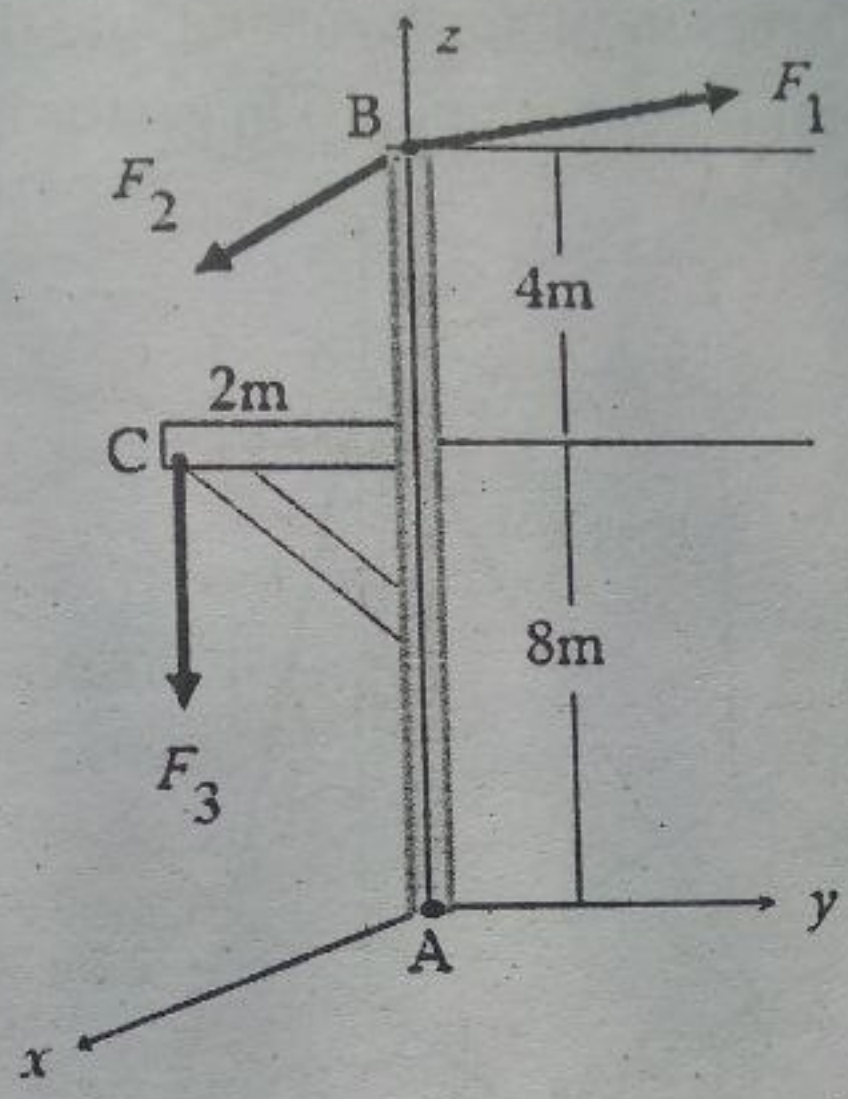
12. Determine the moment of $F_1 = 40i + 30j + 12k$, $F_2 = 10i - 10j - 6k$ and $F_3 = -50k$ at point A acting as shown below.



Solution: To use the vector Method, first identify the position vectors of each force. Using the coordinates of the points B and C, we get the position vectors as follow:

For both F_1 and F_2, $\vec{r}AB = B - A = 12\mathbf{k}$ and for F_3, $\vec{r}AC = -2\mathbf{j} + 8\mathbf{k}$.

Therefore, by Vector-Method, the moment at A is given as follow:

$$M_A = \vec{r}AB \times F_1 + \vec{r}AB \times F_2 + \vec{r}AC \times F_3$$

$$= \begin{vmatrix} i & j & k \\ 0 & 0 & 12 \\ 40 & 30 & 12 \end{vmatrix} + \begin{vmatrix} i & j & k \\ 0 & 0 & 12 \\ 10 & -10 & -6 \end{vmatrix} + \begin{vmatrix} i & j & k \\ 0 & -2 & 8 \\ 0 & 0 & -50 \end{vmatrix}$$

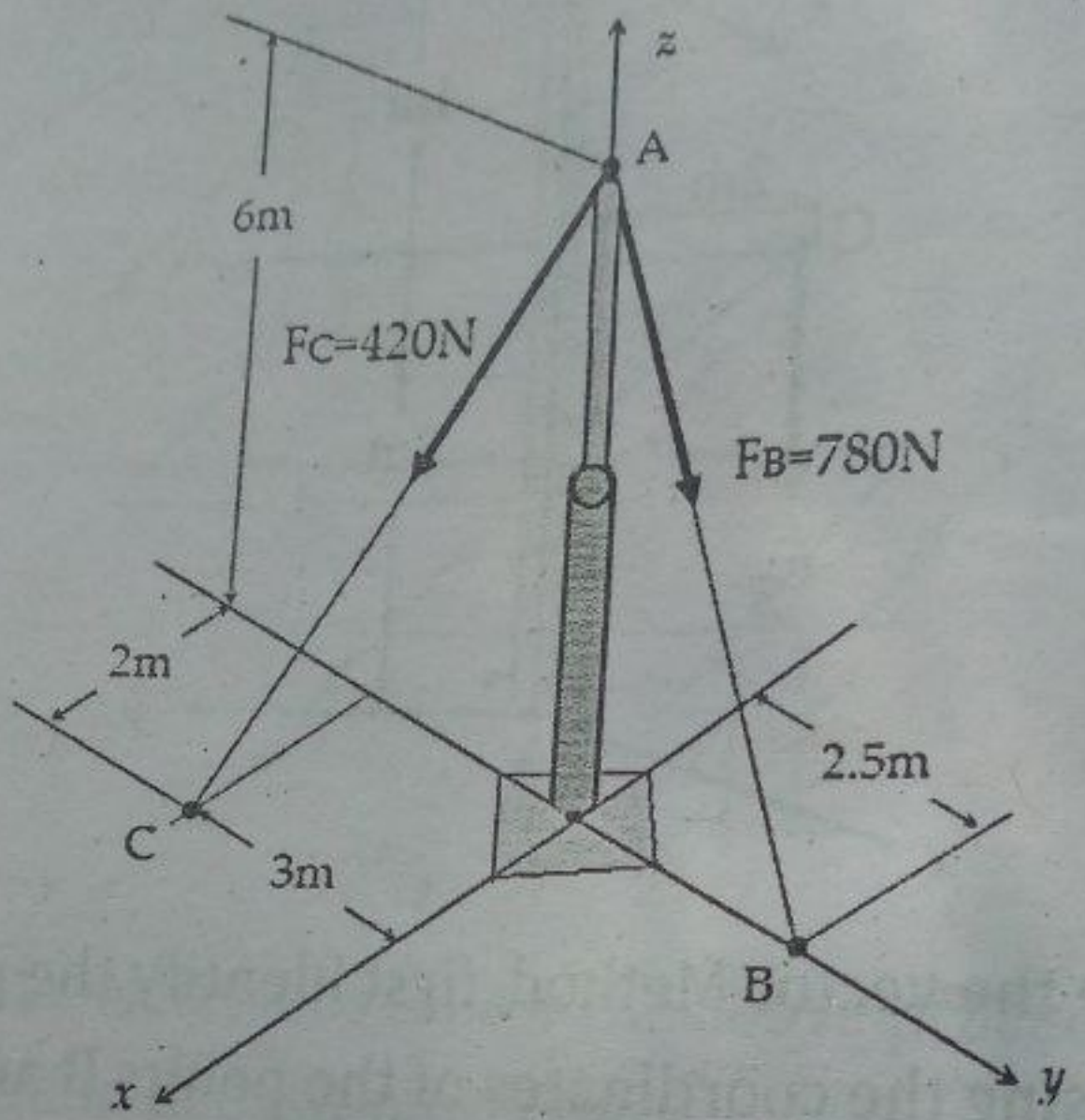
$$= -360i + 480j + 120i + 120j + 100i = -140i + 600j$$

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13. The two tension forces $F_B = 780\text{N}$ and $F_C = 420\text{N}$ are acting at the top of the pole as shown in the diagram below.

a) Express the two forces in vector form and find their resultant

b) Find the resultant moment at point O in vector form



Solution:

a) Here, the coordinates are $A=(0,0,6)$, $B=(0,2.5,0)$, $C=(2,-3,0)$.

i) $\vec{F}_B = F_B.\vec{U}_{\overline{AB}} = 780\text{N}\left(\frac{2.5}{6.5}j - \frac{6}{6.5}k\right) = 300\,\mathbf{j} - 720\,\mathbf{k}$.

ii) $\vec{F}_C = F_C.\vec{U}_{\overline{AC}} = 420\text{N}\left(\frac{2}{7}i - \frac{3}{7}j - \frac{6}{7}k\right) = 120\mathbf{i} - 180\,\mathbf{j} - 360\mathbf{k}$.

Thus, the resultant is the sum of the two forces coordinate wise.

That is $\vec{R} = \vec{F}_B + \vec{F}_C = 120\mathbf{i} + 180\mathbf{j} - 1080\mathbf{k}$.

b) The moment at O is the sum of the moment due to the two forces. It is easily obtained using vector approach as follow:

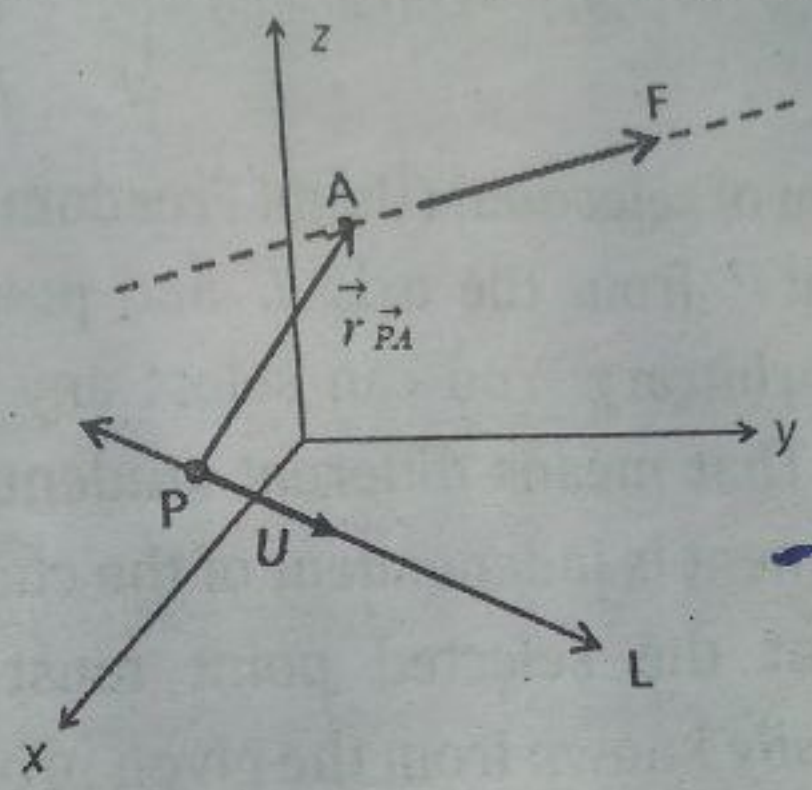
$$\vec{M}_O = \vec{r}_{\overline{OB}} \times \vec{F}_B + \vec{r}_{\overline{OC}} \times \vec{F}_C = \begin{vmatrix} i & j & k \\ 0 & 2.5 & 0 \\ 0 & 300 & -720 \end{vmatrix} + \begin{vmatrix} i & j & k \\ 2 & -3 & 0 \\ 120 & -180 & -360 \end{vmatrix}$$

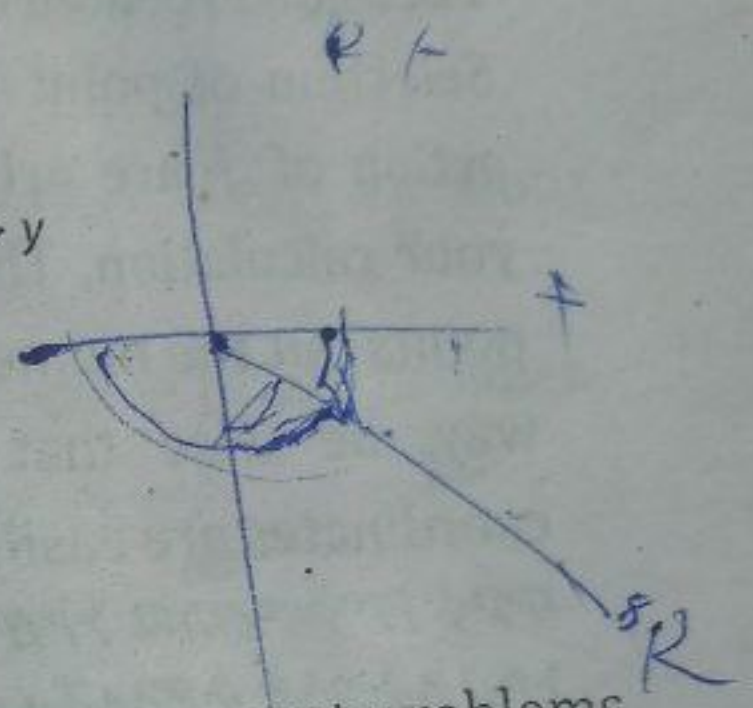
$$= -1800i + 1080i + 720j = -720i + 720j$$

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1.8 Moment about an Axis or a line

Suppose a force F is applied at A and L is any axis or line about which the moment is asked as shown in the diagram below.





Procedures to find Moment about an Axis:

Step-1: Expression forces in vector form: In many moment problems, it is the magnitude of a force that will be given. In such case, using the geometry of the problem, select any two points on the line of action of the force and find a unit vector for the force.

Step-2: Computation of Initial Moment (Point Selection)

Start by selecting a point from the Axis. That is select a point P (with known coordinates) and compute the moment M_P of the force F. using vector approach. That is $\overrightarrow{M}_P = \vec{r}_{\overrightarrow{PA}} \times F = M_x i + M_y j + M_z k$.

Step-3: Determination of a Unit-Vector parallel to moment axis

That is find a unit vector parallel to the axis or the line L about which the moment is asked. Suppose you get $\vec{U} = U_x i + U_y j + U_z k$

Step-4: Apply the Concept of Projection:

Once the direction vector of the line is known, the moment of F about the axis L is the *projection* of the moment M_P on to L.

That is the moment about the given axis or line L becomes

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$\overrightarrow{M_L} = \overrightarrow{U}.\overrightarrow{M_P} = \overrightarrow{U}.(\vec{r}_{\overrightarrow{PA}} \times F)$.

Then, using the scalar triple product, this is reduced into a simple determinant as $\overrightarrow{M_L} = \overrightarrow{U}.\overrightarrow{M_P} = \overrightarrow{U}.(\vec{r}_{\overrightarrow{PA}} \times F) = \begin{vmatrix} U_x & U_y & U_z \\ r_x & r_y & r_z \\ F_x & F_y & F_z \end{vmatrix}$

Focus on Freedom of selection! (Umm! Freedom of selection?)

Selection of point P from the axis L and point A from the line of action of F are arbitrary. You can select any point convenient for your calculation. That means different students can select different points but the moment is independent of the choice of P and A. (Any way, be wise that the selected point must be a point whose coordinates are easily known from the given problem).

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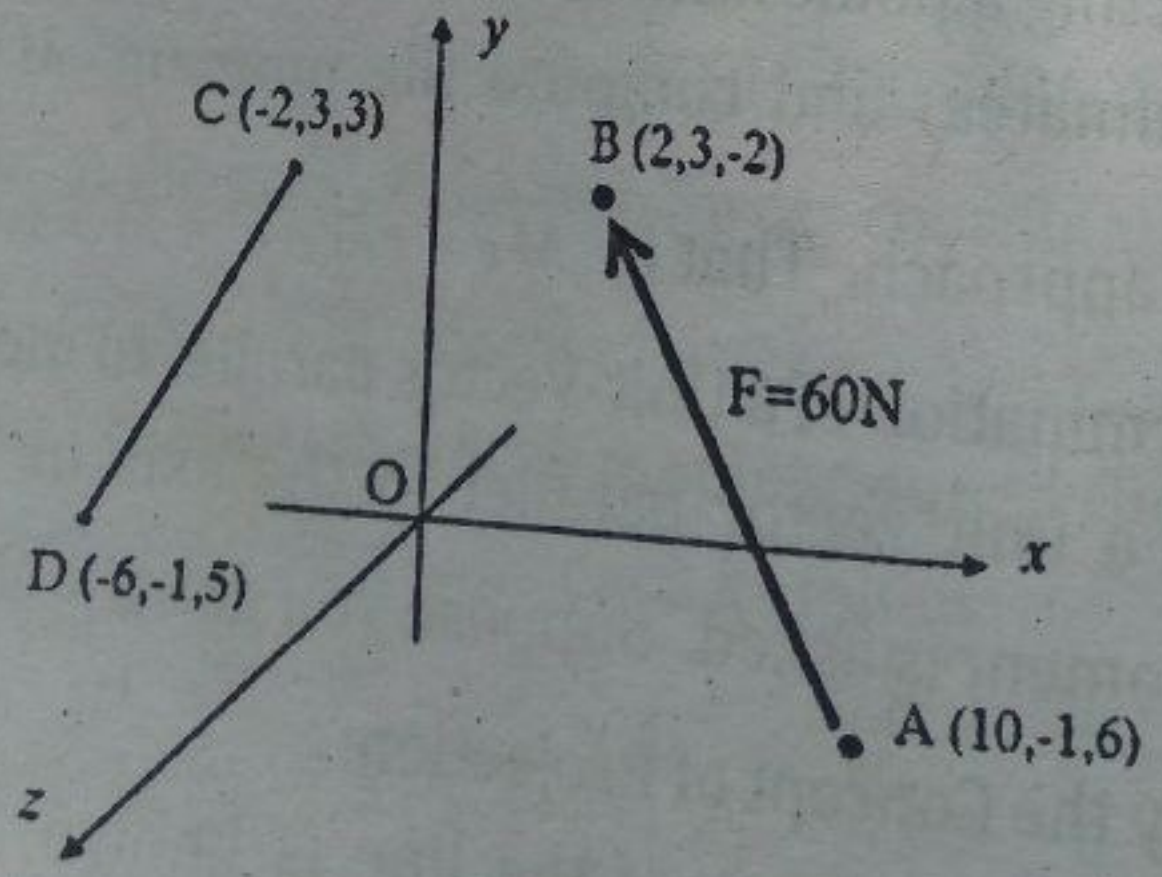
Examples:

1. The force on cable AB is $F = 60\text{N}$. Then,

a) Determine the moment exerted on the coordinate axes.

b) Draw the diagrams showing the moments on each axis

c) Determine the moment exerted on line CD



Solution: First express the force in vector form.

Here, $\vec{U}_{\overrightarrow{AB}} = \dfrac{-8i+4j-8k}{12} = -\dfrac{2}{3}i+\dfrac{1}{3}j-\dfrac{2}{3}k$.

Thus, $\vec{F} = F\vec{U}_{\overrightarrow{AB}} = 60\text{N}.(-\dfrac{2}{3}i+\dfrac{1}{3}j-\dfrac{2}{3}k) = -40i+20j-40k$.

Using vector approach of moment, the moment about point O is given

by $\overrightarrow{Mo} = \vec{r}_{\overrightarrow{OA}} \times \vec{F} = \begin{vmatrix} i & j & k \\ 10 & -1 & 6 \\ -40 & 20 & -40 \end{vmatrix} = -80i+160j+160k$

Besides, we can take the basic unit vectors $\vec{U}_x = \mathbf{i}, \vec{U}_y = \mathbf{j}, \vec{U}_z = \mathbf{k}$ along the coordinate axes.

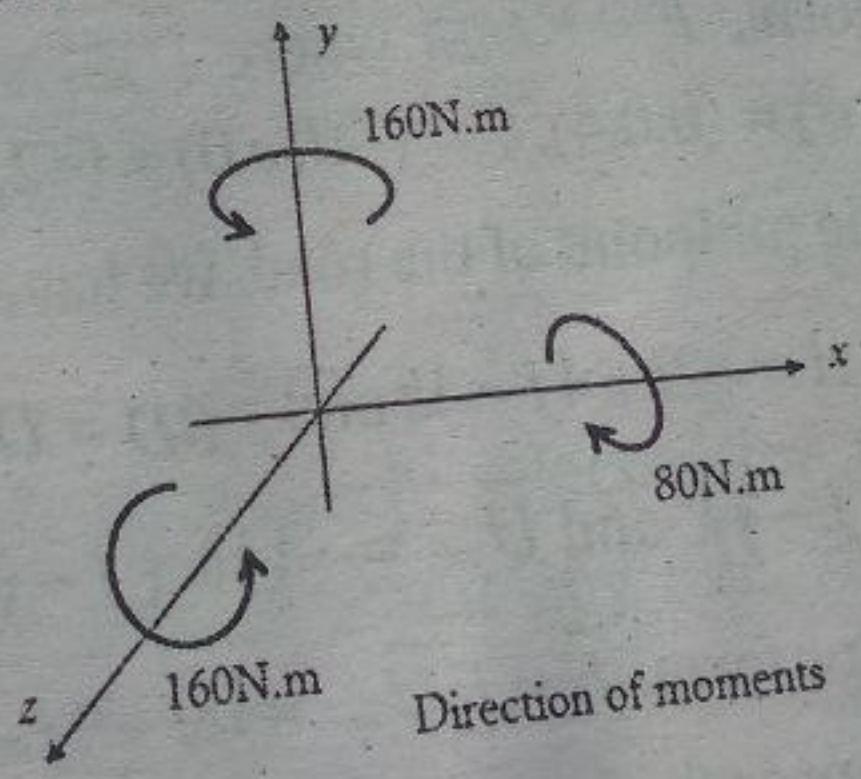
a) The moments about the coordinate axes are taken by dot product.

i) $\overrightarrow{M}_x = \overrightarrow{Mo}.\vec{U}_x = (-80i+160j+160k).\mathbf{i} = -80\text{N.m}$

ii) $\overrightarrow{M}_y = \overrightarrow{Mo}.\vec{U}_y = (-80i+160j+160k).\mathbf{j} = 160\text{N.m}$

ii) $\overrightarrow{M}_z = \overrightarrow{Mo}.\vec{U}_z = (-80i+160j+160k).\mathbf{k} = 160\text{N.m}$

b) The directions of the moment along each axis is shown as follow.



Direction of moments

c) To find the moment about CD, first find the moment about C or D.

That is $\overrightarrow{M}_C = \vec{r}_{\overrightarrow{CA}} \times \vec{F} = \begin{vmatrix} i & j & k \\ 12 & -4 & 3 \\ -40 & 20 & -40 \end{vmatrix} = 100i+360j+80k$.

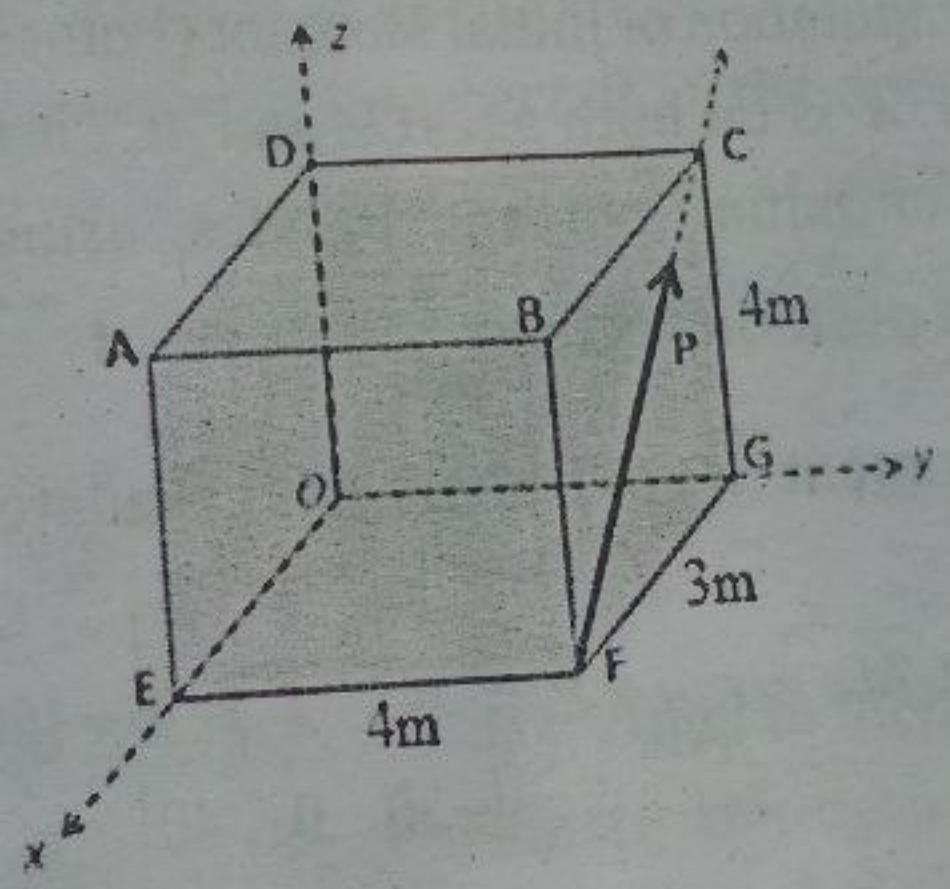
Finally,

$\overrightarrow{M}_C.\vec{U}_{\overrightarrow{CD}} = (100i+360j+80k).(-\dfrac{2}{3}i-\dfrac{2}{3}j+\dfrac{1}{3}k) = -280\text{N.m}$

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3. A force $P = 50\text{N}$ is acting on a rectangular box with dimensions $3\text{m} \times 4\text{m} \times 4\text{m}$ as shown in the diagram below.
Then, determine the moment of the force P

a) About the lines OA and AC. Express the answers in vector form
b) About the three coordinate axes
c) About the lines AB and AF in vector form
d) About the diagonals AG, EC and OB in vector form
e*) Give the perpendicular distance between AG and FC
f) Give the projection of the force P on lines AE and EG



Solution: This example is properly selected for you to understand moment about a given axis or about any line. As we stated above, there are three main steps to find moment about an axis or a line:

Step-1: Expression forces in vector form:
Step-2: Computation of Initial Moment (Point Selection)
Step-3: Determination of Direction Vector
Step-4: Apply the Concept of Projection

a) We need the moment about the lines OA and AC.

Step-1: Expression forces in vector form: To find the moment of a force about a point or about an axis, then first task is to express the

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force in vector form. That means express $P = 50\text{N}$ in vector form. To do so, find a unit vector parallel to P using any two points on its line of action. In this problem, P passes through the points F and C. From the dimensions of the box, the coordinates of these points are $F = (3,4,0)$, $C = (0,4,4)$ so that $\overrightarrow{FC} = C - F = -3i + 4k$.

Then, the unit vector of P is $U_P = \dfrac{\overrightarrow{FC}}{\left\|\overrightarrow{FC}\right\|} = \dfrac{-3i+4k}{5} = -\dfrac{3}{5}\text{i} + \dfrac{4}{5}\text{k}$.

So, the vector expression of P is $\vec{P} = P.U_P = -30\text{i} + 40\text{k}$.

Step-2: Computation of Initial Moment (Point Selection)

Since point A is on both axes OA and AC, it is better to select A as a reference for initial moment. Besides, using $A = (3,0,4)$, $F = (3,4,0)$, we have the position vector $r_{AF} = F - A = 4j - 4k$.

Then, the moment $\overrightarrow{M}_A$ of P about A is calculated as follow:

Moment at A: $\overrightarrow{M}_A = r_{AF} \times \vec{P} = \begin{vmatrix} i & j & k \\ 0 & 4 & -4 \\ -30 & 0 & 40 \end{vmatrix} = 160i + 120j + 120k$.

Step-3: Determination of a Unit-Vector parallel to moment axis

In this problems, since the moment axes are OA and AC, we need two unit vectors parallel to these axes. Hence, the unit vectors are

$$U_{OA} = \frac{\overrightarrow{OA}}{\left\|\overrightarrow{OA}\right\|} = \frac{3}{5}\text{i} + \frac{4}{5}\text{k},\ U_{AC} = \frac{\overrightarrow{AC}}{\left\|\overrightarrow{AC}\right\|} = -\frac{3}{5}\text{j} + \frac{4}{5}\text{k}.$$

Step-4: Apply the Concept of Projection

Compute the moments $\overrightarrow{M}_{AC}$ and $\overrightarrow{M}_{AF}$ using dot product of $\overrightarrow{M}_A$ and with the respective unit vectors as follow:

this problems the respective un e unit vectors of the lines as follow:

Moment about the axis OA: $M_{OA} = \overrightarrow{M}_A . \overrightarrow{U}_{OA} = 192\text{N.m}$

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Moment about the axis AC: $M_{AC} = \overline{M}_A . \overline{U}_{AC} = 0$

To express the moment about OA in vector form, simply multiply the unit vector $\overline{U}_{OA}$ of the line OA by the moment M_{OA}.

