

.2.1 Simple Interest

If interest is paid on the initial amount of money invested or borrowed only and not on subsequently accrued interest, it is called simple interest. The sum of the original amount (principal) and the total interest is the future amount or maturity value or in short amount. Simple interest generally used only on short-term loans or investments -often of duration less than one year. Simple interest is given by the following formula.

$$\text{.....} \text{-----}(1)$$

Where: I = Simple interest

P = principal amount

r = Annual simple interest rate

t = time in years, for which the interest is paid

If any three of the four variables are given, you can solve for the fourth (unknown variable) and their relationship is as follows:

$$\text{Amount (A)} = P + I$$

$$= P + Prt. \text{ factor out the common term } P$$

$$= P (1 + rt) \text{}(2)$$

$$P = I/rt \text{ or } P = \text{}(3)$$

$$r = I/pt \text{}(4)$$

$$t = I/pr \text{}(5)$$

Example 1

Ato Kassahun wanted to buy TV which costs Br. 10, 000. He was short of cash and went to Commercial Bank of Ethiopia (CBE) and borrowed the required sum of money for 9 months at an annual interest rate of 6%. Find the total simple interest and the maturity value of the loan.

Solution:

$$p = \text{Br. } 10,000$$

$$A = P + I$$

$$t^{**} = 9 \text{ months} = 9/12 = \frac{3}{4} \text{ year}$$

$$= P (1 + rt)$$

$$r = 6\% \text{ per year} = 0.06$$

$$= 10,000 (1 + 0.06 \times \frac{3}{4})$$

$$I = ? \quad A = ?$$

$$= 10,000 \times 1.045$$

$$\text{Interest (I)} = Prt$$

$$= \text{Br. } 10,450$$

$$= 10,000 \times 0.06 \times \frac{3}{4}$$

$$= \text{Br. } 450$$

The total amount which will have to be repaid to CBE at the end of the 9th month is Br. 10, 450 (the original borrowed amount plus Br. 450 Interest).

** Note: It is essential that the time period t and r be consistent with each other. That is if r is expressed as a percentage per year, t also should be expressed in number of years (number of months divided by 12 if time is given as a number of months). If time is given as a number of days, then $t = \frac{\text{days}}{360}$. this approach is known as ordinary interest year method which uses a 360 day years, whereas if we use 365 days years the approach is called exact time method.

Example 2

How long will it take if Br. 10, 000 is invested at 5% simple interest to double in value?

Solution

$$\text{Given: } p = \text{Br. } 10,000$$

$$I = prt$$

$$r = 10\% = 0.10$$

$$A = \text{Br. } 20,000 \text{ (} 2 \times 10,000 \text{)}$$

$$t = I^*/pr$$

$$t = ?$$

$$=$$

$$= 10 \text{ Years}$$

$$I^* = \text{Amount (A)} - \text{principal (p)}$$

$$= 20,000 - 10,000$$

$$= 10,000$$

Therefore it will take 20 years for the principal (Br. 10, 000) to double itself in value if it is invested at 10% annual interest rate.

Example 3

How much money you have to deposit in an account today at 3% simple interest rate if you are to receive Br. 5, 000 as an amount in 10 years?

Solution

$$P =$$

$$=$$

$$= \text{Br. } 3,846.15$$

In order to have Br. 5, 000 at the end of the 10th year, you have to deposit Br. 3846.15 in an account that pays 3% per year.

Example 4.

At what interest rate will Br. 5, 000 yield Br. 2, 000 in 8 years time.

Solution:

$$P = \text{Br. } 5,000$$

$$r = I/pt$$

$$I = \text{Br. } 2,000$$

$$=$$

$$t = 8 \text{ years}$$

$$= 0.05 = 5\%$$

$r = ?$

Example 5.

Find the Interest on Br. 5, 000 at 10% for 45 days.

Solution:

$P = \text{Br. } 5, 000$

$I = Prt$

$t = 45 \text{ days} = 45/360 \text{ years}$

$= 5, 000 \times 0.1 \times 45/360$

$r = 10\%$

$= 62.50 \text{ Br.}$

$I = ?$

Try Your Self:

At what interest rate you should invest Br. 5000, if you want to receive an amount of Br. 7,000 in 8 years time.

Answer = 5%

4.2.2 Compound Interest

If the interest, which is due, is added to the principal at the end of each interest period (such as a month, quarter, and year), then this interest as well as the principal will earn interest during the next period. In such a case, the interest is said to be compounded. The result of compounding interest is that starting with the second compounding period, the account earns interest on interest in addition to earning interest on principal during the next payment period. Interest paid on interest reinvested is called compound interest.

The sum of the original principal and all the interest earned is the compound amount. The difference between the compound amount and the original principal is the compound interest.

