ADDIS ABABA EDUCATIN BUREAU

2012/2020 GRADE 12 MATHEMATICS MODEL EXAMINATION

TIME ALLOWED: 3HOURS

GENERAL DIRECTIONS

THIS BOOKLET CONTAINS MATHEMATICS EXAMINATION FOR THE NATURAL SCIENCE CANDIDATES ONLY. IN THIS EXAMINATION, THERE ARE A TOTAL OF 65MULTIPLE CHOICE QUESTIONS.

THERE IS ONLY ONE BEST ANSWER FOR EACH QUESTION. CHOOSE THE BEST ANSWER FROM THE SUGGESTED OPTIONS AND BLACKEN THE LETTER OF YOUR CHOICE ON THE ANSWER SHEET. USE ONLY PENCILE TO MARK YOUR ANSWERS.

YOU WILL BE ALLOWED TO WORK ON THE EXAM FOR 3 HOURS. WHEN TIME IS CALLED, YOU MUST IMMEDIATELY STOP WORKING, PUT YOUR PENCIL DOWN, AND WAIT FOR FURTHER INSTRUCTIONS.

ANY FORM OF CHEATING OR AN ATTEMT TO CHEAT IN THE EXAMINATION WILL RESULT IN AN AUTOMATIC DISMISSAL FROM THE EXAMINATION HALL AND CANCELLATION OF YOUR SCORE(S).

PLEASE MAKE SURE THAT YOU HAVE WRITTEN ALL THE QEQUIRED INFORMATION ON THE ANSWER SHEET BEFORE YOU START TO WORK ON THE EXAMINATION.

DIRECTIONS: For each of the following problems, choose the best answer from the given alternatives and carefully blacken the letter of your best choice on the separate answer sheet provided.

1. Which one of the following is a one –to-one correspondence?

   A. f: $[0,\infty) \to [0,\infty)$, $f(x) = x + 6$
   B. g: $\mathbb{R} \to \mathbb{R}$, $g(x) = 2^x$
   C. h: $[0,1] \to [1,2]$, $h(x) = x + 1$
   D. r: $\mathbb{R} \to \mathbb{R}$, $r(x) = x^2$

2. Let $\{a_n\}$be a sequence such that $a_1 = 1,\ a_2 = 3\ and\ a_n = \frac{a_{n-1}}{a_{n-2}}$ for $n \geq 3$ then, the sixth term of the sequence is equal to:_____

   A. 1
   C. 3
   B. $\frac{1}{3}$
   D. $\frac{1}{9}$

3. If the fourth and the tenth terms of arithmetic sequences are 11 and 35 respectively. Then which one is FALSE about this sequence:

   A. Common difference is 4
   C. $A_{11} = 39$
   B. The first term is -1
   D. $S_{10} = 150$

4. Which one of the following is an even function?

   A. $f(x) = \frac{x}{x^2-1}$
   C. $f(x) = x^3 \cos x$
   B. $f(x) = x^2 \sin x$
   D. $f(x) = x^3\, sgnx$

5. What is the solution set of $\frac{\frac{1}{x^2}}{\frac{1}{x}+\frac{4}{x^2}} + \frac{\frac{4}{x^2}}{\frac{1}{x}-\frac{4}{x^2}} = \frac{1+\frac{16}{x^2}}{1-\frac{16}{x^2}}$?

   A. $\{4, 1\}$
   C. $\{1\}$
   B. $\{-4\}$
   D. $\emptyset$

6. In a geometric sequence, the second term is 3 and the fifth term is $\frac{81}{8}$, then the fourth term of the sequence is equal to:

   A. $\frac{27}{4}$
   C. $\frac{81}{4}$
   B. $\frac{27}{16}$
   D. $\frac{27}{8}$

7. Which one of the following is TRUE about the graph of $f(x) = \frac{x^3 - 3x^2 - 4x}{x^2 + 3x}$?

A. $x = 0$ and $x = -3$ are its vertical asymptotes.
B. $y = -1$ and $y = 4$ are its horizontal asymptotes.
C. $y = x - 6$ is its oblique asymptote.
D. the graph intersects the y – axis at a point, $\left(0, \frac{-4}{3}\right)$.