That is $_{OA} = \overline{M}_A . \overline{U}_{OA} = 192(\frac{3}{5}\mathbf{i} + \frac{4}{5}\mathbf{k}) = 115\mathbf{i} + 154\mathbf{k}$

b) We need the moment about the three coordinate axes: This means about the x, y and z axes. So, let's use the above steps.

Step-1: Expression forces in vector form:

As we did in part (a),in vector form, $\overline{P} = -30\mathbf{i} + 40\mathbf{k}$.

Step-2: Computation of Initial Moment (Point Selection)

Since the three coordinate axes intersect at O, the appropriate point to be selected is the origin (You can select any other points the final answer is the same but you need three points one for each axis).

Moment at O: $\overline{M}_O = \mathbf{r}_{OF} \times \overline{P} = \begin{vmatrix} i & j & k \\ 0 & 3 & 4 \\ -30 & 0 & 40 \end{vmatrix} = 120i - 120j + 90k$

Step-3: Determination of a Unit-Vector parallel to moment axis

Since we are interested on the coordinate axe, their unit vectors are simply $\mathbf{U}_{x\text{-axis}} = \mathbf{i}\ \mathbf{U}_{y\text{-axis}} = \mathbf{j},\ \mathbf{U}_{z\text{-axis}} = \mathbf{k}$.

Step-4: Apply the Concept of Projection

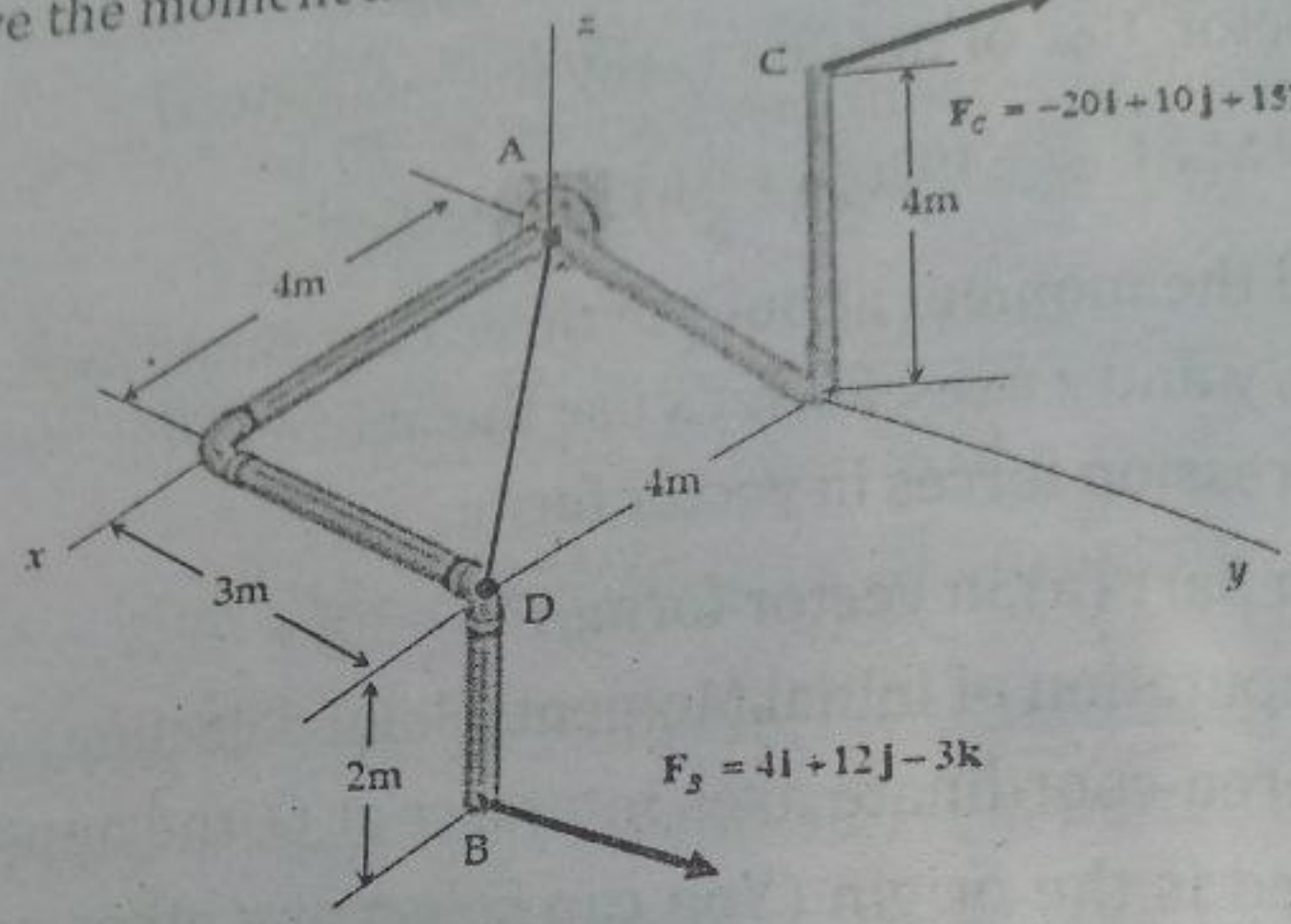
Moment about the x-axis: $M_x = \overline{M}_O . \overline{U}_x = 120\text{N.m}$

Moment about the y-axis: $M_y = \overline{M}_O . \overline{U}_y = -120\text{N.m}$

Moment about the z-axis: $M_z = \overline{M}_O . \overline{U}_z = 90\text{N.m}$

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4. For the assembly below, determine the resultant moment at point A and give the moment about line AD.



Solution: Apply vector approach. That is

$$\vec{M}_A = \vec{r}_{\overrightarrow{AC}} \times \vec{F}_C + \vec{r}_{\overrightarrow{AB}} \times \vec{F}_B = \begin{vmatrix} i & j & k \\ 0 & 3 & 4 \\ -20 & 10 & 15 \end{vmatrix} + \begin{vmatrix} i & j & k \\ 4 & 3 & -2 \\ 4 & 12 & -3 \end{vmatrix} = 35\mathbf{i} - 76\mathbf{j} + 96\mathbf{k}$$

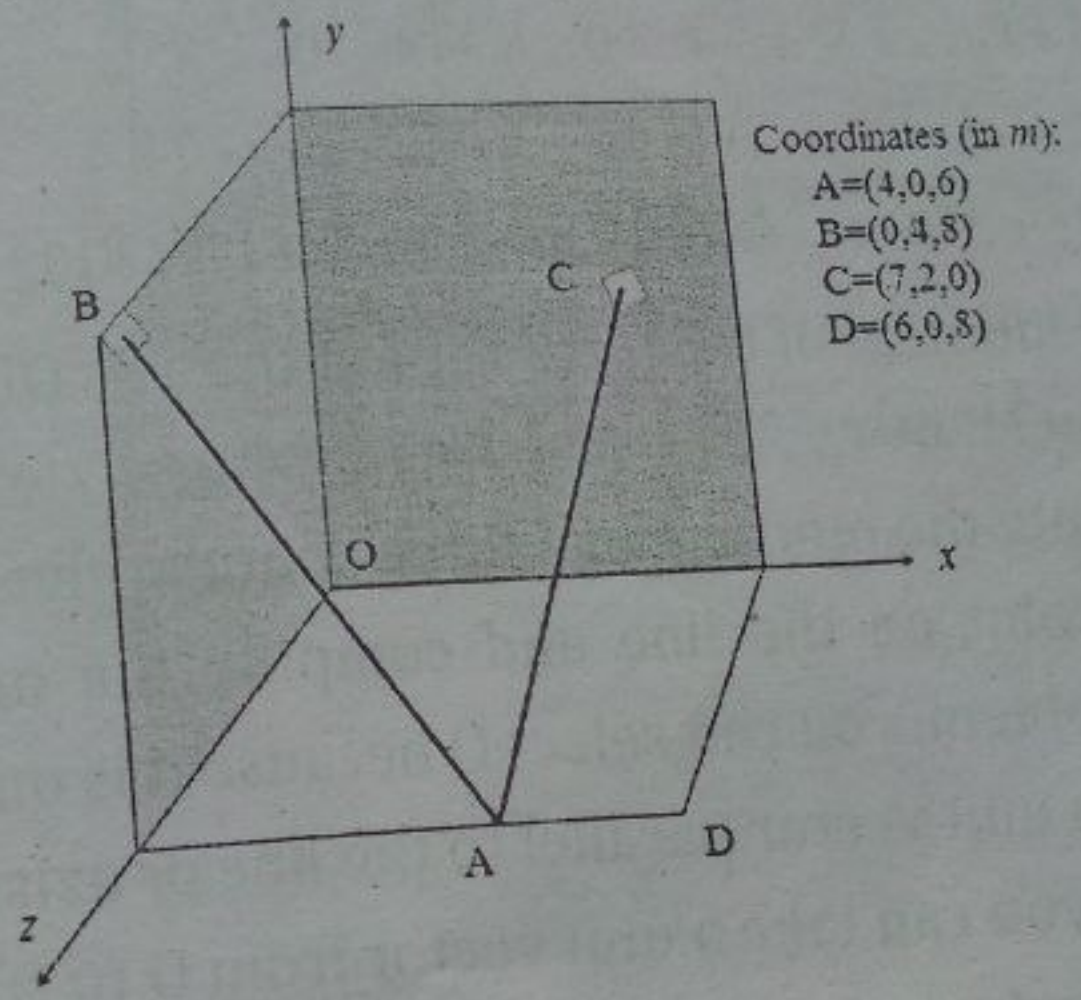
Now, let's determine the moment about axis AD. The moment of forces about an axis is nothing but the dot product of the moment at a point on the axis with the unit vector of that axis.

Here, point A is on the axis AD and we have already, computed the moment about A. So, we are left only to get a unit vector parallel to AD. It is $\vec{U}_{\overrightarrow{AD}} = \frac{4}{5}i + \frac{3}{5}j$. Therefore, $\vec{M}_{AD} = \vec{M}_A . \vec{U}_{\overrightarrow{AD}} = -17.6\text{N.m}$

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5. Suppose the plate is supported by two cables as shown below. If the tensior. forces on the cables are $T_{AB} = 9\text{N}$ and $T_{AC} = 28\text{N}$,then

a) Express the forces in vector form and give the angle between them
b) Determine the moment at O due to the tension forces exerted at A
c) Compute the moment of the two forces about the axis OD
d) Give the projection of the resultant force acting at A onto line OD



Coordinates (in m):
A=(4,0,6)
B=(0,4,8)
C=(7,2,0)
D=(6,0,8)

Solution:

a) To express a force with a given magnitude, always identify a unit vector parallel to the line of action of the force and multiply by the given magnitude. This will give the vector expression of the force. Since two points are given on the line of actions of the forces, we can find their unit vectors and the vector expressions easily as follow.

i) $\overrightarrow{AB} = B - A = -4i + 4j + 2k \Rightarrow \vec{U}_{\overrightarrow{AB}} = \frac{1}{6}(-4i + 4j + 2k) = -\frac{2}{3}i + \frac{2}{3}j + \frac{1}{3}k$

So, in vector form, $\vec{T}_{\overrightarrow{AB}} = T_{AB}.\vec{U}_{\overrightarrow{AB}} = 9\text{N}(-\frac{2}{3}i + \frac{2}{3}j + \frac{1}{3}k) = -6i + 6j + 3k$

ii) $\overrightarrow{AC} = C - A = 3i + 2j - 6k \Rightarrow \vec{U}_{\overrightarrow{AC}} = \frac{1}{7}(3i + 2j - 6k) = \frac{3}{7}i + \frac{2}{7}j - \frac{6}{7}k$

So, in vector form, $\vec{T}_{\overrightarrow{AC}} = T_{AC}.\vec{U}_{\overrightarrow{AC}} = 28\text{N}(\frac{3}{7}i + \frac{1}{7}j - \frac{6}{7}k) = 12i + 8j - 24k$

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Angle between two vectors is obtained from their dot product.

That is $\cos\theta = \dfrac{\vec{T}_{\overrightarrow{AB}}.\vec{T}_{\overrightarrow{AC}}}{T_{AB}.T_{AC}} = -\dfrac{96}{9(28)} = -0.381 \Rightarrow \theta = \cos^{-1}(-0.381) = 112.4^{0}$.

b) The moment at O is simply the sum of the moment due to the two forces. It is easily obtained using vector approach as follow:

$$\overrightarrow{M}_O = \vec{r}_{\overrightarrow{OA}} \times \vec{T}_{\overrightarrow{AB}} + \vec{r}_{\overrightarrow{OA}} \times \vec{T}_{\overrightarrow{AC}} = \begin{vmatrix} i & j & k \\ 4 & 0 & 8 \\ -6 & 6 & 3 \end{vmatrix} + \begin{vmatrix} i & j & k \\ 4 & 0 & 8 \\ 12 & 8 & -24 \end{vmatrix}$$

$$= (-48i - 60j + 24k) + (-64i + 192j + 32k) = -112i + 132j + 56k$$

c) We need the moment of the forces about the axis OD.

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Recall: To compute the moment of a force about a given line or axis:

First: Select a point on the line and compute the moment at that point. (In our problem, you can select O because it is on line OD)

Second: Identify a unit vector parallel to the line or axis.

(In this problem, you can take a unit vector from O to D)

Third: Take dot product of the moment you get in the first step with a unit vector identified in the second step.

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Any way! The basic concept is that you don't need to select a point and compute a moment again because you can use the moment computed in part (b). Look! In part (b), we computed a moment at point O and point O is on line OD (about which you need a moment). So, you need to identify only a unit vector parallel to line OD.

That is $\vec{U}_{\overline{OD}} = \dfrac{3}{5}i + \dfrac{4}{5}k$. Hence, the moment about axis OD is given by

$$\overrightarrow{M}_{OD} = \overrightarrow{M}_O.\vec{U}_{OD} = (-112i + 132j + 56k).(\frac{3}{5}i + \frac{4}{5}k) = -22.4\text{N.m}.$$

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d) The resultant force at A is simply the sum of the two forces. That is $\vec{R}=\vec{T}_{\overrightarrow{AB}}+\vec{T}_{\overrightarrow{AC}}=6i+14j-21k$. Then, the projection of $\vec{R}$ onto line OD is the dot product of $\vec{R}$ with a unit vector parallel to OD. That is $\text{Pr}_{\vec{R}}(\text{on OD})=\vec{R}.\vec{U}_{\overrightarrow{OD}}=-13.2\text{N}$.

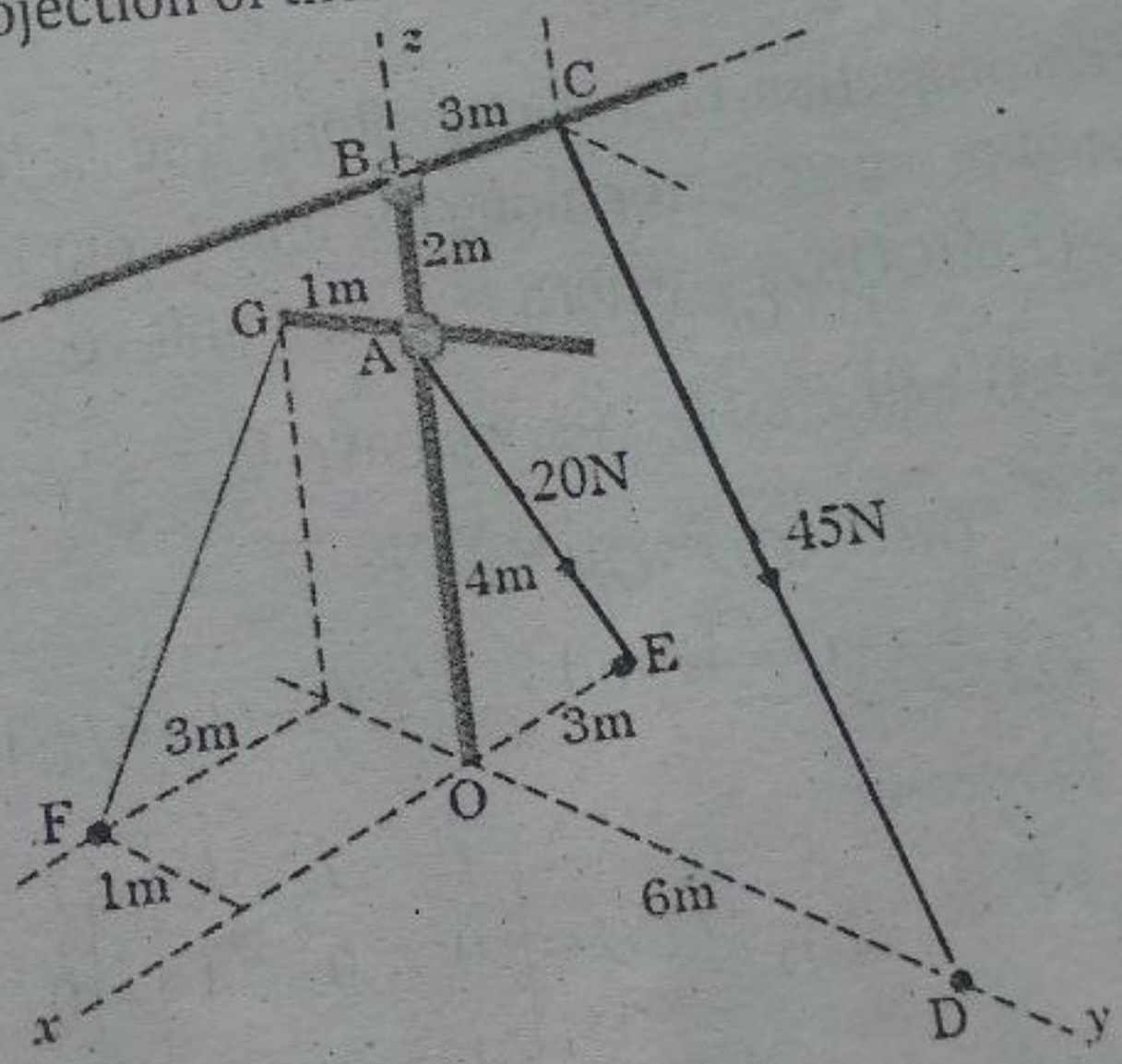
6. The rigid pole and cross-arm assembly is supported by the three cables as shown in the diagram. If the cables CD and AE experience tension forces of 45N and 20N respectively, then,

a) Express the tensions T_{CD} and T_{AE} as a vector using $i-j-k$

b) Determine the projection of T_{CD} onto line GF in vector form.

c) Compute the moment due to the two cable forces at point O

d) Determine the projection of the moment at O along the line AE



Solution:

a) Observe that the vector expression of the tension on CD is simply given by $\vec{T}=T.U_{CD}$ where U_{CD} is a unit vector from C to D. So, the basic task is to find this unit vector first.

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From the diagram, point C is located 3m to the left of the origin along x-axis, 6m above the origin along z-axis. Besides, since it is on the xz-plane, its y-coordinate is zero. Hence, the coordinates of point C is $C=(-3,0,6)$.

Besides, point D is located 6m to the right of the origin on the y-axis. Hence, the coordinates of point D is $D=(0,6,0)$. Then, the directed vector from C to D is $\overrightarrow{CD}=D-C=3i+6j-6k$.

So, the unit vector is $U_{CD}=\frac{1}{9}(3i+6j-6k)=\frac{1}{3}i+\frac{2}{3}j-\frac{2}{3}k$.

Therefore, $\vec{T}=T.U_{CD}=45(\frac{1}{3}i+\frac{2}{3}j-\frac{2}{3}k)=15i+30j-30k$.

Similarly, $\vec{T}AE=T.U_{AE}=20(-\frac{3}{5}i-\frac{4}{5}j)=-12i-16j$.

b) The projection of a vector along line is the *dot product* of the vector with a unit vector along the line. In our case,

$P_{GF}=(T \text{ on } CD).U_{GF}$ where U_{GF} is a unit vector from G to F. From $G=(0,-1,4)$ and $F=(3,-1,0)$, we have $U_{GF}=\frac{3}{5}i-\frac{4}{5}k$.

Therefore, $P_{GF}=T_{CD}.U_{GF}=33\text{N}$.

c) Here, $r_{\overline{OA}}=A-O=4k$ and $r_{\overline{OD}}=D-O=6j$. Therefore,

$$M_O=r_{\overline{OA}}\times\vec{T}AE+r_{\overline{OD}}\times\vec{T}CD=\begin{vmatrix} i & j & k \\ 0 & 0 & 4 \\ -12 & 0 & -16 \end{vmatrix}+\begin{vmatrix} i & j & k \\ 0 & 6 & 0 \\ 15 & 30 & -30 \end{vmatrix}$$

$$=-180i-48j-90k$$

d) The projection of a moment along a line is the dot product of the moment vector by a unit vector along the line. In our case,

P_M on $AE=M_o.U_{AE}$ where U_{AE} is a unit vector from A to E.

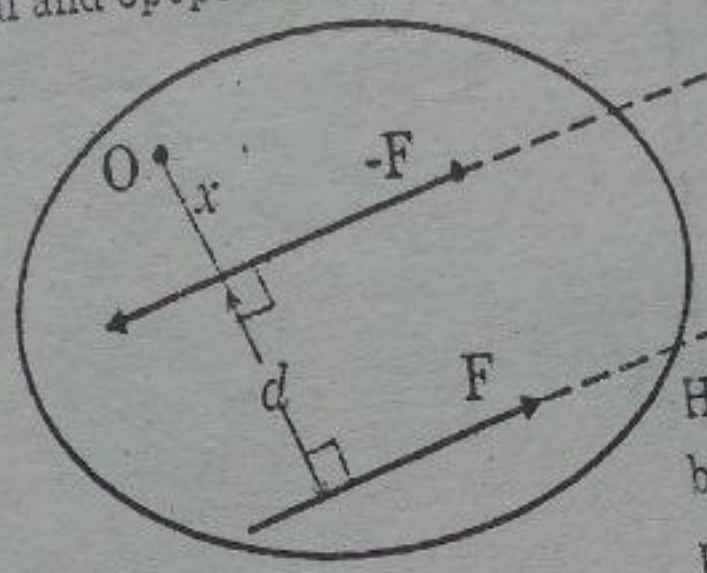
Put $U_{AE}=-\frac{3}{5}i-\frac{4}{5}k$. Therefore, P_M on $AE=M_o.U_{AE}=180\text{N.m}$.

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1.9 Couples and Couple Moments

Couples: What do we mean by a couple?
Two *non-collinear parallel forces* having the same magnitude with opposite direction are called a *couple.* A moment produced by a couple forces is called *couple moment.* In short, a couple moment is simply called a couple. Consider the action of two equal and opposite forces F and $-F$ that are situated at a distance of d apart as shown.

Equal and opoposite forces produce a couple



Here, d is the perpendicular distance between the forces and O is arbitrary point at a distance of x from -F

These two forces cannot be combined into a single force because their sum in in the directions of both forces is zero. As a result, the forces have no translational effect (no movement in the direction of either force). Hence, their effect is to produce a tendency of rotation. That is they simply produce a moment about an axis. Then, the moment of the two forces about an axis normal to their plane and through any point O is what we call a couple M.

Couple moment as free vector: The magnitude of the couple moment in the above diagram is calculated as:

$$M_O = F.(x+d) - F.x = F.x + F.d - F.x = F.d.$$

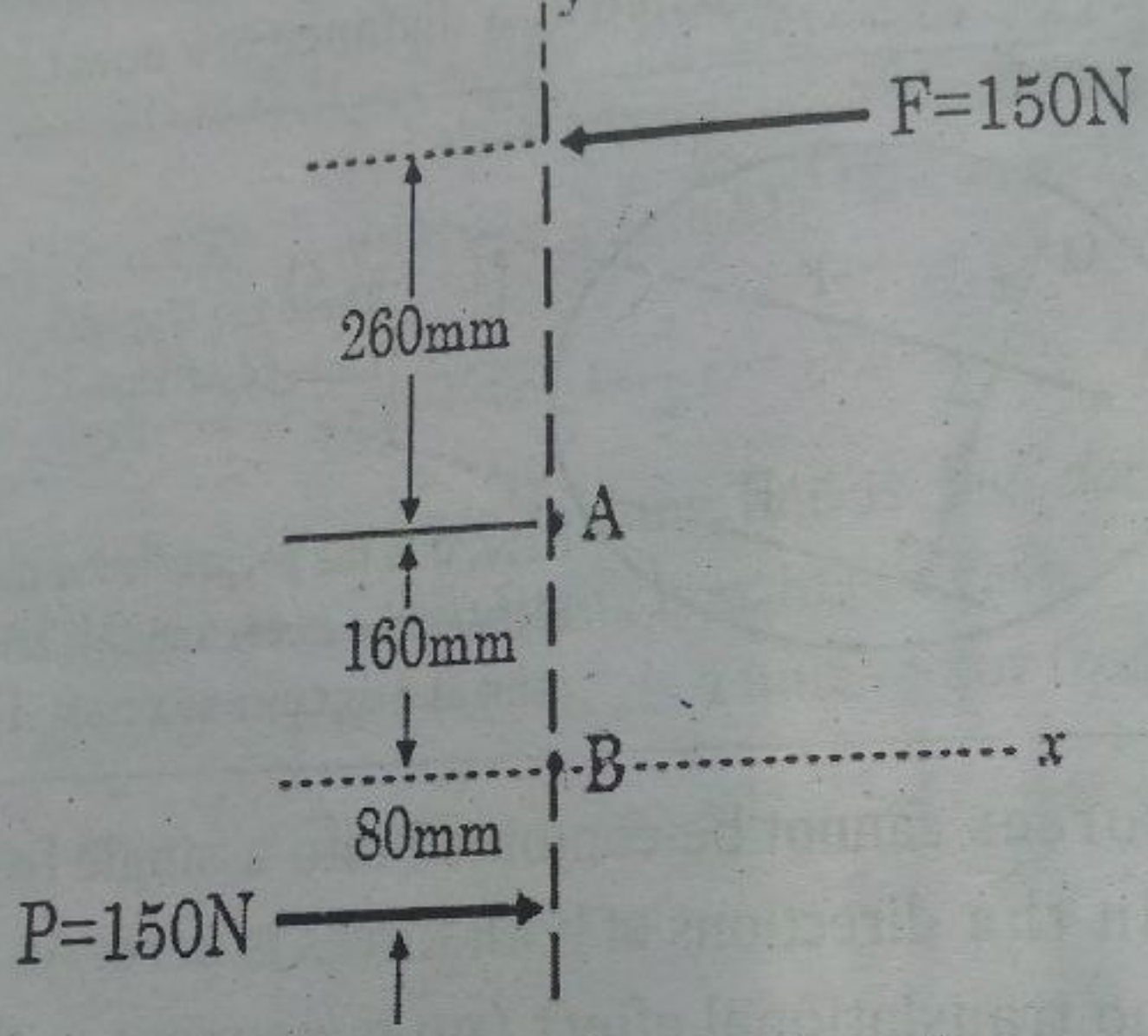
From this result, we can conclude that the magnitude of a couple is independent of the distance of the point from the forces; rather it depends on the distance between the two forces.

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Because of this, a couple moment produced by couple forces has the same value for all moment centers and it can be placed anywhere on the body of the object or at any point in outside the body. As a result, (independence of positions), a couple is categorized as a free vector.

Examples:

1. For the two forces shown in the diagram, find the couple moment at point A and at point B.



Solution: Here, the forces are equal and opposite. So, the moment at any point is a couple exactly equal to the product of the force and the distance between them. From the diagram, the distance between the two forces is $d = 80\text{mm} + 160\text{mm} + 260\text{mm} = 500\text{mm} = 0.5\text{m}$

Therefore, $M_A = M_B = Fd = 150(0.5\text{m}) = 75\text{N.m}$.

Check! You can justify the result or convince yourself by calculating M_A and M_B separately as follow.

a) **About point** A: To find the moments about point A, first find the perpendicular distance of each force from point A.

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As we see from the diagram, F is at a distance of $d_1 = 260\text{mm} = 0.26\text{m}$ and P is at a distance of $d_2 = 80\text{mm} + 160\text{mm} = 240\text{mm} = 0.24\text{m}$.

