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Ethiopia

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MATHS

GRADE 12 AND
UNIVERSITY
COURSE

INTEGRATION

Imdefinite Integration

INDEFINITE INTEGRATION

Integration is the inverse process of differentiation. That is, the process of finding a function, whose differential coefficient is known, is called integration.

If the differential coefficient of $F(x)$ is $f(x)$,

i.e. $\frac{d}{dx}[F(x)] = f(x)$, then we say that the **antiderivative**

or **integral** of $f(x)$ is $F(x)$, written as $\int f(x)dx = F(x)$,

Here $\int dx$ is the notation of integration $f(x)$ is the integrand, x is the variable of integration and dx denotes the integration with respect to x .

1. INDEFINITE INTEGRAL

We know that if $\frac{d}{dx}[F(x)] = f(x)$, then $\int f(x)dx = F(x)$.

Also, for any arbitrary constant C ,

$$\frac{d}{dx}[F(x) + C] = \frac{d}{dx}[F(x)] + 0 = f(x).$$

$$\therefore \int f(x)dx = F(x) + C,$$

This shows that $F(x)$ and $F(x) + C$ are both integrals of the same function $f(x)$. Thus, for different values of C , we obtain different integrals of $f(x)$. This implies that the integral of $f(x)$ is not definite. By virtue of this property $F(x)$ is called the indefinite integral of $f(x)$.

1.1 Properties of Indefinite Integration

$$1. \quad \frac{d}{dx} \left[\int f(x)dx \right] = f(x)$$

$$2. \quad \int f'(x)dx = \int \frac{d}{dx}[f(x)]dx = f(x) + c$$

$$3. \quad \int k f(x)dx = k \int f(x)dx, \text{ where } k \text{ is any constant}$$

$$4. \quad \text{If } f_1(x), f_2(x), f_3(x), \dots \text{ (finite in number) are functions of } x, \text{ then}$$

$$\int [f_1(x) \pm f_2(x) \pm f_3(x) \dots]dx$$

$$= \int f_1(x)dx \pm \int f_2(x)dx \pm \int f_3(x)dx \pm \dots$$

$$5. \quad \text{If } \int f(x)dx = F(x) + c$$

$$\text{then } \int f(ax \pm b)dx = \frac{1}{a} F(ax \pm b) + c$$

1.2 Standard Formulae of Integration

The following results are a direct consequence of the definition of an integral.

$$1. \quad \int x^n dx = \frac{x^{n+1}}{n+1} + C, n \neq -1.$$

$$2. \quad \int \frac{1}{x} dx = \log |x| + C$$

$$3. \quad \int e^x dx = e^x + C$$

$$4. \quad \int a^x dx = \frac{a^x}{\log_e a} + C.$$

$$5. \quad \int \sin x dx = -\cos x + C$$

$$6. \quad \int \cos x dx = \sin x + C$$

$$7. \quad \int \sec^2 x dx = \tan x + C$$

$$8. \quad \int \operatorname{cosec}^2 x dx = -\cot x + C$$

$$9. \quad \int \sec x \tan x \, dx = \sec x + C$$

$$10. \quad \int \operatorname{cosec} x \cot x \, dx = -\operatorname{cosec} x + C.$$

$$11. \quad \int \tan x \, dx = -\log |\cos x| + C = \log |\sec x| + C.$$

$$12. \quad \int \cot x \, dx = \log |\sin x| + C$$

$$13. \quad \int \sec x \, dx = \log |\sec x + \tan x| + C$$

$$14. \quad \int \operatorname{cosec} x \, dx = \log |\operatorname{cosec} x - \cot x| + C$$

$$15. \quad \int \frac{dx}{\sqrt{1-x^2}} = \sin^{-1} x + C ; |x| < 1$$

$$16. \quad \int \frac{dx}{1+x^2} = \tan^{-1} x + C$$

$$17. \quad \int \frac{dx}{x\sqrt{x^2-1}} = \sec^{-1} |x| + C ; |x| > 1$$

2. METHODS OF INTEGRATION

2.1 Method of Substitution

By suitable substitution, the variable x in $\int f(x) \, dx$ is changed into another variable t so that the integrand $f(x)$ is changed into $F(t)$ which is some standard integral or algebraic sum of standard integrals.

There is no general rule for finding a proper substitution and the best guide in this matter is experience.

However, the following suggestions will prove useful.

(i) If the integrand is of the form $f'(ax+b)$, then we

$$\text{put } ax + b = t \text{ and } dx = \frac{1}{a} dt.$$

$$\text{Thus, } \int f'(ax+b) \, dx = \int f'(t) \frac{dt}{a}$$

$$= \frac{1}{a} \int f'(t) \, dt = \frac{f(t)}{a} = \frac{f(ax+b)}{a} + c$$

(ii) When the integrand is of the form $x^{n-1} f'(x^n)$, we put $x^n = t$ and $nx^{n-1} \, dx = dt$.

$$\text{Thus, } \int x^{n-1} f'(x^n) \, dx = \int f'(t) \frac{dt}{n} = \frac{1}{n}$$

$$\int f'(t) \, dt = \frac{1}{n} f(t) = \frac{1}{n} f(x^n) + c$$

(iii) When the integrand is of the form $[f(x)]^n \cdot f'(x)$, we put $f(x) = t$ and $f'(x) \, dx = dt$.

$$\text{Thus, } \int [f(x)]^n f'(x) \, dx = \int t^n \, dt = \frac{t^{n+1}}{n+1} = \frac{[f(x)]^{n+1}}{n+1} + c$$

(iv) When the integrand is of the form $\frac{f'(x)}{f(x)}$, we put

$$f(x) = t \text{ and } f'(x) \, dx = dt.$$

$$\text{Thus, } \int \frac{f'(x)}{f(x)} \, dx = \int \frac{dt}{t} = \log |t| = \log |f(x)| + c$$

2.1.1 Some Special Integrals

$$1. \quad \int \frac{dx}{x^2 + a^2} = \frac{1}{a} \tan^{-1} \frac{x}{a} + C$$

$$2. \quad \int \frac{dx}{x^2 - a^2} = \frac{1}{2a} \log \left| \frac{x-a}{x+a} \right| + C$$

$$3. \quad \int \frac{dx}{a^2 - x^2} = \frac{1}{2a} \log \left| \frac{a+x}{a-x} \right| + C$$

$$4. \quad \int \frac{dx}{\sqrt{a^2 - x^2}} = \sin^{-1} \frac{x}{a} + C$$

$$5. \quad \int \frac{dx}{\sqrt{x^2 + a^2}} = \log \left| x + \sqrt{x^2 + a^2} \right| + C$$

$$6. \int \frac{dx}{\sqrt{x^2 - a^2}} = \log \left| x + \sqrt{x^2 - a^2} \right| + C$$

$$7. \int \sqrt{a^2 - x^2} dx = \frac{x}{2} \sqrt{a^2 - x^2} + \frac{a^2}{2} \sin^{-1} \frac{x}{a} + C$$

$$8. \int \sqrt{x^2 + a^2} dx = \frac{x}{2} \sqrt{x^2 + a^2} + \frac{a^2}{2} \log \left| x + \sqrt{x^2 + a^2} \right| + C$$

$$9. \int \sqrt{x^2 - a^2} dx = \frac{x}{2} \sqrt{x^2 - a^2} - \frac{a^2}{2} \log \left| x + \sqrt{x^2 - a^2} \right| + C$$

2.1.2 Integrals of the Form

$$(a) \int f(a^2 - x^2) dx,$$

$$(b) \int f(a^2 + x^2) dx,$$

$$(c) \int f(x^2 - a^2) dx,$$

$$(d) \int f\left(\frac{a-x}{a+x}\right) dx,$$

Working Rule

Integral

Substitution

$$\int f(a^2 - x^2) dx, \quad x = a \sin \theta \text{ or } x = a \cos \theta$$

$$\int f(a^2 + x^2) dx, \quad x = a \tan \theta \text{ or } x = a \cot \theta$$

$$\int f(x^2 - a^2) dx, \quad x = a \sec \theta \text{ or } x = a \operatorname{cosec} \theta$$

$$\int f\left(\frac{a-x}{a+x}\right) dx \text{ or } \int f\left(\frac{a+x}{a-x}\right) dx \quad x = a \cos 2\theta$$

2.1.3 Integrals of the Form

$$(a) \int \frac{dx}{ax^2 + bx + c}, \quad (b) \int \frac{dx}{\sqrt{ax^2 + bx + c}},$$

$$(c) \int \sqrt{ax^2 + bx + c} dx$$

Working Rule

(i) Make the coefficient of x^2 unity by taking the coefficient of x^2 outside the quadratic.

(ii) Complete the square in the terms involving x , i.e. write $ax^2 + bx + c$ in the form

$$a \left[\left(x + \frac{b}{2a} \right)^2 \right] - \frac{(b^2 - 4ac)}{4a}.$$

(iii) The integrand is converted to one of the nine special integrals.

(iv) Integrate the function.

2.1.4 Integrals of the Form

$$(a) \int \frac{px + q}{ax^2 + bx + c} dx, \quad (b) \int \frac{px + q}{\sqrt{ax^2 + bx + c}} dx,$$

$$(c) \int (px + q) \sqrt{ax^2 + bx + c} dx$$

Integral Working Rule

$\int \frac{px + q}{ax^2 + bx + c} dx$ Put $px + q = \lambda (2ax + b) + \mu$ or $px + q = \lambda$ (derivative of quadratic) $+ \mu$.

Comparing the coefficient of x and constant term on both sides, we get

$$p = 2a\lambda \text{ and } q = b\lambda + \mu \Rightarrow \lambda = \frac{p}{2a} \text{ and } \mu = \left(q - \frac{bp}{2a} \right). \text{ Then}$$

integral becomes

$$\begin{aligned} & \int \frac{px + q}{ax^2 + bx + c} dx \\ &= \frac{p}{2a} \int \frac{2ax + b}{ax^2 + bx + c} dx + \left(q - \frac{bp}{2a} \right) \int \frac{dx}{ax^2 + bx + c} \\ &= \frac{p}{2a} \log | ax^2 + bx + c | + \left(q - \frac{bp}{2a} \right) \int \frac{dx}{ax^2 + bx + c} \end{aligned}$$

$$\int \frac{px + q}{\sqrt{ax^2 + bx + c}} dx \text{ In this case the integral becomes}$$

$$\int \frac{px+q}{\sqrt{ax^2+bx+c}} dx =$$

$$\frac{p}{2a} \int \frac{2ax+b}{\sqrt{ax^2+bx+c}} dx + \left(q - \frac{bp}{2a}\right) \int \frac{dx}{\sqrt{ax^2+bx+c}}$$

$$= \frac{p}{a} \sqrt{ax^2+bx+c} + \left(q - \frac{bp}{2a}\right) \int \frac{dx}{\sqrt{ax^2+bx+c}}$$

$$\int (px+q)\sqrt{ax^2+bx+c} dx$$

The integral in this case is converted to

$$\begin{aligned} \int (px+q)\sqrt{ax^2+bx+c} dx &= \frac{p}{2a} \int (2ax+b)\sqrt{ax^2+bx+c} dx \\ &\quad + \left(q - \frac{bp}{2a}\right) \int \sqrt{ax^2+bx+c} dx \end{aligned}$$

$$= \frac{p}{3a} (ax^2+bx+c)^{3/2} + \left(q - \frac{bp}{2a}\right) \int \sqrt{ax^2+bx+c} dx$$

2.1.5 Integrals of the Form

$$\int \frac{P(x)}{\sqrt{ax^2+bx+c}} dx, \text{ where } P(x) \text{ is a polynomial in } x \text{ of}$$

degree $n \geq 2$.

Working Rule: Write

$$\int \frac{P(x)}{\sqrt{ax^2+bx+c}} dx =$$

$$= (a_0 + a_1x + a_2x^2 + \dots + a_{n-1}x^{n-1}) \sqrt{ax^2+bx+c} + k \int \frac{dx}{\sqrt{ax^2+bx+c}}$$

where $k, a_0, a_1, \dots, a_{n-1}$ are constants to be determined by differentiating the above relation and equating the coefficients of various powers of x on both sides.

2.1.6 Integrals of the Form

$$\int \frac{x^2+1}{x^4+kx^2+1} dx \quad \text{or} \quad \int \frac{x^2-1}{x^4+kx^2+1} dx,$$

where k is a constant positive, negative or zero.

Working Rule

- (i) Divide the numerator and denominator by x^2 .
- (ii) Put $x - \frac{1}{x} = z$ or $x + \frac{1}{x} = z$, whichever substitution, on differentiation gives, the numerator of the resulting integrand.
- (iii) Evaluate the resulting integral in z
- (iv) Express the result in terms of x .

2.1.7 Integrals of the Form

$$\int \frac{dx}{P\sqrt{Q}}, \text{ where } P, Q \text{ are linear or quadratic functions of } x.$$

Integral

Substitution

$$\int \frac{1}{(ax+b)\sqrt{cx+d}} dx$$

$$cx+d = z^2$$

$$\int \frac{dx}{(ax^2+bx+c)\sqrt{px+q}}$$

$$px+q = z^2$$

$$\int \frac{dx}{(px+q)\sqrt{ax^2+bx+c}}$$

$$px+q = \frac{1}{z}$$

$$\int \frac{dx}{(ax^2+b)\sqrt{cx^2+d}}$$

$$x = \frac{1}{z}$$

3. METHOD OF PARTIAL FRACTIONS FOR RATIONAL FUNCTIONS

Integrals of the type $\int \frac{p(x)}{g(x)}$ can be integrated by resolving

the integrand into partial fractions. We proceed as follows:

Check degree of $p(x)$ and $g(x)$.

If degree of $p(x) \geq$ degree of $g(x)$, then divide $p(x)$ by $g(x)$ till its degree is less, i.e. put in the

form $\frac{p(x)}{g(x)} = r(x) + \frac{f(x)}{g(x)}$ where degree of $f(x) <$ degree of

$g(x)$.

CASE 1: When the denominator contains non-repeated linear factors. That is

$$g(x) = (x - \alpha_1)(x - \alpha_2) \dots (x - \alpha_n).$$

In such a case write $f(x)$ and $g(x)$ as:

$$\frac{f(x)}{g(x)} = \frac{A_1}{(x - \alpha_1)} + \frac{A_2}{(x - \alpha_2)} + \dots + \frac{A_n}{(x - \alpha_n)}$$

where $A_1, A_2, \dots, A_n$ are constants to be determined by comparing the coefficients of various powers of x on both sides after taking L.C.M.

CASE 2 : When the denominator contains repeated as well as non-repeated linear factor. That is

$$g(x) = (x - \alpha_1)^2 (x - \alpha_3) \dots (x - \alpha_n).$$

In such a case write $f(x)$ and $g(x)$ as:

$$\frac{f(x)}{g(x)} = \frac{A_1}{x - \alpha_1} + \frac{A_2}{(x - \alpha_1)^2} + \frac{A_3}{x - \alpha_3} + \dots + \frac{A_n}{(x - \alpha_n)}$$

where $A_1, A_2, \dots, A_n$ are constants to be determined by comparing the coefficients of various powers of x on both sides after taking L.C.M.

Note : Corresponding to repeated linear factor $(x - a)^r$ in the denominator, a sum of r partial

fractions of the type $\frac{A_1}{x - a} + \frac{A_2}{(x - a)^2} + \dots + \frac{A_r}{(x - a)^r}$ is taken.

CASE 3 : When the denominator contains a non repeated

quadratic factor which cannot be factorised further:

$$g(x) = (ax^2 + bx + c)(x - \alpha_3)(x - \alpha_4) \dots (x - \alpha_n).$$

In such a case express $f(x)$ and $g(x)$ as:

$$\frac{f(x)}{g(x)} = \frac{A_1x + A_2}{ax^2 + bx + c} + \frac{A_3}{x - \alpha_3} + \dots + \frac{A_n}{x - \alpha_n}$$

where $A_1, A_2, \dots, A_n$ are constants to be determined by comparing the coefficients of various powers of x on both sides after taking L.C.M.

CASE 4 : When the denominator contains a repeated quadratic factor which cannot be factorised further: That is

$$g(x) = (ax^2 + bx + c)^2 (x - \alpha_5)(x - \alpha_6) \dots (x - \alpha_n)$$

In such a case write $f(x)$ and $g(x)$ as

$$\frac{f(x)}{g(x)} = \frac{A_1x + A_2}{ax^2 + bx + c} + \frac{A_3x + A_4}{(ax^2 + bx + c)^2} +$$

$$\frac{A_5}{x - \alpha_5} + \dots + \frac{A_n}{(x - \alpha_n)}$$

where $A_1, A_2, \dots, A_n$ are constants to be determined by comparing the coefficients of various powers of x on both sides after taking L.C.M.

CASE 5 : If the integrand contains only even powers of x

- (i) Put $x^2 = z$ in the integrand.
- (ii) Resolve the resulting rational expression in z into partial fractions
- (iii) Put $z = x^2$ again in the partial fractions and then integrate both sides.

4. METHOD OF INTEGRATION BY PARTS

The process of integration of the product of two functions is known as integration by parts.

For example, if u and v are two functions of x ,

$$\text{then } \int (uv) dx = u \int v dx - \int \left(\frac{du}{dx} \cdot \int v dx \right) dx.$$

In words, integral of the product of two functions = first function $\times$ integral of the second – integral of (differential of first $\times$ integral of the second function).

Working Hints

- (i) Choose the first and second function in such a way that derivative of the first function and the integral of the second function can be easily found.
- (ii) In case of integrals of the form $\int f(x) \cdot x^n dx$, take x^n as the first function and $f(x)$ as the second function.
- (iii) In case of integrals of the form $\int (\log x)^n \cdot 1 dx$, take 1 as the second function and $(\log x)^n$ as the first function.
- (iv) Rule of integration by parts may be used repeatedly, if required.
- (v) If the two functions are of different type, we can choose the first function as the one whose initial comes first in the word "ILATE", where
 I — Inverse Trigonometric function
 L — Logarithmic function
 A — Algebraic function
 T — Trigonometric function
 E — Exponential function.
- (vi) In case, both the functions are trigonometric, take that function as second function whose integral is simple. If both the functions are algebraic, take that function as first function whose derivative is simpler.
- (vii) If the integral consists of an inverse trigonometric function of an algebraic expression in x , first simplify the integrand by a suitable trigonometric substitution and then integrate the new integrand.

4.1 Integrals of the Form

$$\int e^x [f(x) + f'(x)] dx$$

Working Rule

- (i) Split the integral into two integrals.
- (ii) Integrate only the first integral by parts, i.e.

$$\begin{aligned} \int e^x [f(x) + f'(x)] dx &= \int e^x f(x) dx + \int e^x f'(x) dx \\ &= \left[f(x) \cdot e^x - \int f'(x) \cdot e^x dx \right] + \int e^x f'(x) dx \\ &= e^x f(x) + C. \end{aligned}$$

4.2 Integrals of the Form:

Where the initial integrand reappears after integrating by parts.

Working Rule

- (i) Apply the method of integration by parts twice.
- (ii) On integrating by parts second time, we will obtain the given integrand again, put it equal to I.
- (iii) Transpose and collect terms involving I on one side and evaluate I.