8. The sum of the series $\sum_{n=1}^{\infty} \frac{(-1)^n}{3^n}$ is equal to:_____

A. $-\frac{1}{2}$ C. $-\frac{4}{9}$
B. $-\frac{1}{4}$ D. $-\frac{3}{4}$

9. What is the distance between a center of a circle with equation $x^2 + y^2 - 10x - 20y + 110 = 0$ and a line with equation $4x + 3y + 5 = 0$?

A. 11 units C. 9 units
B. 10 units D. 8 units

10. What is the equation of an ellipse with length of minor axis is 8 units and foci at $(0, -1)$ and $(0, 1)$?

A. $4x^2 + y^2 - 8x + 4y - 8 = 0$ C. $\frac{x^2}{4} + y^2 - 1 = 0$
B. $17x^2 + 16y^2 - 272 = 0$ D. $9x^2 + 25y^2 = 225$

11. Which one of the following is equal to $\lim_{x \to 0}\left(\frac{\sqrt{3+x} - \sqrt{3}}{x}\right)$:

A. 0 C. $\frac{1}{\sqrt{3}}$
B. $\frac{1}{2\sqrt{3}}$ D. The limit does not exist

12. What is the length of the latus rectum of a hyperbola whose equation is $25x^2 - 36y^2 - 900 = 0$?

A. $\frac{72}{5} units$ C. $\frac{29}{9} units$
B. $\frac{33}{4} units$ D. $\frac{25}{3} units$

13. Which one of the following compound proposition is a contradiction?

A. $(p \Rightarrow q) \Leftrightarrow (\neg p \vee q)$ C. $(p \Rightarrow q) \Leftrightarrow (p \wedge \neg q)$
B. $(p \Rightarrow q) \Leftrightarrow (\neg q \Rightarrow \neg p)$ D. $p \vee (q \wedge \neg q)$

14.Which of the following is a divergent sequence?

A. $\left\{sin\left(\frac{1+n}{1+3n^4}\right)\right\}$

B. $\left\{\frac{(-1)^n}{3n}\right\}$

C. $\left\{\left(\frac{1}{5}\right)^{1-n}\right\}$

D. $\left\{\frac{n-1}{n^2+1}\right\}$

15.Which one of the following is a valid logical argument?

A. $p \Leftrightarrow q, p \Rightarrow q \vdash q$

B. $p \Rightarrow q, q \vdash p$

C. $\neg p, p \vee q, r \Rightarrow q \vdash r$

D. $\neg p \wedge q, q \Rightarrow r \vdash r$

16.For a real number x and y which one of the following is NOT true?

A. $(\forall x)(\forall y)\ (x + y = 8)$

B. $(\forall x)(\exists y)(x + y = 7)$

C. $(\forall x)(\exists y)\ (x = y)$

D. $(\exists x)(\exists y)\ (x > y)$

17.It is a distribution of marks obtained in a mathematics exam (out of 55).

Mark	5 - 14	15 - 24	25 – 34	35 – 44	45 - 54
Number of student	20	40	70	50	20

If students, whose scores are in the bottom 25% of the marks, are considered as failures, then what is the maximum failing mark?

A. 22

B. 25

C. 15

D. 20

18. $\lim_{x\to -6}\left(\frac{\frac{2x+8}{x^2-12}-\frac{1}{x}}{x+6}\right)$ is equal to:

A. 36

B. $-\frac{1}{36}$

C. -36

D. $\frac{1}{36}$

19.What are the greatest lower bound and the least upper bound of the sequence $\left\{1-\frac{(-1)^n}{n}\right\}_{n=1}^{\infty}$ respectively?