Therefore, the net moment about point A is

$$M_A = Fd_1 + Pd_2 = 150(0.26\text{m}) + 150(0.24\text{m}) = 75\text{Nm}$$

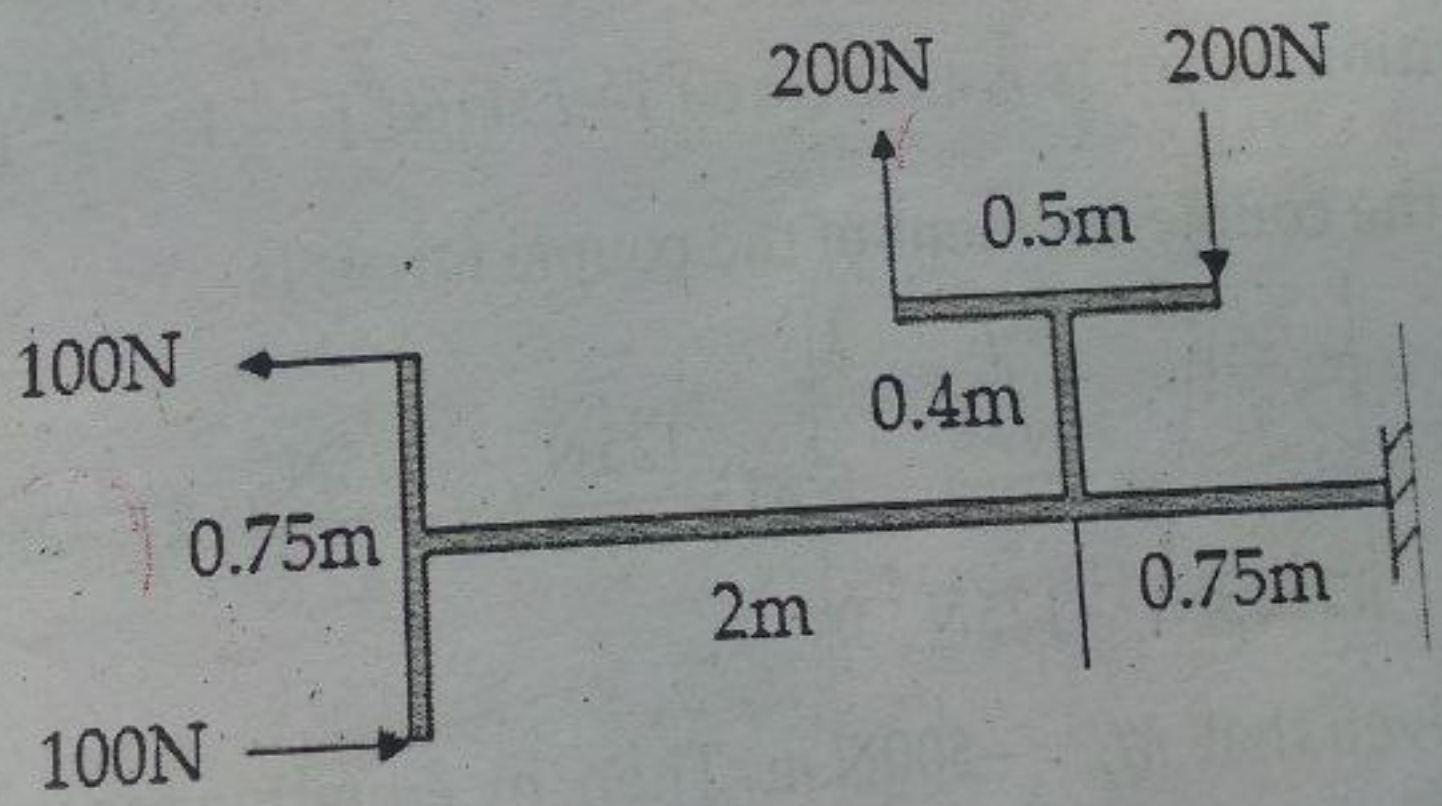
b) **About point** B: As we see from the diagram, F is at a distance of $d_1 = 260\text{mm} + 160\text{mm} = 420\text{mm} = 0.42\text{m}$ and P is at $d_2 = 80\text{mm} = 0.08\text{m}$.

Therefore, $M_B = Fd_1 + Pd_2 = 150(0.42\text{m}) + 150(0.08\text{m}) = 75\text{Nm}$

Question: What do you observe from part a and b ? In both parts, we have got the same couple. This means that a couple depends only on the distance between the pair of forces but it is independent of the moment center (the center selected to compute the couple).

That is why a couple moment is categorized as a free vector.

2. Two couple forces are acting on the cross bar as shown. Find the net couple moment.

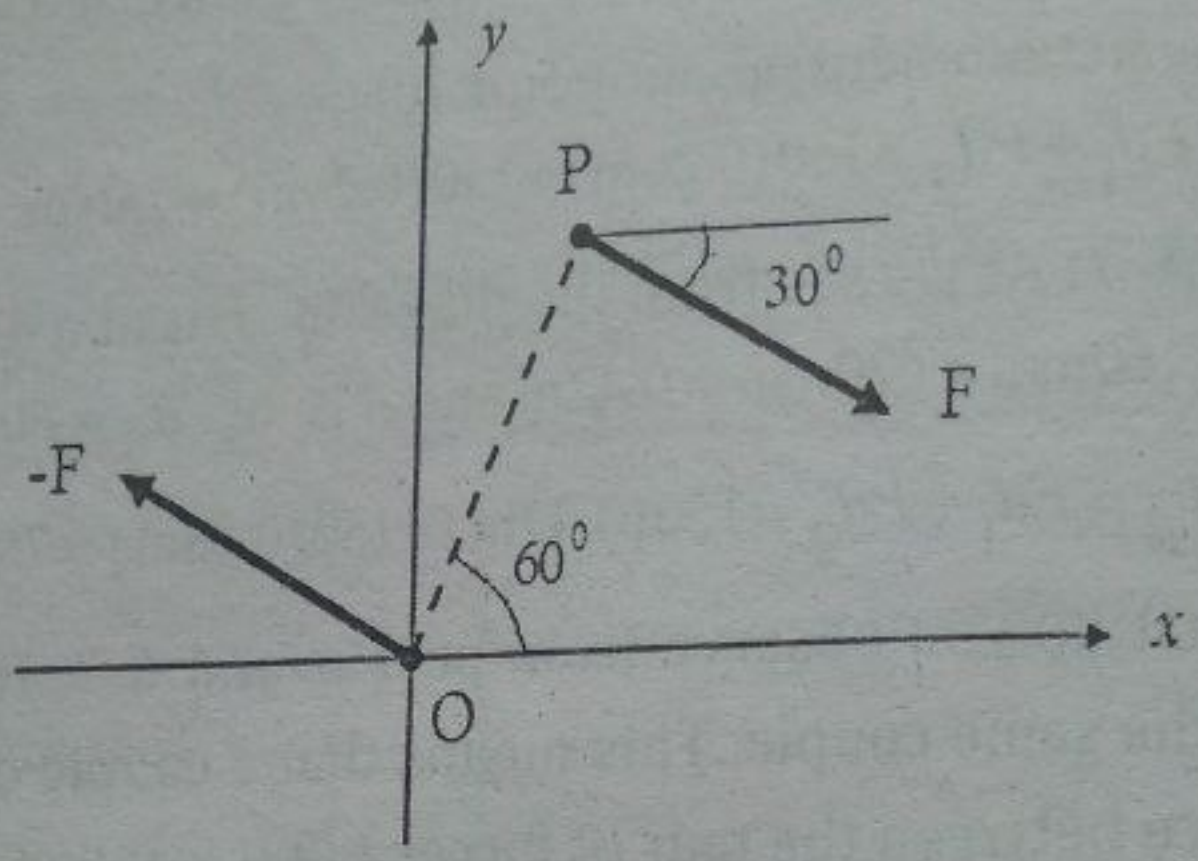


Solution: While the couple on the right produces a clockwise moment which is negative, the couple on the left produces a counterclockwise moment which is positive.

Therefore, $M = 100\text{N}.(0.75\text{m}) - 200\text{N}.(0.5\text{m}) = 75\text{N.m} - 100\text{N.m} = -25\text{N.m}.$

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3. If the couple forces with $F = 250\text{N}$ shown below produce a couple moment of $M_C = -500\text{N.m}$, what are the coordinates of point P?



Solution: First express the couple forces in vector form.

That is $\vec{F} = F\cos 30^0\,\mathbf{i} - F\sin 30^0\,\mathbf{j} = 125\sqrt{3}\text{N}\,\mathbf{i} - 125\text{N}\,\mathbf{j}$.

Now, find the position vector from O to P. Assume the length $\overline{OP} = L$.

Then, the coordinates of P becomes $P = (x, y) = (L\cos 60^0, L\sin 60^0)$.

So, the position vector is $\vec{r}_{\overline{OP}} = L\cos 60^0\,\mathbf{i} + L\sin 60^0\,\mathbf{j} = \frac{L}{2}\mathbf{i} + \frac{\sqrt{3}L}{2}\mathbf{j}$.

Therefore, the couple-moment of the couple forces is

$$\overline{M}_C = \vec{r}_{\overline{OP}} \times \vec{F} = \begin{vmatrix} i & j & k \\ \frac{L}{2} & \frac{\sqrt{3}L}{2} & 0 \\ 125\sqrt{3}\text{N} & -125\text{N} & 0 \end{vmatrix} = (-\frac{125\text{N}}{2}L - \frac{375\text{N}}{2}L).\mathbf{k} = -250\text{N}\,L.\mathbf{k}$$

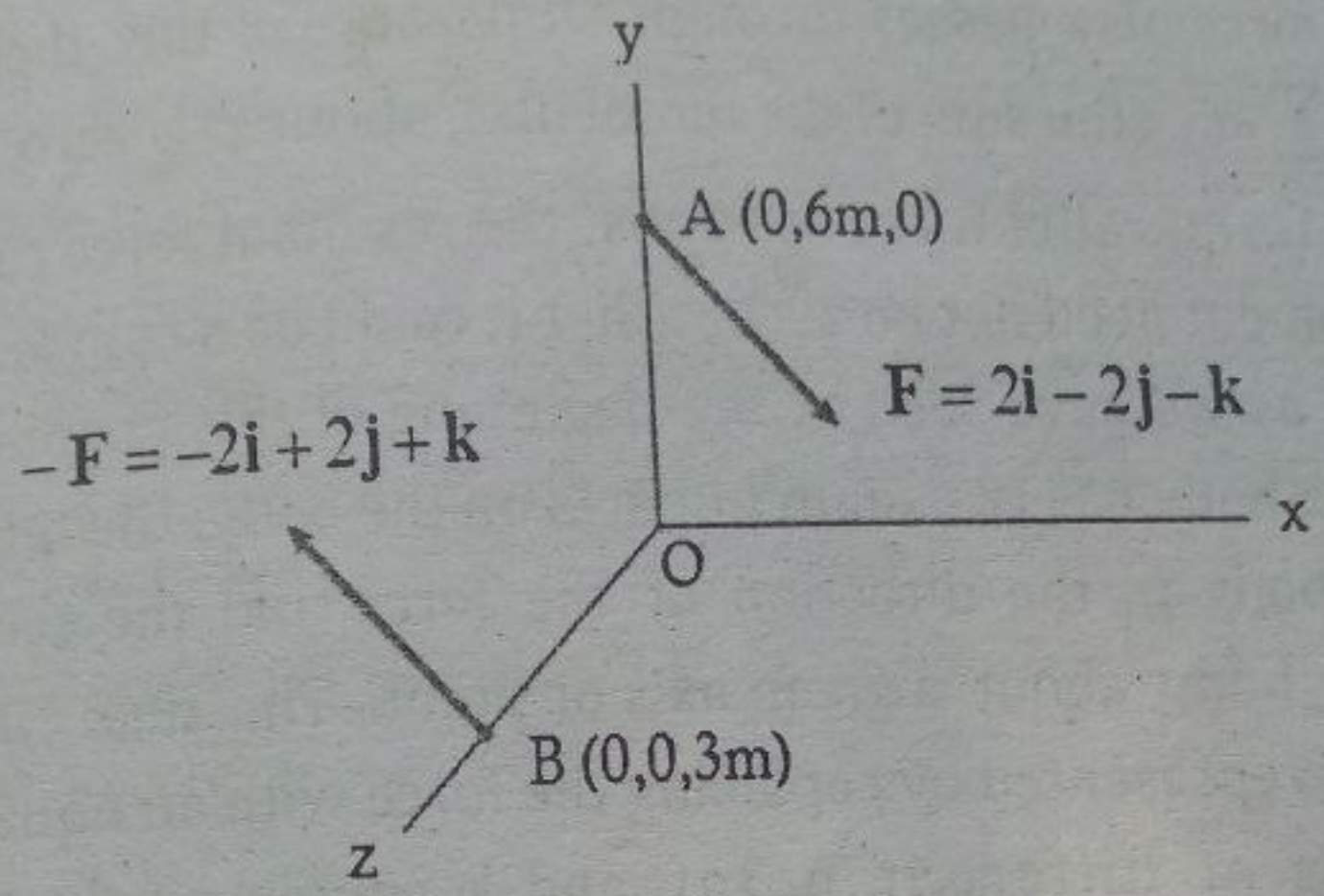
But we are given that $M_C = -500\text{N.m}$. Then, by equating the values,

$M_C = -500\text{N.m} \Rightarrow -250\text{N}\,\text{L} = -500\text{N.m} \Rightarrow 250L = 500\text{m} \Rightarrow L = 2\text{m}$

Therefore, using $\overline{OP} = L = 2\text{m}$ and the angle 60^0, the coordinate of P becomes $P = (x, y) = (L\cos 60^0, L\sin 60^0) = (1\text{m}, \sqrt{3}\text{m})$.

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4. In the diagram below, a couple force system is given in space.
a) What is the couple moment of these couples?
b) Find the perpendicular distance between the lines of action of the two forces



Solution:

a) This problem is simple if we apply the Vector Approach.

That is $\overrightarrow{M} = \vec{r}_{\overrightarrow{BA}} \times \overrightarrow{F} = \begin{vmatrix} i & j & k \\ 0 & 6 & -3 \\ 2 & -2 & -1 \end{vmatrix} = -12\mathbf{i} - 6\mathbf{j} - 12\mathbf{k}$

b) Always if the moment $\overrightarrow{M}$ by a couple forces of magnitude F is known, then the perpendicular distance between the lines of actions of the two forces is given by $d = \dfrac{\left\|\overrightarrow{M}\right\|}{F}$.

In this problem, $d = \dfrac{\left\|\overrightarrow{M}\right\|}{F} = \dfrac{\sqrt{324}}{\sqrt{9}} = \dfrac{18}{3} = 6\text{m}$.

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1.10 Force-Couple System Representations

For a concurrent system of forces where the lines of action of all forces pass through a common point O, the line of action of the resultant force also passes through O. Because of this, the resultant moment, $\sum M_o$ (the sum of the moments), about O is zero.

However, for parallel force systems, the resultant force can be zero but the moment need not be zero rather it could be a couple.

In general, as we have seen at the beginning of this chapter, a force applied on a body has two main impacts on the body: the tendency to move the body in the direction of the force and the tendency to rotate the body about a given axis or point. This dual effect of the force can be represented by replacing the force with an equal parallel force acting at different point and a couple produced by the original force. This equivalent representation or replacement is known as *force-couple system representation.*

Conversely, we can combine a given couple and a force which lies in the plane of the couple to produce a single, equivalent force.

Procedure to produce force-couple systems:

In representing a force system by an equivalent force-couple system, the basic tasks are:

First: Determine the resultant force of the given system

Second: Determine the resultant moment at the required point

Third: Draw a diagram that shows the moment with its direction and the orientation of the resultant force at the new moment center.

Here, you can put the force in two ways. Either by putting the magnitude and direction angle of the force or by putting the force in vector form (In this case, the direction is not a must.)

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Note: Suppose you have got a new force-couple system at A. Again, you may be challenged to determine at what distance d from A or at what point P must the resultant force be placed so as to have the same effect at A as the force-couple representation.
In this case, the distance or the coordinate of a point P is determined from the equation $M_A = F_R.d$ where F_R is the resultant force, A is the new moment center and d is the required distance.

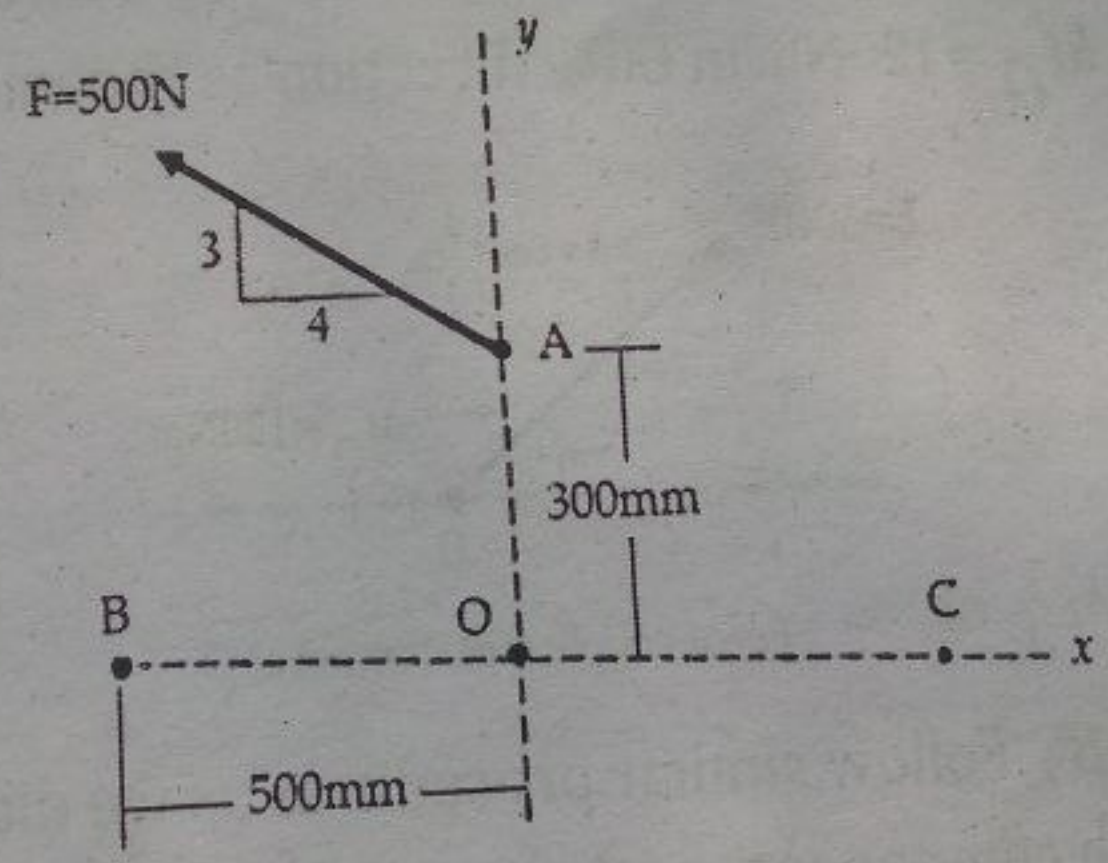
Examples:

1. Given the force $F = 500\text{N}$ acting at A as shown in the diagram.

a) Replace this force by an equivalent force-couple system

i) At point O ii) At point B

b) Find the distance between O and C if the line of action of the force can pass through point C.

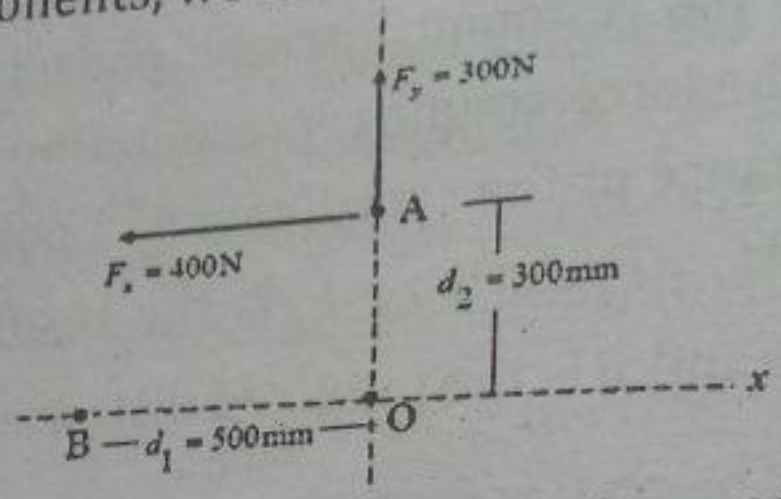


Solution: First: Determine the resultant force of the given system.
Since we are given a single force, the resultant is itself.
Second: Determine the resultant moment at the required point.
First, notice the small triangle is a right triangle where the acute angle near the initial point of the force indicates the inclination angle of the force. This means the given force is acting at A forming an angle of θ with $\tan\theta = \frac{3}{4}$, $\sin\theta = \frac{3}{5}$, $\cos\theta = \frac{4}{5}$.

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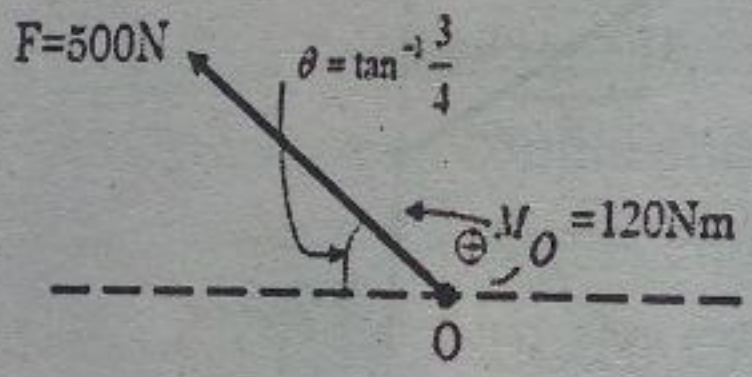
So, $F_x = F\cos\theta = -400\text{N}$, $F_y = F\sin\theta = 300\text{N}$.

Using these components, we have the following diagram.



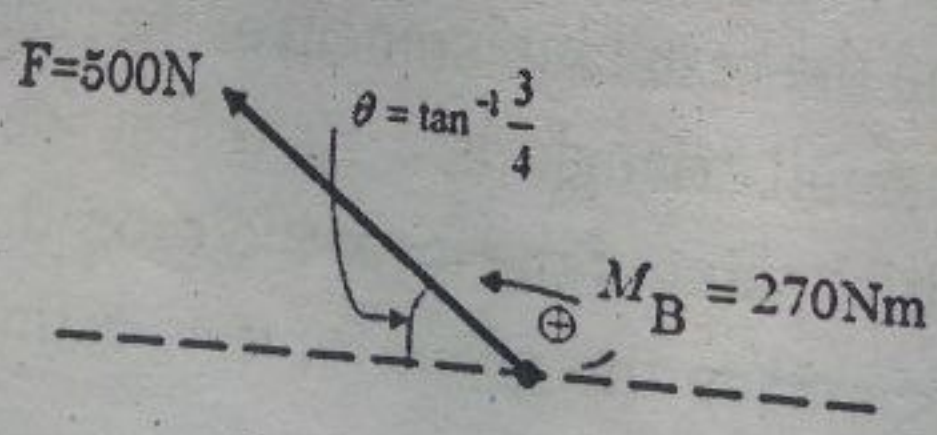
a) About point O: Since the line of action of $F_y = 300\text{N}$ passes through O, it does not produce a moment. Hence, $M_O = F_x d_2 = 120\text{Nm}$.

Third: Draw a diagram that shows the moment with its direction and the orientation of the resultant force at the new moment center. Therefore, at point O the system is replaced by a force $F = 500\text{N}$ and a moment of $M_O = 120\text{Nm}$ in CCW direction as shown below.



Force-couple replacement at O

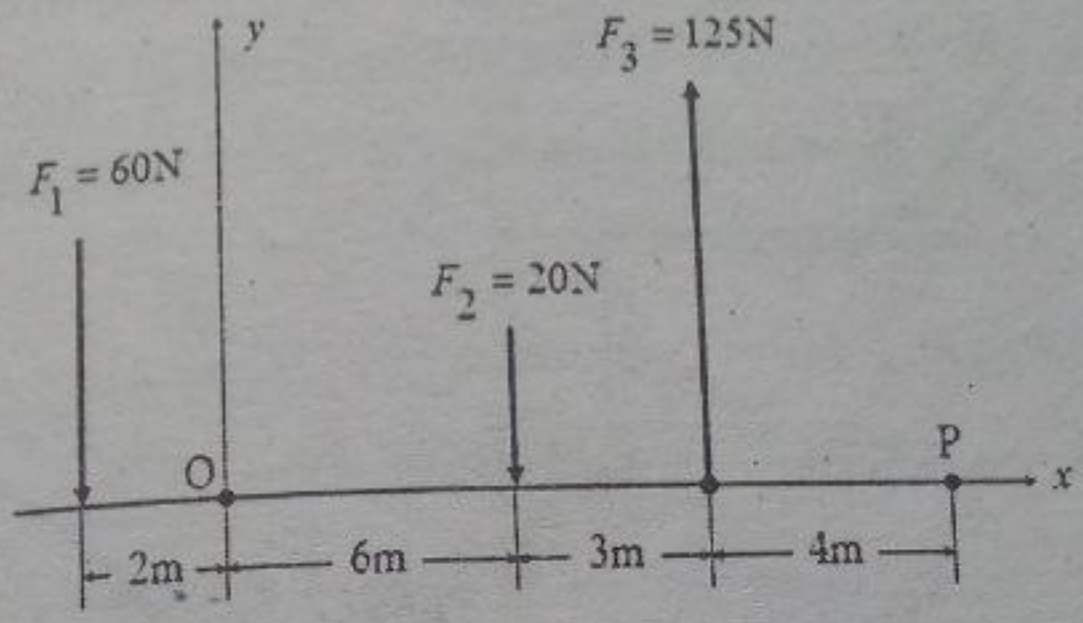
b) About point B: Follow similar procedures as we did above. In this case, both components of the force produce a moment. Hence, $M_B = F_x d_2 + F_y d_1 = 400(0.3\text{m}) + 300(0.5\text{m}) = 270\text{Nm}$



Force-couple system at B

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2. Replace the force system by a force-couple system at P and give the point of application A of the resultant force along x-axis as measured from P. Justify whether A is to the left or to the right of P.



Solution:

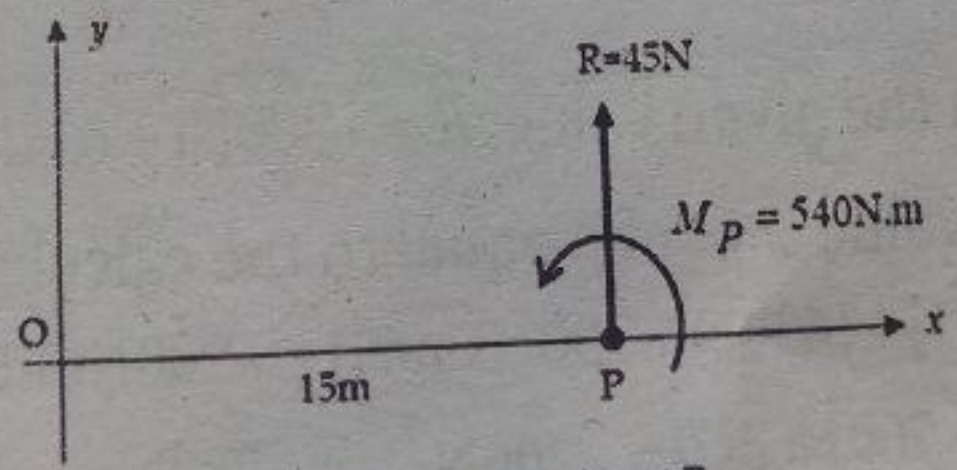
First: Determine the resultant force of the given system

Since all the forces are vertical (along y-axis), the resultant force is $\vec{R} = F_3 - F_1 - F_2 = 125N - 80N = 45N$. That is $\vec{R} = 45N\,\mathbf{j}$.

Second: Determine the resultant moment at point P.

$$M_P = F_1.(2m + 6m + 3m + 4m) + F_2.(3m + 4m) - F_3.(4m)$$
$$= 60N.(15m) + 20N.(7m) - 125N.(4m)$$
$$= 900N.m + 140N.m - 500N.m = 540N.m$$

Third: Draw a diagram that shows the moment with its direction and the orientation of the resultant force at the moment center P.



Force-couple representation at P

For the last part of the problem, we need to find a distance d such that $M_P = R.d$. Therefore, we have $M_P = R.d \Rightarrow 45d = 540 \Rightarrow d = 12$.