5. INTEGRAL OF THE FORM (TRIGONOMETRIC FORMATS)

$$5.1 \quad (a) \int \frac{dx}{a + b \cos x} \quad (b) \int \frac{dx}{a + b \sin x}$$

$$(c) \int \frac{dx}{a + b \cos x + c \sin x}$$

Working Rule

- (i) Put $\cos x = \frac{1 - \tan^2 \frac{x}{2}}{1 + \tan^2 \frac{x}{2}}$ and $\sin x = \frac{2 \tan \frac{x}{2}}{1 + \tan^2 \frac{x}{2}}$ so that the given

integrand becomes a function of $\tan \frac{x}{2}$.

- (ii) Put $\tan \frac{x}{2} = z \Rightarrow \frac{1}{2} \sec^2 \frac{x}{2} dx = dz$
- (iii) Integrate the resulting rational algebraic function of z
- (iv) In the answer, put $z = \tan \frac{x}{2}$.

5.2 Integrals of the Form

$$(a) \int \frac{dx}{a + b \cos^2 x} \quad (b) \int \frac{dx}{a + b \sin^2 x}$$

$$(c) \int \frac{dx}{a \cos^2 x + b \sin x \cos x + c \sin^2 x}$$

Working Rule

- (i) Divide the numerator and denominator by $\cos^2 x$.
- (ii) In the denominator, replace $\sec^2 x$, if any, by $1 + \tan^2 x$.
- (iii) Put $\tan x = z \Rightarrow \sec^2 x \, dx = dz$.
- (iv) Integrate the resulting rational algebraic function of z .
- (v) In the answer, put $z = \tan x$.

5.3 Integrals of the Form

$$\int \frac{a \cos x + b \sin x}{c \cos x + d \sin x} \, dx$$

Working Rule

- (i) Put Numerator = λ (denominator) + μ (derivative of denominator)
 $a \cos x + b \sin x = \lambda (c \cos x + d \sin x) + \mu (-c \sin x + d \cos x)$.
- (ii) Equate coefficients of $\sin x$ and $\cos x$ on both sides and find the values of λ and μ .
- (iii) Split the given integral into two integrals and evaluate each integral separately, i.e.

$$\int \frac{a \cos x + b \sin x}{c \cos x + d \sin x} \, dx =$$

$$\lambda \int 1 \, dx + \mu \int \frac{-c \sin x + d \cos x}{a \cos x + b \sin x} \, dx = \lambda x + \mu \log |a \cos x + b \sin x|.$$

- (iv) Substitute the values of λ and μ found in step 2.

5.4 Integrals of the Form

$$\int \frac{a + b \cos x + c \sin x}{e + f \cos x + g \sin x} \, dx$$

Working Rule

- (i) Put Numerator = l (denominator) + m (derivative of denominator) + n
 $a + b \cos x + c \sin x = l (e + f \cos x + g \sin x) + m (-f \sin x + g \cos x) + n$
- (ii) Equate coefficients of $\sin x$, $\cos x$ and constant term on both sides and find the values of l , m , n .
- (iii) Split the given integral into three integrals and evaluate each integral separately, i.e.

$$\int \frac{a + b \cos x + c \sin x}{e + f \cos x + g \sin x} \, dx$$

$$l \int 1 \, dx + m \int \frac{-f \sin x + g \cos x}{e + f \cos x + g \sin x} \, dx +$$

$$n \int \frac{dx}{e + f \cos x + g \sin x}$$

$$= lx + m \log |e + f \cos x + g \sin x| + n \int \frac{dx}{e + f \cos x + g \sin x} \, dx$$

- (iv) Substitute the values of l , m , n found in Step (ii).

5.5 Integrals of the Form

$$\int \sin^m x \cos^n x \, dx$$

Working Rule

- (i) If the power of $\sin x$ is an odd positive integer, put $\cos x = t$.
- (ii) If the power of $\cos x$ is an odd positive integer, put $\sin x = t$.
- (iii) If the power of $\sin x$ and $\cos x$ are both odd positive integers, put $\sin x = t$ or $\cos x = t$.
- (iv) If the power of $\sin x$ and $\cos x$ are both even positive integers, use De' Moivre's theorem as follows:

Let, $\cos x + i \sin x = z$. Then $\cos x - i \sin x = z^{-1}$

Adding these, we get $z + \frac{1}{z} = 2 \cos x$ and $z - \frac{1}{z} = 2i \sin x$

By De' Moivre's theorem, we have

$$z^n + \frac{1}{z^n} = 2 \cos nx \text{ and } z^n - \frac{1}{z^n} = 2i \sin^n x \dots (1)$$

$$\therefore \sin^m x \cos^n x = \frac{1}{(2i)^m} \cdot \frac{1}{2^n} \left(z + \frac{1}{z} \right)^n \left(z - \frac{1}{z} \right)^m$$

$$= \frac{1}{2^{m+n}} \cdot \frac{1}{i^m} \left(z + \frac{1}{z} \right)^n \left(z - \frac{1}{z} \right)^m.$$

Now expand each of the factors on the R.H.S. using Binomial theorem. Then group the terms equidistant from the beginning and the end. Thus express all such pairs as the sines or cosines of multiple angles. Further integrate term by term.

- (v) If the sum of powers of $\sin x$ and $\cos x$ is an even negative integer, put $\tan x = z$.

SOLVED EXAMPLES

Example – 1

Evaluate : $\int \left(x^3 + 5x^2 - 4 + \frac{7}{x} + \frac{2}{\sqrt{x}} \right) dx$

Sol. $\int \left(x^3 + 5x^2 - 4 + \frac{7}{x} + \frac{2}{\sqrt{x}} \right) dx$

$$= \int x^3 dx + \int 5x^2 dx - \int 4 dx + \int \frac{7}{x} dx + \int \frac{2}{\sqrt{x}} dx$$

$$= \int x^3 dx + 5 \cdot \int x^2 dx - 4 \cdot \int 1 \cdot dx + 7 \cdot \int \frac{1}{x} dx + 2 \cdot \int x^{-1/2} dx$$

$$= \frac{x^4}{4} + 5 \cdot \frac{x^3}{3} - 4x + 7 \log |x| + 2 \left(\frac{x^{1/2}}{1/2} \right) + C$$

$$= \frac{x^4}{4} + \frac{5}{3} x^3 - 4x + 7 \log |x| + 4\sqrt{x} + C$$

Example – 2

Evaluate : $\int e^{x \log a} + e^{a \log x} + e^{a \log a} dx$

Sol. We have,

$$\int e^{x \log a} + e^{a \log x} + e^{a \log a} dx$$

$$= \int e^{\log a^x} + e^{\log x^a} + e^{\log a^a} dx$$

$$= \int (a^x + x^a + a^a) dx$$

$$= \int a^x dx + \int x^a dx + \int a^a dx$$

$$= \frac{a^x}{\log a} + \frac{x^{a+1}}{a+1} + a^a \cdot x + C.$$

Example – 3

Evaluate : $\int \frac{x^4}{x^2 + 1} dx$

Sol. $\int \frac{x^4}{x^2 + 1} dx$

$$= \int \frac{x^4 - 1 + 1}{x^2 + 1} dx = \int \frac{x^4 - 1}{x^2 + 1} + \frac{1}{x^2 + 1} dx$$

$$= \int (x^2 - 1) dx + \int \frac{1}{x^2 + 1} dx = \frac{x^3}{3} - x + \tan^{-1} x + C$$

Example – 4

Evaluate : $\int \frac{2^x + 3^x}{5^x} dx$

Sol. $\int \frac{2^x + 3^x}{5^x} dx$

$$= \int \left(\frac{2^x}{5^x} + \frac{3^x}{5^x} \right) dx$$

$$= \int \left[\left(\frac{2}{5} \right)^x + \left(\frac{3}{5} \right)^x \right] dx = \frac{(2/5)^x}{\log_e 2/5} + \frac{(3/5)^x}{\log_e 3/5} + C$$

Example – 5

Evaluate : $\int x^3 \sin x^4 dx$

Sol. We have

$$I = \int x^3 \sin x^4 dx$$

Let $x^4 = t \Rightarrow d(x^4) = dt$

$$\Rightarrow 4x^3 dx = dt \Rightarrow dx = \frac{1}{4x^3} dt$$

Example – 6

Evaluate $\int \frac{(x^2 - 1) dx}{(x^4 + 3x^2 + 1) \tan^{-1} \left(x + \frac{1}{x} \right)}$

Sol. The given integral can be written as :

$$I = \int \frac{(1 - 1/x^2) dx}{(x^2 + 3 + 1/x^2) \tan^{-1} \left(x + \frac{1}{x} \right)}$$

{dividing N^r. and D^r. by x²}

$$I = \int \frac{(1 - 1/x^2) dx}{\{(x + 1/x)^2 + 1\} \tan^{-1} \left(x + \frac{1}{x} \right)}$$

$$\text{let } x + \frac{1}{x} = t \Rightarrow \left(1 - \frac{1}{x^2} \right) dx = dt$$

$$\therefore I = \int \frac{dt}{(t^2 + 1) \cdot \tan^{-1}(t)} \quad \dots (i)$$

Now make one more substitution

$\tan^{-1} t = u$. Then,

$$\frac{dt}{t^2 + 1} = du$$

$$\therefore (i) \text{ becomes, } I = \int \frac{du}{u} = \log |u| + c$$

$$\Rightarrow I = \log |\tan^{-1} t| + c = \log |\tan^{-1} (x + 1/x)| + c$$

Example – 7

Evaluate : $\int \frac{x}{x^4 + x^2 + 1} dx$

Sol. We have,

$$I = \int \frac{x}{x^4 + x^2 + 1} dx = \int \frac{x}{(x^2)^2 + x^2 + 1} dx$$

Let $x^2 = t$, then, $d(x^2) = dt$

$$\Rightarrow 2x dx = dt$$

$$\Rightarrow dx = \frac{dt}{2x}$$

$$I = \int \frac{x}{t^2 + t + 1} \cdot \frac{dt}{2x}$$

$$= \frac{1}{2} \int \frac{1}{t^2 + t + 1} dt$$

$$= \frac{1}{2} \int \frac{1}{\left(t + \frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2} dt$$

$$= \frac{1}{2} \cdot \frac{1}{\frac{\sqrt{3}}{2}} \tan^{-1} \left(\frac{t + \frac{1}{2}}{\frac{\sqrt{3}}{2}} \right) + C$$

$$= \frac{1}{\sqrt{3}} \tan^{-1} \left(\frac{2t + 1}{\sqrt{3}} \right) + C = \frac{1}{\sqrt{3}} \tan^{-1} \left(\frac{2x^2 + 1}{\sqrt{3}} \right) + C.$$

Example – 8

Evaluate : $\int x \log(1 + x) dx$

Sol. $\int x \log(1 + x) dx$

$$= \log(x + 1) \cdot \frac{x^2}{2} - \int \frac{1}{x + 1} \cdot \frac{x^2}{2} dx$$

$$= \frac{x^2}{2} \log(x + 1) - \frac{1}{2} \int \frac{x^2}{x + 1} dx$$

$$= \frac{x^2}{2} \log(x + 1) - \frac{1}{2} \int \frac{x^2 - 1 + 1}{x + 1} dx$$

$$= \frac{x^2}{2} \log(x + 1) - \frac{1}{2} \int \frac{x^2 - 1}{x + 1} + \frac{1}{x + 1} dx$$

$$= \frac{x^2}{2} \log(x + 1) - \frac{1}{2} \left[\int \left((x - 1) + \frac{1}{x + 1} \right) dx \right]$$

$$= \frac{x^2}{2} \log(x + 1) - \frac{1}{2} \left[\frac{x^2}{2} - x + \log|x + 1| \right] + C$$

Example – 9

Evaluate $\int \frac{\sin^{-1} \sqrt{x} - \cos^{-1} \sqrt{x}}{\sin^{-1} \sqrt{x} + \cos^{-1} \sqrt{x}} dx$

Sol. $\int \frac{\sin^{-1} \sqrt{x} - \cos^{-1} \sqrt{x}}{\sin^{-1} \sqrt{x} + \cos^{-1} \sqrt{x}} dx$

$$= \int \frac{\sin^{-1} \sqrt{x} - (\pi/2 - \sin^{-1} \sqrt{x})}{\pi/2} dx$$

$$\{ \because \sin^{-1} \theta + \cos^{-1} \theta = \pi/2 \}$$

$$\Rightarrow I = \frac{2}{\pi} \int (2 \sin^{-1} \sqrt{x} - \pi/2) dx$$

$$I = \frac{4}{\pi} \int \sin^{-1} \sqrt{x} dx - \int 1 dx$$

$$I = \frac{4}{\pi} \int \sin^{-1} \sqrt{x} dx - x + c \quad \dots (i)$$

Let $x = \sin^2 \theta$, then $dx = 2 \sin \theta \cos \theta d\theta = \sin 2\theta d\theta$

$$\therefore \int \sin^{-1} \sqrt{x} dx = \int \theta \cdot \sin 2\theta d\theta$$

applying integration by parts

$$\int \sin^{-1} \sqrt{x} dx = -\theta \cdot \frac{\cos 2\theta}{2} + \int \frac{1}{2} \cos 2\theta d\theta$$

$$= \frac{-\theta}{2} \cdot \cos 2\theta + \frac{1}{4} \sin 2\theta$$

$$= \frac{-1 \cdot \theta}{2} \cdot (1 - 2 \sin^2 \theta) + \frac{1}{2} \cdot \sin \theta \cdot \sqrt{1 - \sin^2 \theta}$$

$$= \frac{-1}{2} \sin^{-1} \sqrt{x} (1 - 2x) + \frac{1}{2} \cdot \sqrt{x} \sqrt{1 - x} \quad \dots (ii)$$

from (i) and (ii);

$$I = \frac{4}{\pi} \left\{ \frac{-1}{2} (\sin^{-1} \sqrt{x}) (1 - 2x) + \frac{1}{2} \sqrt{x} \sqrt{1 - x} \right\} - x + c$$

$$= \frac{2}{\pi} \{ \sqrt{x - x^2} - (1 - 2x) \sin^{-1} \sqrt{x} \} - x + c$$

Example – 10

Evaluate

$$(i) \int e^x \left(\frac{1 + \sin x \cos x}{\cos^2 x} \right) dx \quad (ii) \int e^{2x} \left(\frac{1 + \sin 2x}{1 + \cos 2x} \right) dx$$

Sol. (i) $I = \int e^x \left(\frac{1 + \sin x \cos x}{\cos^2 x} \right) dx$

$$I = \int e^x \left\{ \frac{1}{\cos^2 x} + \frac{\sin x \cos x}{\cos^2 x} \right\} dx$$

$$I = \int e^x \{ \tan x + \sec^2 x \} dx$$

$$= \int e^x \cdot \tan x dx + \int e^x (\sec^2 x) dx$$

$$= \tan x \cdot e^x - \int \sec^2 x \cdot e^x dx + \int e^x \cdot \sec^2 x dx + c$$

$$I = e^x \tan x + c.$$

(ii) $I = \int e^{2x} \left\{ \frac{1 + \sin 2x}{1 + \cos 2x} \right\} dx$

$$= \int e^{2x} \left\{ \frac{1 + 2 \sin x \cos x}{2 \cos^2 x} \right\} dx$$

$$= \int e^{2x} \left\{ \frac{1}{2 \cos^2 x} + \frac{2 \sin x \cos x}{2 \cos^2 x} \right\} dx$$

$$= \int e^{2x} \left\{ \frac{1}{2} \sec^2 x + \tan x \right\} dx$$

$$= \int e^{2x} \cdot \tan x dx + \frac{1}{2} \int e^{2x} \cdot \sec^2 x dx$$

$$= \tan x \cdot \frac{e^{2x}}{2} - \int \sec^2 x \cdot \frac{e^{2x}}{2} dx + \frac{1}{2} \int e^{2x} \cdot \sec^2 x dx$$

$$I = \frac{1}{2} e^{2x} \cdot \tan x + c.$$

Example – 11

Evaluate : $\int \sqrt{x^2 + 2x + 5} \, dx$

Sol. We have,

$$\begin{aligned} & \int \sqrt{x^2 + 2x + 5} \\ &= \int \sqrt{x^2 + 2x + 1 + 4} \, dx \\ &= \frac{1}{2} (x + 1) \sqrt{(x + 1)^2 + 2^2} + \frac{1}{2} \cdot (2)^2 \log |(x + 1) \\ & \quad + \sqrt{(x + 1)^2 + 2^2}| + C \\ &= \frac{1}{2} (x + 1) \sqrt{x^2 + 2x + 5} + 2 \log |(x + 1) + \sqrt{x^2 + 2x + 5}| + C \end{aligned}$$

Example – 12

Evaluate : $\int \frac{1}{x^2 - x + 1} \, dx$

$$\begin{aligned} \text{Sol. } & \int \frac{1}{x^2 - x + 1} \, dx \\ &= \int \frac{1}{x^2 - x + \frac{1}{4} - \frac{1}{4} + 1} \, dx \\ &= \int \frac{1}{(x - 1/2)^2 + 3/4} \, dx \\ &= \int \frac{1}{(x - 1/2)^2 + (\sqrt{3}/2)^2} \, dx \\ &= \frac{1}{\sqrt{3}/2} \tan^{-1} \left(\frac{x - 1/2}{\sqrt{3}/2} \right) + C \\ &= \frac{2}{\sqrt{3}} \tan^{-1} \left(\frac{2x - 1}{\sqrt{3}} \right) + C. \end{aligned}$$

Example – 13

Evaluate : $\int \frac{1}{\sqrt{9 + 8x - x^2}} \, dx$

$$\begin{aligned} \text{Sol. } & \int \frac{1}{\sqrt{9 + 8x - x^2}} \, dx \\ &= \int \frac{1}{\sqrt{-\{x^2 - 8x - 9\}}} \, dx \\ &= \int \frac{1}{\sqrt{-\{x^2 - 8x + 16 - 25\}}} \, dx \\ &= \int \frac{1}{\sqrt{-\{(x - 4)^2 - 5^2\}}} \, dx \\ &= \int \frac{1}{\sqrt{5^2 - (x - 4)^2}} \, dx = \sin^{-1} \left(\frac{x - 4}{5} \right) + C \end{aligned}$$

Example – 14

Evaluate : $\int \frac{2x + 3}{\sqrt{x^2 + 4x + 1}} \, dx$

$$\begin{aligned} \text{Sol. } & \int \frac{2x + 3}{\sqrt{x^2 + 4x + 1}} \, dx \\ &= \int \frac{(2x + 4) - 1}{\sqrt{x^2 + 4x + 1}} \, dx \\ &= \int \frac{2x + 4}{\sqrt{x^2 + 4x + 1}} \, dx - \int \frac{1}{\sqrt{x^2 + 4x + 1}} \, dx \\ &= \int \frac{dt}{\sqrt{t}} - \int \frac{1}{\sqrt{(x + 2)^2 - (\sqrt{3})^2}} \, dx, \text{ where } t = x^2 + 4x + 1 \\ &= 2 \sqrt{t} - \log |(x + 2) + \sqrt{x^2 + 4x + 1}| + C \\ &= 2 \sqrt{x^2 + 4x + 1} - \log |x + 2 + \sqrt{x^2 + 4x + 1}| + C \end{aligned}$$

