sin 120 \sin 45$

$= -2\left(\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{2}}{2}\right) = -\frac{\sqrt{6}}{2}$

Part two: - write each product as sum or difference involving sine and cosine

1. $\sin 3m \cos 3m = \frac{1}{2}[\sin(3m + m) + \sin(3m - m)] = \frac{1}{2}[\sin 4m + \sin 2m]$
2. $\cos 7A \cos 5A = \frac{1}{2}[\cos(7A + 5A) + \cos(7A - 5A) = \frac{1}{2}[cos12A + cos2A]$
3. $\sin B \sin 3B = \frac{1}{2}[\cos(B - 3B) - \cos(B + 3B) = \frac{1}{2}[cos2B - cos4B]$
4. $\cos 2C \sin 3C = \frac{1}{2}[\sin(2C + 3C) - \sin(2C - 3C) = \frac{1}{2}[sin5C + sinC]$

Part three:- write each difference or sum as product involving sine and cosine

5. $\sin 3t + \sin t = 2\sin\frac{3t+t}{2}\cos\frac{3t-t}{2} = 2\sin 2t \cos t$
6. $\cos 7U + \cos 5U = 2cos\frac{7U+5U}{2}\cos\frac{7U-5U}{2} = 2\cos 6U \cos U$
7. $\cos 5W - \cos 9W = -2\sin\frac{5W+9W}{2}\sin\frac{5W-9W}{2} = 2\sin 7W \sin 2W$
8. $\sin u - \sin 5u = 2\cos\frac{u+5u}{2}\sin\frac{u-5u}{2} = -2\cos 3u \sin 2u$

Part four: - evaluate exactly using an appropriate identity

9. $\cos 285 + \cos 195 = 2\cos\frac{285+195}{2}\cos\frac{285-195}{2} = 2\cos 240 \cos 45 = 2\left(-\frac{1}{2}\right)\left(\frac{\sqrt{2}}{2}\right) = -\frac{\sqrt{2}}{2}$
10. $\sin 195 + \sin 105 = 2cos\frac{195+105}{2}sin\frac{195-105}{2} = 2cos150sin45 = 2\left(-\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{2}}{2}\right) = -\frac{\sqrt{6}}{2}$
11. $Cos\ 15 - cos\ 105 = -2sin\frac{15+105}{2}sin\frac{15-105}{2} = 2sin60sin45 = 2\left(\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{2}}{2}\right) = \frac{\sqrt{6}}{2}$
12. $sin75 * 2\sin 165 = 2 * \frac{1}{2}[\cos(75 - 165) - \cos(75 + 165)] = \cos 90 - \cos 240$
    $= 0 + \frac{1}{2} = \frac{1}{2}$

Section 1.6

Solving trigonometric equations

1. $\text{Sin}\frac{1}{2}\,x = -1$

$\text{Sin}\frac{1}{2}\,x = \sin 270$

$\frac{1}{2}\,x = 270$

$x = 540\ or\ 3\pi$

2. No result

3. $sin2xcosx = sinx$

$2sinxcosxcosx = sinx$

$2sinxcos^2x = sinx$

$2sinxcos^2x - sinx = 0$

$sinx(2cos^2x - 1) = 0$

$sinx = 0$ $\qquad (2cos^2x - 1) = 0$ $\qquad 2cos^2x = 1$ $\qquad cos^2x = \frac{1}{2}$

$x = 0, \pi$ $\qquad cosx = \pm\frac{1}{\sqrt{2}}$ $\qquad x = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$

$x = 0,180$ $\qquad x = 45, 135, 225, 315$

4. $cos2x = -2cos^2x$

$2cos^2x - 1 = -2cos^2x$

$2cos^2x - 1 + 2cos^2x = 0$

$4cos^2x = 1$ $\qquad cos^2x = \frac{1}{4}$ $\qquad cosx = \pm\frac{1}{2}$ $\qquad x = \frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}$

$x = 60, 120, 240, 300$

5. $tan2x - tanx = 0$

$\frac{2tanx}{1-tan^2x} - tanx = 0$

$2tanx - tanx + tan^3x = 0$

$tan^3x + tanx = 0$

$tanx(tan^2x + 1) = 0$ $\quad tanx = 0$ $\quad x = 0, \pi \; or \; 0,180$ $\quad tan^2x + 1 = 0$ $\quad tan^2x = -1$ $\quad tanx = \pm\sqrt{-1}$ $\quad x = No\ real\ solution$

6. $sin\frac{x}{2} + cosx = 1$

$sin\frac{x}{2} = 1 - cosx$

$\left(\sqrt{\frac{1-cosx}{2}}\right)^2 = (1 - cosx)^2$

$\frac{1-cosx}{2} = 1 - 2cosx + cos^2x$

$1 - cosx = 2 - 4cosx + 2cos^2x$

$2cos^2x - 4cosx + cosx + 2 - 1 = 0$

$2cos^2x - 3cosx + 1 = 0$

$2cos^2x - 2cosx - cosx + 1 = 0$

$2cosx(cosx - 1) - 1(cosx - 1) = 0$

$(2cosx - 1)(cosx - 1) = 0$

$(cosx - 1) = 0$ $\quad cosx = 1$ $\quad x = 0\,, 2\pi \; or \; 0\,, 360$ $\quad 2cosx - 1 = 0$ $\quad 2cosx = 1$ $\quad cosx = -\frac{1}{2}$ $\quad x = \frac{\pi}{3}\,, \frac{5\pi}{3} \; or \; 60,300$

7. No result

8. $tan\frac{x}{2} = sinx$

$\frac{1-cosx}{sinx} = sinx$ $1 - cosx = sin^2x$ $1 - cosx - sin^2x = 0$

$1 - cosx - (1 - cos^2x) = 0$

$1 - cosx - 1 + cos^2x = 0$

$cos^2x - cosx = 0$

$cosx(cosx - 1) = 0$

$cosx = 0$ $cosx - 1 = 0$

$x = \frac{\pi}{2}, \frac{3\pi}{2}$ or 90,270 $cosx = 1$ $x = 0,2\pi$ or 0,360

9. $\frac{cos2x}{cos^2x} = 1$

$\frac{2cos^2x-1}{cos^2x} = 1$

$2cos^2x - 1 = cos^2x$

$2cos^2x - cos^2x - 1 = 0$

$cos^2x - 1 = 0$

$cos^2x = 1$

$\sqrt{cos^2x} = \sqrt{1}$

$cosx = \pm 1$ $x = 0, \pi, 2\pi$ or 0,180,360

finding general solution , find the general solution of the equations

10. $cos2x = -1$

$cos2x = cos180$

$2x = 180$

$x = 90 + 2n\pi$ $or\ x = \frac{\pi}{2} + 2n\pi$

11. $sin2x + sinx = 0$

$2sinxcosx + sinx = 0$

$sinx(2cosx + 1) = 0$

$sinx = 0$ $2cosx + 1 = 0$

$x = 0 + 2n\pi$ $2cosx = -1$

$x = \pi + 2n\pi$ $cosx = -\frac{1}{2}$ $x = \frac{2\pi}{3} + 2n\pi$ $x = \frac{4\pi}{3} + 2n\pi$

12. $cos2x - cosx = 0$

$2cos^2x - 1 - cosx = 0$

$2cos^2x - cosx - 1 = 0$

$2cos^2x - 2cosx + cosx - 1 = 0$

$2cosx(cosx - 1) + 1(cosx - 1) = 0$

$(2cosx + 1)(cosx - 1) = 0$

$2cosx + 1 = 0$ $cosx - 1 = 0$

$2cosx = -1$ $cosx = 1$

$Cosx = -\frac{1}{2}$ $x = 0 + 2n\pi$

$x = \frac{2\pi}{3} + 2n\pi$ $x = 2\pi + 2n\pi$

$x = \frac{4\pi}{3} + 2n\pi$

13. $cos\frac{x}{2} - sinx = 0$

$\left(\sqrt{\frac{1+cosx}{2}}\right)^2 = (sinx)^2$

$\frac{1+cosx}{2} = sin^2x$

$1 + cosx = 2sin^2x$

$1 + cosx\text{-}2sin^2x = 0$

$1 - cosx - 2(1 - cos^2x) = 0$

$1 + cosx - 2 + 2cos^2x = 0$

$2cos^2x + cosx - 1 = 0$

$2cos^2x + 2cosx - cosx - 1 = 0$

$2cosx(cosx + 1) - 1(cosx + 1) = 0$

$(2cosx - 1)(cosx + 1) = 0$

$2cosx - 1 = 0$ $\qquad$ $cosx + 1 = 0$

$2cosx = 1$ $\qquad$ $cosx = -1$

$Cosx = \frac{1}{2}$ $\qquad$ $x = 0 + 2n\pi$

$x = \frac{2\pi}{3} + 2n\pi$ $\qquad$ $x = 2\pi + 2n\pi$

$x = \frac{4\pi}{3} + 2n\pi$

14. *No result*

15. $cos2x = 3sinx + 2$

$1 - 2sin^2x = 3sinx + 2$

$2sin^2x + 3sinx + 1 = 0$

$2sin^2x + 2sinx + sinx + 1 = 0$

$2sinx(sinx + 1) + 1(sinx + 1) = 0$

$(2sinx + 1)(sinx + 1) = 0$

$2sinx + 1 = 0$ $\qquad$ $sinx + 1 = 0$

$2sinx = -1$ $\qquad$ $sinx = -1$

$Sinx = -\frac{1}{2}$ $\qquad$ $x = \frac{3\pi}{2} + 2n\pi$

$X = \frac{7\pi}{6} + 2n\pi$

$X = \frac{11\pi}{6} + 2n\pi$

Section 1.7

1) $p = 180$ $\quad L = a$ $\quad A = ?$

$P = a + a + a$ $\quad S = \frac{a+a+a}{2} = \frac{60+60+60}{2} = \frac{180}{2} = 90$

$P = 3a$

$180 = 3a$ $\quad A = \sqrt{s(s-a)(s-a)(s-a)}$

$a = 60$

$A = \sqrt{90(90-60)(90-60)(90-60)}$

$A = \sqrt{90 \times 30 \times 30 \times 30} = 1558.8cm^2$

2) $a = 122m$ $\quad b = 22m$ $\quad c = 120m$

$S = \frac{122+22+120}{2} = \frac{264}{2} = 132m$

$Area\ of\ side\ walls = \sqrt{132(132-122)(132-22)(132-120)}$

$A = \sqrt{132 \times 10 \times 110 \times 12} = 1320m^2$

$if\ 1m^2 \rightarrow 5000\ per\ year$

$1320m^2 \rightarrow X$

$X = 6600000\ per\ year$

$if\ the\ company\ pays\ 6600000\ for\ 12\ months, how\ much\ will\ it\ pay\ for\ 3\ months$

$6600000 \rightarrow 12\ months$

$X \rightarrow 3\ months$

$X = 1650000$

3) *sides of painted in colour are* $a = 15m \quad b = 11m \quad c = 6m$

Then Area painted in colour is $A = \sqrt{s(s-a)(s-b)(s-c)}$

$S = \dfrac{a+b+c}{2} = \dfrac{15+11+6}{2} = 16m$

$A = \sqrt{16(16-15)(16-11)(16-6)} = \sqrt{16 \times 1 \times 5 \times 10} = \sqrt{800} = 28.3m^2$

4) $a = 18cm \qquad b = 10cm \qquad p = 42cm$

$P = a + b + c$

$42 = 18 + 10 + c$

$C = 42 - 28$

$C = 14cm$

$s = \dfrac{a+b+c}{2} = \dfrac{18+10+14}{2} = 21$

$A = \sqrt{21(21-18)(21-10)(21-14)}$

$A = \sqrt{21 \times 3 \times 11 \times 7}$

$A = 69.6cm^2$

5) *sides of triangle in ratio* $12:17:25 \qquad p = 540cm \qquad A = ?$

$P = a + b + c$

$540 = 12x + 17x + 25x$

$540 = 54x$

$X = 10cm$

$a = 12x = 12(10) = 120$

$b = 17x = 17(10) = 170$

$c = 25x = 25(10) = 250$

$s = \dfrac{a+b+c}{2} = \dfrac{120+170+250}{2} = 270cm$

$A = \sqrt{270(270-120)(270-170)(270-250)}$

$A = \sqrt{270 \times 150 \times 100 \times 20} = 9000cm^2$

6. $p = 30cm$ $a = 12cm$ $b = 12cm$ $A = ?$

$P = a + b + c$ $A = \sqrt{15(15-12)(15-12)(15-6)} = \sqrt{15*3*3*9}$

$30 = 12 + 12 + c$ $A = 34.9cm^2$

$C = 30 - 24 = 6cm$

$S = \frac{a+b+c}{2} = \frac{12+12+6}{2} = 15cm$

7. $a)$ $x = 6in$ $y = 4in$ $z = 7in$

$S = \frac{x+y+z}{2} = \frac{6+4+7}{2} = 8.5in$

$A = \sqrt{s(s-x)(s-y)(s-z)}$

$A = \sqrt{8.5(8.5-6)(8.5-4)(8.5-7)} = \sqrt{8.5*2.5*4.5*1.5} = 11.9in^2 \approx 12in^2$

$b)$ $d = 16ft$ $e = 14ft$ $f = 6ft$

$S = \frac{d+e+f}{2} = \frac{16+14+6}{2} = 18ft$

$A = \sqrt{s(s-d)(s-e)(s-f)}$

$A = \sqrt{18(18-16)(18-14)(18-6)} = \sqrt{18*2*4*12}$

$A = 41.6ft^2$

$c)$ $d = 14km$ $e = 16km$ $f = 11km$

$S = \frac{d+e+f}{2} = \frac{14+16+11}{2} = 20.5km$

$A = \sqrt{s(s-d)(s-e)(s-f)}$

$A = \sqrt{20.5(20.5-14)(20.5-16)(20.5-11)} = \sqrt{20.5*6.5*4.5*9.5} = 75.5km^2$

$d)$ $a = 12mi$ $b = 16mi$ $c = 9mi$

$$S = \frac{a+b+c}{2} = \frac{12+16+9}{2} = 18.5mi$$

$$A = \sqrt{s(s-a)(s-b)(s-c)}$$

$$A = \sqrt{18.5(18.5-12)(18.5-16)(18.5-9)} = \sqrt{18.5 * 6.5 * 2.5 * 9.5}$$

$$A = 53.4mi^2$$

$e)$ $d = 12ft$ $e = 11.9ft$ $f = 14ft$

$$S = \frac{d+e+f}{2} = \frac{12+11.9+14}{2} = 18.95ft$$

$$A = \sqrt{s(s-d)(s-e)(s-f)}$$

$$A = \sqrt{18.95(18.95-12)(18.95-11.9)(18.95-14)} = \sqrt{18.95 * 6.95 * 7.05 * 4.95}$$

$$A = 67.8ft^2$$

$f)$ $d = 8ft$ $e = 12ft$ $f = 13ft$

$$S = \frac{d+e+f}{2} = \frac{8+12+13}{2} = 16.5ft$$

$$A = \sqrt{s(s-d)(s-e)(s-f)}$$

$$A = \sqrt{16.5(16.5-8)(16.5-12)(16.5-13)} = \sqrt{16.5 * 8.5 * 4.5 * 3.5}$$

$$A = 47ft^2$$

$g)$ $in\ \triangle CAB$ $c = 6in$ $a = 12.8in$ $b = 13in$

$$s = \frac{a+b+c}{2} = \frac{12.8+13+6}{2} = 15.9in$$

$$A = \sqrt{15.9(15.9-12.8)(15.9-13)(15.9-6)} = \sqrt{15.9 * 3.1 * 2.9 * 9.9} = 37.6in^2$$

$h)\ in\ \Delta DEF \qquad d = 10.3ft \qquad e = 9ft \qquad f = 15ft$

$$s = \frac{d+e+f}{2} = \frac{10.3+9+15}{2} = 17.15ft$$

$$A = \sqrt{17.15(17.15-10.3)(17.15-9)(17.15-15)} \quad = \sqrt{17.15 \times 6.85 \times 8.15 \times 2.15} \ = 45.4ft^2$$