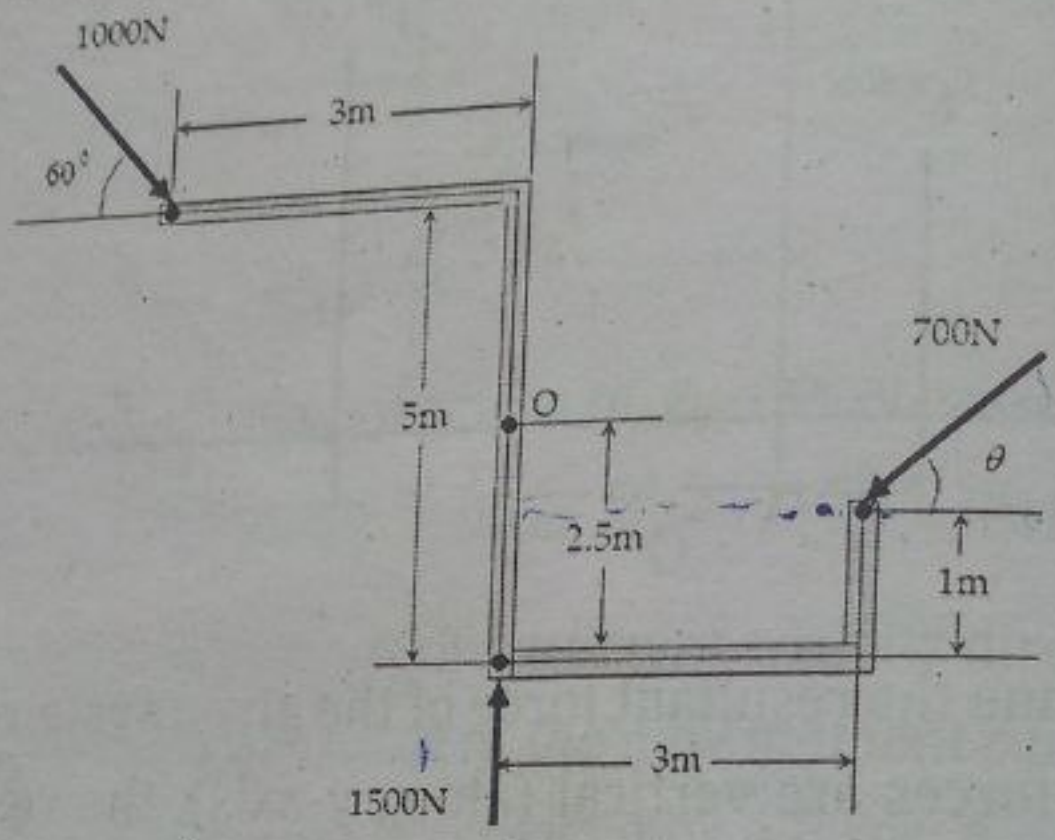
Observe! the point of application A must be $12m$ to the right of P.

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3*. Consider the general force system acting on a interconnected bar.

a) If the resultant force of the system is $\vec{R} = -200\text{Ni} + 634\text{Nj}$, find θ.

b) Using $\theta = 45^0$, replace the system by equivalent force-couple at 0



Solution: Let's denote the forces $F_1 = 1000\text{N}, F_2 = 1500\text{N}, F_3 = 700\text{N}$.

First: Find the resultant force $\vec{R}$. The resultant is $\vec{R} = R_x\mathbf{i} + R_y\mathbf{j}$ where

i) $R_x = 1000\text{N}\cos 60^0 - 700\text{N}\cos\theta = 500\text{N} - 700\text{N}\cos\theta$

ii) $R_y = -1000\text{N}\sin 60^0 + 1500\text{N} - 700\text{N}\sin\theta = 634\text{N} - 700\text{N}\sin\theta$

Thus, the resultant force in terms of unknown angle θ is

$\vec{R} = R_x\mathbf{i} + R_y\mathbf{j} = (500\text{N} - 700\text{N}\cos\theta)\mathbf{i} + (634\text{N} - 700\text{N}\cos\theta)\mathbf{j}$.

Now, equate with the given value $\vec{R} = -200\text{Ni} + 634\text{Nj}$.

That is the given value must be equal to the calculated value:

$$(500\text{N} - 700\text{N}\cos\theta)\mathbf{j} + (634\text{N} - 700\text{N}\sin\theta)\mathbf{j} = -200\text{Ni} + 634\text{Nj}$$

$$\Rightarrow \begin{cases} 500\text{N} - 700\text{N}\cos\theta = -200\text{N} \\ 634\text{N} - 700\text{N}\sin\theta = 634\text{N} \end{cases} \Rightarrow \begin{cases} -700\text{N}\cos\theta = -700\text{N} \\ -700\text{N}\sin\theta = 0 \end{cases}$$

$$\Rightarrow \begin{cases} \cos\theta = 1 \\ \sin\theta = 0 \end{cases} \Rightarrow \begin{cases} \theta = \cos^{-1}(1) = 0^0 \\ \theta = \sin^{-1}(0) = 0^0 \end{cases}$$

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Therefore, the three given forces will have a resultant force of $\vec{R} = -200\text{Ni} + 634\text{Nj}$ only if $\theta = 0^{\circ}$.

b) Given $\theta = 45^{0}$. We need the equivalent force-couple system at O.

First: Find the sum of the moments at O due to the three forces.

By assuming counterclockwise moment as positive,

$$M_o = -F_{1x}.(2.5\text{m}) + F_{1y}.(3\text{m}) - F_{3x}.(1.5\text{m}) - F_{3y}.(3\text{m})$$

$$= -500\text{N}.(2.5\text{m}) + 866\text{N}.(4\text{m}) - 494.9\text{N}.(1.5\text{m}) - 494.9\text{N}.(3\text{m})$$

$$= -1250\text{N.m} + 2598\text{N.m} - 742.35\text{N.m} - 1484.7\text{N.m} = -879.05\text{N.m}$$

Second: Find the resultant force $\vec{R}$.

The resultant force is of the form $\vec{R} = R_x\text{i} + R_y\text{j}$ where

i) $R_x = F_{1x} + F_{2x} + F_{3x} = 1000\text{N}\cos 60^{0} - 700\text{N}\cos 45^{0} = 5.1\text{N}$

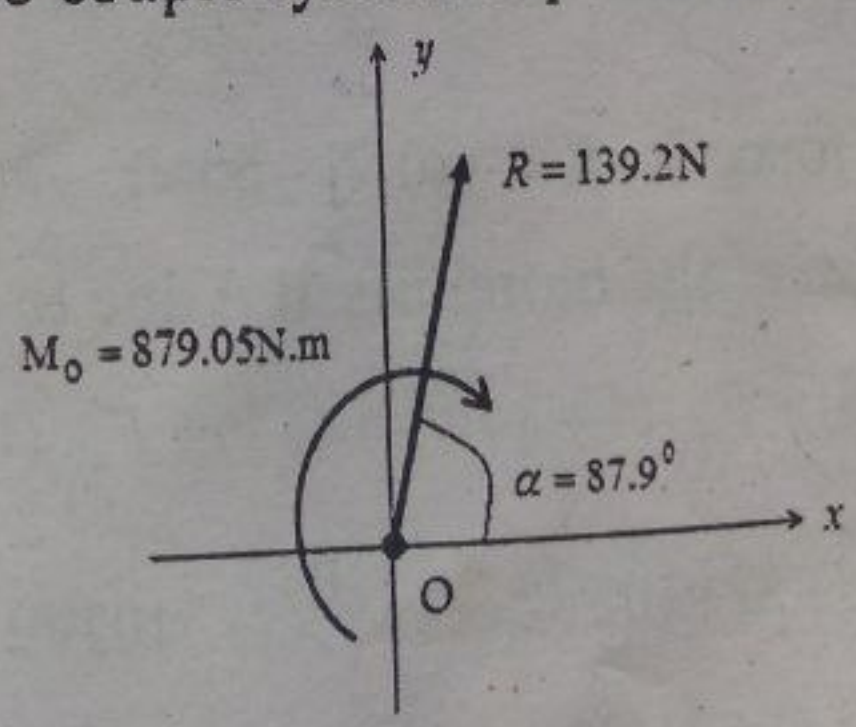
ii) $R_y = F_{1y} + F_{2y} + F_{3y} = -1000\text{N}\sin 60^{0} + 1500\text{N} - 700\text{N}\sin 45^{0} = 139.1\text{N}$

Thus, the resultant force is $\vec{R} = R_x i + R_y j = 5.1\text{Ni} + 139.1\text{j}$.

Direction: The direction of the resultant force is determine by finding the measure of the angle it makes with the positive x-axis.

That is $\tan\theta = \dfrac{R_y}{R_x} = \dfrac{139.1}{5.1} = 87.9^{0}$. Since both components are positive, it is located in the first quadrant.

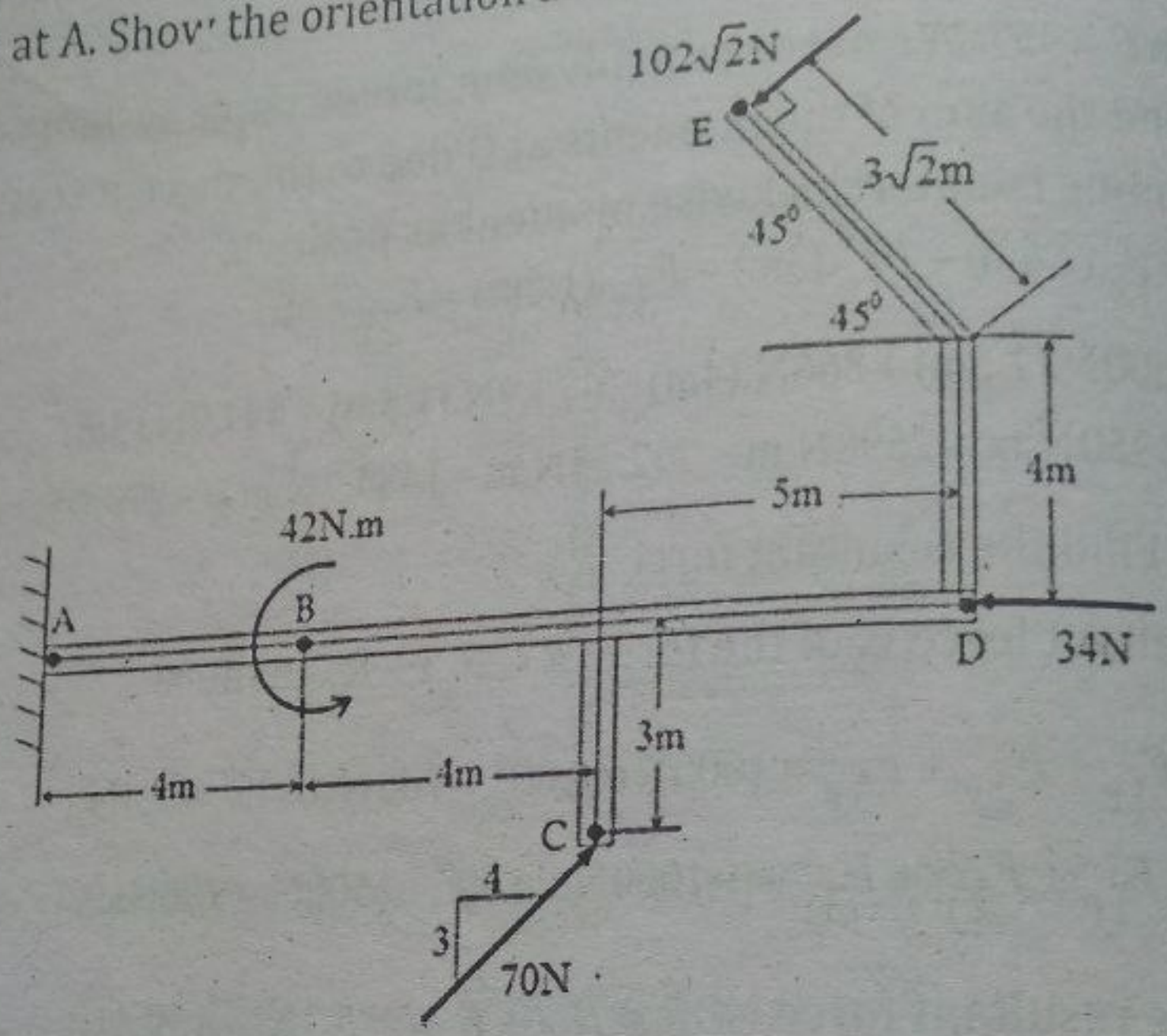
Thus, here is the force-couple system representation diagram.



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4*. Replace the general force system by an equivalent force-couple system at A. Show the orientation and directions diagrammatically.



Solution: Let's denote the three forces and the couple as follow:

$F_1 = 70\text{N}, F_2 = 34\text{N}, F_3 = 102\sqrt{2}\text{N}$ and the couple $M_c = 42\text{N.m}$.

First: Find the resultant force in the form $\vec{R} = R_x\mathbf{i} + R_y\mathbf{j}$.

i) $R_x = F_{1x} - F_2 - F_{3x} = 70\text{N}(\frac{4}{5}) - 34\text{N} - 102\sqrt{2}\text{N}\cos 45^0 = -80\text{N}$

ii) $R_y = F_{1y} - F_{3y} = 70\text{N}(\frac{3}{5}) - 102\sqrt{2}\text{N}\sin 45^0 = -60\text{N}$

Thus, the resultant force is $\vec{R} = -80\text{N}\mathbf{j} - 60\text{N}\mathbf{j}$.

Second: Find the sum of the moments at A due to the three forces.

$$M_A = F_{1x}(3) + F_{1y}(8) + F_{3x}(4 + 3\sqrt{2}\sin 45^0) - F_{3y}(13 - 3\sqrt{2}\cos 45^0)$$

$$= 70(\frac{4}{5})(3) + 70(\frac{3}{5})(8) + 102\sqrt{2}\cos 45^0(7) - 102\sqrt{2}\cos 45^0(10)$$

$$= 168\text{N.m} + 336\text{N.m} + 714\text{N.m} - 1020\text{N.m} = 198\text{N.m}$$

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Besides, the sign of the given couple $M_C = 42\text{N.m}$ is taken to be positive since its arrow shows counterclockwise direction.

Therefore, the resultant moment at A is the sum of the moments $M_A = 198\text{N.m}$ and the given couple $M_C = 42\text{N.m}$.

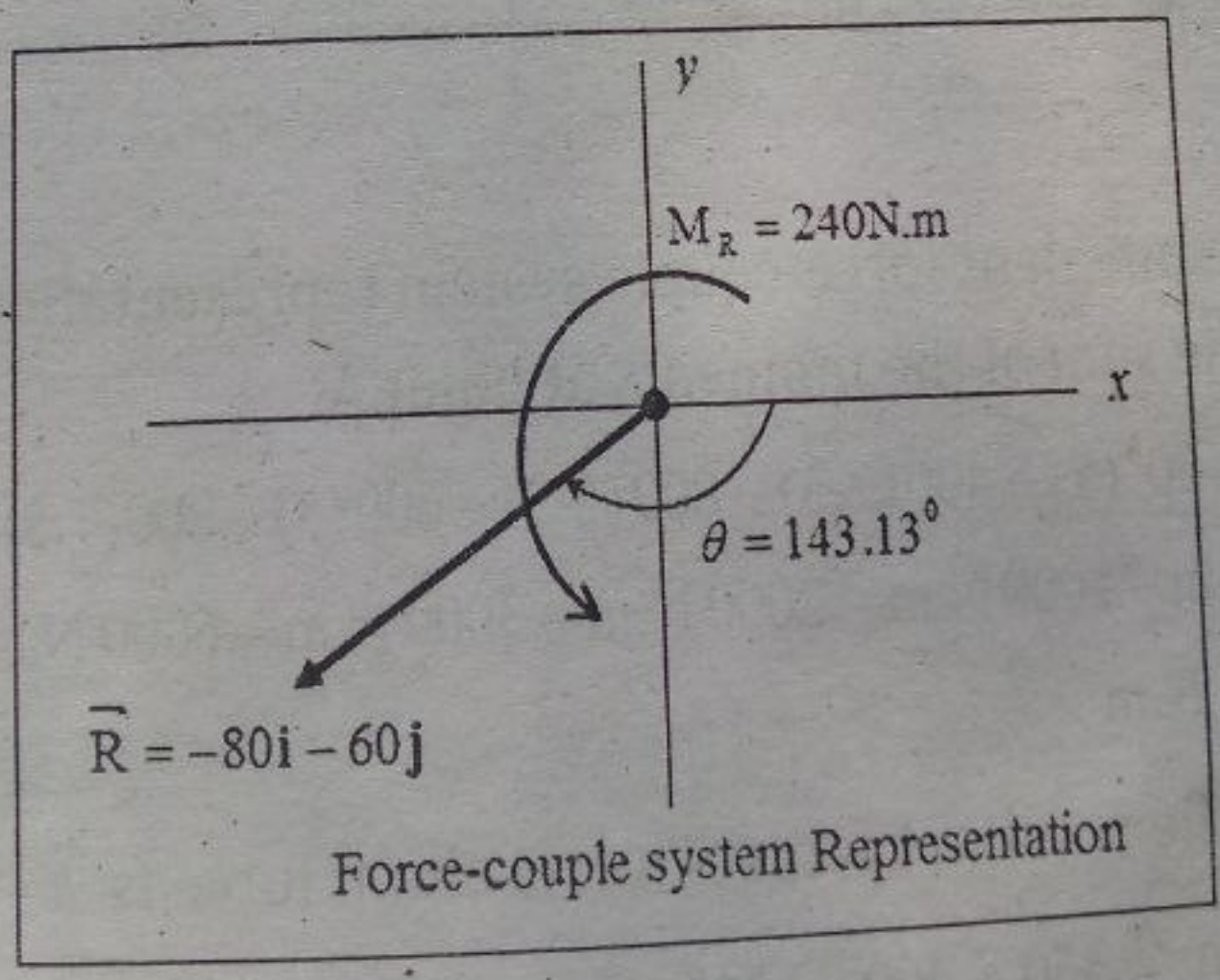
That is $M_R = M_A + M_C = 198\text{N.m} + 42\text{N.m} = 240\text{N.m}$.

Orientation and Directions: For the moment, the direction is determined from the sign of the resultant moment. Since the resultant is positive, it is simply indicated by an arrow pointing counterclockwise. For the force, the direction is determine by finding the measure of the angle it makes with the positive x-axis. It is obtained as $\tan\theta = \frac{-60}{-80} = \frac{3}{4} = 0.75$. Since both components are negative, it is located in the third quadrant.

So, $\tan\theta = 0.75 \Rightarrow \theta = 180^0 + \tan^{-1}(0.75) = 180^0 - 36.87^0 = 216.87^0$.

Or it can be $\theta = 143.13^0$ measured in clockwise direction.

Thus, here is the force-couple system representation diagram.



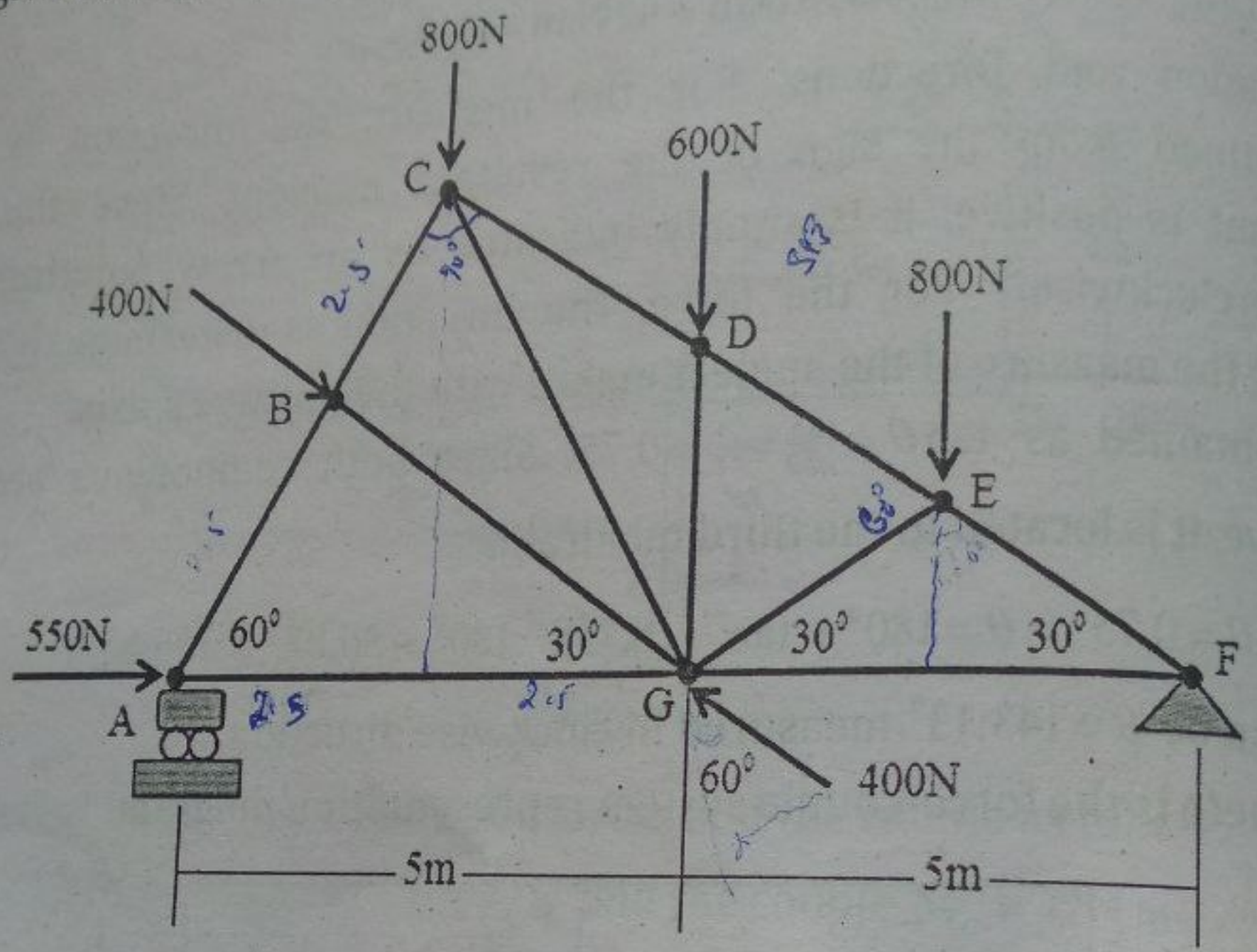
Force-couple system Representation

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5. The roof truss structure is subjected to six forces as shown.

a) Determine the equivalent force-couple system at A

b) Find the equation of the line of action of the resultant force $\overline{R}$ and give its x and y intercepts (points where it crosses the x and y axes)



Solution:

a) We need equivalent force-couple system representation at A:

First: Find the sum of the moments at point A.

$$\overline{M}_A = 400\cos 60^0(5) - 400(2.5) - 800(2.5) - 600(5) - 800(7.5)$$
$$= 1000\,\text{N.m} - 1000\,\text{N.m} - 2000\,\text{N.m} - 3000\,\text{N.m} - 6000\,\text{N.m}$$
$$= -11000\,\text{N.m}$$

Second: Find the resultant force. The resultant force is $\vec{R} = R_x\mathbf{i} + R_y\mathbf{j}$.

$$\text{But } \begin{cases} R_x = \sum F_x = 550 + 400\cos 30^0 - 400\sin 60^0 = 550\,\text{N} \\ R_y = -400\sin 30^0 + 400\cos 60^0 - 800 - 600 - 800 = -2200\,\text{N} \end{cases}$$

Hence, the resultant force becomes $\vec{R} = 550\text{N}\mathbf{i} - 2200\text{N}\mathbf{j}$.

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b) Equation of the line of action of the resultant force:

Let $P(x,y)$ be any point on the line of action of the resultant force. Then, the moment at A from any point $P(x,y)$ on the line of action of the resultant force is given by the vector approach as;

$$\vec{M}_P = \vec{r}_{\overrightarrow{AP}} \times \vec{R} = \begin{vmatrix} i & j & k \\ x & y & 0 \\ 550\text{N} & -2200\text{N} & 0 \end{vmatrix} = (-2200\text{N}x - 550\text{N}y)\mathbf{k}$$

But by applying the principle of moment, we have

$$-2200\text{N}x - 550\text{N}y = -11000\text{N.m}$$

$$\Rightarrow 2200x + 550y = 11000\text{m}$$

$$\Rightarrow 4x + y = 20\text{m}$$

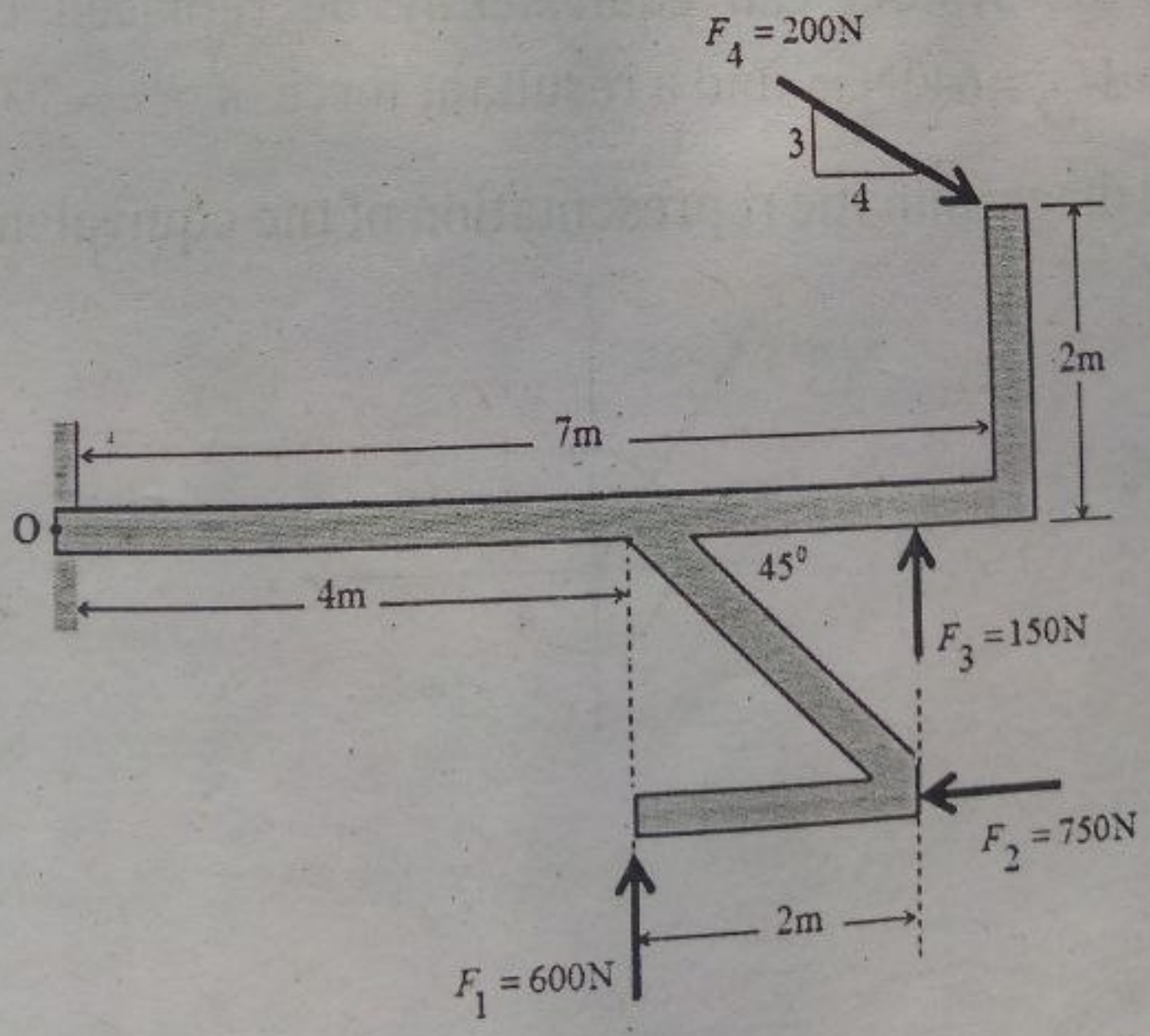
Thus, the equation of the line of action $\vec{R}$ is $4x + y = 20\text{m}$.

x-intercept: Setting $y = 0$, $4x + y = 20\text{m} \Rightarrow 4x = 20\text{m} \Rightarrow x = 5\text{m}$.

y-intercept: Setting $x = 0$, $4x + y = 20\text{m} \Rightarrow y = 20\text{m}$.

Hence, it crosses the x-axis at (5m,0) and the y-axis at (0,20m).

6. Replace the general force system below by an equivalent force-couple system at O and show the representation using diagram.



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Solution: *What are the basic tasks in replacing a given force system by a couple-force system?* (If you don't know the answer to this conceptual question, trying to solve the above problem is like trying to swim in Lake Tana without any skill of swimming.

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Important Tip: To replace a given force system by equivalent force-couple system, we have two basic tasks: To determine the resultant moment at the required point and to find the resultant force. Ok!

By considering CCW moments as positive, we have

$$M_O = F_1.(4\text{m}) - F_2.(2\tan 45^0) + F_3.(6\text{m}) - F_{4x}.(2\text{m}) - F_{4y}.(7\text{m})$$

$$= 600.(4) - 750.(2) + 150.(6) - \tfrac{4}{5}(200).(2) - \tfrac{3}{5}(200).(7)$$

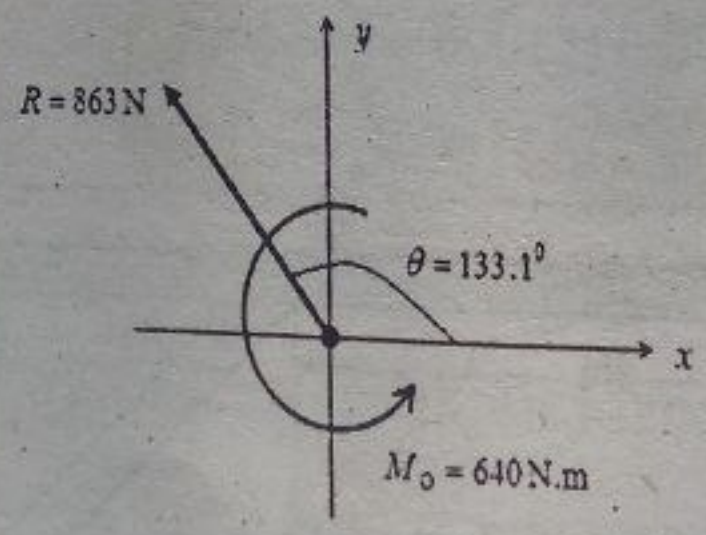
$$= 2400\text{N.m} - 1500\text{N.m} + 900\text{N.m} - 320\text{N.m} - 840\text{N.m} = 640\text{N.m}$$

Besides, the resultant force is of the form $R = R_x\mathbf{i} + R_y\mathbf{j}$.

$$\text{But} \begin{cases} R_x = \sum F_x = F_{4x} - F_2 = \tfrac{4}{5}(200\text{N}) - 750\text{N} = -590\text{N} \\ R_y = \sum F_y = F_1 + F_3 - F_{4y} = 600 + 150 - \dfrac{3}{5}(200) = 630\text{N} \end{cases}$$

Therefore, the system can equivalently be replaced by a couple moment of $M_O = 640\text{N.m}$ and a resultant force of $R = -590\text{Ni} + 630\text{Nj}$.