Example – 15

Evaluate : $\int (x-5)\sqrt{x^2+x} \, dx$

Sol Let $(x-5) = \lambda \cdot \frac{d}{dx} (x^2+x) + \mu$. Then,

$$x-5 = \lambda (2x+1) + \mu.$$

Comparing coefficients of like powers of x, we get

$$1 = 2\lambda \text{ and } \lambda + \mu = -5 \Rightarrow \lambda = \frac{1}{2} \text{ and } \mu = -\frac{11}{2}$$

$$\begin{aligned} & \int (x-5)\sqrt{x^2+x} \, dx \\ &= \int \left(\frac{1}{2}(2x+1) - \frac{11}{2} \right) \sqrt{x^2+x} \, dx \\ &= \int \frac{1}{2}(2x+1) \sqrt{x^2+x} \, dx - \frac{11}{2} \int \sqrt{x^2+x} \, dx \\ &= \frac{1}{2} \int (2x+1) \sqrt{x^2+x} \, dx - \frac{11}{2} \int \sqrt{x^2+x} \, dx \\ &= \frac{1}{2} \int \sqrt{t} \, dt - \frac{11}{2} \int \sqrt{\left(x+\frac{1}{2}\right)^2 - \left(\frac{1}{2}\right)^2} \, dx \text{ where} \\ & t = x^2 + x \\ &= \frac{1}{2} \cdot \frac{t^{3/2}}{3/2} - \frac{11}{2} \left[\frac{1}{2} \left(x+\frac{1}{2}\right) \sqrt{\left(x+\frac{1}{2}\right)^2 - \left(\frac{1}{2}\right)^2} \right. \\ & \quad \left. - \frac{1}{2} \cdot \left(\frac{1}{2}\right)^2 \log \left[\left(x+\frac{1}{2}\right) + \sqrt{\left(x+\frac{1}{2}\right)^2 - \left(\frac{1}{2}\right)^2} \right] \right] + C \\ &= \frac{1}{3} t^{3/2} - \frac{11}{2} \left[\frac{2x+1}{4} \sqrt{x^2+x} - \frac{1}{8} \ell n \left| \left(x+\frac{1}{2}\right) + \sqrt{x^2+x} \right| \right] + C \\ &= \frac{1}{3} (x^2+x)^{3/2} \\ & \quad - \frac{11}{2} \left[\frac{2x+1}{4} \sqrt{x^2+x} - \frac{1}{8} \ell n \left| \left(x+\frac{1}{2}\right) + \sqrt{x^2+x} \right| \right] + C \end{aligned}$$

Example – 16

Evaluate $\int \sin^3 x \cdot \cos^5 x \, dx$

Sol. $I = \int \sin^3 x \cdot \cos^5 x \, dx$

Let $\cos x = t \Rightarrow -\sin x \, dx = dt$

$$I = -\int (1-t^2) \cdot t^5 \cdot dt$$

$$I = \int t^7 dt - \int t^5 dt = \frac{t^8}{8} - \frac{t^6}{6} + c$$

$$I = \frac{\cos^8 x}{8} - \frac{\cos^6 x}{6} + c$$

Aliter : $I = \int R^3(1-R^2)^2 dR$, if $\sin x = R$, $\cos x \, dx = dR$.

$$I = \int R^3 dR - \int 2R^5 dR + \int R^7 dR$$

$$I = \frac{\sin^4 x}{4} - \frac{2\sin^6 x}{6} + \frac{\sin^8 x}{8} + c$$

Example – 17

Evaluate $\int \sin^{-11/3} x \cos^{-1/3} x \, dx$

Sol. Here, $\int \sin^{-11/3} x \cdot \cos^{-1/3} x \, dx$

$$\text{i.e., } \left(-\frac{11}{3} - \frac{1}{3} \right) = -4$$

$$\therefore I = \int \frac{\cos^{-1/3}}{\sin^{-11/3} x \cdot \sin^4 x} dx = \int (\cot^{-1/3} x) (\operatorname{cosec}^2 x)^2 \cdot dx$$

$$I = \int (\cot^{-1/3} x) (1 + \cot^2 x) \operatorname{cosec}^2 x \, dx.$$

{let $\cot x = t$, $-\operatorname{cosec}^2 x \, dx = dt$ }

$$= -\int t^{-1/3} (1+t^2) dt = -\int (t^{-1/3} + t^{5/3}) dt$$

$$= -\left\{ \frac{3}{2} t^{2/3} + \frac{3}{8} t^{8/3} \right\} + c$$

$$= -\left\{ \frac{3}{2} (\cot^{2/3} x) + \frac{3}{8} (\cot^{8/3} x) \right\} + c.$$

Example – 18

Evaluate

$$(i) \int \frac{1}{\sin(x-a) \cos(x-b)} dx$$

$$(ii) \int \frac{1}{\cos(x-a) \cos(x-b)} dx$$

$$\text{Sol. (i) } I = \int \frac{1}{\sin(x-a) \cos(x-b)} dx$$

$$I = \frac{\cos(a-b)}{\cos(a-b)} \cdot \int \frac{dx}{\sin(x-a) \cos(x-b)}$$

$$= \frac{1}{\cos(a-b)} \cdot \int \frac{\cos\{(x-b)-(x-a)\}}{\sin(x-a) \cos(x-b)} dx$$

$$= \frac{1}{\cos(a-b)} \cdot \int \left\{ \frac{\cos(x-b) \cdot \cos(x-a)}{\sin(x-a) \cos(x-b)} + \frac{\sin(x-b) \cdot \sin(x-a)}{\sin(x-a) \cos(x-b)} \right\} dx$$

$$I = \frac{1}{\cos(a-b)} \int \{\cot(x-a) + \tan(x-b)\} dx$$

$$I = \frac{1}{\cos(a-b)} \{\log|\sin(x-a)| - \log|\cos(x-b)|\} + c$$

$$I = \frac{1}{\cos(a-b)} \log_e \left| \frac{\sin(x-a)}{\cos(x-b)} \right| + c$$

$$(ii) \quad I = \int \frac{1}{\cos(x-a) \cos(x-b)} dx$$

$$= \frac{1}{\sin(a-b)} \int \frac{\sin(a-b) dx}{\cos(x-a) \cos(x-b)}$$

$$= \frac{1}{\sin(a-b)} \int \frac{\sin\{(x-b)-(x-a)\}}{\cos(x-a) \cos(x-b)} dx$$

$$= \frac{1}{\sin(a-b)} \int \left\{ \frac{\sin(x-b) \cos(x-a)}{\cos(x-a) \cos(x-b)} - \frac{\cos(x-b) \sin(x-a)}{\cos(x-a) \cos(x-b)} \right\} dx$$

$$= \frac{1}{\sin(a-b)} \int \{\tan(x-b) - \tan(x-a)\} dx$$

$$= \frac{1}{\sin(a-b)} [-\log|\cos(x-b)| + \log|\cos(x-a)|] + c$$

$$= \frac{1}{\sin(a-b)} \left[\log \left| \frac{\cos(x-a)}{\cos(x-b)} \right| \right] + c$$

Example – 19

$$\text{Evaluate : } \int \frac{1}{1 + \sin x + \cos x} dx$$

$$\text{Sol. } I = \int \frac{1}{1 + \sin x + \cos x} dx$$

$$= \int \frac{1}{1 + \frac{2 \tan x/2}{1 + \tan^2 x/2} + \frac{1 - \tan^2 x/2}{1 + \tan^2 x/2}} dx$$

$$= \int \frac{1 + \tan^2 x/2}{1 + \tan^2 x/2 + 2 \tan x/2 + 1 - \tan^2 x/2} dx$$

$$= \int \frac{\sec^2 x/2}{2 + 2 \tan x/2} dx$$

$$\text{Putting } \tan \frac{x}{2} = t \text{ and } \frac{1}{2} \sec^2 \frac{x}{2} dx = dt, \text{ we get}$$

$$I = \int \frac{1}{t+1} dt = \log|t+1| + C = \log \left| \tan \frac{x}{2} + 1 \right| + C$$

Example – 20

Evaluate : $\int \frac{3 \sin x + 2 \cos x}{3 \cos x + 2 \sin x} dx$

Sol. $I = \int \frac{3 \sin x + 2 \cos x}{3 \cos x + 2 \sin x} dx$

Let $3 \sin x + 2 \cos x = \lambda \cdot \frac{d}{dx} (3 \cos x + 2 \sin x) +$

$\mu (3 \cos x + 2 \sin x)$

$\Rightarrow 3 \sin x + 2 \cos x = \lambda (-3 \sin x + 2 \cos x) +$

$\mu (3 \cos x + 2 \sin x)$

Comparing the coefficients of $\sin x$ and $\cos x$ on both sides, we get

$-3\lambda + 2\mu = 3$ and $2\lambda + 3\mu = 2$

$\Rightarrow \mu = \frac{12}{13}$ and $\lambda = -\frac{5}{13}$

$\therefore I = \int \frac{\mu(-3 \sin x + 2 \cos x) + \lambda(3 \cos x + 2 \sin x)}{3 \cos x + 2 \sin x} dx$

$= \lambda \int 1 \cdot dx + \mu \int \frac{-3 \sin x + 2 \cos x}{3 \cos x + 2 \sin x} dx$

$= \lambda x + \mu \int \frac{dt}{t}$, where $t = 3 \cos x + 2 \sin x$

$= \lambda x + \mu \ln |t| + C = \frac{-5}{13} x +$

$\frac{12}{13} \ln |3 \cos x + 2 \sin x| + C$

Example – 21

Evaluate : $\int \frac{3 \cos x + 2}{\sin x + 2 \cos x + 3} dx$

Sol. We have,

$I = \int \frac{3 \cos x + 2}{\sin x + 2 \cos x + 3} dx$

Let $3 \cos x + 2 = \lambda (\sin x + 2 \cos x + 3) +$

$\mu (\cos x - 2 \sin x) + v$

Comparing the coefficients of $\sin x$, $\cos x$ and constant term on both sides, we get

$\lambda - 2\mu = 0$, $2\lambda + \mu = 3$, $3\lambda + v = 2$

$\Rightarrow \lambda = \frac{6}{5}$, $\mu = \frac{3}{5}$ and $v = -\frac{8}{5}$

$\therefore I = \int \frac{\lambda(\sin x + 2 \cos x + 3) + \mu(\cos x - 2 \sin x) + v}{\sin x + 2 \cos x + 3} dx$

$\Rightarrow I = \lambda \int dx + \mu \int \frac{\cos x - 2 \sin x}{\sin x + 2 \cos x + 3} dx +$

$v \int \frac{1}{\sin x + 2 \cos x + 3} dx$

$\Rightarrow I = \lambda x + \mu \log |\sin x + 2 \cos x + 3| + v I_1$, where

$I_1 = \int \frac{1}{\sin x + 2 \cos x + 3} dx$

Putting, $\sin x = \frac{2 \tan x / 2}{1 + \tan^2 x / 2}$, $\cos x = \frac{1 - \tan^2 x / 2}{1 + \tan^2 x / 2}$ we get

$I_1 = \int \frac{1}{\frac{2 \tan x / 2}{1 + \tan^2 x / 2} + \frac{2(1 - \tan^2 x / 2)}{1 + \tan^2 x / 2} + 3} dx$

$= \int \frac{1 + \tan^2 x / 2}{2 \tan x / 2 + 2 - 2 \tan^2 x / 2 + 3(1 + \tan^2 x / 2)} dx$

$= \int \frac{\sec^2 x / 2}{\tan^2 x / 2 + 2 \tan x / 2 + 5} dx$

Example – 22

$$\int \frac{1-x^2}{1+x^2+x^4} dx$$

Sol. $\int \frac{-\left(1-\frac{1}{x^2}\right) dx}{x^2+\frac{1}{x^2}+1}$

Put, $x + \frac{1}{x} = t$

$$\Rightarrow -\int \frac{dt}{t^2-1}$$

$$= -\frac{1}{2} \ln \left| \frac{t-1}{t+1} \right| + C$$

$$= -\frac{1}{2} \ln \left| \frac{x+\frac{1}{x}-1}{x+\frac{1}{x}+1} \right| + C$$

Example – 23

Evaluate : $\int \frac{1}{x^4+1} dx$

Sol. We have,

$$I = \int \frac{1}{x^4+1} dx$$

$$\Rightarrow I = \int \frac{\frac{1}{x^2}}{x^2+\frac{1}{x^2}} dx$$

$$\Rightarrow I = \frac{1}{2} \int \frac{\frac{2}{x^2}}{x^2+\frac{1}{x^2}} dx$$

$$\Rightarrow I = \frac{1}{2} \int \frac{1+\frac{1}{x^2}}{x^2+\frac{1}{x^2}} - \frac{1-\frac{1}{x^2}}{x^2+\frac{1}{x^2}} dx$$

$$\Rightarrow I = \frac{1}{2} \int \frac{1+\frac{1}{x^2}}{x^2+\frac{1}{x^2}} dx - \frac{1}{2} \int \frac{1-\frac{1}{x^2}}{x^2+\frac{1}{x^2}} dx$$

$$\Rightarrow I = \frac{1}{2} \int \frac{1+\frac{1}{x^2}}{\left(x-\frac{1}{x}\right)^2+2} dx$$

$$- \frac{1}{2} \int \frac{1-\frac{1}{x^2}}{\left(x+\frac{1}{x}\right)^2-2} dx$$

Putting $x - \frac{1}{x} = u$ in 1st integral and $x + \frac{1}{x} = v$ in 2nd

integral, we get

$$I = \frac{1}{2} \int \frac{du}{u^2+(\sqrt{2})^2} - \frac{1}{2} \int \frac{dv}{v^2-(\sqrt{2})^2}$$

$$= \frac{1}{2\sqrt{2}} \tan^{-1}\left(\frac{u}{\sqrt{2}}\right) - \frac{1}{2} \frac{1}{2\sqrt{2}} \log \left| \frac{v-\sqrt{2}}{v+\sqrt{2}} \right| + C$$

$$= \frac{1}{2\sqrt{2}} \tan^{-1}\left(\frac{x-1/x}{\sqrt{2}}\right) - \frac{1}{4\sqrt{2}} \log \left| \frac{x+1/x-\sqrt{2}}{x+1/x+\sqrt{2}} \right| + C$$

$$= \frac{1}{2\sqrt{2}} \tan^{-1}\left(\frac{x^2-1}{\sqrt{2}x}\right) - \frac{1}{4\sqrt{2}} \log \left| \frac{x^2-\sqrt{2}x+1}{x^2+x\sqrt{2}+1} \right| + C$$

Example – 24

Evaluate : $\int \frac{x+2}{(x^2+3x+3)\sqrt{x+1}} dx$

Sol. Let $I = \int \frac{x+2}{(x^2+3x+3)\sqrt{x+1}} dx$

Putting $x+1 = t^2$, and $dx = 2t dt$, we get

$$I = \int \frac{(t^2+1) 2t dt}{\{(t^2-1)^2 + 3(t^2-1) + 3\}\sqrt{t^2}}$$

$$\Rightarrow I = 2 \int \frac{(t^2+1)}{t^4+t^2+1} dt = 2 \int \frac{1+\frac{1}{t^2}}{t^2+\frac{1}{t^2}+1} dt$$

$$\Rightarrow I = 2 \int \frac{du}{u^2 + (\sqrt{3})^2} \text{ where } t - \frac{1}{t} = u.$$

$$\Rightarrow I = \frac{2}{\sqrt{3}} \tan^{-1} \left(\frac{u}{\sqrt{3}} \right) + C = \frac{2}{\sqrt{3}} \tan^{-1} \left\{ \frac{t - \frac{1}{t}}{\sqrt{3}} \right\} + C$$

$$\Rightarrow I = \frac{2}{\sqrt{3}} \tan^{-1} \left(\frac{t^2-1}{t\sqrt{3}} \right) + C = \frac{2}{\sqrt{3}} \tan^{-1} \left\{ \frac{x}{\sqrt{3}(x+1)} \right\} + C$$

Example – 25

$$\int \frac{dx}{(x+1)\sqrt{x^2+x+1}}$$

Sol. Putting $x+1 = \frac{1}{t} \Rightarrow dx = -\frac{1}{t^2} dt$

$$I = \int \frac{-dt}{t^2 \left(\frac{1}{t} \right) \sqrt{\left(\frac{1}{t} - 1 \right)^2 + \frac{1}{t}}} = \int \frac{-dt}{t \sqrt{\frac{1}{t^2} - \frac{1}{t} + 1}}$$

$$= \int \frac{-dt}{\sqrt{t^2 - t + 1}}$$

$$= \int \frac{-dt}{\sqrt{\left(t - \frac{1}{2} \right)^2 + \frac{3}{4}}}$$

$$= -\ln \left(t - \frac{1}{2} + \sqrt{\left(t - \frac{1}{2} \right)^2 + \frac{3}{4}} \right) + C$$

Example – 26

$$\int \frac{dx}{(1+x^2)\sqrt{1-x^2}}$$

Sol. Put $x = \frac{1}{t}$

$$\Rightarrow I = \int \frac{-t dt}{(t^2+1)\sqrt{t^2-1}}$$

put $t^2 - 1 = y^2$

$$\Rightarrow I = - \int \frac{y dy}{(y^2+2)y}$$

$$= - \frac{1}{\sqrt{2}} \tan^{-1} \left(\frac{y}{\sqrt{2}} \right) + C$$

$$= - \frac{1}{\sqrt{2}} \tan^{-1} \left(\frac{\sqrt{1-x^2}}{\sqrt{2}x} \right) + C$$

Example – 27

Evaluate $\int \frac{3x-5}{(3x-2)(x+1)^2} dx$

Sol. Let $\frac{3x-5}{(3x-2)(x+1)^2} = \frac{A}{3x-2} + \frac{B}{x+1} + \frac{C}{(x+1)^2}$

$\therefore 3x-5 = A(x+1)^2 + B(3x-2)(x+1) + C(3x-2)$

Putting $x = \frac{2}{3}$, we get $A = -\frac{27}{25}$

Putting $x = -1$, we get $C = \frac{8}{5}$;

Putting $x = 0$, we get $B = \frac{9}{25}$

$\therefore I = \int \left[-\frac{27}{25} \cdot \frac{1}{3x-2} + \frac{9}{25} \cdot \frac{1}{(x+1)} + \frac{8}{5} \cdot \frac{1}{(x+1)^2} \right] dx$
 $= -\frac{9}{25} \log |3x-2| + \frac{9}{25} \log |x+1| - \frac{8}{5} \cdot \frac{1}{(x+1)} + c$

Example – 28

Evaluate $\int \frac{dx}{(x-1)(x^2+1)}$

Sol. Let $\frac{1}{(x-1)(x^2+1)} = \frac{A}{x-1} + \frac{Bx+C}{x^2+1}$

$= \frac{A(x^2+1) + (Bx+C)(x-1)}{(x-1)(x^2+1)}$

or $1 = A(x^2+1) + (Bx+C)(x-1)$

Putting $x = 1$, we get $A = \frac{1}{2}$;

Putting $x = 0$, we get $A - C = 1 \therefore C = A - 1 = -\frac{1}{2}$

Putting $x = -1$, we get $2A - 2(-B+C) = 1$

or $1 + 2B - 2\left(-\frac{1}{2}\right) = 1 \therefore B = -\frac{1}{2}$

Now $\frac{1}{(x-1)(x^2+1)} = \frac{1}{2(x-1)} - \frac{1}{2} \cdot \frac{x+1}{x^2+1}$

$\therefore \int \frac{dx}{(x-1)(x^2+1)} = \int \frac{1}{2(x-1)} dx - \frac{1}{2} \int \frac{x}{x^2+1} dx - \frac{1}{2} \int \frac{dx}{x^2+1}$