The compound interest method is generally used in long-term borrowing unlike that of the simple interest used only for short-term borrowings. The time interval between successive conversions of interest into principal is called the interest period, or conversion period, or Compounding period, and

may be any convenient length of time. The interest rate is usually quoted as an annual rate and must be converted to appropriate rate per conversion period for computational purposes. Hence, the rate per compound period (i) is found by dividing the annual nominal rate (r) by the number of compounding periods per year (m):

$$i = r/m$$

Example if $r = 12\%$, i is calculated as follows:

Conversion period (m)	Rate per compound period (i)
1. Annually (once a year) -----	$i = r/1 = 0.12/1 = 0.12$
2. Semi annually (every 6 months) -----	$i = r/2 = 0.12/2 = 0.06$
3. Quarterly (every 3 months) -----	$i = r/4 = 0.12/4 = 0.03$
4. Monthly -----	$i = r/12 = 0.12/12 = 0.01$

Example 1

Assume that Br. 10, 000 is deposited in an account that pays interest of 12% per year, compounded quarterly. What are the compound amount and compound interest at the end of one year?

Solution

$$P = \text{Br. } 10,000$$

$$r = 12\%$$

$$t = 1 \text{ year}$$

$m = \text{No. of conversion periods} = 4 \text{ times per quarter}$. This means interest will be computed at the end of each three month period and added in to the principal.

$$i = r/m \quad 12\%/4 = 3\%$$

In general, if p is the principal earning interest compounded m times a year at an annual rate of r, then (by repeated use of the simple interest formula, using $i = r/m$, the rate per period, the amount A at the end of each period is:

(1) $A = p(1+i)$ compound amount at the end of first period.

If we are interested in determining the compound amount after two periods, it may be computed using the equation:

(2) Compound amount = Compound amount + Interest earned during
after two periods after one period the 2nd period

$$A = p(1+i) + [P(1+i)](i)$$

Factoring P and (1 + i) from both terms of the right side of the equation gives us:

$$\begin{aligned} A &= P(1+i)(1+i) \\ &= P(1+i)^2 \end{aligned}$$

(3) Compound amount = Compound amount + Interest earned
after three periods after two period during the 3rd period

$$A = P(1+i)^2 + [P(1+i)^2](i)$$

Factor out p and (1 + i)² from the terms on the right side of the equation and it gives you:

$$\begin{aligned} A &= P(1+i)^2(1+i) \\ &= P(1+i)^3 \end{aligned}$$

(4) Compound amount
after nth period

The compound amount formulas developed so far are summarized below:

1. Compound amount after one period = $p(1+i)^1$
2. Compound amount after two periods = $p(1+i)^2$
3. Compound amount after three periods = $p(1+i)^3$
4. Compound amount after nth periods = $p(1+i)^n$

..... * Compound amount formula.

Where: A = amount (future value) at the end of n periods.

P = Principal (present value)

i = r/m = Rate per compounding period.

n = mt = total number of conversion periods

t = total number of years

m = number of compounding/ conversion periods per

Year

r = annual nominal rate of interest

Now let us solve the above problem.

$A = 10,000 (1.03)^1 = \text{Br. } 10,300 \dots\dots 1\text{st quarter}$

$A = [10,000 (1.03)] (1.03) = 10,000 (1.03)^2 = 10,609 \dots\dots 2\text{nd quarter.}$

$A = [10,000 (1.03)^2] (1.03) = 10,000 (1.03)^3 = 10,927.27 \dots\dots 3\text{rd quarter.}$

$A = [(10,000) (1.03)^3] (1.03) = 10,000 (1.03)^4 = 11,255.088 \dots\dots 4\text{th quarter.}$

In general, the compound amount can be found by multiplying the principal by $(1 + i)^n$. So for the above problem the amount at the end of the year, using the general formula, is equal to:

$$A = P (1 + i)^n$$

$$n = mt = 4 \times 1 = 4$$

$$= 10,000 (1.03)^4$$

$$i = r/m = 12\%/4 = 3\%$$

$$= \text{Br. } 11,255.088$$

Compound Interest = Compound amount - original principal

$$= 11,225.088 - 10,000$$

$$= \text{Br. } 1,255.088$$

We may evaluate "A" in several different ways. Among the possible alternatives are:

1. Use a hand-held calculator with a Y^x function key. This is the procedure most often used.

2. Using Logarithms

Restate the equation by finding for $\log A$ (or $\ln A$) and then finding the antilog, using either a hand-held calculator with logarithmic functions or a table of logarithms. Let us illustrate this alternative.