Here, is the diagrammatic representation of the equivalent system.

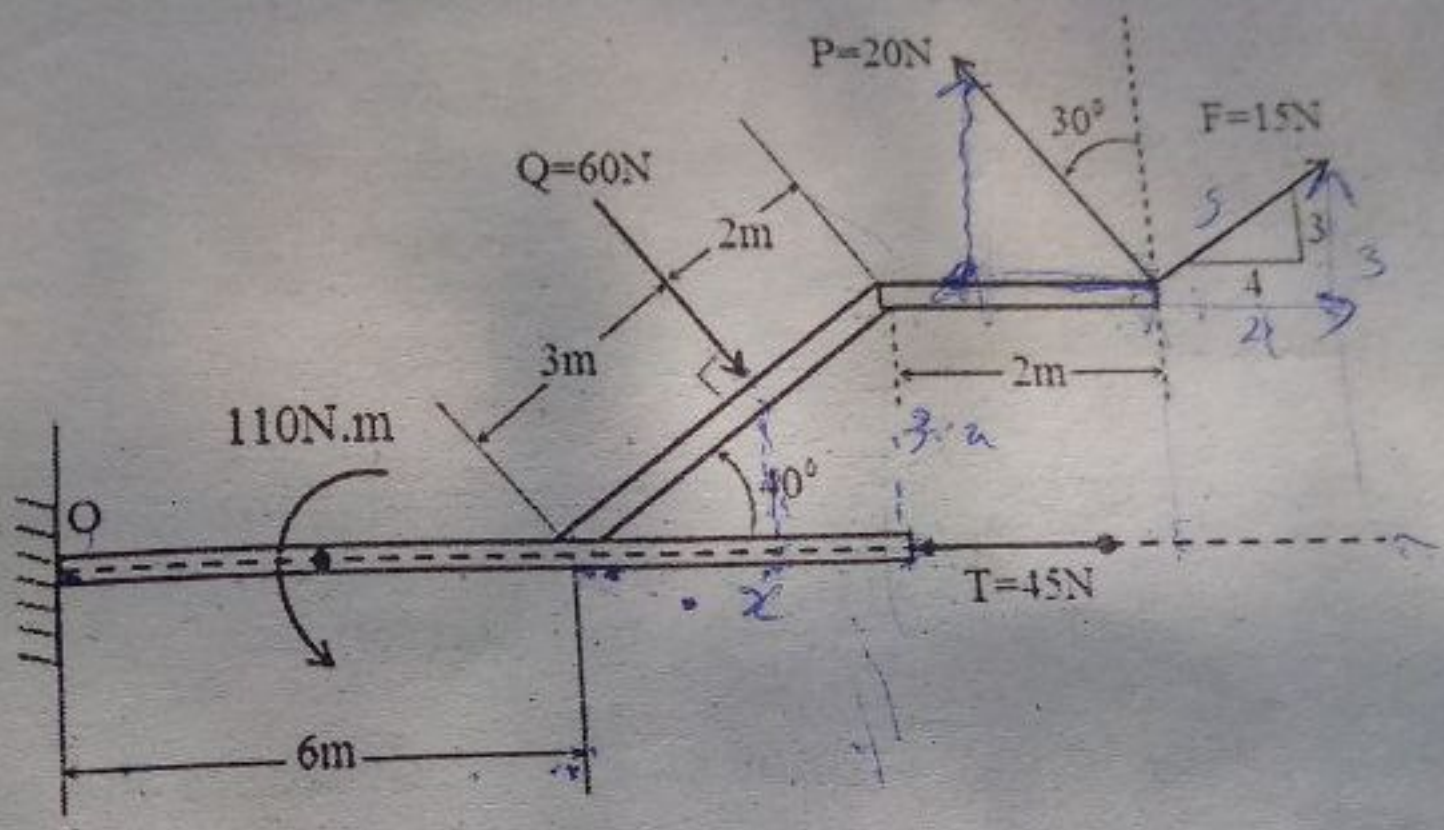


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7*. Consider the general force system acting as shown.

a) Replace the force-system by equivalent force-couple system at O

b) Give the intercept where the resultant force crosses the y-axis



Solution:

a) First, find the moment of each force at O. Then, add the results.

i) Moment by P :

$$M_1 = P_x.(5m\sin 40^0) + P_y.(2m + 5m\cos 40^0 + 6m)$$

$$= 20\sin 30^0(5\sin 40^0) + 20\cos 30(2 + 5\cos 40^0 + 6)$$

$$= 32.1\text{N.m} + 204.9\text{N.m} = 237\text{N.m}$$

ii) Moment by F :

$$M_2 = F_y.(2 + 5\cos 40^0 + 6) - F_x.(5\sin 40^0)$$

$$= 20(\tfrac{3}{5}).(11.83) - 20(\tfrac{4}{5}).(3.21) = 141.96 - 51.34 = 90.6\text{N.m}$$

iii) Moment by Q :

$$M_3 = -Q_x.(3\sin 40^0) - Q_y.(6 + 3\cos 40^0)$$

$$= -60\sin 50^0.(3\sin 40^0) - 60\sin 50^0.(6 + 3\cos 40^0 = -470\text{N.m}$$

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Finally, the net moment is the sum of these three moments and the given couple moment.

That is $M_o = M_c + M_1 + M_2 + M_3$

$= 110\text{N.m} + 237\text{N.m} + 90.6\text{N.m} - 470.$

$= -32.4\text{N.m}$

Besides, the resultant force is obtained as follow:

$$\begin{cases} R_x = Q_x + F_x - P_x - T = 46 + 16\text{N} - 10\text{N} - 40\text{N} = 12\text{N} \\ R_y = F_y + P_x - Q_x = 12\text{N} + 17.3\text{N} - 38.6\text{N} = -9.3\text{N} \end{cases}$$

So, the resultant force is $\mathbf{R} = 12\text{Ni} - 9.3\text{Nj}$. Its magnitude is $R = 15.2\text{N}$

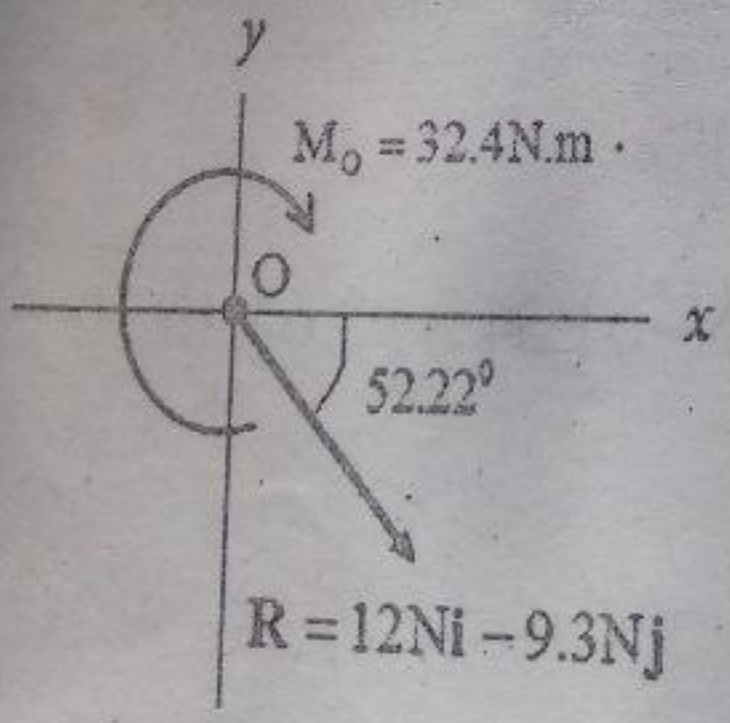
and its direction is $\theta = \tan^{-1}(-\frac{12}{9.3}) = \tan^{-1}(-1.29) = -52.22^0$.

b) Now, we need to find a point on the y-axis where the resultant force passes such that the resultant produces $M_o = -32.4\text{N.m}$.

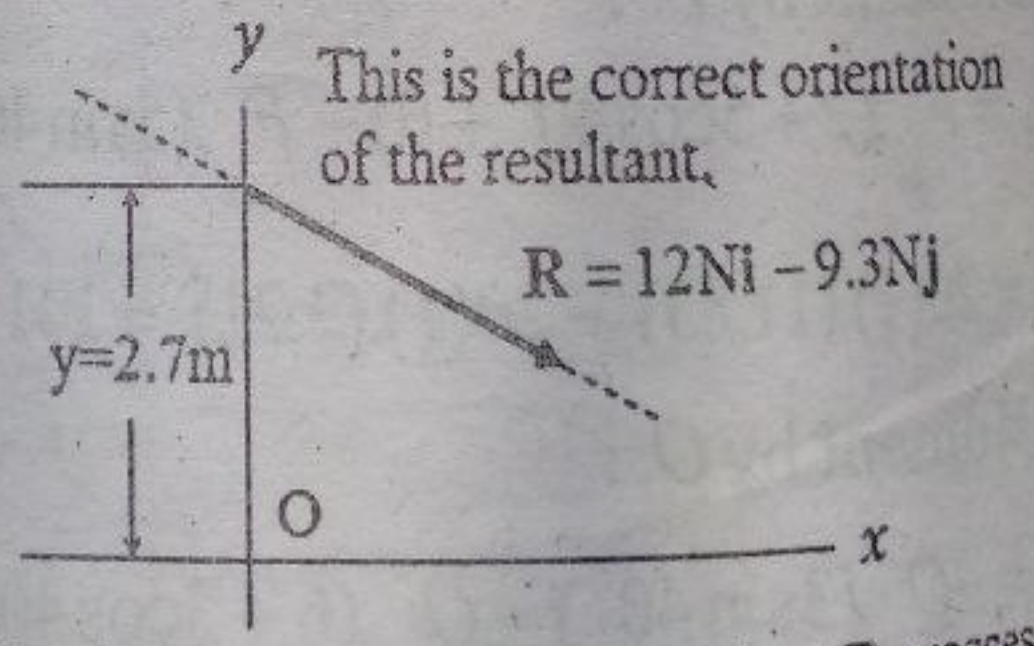
But at any point on y-axis, it is only R_x that produces moment at 0.

Hence, $|R_x|.y = |M_o| \Rightarrow 12y = |-32.4| \Rightarrow 12y = 32.4 \Rightarrow y = 2.7\text{m}$

Conceptual Question: Does the resultant force cross the y-axis above O or below O? Since the resultant moment is negative and the components of the resultant force are also known, it crosses the y-axis at a distance of 2.7m above O with direction as shown below.



a) Force-Couple Representation at O

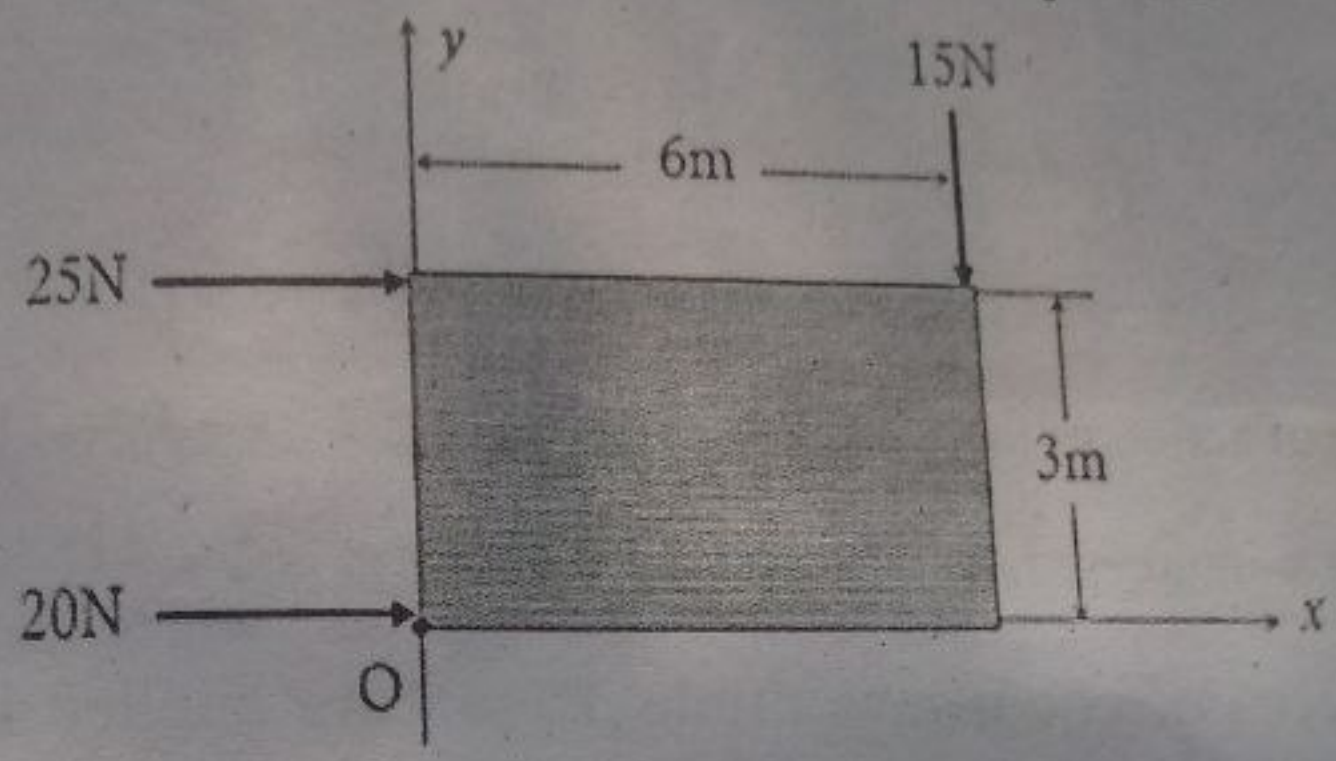


b) The intercept where the resultant R crosses the y-axis so that it gives the same moment at O as the original forces

8. Three forces are applied in a rectangular plate as shown.

a) Replace the forces by a force-couple system at the origin O

b) Determine the equation of the line of action of the resultant force and give the intercepts where it crosses the x and y axes.



Solution:

a) To replace by a force-couple system;

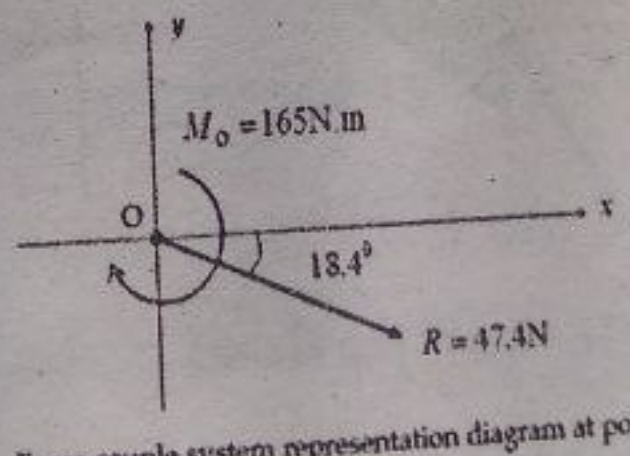
First: Find the resultant force. Here, the resultant is $\vec{R} = 45\mathbf{i} - 15\mathbf{j}$.

Besides, its direction angle θ as measured from the positive x-axis is obtained as $\theta = \tan^{-1}(\frac{Ry}{Rx}) = \tan^{-1}(-\frac{15}{45}) = \tan^{-1}(-\frac{1}{3}) = -18.4^0$.

Second: Compute the resultant moment.

That is $M_o = -25\text{N}(3\text{m}) - 15\text{N}(6\text{m}) = -75\text{N.m} - 90\text{N.m} = -165\text{N.m}$

Third: Draw a diagram that shows the orientations.



Force-couple system representation diagram at point O

b) **Technique to determine equation of the line of action:**

Step-1: Find the resultant force

Step-2: Determine the resultant moment at the origin O

Step-3: Obtain x and y-intercepts of the line of action of the resultant

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Then, write equation of the line using the two points in the 3rd step.

Let $P(x,y)$ be any point on the line of action. Then, the position vector from O to $P(x,y)$ is $\vec{r}_{\overrightarrow{OP}}$. Then, using Vector Approach, the resultant moment is

$$M_O = \vec{r}_{\overrightarrow{OP}} \times \vec{R} = \begin{vmatrix} i & j & k \\ x & y & 0 \\ 45 & -15 & 0 \end{vmatrix} = (-15x - 45y)\mathbf{k}.$$

But $M_O = -165 \Rightarrow (-15x - 45y)k = -165 \Rightarrow x + 3y = 11$

So, the line of action of the resultant force is along $L: x + 3y = 11$.

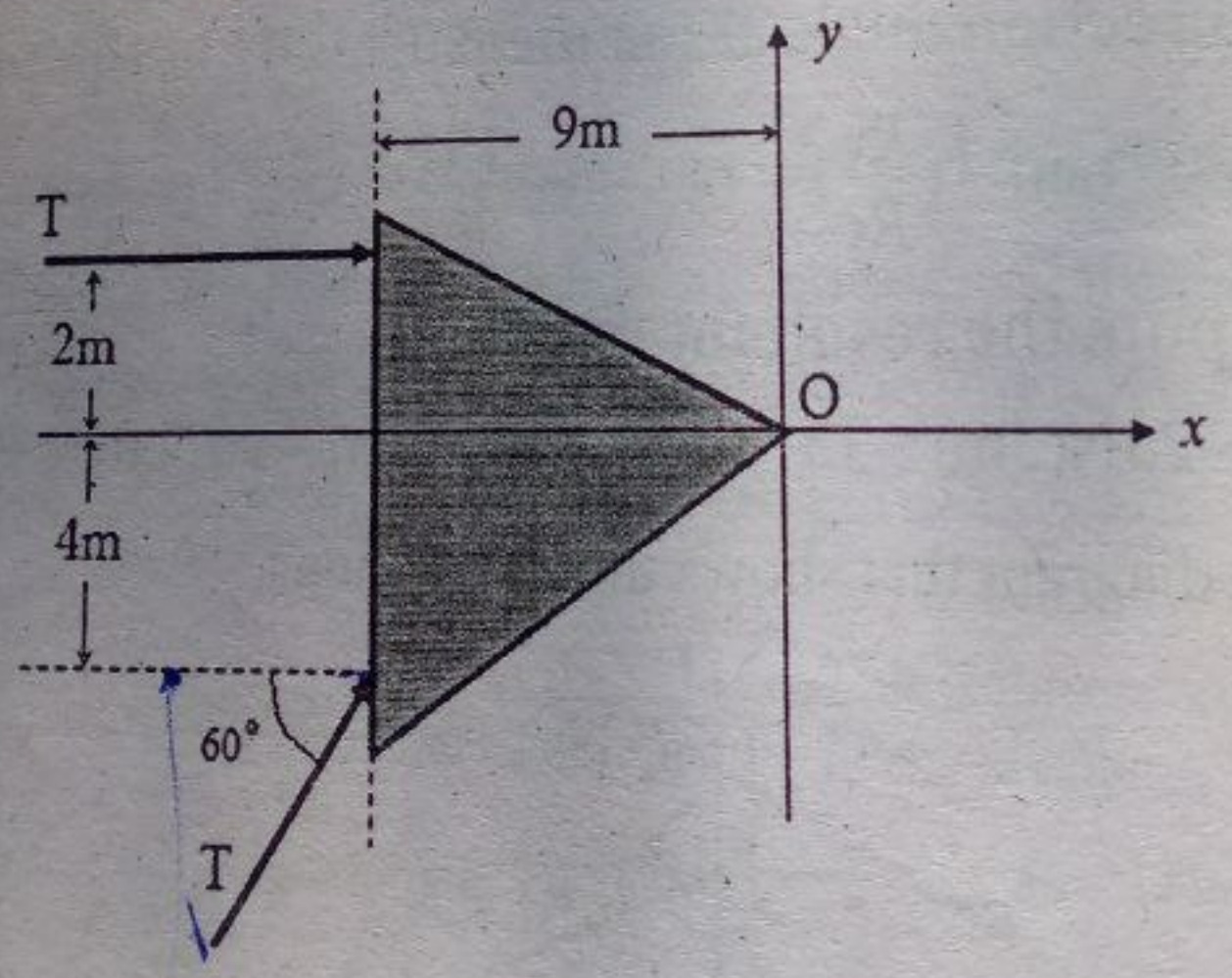
Besides, the x-intercept is $x = 11$ and the y-intercept is $y = \frac{11}{3}$.

9. Two forces of equal magnitude $T = 4\text{N}$ are applied to the plate.

a) Determine the equivalent force-couple system at O

b) Give the equation of the line of action of the resultant force

c*) Find a point H on the line of action of the resultant force that gives a CCW moment of $48\sqrt{3}\text{N.m}$ at the point $G = (0, 4\sqrt{3})$



Solution:

a) We need an equivalent force- couple at O

First, find the resultant force $R = R_x\mathbf{i} + R_y\mathbf{j}$.

Here, $R_x = T + T\cos 60^\circ = 4\text{N} + 2\text{N} = 6\text{N}, R_y = T\sin 60^\circ = 2\sqrt{3}\text{N}$

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Therefore, the resultant is $\vec{R} = R_x\mathbf{i} + R_y\mathbf{j} = 6\text{N}\mathbf{i} + 2\sqrt{3}\text{N}\mathbf{j}$.

Next, compute the resultant moment at O.

$$M_O = T_x.(4\text{m}) - T_y.(9\text{m}) - T.(2\text{m})$$

$$= T\cos 60^\circ.(4\text{m}) - T\sin 60^\circ.(9\text{m}) - T.(2\text{m})$$

$$= 2\text{N}.(4\text{m}) - 2\sqrt{3}\text{N}.(9\text{m}) - 4\text{N}.(2\text{m}) = -18\sqrt{3}\text{N.m}$$

b) Here, we are asked to determine the equation of the line of action of the force. How this is possible?

Technique to determine equation of the line of action:

Step-1: Find the resultant force

Step-2: Determine the resultant moment at the origin O

Step-3: Obtain x and y-intercepts of the line of action of the resultant Then, write equation of the line using the two points in the 3rd step.

Step-1: Find the resultant force. We found in part (a), $\vec{R} = 6\mathbf{i} + 2\sqrt{3}\mathbf{j}$.

Step-2: Determine the resultant moment at the origin O.

As we did in part (a), the moment at O is $M_O = -18\sqrt{3}\text{N.m}$.

Step-3: Obtain x and y-intercepts of the line of action of the resultant

X-intercept: At the x-intercept, the resultant force will give the same moment with the force-couple system at the origin. But at the x-intercept, it is only R_y that contributes a moment at the origin.

So, $|M_O| = R_y.d_x \Rightarrow 2\sqrt{3}.d_x = 18\sqrt{3}\text{N.m} \Rightarrow d_x = 9\text{m}$

Here, $d_x = 9\text{m}$ means the x-intercept is at a distance of 9m from O. It could be to the left or to the right of O. If we assume the x-intercept is on the right of O, the point is $P = (9,0)$ and if we assume the x-intercept is on the left of O, the point is $P = (-9,0)$.

What shall we do to identify the correct point? (Try to identify the correct intercept). Anyway, the correct x-intercept is $P = (-9,0)$.

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b) We need a point P on the y-axis such that the resultant force being at P can produce the same moment at O as the given forces. That is $M_o = R_x.D \Rightarrow D = 1.6m$. The point is either $P = (0,1.6)$ (above O) or $P = (0,-1.6)$ (below O). Which one is the appropriate point?

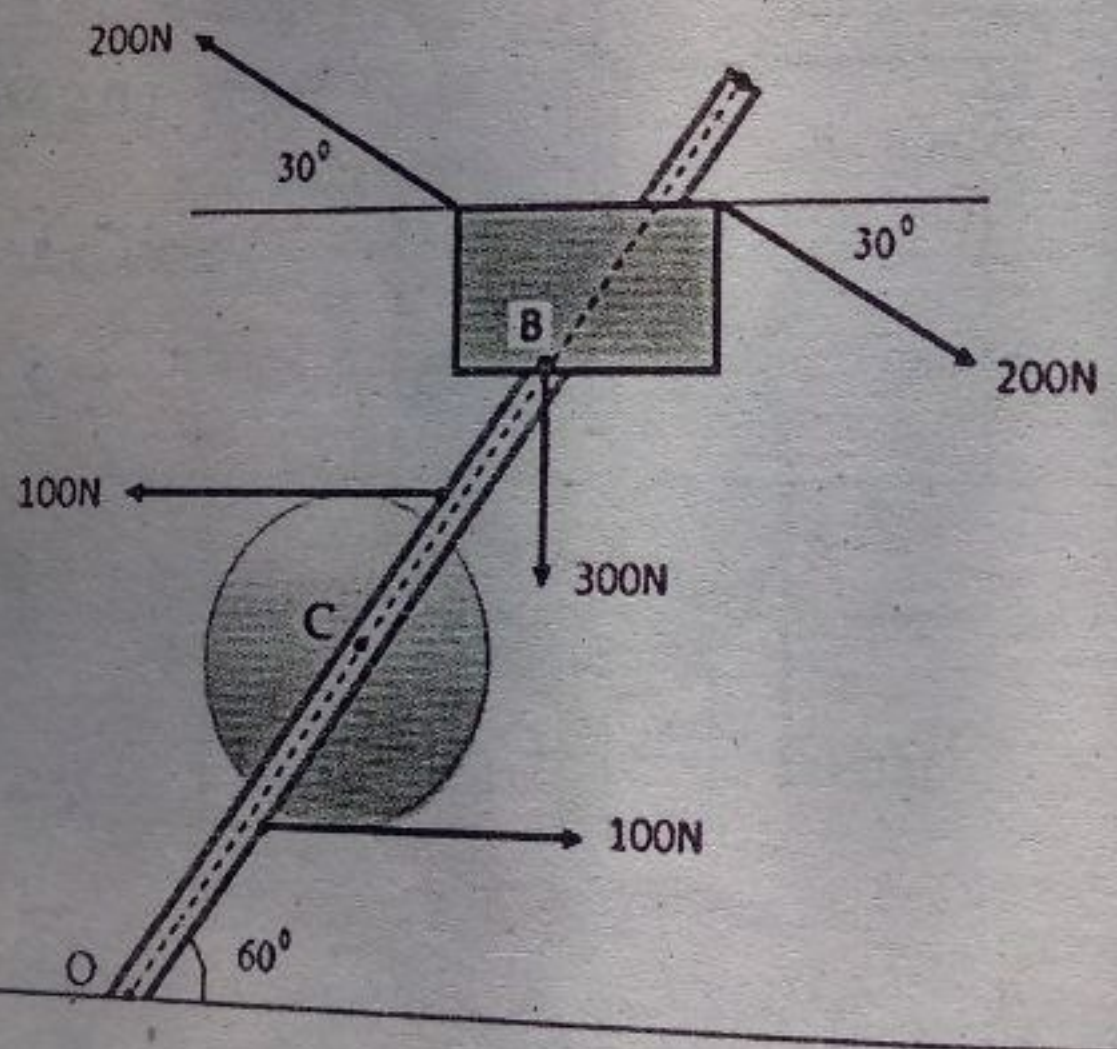
The appropriate point is $P = (0,-1.6)$ (below O). Do you see why?

12. The bar OB of length 600mm is integrated with a disk of radius 80mm and a 100mm × 50mm rectangular plate as shown. The system is designed to carry the effects of two couples and a load of 300N.

a) Replace the system by a couple-force system at O

b) Find a point P on the bar OB so that the force-couple system at O is replaced by a single stand alone force only.

c) If the system exerts a clockwise moment of 69N.m at the center C, find the length CB and give the coordinates of the center of the disc.



Solution: Observe that the two 100N forces tangent to the disc are couples. So, the moment of these forces is a couple moment given by the product of the distance between them with the force magnitude. Besides, since the forces are horizontal and tangent to the disc, the distance between them is $D = 2R$.

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Y-intercept: At the y-intercept, only R_x contributes a moment at O.