$= \frac{1}{2} \log |x-1| - \frac{1}{4} \log (x^2+1) - \frac{1}{2} \tan^{-1} x + c$

Example – 29

Find $\int \frac{x^4}{(x-1)(x^2+1)} dx$

Sol. $x^3 - x^2 + x - 1 \overbrace{x^4}^{x+1}$
 $x^4 - x^3 + x^2 - x,$
 $\quad \quad \quad + \quad - \quad +$
 $\quad \quad \quad x^3 - x^2 + x$
 $x^3 - x^2 + x - 1$
 $\quad \quad \quad + \quad - \quad +$
 $\quad \quad \quad 1$

$\therefore \frac{x^4}{(x-1)(x^2+1)} = x+1 + \frac{1}{(x-1)(x^2+1)}$

$\therefore \int \frac{x^4}{(x-1)(x^2+1)} dx = \int (x+1) dx + \int \frac{dx}{(x-1)(x^2+1)}$

$= \frac{x^2}{2} + x + \frac{1}{2} \log |x-1| - \frac{1}{4} \log (x^2+1)$

$-\frac{1}{2} \tan^{-1} x + c$

Example – 30

Evaluate $\int \frac{dx}{\sin x + \sin 2x}$

Sol. Let the given integral be I, then

$$I = \int \frac{dx}{\sin x + 2 \sin x \cos x} = \int \frac{dx}{\sin x(1 + 2 \cos x)}$$

$$= \int \frac{\sin x \, dx}{\sin^2 x(1 + 2 \cos x)} = \int \frac{\sin x \, dx}{(1 - \cos^2 x)(1 + 2 \cos x)}$$

Put $\cos x = y$ so that $-\sin x \, dx = dy$.

$$\therefore I = \int \frac{-dy}{(1 - y^2)(1 + 2y)}$$

$$= - \int \frac{dy}{(1 - y)(1 + y)(1 + 2y)}$$

Now we break $\frac{1}{(1 - y)(1 + y)(1 + 2y)}$ into partial fractions

$$\text{Let } \frac{1}{(1 - y)(1 + y)(1 + 2y)} = \frac{A}{1 - y} + \frac{B}{1 + y} + \frac{C}{1 + 2y}$$

$$= \frac{A(1 + y)(1 + 2y) + B(1 - y)(1 + 2y) + C(1 - y)(1 + y)}{(1 - y)(1 + y)(1 + 2y)}$$

$$\therefore 1 = A(1 + y)(1 + 2y) + B(1 - y)(1 + 2y) + C(1 - y)(1 + y)$$

$$\text{Putting } y = 1, \text{ we get } A = \frac{1}{6}; \text{ putting } y = -\frac{1}{2}, \text{ we get } C = \frac{4}{3}$$

$$\text{Putting } y = -1, \text{ we get } B = -\frac{1}{2}$$

$$\text{Now } I = - \int \left[\frac{1}{6(1 - y)} - \frac{1}{2(1 + y)} + \frac{4}{3(1 + 2y)} \right] dy$$

$$= \frac{1}{6} \log |1 - y| + \frac{1}{2} \log |1 + y| - \frac{2}{3} \log |1 + 2y| + c$$

$$= \frac{1}{6} \log (1 - \cos x) + \frac{1}{2} \log (1 + \cos x)$$

$$- \frac{2}{3} \log |1 + 2 \cos x| + c$$

Example – 31

Evaluate $\int \frac{x^3 + 3x + 2}{(x^2 + 1)^2(x + 1)} dx$

$$\text{Sol. } I = \int \frac{x(x^2 + 1) + 2(x + 1)}{(x^2 + 1)^2(x + 1)} dx$$

$$= \int \frac{x}{(x^2 + 1)(x + 1)} dx + 2 \int \frac{dx}{(1 + x^2)^2} \quad \dots (1)$$

$$\text{Let } \frac{x}{(1 + x)(1 + x^2)} = \frac{A}{1 + x} + \frac{Bx + C}{1 + x^2}$$

$$\therefore x = A(1 + x^2) + (Bx + C)(1 + x)$$

$$\text{Putting } x = -1, \text{ we get } A = -\frac{1}{2}$$

$$\text{Putting } x = 0, \text{ we get } 0 = A + C \Rightarrow C = -A = \frac{1}{2}$$

$$\text{Putting } x = 1, \text{ we get } 1 = 2A + 2(B + C)$$

$$= 2A + 2B + 2C = -1 + 2B + 1 \therefore B = \frac{1}{2}$$

$$\therefore \int \frac{x}{(1 + x)(1 + x^2)} dx = \int \left(-\frac{1}{2(1 + x)} + \frac{\frac{1}{2}x + \frac{1}{2}}{1 + x^2} \right) dx$$

$$\begin{aligned}
 &= -\frac{1}{2} \log |1+x| + \frac{1}{2} \int \frac{x}{1+x^2} dx + \frac{1}{2} \int \frac{dx}{1+x^2} \\
 &= -\frac{1}{2} \log |1+x| + \frac{1}{4} \log (1+x^2) + \frac{1}{2} \tan^{-1} x \quad \dots (2)
 \end{aligned}$$

To evaluate : $\int \frac{dx}{(1+x^2)^2}$, put $x = \tan \theta$

$$\begin{aligned}
 \text{Then, } \int \frac{dx}{(1+x^2)^2} &= \int \frac{\sec^2 \theta}{(1+\tan^2 \theta)^2} d\theta \\
 &= \int \cos^2 \theta d\theta = \int \frac{1+\cos 2\theta}{2} d\theta \\
 &= \frac{1}{2} \left[\theta + \frac{\sin 2\theta}{2} \right] = \frac{1}{2} [\theta + \sin \theta \cos \theta]
 \end{aligned}$$

$$= \frac{1}{2} \left[\tan^{-1} x + \frac{x}{\sqrt{1+x^2}} \cdot \frac{1}{\sqrt{1+x^2}} \right]$$

$$= \frac{1}{2} \tan^{-1} x + \frac{1}{2} \frac{x}{1+x^2}$$

Now from (1),

$$I = \frac{3}{2} \tan^{-1} x - \frac{1}{2} \log |1+x| + \frac{1}{4} \log (1+x^2) + \frac{x}{1+x^2} + c$$

Example – 32

If $\int f(x) \cos x \, dx = \frac{1}{2} f^2(x) + c$, then $f(x)$ can be

- (a) x (b) 1 (c) $\cos x$ (d) $\sin x$

Sol. Here, $\int f(x) \cos x \, dx = \frac{1}{2} f^2(x) + c$, differentiating both

sides, we get

$$f(x) \cos x = f(x) \cdot f'(x)$$

$$\text{i.e., } \cos x = \frac{d}{dx} f(x)$$

$$\Rightarrow f(x) = \int \cos x \, dx$$

$$\Rightarrow f(x) = \sin x + c.$$

Hence (d) is the correct answer.

EXERCISE - 1 : BASIC OBJECTIVE QUESTIONS

Basic Formats

1. $\int \sin^2(x/2) dx$ equals

(a) $\frac{1}{2}(x + \sin x) + c$ (b) $\frac{1}{2}(x + \cos x) + c$

(c) $\frac{1}{2}(x - \sin x) + c$ (d) none of these

2. $\int \cot^2 x dx$ equals

(a) $-\sec x + x + c$ (b) $-\cot x - x + c$

(c) $-\sin x + x + c$ (d) none of these

3. $\int \frac{5x+7}{x} dx$ equals

(a) $5x + 7 \log|x|$ (b) $7x + 5 \log|x| + C$

(c) $5x + 7 \log|x| + C$ (d) none of these

4. $\int \left(x - \frac{1}{x}\right)^3 dx, (x > 0)$ equals

(a) $\frac{x^3}{3} - \frac{3}{2}x^2 + 3 \log x + \frac{1}{2x^2} + C$

(b) $\frac{x^4}{4} - \frac{3}{2}x^2 + 3 \log x + \frac{1}{2x^2} + C$

(c) $\frac{x^4}{4} + 3 \log x + \frac{1}{2x^2} + C$

(d) none of these

5. The value of $\int \left(\frac{6}{1+x^2} + 10^x\right) dx$ is

(a) $6 \tan^{-1} x + 10^x \log_e 10 + C$

(b) $6 \tan^{-1} x + \frac{10^x}{\log_e 10} + C$

(c) $3 \tan^{-1} x + \frac{10^x}{\log_e 10} + C$

(d) none of these

6. $\int \frac{e^{5 \log_e x} - e^{4 \log_e x}}{e^{3 \log_e x} - e^{2 \log_e x}} dx$ is equal to

(a) $\frac{x^3}{3} + c$ (b) $\frac{x^2}{3} + c$

(c) $\frac{x^4}{3} + c$ (d) $\frac{x^3}{2} + c$

7. $\int \frac{\sin^6 x + \cos^6 x}{\sin^2 x \cos^2 x} dx$ is equal to

(a) $\tan x + \cot x + 3x + c$ (b) $\tan x + \cot x - 3x + c$

(c) $\tan x - \cot x - 3x + c$ (d) $\tan x - \cot x + 3x + c$

8. $\int (\tan x + \cot x)^2 dx$ is equal to

(a) $\tan x - \cot x + c$ (b) $\tan x + \cot x + c$

(c) $\cot x - \tan x + c$ (d) none of these

9. $\int \frac{x^2 + \cos^2 x}{(1+x^2)\sin^2 x} dx$ is equals to

(a) $\tan^{-1} x + \cot x + c$ (b) $\tan^{-1} x - \cot x + c$

(c) $\cot^{-1} x - \tan x + c$ (d) $-\tan^{-1} x - \cot x + c$

10. $\int \left(\frac{2^x - 5^x}{10^x}\right) dx$ is equal to

(a) $\frac{2^{-x}}{\log_e 2} - \frac{5^{-x}}{\log_e 5} + c$ (b) $\frac{2^x}{\log_e 2} - \frac{5^x}{\log_e 5} + c$

(c) $\frac{2^x}{\log_e 2} + \frac{5^x}{\log_e 5} + c$ (d) $\frac{5^{-x}}{\log_e 5} - \frac{2^{-x}}{\log_e 2} + c$

Based on Substitution

11. $\int \frac{(\sin^{-1} x)^3}{\sqrt{1-x^2}} dx$ is equal to

(a) $\frac{(\sin^{-1} x)^2}{2} + c$ (b) $\frac{(\sin^{-1} x)^3}{3} + c$

(c) $\frac{\sin^{-1} x}{x} + c$ (d) $\frac{(\sin^{-1} x)^4}{4} + c$

12. $\int \sec^n x \tan x dx$ is equal to

(a) $\frac{\sec^n x}{n} + c$ (b) $\frac{\sec^2 x}{2} + c$

(c) $\frac{\tan x}{n} + c$ (d) $\frac{(\sec^n x) \tan x}{n} + c$

13. $\int \sqrt[3]{\frac{\sin^n x}{\cos^{n+6} x}} dx$, $n \in \mathbb{N}$ is equal to

(a) $\frac{3}{n} (\tan x)^{\frac{n}{3}+1} + c$ (b) $\frac{3}{3+n} (\tan x)^{\frac{n}{3}+1} + c$

(c) $\frac{3}{n} (\cos x)^{n+1} + c$ (d) none of these

14. $\int \frac{\cos^3 x}{\sin^2 x + \sin x} dx$ is equal to

(a) $\log |\cos x| - \sin x + c$ (b) $\log |\sin x| - \sin x + c$
(c) $\log |\sin x| + \cos x + c$ (d) $\log |\cos x| - \cos x + c$

15. $\int \frac{\sin^4 x}{\cos^8 x} dx$ is equal to

(a) $\frac{(1 + \tan^5 x)}{5} - \frac{\tan^7 x}{7} + c$

(b) $\frac{\tan^5 x}{5} + \frac{\tan^7 x}{7} + c$

(c) $\frac{\tan^5 x}{7} + \frac{\tan^7 x}{5} + c$

(d) none of these

16. If $\int \frac{\cos \sqrt{x}}{\sqrt{x}} dx = a \sin \sqrt{x} + c$, then a equals

(a) $\sqrt{2}$ (b) $\frac{1}{2}$

(c) $\frac{1}{\sqrt{2}}$ (d) 2

17. $\int \frac{dx}{1+e^x} =$

(a) $\log_e \left(\frac{e^x + 1}{e^x} \right) + c$ (b) $\log_e \left(\frac{e^x}{e^x + 1} \right) + c$

(c) $x + \log_e (e^x + 1) + c$ (d) $e^x + x + c$

18. $\int \frac{\log_e x}{x \sqrt{1 + \log_e x}} dx =$

(a) $(1 + \log_e x)^{3/2} + c$

(b) $\frac{2}{3} (1 + \log_e x) (\log_e x - 2) + c$

(c) $\frac{2}{3} (1 + \log_e x)^{1/2} (\log_e x - 5) + c$

(d) $\frac{2}{3} (1 + \log_e x)^{1/2} (\log_e x - 2) + c$

19. $\int \frac{(x-x^3)^{1/3}}{x^4} dx =$

(a) $\frac{3}{8} \left(\frac{1}{x^2} - 1 \right)^{4/3} + c$ (b) $-\frac{3}{8} \left(\frac{1}{x^2} + 1 \right)^{4/3} + c$

(c) $-\frac{3}{8} \left(\frac{1}{x^2} - 1 \right)^{4/3} + c$ (d) $-\frac{3}{4} \left(1 - \frac{1}{x^2} \right)^{4/3} + c$

20. $\int \sec^2 (ax + b) dx$ equals

(a) $\tan (ax + b) + C$ (b) $\frac{1}{2} \tan x + C$

(c) $\frac{1}{a} \tan (ax + b) + C$ (d) none of these

21. $\int \sin 2x \sin 3x \, dx$ equals

- (a) $\frac{1}{2} (\sin x - \sin 5x) + c$
 (b) $\frac{1}{10} (\sin x - \sin 5x) + c$
 (c) $\frac{1}{10} (5 \sin x - \sin 5x) + c$
 (d) none of these

22. $\int \frac{1}{x \log x} \, dx$ is equal to

- (a) $\log |x \log x| + C$ (b) $\log |\log x + x| + C$
 (c) $\log x + C$ (d) $\log |\log x| + C$

23. $\int \sec^2 x \cos(\tan x) \, dx$ equals

- (a) $\sin(\cos x) + C$ (b) $\sin(\tan x) + C$
 (c) $\operatorname{cosec}(\tan x) + C$ (d) none of these

24. $\int \tan^n x \sec^2 x \, dx$ equals

- (a) $\frac{\tan^{n-1} x}{n-1} + C$ (b) $\frac{\tan^{n+1} x}{n+1} + C$
 (c) $\tan^{n+1} x + C$ (d) none of these

25. $\int \frac{\sin 2x}{1 + \cos^4 x} \, dx$ is equal to

- (a) $\cos^{-1}(\cos^2 x) + c$ (b) $\sin^{-1}(\cos^2 x) + c$
 (c) $\cot^{-1}(\cos^2 x) + c$ (d) none of these

26. $\int \sqrt{\frac{1 + \cos x}{1 - \cos x}} \, dx$ equals

- (a) $\log \cos\left(\frac{x}{2}\right) + C$ (b) $2 \log \sin\left(\frac{x}{2}\right) + C$
 (c) $2 \log \sec\left(\frac{x}{2}\right) + C$ (d) none of these

27. $\int \frac{x^2}{1 + x^6} \, dx$ is equal to

- (a) $\tan^{-1} x^3 + c$ (b) $\tan^{-1} x^2 + c$
 (c) $\frac{1}{3} \tan^{-1} x^3 + c$ (d) $3 \tan^{-1} x^3 + c$

28. $\int \sqrt{\frac{1+x}{1-x}} \, dx$ equals

- (a) $\sin^{-1} x + \sqrt{1-x^2} + c$ (b) $\sin^{-1} x + \sqrt{x^2-1} + c$
 (c) $\sin^{-1} x - \sqrt{1-x^2} + c$ (d) $\sin^{-1} x - \sqrt{x^2-1} + c$

29. $\int \frac{dx}{3x^2 + 2x + 1}$ equals

(a) $\frac{1}{\sqrt{2}} \tan^{-1}\left(\frac{3x+1}{\sqrt{2}}\right) + C$

(b) $\frac{1}{\sqrt{2}} \tan^{-1}\left(\frac{3x-1}{\sqrt{2}}\right) + C$

(c) $-\frac{1}{\sqrt{2}} \tan^{-1}\left(\frac{3x-1}{\sqrt{2}}\right) + C$

- (d) none of these

30. $\int \sqrt{1+x-2x^2} \, dx$ equals

(a) $\frac{1}{8}(4x-1)\sqrt{1+x-2x^2} + \frac{9\sqrt{2}}{32} \sin^{-1}\left(\frac{4x-1}{3}\right) + C$

(b) $\frac{1}{8}(4x+1)\sqrt{1+x-2x^2} + \frac{9\sqrt{2}}{32} \sin^{-1}\left(\frac{4x-1}{3}\right) + C$

(c) $\frac{1}{8}(4x-1)\sqrt{1+x-2x^2} + \frac{9\sqrt{2}}{32} \cos^{-1}\left(\frac{4x-1}{3}\right) + C$

- (d) none of these

31. $\int \frac{dx}{\sqrt{3-5x-x^2}}$ equals

(a) $\sin^{-1}\left(\frac{2x+5}{\sqrt{37}}\right) + c$ (b) $\cos^{-1}\left(\frac{2x+5}{\sqrt{37}}\right) + c$

- (c) $\sin^{-1}(2x+5) + c$ (d) none of these

32. $\int \frac{dx}{4\sin^2 x + 4\sin x \cos x + 5\cos^2 x}$ is equal to

(a) $\tan^{-1}\left(\tan x + \frac{1}{2}\right) + c$

(b) $\frac{1}{4}\tan^{-1}\left(\tan x + \frac{1}{2}\right) + c$

(c) $4\tan^{-1}\left(\tan x + \frac{1}{2}\right) + c$

(d) none of these

33. $\int \frac{e^x}{e^{2x} + 5e^x + 6} dx$ equals

(a) $\log\left(\frac{e^x + 3}{e^x + 2}\right) + c$ (b) $\log\left(\frac{e^x + 2}{e^x + 3}\right) + c$

(c) $\frac{1}{2}\log\left(\frac{e^x + 2}{e^x + 3}\right) + c$ (d) none of these

34. $\int \frac{dx}{x + \sqrt{x}}$ equals

(a) $2\log(\sqrt{x} - 1) + c$ (b) $2\log(\sqrt{x} + 1) + c$

(c) $\tan^{-1} x + c$ (d) none of these

35. $\int \frac{x^5}{\sqrt{1+x^3}} dx$ equals

(a) $\frac{2}{9}(x^3 - 2)\sqrt{1+x^3} + c$ (b) $\frac{2}{9}(x^3 + 2)\sqrt{1+x^3} + c$

(c) $(x^3 + 2)\sqrt{1+x^3} + c$ (d) none of these

36. $\int \frac{x^3 + 3}{(x+1)(x^2+1)} dx$ equals.