A. 2 and $\frac{1}{2}$

B. 0 and $\frac{3}{2}$

C. $\frac{1}{2}$ and 2

D. $\frac{3}{2}$ and 2

20. $\lim_{x\to\infty}\left(\frac{3x+4}{3x}\right)^{5x-7}$ is equal to:

A. $e^{\frac{20}{3}}$ C. $e^{-\frac{21}{4}}$

B. $e^{\frac{15}{4}}$ D. $e^{-\frac{28}{3}}$

21. If the standard deviation of $x_1, x_2, x_3, \ldots, x_n$ is 3, then what is the standard deviation of $4x_1 + 5,\ 4x_2 + 5,\ 4x_3 + 5,\ \ldots, 4x_n + 5$?

A. 17 C. 53

B. 48 D. 12

22. Which one of the following is true?

A. $\lim_{n\to\infty}\left(\frac{\sqrt{n^2+1}-1}{\sqrt{n^2+1}+1}\right)=0$

B. $\lim_{x\to 0}\left(\frac{\sin\left(\frac{x}{2}\right)}{x^2+x}\right)=2$

C. $\lim_{x\to 3^-}\left(\frac{x-3}{\sqrt{x^2-6x+9}}\right)=1$

D. $\lim_{x\to 1^+}\left(\frac{x}{x^2-1}\right)=\infty$

23. In a box contains two white balls, three black balls and four red balls. In how many ways three balls can be drawn from the box, if at least one black is to be included in the draw?

A. 64 C. 18

B. 45 D. 15

24. What is the value of *k* so that $f(x)=\begin{cases}\frac{2}{k}+2x\ , & if\ x\le 0\\ xcot(4x)\ , & if\ x>0\end{cases}$ is continuous at $x = 0$?

A. 4 C. 8

B. 2 D. 0

25. What is the coefficient of x^2y^3 in the expansion$(2x + 5y)^5$?

A. 2000 C. 3000

B. 5000 D. 8000

26. Which one of the following is the set of critical number(s) of $f(x) = x^{\frac{1}{3}}(x + 8)$

A. $\{0\ ,\ 2\}$ C. $\{-4\ ,\ 0\}$

B. $\{-2\ ,\ 0\}$ D. $\{-4\ ,\ -2\}$

27.If $f(x) = ln\big(sin(2x)\big)$, then $f"\left(\frac{\pi}{4}\right)$ is equal to:

A. $2\sqrt{2}$
C. -8
B. 2
D. -4

28.If Q_i , D_i and P_i are respectively the i^{th} quartile, deciles and percentile of a data arranged in an increasing order, then which one of the following is necessarily TRUE?

A. $Q_3 > P_{25}$
C. $Q_2 = mean\ of\ the\ data$
B. $P_{25} > Q_1$
D. $Q_2 = \frac{Q_1+Q_2}{3}$

29.If $A = \begin{pmatrix} 0 & -5 & -6 \\ x & 0 & 10 \\ 6 & -10 & 0 \end{pmatrix}$ is askew symmetric matrix, then what is the value of ?

A. 5
C. 10
B. 3
D. 0

30.What is the equation of the tangent line to the graph of $f(x) = 2xsinx$ at$x = \frac{\pi}{2}$?

A. $y = 2x + 2\pi$
C. $y = -2x$
B. $y = 2x$
D. $y = -2x + 2\pi$

31.For any $n \times n$ square matrix A, which one of the following is NOT true?

A. If A is invertible matrix then $(A^{-1})^{-1} = A$
B. For a non – zero scalar k, $(kA)^{-1} = \frac{1}{k}A^{-1}$
C. If A is invertible then, $detA = \det(A^{-1})$
D. For any square matrix A, $detA = \det(A^T)$

32.If $f(x) = \frac{lnx}{x^2}$, which one of the following is equal to $f\,'(x)$?

A. $\frac{1-2lnx}{x^3}$
C. $\frac{1+2lnx}{x^3}$
B. $\frac{1}{2x^2}$
D. $-\frac{2}{x^4}$

33.If $f(x) = ln\left(\frac{x}{\sqrt{x^2+2}}\right)$, then $f'(1)$ is equal to:

A. $\frac{1}{2}ln3$
C. $\frac{2}{3}lne$
B. $-\frac{2}{3}$
D. $ln3$

34.What should be the value of k so that the system of equation

$$\begin{cases} -x+z-t=3 \\ 2x+2y-z-7t=1 \\ 4x-y-9z-5t=k \\ 3x-y-8z-6t=1 \end{cases}$$ has a solution?