$I)\ in\ \Delta FDE \qquad f = 13mi \qquad d = 4mi \qquad e = 13mi$

$$s = \frac{d+e+f}{2} = \frac{4+13+13}{2} = 15mi$$

$$A = \sqrt{15(15-13)(15-4)(15-13)} \ = \sqrt{15 \times 2 \times 11 \times 2} \ = 25.7mi^2$$

$j)\ in\ \Delta XYZ \qquad y = 15km \qquad x = 5km \qquad z = 12km$

$$s = \frac{x+y+z}{2} = \frac{5+15+12}{2} = 16km$$

$$A = \sqrt{16(16-15)(16-5)(15-12)} \ = \sqrt{16 \times 1 \times 11 \times 4} \ = 26.5km^2$$

Answers of chapter two:- complex numbers

Section 2.1

Page 35 exercises 1 − 24 perform the operation and write the result in standard form

1. $(5+i)+(6-2i)=11-i$
2. $(13-2i)+(-5+6i)=8+4i$
3. $(8-i)-(4-i)=4+0i$
4. $(3+2i)-(6+13i)=-3-11i$
5. $(-2+\sqrt{-8})+(5-\sqrt{-50})=(-2+2\sqrt{2}i)+(5-5\sqrt{2}i)=3-3\sqrt{2}i$
6. $(8+\sqrt{-18})-(4+3\sqrt{2}i=(8+3\sqrt{2}i)-(4+3\sqrt{2}i)=4+0i$
7. $13-(14-7i)=13-14+7i=-1+7i$
8. $22+(-5+8i)+10i=22-5+8i+10i=17+18i$
9. $-\left(\frac{3}{2}+\frac{5}{2}i\right)+\left(\frac{5}{3}+\frac{11}{3}i\right)=\left(-\frac{3}{2}-\frac{5}{2}i\right)+\left(\frac{5}{3}+\frac{11}{3}i\right)=\frac{1}{6}+\frac{7}{6}i$
10. $(1.6+3.2i)+(-5.8+4.3i)=-4.2+7.5i$
11. $\sqrt{-6}\cdot\sqrt{-2}=\sqrt{6}i\cdot\sqrt{2}i=\sqrt{12}i^2=-\sqrt{12}$
12. $\sqrt{-5}\cdot\sqrt{-10}=\sqrt{5}i\cdot\sqrt{10}i=\sqrt{50}i^2=-\sqrt{50}$
13. $\left(\sqrt{-10}\right)^2=\left(\sqrt{10}i\right)^2=10i^2=-10$
14. $\left(\sqrt{-75}\right)^2=\left(\sqrt{75}i\right)^2=75i^2=-75$
15. $(1+i)(3-2i)=3-2i+3i-2i^2=3+i+2=5+i$
16. $(6-2i)(2-3i)=12-18i-4i+6i^2=12-22i-6=6-22i$
17. $6i(5-2i)=30i-12i^2=30i+12=12+30i$
18. $-8i(9+4i)=-72i-32i^2=-72i+32=32-72i$
19. $\left(\sqrt{14}+\sqrt{10}i\right)\left(\sqrt{14}-\sqrt{10}i\right)=14-10i^2=14+10=24$
20. $\left(3+\sqrt{-5}\right)\left(7-\sqrt{-10}\right)=\left(3+\sqrt{5}i\right)\left(7-\sqrt{10}i\right)=21-3\sqrt{10}i+7\sqrt{5}i-\sqrt{50}i^2$
    $=\left(21+\sqrt{50}\right)+\left(7\sqrt{5}-3\sqrt{10}\right)i$
21. $(4+5i)^2=16+40i+25i^2=16+40i-25=-9+40i$
22. $(2-3i)^2=4-12i+9i^2=4-12i-9=-5-12i$
23. $(2+3i)^2+(2-3i)^2=4+12i+9i^2+4-12i+9i^2=8+18i^2=8-18=-10$
24. $(1-2i)^2-(1+2i)^2=1-4i+4i^2-1-4i-4i^2=-8i$

Section 2.2

Write the conjugate of the complex number multiply the number and its conjugate .

1. $5+3i$ =?
   the conjugate of $5+3i$ *is* $5-3i$
   $(5+3i)(5-3i) = 25-9i^2 = 25+9 = 34$
2. $-2-\sqrt{5}i$ =?
   The conjugate of $-2-\sqrt{5}i$ *is* $-2+\sqrt{5}i$
   $(-2-\sqrt{5}i\,)(-2+\sqrt{5}i) = 4-5i^2 = 4+5 = 9$
3. $20i$ =?
   The conjugate of $20i$ *is* $-20i$
   $(20i)(-20i) = -400i^2 = 400$
4. $\sqrt{8}$ =?
   the conjugate of $\sqrt{8}$ *is* $\sqrt{8}$
   $\sqrt{8} * \sqrt{8} = 8$
5. $9-12i$ =?
   The conjugate of $9-12i$ *is* $9+12i$
   $(9-12i)(9+12i) = 81-144i^2 = 81+144 = 225$
6. -4+$\sqrt{2}i$ =?
   The conjugate of -4+$\sqrt{2}i$ *is* $-4-\sqrt{2}i$
   $(-4+\sqrt{2}i\,)\left(-4-\sqrt{2}i\right) = 16-2i^2 = 16+2 = 18$
7. $\sqrt{-15} = \sqrt{15}i$ =?
   The conjugate of $\sqrt{15}i$ *is* $-\sqrt{15}i$
   $\sqrt{15}i * -\sqrt{15}i = -15i^2 = 15$

8. $1+\sqrt{8}=?$

   $The\ conjugate\ of\ +1+\sqrt{8}\ is 1+\sqrt{8}$

   $(1+\sqrt{8})*(1+\sqrt{8})=1+\sqrt{8}+\sqrt{8}+8=9+2\sqrt{8}=9+4\sqrt{2}$

Part two: in exercise 1-14 perform the operation and write the result in standard form

1. $\frac{6}{i}=\frac{6}{i}*\frac{-i}{-i}=\frac{-6i}{-i^2}=\frac{-6i}{1}=-6i$
2. $\frac{-10}{2i}=\frac{-10}{2i}*\frac{-2i}{-2i}=\frac{20i}{-4i^2}=\frac{20i}{4}=5i$
3. $\frac{4}{4-5i}=\frac{4}{4-5i}*\frac{4+5i}{4+5i}=\frac{16+20i}{16-25i^2}=\frac{16+20i}{16+25}=\frac{16+20i}{41}=\frac{16}{41}+\frac{20}{41}i$
4. $\frac{3}{1-i}=\frac{3}{1-i}*\frac{1+i}{1+i}=\frac{3+3i}{1-i^2}=\frac{3+3i}{1+1}=\frac{3+3i}{2}=\frac{3}{2}+\frac{3}{2}i$
5. $\frac{2+i}{2-i}=\frac{2+i}{2-i}*\frac{2+i}{2+i}=\frac{4+2i+2i+i^2}{4-i^2}=\frac{4+4i-1}{4+1}=\frac{3+4i}{5}=\frac{3}{5}+\frac{4}{5}i$
6. $\frac{8-7i}{1-2i}=\frac{8-7i}{1-2i}*\frac{1+2i}{1+2i}=\frac{8+16i-7i-14i^2}{1-4i^2}=\frac{8+9i+14}{1+4}=\frac{22+9i}{5}=\frac{22}{5}+\frac{9}{5}i$
7. $\frac{6-7i}{i}=\frac{6-7i}{i}*\frac{-i}{-i}=\frac{-6i+7i^2}{-i^2}=\frac{-6i-7}{1}=-7-6i$
8. $\frac{8+20i}{2i}=\frac{8+20i}{2i}*\frac{-2i}{-2i}=\frac{-16i-40i^2}{-4i^2}=\frac{-16i+40}{4}=\frac{40}{4}-\frac{16}{4}i=10-4i$
9. $\frac{1}{(4-5i)^2}=\frac{1}{16-40i+25i^2}=\frac{1}{16-40i-25}=\frac{1}{-9-40i}$

   $\frac{1}{-9-40i}=\frac{1}{-9-40i}*\frac{-9+40i}{-9+40i}=\frac{-9+40i}{81-1600i^2}=\frac{-9+40i}{81+1600}$

   $=\frac{-9}{1681}+\frac{40}{1681}i$
10. $\frac{(2-3i)(5i)}{2+3i}=\frac{10i-15i^2}{2+3i}=\frac{15+10i}{2+3i}$

    $\frac{15+10i}{2+3i}*\frac{2-3i}{2-3i}=\frac{30-45i+20i-30i^2}{4-9i^2}=\frac{30-25i+30}{4+9}=\frac{60-25i}{13}$

    $=\frac{60}{13}-\frac{25}{13}i$

11. $\frac{2}{1+i}-\frac{3}{1-i}=\frac{2(1-i)-3(1+i)}{(1+i)(1-i)}=\frac{2-2i-3-3i}{1-i^2}=\frac{-1-5i}{1+1}=\frac{-1-5i}{2}=\frac{-1}{2}-\frac{5}{2}i$

12. $\frac{2i}{2+i}+\frac{5}{2-i}=\frac{2i(2-i)+5(2+i)}{(2+i)(2-i)}=\frac{4i-2i^2+10+5i}{4-i^2}=\frac{4i+2+10+5i}{4+1}=\frac{12+9i}{5}=\frac{12}{5}+\frac{9}{5}i$

13. $\frac{i}{3-2i}+\frac{2i}{3+8i}=\frac{i(3+8i)+2i(3-2i)}{(3-2i)(3+8i)}=\frac{3i+8i^2+6i-4i^2}{9+24i-6i-16i^2}=\frac{9i+4i^2}{9+18i+16}=\frac{-4+9i}{25+18i}$

$$\frac{-4+9i}{25+18i}*\frac{25-18i}{25-18i}=\frac{-100+72i+225i-162i^2}{625-324i^2}$$

$$=\frac{-100+297i+162}{625+324}=\frac{62+297i}{949}=\frac{62}{949}+\frac{297}{949}i$$

14. $\frac{1+i}{i}-\frac{3}{4-i}=\frac{(1+i)(4-i)-3(i)}{i(4-i)}=\frac{4-i+4i-i^2-3i}{4i-i^2}=\frac{4+3i+1-3i}{4i+1}=\frac{5}{1+4i}$

$$\frac{5}{1+4i}*\frac{1-4i}{1-4i}=\frac{5-20i}{1-16i^2}=\frac{5-20i}{1+16}=\frac{5-20i}{17}=\frac{5}{17}-\frac{20}{17}i$$

Section 2.3

Part one:-In exercise 2.3 use the quadratic formula to solve the quadratic equation

1. $x^2 - 2x + 2 = 0$

   $a = 1 \qquad b = -2 \qquad c = 2$

   $$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(1)(2)}}{2(1)} = \frac{2 \pm \sqrt{4 - 8}}{2} = \frac{2 \pm \sqrt{-4}}{2}$$

   $$x = \frac{2 \pm 2i}{2} \qquad x = \frac{2}{2} \pm \frac{2i}{2} = 1 \pm i$$

2. $x^2 + 6x + 10 = 0$

   $a = 1 \qquad b = 6 \qquad c = 10$

   $$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-(6) \pm \sqrt{(6)^2 - 4(1)(10)}}{2(1)} = \frac{-6 \pm \sqrt{36 - 40}}{2} = \frac{-6 \pm \sqrt{-4}}{2}$$

   $$x = \frac{-6 \pm 2i}{2} \qquad x = \frac{-6}{2} \pm \frac{2i}{2} = -3 \pm i$$

3. $4x^2 + 16x + 17 = 0$

   $a = 4 \qquad b = 16 \qquad c = 17$

   $$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-(16) \pm \sqrt{(16)^2 - 4(4)(17)}}{2(4)} = \frac{-16 \pm \sqrt{256 - 272}}{8}$$

   $$x = \frac{-16 \pm \sqrt{-16}}{8} = \frac{-16 \pm 4i}{8} \qquad x = \frac{-16}{8} \pm \frac{4i}{8} = -2 \pm \frac{1}{2}i$$

4. $9x^2 - 6x + 37 = 0$

   $a = 9 \qquad b = -6 \qquad c = 37$

   $$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-(-6) \pm \sqrt{(-6)^2 - 4(9)(37)}}{2(9)} = \frac{6 \pm \sqrt{36 - 1332}}{18}$$

   $$x = \frac{6 \pm \sqrt{-1296}}{18} = \frac{6 \pm 36i}{18} \qquad x = \frac{6}{18} \pm \frac{36}{18}i = \frac{1}{3} \pm 2i$$

5. $4x^2 + 16x + 15 = 0$

   $a = 4 \qquad b = 16 \qquad c = 15$

   $$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-(16) \pm \sqrt{(16)^2 - 4(4)(15)}}{2(4)} = \frac{-16 \pm \sqrt{256 - 240}}{8}$$

$$x = \frac{-16 \pm \sqrt{16}}{8} = \frac{-16 \pm 4}{8}$$

$$x = \frac{-16}{8} \pm \frac{4}{8} = -2 \pm \frac{1}{2} \qquad x_{1=} -2 + \frac{1}{2} = \frac{-3}{2} \qquad x_{2=} -2 - \frac{1}{2} = \frac{-5}{2}$$

6. $9x^2 - 6x - 35 = 0$

$a = 9 \qquad b = -6 \qquad c = -35$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-(-6) \pm \sqrt{(-6)^2 - 4(9)(-35)}}{2(9)} = \frac{6 \pm \sqrt{36 + 1260}}{18}$$

$$x = \frac{6 \pm \sqrt{1296}}{18} = \frac{6 \pm 36}{18}$$

$$x = \frac{6}{18} \pm \frac{36}{18} = \frac{1}{3} \pm 2 \qquad x_{1=} \frac{1}{3} + 2 = \frac{7}{3} \qquad x_{2=} \frac{1}{3} - 2 = \frac{-5}{3}$$

7. $16t^2 - 4t + 3 = 0$

$a = 16 \qquad b = -4 \qquad c = 3$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(16)(3)}}{2(16)} = \frac{4 \pm \sqrt{16 - 192}}{32}$$

$$x = \frac{4 \pm \sqrt{-176}}{32} = \frac{4 \pm \sqrt{-16 * 11}}{32} = \frac{4 \pm 4i\sqrt{11}}{32}$$

$$x = \frac{4}{32} \pm \frac{4i\sqrt{11}}{32} = \frac{1}{8} \pm \frac{\sqrt{11}}{8}i$$

8. $5s^2 + 6s + 3 = 0$

$a = 5 \qquad b = 6 \qquad c = 3$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-(6) \pm \sqrt{(6)^2 - 4(5)(3)}}{2(5)} = \frac{-6 \pm \sqrt{36 - 60}}{10} = \frac{-6 \pm \sqrt{-24}}{10}$$

$$= \frac{-6 \pm \sqrt{-4 * 6}}{10} = \frac{-6 \pm 2i\sqrt{6}}{10}$$

$$x = \frac{-6}{10} \pm \frac{2i\sqrt{6}}{10} = \frac{-3}{5} \pm \frac{\sqrt{6}}{5}i$$

Part two:- in exercise 1-8 simplify the complex number and write it in standard form

1. $-6i^3 + i^2 = -6(-i) + (-1) = 6i - 1 = -1 + 6i$
2. $4i^2 - 2i^3 = 4(-1) - 2(-i) = -4 + 2i$
3. $-5i^5 = -5(i) = 0 - 5i$
4. $(-i)^3 = -i^3 = -(-i) = 0 + i$

5. $\left(\sqrt{-75}\right)^3 = \left(\sqrt{-25*3}\right)^3 = \left(5i\sqrt{3}\right)^3 = 125i^3\left(3\sqrt{3}\right) = 0 - 375\sqrt{3}i$
6. $\left(\sqrt{-2}\right)^6 = \left(i\sqrt{2}\right)^6 = (i)^6\left(\sqrt{2}\right)^6 = (-1)(8) = -8 = -8 = -8 + 0i$
7. $\frac{1}{i^3} = \frac{1}{i^3} * \frac{-i^3}{-i^3} = \frac{-i^3}{-i^6} = \frac{i^3}{i^6} = \frac{-i}{-1} = i{=}0 + i$
8. $\frac{1}{(2i)^3} = \frac{1}{8i^3} = \frac{1}{8(-i)} = -\frac{1}{8i} = -\frac{1}{8i} * \frac{-8i}{-8i} = \frac{-8i}{-64i^2} = \frac{-8i}{-64(-1)} = \frac{-8i}{64} = -\frac{1}{8}i$