So, $|M_O| = R_x.d_y \Rightarrow 6\text{N}.d_y = 18\sqrt{3}\text{N.m} \Rightarrow d_y = 3\sqrt{3}\text{m}$

For similar reasons as above, we get $Q=(0,3\sqrt{3})$ or $Q=(0,-3\sqrt{3})$.

Hence, the correct y-intercept is $Q=(0,-3\sqrt{3})$.

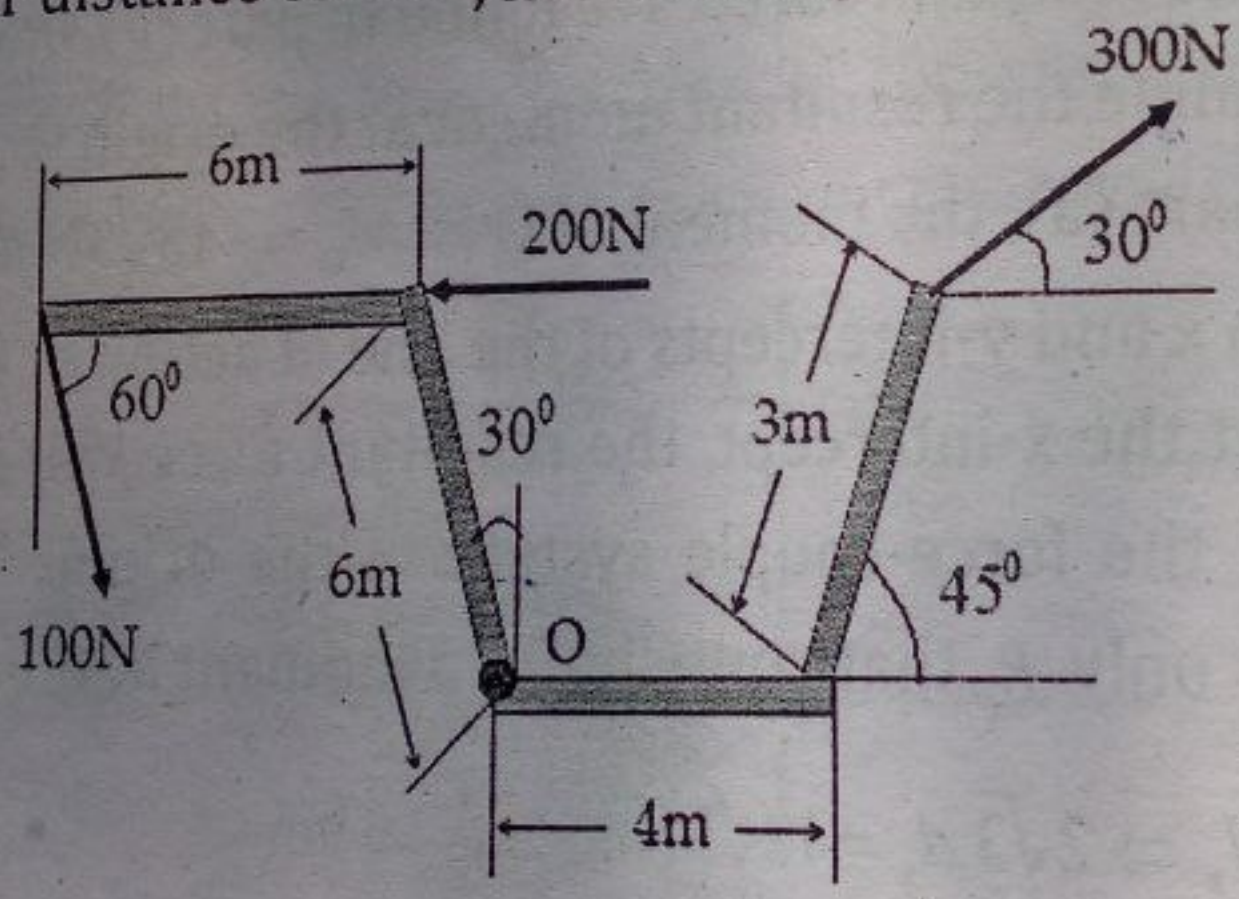
Equation: Using $P=(-9,0)$ and $Q=(0,-3\sqrt{3})$, write the equation.

So, using point-slope form of equation of a line, the line of action L_A becomes $L_A : \frac{y-0}{x+9} = \frac{3\sqrt{3}}{9} \Rightarrow L_A : \frac{y}{x+9} = \frac{\sqrt{3}}{3} \Rightarrow L_A : y = \frac{\sqrt{3}}{3}x + 3\sqrt{3}$.

10. Consider the general force system acting on a connected bars.

a) Find the resultant moment of the forces at point O

b) If the system can be represented by a couple force separated by a perpendicular distance of $20m$, find the magnitude of the force



Solution:

a) Let's denote the forces $F_1 = 100\text{N}, F_2 = 200\text{N}, F_3 = 300\text{N}$. Then,

$$M_o = F_{1y}.(6\text{m} + 6\text{msin}30^0) - F_{1x}.(6\text{mcos}30^0) + F_2.(6\text{mcos}30^0)$$
$$- F_{3x}.(3\text{msin}45^0) + F_{3y}.(4\text{m} + 3\text{mcos}45^0)$$
$$= 86.6(9) - 50(5.2) + 200(5.2) - 259.8(2.12) + 150(6.12)$$
$$= 779.4 - 260 + 1040 - 550.8 + 918 = 1926.6\text{N.m}$$

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b) We need to replace the system by couples F and $-\mathbf{F}$ that are separated by $20m$. But if the separation (the perpendicular distance) between F and $-\mathbf{F}$ is D, the couple moment is simply the product of the norm of one of the couple and D.

That is $M_C = F.D$. But to be equivalent, we must get the same moment by the couples and by the given three forces.

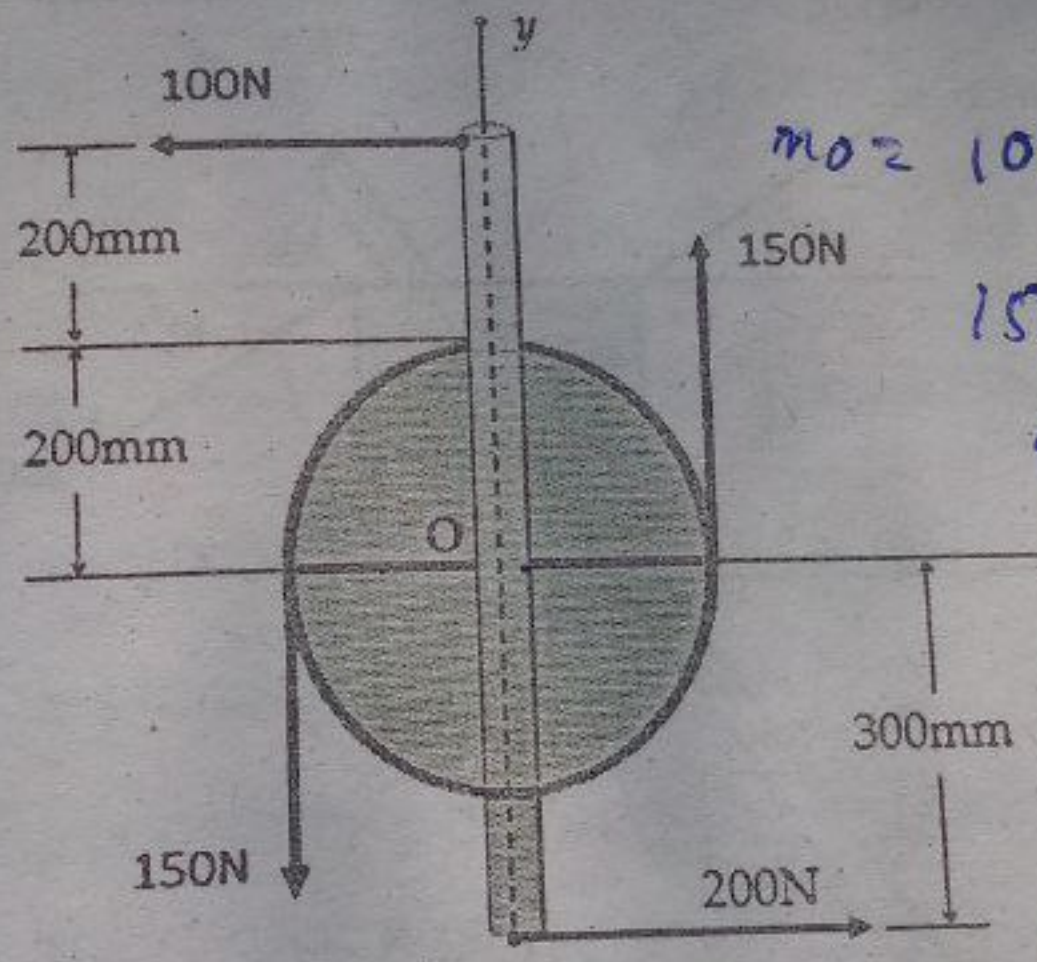
Hence, $M_C = M_o \Rightarrow F.D = 1926.6\text{N.m}$

$$\Rightarrow 20\text{m}F = 1926.6\text{N.m} \Rightarrow F = 96.33\text{N}$$

11. A rigidly integrated pulley and a rotating cylinder are assembled for special purpose with the loads as shown in the diagram.

a) Replace the system by a force-couple system at O

b) Find a point where the line of action of the resultant crosses y-axis



$M_O = 150N \times 0.4 + 150N \times 0.2$

$150N \times 0.2 + 200 \times 0.$

$40 + 30 + 30 + 60$

$= -20\,N.m$

$20\,N.m\,CW$

$\Rightarrow$

$R = (-100 + 200)\hat{i} =$

Solution:

a) Here, by assuming clockwise moment as negative, we have

$$M_o = 200\text{N}(0.3\text{m}) + 150\text{N}(0.2\text{m}) + 150\text{N}(0.2\text{m}) + 100\text{N}(0.4\text{m})$$

$$= 60\text{N.m} + 30\text{N.m} + 30\text{N.m} + 40\text{N.m} = 160\text{N.m}$$

Besides, the resultant force is $\vec{R} = 100\text{Ni}$.

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Similarly, the two 200N forces applied at the corner of the rectangle are couples. But the distance between them is not clear.

So, it is better to decompose into components. If we do so, the distance between their y-component is equal to the length of the rectangle where as the distance between their x-component is zero. Hence, by assuming counterclockwise moment as positive, we have

$$a)\ M_o = 100(2R) - 300(\overline{OB}\cos 60^0) - 200\sin 30^0.(L)$$

$$= 100(0.16\text{m}) - 300(0.6\text{m}\cos 60^0) - 200\sin 30^0(0.1m) = -84\text{N.m}$$

Besides, the resultant force is $F_R = -300\text{Nj}$.

Therefore, the force-couple system at O consists of a clockwise couple moment of $M_o = 84\text{N.m}$ with a resultant force of $F_R = -300\text{Nj}$.

b) We need a point P on the segment CB of the bar such that $\overrightarrow{OP} \times \vec{F}_R = M_o$. But $\overrightarrow{OP} = \overline{OP}\cos 60^0\,\mathbf{i} + \overline{OP}\sin 60^0\,\mathbf{j}$.

$$\overrightarrow{OP} \times \vec{F}_R = M_o \Rightarrow (\overline{OP}\cos 60^0\,\mathbf{i} + \overline{OP}\sin 60^0\,\mathbf{j}) \times \vec{F}_R = M_o$$

$$\Rightarrow (\overline{OP}\cos 60^0\,\mathbf{i} + \overline{OP}\sin 60^0\,\mathbf{j}) \times -300\text{Nj} = M_o$$

$$\Rightarrow -300\text{N}\overline{OP}\cos 60^0\,\mathbf{k} = -84\text{N.m}\mathbf{k}$$

$$\Rightarrow 150\overline{OP} = 84\text{m} \Rightarrow \overline{OP} = 0.56\text{m} = 560\,\text{mm}$$

Furthermore, the coordinate of P is obtained from length $\overline{OP} = 560\text{mm}$ as $P = (x, y) = (\overline{OP}\cos 60^0, \overline{OP}\sin 60^0) = (280\text{mm}, 280\sqrt{3}\text{mm})$.

c) We need the length OB such that $M_c = -69\text{N.m}$. That is

$$M_c = -69 \Rightarrow 100(2R) - 300(\overline{CB}\cos 60^0) - 200\sin 30^0(0.1) = -69$$

$$\Rightarrow 16 - 300(\overline{CB}\cos 60^0) - 10 = -69 \Rightarrow \overline{CB} = 0.5\text{m} = 500\,\text{mm}$$

Besides, $\overline{OC} = \overline{OB} - \overline{CB} = 600\text{mm} - 500\text{mm} = 100\text{mm}$.

So, the center is $C = (a, b) = (\overline{OC}\cos 60^0, \overline{OC}\sin 60^0) = (50\text{mm}, 50\sqrt{3}\text{mm})$.

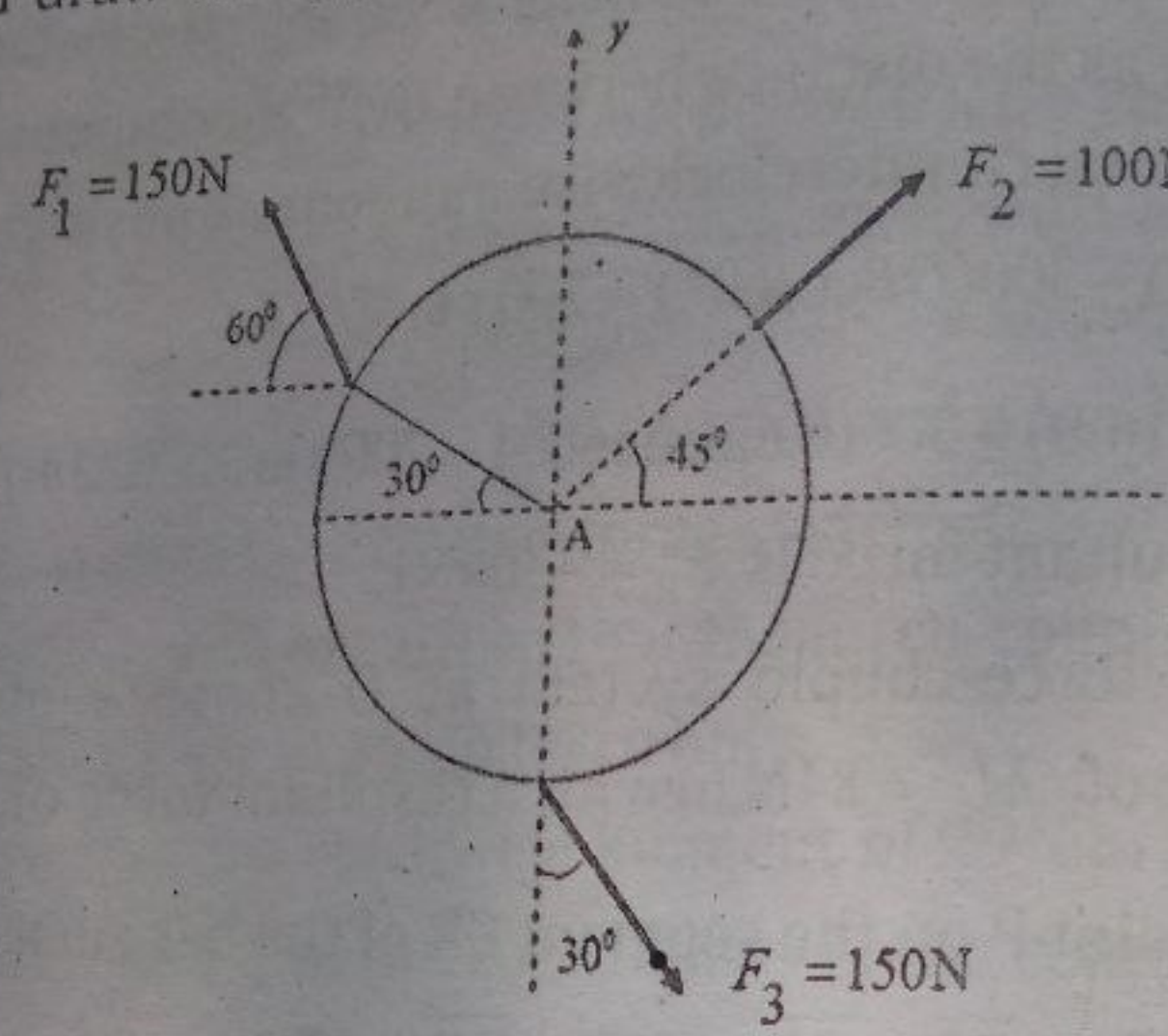
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13. Three forces are exerted on a dick of radius $r = 200\text{mm}$ as shown.

a) Replaces the three force by a force-couple system at A

b) Specify or draw the application point of the resultant force



Solution:

a) $M_A = F_{1x}.(r\sin 30^0) - F_{1y}.(r\cos 30^0) + F_{3x}.r$

$= 150\text{N}\cos 60^0.(0.1\text{m}) - 150\text{N}\sin 60^0.(0.\sqrt{3}\text{m}) + 150\text{N}\sin 30^0.(0.2\text{m})$

$= 7.5\text{N.m} - 22.5\text{N.m} + 15\text{N.m} = 0$

Now, find the resultant force. It is of the form $\vec{R} = R_x i + R_y j$

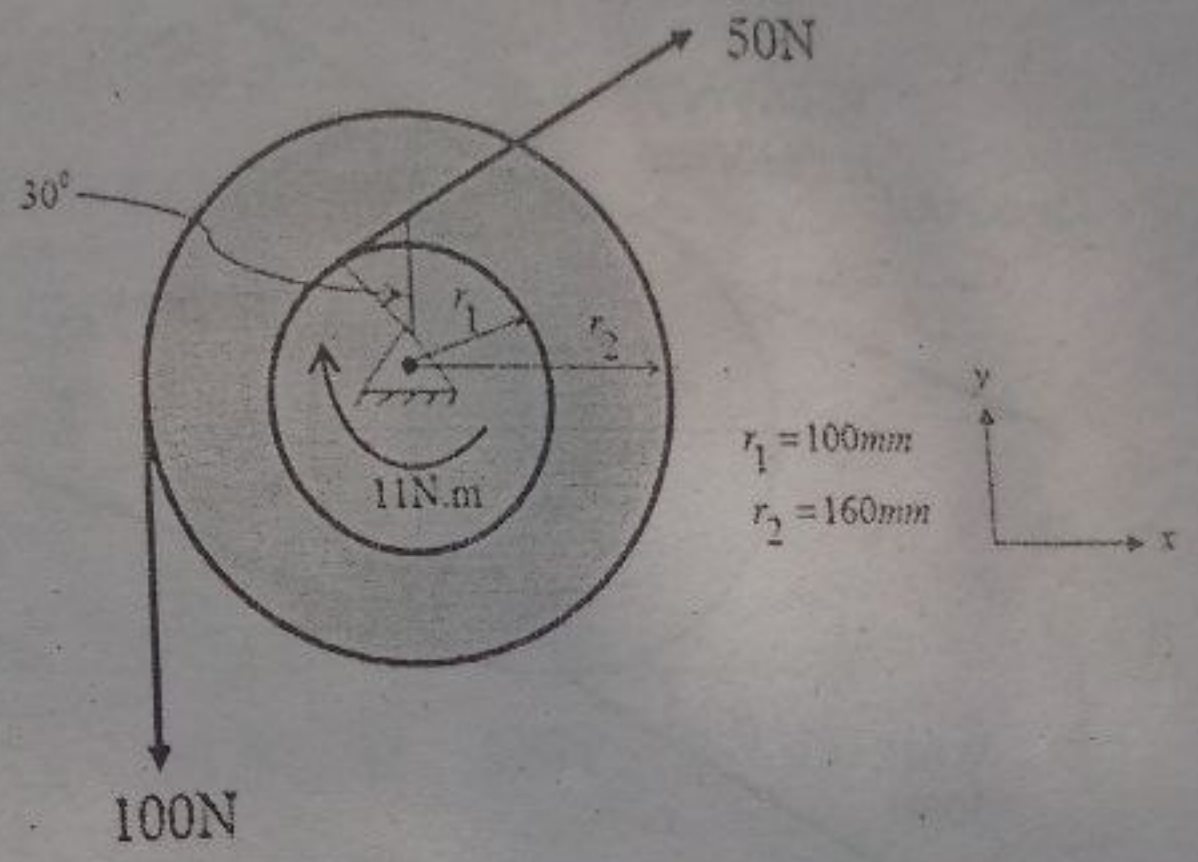
But $\begin{cases} R_x = -150\text{N}.\cos 60^0 + 100\text{N}.\cos 45^0 + 150\text{N}.\sin 30^0 = 50\sqrt{2}\text{N} \\ R_y = 150\text{N}.\sin 60^0 + 100\text{N}.\sin 45^0 - 150\text{N}.\cos 30^0 = 50\sqrt{2}\text{N} \end{cases}$

Therefore, $\vec{R} = R_x i + R_y j = 50\sqrt{2}\text{Ni} + 50\sqrt{2}\text{Nj}$. Besides, the angle it makes with the x-axis satisfies $\tan\theta = \dfrac{50\sqrt{2}}{50\sqrt{2}} = 1 \Rightarrow \theta = \tan^{-1}(1) = 45^\circ$.

Since $M_A = 0$, the system consists of only a force applied at A.

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14. The unit consisting of two rigidly connected pulleys is acted by a couple and two tensions as shown. The tension forces exerted by the belts are securely wrapped onto the two pulley surfaces. Determine equivalent force force-couple system at the pulley center "O".



Solution: We need to find the force-couple representation at O.

$$M_O = F_2.r_2 - F_1.r_1 - M = 100\text{N}.(0.16\text{m}) - 50\text{N}(.1\text{m}) - 11\text{N.m} = 0$$

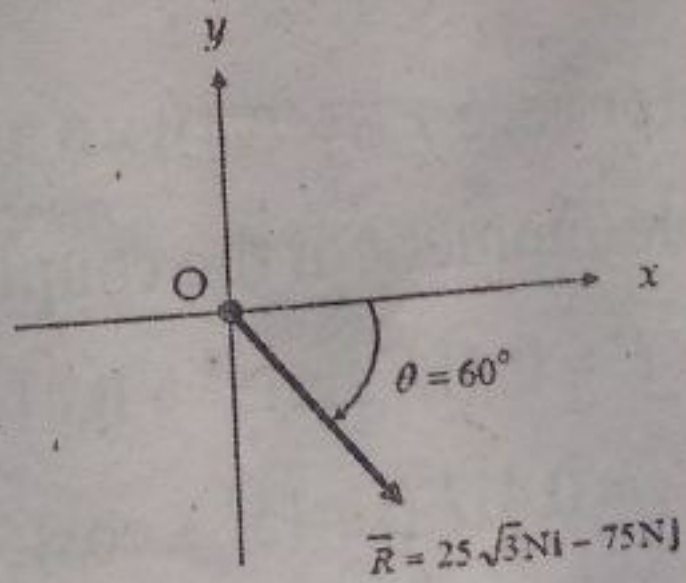
Besides, $R_x = 50\text{N}\cos 30^\circ = 25\sqrt{3}\text{N}, R_y = 50\text{N}\sin 30^\circ - 100\text{N} = -75\text{N}$

Hence, the resultant force is $\vec{R} = 25\sqrt{3}\text{Ni} - 75\text{Nj}$.

Besides, the direction of the resultant force (as measured from the positive x-axis to the resultant in clockwise direction) is given by

$$\tan\theta = -\frac{75}{25\sqrt{3}} = -\frac{3}{\sqrt{3}} = -\sqrt{3} \Rightarrow \theta = \tan^{-1}(-\sqrt{3}) = 60^\circ$$

Therefore, representation consists of a force only as shown.

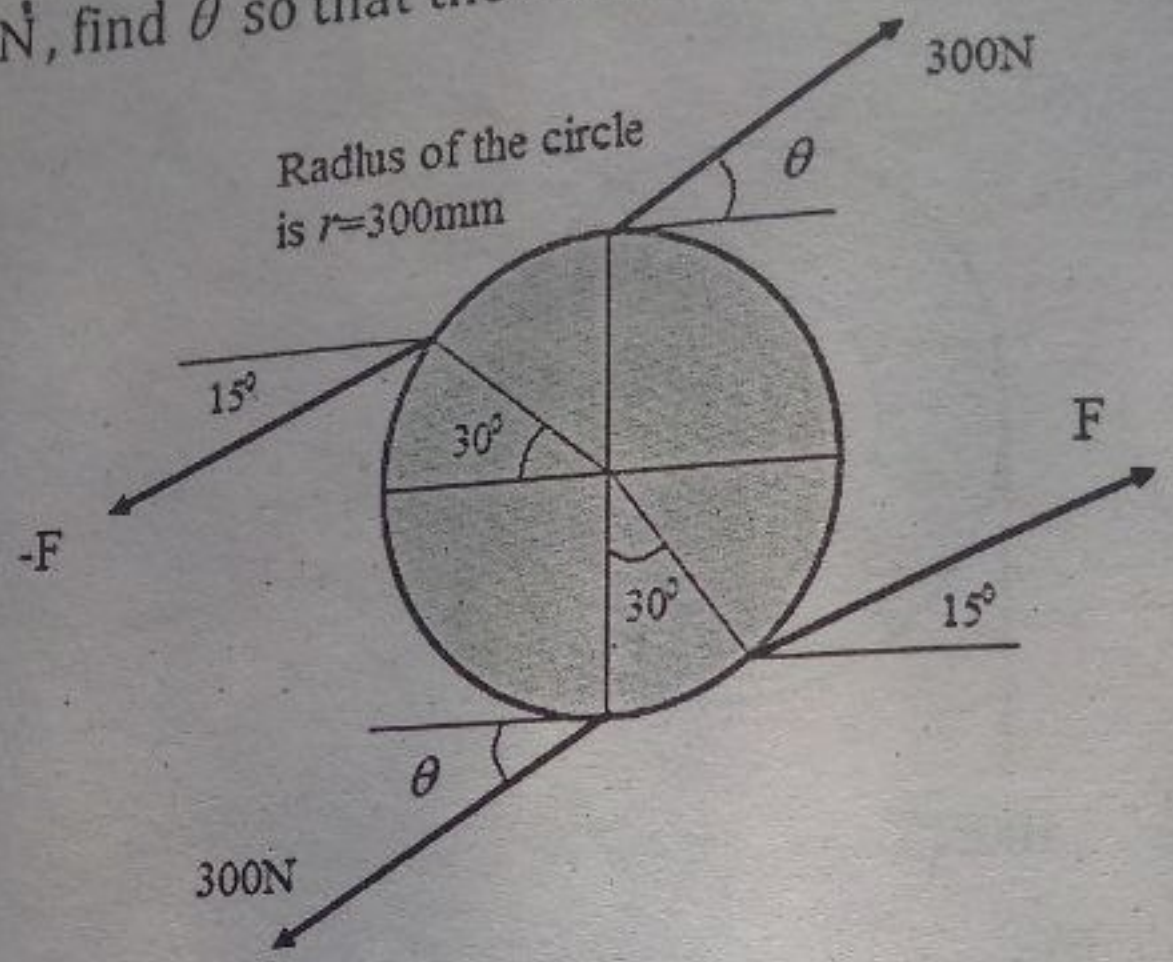


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15*. Two couple forces are acting on a disc as shown below.

a) If the resultant couple of the force system is $M_C = 100\text{N.m(CW)}$ when $\theta = 30^0$, determine the magnitude of the force F.

b) If $F = 200\text{N}$, find θ so that the resultant couple moment is zero



Solution:

a) First express the couple forces in vector form. That is

$$\begin{cases} \vec{F} = F\cos 15^0\,\mathbf{i} + F\sin 15^0\,\mathbf{j} \\ \vec{P} = 300\cos 30^0\,\mathbf{i} + 300\sin 30^0\,\mathbf{j} = 150\sqrt{3}\mathbf{i} + 150\mathbf{j} \end{cases}$$

Now, find the position vectors from **-F** to **F** and from **–P** to **P**.

Here, the coordinates are obtained using trigonometric as;

$$A = (-0.3\cos 30^0, 0.3\sin 30^0) = (-0.26.0.15)$$

$$B = (0.3\sin 30^0, -0.3\cos 30^0) = (0.15, -0.26)$$

$$C = (0, -0.3),\ D = (0, 0.3)$$

So, the position vectors are $\vec{r}_{\overline{AB}} = 0.41\mathbf{i} - 0.41\mathbf{j}$ and $\vec{r}_{\overline{CD}} = 0.6\mathbf{j}$.