A. 0
C. 2
B. 1
D. -2

35.What is the simplified form of $(1+2i)i-\frac{3+2i}{1-i}$?

A. $-\frac{5}{2}-\frac{3i}{2}$
C. $\frac{4}{3}+\frac{5i}{3}$
B. $-\frac{7}{2}+\frac{3i}{4}$
D. $\frac{6}{7}+\frac{2i}{7}$

36.Let $f(x)=|x|x^2$, then $f"(-3)$ is equal to:_____

A. -18
C. 0
B. 18
D. $Does\ not\ exist$

37. If $f(3)=2\,, f'(3)=-1\,, g(2)=3, g'(2)=1\ and\ G(x)=f(x^2+2x).g(2x)$, then what is the value of $G'(1)$?

A. -10
C. -8
B. 2
D. 8

38. Which one of the following is not true about the function $f(x)=4x^3-6x^2-9x$

A. f is decreasing on $\left[-\frac{1}{2}\,,\frac{3}{2}\right]$ and increasing on $\left(-\infty\,,-\frac{1}{2}\right]\cup[\frac{3}{2}\,,\infty)$

B. f has local maximum value at $\frac{3}{2}$ and local minimum value at $-\frac{1}{2}$

C. $f$is concave downward on $\left(-\infty\,,\frac{1}{2}\right)$ and concave upward on $\left(\frac{1}{2}\,,\infty\right)$

D. $\left(\frac{1}{2}\,,-\frac{11}{2}\right)$ is the point of inflection of f

39.What is the polar form of $2+2i$?

A. $2\sqrt{2}(\cos\frac{\pi}{2}+i\sin\frac{\pi}{2})$
C. $\sqrt{2}(\cos\frac{\pi}{4}+i\sin\frac{\pi}{4})$
B. $\sqrt{2}(\cos\frac{\pi}{2}+i\sin\frac{\pi}{2})$
D. $2\sqrt{2}(\cos\frac{\pi}{4}+i\sin\frac{\pi}{4})$

40.What is the absolute maximum value of $f(x)=x^2-x^3$ on $\left[-\frac{1}{2},\frac{3}{2}\right]$?

A. $-\frac{9}{8}$
C. $\frac{3}{8}$
B. $\frac{27}{4}$
D. 0

41. The area of the region bounded by the graph of $f(x) = x^2 - 3x + 2$ and the $x-$ axis between $x = 0$ to $x = 3$ is_____ square units.

A. $\frac{3}{2}$ C. $\frac{1}{6}$

B. $\frac{11}{6}$ D. $\frac{5}{6}$

42. An open top square base rectangular box container is to have a volume of $32cm^3$. What should be the height of the box to obtain the maximum possible surface area?

A. $8cm$ C. $4cm$

B. $16cm$ D. $2cm$

43. Which of the following statement is true?

A. If a function is continuous at a point, then it is differentiable at that point

B. If a function is not differentiable at some point in its domain then it is not continuous

C. Every function has a unique limit at a point in its domain, provided that the limit exists.

D. If a function is continuous on open interval, then both the absolute maximum and minimum value must occur.

44. A spherical balloon is inflated with gas at the rate of $20cm^3/min$. How fast is the radius of the balloon changing when the radius is $2cm$?

A. $\frac{5}{4\pi}cm/min$ C. $\frac{5\pi}{4}cm/min$

B. $\frac{4}{5\pi}cm/min$ D. $\frac{4\pi}{5}cm/min$

45. What is the value of $\int_1^2 \frac{1}{x\left(x^2+1\right)}dx$?

A. $ln\left(\frac{2\sqrt{2}}{\sqrt{5}}\right)$ C. $ln\left(\frac{3}{\sqrt{2}}\right)$

B. $\ln\left(\frac{2}{\sqrt{5}}\right)$ D. $ln\left(\frac{1}{\sqrt{2}}\right)$

46. Which one of the following is equal to $\int x\cos(2x)dx$?

A. $\frac{x}{2}\sin(2x) + \cos(2x) + c$ C. $\frac{x}{2}\sin(2x) + \frac{1}{4}\cos(2x) + c$

B. $\frac{x}{2}\sin(2x) + 2cosx + c$ D. $xsin(2x) + \frac{1}{4}\cos(2x) + c$

47. The volume of the regions bounded by $f(x) = x$ and $g(x) = x^2$ between $x = 0$ and $x = 1$ revolved about the *x-axis* is _____ cubic units.