Section 2.4

Exercise 2.4 in exercise 1-6 plot the complex number and find its absolute value

1. $-5i$ =? $a = 0$ $b = -5$
   the absolute value of $-5i$ *is* $= \sqrt{a^2 + b^2} = \sqrt{0 + 25} = \sqrt{25} = 5$
2. -5 =? $a = -5$ $b = 0$
   the absolute value of -5 *is* $= \sqrt{a^2 + b^2} = \sqrt{25 + 0} = \sqrt{25} = 5$
3. $-4 + 4i$ =? $a = -4$ $b = 4$
   the absolute value of $-4 + 4i$ *is* $= \sqrt{a^2 + b^2} = \sqrt{16 + 16} = \sqrt{32} = 4\sqrt{2}$
4. $5 - 12i$ =? $a = 5$ $b = -12$
   the absolute value of $5 - 12i$ *is* $= \sqrt{a^2 + b^2} = \sqrt{25 + 144} = \sqrt{169} = 13$
5. $6 - 7i$ =? $a = 6$ $b = -7$
   the absolute value of $6 - 7i$ *is* $= \sqrt{a^2 + b^2} = \sqrt{36 + 49} = \sqrt{85}$
6. $-8 + 3i$ =? $a = -8$ $b = 3$
   the absolute value of $-8 + 3i$ *is* $= \sqrt{a^2 + b^2} = \sqrt{64 + 9} = \sqrt{73}$

Part two: - in exercise 1-8 represent the complex number graphically and find the polar form of the number

1. $3 - 3i$ =? $a = 3$ $b = -3$
   $r = \sqrt{a^2 + b^2} = \sqrt{9 + 9} = \sqrt{18} = 3\sqrt{2}$
   $\theta = \tan^{-1}\left(\frac{-3}{3}\right) = \tan^{-1}(-1) = -45 = 315$
   $\therefore 3\sqrt{2}(cos315 + isin315)$
2. $2 + 2i$ =? $a = 2$ $b = 2$
   $r = \sqrt{a^2 + b^2} = \sqrt{4 + 4} = \sqrt{8} = 2\sqrt{2}$
   $\theta = \tan^{-1}\left(\frac{2}{2}\right) = \tan^{-1}(1) = 45$
   $\therefore 2\sqrt{2}(cos45 + isin45)$
3. $\sqrt{3} + i$ =? $a = \sqrt{3}$ $b = 1$
   $r = \sqrt{a^2 + b^2} = \sqrt{3 + 1} = \sqrt{4} = 2$
   $\theta = \tan^{-1}\left(\frac{1}{\sqrt{3}}\right) = 30$
   $\therefore 2(cos30 + isin30)$
4. $-1 + \sqrt{3}i$ =? $a = -1$ $b = \sqrt{3}$
   $r = \sqrt{a^2 + b^2} = \sqrt{1 + 3} = \sqrt{4} = 2$
   $\theta = \tan^{-1}\left(\frac{\sqrt{3}}{-1}\right) + \pi = \frac{5\pi}{3} + \pi = \frac{8\pi}{3}$ *or* $\frac{2\pi}{3}$
   $\therefore 2\left(cos\frac{2\pi}{3} + isin\frac{2\pi}{3}\right)$
5. $-2(1 + \sqrt{3}i) = -2 - 2\sqrt{3}i$ =? $a = -2$ $b = -2\sqrt{3}$
   $r = \sqrt{a^2 + b^2} = \sqrt{4 + 12} = \sqrt{16} = 4$

$\theta = \tan^{-1}\left(\frac{-2\sqrt{3}}{-2}\right) + \pi = \frac{\pi}{3} + \pi = \frac{4\pi}{3}$

$\therefore 4\left(cos\frac{4\pi}{3} + isin\frac{4\pi}{3}\right)$

6. $\frac{5}{2}(\sqrt{3} - i) = \frac{5\sqrt{3}}{2} - \frac{5}{2}i = ?$ $\quad a = \frac{5\sqrt{3}}{2}$ $\quad b = -\frac{5}{2}$

$r = \sqrt{a^2 + b^2} = \sqrt{\frac{75}{4} + \frac{25}{4}} = \sqrt{\frac{100}{4}} = \sqrt{25} = 5$

$\theta = \tan^{-1}\left(\frac{\frac{-5}{2}}{\frac{5\sqrt{3}}{2}}\right) = \tan^{-1}\left(\frac{-1}{\sqrt{3}}\right) = \frac{11\pi}{6}$

$\therefore 5\left(cos\frac{11\pi}{6} + isin\frac{11\pi}{6}\right)$

7. $6i = ?$ $\quad a = 0$ $\quad b = 6$

$r = \sqrt{a^2 + b^2} = \sqrt{0 + 36} = \sqrt{36} = 6$

$\theta = \tan^{-1}\left(\frac{6}{0}\right) = 90\ or\ \frac{\pi}{2}$

$\therefore 6\left(cos\frac{\pi}{2} + isin\frac{\pi}{2}\right)$

8. $4 = ?$ $\quad a = 4$ $\quad b = 0$

$r = \sqrt{a^2 + b^2} = \sqrt{16 + 0} = \sqrt{16} = 4$

$\theta = \tan^{-1}\left(\frac{0}{4}\right) = 0\ or\ 180$

$\therefore 4(cos\pi + isin\pi)$

Part three: - in exercise 1-6 represent the complex number graphically and find the standard form of the result

1. $2(cos150 + isin150) - ?$ $\quad r = 2$ $\quad \theta = 150$

$a = rcos\theta = 2cos150 = 2\left(-\frac{\sqrt{3}}{2}\right) = -\sqrt{3}$

$b = rsin\theta = 2sin150 = 2\left(\frac{1}{2}\right) = 1$

$\therefore the\ standard\ form\ of\ complex\ number\ is\ = a + bi = -\sqrt{3} + i$

2. $5(cos135 + isin135) = ?$ $\quad r = 5$ $\quad \theta = 135$

$a = rcos\theta = 5cos135 = 5\left(-\frac{\sqrt{2}}{2}\right) = -\frac{5\sqrt{2}}{2}$

$b = rsin\theta = 5sin135 = 5\left(\frac{\sqrt{2}}{2}\right) = \frac{5\sqrt{2}}{2}$

$\therefore the\ standard\ form\ of\ complex\ number\ is\ = a + bi = -\frac{5\sqrt{2}}{2} + \frac{5\sqrt{2}}{2}i$

3. $\frac{3}{2}(cos300 + isin300) = ?$ $\quad r = \frac{3}{2}$ $\quad \theta = 300$

$a = rcos\theta = \frac{3}{2}cos300 = \frac{3}{2}\left(\frac{1}{2}\right) = \frac{3}{4}$

$b = rsin\theta = \frac{3}{2}sin300 = \frac{3}{2}\left(\frac{-\sqrt{3}}{2}\right) = \frac{-3\sqrt{3}}{4}$

$\therefore$ *the standard form of complex number is* $= a + bi = \frac{3}{4} - \frac{3\sqrt{3}}{4}i$

4. $\frac{3}{4}(cos315 + isin315) = ?$ $\quad r = \frac{3}{4}$ $\quad \theta = 315$

$a = rcos\theta = \frac{3}{4}cos315 = \frac{3}{4}\left(\frac{\sqrt{2}}{2}\right) = \frac{3\sqrt{2}}{8}$

$b = rsin\theta = \frac{3}{4}sin315 = \frac{3}{4}\left(\frac{-\sqrt{2}}{2}\right) = \frac{-3\sqrt{2}}{8}$

$\therefore$ *the standard form of complex number is* $= a + bi = \frac{3\sqrt{2}}{8} - \frac{3\sqrt{2}}{8}i$

5. $3.75\left(cos\frac{3\pi}{4} + isin\frac{3\pi}{4}\right) = ?$ $\quad r = 3.75$ $\quad \theta = \frac{3\pi}{4}$

$a = rcos\theta = 3.75cos\frac{3\pi}{4} = \frac{15}{4}\left(-\frac{\sqrt{2}}{2}\right) = \frac{-15\sqrt{2}}{8}$

$b = rsin\theta = \frac{15}{4}sin\frac{3\pi}{4} = \frac{15}{4}\left(\frac{\sqrt{2}}{2}\right) = \frac{15\sqrt{2}}{8}$

$\therefore$ *the standard form of complex number is* $= a + bi = \frac{-15\sqrt{2}}{8} + \frac{15\sqrt{2}}{8}i$

6. $8\left(cos\frac{\pi}{12} + isin\frac{\pi}{12}\right) = ?$ $\quad r = 8$ $\quad \theta = \frac{\pi}{12}$

$a = rcos\theta = 8cos\frac{\pi}{12} = 8\left(\frac{\sqrt{6}+\sqrt{2}}{4}\right) = 2(\sqrt{6}+\sqrt{2})$

$b = rsin\theta = 8sin\frac{\pi}{12} = 8\left(\frac{\sqrt{6}-\sqrt{2}}{4}\right) = 2(\sqrt{6}-\sqrt{2})$

$\therefore$ *the standard form of complex number is* $= a + bi = 2(\sqrt{6}+\sqrt{2}) + 2(\sqrt{6}-\sqrt{2})i$

Section 2.5

In exercise 1-4 perform the operation and leave the result in standard form

1. $\left|3\left(cos\frac{\pi}{3}+isin\frac{\pi}{3}\right)\right|\left|4\left(cos\frac{\pi}{6}+isin\frac{\pi}{6}\right)\right| = 12\left|cos\left(\frac{\pi}{3}+\frac{\pi}{6}\right)+isin\left(\frac{\pi}{3}+\frac{\pi}{6}\right)\right| = 12\left(cos\frac{\pi}{2}+isin\frac{\pi}{2}\right)$
2. $\left|\frac{3}{2}\left(cos\frac{\pi}{2}+isin\frac{\pi}{2}\right)\right|\left|6\left(cos\frac{\pi}{4}+isin\frac{\pi}{4}\right)\right| = 9\left|cos\left(\frac{\pi}{2}+\frac{\pi}{4}\right)+isin\left(\frac{\pi}{2}+\frac{\pi}{4}\right)\right| = 9\left(cos\frac{3\pi}{4}+isin\frac{3\pi}{4}\right)$
3. $\left|\frac{5}{3}(cos140+isin140)\right|\left|\frac{2}{3}(cos60+isin60)\right| = \frac{10}{9}[cos(140+60)+isin(140+60)] = \frac{10}{9}(cos200+isin200)$
4. $\frac{\cos\left(\frac{5\pi}{3}\right)+isin\left(\frac{5\pi}{3}\right)}{cos\pi+isin\pi} = \cos\left(\frac{5\pi}{3}-\pi\right)+isin\left(\frac{5\pi}{3}-\pi\right) = cos\frac{2\pi}{3}+isin\frac{2\pi}{3}$

Part two: - in exercise 1-8 use demoivre's theorem to find indicated power of the complex number express the result in standard form

1. $(1+i)^5 = ?$ $\quad a=1$ $\quad b=1$

   $r = \sqrt{a^2+b^2} = \sqrt{1+1} = \sqrt{2}$

   $\theta = \tan^{-1}\left(\frac{1}{1}\right) = 45$

   $\left[\sqrt{2}(cos45+isin45)\right]^5 = \left(\sqrt{2}\right)^5[\cos(45*5)+isin(45*5)]$

   $= 4\sqrt{2}(cos225+isin225)$

   The standard form of $4\sqrt{2}(cos225+isin225) = -4-4i$
2. $(2+2i)^6 = ?$ $\quad a=2$ $\quad b=2$

   $r = \sqrt{a^2+b^2} = \sqrt{4+4} = \sqrt{8} = 2\sqrt{2}$

   $\theta = \tan^{-1}\left(\frac{2}{2}\right) = \tan^{-1}(1) = 45$

   $\left[2\sqrt{2}(cos45+isin45)\right]^6 = \left(2\sqrt{2}\right)^6[\cos(45*6)+isin(45*6)]$

   $= 512(cos270+isin270)$

   The standard form of $512(cos270+isin270) = 0-512i$
3. $(-1+i)^{10} = ?$ $\quad a=-1$ $\quad b=1$

   $r = \sqrt{a^2+b^2} = \sqrt{1+1} = \sqrt{2}$

$\theta = \tan^{-1}\left(\frac{1}{-1}\right) + \pi = \frac{7\pi}{4} + \pi = \frac{11\pi}{4}\ 0r\ \frac{3\pi}{4}$

$\left[\sqrt{2}\left(cos\frac{3\pi}{4} + isin\frac{3\pi}{4}\right)\right]^{10} = (\sqrt{2})^{10}\left[\cos\left(\frac{3\pi}{4} * 10\right) + isin\left(\frac{3\pi}{4} * 10\right)\right]$

$= 32\left(cos\frac{15\pi}{2} + isin\frac{15\pi}{2}\right)$

The standard form of $32\left(cos\frac{15\pi}{2} + isin\frac{15\pi}{2}\right) = 0 - 32i$

4. $(1-i)^{12} = ?$ $a = 1$ $b = -1$

$r = \sqrt{a^2 + b^2} = \sqrt{1+1} = \sqrt{2}$

$\theta = \tan^{-1}\left(\frac{-1}{1}\right) = 315$

$\left[\sqrt{2}(cos315 + isin315)\right]^{12} = (\sqrt{2})^{12}[\cos(315 * 12) + isin(315 * 12)]$

$= 64(cos3780 + isin3780)$

The standard form of $64(cos3780 + isin3780) = -64 + 0i$

5. $2(\sqrt{3} + i)^7 = ?$ $a = \sqrt{3}$ $b = 1$

$r = \sqrt{a^2 + b^2} = \sqrt{3+1} = \sqrt{4} = 2$

$\theta = \tan^{-1}\left(\frac{1}{\sqrt{3}}\right) = 30$

$2[2(cos30 + isin30)]^7 = 2\{(2)^7[\cos(30 * 7) + isin(30 * 7)]\}$

$= 256(cos210 + isin210)$

The standard form of $256(cos210 + isin210) = -128\sqrt{3} - 128i$

6. $4(1 - \sqrt{3}i)^3 = ?$ $a = 1$ $b = -\sqrt{3}$

$r = \sqrt{a^2 + b^2} = \sqrt{1+3} = \sqrt{4} = 2$

$\theta = \tan^{-1}\left(\frac{-\sqrt{3}}{1}\right) = 300$

$4[2(cos300 + isin300)]^3 = 4\{(2)^3[\cos(300 * 3) + isin(300 * 3)]\}$

$= 32(cos900 + isin900)$

The standard form of $32(cos900 + isin900) = -32 + 0i$

7. $\left(cos\frac{5\pi}{4}+isin\frac{5\pi}{4}\right)^{10}=(1)^{10}\left[\cos\left(\frac{5\pi}{4}*10\right)+isin\left(\frac{5\pi}{4}*10\right)\right]$
$=cos\frac{25\pi}{2}+isin\frac{25\pi}{2}$
The standard form of $\left(cos\frac{25\pi}{2}+isin\frac{25\pi}{2}\right)=0+i$
8. $\left[2\left(cos\frac{\pi}{2}+isin\frac{\pi}{2}\right)\right]^{8}=(2)^{8}\left[\cos\left(\frac{\pi}{2}*8\right)+isin\left(\frac{\pi}{2}*8\right)\right]$
$=256(cos4\pi+isin4\pi)$
The standard form of $256\,(cos4\pi+isin4\pi)=256+0i$

Part three:–

in exercise 1_6 a) use theorem A.5 on page A64 to find indicated roots of the complex number b)re