Therefore, the couple-moment of the couples becomes

$$\overrightarrow{M_C} = \vec{r}_{\overline{AB}} \times \vec{F} + \vec{r}_{\overline{CD}} \times \vec{P} = 0.41F\sin 15^0 + 0.41F\cos 15^0 - 155.8$$

$$= 0.41F(\sin 15^0 + \cos 15^0) - 155.8$$

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But $M_C = 100\text{N.m}$ clockwise. Then, by equating the values, we have

$$M_C = -100\text{N.m} \Rightarrow 0.41F(\sin 15^0 + \cos 15^0) - 155.8\text{N} = -100\text{N}$$

$$\Rightarrow 0.41F(\sin 15^0 + \cos 15^0) = 55.88\text{N}$$

$$\Rightarrow 0.504F = 55.88\text{N} \Rightarrow F = 111\text{N}$$

b) Given $F = 200\text{N}$ and $P = 300\text{N}$. In vector form,

$\vec{F} = 200\cos 15^0\,\mathbf{i} + 200\sin 15^0\,\mathbf{j} = 193.2\mathbf{i} + 51.8\mathbf{j}$, $\vec{P} = 300\cos\theta\,\mathbf{i} + 300\sin\theta\,\mathbf{j}$

Using similar procedure, we have

$$\vec{M}_C = 0 \Rightarrow \vec{r}_{\overrightarrow{AB}} \times \vec{F} + \vec{r}_{\overrightarrow{CD}} \times \vec{P} = 0$$

$$\Rightarrow \begin{vmatrix} i & j & k \\ 0.41 & -0.41 & 0 \\ 193.2 & 51.8 & 0 \end{vmatrix} + \begin{vmatrix} i & j & k \\ 0 & 0.6 & 0 \\ 300\cos\theta & 300\sin\theta & 0 \end{vmatrix} = 0$$

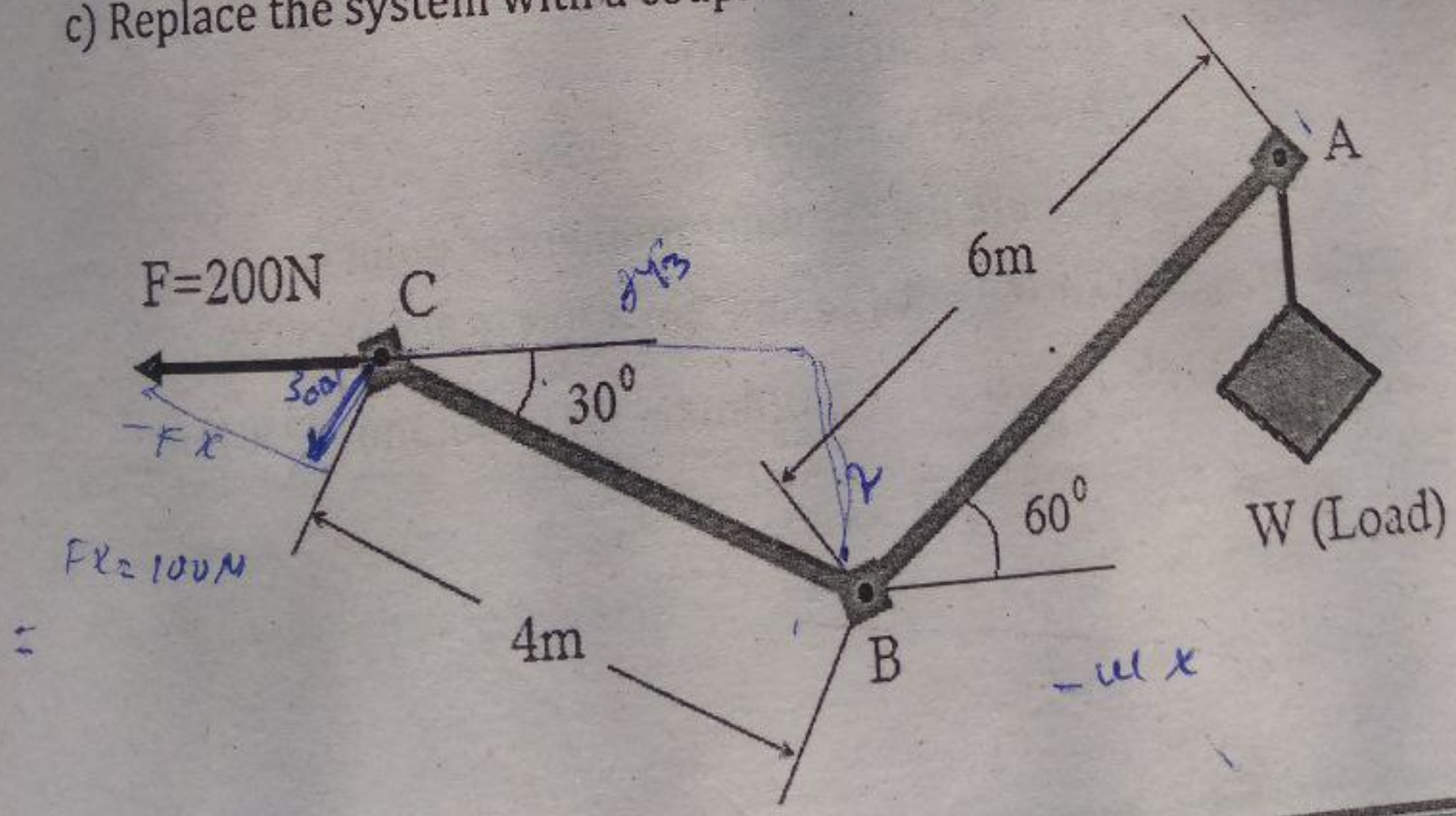
$$\Rightarrow 180\cos\theta = 100.45 \Rightarrow \theta = \cos^{-1}(0.558) = 56.1^0$$

16. The force $F = 200\text{N}$ and the load W produced a resultant moment of $M_B = 250\text{N.m}$ in a counterclockwise direction at B. Then,

a) Find the allowable value of the load W

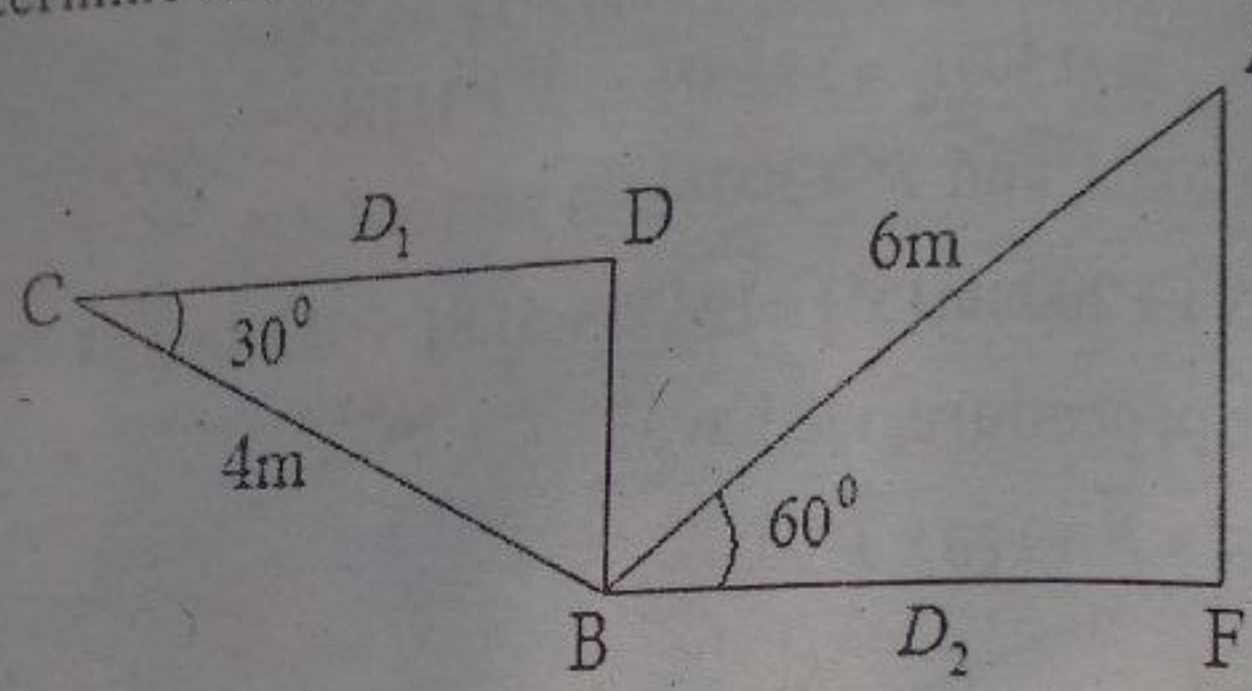
b) What is the resultant moment at point C?

c) Replace the system with a couple moment and a force at point B



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Solution: First of all, the weight itself is not given. So, the role of $M_B = 250\text{N.m}$ is to determine the weight W. Consider the following FBD to determine the distances D_1 and D_2.



By applying trigonometric on the two right angle triangles, we have

i) $\cos 30^0 = \dfrac{D_1}{CB} \Rightarrow D_1 = CB\cos 30^0 = 2\sqrt{3}\text{m}$

ii) $\cos 60^0 = \dfrac{D_2}{AB} \Rightarrow D_2 = AB\cos 60^0 = 3\text{m}$

a) Then, using moment at point B, we get the weight W as follow:

$M_B = F.(4\text{m}.\sin 30^0) - W.(6\text{m}.\cos 60^0)$

$\Rightarrow F.(4\text{m}.\sin 30^0) - W.(6\text{m}.\cos 60^0) = 250\,\text{N.m}$

$\Rightarrow 200\,\text{N}.(2\text{m}) - W.(3\text{m}) = 250\,\text{N.m}$

$\Rightarrow 3W = 400\,\text{N} - 250\,\text{N} = 150\,\text{N} \Rightarrow W = 50\,\text{N}$

b) Now we can compute the resultant moment at point C.

That is $M_C = W.(D_1 + D_2) = 50.(2\sqrt{3}\text{m} + 3\text{m}) = 50\text{N}.(6.46\text{m}) = 323\text{N.m}$

c) $M_B = F.DB\sin 30^0 - W.D_2 = 200(4\text{m}\sin 30^0) - 50(3\text{m}) = 250\,\text{N.m}$

Besides, the resultant force is $R = -200\mathbf{i} - 50\mathbf{j}$.

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First: Express the two forces in vector form. To express in vector form, always identify a unit vector parallel to the line of action of the force. Since two points are given on the line of actions of the forces, we can find their vector expressions easily as follow. If you examine the coordinates of the points from the diagram carefully, you will get the points to be $A=(0,0,8), B=(0,6,0), C=(0,0,6), D=(2,-3,0)$.

i) $\overrightarrow{AB}=B-A=6j-8k \Rightarrow \vec{U}_{\overrightarrow{AB}}=\frac{1}{10}(6j-8k)=\frac{3}{5}j-\frac{4}{5}k$

So, in vector form, $\vec{F}_B=F_B.\vec{U}_{\overrightarrow{AB}}=5\text{kN}(\frac{3}{5}j-\frac{4}{5}k)=3\text{kN}\mathbf{j}-4\text{kN}\mathbf{k}$

ii) $\overrightarrow{CD}=D-C=2i-3j-6k \Rightarrow \vec{U}_{\overrightarrow{CD}}=\frac{1}{7}(2i-3j-6k)=\frac{2}{7}i-\frac{2}{7}j-\frac{6}{7}k$

So, $\vec{F}_D=F_D.\vec{U}_{\overrightarrow{CD}}=7\text{kN}(\frac{2}{7}i-\frac{3}{7}j-\frac{6}{7}k)=2\text{kN}\mathbf{i}-3\text{kN}\mathbf{j}-6\text{kN}\mathbf{k}$

Second: Find the resultant moment.

The moment at O is simply the sum of the moment due to the two forces. It is easily obtained using vector approach as follow:

$$\vec{M}_O=\vec{r}_{\overrightarrow{OB}}\times\vec{F}_B+\vec{r}_{\overrightarrow{OD}}\times\vec{F}_D=\begin{vmatrix} i & j & k \\ 0 & 6 & 0 \\ 0 & 3 & -4 \end{vmatrix}+\begin{vmatrix} i & j & k \\ 2 & -3 & 0 \\ 2 & -3 & -6 \end{vmatrix}$$

$$=-24i+18i+12j=-6i+12j$$

Third: Find the resultant force and its direction.

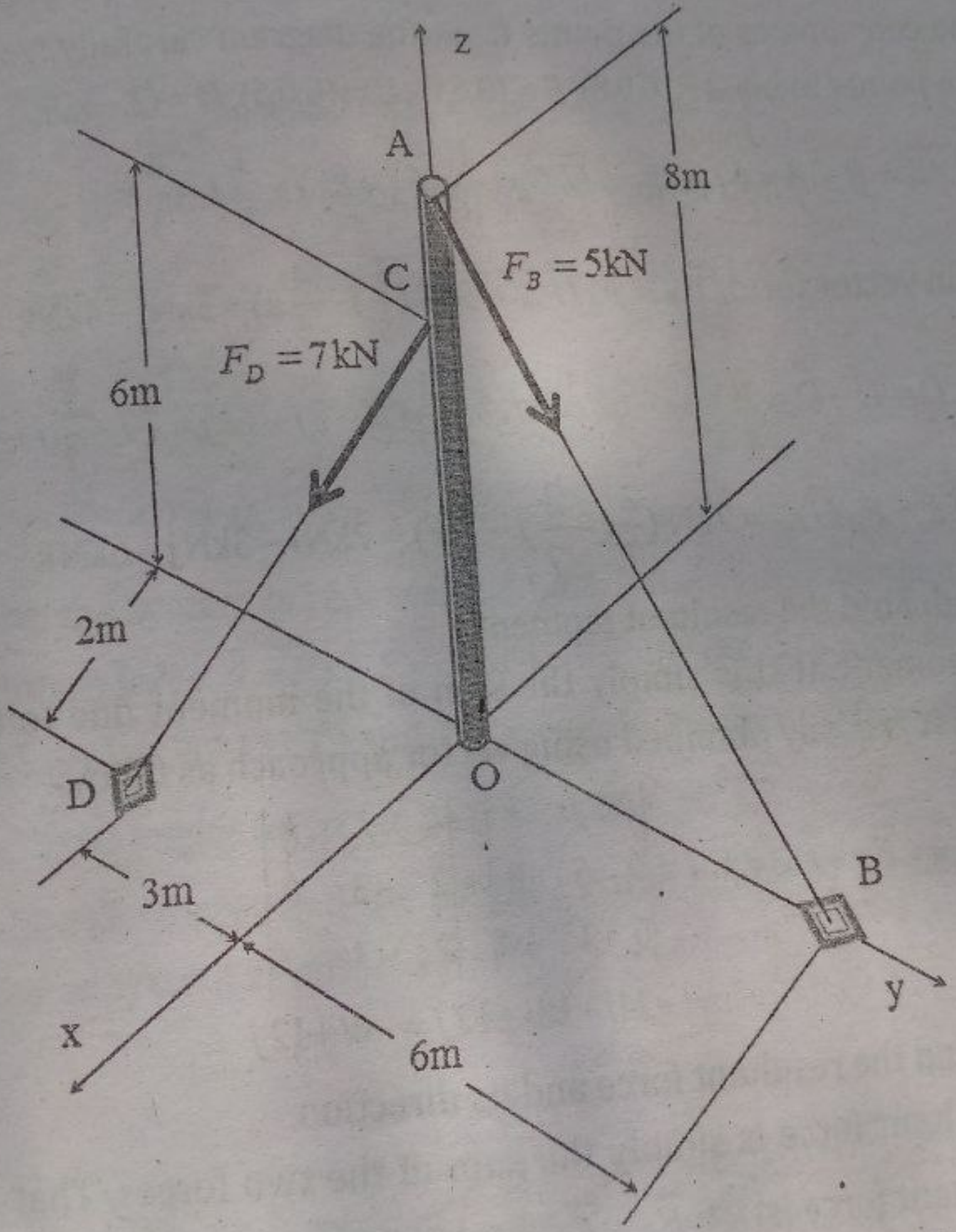
The resultant force is simply the sum of the two forces. That means the resultant force is $\vec{R}=\vec{F}_B+\vec{F}_D=5\text{kN}\mathbf{i}-10\text{kN}$.

Vector expressions of the results: Notice that we have already expressed the force and the moment in vector forms.

Directions of each vector: You can specify the direction of any vector space by finding its direction angles.

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18. Replace the two forces acting on the post by a resultant force and coupe moment at point O. Express both the resultant vector and couple moment in vector form using i, j, k notations. Specify the direction of both resultant vectors.



Solution: Given the magnitudes of two forces and the line of actions of each force is specified by two points. We need to find the force-couple representation at O in vector forms with specified directions.

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Solution: To solve this problem, we proceed as follow.

First: Express the forces $F = 20\text{N}$, $P = 40\text{N}$ in vector form.

Observe that the force $P = 40\text{N}$ is on the yz-plane. That is its decomposition is $\vec{P} = P_y j + P_z k = P\cos\theta.\mathbf{j} + P\sin\theta.\mathbf{k}$. But from the sides of the box, using $\theta = \angle CBG$, we have $\cos\theta = \frac{4}{5}$, $\sin\theta = \frac{3}{5}$.

So, $\vec{P} = P_y j + P_z k = -32j + 24k$ and $\vec{F} = -20\mathbf{i}$.

Hence, the resultant force is $\vec{R} = -20\mathbf{i} - 32\mathbf{j} + 24\mathbf{k}$.

Second: Find the position vectors from point A to any point on the line of actions of the two forces (Here, it is simple if we take the vectors from A to the points of applications). Here, the coordinates of point A is $A = (2,0,0)$. Besides, the 20N force is applied at the vertex of the box which is $D = (2,0,-3)$ and that of the 40N force is applied at the vertex of the box which is $C = (0,4,-3)$. So, we get the vectors $\vec{r}_{AD} = -3k$, $\vec{r}_{AC} = -2i + 4j - 3k$.

Third: Apply the vector approach.

$$M_A = \vec{r}_{AD} \times F + \vec{r}_{AC} \times P - Mk$$

$$= \begin{vmatrix} i & j & k \\ 0 & 0 & -3 \\ -20 & 0 & 0 \end{vmatrix} + \begin{vmatrix} i & j & k \\ -2 & 4 & -3 \\ 0 & -32 & 24 \end{vmatrix} - 35k = 108j + 29k$$

Therefore, the force-couple system at A consists of a resultant force $\vec{R} = -20\mathbf{i} - 32\mathbf{j} + 24\mathbf{k}$ and a couple moment $\vec{M} = 108\mathbf{j} + 29\mathbf{k}$.

So, for the force vector, its direction is specified as follow:

i) $\cos\theta_x = \frac{R_x}{R} = \frac{5}{11.2} = 0.446 \Rightarrow \theta_x = \cos^{-1}(0.446) = 63.5^0$

ii) $\cos\theta_y = \frac{R_y}{R} = 0 \Rightarrow \theta_y = \cos^{-1}(0) = 90^0$

iii) $\cos\theta_z = \frac{R_z}{R} = -\frac{10}{11.2} = -0.893 \Rightarrow \theta_z = \cos^{-1}(-0.893) = 153.3^0$

Similarly, for the moment vector, using cosine directions;

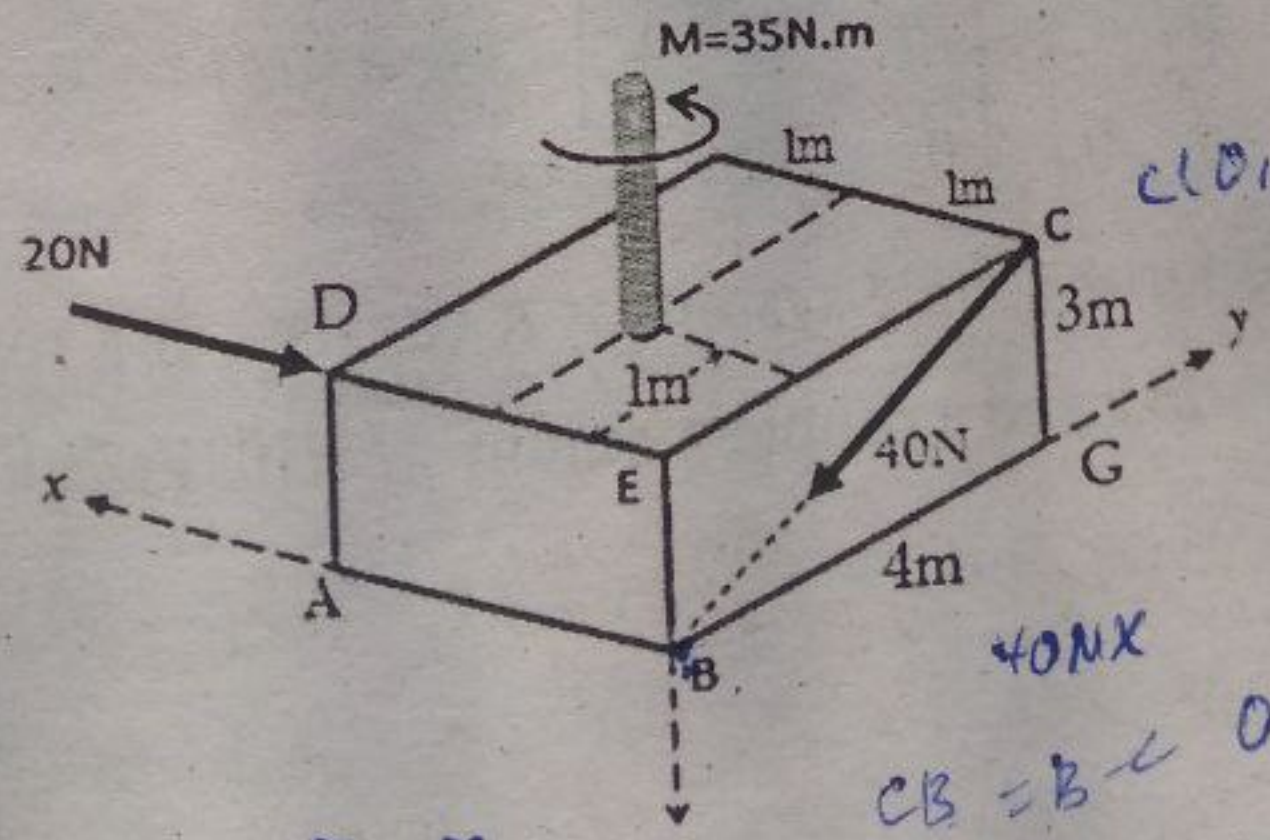
i) $\cos\alpha_x = \frac{M_x}{M} = -\frac{6}{13.4} = -0.448 \Rightarrow \alpha_x = \cos^{-1}(-0.448) = 116.6^0$

ii) $\cos\alpha_y = \frac{M_y}{M} = \frac{12}{13.4} = 0.896 \Rightarrow \alpha_y = \cos^{-1}(0.896) = 26.4^0$

iii) $\cos\alpha_z = \frac{M_z}{M} = 0 \Rightarrow \alpha_z = \cos^{-1}(0) = 90^0$

Therefore, the system is represented by a force-couple moment at O by a resultant force $\vec{R} = \vec{F}_B + \vec{F}_D = 5\text{kN}i - 10\text{kN}$ in the direction of $(\theta_x, \theta_y, \theta_z) = (63.5^0, 90^0, 153.3^0)$ and by a couple moment $\vec{M}_O = -6i + 12j$ with direction angles $(\alpha_x, \alpha_y, \alpha_z) = (116.6^0, 26.4^0, 90^0)$.

19. Replace the forces and the couple system below by an equivalent force-couple system at point A.

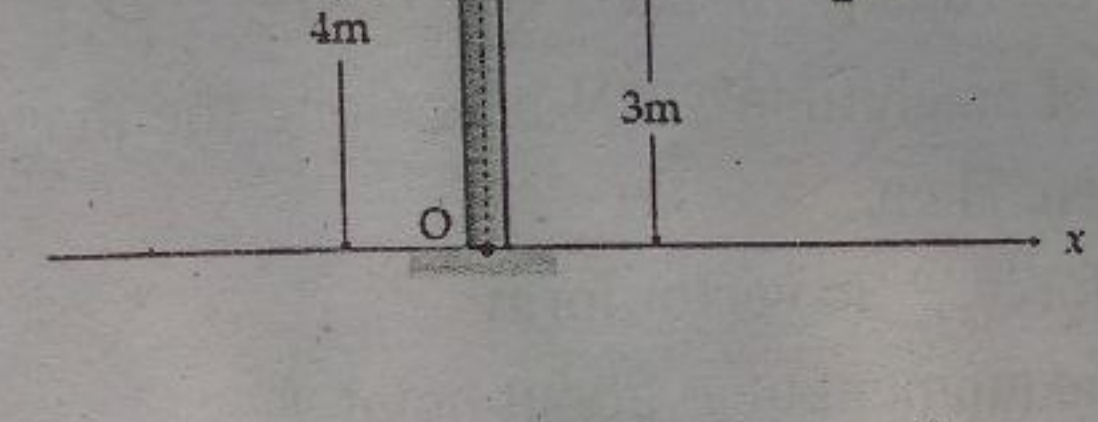


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20. Consider the forces $F_1 = 30\text{N}, F_2 = 40\text{N}, F_3 = 60\text{N}$ acting as shown.

a) Replace the forces by a force-couple system both at O and P

b) Find the coordinates of point Q if the force-couple system at P can be replaced by a single force applied at point Q



Solution:

a) We need to replace the forces by a force-couple system at O

First: Determine the resultant force of the given force system.

$$\text{Here,} \begin{cases} F_{1x} = 0, F_{2x} = 42\text{N}, F_{3x} = -F_3\cos\theta = -\frac{3}{5}(60\text{N}) = -36\text{N} \\ F_{1y} = -52\text{N}, F_{2y} = 0, F_{3y} = -F_3\sin\theta = -\frac{4}{5}(60\text{N}) = -48\text{N} \end{cases}$$

$$\text{Hence, } \vec{R} = R_x i + R_y j = 6\text{N}\mathbf{i} - 100\text{N}\mathbf{j}.$$

Second: We need the resultant moment at O and P.

At O;

$$M_o = F_{3x}.(3\text{m} + 4\text{m}) - F_2.(3\text{m})$$
$$= 36\text{N}.(7\text{m}) - 42\text{N}(3\text{m})$$
$$= 126\text{N.m}$$

At P;

$$M_P = F_2.(4\text{m} + 1\text{m}) - F_{3x}.(1\text{m})$$
$$= 42\text{N}(5\text{m}) - 36\text{N}.(1\text{m})$$
$$= 174\text{N.m}$$

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Solution:

a) To express P in vector form, first find a unit vector along its line of action, that is along BC. Here, $B=(0,225,0)$, $C=(0,0,300)$

Here, a unit vector along the diagonal BC is obtained as follow:

That is $\overrightarrow{BC}=-0.225\mathbf{j}+0.3\mathbf{k}, \left\|\overrightarrow{BC}\right\|=0.375$. So, $\vec{U}_{\overline{BC}}=-\frac{3}{5}\mathbf{j}+\frac{4}{5}\mathbf{k}$

Therefore, in vector form, $\vec{P}=P.\vec{U}_{\overline{BC}}=200(-\frac{3}{5}\mathbf{j}+\frac{4}{5}\mathbf{k})=-120\mathbf{j}+160\mathbf{k}$

b) To determine the moment of P about point E, first find the position vector from E to any known point along the line of action of the force. In our case, point $B=(0,225,0)$ is appropriate because it is on the line of action plus its coordinates are known.

Thus, the position vector becomes $\vec{r}_{\overline{EB}}=-0.4\mathbf{i}+0.225\mathbf{j}$.

Therefore, by vector analysis the moment is given by

$$M_E=\vec{r}_{\overline{EB}}\times\vec{P}=\begin{vmatrix} i & j & k \\ -0.4 & 0.225 & 0 \\ 0 & -120 & 160 \end{vmatrix}=36\mathbf{i}+64\mathbf{j}+48\mathbf{k}$$

c) The moment of P about CE is simply the projection of the moment M_E on a unit vector along CE. So, let's find the unit vector along the diagonal CE first. It is $\vec{U}_{\overline{CE}}=\frac{\overrightarrow{CE}}{\left\|\overrightarrow{CE}\right\|}=\frac{4}{5}\mathbf{i}-\frac{3}{5}\mathbf{k}$.

Therefore, the moment of P about the axis EC becomes

$$M_{CE}=\vec{U}_{\overline{CE}}.M_E=(\frac{4}{5}\mathbf{i}-\frac{3}{5}\mathbf{k}).(36\mathbf{i}+64\mathbf{j}+48\mathbf{k})=\frac{144}{5}-\frac{144}{5}=0$$

d) The projection of P on the diagonal OD is simply the dot product of P with a unit vector $\vec{U}_{\overline{OD}}$ along OD. But $\vec{U}_{\overline{OD}}=\frac{4}{5}\mathbf{i}+\frac{3}{5}\mathbf{k}$.

Hence, $P_{OD}=\vec{P}.\vec{U}_{\overline{OD}}=(-120\mathbf{j}+160\mathbf{k}).(\frac{4}{5}\mathbf{i}+\frac{3}{5}\mathbf{k})=96\text{N}$.

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b) The task is to find the coordinates of point Q such that the force-couple system at P is replaced by a single force applied at Q.

So, how can we start? First of all, the coordinates of point Q is of the form $Q=(x,y)$. Then, our task is to find the values of x and y.