$$A = 10,000 (1.03)^4$$

$$\log A = \log 10,000 + \log (1.03)^4$$

$$= \log 10,000 + 4 \log 1.03$$

$$= \log 10^4 + 4 \log 1.03$$

$$= 4 + 4(0.01284)$$

$$= 4 + 0.05135$$

$$\log A = 4.05135$$

$$A = \text{Antilog } 4.05135$$

$$= \text{Br. } 11255.117$$

Basic rules of logarithm

$$1. \quad ax = b \quad \text{Eg. } 2x = 5$$

$$\log ax = \log b \quad \log 2x = \log 5$$

$$x \log a = \log b \quad x \log 2 = \log 5$$

$$x = \log b / \log a \quad x = \log 5 / \log 2$$

$$x = 2.322$$

$$2. \quad abx + c = d$$

$$abx = d - c$$

$$bx = d - c/a$$

$$\log bx = \log \quad \text{let: } = K$$

$$X \log b = \log K \quad \text{divide both side of the equality by } \log b.$$

$$X =$$

$$\text{Eg. } 4(3x) + 10 = 18 \quad \text{find the value of } X$$

$$4(3x) = 18 - 10$$

$$4(3x) = 8 \quad (\text{Divide by } 4)$$

$$3x = 2$$

$$\log 3x = \log 2$$

$$X \log 3 = \log 2$$

$$X =$$

$$= 0.63093$$

$$3. \quad X^3 = a$$

$$\log X^3 = \log a$$

$$3 \log x = \log a$$

$$\log x = \quad ; \text{ Let } = K$$

$$X = \text{Antilog } K$$

$$\text{Eg. } X^3 = 1,000$$

$$\log X^3 = \log 1,000$$

$$3 \log X = \log 10^3$$

$$\log X = 3/3$$

$$\log X = 1$$

$$X = \text{Antilog } 1$$

$$X = 10$$

$$4. \quad a = b(c + x)^d \quad \text{find the value of } X$$

$$(c + x)^d = a/b$$

$$\log (c + X)^d = \log a - \log b$$

$$d \log c + X = \log a = \log b$$

$$\log c + X = \quad \text{Let: } = K$$

$$\log c + x = K$$

$$c + x = \text{Antilog } K$$

$$X = \text{Antilog } K - C$$

$$\text{Eg. } 1,000 = 250 (5 + X)^5$$

$$(5 + X)^5 = 4$$

$$\log (5 + X)^5 = \log 4$$

$$5 \log 5 + x = \log 4$$

$$\log 5 + X =$$

$$\log 5 + X = 0.120412$$

$$X = \text{antilog } 0.120412 - 5$$

$$= 1.319508 - 5$$

$$= -3.6805$$

3. The third way of finding the compound amount is using specially prepared tables which provide values of $(1 + i)^n$ for selected values of i and n .

Therefore, to calculate the value of "A" or other variables in the compound interest formula, you can use any of these three approaches which ever convenient to you.

Example 2. Find the compound amount and compound interest after 10 years if Br. 15, 000 were invested at 8% interest;

a) If compounded annually

Compounding annually means that there is one interest payment period per year. Thus

$$t = 10 \text{ years}$$

$$m = 1$$

$$n = mt = 1 \times 10 = 10$$

$$i = r/m = 8\%/1 = 8\% = 0.08$$

The compound amount will be:

$$A = 15,000 (1.08)^{10}$$

$$= 15,000 (2.158925)$$

$$= \text{Br. } 32,383.875$$

" Compound Interest = compound amount (A) - Principal (P)

$$= 32,383.875 - 15,000$$

$$= \text{Br. } 17,383.875$$

b) If compounded semiannually

Compounding semiannually means that there are two interest payment periods per year. Thus, the number of payment periods in 10 years $n = 2 \times 10 = 20$ and the interest rate per conversion period will be $i = r/m = 8\%/2 = 4\%$. The compound amount then will be:

$$A = P (1 + i)^n$$

$$= 15,000 (1.04)^{20}$$

$$= 15,000 (2.191123)$$

$$= \text{Br. } 32,866.85$$

Compound Interest = A - P

$$= 32,866.85 - 15,000$$

$$= \text{Br. } 17,866.85$$

c) If Compounded quarterly

If compounding takes place quarterly (four times a year), then an 8% annual interest rate, the interest rate per conversion period will be $i = 0.08/4 = 0.02$, there will be a total of $n = 4 \times 10 = 40$ conversion periods over the 10 years. The compound amount will be:

$$\begin{aligned} A &= 15,000 (1.02)^{40} \\ &= 15,000 (2.208039) \\ &= \text{Br. } 33,120.60 \end{aligned}$$

d) If compound monthly

$$\begin{aligned} p &= 15,000 \\ t &= 10 \text{ years} \\ m &= 12 \text{ (12 payment periods per year)} \\ n &= 12 \times 10 = 120 \text{ payment periods over the 10 years} \\ i &= r/m = 8\%/12 = 0.667\% = 0.00667 \end{aligned}$$

Under these conditions:

$$\begin{aligned} A &= 15,000 (1.00667)^{120} \\ &= 15,000 (2.220522) \\ &= \text{Br. } 33,307.84 \end{aligned}$$

$$\text{Interest} = \text{Br. } 18,307.84 (33,307.84 - 15,000)$$

e) If compounded weekly

$$\begin{aligned} m &= 52 \\ n &= 10 \times 52 = 520 \\ i &= 8\%/52 = 0.154\% = 0.00154, \text{ then} \end{aligned}$$

$$\begin{aligned} A &= 15,000 (1.00154)^{520} \\ &= \text{Br. } 33,362.60 \end{aligned}$$

$$\begin{aligned} \text{Interest} &= 33,362.60 - 15,000 \\ &= 18362.60 \end{aligned}$$