1. *Square roots of* $5(cos120+isin120)=[\,5(cos120+isin120)]^{\frac{1}{2}}$
$k=0,1$
When $k=0$ $(5)^{\frac{1}{2}}\left(cos\frac{120+360(0)}{2}+isin\frac{120+360(0)}{2}\right)$
$=\sqrt{5}(cos60+isin60)$
The standard form of $\sqrt{5}\,(cos60+isin60)=\frac{\sqrt{5}}{2}+\frac{\sqrt{15}}{2}i$

When $k=1$ $\sqrt{5}\left(cos\frac{120+360(1)}{2}+isin\frac{120+360(1)}{2}\right)$

$=\sqrt{5}(cos240+isin240)$

The standard form of $\sqrt{5}\,(cos240+isin240)=-\frac{\sqrt{5}}{2}-\frac{\sqrt{15}}{2}i$

Square roots of $16(cos60+isin60)=[16\,(cos60+isin60)]^{\frac{1}{2}}$
$k=0,1$
When $k=0$ $(16)^{\frac{1}{2}}\left(cos\frac{60+360(0)}{2}+isin\frac{60+360(0)}{2}\right)=4(cos30+isin30)$

The standard form of $4(cos30+isin30)=2\sqrt{3}+2i$

When $k=1$ $4\left(cos\frac{60+360(1)}{2}+isin\frac{60+360(1)}{2}\right)=5(cos210+isin210)$
The standard form of $4(cos210+isin210)=-2\sqrt{3}-2i$

2. $fourth\ roots\ of\ 16\left(cos\frac{4\pi}{3}+isin\frac{4\pi}{3}\right)=\left[16\ \left(cos\frac{4\pi}{3}+isin\frac{4\pi}{3}\right)\right]^{\frac{1}{4}}=$

$k=0,1,2,3$

$When\ k=0 \quad (16)^{\frac{1}{4}}\left(cos\frac{\frac{4\pi}{3}+2\pi(0)}{4}+isin\frac{\frac{4\pi}{3}+2\pi(0)}{4}\right)=2\left(cos\frac{\pi}{3}+isin\frac{\pi}{3}\right)$

$The\ standard\ form\ of \quad 2\left(cos\frac{\pi}{3}+isin\frac{\pi}{3}\right)=1+\sqrt{3}i$

$When\ k=1 \quad (16)^{\frac{1}{4}}\left(cos\frac{\frac{4\pi}{3}+2\pi(1)}{4}+isin\frac{\frac{4\pi}{3}+2\pi(1)}{4}\right)$

$=2\left(cos\frac{5\pi}{6}+isin\frac{5\pi}{6}\right)$

$The\ standard\ form\ of \quad 2\left(cos\frac{5\pi}{6}+isin\frac{5\pi}{6}\right)=-\sqrt{3}+i$

$When\ k=2 \quad 0 \quad (16)^{\frac{1}{4}}\left(cos\frac{\frac{4\pi}{3}+2\pi(2)}{4}+isin\frac{\frac{4\pi}{3}+2\pi(2)}{4}\right)$

$=2\left(cos\frac{4\pi}{3}+isin\frac{4\pi}{3}\right)$

$The\ standard\ form\ of \quad 2\left(cos\frac{4\pi}{3}+isin\frac{4\pi}{3}\right)=-1-\sqrt{3}i$

$When\ k=3 \quad (16)^{\frac{1}{4}}\left(cos\frac{\frac{4\pi}{3}+2\pi(3)}{4}+isin\frac{\frac{4\pi}{3}+2\pi(3)}{4}\right)$

$=2\left(cos\frac{11\pi}{6}+isin\frac{11\pi}{6}\right)$

$The\ standard\ form\ of \quad 2\left(cos\frac{11\pi}{6}+isin\frac{11\pi}{6}\right)=\sqrt{3}-i$

3. $fifth\ roots\ of\ 32\left(cos\frac{5\pi}{6}+isin\frac{5\pi}{6}\right)=\left[32\ \left(cos\frac{5\pi}{6}+isin\frac{5\pi}{6}\right)\right]^{\frac{1}{5}}=$

$k=0,1,2,3$

$When\ k=0 \quad (32)^{\frac{1}{5}}\left(cos\frac{\frac{5\pi}{6}+2\pi(0)}{5}+isin\frac{\frac{5\pi}{6}+2\pi(0)}{5}\right)=2\left(cos\frac{\pi}{6}+isin\frac{\pi}{6}\right)$

$The\ standard\ form\ of \quad 2\left(cos\frac{\pi}{6}+isin\frac{\pi}{6}\right)=\sqrt{3}+i$

$When\ k=1 \quad (32)^{\frac{1}{5}}\left(cos\frac{\frac{5\pi}{6}+2\pi(1)}{5}+isin\frac{\frac{5\pi}{6}+2\pi(1)}{5}\right)$

$=2\left(cos\frac{17\pi}{30}+isin\frac{17\pi}{30}\right)$

$The\ standard\ form\ of \qquad 2\left(cos\frac{17\pi}{30}+isin\frac{17\pi}{30}\right)=-0.42+1.96i$

$When\ k=2 \quad 0 \quad (32)^{\frac{1}{5}}\left(cos\frac{\frac{5\pi}{6}+2\pi(2)}{5}+isin\frac{\frac{5\pi}{6}+2\pi(2)}{5}\right)$

$=2\left(cos\frac{29\pi}{30}+isin\frac{29\pi}{30}\right)$

$The\ standard\ form\ of \qquad 2\left(cos\frac{29\pi}{30}+isin\frac{29\pi}{30}\right)=-1.99+0.2i$

$When\ k=3 \qquad (32)^{\frac{1}{5}}\left(cos\frac{\frac{5\pi}{6}+2\pi(3)}{5}+isin\frac{\frac{5\pi}{6}+2\pi(3)}{5}\right)$

$=2\left(cos\frac{41\pi}{30}+isin\frac{41\pi}{30}\right)$

$The\ standard\ form\ of \qquad 2\left(cos\frac{41\pi}{30}+isin\frac{41\pi}{30}\right)=-1-1.8i$

$when\ k=4 \qquad (32)^{\frac{1}{5}}\left(cos\frac{\frac{5\pi}{6}+2\pi(4)}{5}+isin\frac{\frac{5\pi}{6}+2\pi(4)}{5}\right)$

$=2\left(cos\frac{53\pi}{30}+isin\frac{53\pi}{30}\right)$

$The\ standard\ form\ of \qquad 2\left(cos\frac{53\pi}{30}+isin\frac{53\pi}{30}\right)=1.49-1.34i$

4. $cube\ roots\ of -\frac{125}{2}\left(1+\sqrt{3}i\right)=\left(-\frac{125}{2}-\frac{125\sqrt{3}}{2}i\right)^{\frac{1}{3}}=[216\ (cos180+isin180)]^{\frac{1}{3}}=$

$k=0,1,2$

$When\ k=0 \qquad (216)^{\frac{1}{3}}\left(cos\frac{180+360(0)}{3}+isin\frac{180+360(0)}{3}\right)$

$=6(cos60+isin60)$

$The\ standard\ form\ of \qquad 6(cos60+isin60)=3+5.2i$

$When\ k=1 \quad (216)^{\frac{1}{3}}\left(cos\frac{180+360(1)}{3}+isin\frac{180+360(1)}{3}\right)$

$=6(cos180+isin180)$

$The\ standard\ form\ of \qquad 6(cos180+isin180)=-6+0i$

$When\ k = 2 \quad (216)^{\frac{1}{3}}\left(cos\frac{180+360(2)}{3} + isin\frac{180+360(2)}{3}\right)$

$= 6(cos180 + isin180)$

$The\ standard\ form\ of \qquad 6(cos180 + isin180) = -6 + 0i$

5. $cube\ roots\ of\ -4\sqrt{2}(1-i) = (-4\sqrt{2} + 4\sqrt{2}i)^{\frac{1}{3}} = [8\ (cos135 + isin135)]^{\frac{1}{3}} =$
$k = 0,1,2$

$When\ k = 0 \quad (8)^{\frac{1}{3}}\left(cos\frac{135+360(0)}{3} + isin\frac{135+360(0)}{3}\right) = 2(cos45 + isin45)$

$The\ standard\ form\ of \qquad 2(cos45 + isin45) = \sqrt{2} + \sqrt{2}i$

$When\ k = 1 \quad (8)^{\frac{1}{3}}\left(cos\frac{135+360(1)}{3} + isin\frac{135+360(1)}{3}\right)$

$= 2(cos165 + isin165)$

$The\ standard\ form\ of \qquad 2(cos165 + isin165) = -1.9 + 0.5i$

$When\ k = 2 \quad (8)^{\frac{1}{3}}\left(cos\frac{135+360(2)}{3} + isin\frac{135+360(2)}{3}\right)$

$= 6(cos285 + isin285)$

$The\ standard\ form\ of \qquad 6(cos285 + isin285) = 1.6 - 5.8i$

part four: −in exercises 1 8 use theorem A5 on page A64 to find all the solutions of the equation an

1. $x^4 - i = 0 \qquad x^4 = i$

$x = (i)^{\frac{1}{4}} = (cos90 + isin90)^{\frac{1}{4}}$

$When\ k = 0 \quad (1)^{\frac{1}{4}}\left(cos\frac{90+360(0)}{4} + isin\frac{90+360(0)}{4}\right) = (cos22.5 + isin22.5)$

$= 0.92 + 0.38i$

$When\ k = \ (1)^{\frac{1}{4}}\left(cos\frac{90+360(1)}{4} + isin\frac{90+360(1)}{4}\right) = (cos112.5 + isin112.5)$

$= -0.38 + 0.92i$

$When\ k = 2 \quad (1)^{\frac{1}{4}}\left(cos\frac{90+360(2)}{4} + isin\frac{90+360(2)}{4}\right) = (cos202.5 + isin202.5)$

$= -0.92 - 0.38i$

$When\ k=3\quad (1)^{\frac{1}{4}}\left(cos\frac{90+360(3)}{4}+isin\frac{90+360(3)}{4}\right)=(cos292.5+isin292.5)$

$=0.38-0.92i$

2. $x^5+243=0$ $\qquad x^5=-243$

$x=(-243)^{\frac{1}{5}}=[243(cos180+isin180)]^{\frac{1}{5}}$

$When\ k=0\quad (243)^{\frac{1}{5}}\left(cos\frac{180+360(0)}{5}+isin\frac{180+360(0)}{5}\right)=3(cos36+isin36)$

$=2.43+1.76i$

$When\ k=1\quad (243)^{\frac{1}{5}}\left(cos\frac{180+360(1)}{5}+isin\frac{180+360(1)}{5}\right)=3(cos108+isin108)$

$=-0.92+2.85i$

$When\ k=2\quad (243)^{\frac{1}{5}}\left(cos\frac{180+360(2)}{5}+isin\frac{180+360(2)}{5}\right)=3(cos180+isin180)$

$=-3+0i$

$When\ k=3\ (243)^{\frac{1}{5}}\left(cos\frac{180+360(3)}{5}+isin\frac{180+360(3)}{5}\right)$

$=3(cos252+isin252)=-0.92-2.85i$

$When\ k=4\quad (243)^{\frac{1}{5}}\left(cos\frac{180+360(4)}{5}+isin\frac{180+360(4)}{5}\right)=3(cos324+isin324)$

$=2.43-1.76i$

3. $x^3+64i=0$ $\qquad x^5=-64i$

$x=(-64i)^{\frac{1}{3}}=[64(cos270+isin270)]^{\frac{1}{3}}$

$When\ k=0\quad (64)^{\frac{1}{3}}\left(cos\frac{270+360(0)}{3}+isin\frac{270+360(0)}{3}\right)=4(cos90+isin90)$

$=0+4i$

$When\ k = 1 \qquad (64)^{\frac{1}{3}}\left(cos\frac{270+360(1)}{3} + isin\frac{270+360(1)}{3}\right) = 4(cos210 + isin210)$

$= -3.46 - 2i$

$When\ k = 2 \qquad (64)^{\frac{1}{3}}\left(cos\frac{270+360(2)}{3} + isin\frac{270+360(2)}{3}\right) = 4(cos330 + isin330)$

$= 3.46 - 2i$

4. $x^3 - (1 - i) = 0 \qquad x^3 = 1 - i$

$x = (1 - i)^{\frac{1}{3}} = \left[\sqrt{2}(cos315 + isin315)\right]^{\frac{1}{3}}$

$When\ k = 0 \qquad \left(\sqrt{2}\right)^{\frac{1}{3}}\left(cos\frac{315+360(0)}{3} + isin\frac{315+360(0)}{3}\right) = \sqrt[3]{2}(cos105 + isin105)$

$= -0.3 + 1.2i$

$When\ k = 1 \qquad \left(\sqrt{2}\right)^{\frac{1}{3}}\left(cos\frac{315+360(1)}{3} + isin\frac{315+360(1)}{3}\right) = \sqrt[3]{2}(cos225 + isin225)$

$= -0.9 - 0.9i$

$When\ k = 2 \qquad \left(\sqrt{2}\right)^{\frac{1}{3}}\left(cos\frac{315+360(2)}{3} + isin\frac{315+360(2)}{3}\right) = \sqrt[3]{2}(cos345 + isin345)$

$= 1.2 - 0.3i$

5. $x^3 + 1 = 0 \qquad x^3 = -1$

$x = (-1)^{\frac{1}{3}} = (cos180 + isin180)^{\frac{1}{3}}$

$When\ k = 0 \qquad (1)^{\frac{1}{3}}\left(cos\frac{180+360(0)}{3} + isin\frac{180+360(0)}{3}\right) = (cos60 + isin60) = \frac{1}{2} + \frac{\sqrt{3}}{2}i$

$When\ k = 1 \qquad (1)^{\frac{1}{3}}\left(cos\frac{180+360(1)}{3} + isin\frac{180+360(1)}{3}\right) = (cos180 + isin180) = -1 + 0i$

$When\ k = 2 \qquad (1)^{\frac{1}{3}}\left(cos\frac{180+360(2)}{3} + isin\frac{180+360(2)}{3}\right) = (cos300 + isin300) = \frac{1}{2} - \frac{\sqrt{3}}{2}i$

6. $x^4 - 81 = 0 \qquad x^4 = 81$

$x = (81)^{\frac{1}{4}} = [81(cos0 + isin0)]^{\frac{1}{4}}$

$When\ k = 0 \quad (81)^{\frac{1}{4}}\left(cos\frac{0+360(0)}{4} + isin\frac{0+360(0)}{4}\right) = 3(cos0 + isin0) = 3 + 0i$

$When\ k = 1 \quad (81)^{\frac{1}{4}}\left(cos\frac{0+360(1)}{4} + isin\frac{0+360(1)}{4}\right) = 3(cos90 + isin90) = 0 + 3i$

$When\ k = 2 \quad (81)^{\frac{1}{4}}\left(cos\frac{0+360(2)}{4} + isin\frac{0+360(2)}{4}\right) = 3(cos180 + isin180) = -3 + 0i$

$When\ k = 3 \quad (81)^{\frac{1}{4}}\left(cos\frac{0+360(3)}{4} + isin\frac{0+360(3)}{4}\right) = 3(cos270 + isin270)$

$= 0 - 3i$

$7.\ x^6 - 64i = 0 \qquad x^6 = 64i$

$x = (64i)^{\frac{1}{6}} = [64(cos90 + isin90)]^{\frac{1}{6}}$

$When\ k = 0 \quad (64)^{\frac{1}{6}}\left(cos\frac{90+360(0)}{6} + isin\frac{90+360(0)}{6}\right) = 2(cos15 + isin15) = 1.9 + 0.5i$

$When\ k = 1 \quad (64)^{\frac{1}{6}}\left(cos\frac{90+360(1)}{6} + isin\frac{90+360(1)}{6}\right) = 2(cos75 + isin75) = 0.5 + 1.9i$

$When\ k = 2 \quad (64)^{\frac{1}{6}}\left(cos\frac{90+360(2)}{6} + isin\frac{90+360(2)}{6}\right) = 2(cos135 + isin135) = -\sqrt{2} + 0i$

$When\ k = 3 \quad (64)^{\frac{1}{6}}\left(cos\frac{90+360(3)}{6} + isin\frac{90+360(3)}{6}\right) = 3(cos195 + isin195)$