A. $\frac{8}{15}\pi$ C. $\frac{15}{2}\pi$

B. $\frac{15}{8}\pi$ D. $\frac{2}{15}\pi$

48. Which of the following is an anti-derivative of $f(x) = \frac{1}{x^3\sqrt{x}}$?

A. $\frac{-5}{2x^2\sqrt{x}} + c$ C. $\frac{2}{9x^4\sqrt{x}} + c$

B. $\frac{-2}{5x^2\sqrt{x}} + c$ D. $\frac{9}{2x^4\sqrt{x}} + c$

49. What is the value of $\int_0^1 \frac{3x^2 - 1}{e^{x^3 - x}} dx$

A. 0 C. e^{-2}

B. 2 D. e^2

50. If $f(x) = \begin{cases} x^3 - 2x^2 \text{ , } for -1 \le x < 0 \\ sinx \quad\quad \text{ , } for\ x \ge 0 \end{cases}$, then which of the following is equal to $\int_{-1}^{\frac{\pi}{2}} f(x)dx$?

A. $-\frac{11}{12}$ C. $\frac{23}{12}$

B. $\frac{1}{12}$ D. 1

51. If $A(0\ ,0\ ,0)$ and $B(4\ ,6,0)$ are end points of a diameter of a sphere, then which one of the following is true about the sphere?

A. The radius of the sphere is $2\sqrt{13}$

B. The equation of the sphere is $(x-2)^2 + (y-3)^2 + z^2 = 52$

C. The equation of the sphere is $(x-2)^2 + (y-3)^2 + z^2 = 26$

D. P$(-1\ ,2,\ 3)$ is a point outside of sphere.

52. Let $\vec{u} = -i + j + k$ and $\vec{v} = i + 2j + 2k$ be vectors in space. Which one of the following is the cosine of the angle between $\vec{u}$ and $\vec{u} + \vec{v}$?

A. $\frac{\sqrt{6}}{3}$ C. $-\frac{\sqrt{6}}{3}$

B. $\frac{\sqrt{6}}{2}$ D. $\frac{6}{3\sqrt{2}}$

53. What is the induction hypothesis assumption for the inequality $m! > 2^m$ where $m \geq 4$?

A. For $m = k$, $(k+1)! > 2^k$ holds
B. For $m = k$, $k! > 2^k$ holds
C. For $m = k$, $k! > 3^k$ holds
D. For $m = k$, $k! > 2^{k+1}$ holds

54. What is the image of a circle $(x-5)^2 + (y-2)^2 = 1$, when it is rotated through $\frac{2\pi}{3}$ about $(3, -2)$?

A. $(x - 1 - \sqrt{3})^2 + (y + 1 + 2\sqrt{3})^2 = 1$
B. $(x + 1 - \sqrt{3})^2 + (y - 1 - 2\sqrt{3})^2 = 1$
C. $(x - 2 + \sqrt{3})^2 + (y + 4 + \sqrt{3})^2 = 1$
D. $(x + 2 - \sqrt{3})^2 + (y - 4 - \sqrt{3})^2 = 1$

55. If $\mathrm{A} = \sin^{-1}(\frac{1}{2})$ then, what is the value of $\tan 2A$?

A. $\frac{\sqrt{3}}{2}$
B. $\frac{\sqrt{3}}{5}$
C. $\frac{1}{2}$
D. $\sqrt{3}$

56. Consider the following statements "if $3n + 4$ is odd, then n is odd" for a natural number n. To show this, a person constructed the following proof.

