***Bahir Dar University, Institute of Technology***

***Applied Mathematics II Worksheet III & IV***

1. *Find the domain and range of the following functions*

   *a)* $f(x,y)=\sqrt{x^2+y^2-1}$ *b)* $f(x,y,z)=\ln\left(x^2+y^2+z^2-4\right)$ *c)* $f(x,y)=\dfrac{xy}{x^2+y^2}$

2. *Compute the following limits if exist*

   *a)* $\lim\limits_{(x,y,z)\to(0,0,0)}\dfrac{xyz}{x^2+y^2+z^2}$ *b)* $\lim\limits_{(x,y)\to(0,0)}\dfrac{xy}{\sqrt{x^2+y^2}}$ *c)* $\lim\limits_{(x,y)\to(0,0)}\left(y\sin\dfrac{1}{x}+x\sin\dfrac{1}{y}\right)$

   *d)* $\lim\limits_{(x,y)\to(0,0)}(1+xy)^{\frac{1}{xy}}$ *e)* $\lim\limits_{(x,y,z)\to(0,0,0)}\dfrac{xyz}{x^2+y^4+z^4}$

3. *Find the partial derivatives of the following functions*

   *a)* $f(x,y)=\sqrt{4x^2+y^2}$, *at* $(2,-3)$ *c)* $f(x,y,z)=x^3e^{zy}$ *e)* $f(x,y)=\int\limits_{\pi}^{x^2+y^2}\sin t^2\,dt$

   *b)* $f(x,y)=\begin{cases}\dfrac{x^2y^3}{x^2+4y^3} & at\ (0,0)\\ 0\end{cases}$ *d)* $f(x,y,z)=\arcsin\dfrac{1}{1+xyz^2}$

4. *Let* $u(x,t)=\sin(x-ct)$. *Show that* $\dfrac{\partial^2u}{\partial t^2}=c^2\dfrac{\partial^2u}{\partial x^2}$

5. *Given the following known equations*

   $\dfrac{\partial^2z}{\partial x^2}+\dfrac{\partial^2z}{\partial y^2}=0$ *…….Laplace's equation &* $\dfrac{\partial z}{\partial t}=c^2\dfrac{\partial^2z}{\partial x^2}$ *…… heat equation*

   *a) Show that* $z=\ln\left(x^2+y^2\right)+2\tan^{-1}\left(\dfrac{y}{x}\right)$ *satisfies Laplace's equation*

   *b) Show that* $z=e^{-t}\sin\dfrac{x}{c}$ *satisfies heat equation*

6. *Find* $\dfrac{dz}{dt}$ *the followings. a)* $z=3\cos x-\sin xy,\ x=\dfrac{1}{t},\ y=3t$ *b)* $z=e^{1-xy},\ x=t^{\frac{1}{3}},\ y=t^3$

7. *Let* $z=x^ce^{-\frac{y}{x}}$, *where c is a constant. Find the value of c such that* $\dfrac{\partial z}{\partial x}=y\dfrac{\partial^2z}{\partial y^2}+\dfrac{\partial z}{\partial y}$

8. *Find the directional derivative of* $f(x,y)=\tan(2x+y)$ *at* $\left(\dfrac{\pi}{6},\dfrac{\pi}{3}\right)$ *in the direction of a vector which makes an angle of* $\dfrac{7\pi}{4}$ *with the positive x-axis in counterclockwise direction.*

9. *Suppose that* $D_uf(1,2)=-5$ *and* $D_vf(1,2)=10$, *where* $u=\dfrac{3}{5}i-\dfrac{4}{5}j$ *and* $v=\dfrac{4}{5}i+\dfrac{3}{5}j$. *Find*

   *a)* $f_x(1,2)$ *b)* $f_y(1,2)$

10. *Suppose* $D_vf(3,-2,1)=-5$ *and* $\|\nabla f(3,-2,1)\|=5$. *Find* $\nabla f(3,-2,1)$ *if* $v=2i-j-2k$.

11. *Find the equation the plane tangent to the surface at the specific point*

*a)* $z = \sin(x+y),\ at\ (1,-1,0)$ *b)* $z = \ln(2x+y),\ at\ (-1,3,0)$

12. *Approximate* *a)* $\sqrt{(3.012)^2 + (3.997)^2}$ *b)* $\sqrt{3.99}\ \sqrt[3]{8.01}$
13. *Find the relative extreme values and saddle points of the following functions*
    *a)* $f(x,y) = x^4 - 8x^2 + 6y^2 - 12y + 3$ *b)* $f(x,y) = 2x^3 - xy^2 + 5x^2 + y^2$
14. *Find the extreme values of* $f(x,y) = x^2 + 2y^2 - x$ *constrained to the circular region* $x^2 + y^2 \le 4$.
15. *Find the points on the circle* $x^2 + y^2 = 45$ *that are closest and furthest from* $(1,2)$
16. *The temperature at each point of the rectangular region with vertices* $(3,2), (0,2), (0,-3),$ *and* $(3,-3)$*is given by* $T(x,y) = x^2 + 2y^2 + xy - 3x + 2y + 6$. *Find the hottest and coldest points of the region.*
17. *Evaluate the following iterated integrals*

*a)* $\int_{-2}^{2}\int_{0}^{\sqrt{4-x^2}}(x^2+y^2+2x)dydx$ *b)* $\int_0^1\int_0^{\sqrt{1-x^2}}\int_0^{\sqrt{1-x^2-y^2}} e^{\sqrt{x^2+y^2+z^2}}\,dzdydx$

*c)* $\int_0^{\sqrt{\pi}}\int_x^{\sqrt{\pi}} cos(y^2)dydx$ *d)* $\int_0^1\int_{y^2}^1 ye^{x^2}dxdy$ *e)* $\int_0^{\frac{3\pi}{2}}\int_0^{\frac{\pi}{3}}\int_{sec\,\emptyset}^2 3\rho^2 \sin\emptyset\, d\rho d\emptyset d\theta$

18.. *a) Evaluate* $\iint_R xy^2 dA$ *, where R is the region enclosed by* $y = x^2$ *and* $y = -x^2 + 8x - 10$.

*b) Evaluate* $\iint_R ydA$ *, where R is the region bounded by the circle* $r = 2\sin\theta$.

*c) Evaluate* $\iint_R (x+y)dA$ *, where R is the region in the first quadrant bounded by the circle* $r = 2$ *and the lines* $y = 0$ *,* $y = \sqrt{3}x$.

*d) Find the volume of the solid region bounded below by* $z = 0$ *and above by* $z = 4 - x^2 - y^2$ *,and laterally by the cylinder* $x^2 + y^2 = 4$.

19. *Evaluate :*

*a)* $\iiint_D z\,dv$*, where D-is a solid region bounded by the cylinder* $x^2 + y^2 = 9$*, and the planes* $z + y = 8$ *, and* $z = 0$

*b)* $\iiint_D \sqrt{x^2+y^2+2}\,dv$*, where D-is a solid region enclosed by the paraboloids* $z = 9 - x^2 - y^2$ *and* $z = 2x^2 + 2y^2$

*c)* $\iiint_D 3z\,dv$*, where D-is a solid region enclosed by the spheres* $x^2 + y^2 + z^2 = 1$ *and* $x^2 + y^2 + z^2 = 4$.

*d)* $\iiint_D dv$*, where D-is a solid region bounded by the sphere* $x^2 + y^2 + z^2 = 8$ *and the cone* $z = \sqrt{x^2+y^2}$.