(a) $x + \log_e |x+1| - \log_e (x^2+1) + \cot^{-1} x + c$

(b) $x - \log_e |x+1| + \log_e (x^2+1) + \tan^{-1} x + c$

(c) $x + \log_e |x+1| - \log_e (x^2+1) + \tan^{-1} x + c$

(d) $x - \log_e |x+1| - \log_e (x^2+1) - \tan^{-1} x + c$

Integration by Parts :

37. $\int x^2 \sin x dx$ is equal to

(a) $-x^2 \cos x + 2[x \sin x + \sin x] + c$

(b) $-x^2 \cos x + \frac{1}{2}[x \sin x + \sin x] + c$

(c) $-x^2 \cos x + 2[x \sin x + \cos x] + c$

(d) $-x^2 \cos x + \frac{1}{2}[x \sin x + \cos x] + c$

38. $\int x^2 e^x dx$ is equal to

(a) $x^2 e^x - 2[e^{2x} - x e^x] + c$

(b) $x^2 e^x - 2[e^x - x e^x] + c$

(c) $x^2 e^x - 2[x e^{2x} - e^x] + c$

(d) $x^2 e^x - 2[x e^x - e^x] + c$

39. $\int \log x dx$ is equal to

(a) $x(\log x) - 1 + c$ (b) $x(\log x) - x + c$

(c) $x(\log x) - 1/x + c$ (d) $x(\log x) + c$

40. $\int (\log x)^2 dx$ is equal to

(a) $x(\log x)^2 - 2[x \log x - x] + c$

(b) $x(\log x)^2 - 2[\log x - x] + c$

(c) $x(\log x)^2 - 2[\log x^2 - x] + c$

(d) $x(\log x)^2 - 2[\log x - 2x] + c$

41. $\int \sin^{-1} x dx$ is equal to

(a) $x \sin^{-1} x + \sqrt{\sin^2 x - 1} + c$

(b) $x \sin^{-1} x + \sqrt{1 - \sin^2 x} + c$

(c) $x \sin^{-1} x + \sqrt{1 - x^2} + c$

(d) none of these

42. $\int \sec^{-1} x \, dx$ is equal to

- (a) $x(\sec x) - \log|x + \sqrt{x^2 - 1}| + c$
 (b) $x(\sec^{-1} x) - \log|x + \sqrt{x^2 - 1}| + c$
 (c) $x(\sec x) - \log|\tan x + \sqrt{x^2 - 1}| + c$
 (d) none of these

43. $\int \frac{\log x}{x^2} \, dx$ is equal to

- (a) $-\frac{1}{x}(2 + \log x) + c$ (b) $-\frac{1}{x}(1 + \log 2x) + c$
 (c) $-\frac{1}{x}(1 + \log x) + c$ (d) none of these

44. $\int \frac{x - \sin x}{1 - \cos x} \, dx$ is equal to

- (a) $-x \cot \frac{x}{2} + c$ (b) $\cot \frac{x}{2} + c$
 (c) $-\cot \frac{x}{2} + c$ (d) none of these

45. $\int \sec^3 x \, dx$ is equal to

- (a) $\sec x \tan x + \log|\sec x + \tan x| + c$
 (b) $\frac{1}{2} \sec x \tan x + \frac{1}{2} \log|\sec x + \tan x| + c$
 (c) $\sec^3 x \tan x + \log|\sec^3 x + \tan x| + c$
 (d) none of these

46. $\int \frac{\sin^{-1} x}{(1 - x^2)^{3/2}} \, dx$ is equal to

- (a) $\frac{x}{\sqrt{1 - x^2}} \sin^{-1} x + \frac{1}{2} \log|\sqrt{1 - x^2}| + c$
 (b) $\frac{x}{\sqrt{1 - x^2}} \sin^{-1} x + \frac{1}{2} \log|1 - x^2| + c$
 (c) $\frac{x}{\sqrt{1 - x^2}} \sin^{-1} x + \frac{1}{2} \log|\sqrt{1 - x^2}| + c$
 (d) none of these

47. $\int e^x \frac{(1 - x)^2}{(1 + x^2)^2} \, dx$ is equal to

- (a) $\frac{1}{x^2 + 1} + c$ (b) $\frac{e^x}{x^2 + 1} + c$
 (c) $\frac{e^x - 1}{x^2 + 1} + c$ (d) $\frac{1 - e^x}{x^2 + 1} + c$

48. $\int \frac{x^2}{x^2 - 1} \, dx$ equals

- (a) $x + \log \sqrt{\frac{x-1}{x+1}} + c$ (b) $x + \log \sqrt{\frac{x+1}{x-1}} + c$
 (c) $x + \log \left| \frac{x-1}{x+1} \right| + c$ (d) $x + \log \left| \frac{x+1}{x-1} \right| + c$

49. $\int x \tan^{-1} x \, dx$ is equal to

- (a) $\frac{1}{2}(x^2 + 1) \tan^{-1} x - x + c$
 (b) $\frac{1}{2}(x^2 + 1) \tan^{-1} x + x + c$
 (c) $\frac{1}{2}(x^2 + 1) \tan^{-1} x - \frac{1}{2}x + c$
 (d) $\frac{1}{2}(x^2 - 1) \tan^{-1} x - \frac{1}{2}x + c$

50. The primitive of $\log x$ will be

- (a) $x \log(e + x) + C$ (b) $x \log\left(\frac{e}{x}\right) + C$
 (c) $x \log\left(\frac{x}{e}\right) + C$ (d) $x \log(ex) + C$

51. $\int \sin(\log x) \, dx$ equals

- (a) $\frac{x}{\sqrt{2}} \sin\left(\log x + \frac{\pi}{8}\right) + c$ (b) $\frac{x}{\sqrt{2}} \sin\left(\log x - \frac{\pi}{4}\right) + c$
 (c) $\frac{x}{\sqrt{2}} \cos\left(\log x - \frac{\pi}{4}\right) + c$ (d) none of these

52. $\int \frac{xe^x}{(x+1)^2} dx$ is equal to

- (a) $\frac{e^x}{(x+1)^2} + c$ (b) $\frac{e^x}{x+1} + c$
(c) $\frac{e^x}{(x+1)^2} + c$ (d) none of these

53. $\int x^3 (\log x)^2 dx$ equals

- (a) $\frac{1}{32} x^4 [8(\log x)^2 - 4 \log x + 1] + c$
(b) $\frac{1}{32} x^4 [8(\log x)^2 - 4 \log x - 1] + c$
(c) $\frac{1}{32} x^4 [8(\log x)^2 + 4 \log x + 1] + c$
(d) none of these

54. The value of $\int x \sec x \tan x dx$ is

- (a) $x \sec x + \log |\sec x + \tan x| + c$
(b) $x \sec x - \log |\sec x + \tan x| + c$
(c) $x \sec x + \log |\sec x - \tan x| + c$
(d) none of these

55. $\int \frac{x + \sin x}{1 + \cos x} dx$ equals

- (a) $\frac{x}{2} \tan\left(\frac{x}{2}\right) + C$ (b) $\frac{x}{2} \tan x + C$
(c) $x \tan\left(\frac{x}{2}\right) + C$ (d) $x \tan x + C$

56. $\int e^x \frac{x-1}{(x+1)^3} dx$ equals

- (a) $-\frac{e^x}{x+1} + C$ (b) $\frac{e^x}{x+1} + C$
(c) $\frac{e^x}{(x+1)^2} + C$ (d) $-\frac{e^x}{(x+1)^2} + C$

57. $\int \frac{x^2+1}{(x-1)(x-2)} dx$ equals

- (a) $\log \left| \frac{(x-2)^5}{(x-1)^2} \right| + C$ (b) $x + \log \left| \frac{(x-2)^5}{(x-1)^2} \right| + C$
(c) $x + \log \left| \frac{(x-1)^5}{(x-2)^5} \right| + C$ (d) none of these

58. The value of $\int \frac{x^2 dx}{(x^2+a^2)(x^2+b^2)}$ is

- (a) $\frac{1}{b^2-a^2} \left[b \tan^{-1} \frac{x}{b} - a \tan^{-1} \frac{x}{a} \right] + C$
(b) $\frac{1}{b^2-a^2} \left[a \tan^{-1} \frac{x}{b} - b \tan^{-1} \frac{x}{a} \right] + C$
(c) $\frac{1}{b^2-a^2} \left[b \tan^{-1} \frac{x}{b} + a \tan^{-1} \frac{x}{a} \right] + C$
(d) none of these

59. $\int \left(\frac{1-x}{1+x} \right)^2 dx$ is equal to

- (a) $x - 4 \log |x+1| + \frac{4}{x+1} + c$
(b) $x - \log |x+1| + \frac{4}{x+1} + c$
(c) $x - 4 \log |x+1| - \frac{4}{x+1} + c$
(d) $x + \log |x+1| - \frac{4}{x+1} + c$

60. $\int \frac{\sin^{-1} \sqrt{x}}{\sqrt{1-x}} dx$ equals

- (a) $2 \left[\sqrt{x} - \sqrt{1-x} \sin^{-1} \sqrt{x} \right] + c$
(b) $2 \left[\sqrt{x} + \sqrt{1-x} \sin^{-1} \sqrt{x} \right] + c$
(c) $\left[\sqrt{x} - \sqrt{1-x} \sin^{-1} \sqrt{x} \right] + c$
(d) none of these

61. If $\int \frac{2x+3}{(x-1)(x^2+1)} dx$

$$= \log \left[|x-1|^{5/2} (x^2+1)^a \right] - \frac{1}{2} \tan^{-1} x + A$$

where A is any arbitrary constant, then a is equal to

- (a) 5/4 (b) -5/3
(c) -5/6 (d) -5/4

Integration of Trigonometric Forms

62. $\int \sin^5 x \cdot \cos^3 x dx$ is equal to

- (a) $\frac{\sin^6 x}{6} - \frac{\sin^8 x}{8} + c$ (b) $\frac{\cos^6 x}{6} - \frac{\cos^8 x}{8} + c$
(c) $\frac{\cos^6 x}{6} - \frac{\sin^8 x}{8} + c$ (d) none of these

63. If $\int \frac{dx}{1+\tan x} = px + q \log_e |\cos x + \sin x| + c$

then p + q equals.

- (a) $\frac{3}{2}$ (b) 2
(c) $\frac{1}{2}$ (d) 1

64. If $\int \frac{\sin^4 x}{\cos x} dx = \frac{1}{2} \log_e \left(\frac{1+\sin x}{1-\sin x} \right) - g(x) + c$ where g(x) equals.

- (a) $\frac{1}{3} \sin^3 x + \sin x$ (b) $\frac{1}{3} \cos^3 x + \cos x$
(c) $\frac{1}{3} \sin^3 x - \sin x$ (d) $\frac{1}{3} \cos^3 x - \cos x$

65. $\int \frac{dx}{\sin^2 x \cos^2 x}$ is equal to

- (a) $\tan x + \cot x + C$
(b) $(\tan x + \cot x)^2 + C$
(c) $\tan x - \cot x + C$
(d) $(\tan x - \cot x)^2 + C$

66. $\int \frac{\cos 2x - \cos 2\theta}{\cos x - \cos \theta} dx$ is equal to

- (a) $2(\sin x + x \cos \theta) + C$
(b) $2(\sin x - x \cos \theta) + C$
(c) $2(\sin x + 2x \cos \theta) + C$
(d) $2(\sin x - 2x \cos \theta) + C$

67. $\int \frac{1}{\sin(x-a)\sin(x-b)} dx$ is equal to

- (a) $\sin(b-a) \log \left| \frac{\sin(x-b)}{\sin(x-a)} \right| + C$
(b) $\operatorname{cosec}(b-a) \log \left| \frac{\sin(x-a)}{\sin(x-b)} \right| + C$
(c) $\operatorname{cosec}(b-a) \log \left| \frac{\sin(x-b)}{\sin(x-a)} \right| + C$
(d) $\sin(b-a) \log \left| \frac{\sin(x-a)}{\sin(x-b)} \right| + C$

68. $\int \frac{1 - \cos 2x}{1 + \cos 2x} dx =$

- (a) $\tan x - x + C$ (b) $x + \tan x + C$
(c) $x - \tan x + C$ (d) $-x - \cot x + C$

69. If $\int \tan^4 x dx = \lambda \tan^3 x + \mu \tan x + x + C$, then

- (a) $\lambda = \frac{1}{3}$ (b) $\mu = 1$
(c) $\lambda = -\frac{1}{3}$ (d) none of these

70. $\int \frac{\sin^{10} x}{\cos^{12} x} dx =$

- (a) $10 \tan^9 x + C$ (b) $\frac{\tan^{11} x}{11} + C$
(c) $\frac{\tan^{11} x}{11} + C$ (d) none of these

D_Marv Online Objective Question

1. If $\int \frac{\sin x}{\sin(x-\alpha)} dx = Ax + B \log \sin(x-\alpha) + C$, then value of (A, B) is Online

- (a) $(-\sin \alpha, \cos \alpha)$ (b) $(\cos \alpha, \sin \alpha)$
(c) $(\sin \alpha, \cos \alpha)$ (d) $(-\cos \alpha, \sin \alpha)$

2. $\int \frac{dx}{\cos x - \sin x}$ is equal to Online

(a) $\frac{1}{\sqrt{2}} \log \left| \tan \left(\frac{x}{2} - \frac{3\pi}{8} \right) \right| + C$

(b) $\frac{1}{\sqrt{2}} \log \left| \cot \left(\frac{x}{2} \right) \right| + C$

(c) $\frac{1}{\sqrt{2}} \log \left| \tan \left(\frac{x}{2} - \frac{\pi}{8} \right) \right| + C$

(d) $\frac{1}{\sqrt{2}} \log \left| \tan \left(\frac{x}{2} + \frac{3\pi}{8} \right) \right| + C$

3. $\int \left\{ \frac{(\log x - 1)}{1 + (\log x)^2} \right\}^2 dx$ is equal to Online

(a) $\frac{x}{x^2 + 1} + C$ (b) $\frac{\log x}{(\log x)^2 + 1} + C$

(c) $\frac{x}{(\log x)^2 + 1} + C$ (d) $\frac{xe^x}{1 + x^2} + C$

4. The value of is $\sqrt{2} \int \frac{\sin x dx}{\sin \left(x - \frac{\pi}{4} \right)}$ Online

(a) $x - \log \left| \cos \left(x - \frac{\pi}{4} \right) \right| + C$

(b) $x + \log \left| \cos \left(x - \frac{\pi}{4} \right) \right| + C$

(c) $x - \log \left| \sin \left(x - \frac{\pi}{4} \right) \right| + C$

(d) $x + \log \left| \sin \left(x - \frac{\pi}{4} \right) \right| + C$

5. $\int \frac{dx}{\cos x + \sqrt{3} \sin x}$ equals Online

(a) $\log \tan \left(\frac{x}{2} + \frac{\pi}{12} \right) + C$ (b) $\log \tan \left(\frac{x}{2} - \frac{\pi}{12} \right) + C$

(c) $\frac{1}{2} \log \tan \left(\frac{x}{2} + \frac{\pi}{12} \right) + C$ (d) $\frac{1}{2} \log \tan \left(\frac{x}{2} - \frac{\pi}{12} \right) + C$

6. If the integral $\int \frac{5 \tan x}{\tan x - 2} dx = x + a \ln |\sin x - 2 \cos x| + k$, then a is equal to Online

- (a) -1 (b) -2
(c) 1 (d) 2

7. If $\int f(x) dx = \psi(x)$, then $\int x^5 f(x^3) dx$ is equal to Online

(a) $\frac{1}{3} \left[x^3 \psi(x^3) - \int x^2 \psi(x^3) dx \right] + C$

(b) $\frac{1}{3} x^3 \psi(x^3) - 3 \int x^3 \psi(x^3) dx + C$

(c) $\frac{1}{3} x^3 \psi(x^3) - \int x^2 \psi(x^3) dx + C$

(d) $\frac{1}{3} \left[x^3 \psi(x^3) - \int x^3 \psi(x^3) dx \right] + C$

8. The integral $\int \left(1 + x - \frac{1}{x} \right) e^{\frac{x+1}{x}} dx$ is equal to : Online

(a) $-xe^{\frac{x+1}{x}} + c$ (b) $(x-1)e^{\frac{x+1}{x}} + c$

(c) $xe^{\frac{x+1}{x}} + c$ (d) $(x+1)e^{\frac{x+1}{x}} + c$

9. $\int \frac{\sin^8 x - \cos^8 x}{(1 - 2 \sin^2 x \cos^2 x)} dx$ is equal to: Online

(a) $\frac{1}{2} \sin 2x + c$ (b) $-\frac{1}{2} \sin 2x + c$

(c) $-\frac{1}{2} \sin x + c$ (d) $-\sin^2 x + c$

10. The integral $\int x \cos^{-1} \left(\frac{1-x^2}{1+x^2} \right) dx$ ($x > 0$) is equal to:

Online

- (a) $-x + (1+x^2) \tan^{-1} x + c$
 (b) $x - (1+x^2) \cot^{-1} x + c$
 (c) $-x + (1+x^2) \cot^{-1} x + c$
 (d) $x - (1+x^2) \tan^{-1} x + c$

11. The integral $\int \frac{\sin^2 x \cos^2 x}{(\sin^3 x + \cos^3 x)^2} dx$ equal to:

Online

- (a) $\frac{1}{(1+\cot^3 x)^2} + c$ (b) $-\frac{1}{3(1+\tan^3 x)^2} + c$
 (c) $\frac{\sin^3 x}{(1+\cos^3 x)^2} + c$ (d) $-\frac{\cos^3 x}{3(1+\sin^3 x)^2} + c$

12. If m is a non-zero number and

$$\int \frac{x^{5m-1} + 2x^{4m-1}}{(x^{2m} + x^m + 1)^3} dx = f(x) + c, \text{ Then } f(x) \text{ is}$$

Online

- (a) $\frac{x^{5m}}{2m(x^{2m} + x^m + 1)^2}$ (b) $\frac{x^{4m}}{2m(x^{2m} + x^m + 1)^2}$
 (c) $\frac{2m(x^{5m} + x^{4m})}{(x^{2m} + x^m + 1)^2}$ (d) $\frac{(x^{5m} - x^{4m})}{2m(x^{2m} + x^m + 1)^2}$

13. The integral $\int \frac{dx}{x^2(x^4 + 1)^{3/4}}$ equal

Online

- (a) $-(x^4 + 1)^{1/4} + c$ (b) $-\left(\frac{x^4 + 1}{x^4}\right)^{1/4} + c$
 (c) $\left(\frac{x^4 + 1}{x^4}\right)^{1/4} + c$ (d) $(x^4 + 1)^{1/4} + c$

14. The integral $\int \frac{dx}{(x+1)^{3/4}(x-2)^{5/4}}$ is equal to

Online

- (a) $4\left(\frac{x+1}{x-2}\right)^{1/4} + C$ (b) $-\frac{4}{3}\left(\frac{x+1}{x-2}\right)^{1/4} + C$
 (c) $-\frac{4}{3}\left(\frac{x-2}{x+1}\right)^{1/4} + C$ (d) $4\left(\frac{x-2}{x+1}\right)^{1/4} + C$

15. If $\int \frac{\log(t + \sqrt{1+t^2})}{\sqrt{1+t^2}} dt = \frac{1}{2} (g(t))^2 + C$, where C is constant, then $g(2)$ is equal to :

Online

- (a) $\frac{1}{\sqrt{5}} \log(2 + \sqrt{5})$ (b) $\frac{1}{2} \log(2 + \sqrt{5})$
 (c) $2 \log(2 + \sqrt{5})$ (d) $\log(2 + \sqrt{5})$

16. The integral $\int \frac{2x^{12} + 5x^9}{(x^5 + x^3 + 1)^3} dx$ is equal to : *Online*

- (a) $\frac{x^{10}}{2(x^5 + x^3 + 1)^2} + C$ (b) $\frac{x^5}{2(x^5 + x^3 + 1)^2} + C$
 (c) $\frac{-x^{10}}{2(x^5 + x^3 + 1)^2} + C$ (d) $\frac{-x^5}{(x^5 + x^3 + 1)^2} + C$

Where C is an arbitrary constant.