$= -2.9 - 0.8i$

$When\ k = 4 \quad (64)^{\frac{1}{6}}\left(cos\frac{90+360(4)}{6} + isin\frac{90+360(4)}{6}\right) = 2(cos255 + isin255) = -0.5 - 1.9i$

$When\ k = 5 \quad (64)^{\frac{1}{6}}\left(cos\frac{90+360(5)}{6} + isin\frac{90+360(5)}{6}\right) = 2(cos315 + isin315) = \sqrt{2} - \sqrt{2}i$

$8.\ x^4 + (1+i) = 0 \qquad x^4 = -1 - i$

$x = (-1-i)^{\frac{1}{4}} = \left[\sqrt{2}(cos225 + isin225)\right]^{\frac{1}{4}}$

$When\ k = 0 \quad (\sqrt{2})^{\frac{1}{4}}\left(cos\frac{225+360(0)}{4} + isin\frac{225+360(0)}{4}\right)$

$= \sqrt[4]{2}(cos56.3 + isin56.3) = 0.7 + 0.9i$

Chapter three: - Probability

Page 57 Exercise 3.1

1. a) permutation b) combination c) permutation d) combination
2. a) RBG,RGB b) BRG,BGR c) GRB,GBR d) 6 possible arrangements
3. a) 5 people b) 4 people c) 3 people d) 1 people e) 5!= 120
4. a) 5!= 120 b) C(5,3)= 5

page 58 Exercise 3.2

1. s={ 1,2,3,4,5,6}
   a) $p(1)=\frac{1}{6}$ $p(2)=\frac{1}{6}$ $p(3)=\frac{1}{6}$ $p(4)=\frac{1}{6}$ $p(5)=\frac{1}{6}$ $p(6)=\frac{1}{6}$
   so these outcomes are eqully likely outcomes b/c they has same probability
2. s={ HHH,HHT,HTH,HTT,THH,THT,TTH,TTT}
   $p(3\text{ heads})=\frac{1}{8}$ $p(3\text{ tails})=\frac{1}{8}$
   these two events are equally likely outcomes
   $p(2\text{ heads and }1\text{ tails})=\frac{3}{8}$ $p(2\text{ tails and }1\text{ heads})=\frac{3}{8}$
   also these two events are equally likely outcomes

page 60 Exercise 3.3

1. a) two tails ={ TTH,THT,HTT}
   b) At least two tails = { TTH,THT,HTT,TTT}
   c) At most two tails= { THH,HTH,THT,HHT,TTH,HTT}

Exercise 3.4

a) two heads are obtained :-{ HHT,THH,HTH}
   complementary :-{ HHH,THT,TTH,HTT,TTT}

b) at least two heads are obtained :-{ HHT,THH,HTH,HHH}
   complementary :- { THT,TTH,HTT,TTT}

c) at most two heads are obtained :-{ HTT,TTH,THT,HHT,THH,HTH}
   complementary :-{ HHH,TTT}

page 65 Exercise 3.5

I) 1.dependent 2. Independent 3. Independent
   4. independent 5. Dependent

II) 6. $P(A)=P(1^{st}\text{ will show }3)=\frac{1}{6}$
   $P(B)=P(2^{nd}\text{ will show }3)=\frac{1}{6}$

P(A and B) $= \frac{1}{6} * \frac{1}{6} = \frac{1}{36}$

7. p(1st throw will show 6) $= \frac{1}{6}$

P(2nd throw will show 6)$= \frac{1}{6}$

P(3rd throw will show 6) $= \frac{1}{6}$

P(all three times will show 6)$= \frac{1}{6} * \frac{1}{6} * \frac{1}{6} = \frac{1}{216}$

8. red= 5 white= 4 blue= 7

a) no replacement occurs :-

p(1st red) $= \frac{5}{16}$ p(2nd white)$= \frac{4}{15}$ p(3rd blue)$= \frac{7}{14}$

p(red) * p(white following red) * p(blue following red and white $= \frac{5}{16} * \frac{4}{15} * \frac{7}{14} = \frac{1}{24}$

b) replacement occurs each time:-

p(1st red) $= \frac{5}{16}$ p(2nd white)$= \frac{4}{16}$ p(3rd blue)$= \frac{7}{16}$

p(red) * p(white) *p(blue) $= \frac{5}{16} * \frac{4}{16} * \frac{7}{16} = \frac{35}{1024}$

10. a) p(A)= p(green neither shows at 2) $= \frac{5}{6}$

P(B)= p(red neither shows at 2)$= \frac{5}{6}$

P(A and B) $= \frac{5}{6} * \frac{5}{6} = \frac{25}{36}$

b) same numbers in a die are :-{ (1,1)(2,2)(3,3)(4,4)(5,5)(6,6) }

p(both shows same number) $= \frac{6}{36} = \frac{1}{6}$

c) different numbers in a die are :-{ (1,2)(1,3)(1,4)(1,5) (1,6) (2,1) (2,3)(2,4)(2,5) (2,6)(3,1)(3,2)(3,4)(3,5)(3,6)(4,1)(4,2)(4,3)(4,5)(4,6)(5,1)(5,2)(5,3)(5,4)(5,6)(6,1)(6,2)(6,3)(6,4) (6,5)

P(both shows different numbers):- $\frac{30}{36} = \frac{5}{6}$

d) P(A)=p(red shows2) $= \frac{1}{6}$

P(B)=P(green shows any other number) $= \frac{5}{6}$

P(A and B) $= \frac{1}{6} * \frac{5}{6} = \frac{5}{36}$

12. drawer 1 = 6 pairs of block socks and 6 coloured handkerchiefs drawer 2= 11 white handkerchiefs

P(A)= P(a pair of block socks)$= \frac{2}{6} = \frac{1}{3}$

P(B)= p(white handkerchiefs)$= \frac{11}{17}$

P(A and B) $= \frac{1}{3} * \frac{11}{17} = \frac{11}{51}$

13. S= 100 clocks 4 damaged 6 buys

P (6 clocks that declare bought can 4 of them is damaged in a shipment)=

$\frac{C(6,4)}{C(100,4)} = \frac{1}{261415}$

P (6 clocks that declare bought cant be damaged in a shipment) = $1 - \frac{1}{261415} = 0.99$

Page 71 more exercises

Exercise a

1. Bag A = 4 white and 6 red bag B = 2 white and 3 red

Ⅰ. P(white) $= \frac{6}{15} = \frac{2}{5}$

Ⅱ. P(not white)= $\frac{9}{15} = \frac{3}{5}$

Exercise b

1. Bag A = 3 red and 7 white bag B= 2 red and 3 white

P (white) $= \frac{10}{15} = \frac{2}{3}$

p(red)$= \frac{5}{15} = \frac{1}{3}$

P (white or red) $= \frac{2}{3} + \frac{1}{3} = 1$

2. $p\ (\ at\ least\ two\ diamonds\ ) = \frac{C(13,2)}{C(52,2)} + \frac{C(13,3)}{C(52,3)} + \frac{C(13,4)}{C(52,4)} + \frac{C(13,5)}{C(52,5)} + \frac{C(13,6)}{C(52,6)} + \frac{C(13,7)}{C(52,7)} + \frac{C(13,8)}{C(52,8)} + \frac{C(13,9)}{C(52,9)} + \frac{C(13,10)}{C(52,10)} + \frac{C(13,11)}{C(52,11)} + \frac{C(13,12)}{C(52,12)} + \frac{C(13,13)}{C(52,13)} = 0.075$

3.

Age(yrs)	46	47	48
Number	10	18	2

i) p(47 yrs old) $= \frac{18}{30} = \frac{3}{5}$ ii)p(not 47 yrs old) $= \frac{12}{30} = \frac{2}{5}$

iii)p(over 47 yrs old) $= \frac{2}{30} = \frac{1}{15}$

4. p(A) =p(servant) = $\frac{7}{26}$ p(B) = p(boy) =$\frac{3}{26}$

p(A or B) =$\frac{7}{26}+\frac{3}{26}=\frac{10}{26}$

5.a) p(red card) = $\frac{26}{52}=\frac{1}{2}$ b) p(black queen) =$\frac{2}{52}=\frac{1}{26}$

c) p(a king)= $\frac{4}{52}=\frac{1}{13}$ d) p(king of spades) = $\frac{1}{52}$

6.

grade	A	A-	B+	B	B-	C+	C	C-	D+	D	D-	E
No of students	2	6	7	8	8	14	15	9	7	5	3	1

i) P(B+ and above) = $\frac{15}{85}=\frac{3}{17}$ ii) p(will make it to the university) = $\frac{45}{85}=\frac{9}{17}$

iii) p(poor students) = $\frac{9}{85}$

7. crate of soda = 16 bottles of Fanta and 8 bottles of sprites

P(Fanta) = $\frac{16}{24}=\frac{2}{3}$ p(sprites) = $\frac{8}{24}=\frac{1}{3}$

8. s= 10

i) number pairs their sum is 6 = { (0,6)(1,5)(4,2) }

P(sum is 6) = $\frac{3}{10}$

ii) number pairs their sum is 9= {(0,9)(1,8) (2,7)(3,6)(4,5)}

P(sum is 9)= $\frac{5}{10}=\frac{1}{2}$

iii) number pairs their sum is 12={(3,9)(4,8)(5,7)}

P (sum is 12) = $\frac{3}{10}$

9. a box contains 3red ,4 blue , 3 green s= 10

i) p(blue bead)= $\frac{4}{10}=\frac{2}{5}$ ii) p(red bead) = $\frac{3}{10}$

iii) p(blue or red bead) = $\frac{4}{10}+\frac{3}{10}=\frac{7}{10}$

10. i) s= 36

ii) number pairs both are odd= {(1,1)(1,3)(1,4)(3,1)(3,3)(3,5)(5,1)(5,3)(5,5)}

P(both numbers are odd) = $\frac{9}{36} = \frac{1}{4}$

iii) number pairs both are even={(2,2)(2,4)(2,6)(4,2)(4,4)(4,6)(6,2)(6,4)(6,6)}

P(both numbers are even)= $\frac{9}{36} = \frac{1}{4}$

iv) number pairs their sum is odd =

{(1,2)(2,1)(1,4)(4,1)(1,6)(6,1)(2,3)(3,2)(3,4)(4,3)(2,5)(5,2)(4,5)(5,4)(5,6)(6,5)(3,6)(6,3)}

P(that their sum is odd)= $\frac{18}{36} = \frac{1}{2}$

v) number pairs their sum is greater is 7 =

{(2,6)(3,5)(4,4)(5,3)(6,2)(3,6)(4,5)(5,4)(6,3)(4,6)(6,4)(5,5)(6,5)(5,6)(6,6)}

P(sum is greater than 7)= $\frac{15}{36}$

vi)p(A)= p(1st throw appears 3) = $\frac{1}{6}$ P(B)=p(1st throw appears 4)= $\frac{1}{6}$

p(A or B)= $\frac{1}{6} + \frac{1}{6} = \frac{2}{6} = \frac{1}{3}$

p(A)= p(2nd throw appears 3) = $\frac{1}{6}$ P(B)=p(2nd throw appears 4)= $\frac{1}{6}$

p(A or B)= $\frac{1}{6} + \frac{1}{6} = \frac{2}{6} = \frac{1}{3}$

so p(A or B) on 1st throw and p(A or B) on 2nd throw is = $\frac{1}{3} * \frac{1}{3} = \frac{1}{9}$

Answers of chapter four

Section 4.1

1.

Class limit	10-19	20-29	30-39	40-49	50-59
Frequency	4	3	2	5	6

a) The upper class limit of the first class interval (10-19) is 19

b)

Class boundary	9.5-19.5	19.5-29.5	29.5-39.5	39.5-49.5	49.5-59.5

$$\therefore \textit{the lower class boundry of the last class interval}$$
$$(49.5 - 59.5) is\ 49.5$$

c) The class size:-[(upper class limit –lower class limit)+1]=[(19-10)+1]=10
So the class size is 10

d)

Class marks	14.5	24.5	34.5	44.5	54.5

$$\therefore \textit{the midpoints}(\textit{class marks})\textit{are}: -14.5,24.5,34,5,44.5, \textit{and}\ 54.5$$

2.

Class limit	1.10-1.49	1.50-1.89	1.90-2.29

a)

Class boundaries	1.095-1.495	1.495-1.895	1.895-2.295

The class boundaries of the three class intervals are 1.095-1.495, 1.495-1.895, 1.895-2.295.

b) The unit is 0.01

c) The class size is 0.4

Section 4.2

1. a) 31,36,32,39,40,32,37
   First arrange the data in (descending or in ascending)
   31,32,32,36,37,39,40
   - The mode is 32
   - The Median is 36
   - The mean = $\frac{\sum x}{n} = \frac{31+32+32+36+37+39+40}{7} = 35.28$

   b) 14, 20,17,17,15,19,18,14

   First arrange the data in (descending or in ascending)

   14, 14,15,17,17,18,19,20

   - The mode is 14 and 17
   - The median is $\frac{17+17}{2} = 17$
   - The mean = $\frac{\sum x}{n} = \frac{14+14+15+17+17+18+19+20}{8} = 16.75$

   c)

   i.

Number of family members	2	3	4	5	6	7
Frequency	3	6	9	8	7	2

   - The mode is 4

   ii.

Number of family members	2	3	4	5	6	7
Frequency	3	6	9	8	7	2
Cumulative frequency	3	9	18	26	33	35

   - The median is 18th place which is 4

Number of family members	Frequency	FX
2	3	6
3	6	18
4	9	36
5	8	40
6	7	42
7	2	14
	$\sum F = 35$	$\sum FX = 156$

- The mean is $\frac{\sum FX}{\sum F} = \frac{156}{35} = 4.46$

2.

Class	30-39	40-49	50-59	60-69	70-79
frequency	3	13	14	6	4

I) Modal class(class with highest frequency)= 50-59

II) Median mark =L_m+$\left(\frac{\frac{N}{2}-FB}{Fm}\right)I$

L_m (lower class boundaries of median class) =49.5 N ($\sum F$) $= 40$

FB ($\sum F\ before\ median\ class$) $= 16$

F_m (frequency of median class) =14 $I(class\ interval) = 10$

- Median mark =49.5+$\left(\frac{\frac{40}{2}-16}{14}\right)10$ =52.36

III)

Class limit	Frequency(f)	Class mark(x)	FX
30-39	3	34.5	103.5
40-49	13	44.5	578.5
50-59	14	54.5	763
60-69	6	64.5	387
70-709	4	74.5	298
	$\sum F = 40$		$\sum FX = 2130$

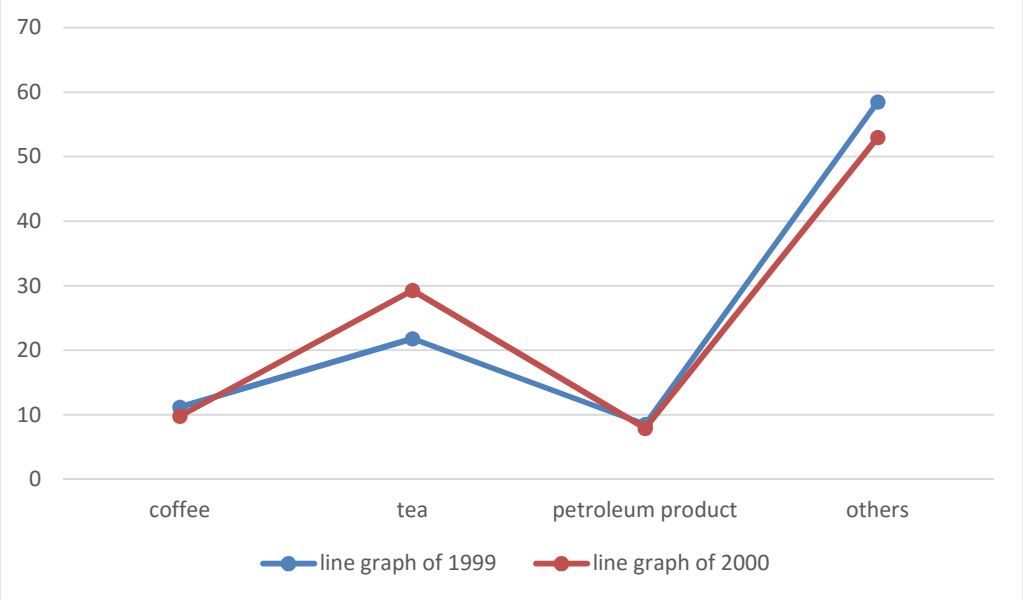
- The mean score is =$\frac{\sum FX}{\sum F} = \frac{2130}{40} = 53.25$

