3- اذا كانت $f(x)=\sin(ax)$ و a عدد فأن

$$L\{\sin(ax)\}=\int_0^{\infty} e^{-px}\sin axdx=\frac{a}{p^2+a^2}$$

البرهان:

$$L\{\sin ax\}=\int_0^{\infty} \mathrm{e}^{-px}\sin axdx$$

$$=\frac{e^{-px}}{p^2+a^2}[-p\sin ax-a\cos ax]_0^{\infty}$$

$$=0-(\frac{1}{p^2+a^2})[0-a]$$

$$=\frac{a}{p^2+a^2}$$

4- اذا كانت $f(x)=\cos ax$ و a عدد فأن

$$L\{\cos ax\}=\int_0^{\infty} e^{-px}\cos axdx=\frac{p}{p^2+a^2}$$

البرهان:

$$L\{\cos ax\}=\int_0^{\infty} e^{-px}\cos axdx$$

$$=\left[\frac{e^{-px}}{p^2+a^2}(-p\cos ax+a\sin ax)\right]_0^{\infty}$$

$$=\left[0-\frac{1}{p^2+a^2}(-p+0)\right]$$

$$=\frac{p}{p^2+a^2}$$

5- اذا كانت $f(x)=x^n$ و $n=1,2,3,.....$ فأن

$$L\{x^n\}=\int_0^{\infty} e^{-px}x^n dx=\frac{n!}{p^{n+1}} \qquad ,p>0$$

البرهان:

$$L\{x^n\}=\int_0^{\infty}e^{-px}x^n dx$$

$$u=x^n, dv=e^{-px}$$

$$du=nx^{n-1}, v=-\frac{1}{p}e^{-px}$$

$$L\{x^n\}=\int_0^{\infty}e^{-px}x^n dx=\left[-\frac{1}{p}e^{-px}x^n\right]_0^{\infty}+\frac{n}{p}\int_0^{\infty}e^{-px}x^{n-1}dx=0+\frac{n}{p}\int_0^{\infty}e^{-px}x^{n-1}dx=\frac{n}{p}\int_0^{\infty}e^{-px}x^{n-1}dx$$

$$L\{x^n\}=\frac{n}{p}L\{x^{n-1}\} \qquad , p>0$$

$$L\{x^n\}=\frac{n}{p}L\{x^{n-1}\}=\frac{n}{p}.\frac{n-1}{p}L\{x^{n-2}\} \qquad , p>0$$

$$L\{x^n\}=\frac{n}{p}L\{x^{n-1}\}=\frac{n}{p}.\frac{n-1}{p}L\{x^{n-2}\}=\frac{n}{p}.\frac{n-1}{p}.\frac{n-2}{p}L\{x^{n-3}\} \qquad , p>0$$

$$L\{x^n\}=\frac{n}{p}.\frac{n-1}{p}.\frac{n-2}{p}.\frac{n-3}{p}.....\frac{2}{p}.\frac{1}{p}L\{x^{n-n}\} \qquad , p>0$$

$$L\{x^n\}=\frac{n}{p}.\frac{n-1}{p}.\frac{n-2}{p}.\frac{n-3}{p}.....\frac{2}{p}.\frac{1}{p}L\{1\} \qquad , p>0$$

$$L\{x^n\}=\frac{n}{p}.\frac{n-1}{p}.\frac{n-2}{p}.\frac{n-3}{p}.....\frac{2}{p}.\frac{1}{p}.\frac{1}{p} \qquad , p>0$$

$$L\{x^n\}=\frac{n(n-1)(n-2)(n-3)...2.1}{p^n}.\frac{1}{p}$$

$$L\{x^n\}=\frac{n!}{p^{n+1}} \qquad , p>0$$

6- اذا كانت $f(x)=\sinh ax$ و a عدد فأن

$$L\{\sinh ax\}=\int_0^{\infty}e^{-px}\sinh axdx=\frac{a}{p^2-a^2}$$

البرهان:

$$L\{\sinh ax\} = L\{\frac{e^{ax} - e^{-ax}}{2}\} = \frac{1}{2} L\{e^{ax}\} - \frac{1}{2} L\{e^{-ax}\}$$
$$= \frac{1}{2} \cdot \frac{1}{p-a} - \frac{1}{2} \cdot \frac{1}{p+a}$$
$$= \frac{1}{2} (\frac{p+a-p+a}{p^2-a^2})$$
$$= \frac{1}{2} \cdot \frac{2a}{p^2-a^2}$$
$$= \frac{a}{p^2-a^2}$$

7- اذا كانت $f(x) = L\{\cosh ax\}$ و a عدد فأن

$$L\{\cosh ax\} = \int_0^\infty e^{-px} \cosh ax dx = \frac{p}{p^2-a^2}$$

البرهان:

$$L\{\cosh ax\} = L\{\frac{e^{ax} + e^{-ax}}{2}\} = \frac{1}{2} L\{e^{ax}\} + \frac{1}{2} L\{e^{-ax}\}$$
$$= \frac{1}{2} \cdot \frac{1}{p-a} + \frac{1}{2} \cdot \frac{1}{p+a}$$
$$= \frac{1}{2} (\frac{p+a+p-a}{p^2-a^2})$$
$$= \frac{1}{2} \cdot \frac{2p}{p^2-a^2}$$
$$= \frac{p}{p^2-a^2}$$