17. If $\int \frac{dx}{\cos^3 x \sqrt{2 \sin 2x}} = (\tan x)^A + C(\tan x)^B + k$, where k is a constant of integration, then $A+B+C$ equals :

Online

- (a) $\frac{21}{5}$ (b) $\frac{16}{5}$
 (c) $\frac{10}{7}$ (d) $\frac{27}{10}$

18. The integral $\int \frac{dx}{(1+\sqrt{x})\sqrt{x-x^2}}$ is equal to :

(where C is a constant of integration.)

Online

(a) $-2\sqrt{\frac{1+\sqrt{x}}{1-\sqrt{x}}} + C$ (b) $-2\sqrt{\frac{1-\sqrt{x}}{1+\sqrt{x}}} + C$

(c) $-\sqrt{\frac{1-\sqrt{x}}{1+\sqrt{x}}} + C$ (d) $2\sqrt{\frac{1+\sqrt{x}}{1-\sqrt{x}}} + C$

19. Let $I_n = \int \tan^n x dx, (n > 1)$. If $I_4 + I_6 = a \tan^5 x + bx^5 + C$, where C is a constant of integration, then the ordered pair (a, b) is equal to:

Online

(a) $\left(-\frac{1}{5}, 1\right)$ (b) $\left(\frac{1}{5}, 0\right)$

(c) $\left(\frac{1}{5}, -1\right)$ (d) $\left(-\frac{1}{5}, 0\right)$

20. The integral

$\int \sqrt{1+2\cot x(\operatorname{cosec} x + \cot x)} dx \left(0 < x < \frac{\pi}{2}\right)$ is equal to (where C is a constant of integration)

Online

(a) $4\log\left(\sin \frac{x}{2}\right) + C$ (b) $2\log\left(\sin \frac{x}{2}\right) + C$

(c) $2\log\left(\cos \frac{x}{2}\right) + C$ (d) $4\log\left(\cos \frac{x}{2}\right) + C$

21. If $f\left(\frac{3x-4}{3x+4}\right) = x+2, x \neq -\frac{4}{3}$, and

$\int f(x) dx = A \log |1-x| + Bx + C$, then the ordered pair (A, B) is equal to :

(where C is a constant of integration)

Online

(a) $\left(\frac{8}{3}, \frac{2}{3}\right)$ (b) $\left(-\frac{8}{3}, \frac{2}{3}\right)$

(c) $\left(-\frac{8}{3}, -\frac{2}{3}\right)$ (d) $\left(\frac{8}{3}, -\frac{2}{3}\right)$

22. The integral

$\int \frac{\sin^2 x \cos^2 x}{(\sin^5 x + \cos^3 x \sin^2 x + \sin^3 x \cos^2 x + \cos^5 x)^2} dx$

Online

(a) $\frac{-1}{1+\cot^3 x} + C$ (b) $\frac{1}{3(1+\tan^3 x)} + C$

(c) $\frac{-1}{3(1+\tan^3 x)} + C$ (d) $\frac{1}{1+\cot^3 x} + C$

(Where C is a constant of integration)

23. If $f\left(\frac{x-4}{x+2}\right) = 2x+1, (x \in \mathbb{R} - \{-2\})$, the $\int f(x) dx$ is equal to

(where C is a constant of integration)

Online

(a) $12 \log_e |1-x| + 3x + C$

(b) $-12 \log_e |1-x| - 3x + C$

(c) $12 \log_e |1-x| - 3x + C$

(d) $-12 \log_e |1-x| + 3x + C$

24. If $\int \frac{2x+5}{\sqrt{7-6x-x^2}} dx = A\sqrt{7-6x-x^2} + B\sin^{-1}\left(\frac{x+3}{4}\right) + C$

(where C is a constant of integration), then the ordered pair (A, B) is equal to :

Online

(a) (2, 1) (b) (-2, -1)

(c) (-2, 1) (d) (2, -1)

25. If

$\int \frac{\tan x}{1+\tan x+\tan^2 x} dx = x - \frac{K}{\sqrt{A}} \tan^{-1}\left(\frac{K \tan x + 1}{\sqrt{A}}\right) + C$, (C is a

constant of integration), then the ordered pair (K, A) is equal to :

Online

(a) (2, 1) (b) (-2, 3)

(c) (2, 3) (d) (-2, 1)

EXERCISE - 3 : ADVANCED OBJECTIVE QUESTIONS

Single Answer Type Question

1. If $f(x)$ is a polynomial function of the n th degree, then $\int e^x f(x) dx$ is equal to

- (a) $e^x \{f(x) - f'(x) - f''(x) - f'''(x) - \dots - (-1)^n f^{(n)}(x)\}$
 (b) $e^x \{f(x) - f'(x) + f''(x) - f'''(x) + \dots + (-1)^n f^{(n)}(x)\}$
 (c) $e^x \{f(x) - f'(x) + f''(x) - f'''(x) + \dots + (1)^n f^{(n)}(x)\}$
 (d) none of these

2. $\int \sqrt{e^{2x} - 1} dx$ is equal to

- (a) $\sqrt{e^{2x} - 1} + \sec^{-1} e^{2x} + c$
 (b) $\sqrt{e^{2x} - 1} - \sec^{-1} e^{2x} + c$
 (c) $\sqrt{e^{2x} - 1} - \sec^{-1} e^x + c$
 (d) none of these

3. The value of $\int \frac{(x^2 - 1) dx}{x^3 \sqrt{2x^4 - 2x^2 + 1}}$ is

- (a) $2\sqrt{2 - \frac{2}{x^2} + \frac{1}{x^4}} + c$ (b) $2\sqrt{2 + \frac{2}{x^2} + \frac{1}{x^4}} + c$
 (c) $\frac{1}{2}\sqrt{2 - \frac{2}{x^2} + \frac{1}{x^4}} + c$ (d) none of these

4. Let $f(x) = \frac{x}{(1+x^n)^{1/n}}$ for $n \geq 2$ and

$g(x) = \underbrace{(\text{fofo} \dots \text{of})}_{f \text{ occurs } n \text{ times}}(x)$. Then $\int x^{n-2} g(x) dx$

equals

- (a) $\frac{1}{n(n-1)} (1+nx^n)^{1-\frac{1}{n}} + k$
 (b) $\frac{1}{n-1} (1+nx^n)^{1-\frac{1}{n}} + k$
 (c) $\frac{1}{n(n+1)} (1+nx^n)^{1+\frac{1}{n}} + k$
 (d) $\frac{1}{n+1} (1+nx^n)^{1+\frac{1}{n}} + k$

5. If $I = \int \frac{dx}{(a^2 - b^2 x^2)^{3/2}}$

- (a) $\frac{x}{\sqrt{a^2 - b^2 x^2}} + c$ (b) $\frac{x}{a^2 \sqrt{a^2 - b^2 x^2}} + c$
 (c) $\frac{ax}{\sqrt{a^2 - b^2 x^2}} + c$ (d) none of these

6. If $I = \int \frac{x^3 + x}{x^4 - 9} dx$ then I equals :

- (a) $\frac{1}{4} \log |x^4 - 9| + \frac{1}{12} \log \left| \frac{x^2 + 3}{x^2 - 3} \right| + c$
 (b) $\frac{1}{4} \log |x^4 - 9| + \frac{1}{12} \log \left| \frac{x^2 - 3}{x^2 + 3} \right| + c$
 (c) $\frac{1}{4} \log |x^4 - 9| - \frac{1}{12} \log \left| \frac{x-3}{x+3} \right| + c$
 (d) None

7. If $I = \int e^x (x \cos x + \sin x) dx$ then I equals :

(a) $\frac{1}{2} e^x (x \sin x - \cos x) + c$

(b) $\frac{1}{2} e^x (x \sin x + \cos x) + c$

(c) $\frac{1}{2} e^x (x \cos x - \sin x) + c$

(d) None

8. Evaluate $I = \int \frac{dx}{\sin^3 x + \cos^3 x}$

(a) $\frac{1}{3\sqrt{2}} \log \left| \frac{\sqrt{2}+t}{\sqrt{2}-t} \right| + \tan^{-1} t + c$

(b) $\frac{1}{3\sqrt{2}} \log \left| \frac{\sqrt{2}-t}{\sqrt{2}+t} \right| + \frac{1}{3} \tan^{-1} t + c$

(c) $\frac{1}{3\sqrt{2}} \log \left| \frac{\sqrt{2}+t}{\sqrt{2}-t} \right| + \frac{2}{3} \tan^{-1} t + c$

(d) None of these

where $t = \sin x - \cos x$

9. If $\int \log (x^2 + x) dx = x \log |x| + (x+1) \log |x+1| + k$, then k equals

(a) $2x + \log |x+1| + c$

(b) $2x - \log |x+1| + c$

(c) constant

(d) None

10. If $I = \int \sin^7 x dx$, then I equals

(a) $-\cos x + \cos^3 x - \cos^5 x + \frac{1}{7} \cos^7 x + c$

(b) $-\cos x + \cos^3 x - \frac{3}{5} \cos^5 x + \frac{1}{7} \cos^7 x + c$

(c) $\tan x - \tan^3 x + \sin x - \frac{1}{3} \cos^3 x + c$

(d) $-\cos x + \cos^5 x - \frac{1}{3} \cos^3 x + \frac{1}{7} \cos^7 x + c$

11. If $I = \int \cos \theta \log \left(\tan \frac{\theta}{2} \right) d\theta$, then I equals

(a) $\sin \theta \log (\tan \theta/2) + \theta + c$

(b) $\cos \theta \log (\tan \theta/2) + \theta + c$

(c) $\sin \theta \log (\tan \theta/2) - \theta + c$

(d) None of these

12. If $I = \int \frac{x^2 + a^2}{x^4 - a^2 x^2 + a^4} dx$, then I =

(a) $\frac{1}{a} \tan^{-1} \left(\frac{ax}{x^2 - a^2} \right) + c$

(b) $\frac{1}{a} \tan^{-1} \left(\frac{x^2 - a^2}{ax} \right) + c$

(c) $\log |x + \sqrt{x^2 - a^2}| + c$

(d) None of these

13. If $I = \int \frac{dx}{(1 + \sqrt{x}) \sqrt{x - x^2}}$, then I equals

(a) $\frac{2(\sqrt{x} + 1)}{\sqrt{1-x}} + c$

(b) $\frac{2(\sqrt{x} - 1)}{\sqrt{1-x}} + c$

(c) $\frac{2(1 - \sqrt{x})}{\sqrt{x(1-x)}} + c$

(d) $\frac{2(\sqrt{x} - 1)}{\sqrt{x(1-x)}} + c$

14. Evaluate $I = \int \left(\sqrt{\frac{a+x}{a-x}} + \sqrt{\frac{a-x}{a+x}} \right) dx$

(a) $2 \sin^{-1} \left(\frac{x}{a} \right) + c$

(b) $2a \sin^{-1} \left(\frac{x}{a} \right) + c$

(c) $2 \cos^{-1} \left(\frac{x}{a} \right) + c$

(d) $2a \cos^{-1} \left(\frac{x}{a} \right) + c$

15. If $I = \int \frac{(2x+3)dx}{(x^2+2x+3)\sqrt{x^2+2x+4}}$, then I equals

(a) $\log \left| \frac{\sqrt{x^2+2x+4}-1}{\sqrt{x^2+2x+4}+1} \right| + c$

(b) $\log \left| \frac{\sqrt{x^2+2x+4}-1}{\sqrt{x^2+2x+4}+1} \right| + \tan^{-1} \left(\frac{x+2}{3} \right) + c$

(c) $\log \tan^{-1} \left(\frac{\sqrt{x+3}}{2} \right) + c$

(d) None

16. If $I = \int \frac{\sqrt{\sin^3 2x}}{\sin^5 x} dx$, and $f(x) = (\cot x)^{3/2}$, $g(x) = (\cot x)^{5/2}$, then

I equals

(a) $\frac{2\sqrt{3}}{3} f(x) - \frac{1}{5} g(x) + c$ (b) $-\frac{4\sqrt{2}}{5} g(x) + c$

(c) $\frac{1}{2\sqrt{3}} f(x) + c$ (d) $\frac{2\sqrt{2}}{3} f(x) + \frac{1}{5} g(x) + c$

17. Evaluate : $\int \frac{\sin x dx}{\sqrt{3\sin^2 x + 4\cos^2 x}}$

(a) $-\ln(\cos x + \sqrt{3 + \cos^2 x}) + c$

(b) $\ln(\cos x + \sqrt{3 + \cos^2 x}) + c$

(c) $-\ln(\sin x + \sqrt{3 + \sin^2 x}) + c$

(d) None of these

18. Evaluate : $\int \frac{\sec x dx}{\sqrt{\sin(2x+A) + \sin A}}$

(a) $2 \cos A \sqrt{2 \cos A + \sin A \tan x} + c$

(b) $\sqrt{2} \sec A \sqrt{2 \cos A \tan x + 2 \sin A} + c$

(c) $\sqrt{2} \sec A \sqrt{\cos A \tan x + \sin A} + c$

(d) None of these

19. $\int \sqrt{\sec x - 1} dx$

(a) $2 \ln(\sqrt{\cos x} + \sqrt{1 + \cos x}) + c$

(b) $2 \ln(\sqrt{\sin x} + \sqrt{1 + \sin x}) + c$

(c) $-2 \ln(\sqrt{\cos x} + \sqrt{1 + \cos x}) + c$

(d) None of these

20. $\int \sqrt{\frac{e^x + a}{e^x - a}} dx$

(a) $\ln(e^x + \sqrt{e^{2x} - a^2}) - \sin^{-1}(ae^{-x}) + c$

(b) $\ln(e^x - \sqrt{e^{2x} - a^2}) + \cos^{-1}(ae^{-x}) + c$

(c) $\ln(e^x + \sqrt{e^{2x} - a^2}) + \sin^{-1}(ae^x) + c$

(d) None

21. $\int \frac{\sqrt{x} - \sqrt{a}}{\sqrt{x+a}} dx$

(a) $\sqrt{ax+x^2} - 2\sqrt{a^2+ax} - a \ln(\sqrt{x} + \sqrt{a+x}) + c$

(b) $\sqrt{ax+x^2} - 2\sqrt{a+x} - a \ln(\sqrt{x} + \sqrt{a+x}) + c$

(c) $\sqrt{ax+x^2} - 2\sqrt{a^2+ax} + a \ln(\sqrt{x} + \sqrt{a-x}) + c$

(d) None of these

22. Let $I = \int \frac{e^x}{e^{4x} + e^{2x} + 1} dx$, $J = \int \frac{e^{-x}}{e^{-4x} + e^{-2x} + 1} dx$

Then for any arbitrary constant c , the value of $J - I$ equals.

(a) $\frac{1}{2} \log \left(\frac{e^{4x} - e^{2x} + 1}{e^{4x} + e^{2x} + 1} \right) + c$

(b) $\frac{1}{2} \log \left(\frac{e^{2x} + e^x + 1}{e^{2x} - e^x + 1} \right) + c$

(c) $\frac{1}{2} \log \left(\frac{e^{2x} - e^x + 1}{e^{2x} + e^x + 1} \right) + c$

(d) $\frac{1}{2} \log \left(\frac{e^{4x} + e^{2x} + 1}{e^{4x} - e^{2x} + 1} \right) + c$

23. $\int \frac{x^2 - 1}{(x^4 + 3x^2 + 1) \tan^{-1} \left(\frac{x^2 + 1}{x} \right)} dx$ is equal to

(a) $\tan^{-1} \left(x + \frac{1}{x} \right) + c$

(b) $\log_e \left| \tan^{-1} \left(x + \frac{1}{x} \right) \right| + c$

(c) $\log_e \left| \tan \left(\frac{x^2 + 1}{x} \right) \right| + c$

(d) $\left(x + \frac{1}{x} \right) \tan^{-1} \left(x + \frac{1}{x} \right) + c$

24. $\int \frac{x-1}{(x+1)\sqrt{x(x^2+x+1)}} dx$ is

(a) $\tan^{-1} \left(\frac{x^2 + x + 1}{x} \right) + c$

(b) $2 \tan^{-1} \left(\frac{x^2 + x + 1}{x} \right) + c$

(c) $\tan^{-1} \left(\frac{\sqrt{x^2 + x + 1}}{x} \right) + c$

(d) $2 \tan^{-1} \sqrt{x + \frac{1}{x}} + c$

25. If $\int \left(\frac{1 - \sqrt{x}}{1 + \sqrt{x}} \right)^{1/2} \frac{dx}{x} = 2 \cos^{-1} \sqrt{x} - \phi(x) + c$, then $\phi(x)$ equals.