Section 4.3

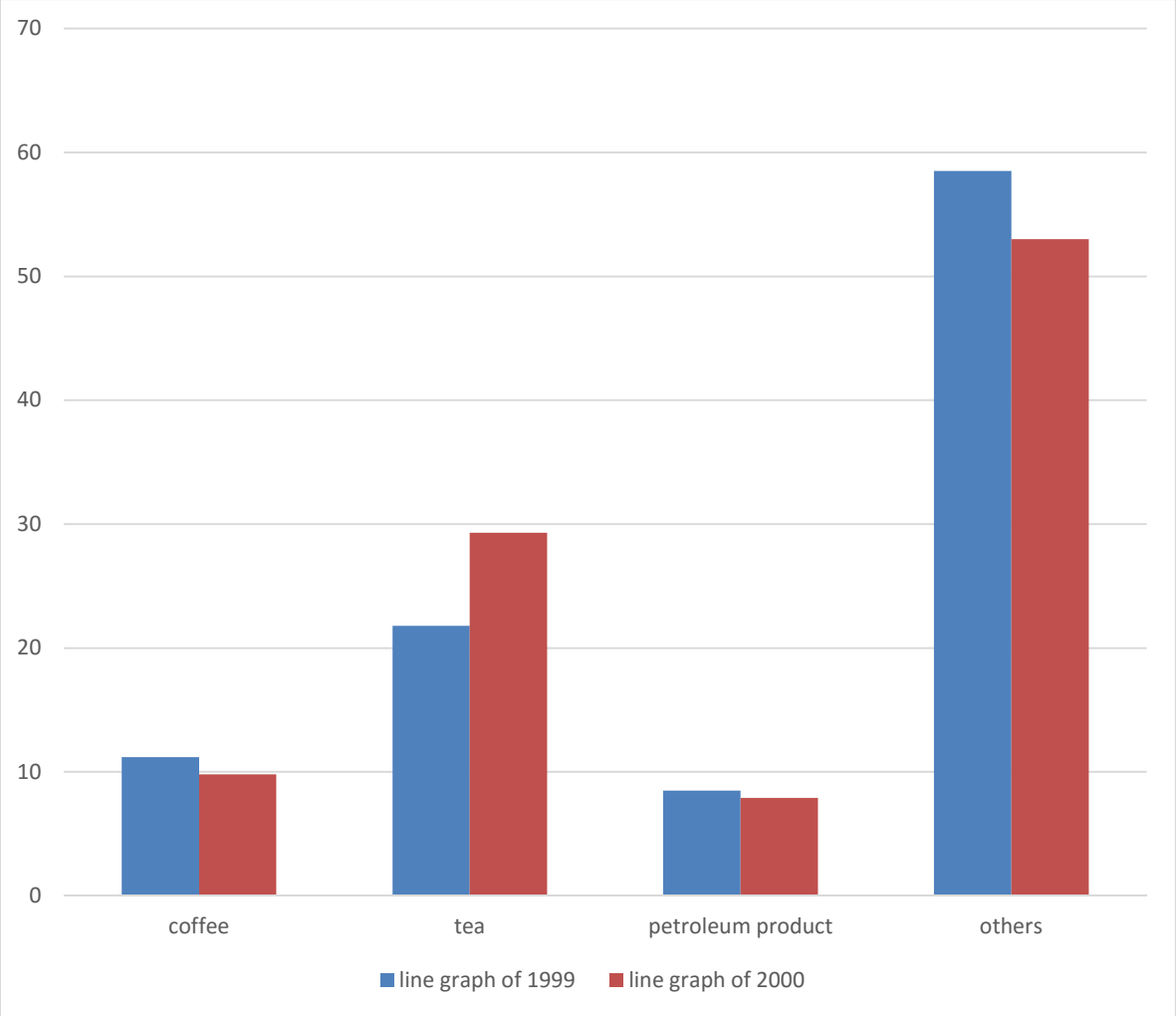
1.

Year	Coffee	Tea	Petroleum Product	Others
1999	11.2	218	8.5	58.5
2000	9.8	29.3	7.9	53

A. Line Graph

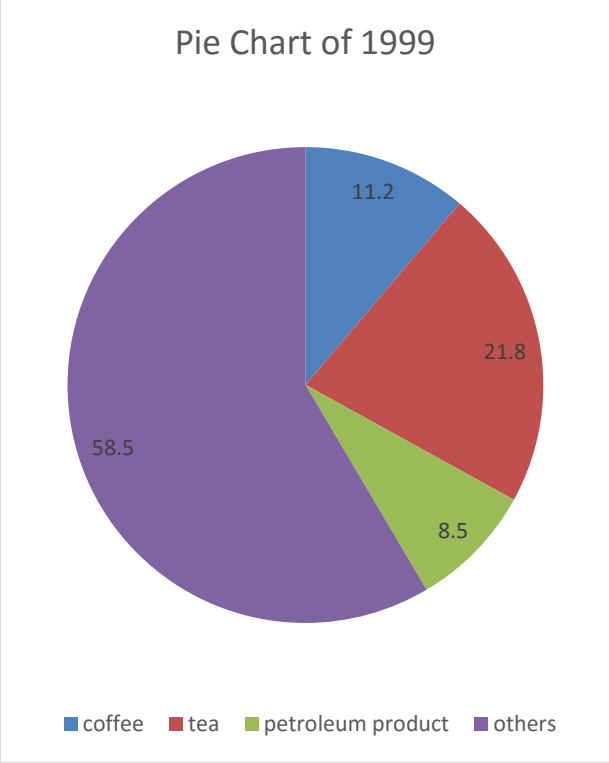


B. Bar Graph


70
60
50
40
30
20
10
0
coffee
tea
petroleum product
others
line graph of 1999
line graph of 2000

C.1. Pie Chart

- $11.2\% = \frac{11.2}{100} \times 360 = 40.32$
- $21.8\% = \frac{21.8}{100} \times 360 = 78.48$
- $8.5\% = \frac{8.5}{100} \times 360 = 30.6$
- $58.5\% = \frac{58.5}{100} \times 360 = 210.6$



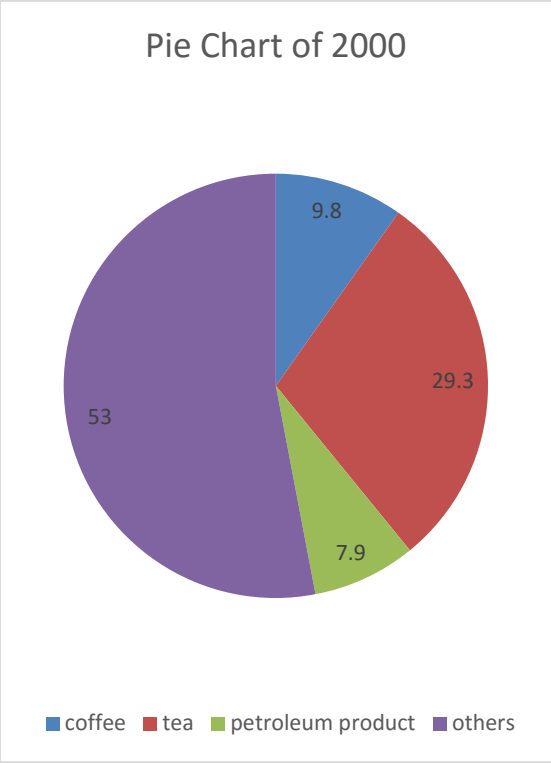
C.2. Pie Chart

$9.8\% = \frac{9.8}{100} \times 360 = 35.28$

$29.3\% = \frac{29.3}{100} \times 360 = 105.48$

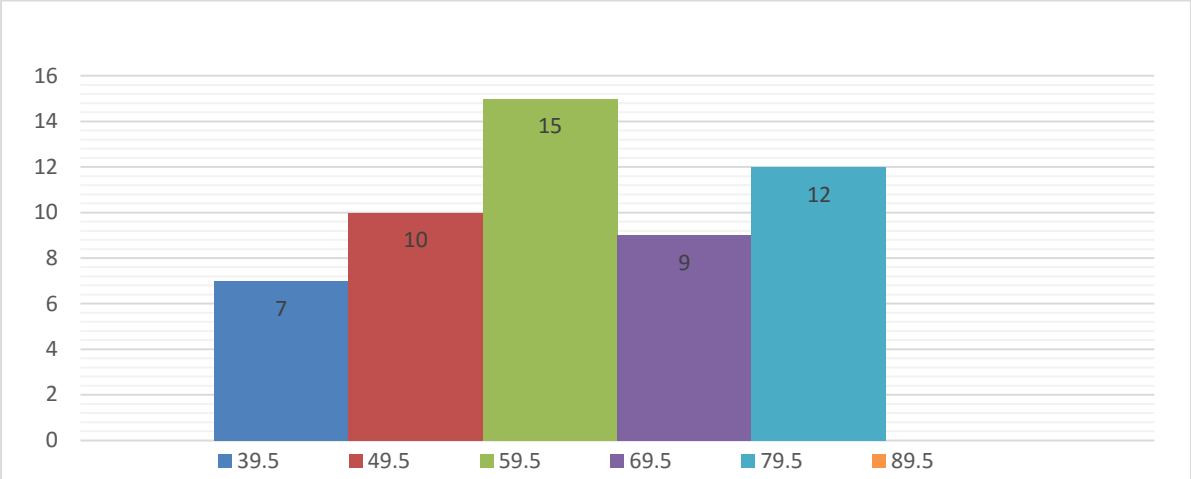
$7.9\% = \frac{7.9}{100} \times 360 = 28.44$

$53\% = \frac{53}{100} \times 360 = 190.8$



Section 4.4

Class	40-49	50-59	60-69	70-79	80-89
Frequency	7	10	15	9	12



Section 4.5

1.

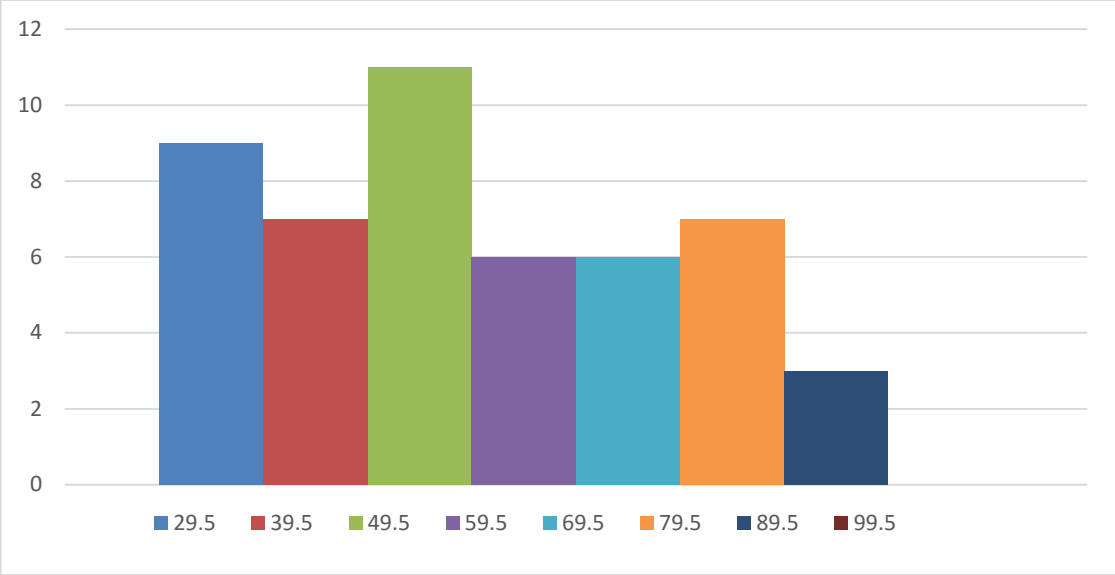
I)

Class	Frequency
30-39	9
40-49	7
50-59	11
60-69	6
70-79	6
80-89	7
90-99	3

II)

Class	Frequency(F)	Class mark(x)	FX
30-39	9	34.5	310.5
40-49	7	44.5	311.5
50-59	11	54.5	599.5
60-69	6	64.5	387
70-79	6	74.5	447
80-89	7	84.5	591.5
90-99	3	94.5	283.5
	$\sum F = 49$		$\sum FX = 2930.5$

III)



2. a) 14,10,11,15,19,10,12
   First arrange the data in ascending or in descending
   10,10,11,12,14,15,19
   The median is 12
   b) 55, 50, 51,60,55,54
   First arrange the data in ascending or in descending
   50, 51, 54,55,55,60
   The median is $\frac{54+55}{2} = 54.5$
3.

X	5	6	7	8	9	10
frequency	7	12	4	5	8	4

i)

X	Frequency(F)	FX
5	7	35
6	12	72
7	4	28
8	5	40
9	8	72
10	4	40
	$\sum F = 40$	$\sum FX = 287$

- mean $=\frac{\sum FX}{\sum F} = \frac{287}{40} = 7.175$

i) mode =6

ii)

X	Frequency(F)	Cumulative frequency
5	7	7
6	12	19
7	4	23
8	5	28
9	8	36
10	4	40

- Median = $\frac{20th+21th}{2} = \frac{7+7}{2} = 7$

4.

Class	30-34	35-39	40-44

i) Upper class limit of first class (30-34) is 34

ii) Lower class boundary of second class (35-39) is 34.5

iii) The class width is 5

Section 4.6

class	Frequency(F)	Class mark(X)	d(x-A)	Fd
115-119	23	117	-10	-230
120-124	17	122	-5	-85
125-129	20	127	0	0
130-134	16	132	5	80
	$\sum F = 76$			$\sum Fd = -235$

Mean=A+$\frac{\sum Fd}{\sum F} = 127 + \frac{-235}{76} = 123.9$

Section 4.7

Mass(g)	470-479	480-489	490-499	500-509	510-519	520-529	530-539
Number of loaves	5	10	16	2	14	9	6

I) By assumed mean (take A=504.5)

Class	Frequency(F)	Class mark(X)	d(x-A)	Fd
470-479	5	474.5	-30	-150
480-489	10	484.5	-20	-200
490-499	16	494.5	-10	-160
500-509	2	504.5	0	0
510-519	14	514.5	10	140
520-529	9	524.5	20	180
530-539	6	534.5	30	180
	$\sum F = 62$			$\sum Fd = -10$

Mean =$A + \frac{\sum Fd}{\sum F}$= 504.5+$\frac{-10}{62} = 504.3$

II) By coding method (take A=504.5)

Class	Frequency(F)	Class mark(X)	T=$\frac{(x-A)}{c}$	Ft
470-479	5	474.5	-3	-15
480-489	10	484.5	-2	-20
490-499	16	494.5	-1	-16
500-509	2	504.5	0	0
510-519	14	514.5	1	14
520-529	9	524.5	2	18
530-539	6	534.5	3	18
	$\sum F = 62$			$\sum Ft = -1$

$$mean = A + \frac{c\sum Ft}{\sum F}$$

$$mean = 504.5 + \frac{10(-1)}{62} = 504.3$$

Section *4.8*

Marks		30-39	40-49	50-59	60-69	70-79	80-89	90-99
Number of students	4A	3	4	9	3	8	5	3
	4B	2	6	12	10	5	4	1

i) Mean and standard deviation of 4A

Class	Frequency(F)	Class mark(x)	FX	$(x-\bar{x})^2$	$F(x-\bar{x})^2$
30-39	3	34.5	103.5	918.09	2754.27
40-49	4	44.5	178	412.09	1648.36
50-59	9	54.5	490.5	106.09	954.81
60-69	3	64.5	193.5	0.09	0.27
70-79	8	74.5	596	94.09	752.72
80-89	5	84.5	422.5	388.09	1940.45
90-99	3	94.5	283.5	882.09	2646.27
	$\sum F = 35$		$\sum FX = 2267.5$		$\sum F(x-\bar{x})^2 = 10697.15$