"Proof: - Let n is not odd

$\Longrightarrow n$ is even
$\Longrightarrow n = 2k$, for some $k \epsilon \mathbb{N}$
$\Longrightarrow 3n + 4 = 3(2k) + 4 = 2(3k + 2)$
$\Longrightarrow 3n + 4$ is even

Therefore, $3n + 4$ is odd $\Longrightarrow n$ is odd."

A. The proof is correct by method of contradiction
B. The proof is correct by method of indirect proof.
C. The proof is correct by principle of mathematical induction
D. The proof is correct by method of direct proof

57. If $\vec{a}$ and $\vec{b}$ are vectors such that $|\vec{a}| = 2$, $\left|\vec{b}\right| = \sqrt{3}$, and the angle between vectors $\vec{a}$ and $\vec{b}$ is $\frac{\pi}{6}$, then what is the value of $\left|3\vec{a} - 2\vec{b}\right|$?

A. 5
B. $-2\sqrt{3}$
C. 12
D. $2\sqrt{3}$

58. If $\vec{U} = 3i + 2j$ and $\vec{V} = 5i + 4j$ then, what is a unit vector in the direction of $\frac{\vec{U}-\vec{V}}{|\vec{U}+\vec{V}|}$?

A. $\frac{2i}{\sqrt{5}} + \frac{j}{\sqrt{5}}$

B. $-\frac{i}{\sqrt{2}} - \frac{j}{\sqrt{2}}$

C. $\frac{3i}{\sqrt{13}} + \frac{2j}{\sqrt{13}}$

D. $-\frac{2i}{\sqrt{5}} - \frac{j}{\sqrt{5}}$

59. What is the equation of the curve $2x^2 + 3y^2 - 8x + 6y = 7$, when the origin is translated to the point $A(2, -1)$?

A. $2x^2 + 3y^2 - 16x + 12y + 26 = 0$

B. $7x^2 + 5y^2 - 16x + 5y - 8 = 0$

C. $x^2 - y^2 + 5x - 2y + 5 = 0$

D. $4x^2 - 5x^2 + 6x - y + 9 = 0$

60. Which of the following is the image of the point $P(-1\ ,\ 5)$ when reflected along the line $y = x + 2$?

A. $(3\ , -3)$

B. $(-5\ , 1)$

C. $(3\ , 1)$

D. $(3\ , -1)$

61. A parametric vector equation of the line through the points $(1\ , 3) and (-1, -1)$ is :

A. $(x\ , y) = (1\ , 3) + \lambda(-1\ , -1)$

B. $(x\ , y) = (1\ , 3) + \lambda(2\ , 4)$

C. $(x\ , y) = (-1\ , -1) + \lambda(1\ , 3)$

D. $(x\ , y) = (1\ , 3) + \lambda(-2\ , -4)$

62. What is the solution set of $5^{sin^2 x}\sqrt{5} = 5^{cos^2 x}$ in the interval $(\frac{\pi}{6},\ \pi]$?

A. $\frac{5\pi}{6}$

B. $\frac{\pi}{3}$

C. $\frac{\pi}{4}$

D. $\frac{\pi}{2}$

63. Which one of the following is NOT true about the graph $(x) = 6(\sin\frac{1}{4}x - \frac{\pi}{6}) + 1$?

A. Its period is 8π

B. Its amplitude is $6\ units$

C. Its phase shift is $\frac{2\pi}{5}$

D. The graph completes full cycle on $\left[\frac{2\pi}{3},\ \frac{26\pi}{3}\right]$

64. Which one of the following is equal to $\frac{1-\cos 2\theta}{\sin 2\theta}$?

A. $\csc\theta$

B. $\cot\theta$

C. $\sec\theta$

D. $\tan\theta$

65. The angle of elevation of an air craft from a point A on the ground is 60°. After a flight of $15\ seconds$ horizontally, the angle of elevation changes to 30°. If the air craft is flying at a speed of $200\ m/s$ then, what is the height at which the air craft is flying?

A. $3000\sqrt{3}\ m$
B. $2500\sqrt{3}\ m$
C. $1500\sqrt{3}\ m$
D. $1000\sqrt{3}\ m$