(a) $\log_e \left(\frac{1 + \sqrt{1-x}}{\sqrt{x}} \right)$ (b) $\frac{1}{2} \log_e \left(\frac{1 + \sqrt{1-x}}{\sqrt{x}} \right)$

(c) $2 \log_e \left(\frac{1 - \sqrt{1-x}}{\sqrt{x}} \right)$ (d) $2 \log_e \left(\frac{1 + \sqrt{1-x}}{\sqrt{x}} \right)$

26. If $y = \sqrt{x^2 - x + 1}$ and for $n \geq 1$, $I_n = \int x^n / y dx$ and $aI_3 + bI_2 + cI_1 = x^2y$, then (a, b, c) is equal to

(a) $\left(\frac{3}{2}, \frac{1}{2}, -1 \right)$ (b) $(1, -1, 1)$

(c) $\left(3, -\frac{5}{2}, 2 \right)$ (d) $\left(\frac{1}{2}, -\frac{1}{2}, 1 \right)$

27. $\int \frac{(1+x)\sin x}{(x^2+2x)\cos^2 x - (1+x)\sin 2x} dx$

$$= \frac{1}{2} \log_e \left| \frac{t+1}{t-1} \right| + c$$

where t is

(a) $(x+1)\cos x - \sin x$ (b) $(x+1)\sin x - \cos x$

(c) $(x+1)\sin x + \cos x$ (d) $(x+1)\cos x + \sin x$

28. $\int \sqrt[3]{x} (\log_e x)^2 dx =$

(a) $\frac{3}{4}(x)^{4/3} \left[(\log_e x)^2 - \frac{3}{2} \log_e x - \frac{9}{8} \right] + c$

(b) $\frac{3}{4}(x)^{4/3} \left[(\log_e x)^2 + \frac{3}{2} \log_e x + \frac{9}{8} \right] + c$

(c) $\frac{3}{4}(x)^{4/3} \left[(\log_e x)^2 - \frac{3}{2} \log_e x + \frac{9}{8} \right] + c$

(d) $\frac{3}{4}(x)^{1/3} \left[(\log_e x)^2 + \frac{3}{2} \log_e x - \frac{9}{8} \right] + c$

29. If $\lim_{x \rightarrow 0} \frac{f(x)}{x^2}$ exists finitely and

$$\lim_{x \rightarrow 0} \left(1 + x + \frac{f(x)}{x} \right)^{1/x} = e^3, \text{ where } f(x) = ax^2 + bx + c$$

then $\int f(x) \log_e x dx$ is equal to

(a) $\frac{2}{3} x^3 \left(\log_e x - \frac{1}{3} \right) + c$ (b) $\frac{x^3}{3} \left(\log_e x - \frac{1}{3} \right) + c$

(c) $\frac{2}{3} x^3 (\log_e x + 1) + c$ (d) $\frac{2}{3} x^3 (\log_e x - 1) + c$

30. For $0 < x < 1$, let

$$f(x) = \lim_{n \rightarrow \infty} (1+x)(1+x^2)(1+x^4) \dots (1+x^{2^n})$$

then $\int \frac{f(x)}{1-x} \log_e x dx$ equals

(a) $\log_e \left(\frac{x}{1-x} \right) + c$

(b) $-\log_e \left(\frac{x}{1-x} \right) + \frac{\log_e x}{1-x} + c$

(c) $\frac{\log_e x}{1-x} + \log_e (1-x) + c$

(d) $x \log_e x + \log_e (1-x) + c$

31. $\int (\sin 4x) e^{\tan^2 x} dx =$

(a) $-2e^{\tan^2 x} \cos^4 x + c$ (b) $2e^{\tan^2 x} \sec^4 x + c$

(c) $-2e^{\tan^2 x} \sec^2 x + c$ (d) $2e^{\tan^2 x} \cos^2 x + c$

32. $\int \frac{x^m - 1}{x^{m+1} \sqrt{1-2x^m + mx^{2m}}} dx = \frac{f(x)}{mx^m} + c$ where $f(x)$ is equal to

(a) $\sqrt{1-2x^m + mx^{2m}}$ (b) $\sqrt{1-x^m + x^{2m}}$

(c) $\sqrt{x^m + \frac{1}{x^m}}$ (d) $\sqrt{x^{2m} + \frac{1}{x^{2m}}}$

33. If $\int \frac{dx}{(x^2-4)\sqrt{x+1}} = \frac{1}{k} \log_2 \left| \frac{\sqrt{x+1}-\sqrt{3}}{\sqrt{x+1}+\sqrt{3}} \right|$

$-\frac{1}{2} \tan^{-1} \sqrt{x+1} + c$ then k equals

(a) $2\sqrt{3}$

(b) $4\sqrt{3}$

(c) $\frac{1}{4\sqrt{3}}$

(d) none of these

34. $\int \frac{dx}{\tan x + \cot x + \sec x + \operatorname{cosec} x}$ is equal to

(a) $\frac{1}{2} (\sin x - \cos x + x) + c$

(b) $\frac{1}{2} (\sin x - \cos x - \tan x + \cot x) + c$

(c) $\frac{1}{2} (\sin x - \cos x - x) + c$

(d) $\frac{1}{2} (\sin x + \cos x - \tan x - \cot x + x) + c$

35. $\int \frac{x(x-1)}{(x^2+1)(x+1)\sqrt{x^3+x^2+x}} dx$ is equal to

(a) $\frac{1}{2} \log_e \left(\frac{\sqrt{x^2+x+1}-\sqrt{x}}{\sqrt{x^2+x+1}+\sqrt{x}} \right) + c$

(b) $\frac{1}{2} \log_e \left(\frac{\sqrt{x^2+x+1}-\sqrt{x}}{\sqrt{x^2+x+1}+\sqrt{x}} \right) - \tan^{-1} \sqrt{\frac{x^2+x+1}{x}} + c$

(c) $\frac{1}{4} \log_e \left(\frac{x^2+x+1}{x^2-x+1} \right) + \tan^{-1} \left(\frac{x^2+x+1}{x} \right) + c$

(d) $\frac{1}{2} \log_e \left| \frac{x^2-1}{x^2+1} \right| - \tan^{-1} \left(\frac{x^2+1}{x^2-1} \right) + c$

36. If $f: \mathbb{R} \rightarrow \mathbb{R}$ is a function satisfying the following :

(i) $f(-x) = -f(x)$

(ii) $f(x+1) = f(x) + 1$

(iii) $f\left(\frac{1}{x}\right) = \frac{f(x)}{x^2} \quad \forall x \neq 0$

then $\int e^x f(x) dx$ is equal to

(a) $e^x (x-1) + c$ (b) $e^x \log x + c$

(c) $\frac{e^x}{x} + c$ (d) $\frac{e^x}{x+1} + c$

37. $\int \frac{\sin x - \cos x}{(\sin x + \cos x) \sqrt{\sin x \cos x + \sin^2 x \cos^2 x}} dx =$

(a) $-\sin(\sin 2x + 1) + c$

(b) $\operatorname{cosec}(\sin 2x + 1)$

(c) $-\sec^{-1}(\sin 2x + 1) + c$

(d) $\tan^{-1}(\sin 2x + 1) + c$

38. $\int \frac{x^2}{(x \sin x + \cos x)^2} dx$ equals

(a) $\frac{\cos x - x \sin x}{x \sin x + \cos x} + c$ (b) $\frac{\cos x + x \sin x}{x \cos x - \sin x} + c$

(c) $\frac{\sin x - x \cos x}{x \sin x + \cos x} + c$ (d) $\frac{\sin x + x \cos x}{x \sin x - \cos x} + c$

39. $\int \tan 2x \tan 3x \tan 5x dx$ is equal to

(a) $\frac{1}{2} \log |\sec 2x| - \frac{1}{3} \log |\sec 3x| - \frac{1}{5} \log |\sec 5x| + c$

(b) $\frac{1}{2} \log |\sec 2x| + \frac{1}{3} \log |\sec 3x| + \frac{1}{5} \log |\sec 5x| + c$

(c) $\frac{1}{5} \log |\sec 5x| - \frac{1}{2} \log |\sec 2x| - \frac{1}{3} \log |\sec 3x| + c$

(d) none of these

40. $\int \frac{e^{2 \tan^{-1} x} (1+x)^2}{(1+x^2)} dx$ is equal to

(a) $x e^{\tan^{-1} x} + c$

(b) $x e^{2 \tan^{-1} x} + c$

(c) $2x e^{2 \tan^{-1} x} + c$

(d) none of these

41. If $I = \int \cos^{-1} \sqrt{x} dx$ and

$J = \int \frac{\sin^{-1} \sqrt{x} - \cos^{-1} \sqrt{x}}{\sin^{-1} \sqrt{x} + \cos^{-1} \sqrt{x}} dx$ then J equals

(a) $x - 4I$

(b) $x + I$

(c) $x - \frac{4}{\pi} I$

(d) $\frac{\pi}{4}$

42. If $\int g(x) dx = g(x)$ then $\int g(x)(f(x) + f'(x)) dx$ is equal to

(a) $g(x)f(x) - g(x)f'(x) + c$ (b) $g(x)f'(x) + c$

(c) $g(x)f(x) + c$

(d) $g(x)f^2(x) + c$

43. $\int \frac{(x^2 - 1) dx}{(x^4 + 3x^2 + 1) \tan^{-1}\left(\frac{x^2 + 1}{x}\right)} = \ln |f(x)| + c$, then $f(x)$ is

- (a) $\ln\left(x + \frac{1}{x}\right)$ (b) $\tan^{-1}\left(x + \frac{1}{x}\right)$
 (c) $\cot^{-1}\left(x + \frac{1}{x}\right)$ (d) $\ln\left(\tan^{-1}\left(x + \frac{1}{x}\right)\right)$

44. Which of the following could be true if $f''(x) = x^{1/3}$

(1) $f(x) = \frac{9}{28}x^{7/3} + 9$ (2) $f'(x) = \frac{9}{28}x^{7/3} - 2$

(3) $f'(x) = \frac{3}{4}x^{4/3} + 6$ (4) $f(x) = \frac{3}{4}x^{4/3} - 4$

- (a) (1) only (b) (3) only
 (c) (2) and (4) only (d) (1) and (3) only

45. $\int \sec^2 \theta (\sec \theta + \tan \theta)^2 d\theta$

- (a) $\frac{\sec \theta + \tan \theta}{2} [2 + \tan \theta (\sec \theta + \tan \theta)] + c$
 (b) $\frac{\sec \theta + \tan \theta}{3} [2 + 4 \tan \theta (\sec \theta + \tan \theta)] + c$
 (c) $\frac{(\sec \theta + \tan \theta)}{3} [2 + \tan \theta (\sec \theta + \tan \theta)] + c$
 (d) $\frac{3(\sec \theta + \tan \theta)}{2} [2 + \tan \theta (\sec \theta + \tan \theta)] + c$

46. If $\int (x^3 - 2x^2 + 5) e^{3x} dx = e^{3x} (Ax^3 + Bx^2 + Cx + 13/9)$ then which of the following statement is incorrect :

- (a) $3C + 13 = 15$ (b) $A + B + \frac{2}{3} = 0$
 (c) $C + 2B = 0$ (d) $A + B + C = 0$

47. The evaluation of $\int \frac{px^{p+2q-1} - qx^{q-1}}{x^{2p+2q} + 2x^{p+q} + 1} dx$ is

- (a) $-\frac{x^p}{x^{p+q} + 1} + c$ (b) $\frac{x^q}{x^{p+q} + 1} + c$
 (c) $-\frac{x^q}{x^{p+q} + 1} + c$ (d) $\frac{x^p}{x^{p+q} + 1} + c$

48. Which of the following is TRUE ?

- (a) $x \int \frac{dx}{x} = x \ln |x| + c$
 (b) $x \int \frac{dx}{x} = x \ln |x| + cx$
 (c) $\frac{1}{\cos x} \int \cos x dx = \tan x + c$
 (d) $\frac{1}{\cos x} \int \cos x dx = x + c$

49. Evaluate $\int \frac{(x+2) dx}{(x^2 + 3x + 3) \sqrt{x+1}}$

- (a) $\frac{2}{\sqrt{3}} \tan^{-1}\left(\frac{\sqrt{3} \sqrt{x+1}}{-x}\right) + c$
 (b) $\frac{2}{\sqrt{3}} \tan^{-1}\left(\frac{\sqrt{3} \sqrt{x+1}}{x}\right) + c$
 (c) $\frac{1}{\sqrt{3}} \tan^{-1}\left(\frac{\sqrt{3} \sqrt{x+1}}{-x}\right) + c$
 (d) $\frac{1}{\sqrt{3}} \tan^{-1}\left(\frac{\sqrt{3} \sqrt{x+1}}{x}\right) + c$

50. If $I_n = \int (\ln x)^n dx$ then $I_n + n I_{n-1} =$

- (a) $(\ln x)^n$ (b) $x (\ln x)^n$
 (c) $x^n \ln x$ (d) $x (\ln x)^{n-1}$

51. Evaluate : $\int \sqrt{\frac{1+x^{2n}}{x^{2n}}} \cdot \frac{\ln(1+x^{2n}) - 2n \ln x}{x^{2n+1}} dx$

- (a) $\frac{2P^3}{9n} (1 - 3 \ln P) + C$ (b) $\frac{P^3}{3n} (3 \ln P - 1) + C$
 (c) $\frac{2P^3}{3n} (3 \ln P - 1) + C$ (d) None of these

where $P = \left(1 + \frac{1}{x^{2n}}\right)^{1/2}$

52. Evaluate : $\int \frac{\sqrt{x-1}}{x\sqrt{x+1}} dx$

(a) $\ln(x + \sqrt{x^2+1}) + \sec^{-1} x + c$

(b) $\ln(x - \sqrt{x^2+1}) - \sec^{-1} x + c$

(c) $\ln(x + \sqrt{x^2-1}) - \sec^{-1} x + c$

(d) None of these

53. Evaluate : $\int \sqrt{x + \sqrt{x^2+2}} dx$

(a) $\frac{2}{3}(x + \sqrt{x^2+2})^{3/2} - \frac{1}{\sqrt{x + \sqrt{x^2+2}}} + c$

(b) $\frac{1}{3}(x + \sqrt{x^2+2})^{3/2} - \frac{2}{\sqrt{x + \sqrt{x^2+2}}} + c$

(c) $\frac{1}{3}(x + \sqrt{x^2+2})^{1/2} - \frac{2}{\sqrt{x + \sqrt{x^2+2}}} + c$

(d) None of these

54. If $I_n = \int \cos^n x dx$ then $I_n - \left(\frac{n-1}{n}\right)I_{n-2} =$

(a) $\cos^{n-1} x \cdot \sin x + c$

(b) $\frac{1}{n}(\cos^{n-1} x \sin x) + c$

(c) $\frac{1}{n}(\cos^n x \sin x) + c$

(d) None of these

55. Evaluate : $\int \frac{(1+x)}{x(1+xe^x)^2} dx$

(a) $\ln\left(\frac{xe^x}{1+xe^x}\right) + \frac{1}{1+xe^x} + c$

(b) $\ln\left(\frac{1+xe^x}{xe^x}\right) + \frac{1}{xe^x+1} + c$

(c) $\ln\left(\frac{xe^x}{xe^x+1}\right) - \frac{1}{xe^x+1} + c$

(d) None of these

56. Evaluate : $\int \sin^{-1}\left(\frac{2x+2}{\sqrt{4x^2+8x+13}}\right) dx; |x| < 1$

(a) $(x+1) \tan^{-1}\left(\frac{2x+2}{3}\right) - \frac{3}{4} \ln\left(1 + \left(\frac{2x+2}{3}\right)^2\right) + c$

(b) $(x+1) \tan^{-1}\left(\frac{2x+2}{3}\right) - \frac{3}{2} \ln\left(1 + \left(\frac{2x+2}{3}\right)^2\right) + c$

(c) $\frac{3}{2}(x+1) \tan^{-1}\left(\frac{2x+2}{3}\right) - \frac{3}{4} \ln\left(1 + \frac{(2x+2)^2}{3}\right) + c$

(d) None of these

57. $\int \frac{(x^2-1)dx}{2x\sqrt{x^4+4x^3-6x^2+4x+1}}$ is

(a) $\frac{1}{2} \ln\left|x + \frac{1}{x} + 2 + \sqrt{\left(x + \frac{1}{x} + 2\right)^2 - 12}\right| + c$

(b) $\frac{1}{2} \ln\left|x - \frac{1}{x} + 2 + \sqrt{\left(x - \frac{1}{x} + 2\right)^2 - 12}\right| + c$

(c) $\frac{1}{2} \ln\left|x + \frac{1}{x} - 2 + \sqrt{\left(x + \frac{1}{x} - 2\right)^2 - 12}\right| + c$

(d) None of these

58. $\int \frac{\sqrt{\cot x} - \sqrt{\tan x}}{1 + 3 \sin 2x} dx$ is

(a) $\tan^{-1} \left(\frac{\sqrt{2 \sin 2x}}{\sin x + \cos x} \right) + c$

(b) $\tan^{-1} \left(\frac{2 \sin 2x}{\sin x + \cos x} \right) + c$

(c) $\tan^{-1} \left(\frac{\sqrt{\sin 2x}}{\sin x - \cos x} \right) + c$

(d) None of these

59. Let $g(x)$ be an antiderivative of $f(x)$. Then $\ln(1+(g(x))^2)$ is an antiderivative for :

(a) $\frac{2f(x) \cdot g(x)}{1+(f(x))^2}$ (b) $\frac{2f(x) \cdot g(x)}{1+(g(x))^2}$

(c) $\frac{2f(x)}{1+(f(x))^2}$ (d) None

Multiple Option Correct Type :

60. If $\int e^u \sin 2x dx$ can be found in terms of known functions of x then u can be

- (a) x (b) $\sin x$
(c) $\cos x$ (d) $\cos 2x$

61. $\int \frac{1}{x^2-1} \ln \left(\frac{x-1}{x+1} \right) dx$ equals

(a) $\frac{1}{2} \ln \left(\frac{x-1}{x+1} \right) + c$ (b) $\frac{1}{4} \ln^2 \left(\frac{x-1}{x+1} \right) + c$

(c) $\frac{1}{2} \ln^2 \left(\frac{x+1}{x-1} \right) + c$ (d) $\frac{1}{4} \ln^2 \left(\frac{x+1}{x-1} \right) + c$

62. If $f(x) = \lim_{n \rightarrow \infty} e^{x \tan(1/n) \log(1/n)}$ and $\int \frac{f(x) dx}{\sqrt[3]{\sin^{11} x \cos x}} = g(x) + c$

then

(a) $g\left(\frac{\pi}{4}\right) = \frac{3}{2}$

(b) $g(x)$ is continuous for all x

(c) $g\left(\frac{\pi}{4}\right) = \frac{-15}{8}$

(d) $g(x)$ is non-differentiable at infinitely many points

63. If primitive of $\sin(\log x)$ is $f(x)$ ($\sin(g(x)) - \cos(h(x)) + c$) then

(a) $\lim_{x \rightarrow 2} f(x) = 1$ (b) $\lim_{x \rightarrow 1} \frac{g(x)}{h(x)} = 1$

(c) $g(e^3) = 3$ (d) $h(e^5) = 5$

64. $\int \frac{dx}{5+4 \cos x} = \lambda \tan^{-1} \left(m \tan \frac{x}{2} \right) + c$ then

(a) $\lambda = \frac{2}{3}$ (b) $m = \frac{1}{3}$

(c) $\lambda = \frac{1}{3}$ (d) $m = \frac{2}{3}$

Assertion-Reason Type :

- (A) Both A and R are true and R is correct reason of A.
(B) Both A and R are true and R is not the correct reason of A
(C) A is true and R is false.
(D) A is false and R is true.

65. **Assertion :** $\int \frac{x^2-1}{x^2} e^{\frac{x^2+1}{x}} dx = e^{\frac{x^2+1}{x}} + c$

Reason : $\int f(x) e^{f(x)} dx = e^{f(x)} + c$

66. Assertion : The indefinite integration of an odd function is an even function.

Reason : For an odd function $f(x), f(-x) = -f(x)$

67. Assertion : $\int e^{x^2} dx = e^{x^2} + c$

Reason : $\int e^x dx = e^x + c$

68. Assertion : The function $f(x) = \int \sin^2 x dx$ satisfies

$$f(x + \pi) = f(x) \quad \forall x \in \mathbb{R}$$

Reason : $\sin^2(x + \pi) = \sin^2 x \quad \forall x \in \mathbb{R}$

69. Assertion : If $\int \frac{1}{f(x)} dx = 2 \log |f(x)| + c$, then $f(x) = \frac{x}{2}$

Reason : When $f(x) = \frac{x}{2}$, then

$$\int \frac{1}{f(x)} dx = \int \frac{2}{x} dx = 2 \log |x| + c$$

Comprehension Type :

Passage-1

We can derive reduction formula for the integrat of the form $\int \sin^n x dx, \int \cos^n x dx, \int \tan^n x dx$ and other integrals of these form using integration by parts. In turn these reduction formulas can be used to compute integrals of higher power of $\sin x$ and $\cos x$.