- Mean=$\frac{\sum Fx}{\sum F} = \frac{2267.5}{35} = 64.8$
- Variance=$\frac{\sum F(x-\bar{x})^2}{\sum F} = \frac{10697.15}{35}$=305.6
- Standard deviation=$\sqrt{variance} = \sqrt{305.6} = 17.5$

ii) Mean and standard deviation of 4B

Class	Frequency(F)	Class mark(x)	FX	$(x-\bar{x})^2$	$F(x-\bar{x})^2$
30-39	2	34.5	69	702.25	1404.5
40-49	6	44.5	267	272.25	1633.5
50-59	12	54.5	654	42.25	507
60-69	10	64.5	645	12.25	122.5
70-79	5	74.5	372.5	182.25	911.25
80-89	4	84.5	338	552.25	2209
90-99	1	94.5	94.5	1122.25	1122.25
	$\sum F = 40$		$\sum FX = 2440$		$\sum F(x-\bar{x})^2 = 7910$

- Mean=$\frac{\sum Fx}{\sum F} = \frac{2440}{40} = 61$
- Variance=$\frac{\sum F(x-\bar{x})^2}{\sum F} = \frac{7910}{40}$=197.75
- Standard deviation=$\sqrt{variance} = \sqrt{197.75} = 14.1$

1. 6,9,10,13,15,17
   - Mean:- $\frac{\sum x}{n} = \frac{6+9+10+13+15+17}{6} = 11.7$
   - Variance=$\frac{\sum(x-\bar{x})^2}{n}$

$$= \frac{(6-11.7)^2 + (9-11.7)^2 + (10-11.7)^2 + (13-11.7)^2 + (15-11.7)^2 + (17-11.7)^2}{6}$$

$$= \frac{32.49+7.29+2.89+1.69+10.89+28.09}{6} = 13.89$$

- Standard deviation=$\sqrt{variance} = \sqrt{13.89} = 3.7$

a) 26,29,30,33,35,37

- Mean:- $\frac{\sum x}{n} = \frac{26+29+30+33+35+37}{6} = 31.7$

- Variance$=\frac{\sum(x-\bar{x})^2}{n}$

$$=\frac{(26-31.7)^2+(29-31.7)^2+(30-31.7)^2+(33-31.7)^2+(35-31.7)^2+(37-31.7)^2}{6}$$

$$=\frac{32.49+7.29+2.89+1.69+10.89+28.09}{6}=13.89$$

- Standard deviation$=\sqrt{variance}=\sqrt{13.89}=3.7$

b) 30,45,50,65,75,85

- Mean:- $\frac{\sum x}{n}=\frac{30+45+50+65+75+85}{6}=58.3$
- Variance$=\frac{\sum(x-\bar{x})^2}{n}$

$$=\frac{(30-58.3)^2+(45-58.3)^2+(50-58.3)^2+(65-58.3)^2+(75-58.3)^2+(85-58.3)^2}{6}$$

$$=\frac{800.89+176.89+68.89+44.89+278.89+712.89}{6}=347.2$$

- Standard deviation$=\sqrt{variance}=\sqrt{347.2}=18.6$

Section 4.9

1. 105,109,113,101,120 (A=100)

- Mean $= \frac{\sum(x-A)}{n}$

$$= \frac{(105-100)+(109-100)+(113-100)+(101-100)+(120-100)}{5}$$

$$= \frac{5+9+13+1+20}{5} = 9.6$$

2. Assumed mean (A=439.5)

Class	Frequency(F)	Class mark(X)	d(x-A)	Fd
415-424	13	419.5	-20	-260
425-434	24	429.5	-10	-240
435-444	12	439.5	0	0
445-454	5	449.5	10	50
455-464	17	459.5	20	340
	$\sum F = 71$			$\sum Fd = -110$

- Mean $= A + \frac{\sum Fd}{\sum F} = 439.5 + \frac{-110}{71} = 437.9$

4. Mean and standard deviation

Class	Frequency(F)	Class mark(x)	FX	$(x-\bar{x})^2$	$F(x-\bar{x})^2$
65-69	6	67	402	151.29	907.74
70-74	10	72	720	53.29	532.9
75-79	12	77	924	5.29	63.48
80-84	9	82	738	7.29	65.61
85-89	11	87	957	59.29	652.19
90-94	5	92	460	161.29	806.45
	$\sum F = 53$		$\sum FX = 4201$		$\sum F(x-\bar{x})^2 = 3028.37$

- Mean=$\frac{\sum Fx}{\sum F} = \frac{4201}{53} = 79.3$
- Variance=$\frac{\sum F(x-\bar{x})^2}{\sum F} = \frac{3028.37}{53}$=57.14
- Standard deviation=$\sqrt{variance} = \sqrt{57.14} = 7.56$

5. Mean $= \frac{\sum x}{n}$ Variance$= \frac{\sum(x-\bar{x})^2}{n}$

$= \frac{x+13+10+y+9}{5} = 12$ $\frac{(x-12)^2+(13-12)^2+(10-12)^2+(y-12)^2+(9-12)^2}{5} = 6$

$= \frac{x+y+32}{5} = 12$ $\frac{x^2-24x+144+1+4+y^2-24y+144+9}{5} = 6$

$= x + y + 32 = 60$ $\frac{x^2+y^2-24x-24y+302}{5} = 6$

$= x + y = 60 - 32$ $x^2 + y^2 - 24x - 24y + 302 = 30$

$= x + y = 28$ $x^2 + y^2 - 24x - 24y = -272$

$= y = 28 - x$ $x^2 + (28 - x)^2 - 24x - 24(28 - x) = -272$

$x^2 + 784 - 56x + x^2 - 24x - 672 + 24x = -272$

$2x^2 - 56x + 384 = 0$

$X_1=12$ $X_2=16$

If x= 16 then y= 28-16 =12

$\therefore$ *so we get* $x = 16$ *becouse of the condition if* $x > y$ *and then* $y = 12$

6. A) 9, 11,13,16,17
   - Mean:- $\frac{\sum x}{n} = \frac{9+11+13+16+17}{5} = 13.2$
   - Variance$=\frac{\sum(x-\bar{x})^2}{n}$

$= \frac{(9 - 13.2)^2 + (11 - 13.2)^2 + (13 - 13.2)^2 + (16 - 13.2)^2 + (17 - 13.2)^2}{5}$

$= \frac{17.64+4.84+0.04+7.84+14.4}{5} = 8.96$

- Standard deviation=$\sqrt{variance} = \sqrt{8.96} = 2.99$

b) 21, 23,25,28,29

- Mean:- $\frac{\sum x}{n} = \frac{21+23+25+28+29}{5} = 25.2$
- Variance=$\frac{\sum(x-\bar{x})^2}{n}$

$$= \frac{(21-25.2)^2 + (23-25.2)^2 + (25-25.2)^2 + (28-25.2)^2 + (29-25.2)^2}{5}$$

$$= \frac{17.64+4.84+0.04+7.84+14.4}{5} = 8.96$$

- Standard deviation=$\sqrt{variance} = \sqrt{8.96}$ =2.99

c) 108, 132,156,192,204

- Mean:- $\frac{\sum x}{n} = \frac{108+132+156+192+204}{5} = 158.4$
- Variance=$\frac{\sum(x-\bar{x})^2}{n}$

$$= \frac{(108-158.4)^2 + (132-158.4)^2 + (156-158.4)^2 + (192+158.4) + (204-158.4)^2}{5}$$

$$= \frac{2540.16+696.96+5.76+1128.96+2079.36}{5} = 1290.24$$

- Standard deviation=$\sqrt{variance} = \sqrt{1290.24}$ =35.92

Answers of Chapter five: - LOGIC

1. Construct a truth table of each of the following

$a)\ [p \Longrightarrow (q \wedge \phi \sim q)] \Leftrightarrow \sim p$

P	q	$\sim P$	$\sim q$	$q \wedge \sim q$	$p \Longrightarrow (q \wedge \sim q)$	$p \Longrightarrow (q \wedge \sim q) \Leftrightarrow \sim p$
T	T	F	F	F	F	T
F	F	F	T	F	F	T
F	T	T	F	F	F	T
F	F	T	T	F	T	T

$b)\ \ (q \Longrightarrow \sim p) \Leftrightarrow\ (p \wedge q)$

P	q	$\sim P$	$p \wedge q$	$q \Longrightarrow \sim p$	$(q \Longrightarrow \sim p) \Leftrightarrow (p \wedge q)$
T	T	F	T	F	F
T	F	F	F	T	F
F	T	T	F	T	F
F	F	T	F	T	F

c) $(p \wedge q) \Leftrightarrow (q \wedge p)$

P	Q	$p \wedge q$	$q \wedge p$	$(p \wedge q) \Leftrightarrow (q \wedge p)$
T	T	T	T	T
T	F	F	F	T
F	T	F	F	T
F	F	F	F	T

d) $(p \vee q) \Leftrightarrow (q \vee p)$

P	Q	$p \vee q$	$q \vee p$	$(p \vee q) \Leftrightarrow (q \vee p)$
T	T	T	T	T
T	F	T	T	T
F	T	T	T	T
F	F	F	F	T

e) $[p \wedge (q \wedge r)] \Leftrightarrow [(p \wedge q) \wedge r]$

P	q	r	q∧r	$p \wedge (q \wedge r)$	p∧q	$(p \wedge q) \wedge r$	$[p \wedge (q \wedge r)] \Leftrightarrow [(p \wedge q) \wedge r]$
T	T	T	T	T	T	T	T
T	T	F	F	F	T	F	T
T	F	T	F	F	F	F	T
T	F	F	F	F	F	F	T
F	T	T	T	F	F	F	T
F	T	F	F	F	F	F	T
F	F	T	F	F	F	F	T
F	F	F	F	F	F	F	T

f) $[p \vee (q \vee r)] \Leftrightarrow [(p \vee q) \vee r]$

P	q	r	q ∨ r	$p \vee (q \vee r)$	p ∧ q	$(p \vee q) \vee r$	$[p \vee (q \vee r)] \Leftrightarrow [(p \vee q) \vee r]$
T	T	T	T	T	T	T	T
T	T	F	T	T	T	T	T
T	F	T	T	T	T	T	T
T	F	F	F	T	T	T	T
F	T	T	T	T	T	T	T
F	T	F	T	T	T	T	T
F	F	T	T	T	F	T	T
F	F	F	F	F	F	F	T

g) $[p \wedge (q \vee r)] \Leftrightarrow [(p \wedge q) \vee (p \wedge r)]$

P	Q	r	$q \vee r$	$p \wedge (q \vee r)$	$p \wedge q$	$p \wedge r$	$(p \wedge q) \vee (p \wedge r)$	$[p \wedge (q \vee r)] \Leftrightarrow [(p \wedge q) \vee (p \wedge r)]$
T	T	T	T	T	T	T	T	T
T	T	F	T	T	T	F	T	T
T	F	T	T	T	F	T	T	T
T	F	F	F	F	F	F	F	T
F	T	T	T	F	F	F	F	T
F	T	T	T	F	F	F	F	T
F	F	T	T	F	F	F	F	T
F	F	F	F	F	F	F	F	T

h) $[p \vee (q \wedge r)] \Leftrightarrow [(p \vee q) \wedge (p \vee r)]$

P	q	r	$q \wedge r$	$p \vee (q \wedge r)$	$p \vee q$	$p \vee r$	$(p \vee q) \wedge (p \vee r)$	$[p \vee (q \wedge r)] \Leftrightarrow [(p \vee q) \wedge (p \vee r)]$
T	T	T	T	T	T	T	T	T
T	T	F	F	T	T	T	T	T
T	F	T	F	T	T	T	T	T
T	F	F	F	T	T	F	F	F
F	T	T	T	T	T	T	T	T
F	T	T	F	F	T	T	T	F
F	F	T	F	F	F	T	F	T
F	F	F	F	F	F	F	F	T

2. De Morgan's law prove that $\sim (p \vee q)$ is equivalent to $(\sim p) \wedge (\sim q)$

P	Q	$p \vee q$	$\sim (p \vee q)$	$\sim p$	$\sim q$	$\sim p \vee \sim q$
T	T	T	F	F	F	F
T	F	T	F	F	T	F
F	T	T	F	T	F	F
F	F	F	T	T	T	T

So $\sim (p \vee q)$ is equivalent to $(\sim p) \wedge (\sim q)$ because they have same truth table

1. *Establish the logical equivalence of these compound statements*

A.

P	Q	$\sim q$	$(P \Longrightarrow q)$	$\sim (P \Longrightarrow q)$	$(P \wedge \sim q)$
T	T	F	T	F	F
T	F	T	F	T	T
F	T	F	T	F	F
F	F	T	T	F	F

B.

P	Q	$P \Longrightarrow q$	$q \Longrightarrow P$
T	T	T	T
T	F	F	T
F	T	T	F
F	F	T	T

4. *show* $P \vee (Q \wedge R)$ *and* $(p \vee Q) \wedge (p \vee R)$ are logically equivalent

P	Q	R	$Q \wedge R$	$P \vee (Q \wedge R)$	$(p \vee Q)$	$(p \vee R)$	$(p \vee Q) \wedge (p \vee R)$
T	T	T	T	T	T	T	T
T	T	F	F	T	T	T	T
T	F	T	F	T	T	T	T
T	F	F	F	T	T	T	T
F	T	T	T	T	T	T	T
F	F	T	F	F	F	T	F
F	F	F	F	F	F	F	F

So $P \vee (Q \wedge R)$ *and* $(p \vee Q) \wedge (p \vee R)$ are logically equivalent because they have same truth table.

Answers of chapter six: - calculus

Estimating a limit numerically In Exercises 5-8 complete the table and use the result to estimate the limit numerically. Determine whether the limit can be reached.

5. $\lim_{x\to 2}(5x+4)$

$\lim_{x\to 2}(5(2)+4)$

$= 10 + 4$

$= \ 14$

X	1.9	1.99	1.999	2	2.001	2.01	2.1
F(x)	13.5	13.95	13.995	14	14.005	14.05	14.5

6. $\lim_{x\to 1}(2x^2+x-4)$

$\lim_{x\to 1}(2(1)^2+1-4)$

$= (2+1-4)$

$= 3-4$

$= \ -1$

X	0.9	0.99	0.999	1	1.001	1.01	1.1
F(x)	-1.48	-1.04	-1.004	-1	-0.99	-0.94	-0.48

7. $\lim_{x\to -1}\frac{x+1}{x^2-x-2}$

$\lim_{x\to -1}\ \frac{-1+1}{(-1)^2-(-1)-2}$

$= \ \frac{0}{1+1-2}$

$= \ \frac{0}{0}$

$\lim_{x\to -1}\frac{x+1}{(x+1)(x-2)}$

$\lim_{x\to -1}\frac{1}{(x-2)}$

X	-1.1	-1.01	-1.001	-1	-0.999	-0.99	-0.9
F(x)	$-\frac{10}{13}$	$-\frac{100}{301}$	$-\frac{1000}{3001}$	$-\frac{1}{3}$	$-\frac{1000}{2999}$	$-\frac{100}{299}$	$-\frac{10}{29}$

$$\lim_{x\to -1}\frac{1}{(-1-2)}$$

$$=-\frac{1}{3}$$

8. $\lim_{x\to 0}\frac{\sin 2x}{x}$

$$\lim_{x\to 0}\frac{2\sin 2x}{2x}=2$$

x	−0.1	−0.01	−0.001	0	0.001	0.01	0.1
F(x)	1.9	1.99	1.999	2	1.9999	1.99	1.986

9. $\lim_{x\to 0}\frac{\tan x}{2x}$

$$\lim_{x\to 0}\frac{\frac{\sin x}{\cos x}}{2x}=\lim_{x\to 0}\frac{\sin x}{2x\cos x}=\lim_{x\to 0}\frac{\sin x}{x}*\frac{1}{2\cos x}$$

$$\lim_{x\to 0}\frac{1}{2\cos x}=\frac{1}{2(1)}=\frac{1}{2}$$

-0.1	-0.01	-0.001	0	0.001	0.01	0.1
0.501	0.500	0.5000	0.5	0.50000	0.5000	0.501

Evaluating Limits In Exercises 11 and 14, use the given information to evaluate each limit.

