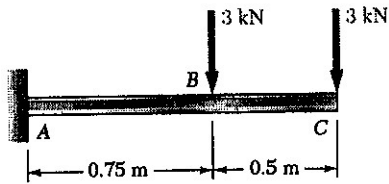


PROBLEM 9.79

9.79 For the cantilever beam shown, determine the slope and deflection at end C. Use $E = 200 \text{ GPa}$.



 $S100 \times 11.5$

SOLUTION

Units: Forces in kN, lengths in m.

Loading I: Concentrated load at B

Case 1 of Appendix D applied to portion AB.

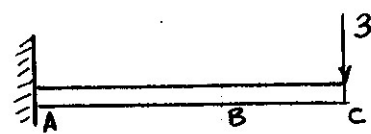
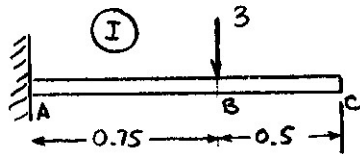
$$\theta'_B = -\frac{PL^2}{2EI} = -\frac{(3)(0.75)^2}{2EI} = -\frac{0.84375}{EI}$$

$$y'_B = -\frac{PL^3}{3EI} = -\frac{(3)(0.75)^3}{3EI} = -\frac{0.421875}{EI}$$

Portion BC remains straight

$$\theta'_C = \theta'_B = -\frac{0.84375}{EI}$$

$$y'_C = y'_B - (0.5)\theta'_B = -\frac{0.84375}{EI}$$



Loading II: Concentrated load at C. Case 1 of Appendix D.

$$\theta''_A = -\frac{PL^2}{2EI} = -\frac{(3)(1.25)^2}{2EI} = -\frac{2.34375}{EI}$$

$$y''_A = -\frac{PL^3}{3EI} = -\frac{(3)(1.25)^3}{3EI} = -\frac{1.953125}{EI}$$

By superposition: $\theta_A = \theta'_A + \theta''_A = -\frac{3.1875}{EI}$

$$y_A = y'_A + y''_A = -\frac{2.796875}{EI}$$

Data: $E = 200 \times 10^9 \text{ Pa}$, $I = 2.53 \times 10^6 \text{ mm}^4 = 2.53 \times 10^{-6} \text{ m}^4$

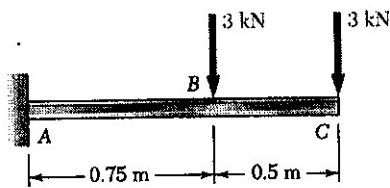
$$EI = (200 \times 10^9)(2.53 \times 10^{-6}) = 506 \times 10^3 \text{ N}\cdot\text{m}^2 = 506 \text{ kN}\cdot\text{m}^2$$

Slope at C $\theta_C = -\frac{3.1875}{506} = -6.30 \times 10^{-3} \text{ rad} = 6.30 \times 10^{-3} \text{ rad} \nabla$

Deflection at C $y_C = -\frac{2.796875}{506} = -5.53 \times 10^{-3} \text{ m} = 5.53 \text{ mm} \downarrow$

PROBLEM 9.80

9.80 For the cantilever beam shown, determine the slope and deflection at point B. Use $E = 200 \text{ GPa}$.



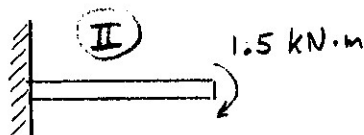
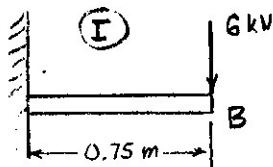
I
S100 × 11.5

SOLUTION

Units: Forces in kN, lengths in m.

The slope and deflection at B depend only on the deformation of portion AB.

Reducing the force at C to an equivalent force-couple system at B and adding the force already at B gives the loadings I and II shown.



Loading I: Case 1 of Appendix D

$$\theta_B' = -\frac{PL^2}{2EI} = -\frac{(6)(0.75)^2}{2EI} = -\frac{1.6875}{EI}$$

$$y_B' = -\frac{PL^3}{3EI} = -\frac{(6)(0.75)^3}{3EI} = -\frac{0.84375}{EI}$$

Loading II: Case 3 of Appendix D

$$\theta_B'' = -\frac{ML}{EI} = -\frac{(1.5)(0.75)}{EI} = -\frac{1.125}{EI}$$

$$y_B'' = -\frac{ML^2}{2EI} = -\frac{(1.5)(0.75)^2}{2EI} = -\frac{0.421875}{EI}$$

By superposition

$$\theta_B = \theta_B' + \theta_B'' = -\frac{2.8125}{EI}$$

$$y_B = y_B' + y_B'' = -\frac{1.265625}{EI}$$

Data: $E = 200 \times 10^9 \text{ Pa}$, $I = 2.53 \times 10^6 \text{ mm}^4 = 2.53 \times 10^{-6} \text{ m}^4$

$$EI = (200 \times 10^9)(2.53 \times 10^{-6}) = 506 \times 10^3 \text{ N} \cdot \text{m}^2 = 506 \text{ kN} \cdot \text{m}^2$$

Slope at B $\theta_B = -\frac{2.8125}{506} = -5.56 \times 10^{-3} \text{ rad} = 5.56 \text{ rad} \swarrow$

Deflection at B $y_B = -\frac{1.265625}{506} = -2.50 \times 10^{-3} \text{ m} = 2.50 \text{ mm} \downarrow$