70. If $\int \sin^5 x dx = -\frac{1}{5} \sin^4 x \cos x + A \sin^2 x \cos x - \frac{8}{15} \cos x + c$ then A is equal to

(a) $-\frac{2}{15}$ (b) $-\frac{3}{5}$

(c) $-\frac{4}{15}$ (d) $-\frac{1}{15}$

71. If $\int \tan^6 x dx = \frac{1}{5} \tan^5 x + A \tan^3 x + \tan x - x + c$ then A is equal to

(a) $\frac{1}{3}$ (b) $\frac{2}{3}$

(c) $-\frac{2}{3}$ (d) $-\frac{1}{3}$

72. If $I = \int \sec^6 x dx = \frac{1}{5} \tan^5 x + A \tan^3 x + \tan x + c$ then A is equal to

(a) $\frac{1}{3}$ (b) $\frac{2}{3}$

(c) $-\frac{1}{3}$ (d) $-\frac{2}{3}$

73. If $\int \operatorname{cosec}^n x dx = -\frac{\operatorname{cosec}^{n-2} x \cot x}{n-1} + A \int \operatorname{cosec}^{n-2} x dx$ then A is equal to

(a) $\frac{1}{n-2}$ (b) $\frac{n}{n-2}$

(c) $\frac{n-1}{n-2}$ (d) $\frac{n-2}{n-1}$

Passage – 2

If the integrand is a rational function of x and fractional powers of a linear fractional function of the form $\frac{ax+b}{cx+d}$. Then rationalization of the integral is affected by the substitution $\frac{ax+b}{cx+d} = t^m$, where m is the L.C.M. of fractional powers of $\frac{ax+b}{cx+d}$.

74. If $I = \int \frac{(2x-3)^{1/2}}{(2x-3)^{1/3}+1} dx = 3 \left[\frac{1}{7} (2x-3)^{7/6} - \frac{1}{5} (2x-3)^{5/6} + \frac{1}{3} (2x-3)^{1/2} - (2x-3)^{1/6} + g(x) \right] - 1$ then $g(x)$ is equal to
- (a) $\tan^{-1}(2x-3)^{1/6}$ (b) $(2x-3)^{1/2}$
 (c) $3 \tan^{-1}(2x-3)^{1/6}$ (d) $4(2x-3)^{1/6}$

75. If $I = \int \frac{dx}{\sqrt[4]{(x-1)^3(x+2)^5}} = A \sqrt[4]{\frac{x-1}{x+2}} + c$ then A is equal to
- (a) $\frac{1}{3}$ (b) $\frac{2}{3}$
 (c) $\frac{3}{4}$ (d) $\frac{4}{3}$

76. If $I = \int \frac{dx}{\sqrt[3]{(x+1)^2(x-1)^4}} = k \sqrt[3]{\frac{1+x}{1-x}} + c$ then k equals
- (a) $\frac{2}{3}$ (b) $\frac{3}{2}$
 (c) $\frac{1}{3}$ (d) $\frac{1}{2}$

Match the column :

77. $\int f(x) dx$ when

- (a) $f(x) = \frac{1}{e^x + 1}$ (p) $2 \log(e^{x/2} + e^{-x/2}) + c$
 (b) $f(x) = e^{e^x + x}$ (q) $\log \frac{e^x}{e^x + 1} + c$
 (c) $f(x) = \frac{e^x - 1}{e^x + 1}$ (r) $\frac{1}{4} e^{2x^2} + c$
 (d) $f(x) = e^{2x^2 + \log x}$ (s) $e^{e^x} + c$

78. $\int f(x) dx$, when

- (a) $f(x) = \sqrt{\frac{1-\sqrt{x}}{1+\sqrt{x}}}$ (p) $c - \frac{2}{\sqrt{15}} \log \left| \frac{x+6+\sqrt{60x-15x^2}}{2x-3} \right|$
 (b) $f(x) = \sqrt{\frac{1-x}{1+x}} \cdot \frac{1}{x}$ (q) $\log \left| \frac{\sqrt{x+1}-\sqrt{1-x}}{\sqrt{1+x}+\sqrt{1-x}} \right| + 2 \tan^{-1} \sqrt{\frac{1-x}{1+x}} + c$
 (c) $f(x) = \frac{1}{\left(x - \frac{3}{2}\right) \sqrt{4x-x^2}}$ (r) $(\sqrt{x}-2) \sqrt{1-x} - \sin^{-1} \sqrt{x} + c$
 (d) $f(x) = \sqrt{\frac{1+\sqrt{x}}{x}}$ (s) $\frac{4}{3} (1+\sqrt{x})^{3/2} + c$

Integer Type :

79. If $\int \frac{x^2-1}{(x^2+1)\sqrt{x^4+1}} dx$ is equal to

$$\frac{\sqrt{2}}{4} A \tan^{-1} \left(\frac{1}{\sqrt{2}} \sqrt{x^2 + \frac{1}{x^2}} \right) + c \text{ then } A \text{ is equal to}$$

80. If the graph of the antiderivative $F(x)$ of $f(x) = \log(\log x) + (\log x)^{-2}$ passes through (e, 1998-e) then the term independent of x in $F(x)$ is

81. Let $F(x)$ be the antiderivative of

$$f(x) = \frac{1}{(3+5\sin x + 3\cos x)} \text{ whose graph passes through}$$

the point (0, 0) then the value of $F\left(\frac{\pi}{2}\right) - \frac{1}{5} \log \frac{8}{3} + 1982$ is equal to

82. Let f be a function satisfying $f''(x) = x^{-3/2}$, $f'(4) = 2$ and $f(0) = 0$ then $f(784)$ is equal to

D_Marv Online Advanced Question

Single Answer Type Question

1. The value of the integral $\int \frac{\cos^3 x + \cos^5 x}{\sin^2 x + \sin^4 x} dx$ is

Online

- (a) $\sin x - 6 \tan^{-1}(\sin x) + c$
 (b) $\sin x - 2(\sin x)^{-1} + c$
 (c) $\sin x - 2(\sin x)^{-1} - 6 \tan^{-1}(\sin x) + c$
 (d) $\sin x - 2(\sin x)^{-1} + 5 \tan^{-1}(\sin x) + c$

2. Let $f(x) = \int e^x (x-1)(x-2) dx$, then f decrease in the interval

Online

- (a) $(-\infty, -2)$ (b) $(-2, -1)$
 (c) $(1, 2)$ (d) $(2, \infty)$

3. $\int \frac{x^2 - 1}{x^3 \sqrt{2x^4 - 2x^2 + 1}} dx$ is equal to

Online

- (a) $\frac{\sqrt{2x^4 - 2x^2 + 1}}{x^2} + c$ (b) $\frac{\sqrt{2x^4 - 2x^2 + 1}}{x^3} + c$
 (c) $\frac{\sqrt{2x^4 - 2x^2 + 1}}{x} + c$ (d) $\frac{\sqrt{2x^4 - 2x^2 + 1}}{2x^2} + c$

4. Let $f(x) = \frac{x}{(1+x^n)^{1/n}}$ for $n \geq 2$ and

$$g(x) = \underbrace{(f \circ f \circ \dots \circ f)}_{f \text{ occurs } n \text{ times}}(x)$$

Then $\int x^{n-2} g(x) dx$ equals

Online

- (a) $\frac{1}{n(n-1)}(1+nx^n)^{1-\frac{1}{n}} + c$
 (b) $\frac{1}{n-1}(1+nx^n)^{1-\frac{1}{n}} + c$
 (c) $\frac{1}{n(n+1)}(1+nx^n)^{1+\frac{1}{n}} + c$
 (d) $\frac{1}{n+1}(1+nx^n)^{1+\frac{1}{n}} + c$

5. Let $I = \int \frac{e^x}{e^{4x} + e^{2x} + 1} dx$, $J = \int \frac{e^{-x}}{e^{-4x} + e^{-2x} + 1} dx$

Then, for an arbitrary constant c , the value of $J-I$ equals

Online

- (a) $\frac{1}{2} \log \left(\frac{e^{4x} - e^{2x} + 1}{e^{4x} + e^{2x} + 1} \right) + c$
 (b) $\frac{1}{2} \log \left(\frac{e^{2x} + e^x + 1}{e^{2x} - e^x + 1} \right) + c$
 (c) $\frac{1}{2} \log \left(\frac{e^{2x} - e^x + 1}{e^{2x} + e^x + 1} \right) + c$
 (d) $\frac{1}{2} \log \left(\frac{e^{4x} + e^{2x} + 1}{e^{4x} - e^{2x} + 1} \right) + c$

6. The integral $\int \frac{\sec^2 x}{(\sec x + \tan x)^{9/2}} dx$ equals to (for some arbitrary constant C)

Online

- (a) $\frac{-1}{(\sec x + \tan x)^{11/2}} \left\{ \frac{1}{11} - \frac{1}{7} (\sec x + \tan x)^2 \right\} + C$
 (b) $\frac{1}{(\sec x + \tan x)^{11/2}} \left\{ \frac{1}{11} - \frac{1}{7} (\sec x + \tan x)^2 \right\} + C$
 (c) $\frac{-1}{(\sec x + \tan x)^{11/2}} \left\{ \frac{1}{11} + \frac{1}{7} (\sec x + \tan x)^2 \right\} + C$
 (d) $\frac{1}{(\sec x + \tan x)^{11/2}} \left\{ \frac{1}{11} + \frac{1}{7} (\sec x + \tan x)^2 \right\} + C$

Assertion Reason

- (A) If ASSERTION is true, REASON is true, REASON is a correct explanation for ASSERTION.
- (B) If ASSERTION is true, REASON is true, REASON is not a correct explanation for ASSERTION.
- (C) If ASSERTION is true, REASON is false
- (D) If ASSERTION is false, REASON is true
- (E) If both ASSERTION and REASON are false.

7. Let $F(x)$ be an indefinite integral of $\sin^2 x$.

Assertion : The function $F(x)$ satisfies $F(x + \pi) = F(x)$ for all real x .

Reason : $\sin^2(x + \pi) = \sin^2 x$ for all real x . *Online*

Fill in the Blanks

8. If $\int \frac{4e^x + 6e^{-x}}{9e^x - 4e^{-x}} dx = Ax + B \log(9e^{2x} - 4) + C$, then

$A = \dots$, $B = \dots$ and $C = \dots$ *Online*

Subjective Questions

9. Integrate $\frac{1}{1 - \cot x}$ or $\frac{\sin x}{\sin x - \cos x}$. *Online*

10. Integrate the curve $\frac{x}{1 + x^4}$ *Online*

11. Integrate $\sin x \cdot \sin 2x \cdot \sin 3x + \sec^2 x \cdot \cos^2 2x + \sin^4 x \cdot \cos^4 x$. *Online*

12. Integrate $\frac{x^2}{(a + bx)^2}$. *Online*

13. $f(x)$ is the integral of $\frac{2 \sin x - \sin 2x}{x^3}$, $x \neq 0$ find

$$\lim_{x \rightarrow 0} f'(x).$$

Online

14. Evaluate the following integrals

(a) $\int \sqrt{1 + \sin\left(\frac{1}{2}x\right)} dx$ (b) $\int \frac{x^2 dx}{\sqrt{1-x}}$

15. Evaluate $\int (e^{\log x} + \sin x) \cos x dx$.

16. Evaluate $\int \frac{(x-1)e^x}{(x+1)^3} dx$.

17. Evaluate the following $\int \frac{dx}{x^2(x^4 + 1)^{3/4}}$

18. $\int \sqrt{\frac{1-\sqrt{x}}{1+\sqrt{x}}} dx$.

19. Evaluate $\int \frac{\sin^{-1} \sqrt{x} - \cos^{-1} \sqrt{x}}{\sin^{-1} \sqrt{x} + \cos^{-1} \sqrt{x}} dx$

20. Evaluate $\int \frac{(\cos 2x)^{1/2}}{\sin x} dx$

21. Evaluate $\int (\sqrt{\tan x} + \sqrt{\cot x}) dx$

Online

22. Find the indefinite integral

$$\int \left(\frac{1}{\sqrt[3]{x} + \sqrt[4]{x}} + \frac{\ln(1 + \sqrt[6]{x})}{\sqrt[3]{x} + \sqrt{x}} \right) dx$$

23. Find the indefinite integral

$$\int \cos 2\theta \log \left(\frac{\cos \theta + \sin \theta}{\cos \theta - \sin \theta} \right) d\theta$$

24. Evaluate $\int \frac{(x+1)}{x(1+xe^x)^2} dx$

25. Integrate the following $\int \left(\frac{1-\sqrt{x}}{1+\sqrt{x}} \right)^{1/2} \frac{dx}{x}$

Online

26. Integrate $\int \frac{x^3 + 3x + 2}{(x^2 + 1)^2 (x + 1)} dx$

27. Evaluate $\int \sin^{-1} \left(\frac{2x + 2}{\sqrt{4x^2 + 8x + 13}} \right) dx$

28. For any natural number m, evaluate

$$\int (x^{3m} + x^{2m} + x^m) (2x^{2m} + 3x^m + 6)^{1/m} dx, x > 0$$

Online

29. Let $f: \mathbb{R} \rightarrow \mathbb{R}$ be a differentiable function such that

$$f(0) = 0, f\left(\frac{\pi}{2}\right) = 3 \text{ and } f'(0) = 1. \text{ If}$$

$$g(x) = \int_x^{\frac{\pi}{2}} [f'(t) \operatorname{cosec} t - \cot t \operatorname{cosec} t f(t)] dt$$

For $x \in (0, \frac{\pi}{2}]$, then $\lim_{x \rightarrow 0} g(x) =$

Online

ANSWER KEY

EXERCISE - 1 : BASIC OBJECTIVE QUESTIONS

1. (c)	2. (b)	3. (c)	4. (b)	5. (b)	6. (a)	7. (c)	8. (a)	9. (d)	10. (a)
11. (d)	12. (a)	13. (b)	14. (b)	15. (b)	16. (d)	17. (b)	18. (d)	19. (c)	20. (c)
21. (c)	22. (d)	23. (b)	24. (b)	25. (c)	26. (b)	27. (c)	28. (c)	29. (a)	30. (a)
31. (a)	32. (b)	33. (d)	34. (b)	35. (a)	36. (c)	37. (c)	38. (d)	39. (b)	40. (a)
41. (c)	42. (b)	43. (c)	44. (a)	45. (b)	46. (b)	47. (b)	48. (a)	49. (c)	50. (c)
51. (b)	52. (b)	53. (a)	54. (b)	55. (c)	56. (c)	57. (b)	58. (a)	59. (c)	60. (a)
61. (d)	62. (a)	63. (d)	64. (a)	65. (c)	66. (a)	67. (c)	68. (a)	69. (a)	70. (b)

D_Marv Online Objective Answer

1. (b)	2. (d)	3. (c)	4. (d)	5. (c)	6. (d)	7. (c)	8. (c)	9. (b)
10. (a)	11. (b)	12. (c)	13. (b)	14. (b)	15. (c)	16. (a)	17. (b)	18. (b)
19. (b)	20. (b)	21. (b)	22. (c)	23. (b)	24. (b)	25. (c)		

EXERCISE - 3 : ADVANCED OBJECTIVE QUESTIONS

1. (b)	2. (c)	3. (c)	4. (a)	5. (b)	6. (b)	7. (d)	8. (c)	9. (d)	10. (b)
11. (c)	12. (b)	13. (b)	14. (b)	15. (d)	16. (b)	17. (a)	18. (c)	19. (c)	20. (a)
21. (a)	22. (c)	23. (b)	24. (d)	25. (d)	26. (c)	27. (a)	28. (c)	29. (a)	30. (b)
31. (a)	32. (a)	33. (b)	34. (c)	35. (b)	36. (a)	37. (c)	38. (c)	39. (c)	40. (b)
41. (c)	42. (c)	43. (b)	44. (d)	45. (c)	46. (c)	47. (c)	48. (b)		
49. (a)	50. (b)	51. (a)	52. (c)	53. (b)	54. (b)	55. (a)	56. (a)	57. (a)	58. (a)
59. (b)	60. (a,b,c,d)	61. (b,d)	62. (c,d)	63. (a,b,c,d)	64. (a,b)	65. (c)	66. (a)	67. (d)	68. (d)
69. (d)	70. (c)	71. (d)	72. (b)	73. (d)	74. (a)	75. (d)	76. (b)		
77. a-q; b-s; c-p; d-r	78. a-r; b-q; c-p; d-s	79. (2)	80. (1998)	81. (1982)	82. (2240)				

D_Marv Online Advanced Answer

1. (c) 2. (c) 3. (d) 4. (a) 5. (c) 6. (c) 7. (d)
8. $A = -\frac{3}{2}$, $B = \frac{35}{36}$ and $C \in \mathbb{R}$ 9. $\frac{1}{2} \log |\sin x - \cos x| + \frac{1}{2}x + c$ 10. $\frac{1}{2} \tan^{-1}(x^2) + c$,
11. $\frac{-\cos 2x}{8} - \frac{\cos 4x}{16} + \frac{\cos 6x}{24} + \sin 2x + \tan x - \left(\frac{253}{128}\right)x - \frac{1}{128} \sin 4x + \frac{1}{1024} \sin 8x$
12. $\frac{1}{b^3} \left(a + bx - 2a \log |a + bx| - \frac{a^2}{a + bx} + c \right)$ 13. 1
14. (a) $4 \sin \frac{x}{4} - 4 \cos \frac{x}{4} + c$; (b) $-2 \left\{ \sqrt{1-x} - \frac{2}{3}(1-x)^{3/2} + \frac{1}{5}(1-x)^{5/2} \right\} + c$ 15. $x \sin x + \cos x - \frac{\cos 2x}{4} + c$
16. $\frac{e^x}{(x+1)^2} + c$ 17. $-\frac{(x^4+1)^{1/4}}{x} + c$ 18. $-2\sqrt{1-x} + \cos^{-1} \sqrt{x} + \sqrt{x(1-x)} + c$
19. $\frac{2}{\pi} \left\{ \sqrt{x-x^2} - (1-2x) \sin^{-1} \sqrt{x} \right\} - x + c$ 20. $-\log \left| \cot x + \sqrt{\cot^2 x - 1} \right| + \frac{1}{\sqrt{2}} \log \left| \frac{\sqrt{2} + \sqrt{1 - \tan^2 x}}{\sqrt{2} - \sqrt{1 - \tan^2 x}} \right| + c$
21. $\sqrt{2} \tan^{-1} \left(\frac{\sqrt{\tan x} - \sqrt{\cot x}}{\sqrt{2}} \right) + c$
22. $\frac{3}{2}x^{2/3} - \frac{12}{7}x^{7/12} + \frac{4}{3}x^{1/2} - \frac{12}{5}x^{5/12} + \frac{11}{2}x^{1/3} - 4x^{1/4} - 5x^{1/6} - 12x^{1/12}$
 $+ (2x^{1/2} - 3x^{1/3} + 6x^{1/6} + 11) \ln(1+x^{1/6}) + 12 \ln(1+x^{1/12}) - 3 \left[\ln(1+x^{1/6}) \right]^2 + c$
23. $\frac{1}{2} \sin 2\theta \ln \left(\frac{\cos \theta + \sin \theta}{\cos \theta - \sin \theta} \right) + \frac{1}{2} \ln(\cos 2\theta) + c$ 24. $\ln \left| \frac{xe^x}{1+xe^x} \right| + \frac{1}{1+xe^x} + c$
25. $2 \left\{ \cos^{-1} \sqrt{x} - \log |1 + \sqrt{1-x}| + \frac{1}{2} \log |x| \right\} + c$ 26. $-\frac{1}{2} \log |x+1| + \frac{1}{4} \log(x^2+1) + \frac{3}{2} \tan^{-1} x + \frac{x}{x^2+1} + c$
27. $(x+1) \tan^{-1} \left(\frac{2x+2}{3} \right) - \frac{3}{4} \log(4x^2+8x+13) + c$ 28. $\frac{1}{6(m+1)} (2x^{3m} + 3x^{2m} + 6x^m)^{\frac{m+1}{m}} + c$
29. (2)

Dream on !!