11. $\lim_{x\to c} f(x) = 4$ $\lim_{x\to c} g(x) = 8$

(A) $\lim_{x\to c}[-2g(x)]$

$\lim_{x\to c}[-2(8)]$

$= -16$

(B) $\lim_{x\to c}[f(x) + g(x)]$

$\lim_{x\to c}[4 + 8]$

$= 12$

(C) $\lim_{x\to c} \frac{f(x)}{g(x)}$

$\lim_{x\to c} \frac{4}{8}$

$= \frac{1}{2}$

(D) $\lim_{x\to c} \sqrt{f(x)}$

$\lim_{x\to c} \sqrt{4}$

$= 2$

12. $\lim_{x\to c} f(x) = 3$ $\lim_{x\to c} g(x) = -1$

(A) $\lim_{x\to c}[f(x) + g(x)]^2$

$\lim_{x\to c}[3 + (-1)]^2$

$= [2]^2$

$= 4$

(B) $\lim_{x\to c}[6f(x)g(x)]$

$\lim_{x\to c}[6(3)(-1)]$

$= [(18)(-1)]$

$= -18$

(C) $\lim_{x\to c} \frac{5g(x)}{4f(x)}$

$\lim_{x\to c} \frac{5(-1)}{4(3)}$

$= -\frac{5}{12}$

(D) $\lim_{x\to c} \frac{1}{\sqrt{f(x)}}$

$\lim_{x\to c} \frac{1}{\sqrt{3}}$

$= \frac{1}{\sqrt{3}}$

13. $f(x) = x^3$ $g(x) = \frac{\sqrt{x^2+5}}{2x^2}$

(A) $\lim_{x\to 2} f(x)$

(B) $\lim_{x\to 2} g(x)$

$\lim_{x\to 2} x^3$

$\lim_{x\to 2} 2^3$

$= 8$

$\lim_{x\to 2} \frac{\sqrt{x^2+5}}{2x^2}$

$= \frac{\sqrt{(2)^2+5}}{2(2)^2}$

$= \frac{\sqrt{4+5}}{2(4)}$

$= \frac{3}{8}$

(C) $\lim_{x\to 2}[g(x).f(x)]$

$\lim_{x\to 2}\left[\frac{\sqrt{x^2+5}}{2x^2}.x^3\right]$

$\lim_{x\to 2}\left[\frac{x^3\sqrt{x^2+5}}{2x^2}\right]$

$= \left[\frac{(2)^3\sqrt{(2)^2+5}}{2(2)^2}\right]$

$= \frac{8\sqrt{9}}{8}$

$= \frac{8(3)}{8}$

$= \frac{24}{8}$

$= 3$

(D) $\lim_{x\to 2}[g(x) - f(x)]$

$\lim_{x\to 2}\left[\frac{\sqrt{x^2+5}}{2x^2} - x^3\right]$

$\lim_{x\to 2}\left[\frac{\sqrt{(2)^2+5}}{2(2)^2} - (2)^3\right]$

$= \left[\frac{\sqrt{4+5}}{2(4)} - 8\right]$

$= \left[\frac{3}{8} - 8\right]$

$= -\frac{61}{8}$

14. $f(x) = \frac{x}{3-x}$ $\quad g(x) = \sin \pi x$

(A) $\lim_{x\to 2} f(x)$

$\lim_{x\to 2} \frac{x}{3-x}$

$\lim_{x\to 2} \frac{2}{3-2}$

$= \frac{2}{1}$

$= 2$

(B) $\lim_{x\to 2} g(x)$

$\lim_{x\to 2} \sin \pi x$

$\lim_{x\to 2} \sin 2\pi$

$= \sin 2\pi$

$= 0$

(C) $\lim_{x\to 2}[f(x).g(x)]$

$\lim_{x\to 2}\left[\frac{x}{3-x}.\sin \pi x\right]$

$\lim_{x\to 2}\left[\frac{x\sin \pi x}{3-x}\right]$

$=\left[\frac{2\sin 2\pi}{3-2}\right]$

$=\left[\frac{2\sin 2\pi}{1}\right]$

$=2(0)$

$=0$

(D) $\lim_{x\to 2}[g(x)-f(x)]$

$\lim_{x\to 2}\left[\sin \pi x-\frac{x}{3-x}\right]$

$=\sin 2\pi-\frac{2}{3-2}$

$=0-\frac{2}{1}$

$=0-2$

$=-2$

Evaluating a limit by direct substitution.

16. $\lim_{x\to 5}(10-x^2)$

$\lim_{x\to 5}(10-(5)^2)$

$=(10-25)$

$=-15$

17. $\lim_{x\to -3}(2x^2+4x+1)$

$\lim_{x\to -3}(2(-3)^2+4(-3)+1)$

$=2(9)-18+1$

$=7$

18. $\lim_{x\to -3}\frac{3x}{x^2+1}$

$\lim_{x\to -3}\frac{3(-3)}{(-3)^2+1}$

$=\frac{-9}{9+1}$

$=\frac{-9}{10}$

19. $\lim_{x\to -2}\frac{5x+3}{2x-9}$

$\lim_{x\to -2}\frac{5(-2)+3}{2(-2)-9}$

$=\frac{-10+3}{-4-9}$

$=\frac{7}{13}$

20. $\lim_{x\to -1}\sqrt{x+2}$

$\lim_{x\to -1}\sqrt{-1+2}$

$=\sqrt{1}$

$=1$

21. $\lim_{x\to 3}e^{x}$

$\lim_{x\to 3}e^{3}$

$=e^{3}$

$=20.08$

22. $\lim_{x\to \pi}\sin 2x$

$\lim_{x\to \pi}\sin 2\pi$

$=\sin 2\pi$

$=0$

23. $\lim_{x\to \frac{1}{2}} arc\sin x$

$=arc\sin(\frac{1}{2})$

$=\sin^{-1}(\frac{1}{2})$

$=30$

24. $\lim_{x\to -2}\left[\frac{1}{2}x^3-5x\right]$

$\lim_{x\to -2}\left[\frac{1}{2}(-2)^3-5(-2)\right]$

$=\frac{1}{2}(-8)+10$

$=-4+10$

$=6$

25. $\lim_{x\to -2}[x^3-6x+5]$

$\lim_{x\to -2}[(-2)^3-6(-2)+5]$

$=[-8+12+5]$

$=[-8+17]$

$=9$

26. $\lim_{x\to 4}\frac{x-1}{x^2+2x+3}$

$\lim_{x\to 4}\frac{4-1}{(4)^2+2(4)+3}$

$=\frac{3}{16+8+3}$

$=\frac{3}{27}$

$=\frac{1}{9}$

27. $\lim_{x\to 3}\frac{x^2+1}{x}$

$\lim_{x\to 3}\frac{(3)^2+1}{3}$

$=\frac{9+1}{3}$

$=\frac{10}{3}$

28. $\lim_{x\to 3}\sqrt[3]{x^2-1}$

$\lim_{x\to 3}\sqrt[3]{(3)^2-1}$

$=\sqrt[3]{9-1}$

$=\sqrt[3]{8}$

$=2$

29. $\lim_{x\to e}\ln x$

$\lim_{x\to e}\ln e$

$=\ln e$

$=1$

30. $\lim_{x\to \pi}\tan x$

$\lim_{x\to \pi}\tan \pi$

$=\tan \pi$

$=0$

31. $\lim_{x\to 1} arc\cos\frac{x}{2}$

$=arc\cos\frac{1}{2}$

$=\cos^{-1}(\frac{1}{2})$

$=60$

Section 6.2

A. Find the derivative of each function.

1. $y = x^8$

$y' = 8x^{8-1}$

$y' = 8x^7$

2. $y = \sqrt[3]{x}$

$y = (x)^{\frac{1}{3}}$

$y' = \frac{1}{3}(x)^{\frac{1}{3}-1}$

$y' = \frac{1}{3}(x)^{-\frac{2}{3}}$

3. $y = x^{-\frac{2}{5}}$

$y' = -\frac{2}{5}x^{-\frac{2}{5}-1}$

$y' = -\frac{2}{5}x^{-\frac{7}{5}}$

4. $v(r) = \frac{4}{3}\pi r^3$

$v'(r) = \frac{4}{3}(3)\pi r^{3-1}$

$v'(r) = 4\pi r^2$

5. $y(t) = 6t^{-9}$

$y'(t) = 6(-9)t^{-9-1}$

$y'^{(t)} = -54t^{-10}$

6. $f(x) = x^3 - 10x + 100$

$f'(x) = 3x^{3-1} - 10$

$f'(x) = 3x^2 - 10$

7. $g(x) = x^{100} + 50x + 1$

$g'(x) = 100x^{100-1} + 50$

$g'(x) = 100x^{99} + 50$

8. $f(x) = (2x)^3$

$f'(x) = 3(2x)^{3-1}$

$f'(x) = 3(2x)^2$

$f'(x) = 6x^2$

9. $g(x) = x^2 + \frac{1}{x^2}$

$g(x) = x^2 + x^{-2}$

$g'(x) = 2x^{2-1} + (-2)x^{-2-1}$

$g'(x) = 2x - 2x^{-3}$

$g'^{(x)} = 2x - \frac{2}{x^3}$

10. $s(t) = t^8 + 6t^7 - 18t^2 + 2t$

$s'(t) = 8t^{8-1} + 6(7)t^{7-1} - 18(2)t^{2-1} + 2$

$s'(t) = 8t^7 + 42t^6 - 36t + 2$

11. $y = \frac{x^2+4x+3}{x}$

$$y' = \frac{x(2x+4)-x^2+4x+3(1)}{x^2}$$

$$y' = \frac{2x^2+4x-x^2-4x-3}{x^2}$$

$$y' = \frac{x^2-3}{x^2}$$

$$y' = 1 - \frac{3}{x^2}$$

12. $f(x) = x - 3x^{\frac{1}{3}}$

$$f'(x) = 1 - 3(\frac{1}{3})x^{\frac{1}{3}-1}$$

$$f'(x) = 1 - x^{-\frac{2}{3}}$$

13. $y = 5x^{-4} - \frac{7}{8}x^{-2} + 3x^2 - 6$

$$y' = 5(-4)x^{-4-1} - \frac{7}{8}(-2)x^{-2-1} + 3(2)x^{2-1}$$

$$y' = -20x^{-5} + \frac{7}{4}x^{-3} + 6x$$

$$y' = -20\frac{1}{x^5} + \frac{7}{4}\frac{1}{x^3} + 6x$$

14. $y = \frac{x^{12}-2x^9+5x^{-7}}{4}$

$$y' = \frac{4(12x^{12-1}-2(9)x^{9-1}+5(-7)x^{-7-1})-(x^{12}-2x^9+5x^{-7})(0)}{(4)^2}$$

$$y' = \frac{4(12x^{11}-18x^8-35x^{-8})-0}{(4)^2}$$

$$y' = \frac{(48x^{11}-72x^8-140x^{-8})}{16}$$

$$y' = \frac{48x^{11}}{16} - \frac{72x^8}{16} - \frac{140x^{-8}}{16}$$

$$y' = 3x^{11} - \frac{9}{2}x^8 - \frac{35}{4x^8}$$

B. Differentiate the following:

1. $f(x) = x^3 - 7$

$f'(x) = 3x^{3-2}$

$f'(x) = 3x^2$

2. $f(x) = 5\sqrt{x} + x$

$f(x) = 5(x)^{\frac{1}{2}} + x$

$f'(x) = 5(\frac{1}{2})(x)^{\frac{1}{2}-1} + 1$

$f'(x) = \frac{5}{2}(x)^{-\frac{1}{2}} + 1$

3. $h(x) = \frac{1}{x} - x^4$

$h(x) = x^{-1} - x^4$

$h'(x) = -x^{-1-1} - 4x^{4-1}$

$h'(x) = -x^{-2} - 4x^3$

4. $r(y) = 11y^3 - 2y + 7$

$r'(y) = 11(3)y^{3-1} - 2$

$r'(y) = 33y^2 - 2$

5. $s(t) = t^3 - 3t^2 + 2t - \sqrt{6}$

$s'(t) = 3t^{3-1} - 3(2)t^{2-1} + 2$

$s'(t) = 3t^2 - 6t + 2$

6. $f(x) = \frac{x-1}{x}$

$f'(x) = \frac{x(1)-(x-1)(1)}{x^2}$

$f'(x) = \frac{x-x+1}{x^2}$

$f'(x) = \frac{1}{x^2}$

7. $f(x) = (x + 6x^2)(3\sqrt{x} - 2)$

$f(x) = (x + 6x^2)(3x^{\frac{1}{2}} - 2)$

$f'(x) = (1 + 6(2)x^{2-1})(3x^{\frac{1}{2}} - 2) + (x + 6x^2)(3\left(\frac{1}{2}\right)x^{\frac{1}{2}-1})$

$f'(x) = (1 + 12x)(3x^{\frac{1}{2}} - 2) + (x + 6x^2)(\frac{3}{2}x^{-\frac{1}{2}})$

$f'(x) = 3x^{\frac{1}{2}} + 36x^{\frac{3}{2}} - 2 - 24x + \frac{3}{2}x^{\frac{1}{2}} + 9x^{\frac{3}{2}}$

$f'(x) = \frac{9}{2}x^{\frac{1}{2}} + 45x^{\frac{3}{2}} - 24x - 2$

8. $f(x) = (x^5 - 3)^2$

$f'(x) = 2(x^5 - 3)(5x^4)$

$f'(x) = (x^5 - 3)(10x^4)$

$f'(x) = 10x^9 - 30x^4$

9. $y = -2x^3 + x^2 - 4x + 7$ $\quad at\ x = 2$

$first\ we\ find\ the\ value\ of\ y\ by\ substituation\ x - value\ to\ the\ function$

$y = -2(2)^3 + (2^2) - 4(2) + 7$

$y = -16 + 4 - 8 + 7$

$y = -13$

$so\ we\ have\ a\ point\ (x, y) = (2, -13)$

$and\ then\ we\ find\ slope(m)\ by\ finding\ the\ derivitive\ of\ the\ function\ and\ then\ subtitue\ x - value\ to\ derivitive\ of\ the\ function$

$y' = -6x^2 + 2x - 4$

$slope(m) = y'(x) = -6(2)^2 + 2(2) - 4$

$m = -24 + 4 - 4 = -24$

$and\ then\ we\ find\ equatin\ of\ tangent\ line = \ y - y_1 = m(x - x_1)$

$y - (-13) = -24(x - 2)$

$y + 13 = -24x + 48$

$y = -24x + 48 - 13$

$y = -24x + 35$

10. $y = 2\sqrt{x} - \frac{1}{x^2} + \frac{1}{x} + 4$ $\quad at\ x = 1$

$first\ we\ find\ the\ value\ of\ y\ by\ substituation\ x - value\ to\ the\ function$

$y = 2\sqrt{1} - \frac{1}{(1)^2} + \frac{1}{1} + 4$

$y = 2 - 1 + 1 + 4$

$y = 6$

$so\ we\ have\ a\ point\ (x, y) = (1,6)$

$and\ then\ we\ find\ slope(m)\ by\ finding\ the\ derivitive\ of\ the\ function\ and\ then\ subtitue\ x - value\ to\ derivitive\ of\ the\ function$

$y' = 2 * \frac{1}{2\sqrt{x}} + \frac{2}{x^3} - \frac{1}{x^2} = \frac{1}{\sqrt{x}} + \frac{2}{x^3} - \frac{1}{x^2}$

$slope(m) = y'(x) = \frac{1}{\sqrt{1}} + \frac{2}{(1)^3} - \frac{1}{(1)^2}$

$m = 1 + 2 - 1 = 2$

and then we find equatin of tangent line $= y - y_1 = m(x - x_1)$

$y - 6 = 2(x - 1)$

$y - 6 = 2x - 2$

$y = 2x - 2 + 6$

$y = 2x + 4$