But from the diagram, we have $y=4m$. Now, calculate the moment of the resultant force at P in terms of x. That is

$$M_P = R_x.(3m+1m) + R_y.(x) = 6N.(4m) + 100N.x = 100N.x + 24N.m$$

Here, if the force-couple system at P is represented by a single force at Q, the single force must produce the same moment at P. That means for the system to be equivalent the moments must be equal.

Hence, $100N.x + 24N.m = 174N.m \Rightarrow 100x = 150m \Rightarrow x = 1.5m$.

Therefore, the coordinates of point Q is $Q=(-1.5,4)$.

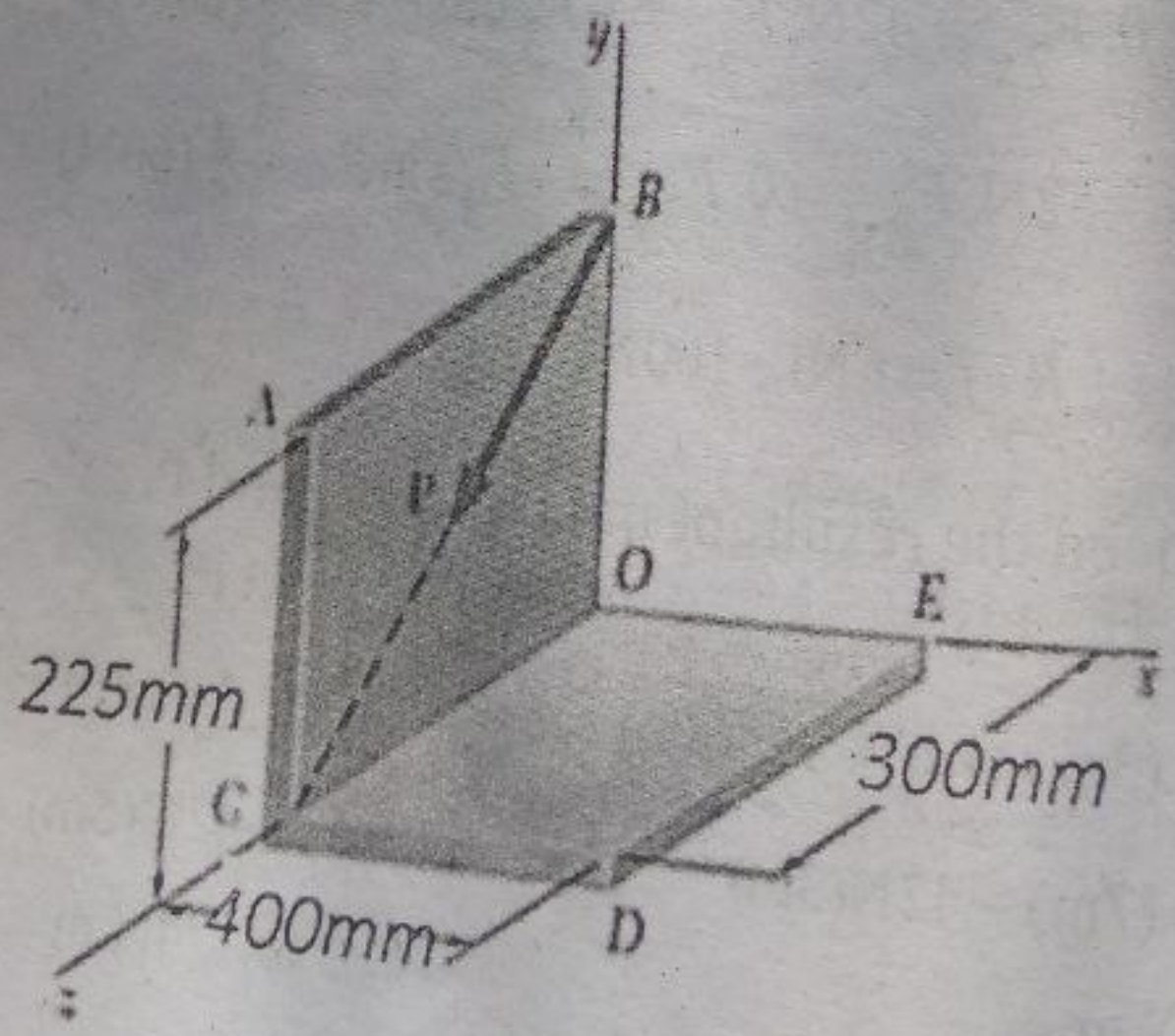
21. A force P of magnitude 200N acts along the diagonal BC of the bent plate shown. Then,

a) Express the force P in vector form

b) Determine the moment of P about point E

c) Determine the moment of P about the axis CE and OA

d) Give the projection of P on the diagonal OD



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Or you can use the position vector $\vec{r}_{OB}$. That is

$$\vec{M}_O = \vec{r}_{OB} \times \vec{T} = \begin{vmatrix} i & j & k \\ 1.6 & -0.8 & 0.8 \\ -480 & 450 & 360 \end{vmatrix} = -648\mathbf{i} - 960\mathbf{j} + 336\mathbf{k}\,.$$

c) The moment of the force about the coordinate axes.

Since the point O lies on the three coordinate axes, the moment about these axes is simply the dot product of the moment at O with the respective unit vectors $\vec{U}_x = \mathbf{i}$, $\vec{U}_y = \mathbf{j}$, $U_z = \mathbf{k}$.

i) Moment about x-axis: $\vec{M}_x = \vec{M}_O.\vec{U}_x = -648\text{N.m}$.

ii) Moment about y-axis: $\vec{M}_y = \vec{M}_O.\vec{U}_y = -960\text{N.m}$.

ii) Moment about z-axis: $\vec{M}_z = \vec{M}_O.\vec{U}_z = 336\text{N.m}$.

d) The directions of a vector means the direction angles it makes with the coordinate axes measured from the positive axes to the vector. Thus, for the force vector, using cosine directions;

i) $\cos\theta_x = \dfrac{T_x}{T} = -\dfrac{480}{750} = -0.64 \Rightarrow \theta_x = \cos^{-1}(-0.64) = 129.8^0$

ii) $\cos\theta_y = \dfrac{T_y}{T} = \dfrac{450}{750} = 0.6 \Rightarrow \theta_y = \cos^{-1}(0.6) = 53^0$

iii) $\cos\theta_z = \dfrac{T_z}{T} = \dfrac{360}{750} = 0.48 \Rightarrow \theta_z = \cos^{-1}(0.48) = 61.3^0$

Similarly, for the moment vector, using cosine directions:

i) $\cos\theta_x = \dfrac{M_x}{M} = -\dfrac{648}{1206} = -0.537 \Rightarrow \theta_x = \cos^{-1}(-0.537) = 122.5^0$

ii) $\cos\theta_y = \dfrac{M_y}{M} = -\dfrac{960}{1206} = -0.796 \Rightarrow \theta_y = \cos^{-1}(-0.796) = 142.7^0$

iii) $\cos\theta_z = \dfrac{M_z}{M} = \dfrac{336}{1206} = 0.279 \Rightarrow \theta_z = \cos^{-1}(0.279) = 73.8^0$

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22. The forces $F = 260\text{N}$ and $P = 20\text{N}$ are applied on a parallelepiped as shown in the diagram below. Then,

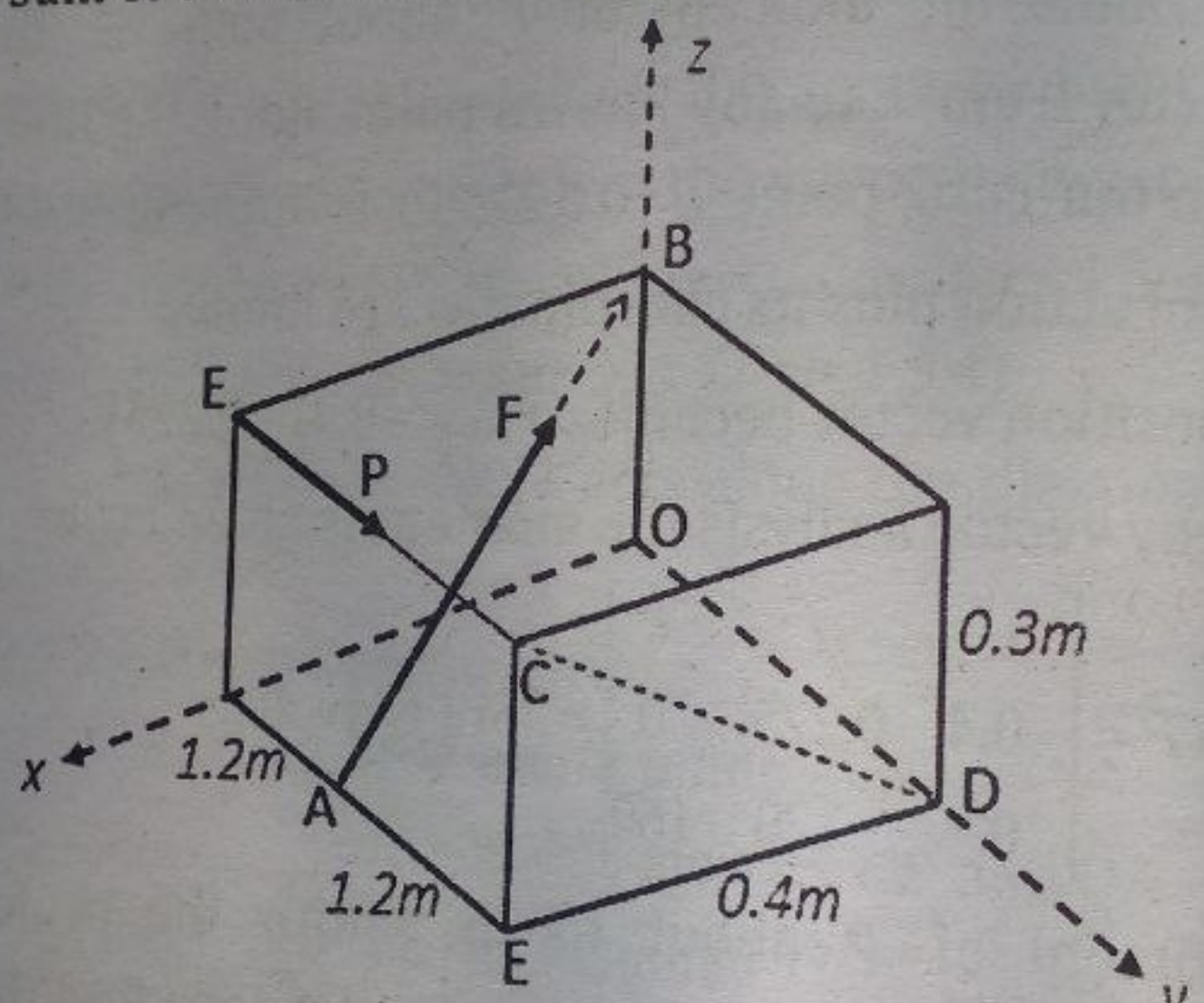
a) Express F in vector form and identify its x, y and z components

b) Determine the projection of the force F on the line CD and give the angle between the force F and the line CD

c) Compute the moment of the force F about the line CD

d) Determine the sum of the moments of the forces F and P about O

e) Give the sum of the moments of F and P about the axis OE



Solution:

a) To express in vector form, first find a unit vector from A to B. Here, the coordinates of A and B are $A=(0.4,1.2,0)$ and $B=(0,0,0.3)$.

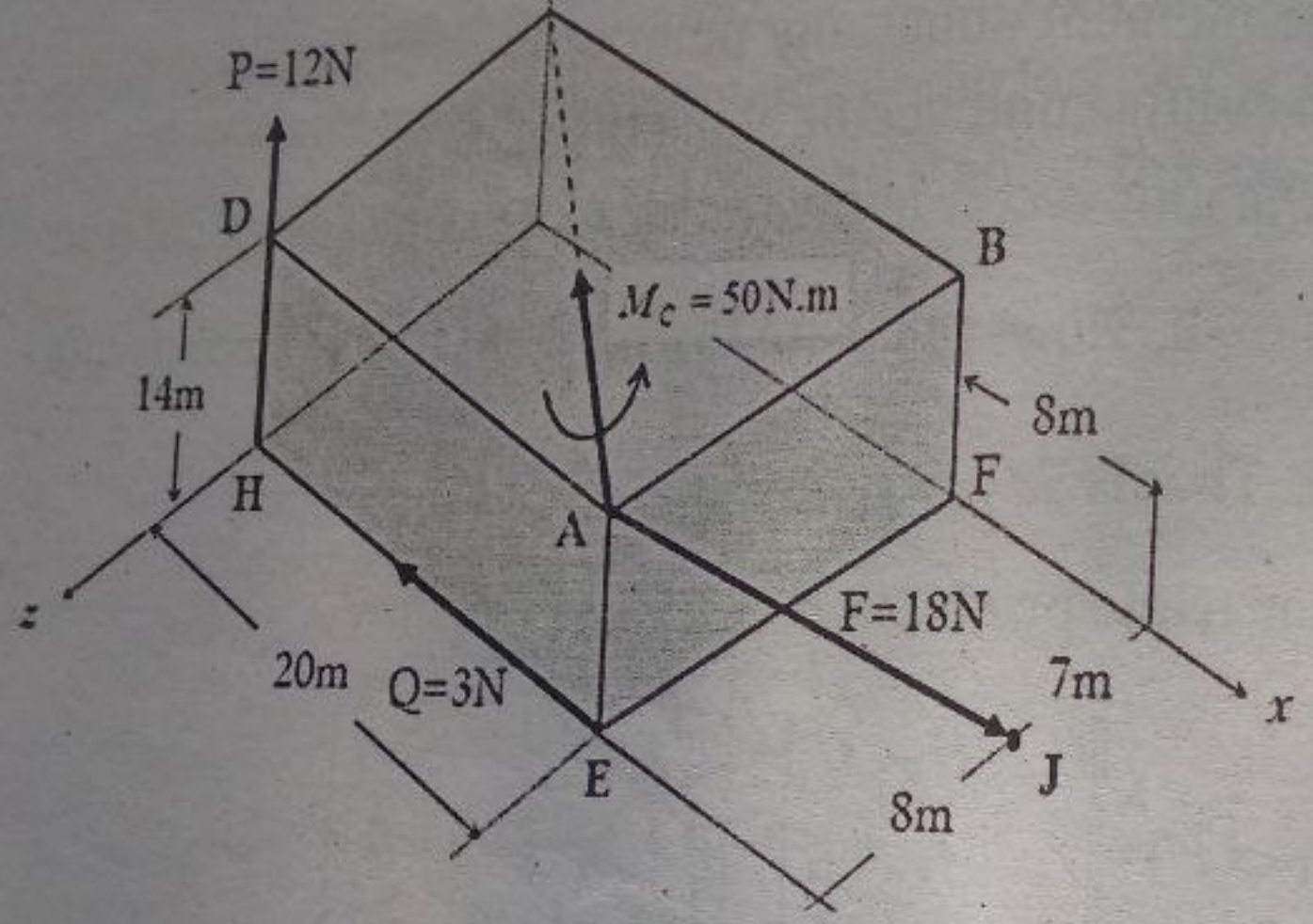
Then, $\vec{U}_{\overline{AB}} = \dfrac{\overrightarrow{AB}}{\left\|\overrightarrow{AB}\right\|} = \dfrac{-0.4\mathbf{i}-1.2\mathbf{j}+0.3\mathbf{k}}{1.3} = -\dfrac{4}{13}\mathbf{i}-\dfrac{12}{13}\mathbf{j}+\dfrac{3}{13}\mathbf{k}.$

So, in vector form $\vec{F} = F\vec{U}_{\overline{AB}} = -80\mathbf{i}-240\mathbf{j}+60\mathbf{k}$

b) To determine the projection of the force F on the line CD, first find a unit vector from C to D and take dot product with F. Here, $C=(0.4,2.4,0.3)$ and $D=(0,2.4,0)$.

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23*. Three forces of magnitude $F=18\text{N}$, $P=12\text{N}$, $Q=3\text{N}$ and a couple $M_C=50\text{N.m}$ are applied on the block as shown. Replace the force-couple system by an equivalent force-couple system at point H.



Solution:

First express the couple M_C and the force F in vector form.

That is $\vec{F}=F.\vec{U}_{\overrightarrow{AJ}}$ and $\vec{M}_C=M_C.\vec{U}_{\overrightarrow{AC}}$.

$$\text{But}\begin{cases}\overrightarrow{AJ}=J-A=8i-14j-8k\Rightarrow\vec{U}_{\overrightarrow{AJ}}=\dfrac{1}{18}(8i-14j-8k)=\dfrac{4}{9}i-\dfrac{7}{9}j-\dfrac{4}{9}k\\ \overrightarrow{AC}=C-A=-20i-15k\Rightarrow\vec{U}_{\overrightarrow{AC}}=\dfrac{1}{25}(-20i-15k)=-\dfrac{4}{5}i-\dfrac{3}{5}k\end{cases}$$

Hence, the force and the couple are expressed in vector form as

$$\begin{cases}\vec{F}=F.\vec{U}_{\overrightarrow{AJ}}=18.(\dfrac{4}{9}i-\dfrac{7}{9}j-\dfrac{4}{9}k)=8i-14j-8k\\ \vec{M}_C=M_C.\vec{U}_{\overrightarrow{AC}}=50(-\dfrac{4}{5}i-\dfrac{3}{5}k)=-40i-30k\end{cases}$$

Besides, in vector form as $\vec{P}=12\text{Nj}$, $\vec{Q}=-3\text{Nk}$.

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Then, $\vec{U}_{\overline{CD}} = \dfrac{\overline{CD}}{\|\overline{CD}\|} = \dfrac{-0.4\mathbf{i} - 0.3\mathbf{k}}{0.5} = -\dfrac{4}{5}\mathbf{i} - \dfrac{3}{5}\mathbf{k}$.

So, $P_{\overline{F}} = F.\vec{U}_{\overline{CD}} = (-80\mathbf{i} - 240\mathbf{j} + 60\mathbf{k}).(-\dfrac{4}{5}\mathbf{i} - \dfrac{3}{5}\mathbf{k}) = 64\text{N} - 36\text{N} = 28\text{N}$

c) To compute the moment of the force F on the axis about the line CD, first find the moment about any point on the axis CD and then take a dot product with a unit vector along the axis.

In this problem the unit vector along the axis CD is already calculated to be $\vec{U}_{\overline{CD}} = -\dfrac{4}{5}\mathbf{i} - \dfrac{3}{5}\mathbf{k}$. Now, calculate the moment of the force at any point on the axis. That is $M_D = \vec{r}_{\overline{DA}} \times F$.

Thus, using scalar triple product,

$$M_{CD} = \vec{U}_{\overline{CD}}.(\vec{r}_{\overline{DA}} \times F) = \begin{vmatrix} U_x & U_y & U_z \\ r_x & r_y & r_z \\ F_x & F_y & F_z \end{vmatrix} = \begin{vmatrix} -\frac{4}{5} & 0 & -\frac{3}{5} \\ 0.4 & -1.2 & -0.3 \\ -80 & -240 & 60 \end{vmatrix}$$

$$= 115.2\text{N.m} + 115.2\text{N.m} = 230.4\text{N.m}$$

d) The resultant moment of the two forces about point O is

$$M_O = \vec{r}_{\overline{OB}} \times F + \vec{r}_{\overline{OE}} \times P = \begin{vmatrix} i & j & k \\ 0 & 0 & 0.3 \\ -80 & -240 & 60 \end{vmatrix} + \begin{vmatrix} i & j & k \\ 0.4 & 0 & 0.3 \\ 0 & 20 & 0 \end{vmatrix}$$

$$= 72i - 24j - 6i + 8k = 66i - 24j + 8k$$

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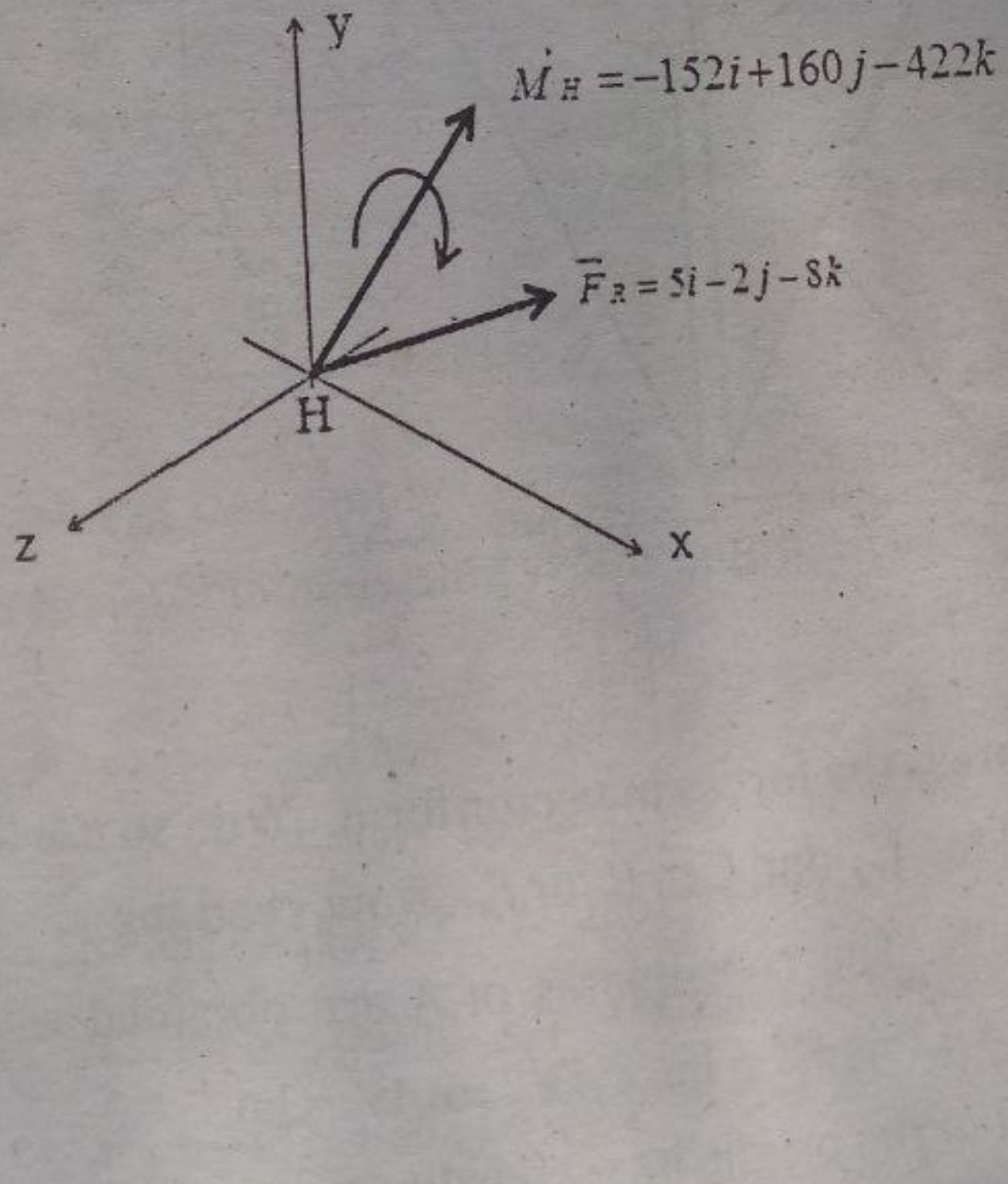
To compute the resultant moment, first of all notice that the two forces $P = 12\text{N}$ and $Q = 3\text{N}$ do not produce moment at H. (Why?) This means the resultant moment is the moment due to $F = 18\text{N}$ only. Therefore, the resultant moment at point H becomes

$$\vec{M}_H = \vec{r}_{HA} \times \vec{F} + \vec{M}_C = \begin{vmatrix} i & j & k \\ 20 & 14 & 0 \\ 8 & -14 & -8 \end{vmatrix} - 40i - 30k$$

$$= -112i + 160j - 392k - 40i - 30k = -152i + 160j - 422k$$

Now, find the resultant force of the system. The resultant force acting on the block is $\vec{F}_R = \vec{F} + \vec{P} + \vec{Q} = 8i - 14j - 8k + 12j - 3i = 5i - 2j - 8k$

Therefore, at point H the force-couple system representation is given by putting the resultant force and the moment $\vec{M}_H$ as shown.



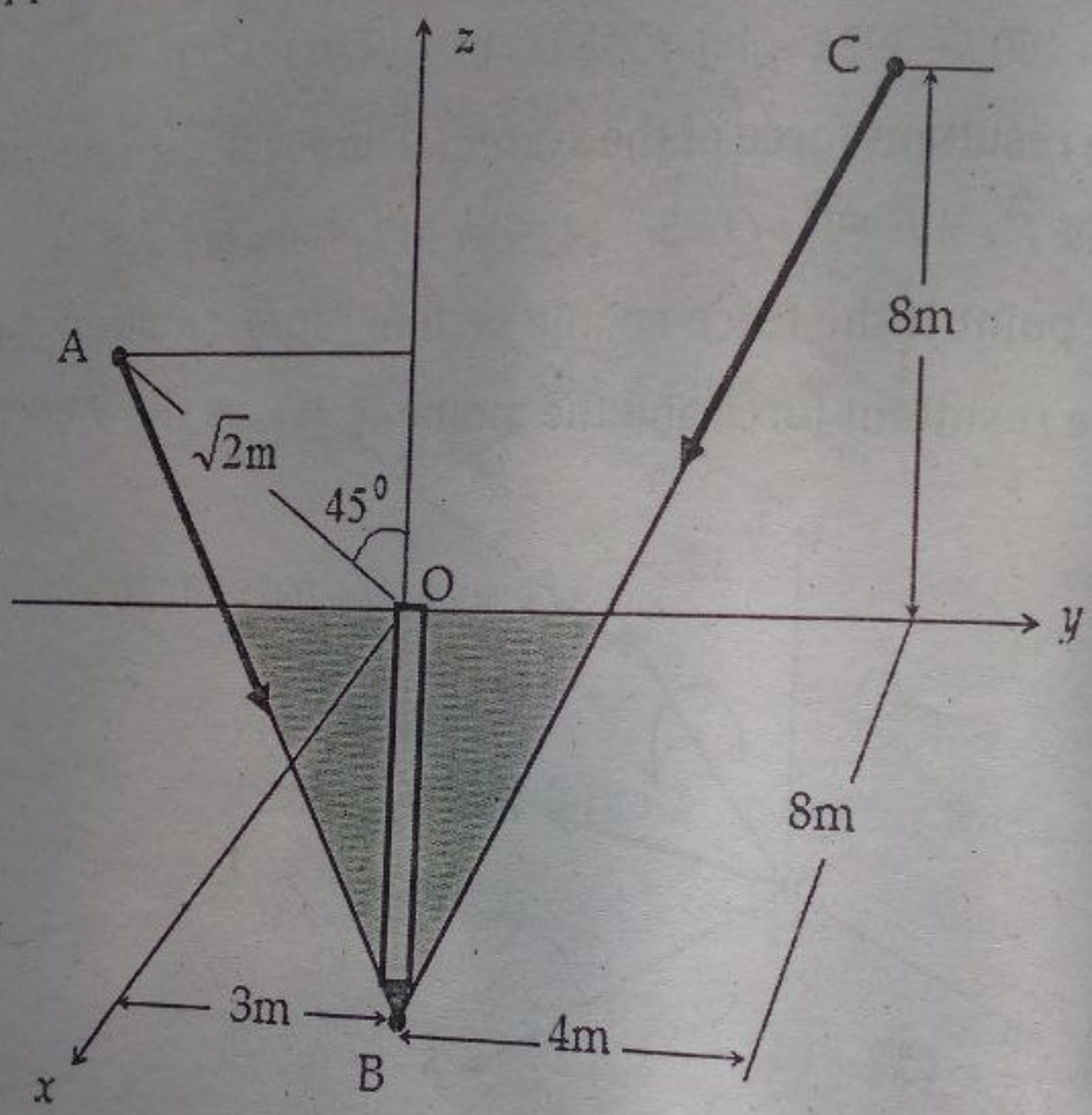
24. Two tension forces $F_A = 36\text{N}$ and $F_C = 60\text{N}$ are acting on a bar OB lying horizontally as shown.

a) Express the forces in vector form and give the angle between them

b) Find the magnitude and directions of the resultant force

c) Replace the force system by an equivalent force-couple at point O

d) At what point P on the z-axis an additional force of $F_P = 56\text{N}$ from P to B is applied so that the net moment at O is zero?



Solution:

a) We need to express the forces in vector form. To do so use the unit vector from A to B for F_A and C to B for F_C. Now, read the coordinates carefully. The coordinates of A are obtained using the trigonometry as $y = \sqrt{2}\sin 45^0 = 1\text{m}$, $z = \sqrt{2}\cos 45^0 = 1\text{m}$.

Since A is to the left of the origin, its y-coordinate is negative.

Hence, the coordinates are $A = (0,-1,1)$, $B = (8,3,0)$, $C = (0,7,8)$.

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Thus, the forces are expressed in vector form as follow;

i) $\vec{F}_A = F_A.\vec{U}_{\overrightarrow{AB}} = F_A.\dfrac{\overrightarrow{AB}}{\overline{AB}} = 36\text{N}\,(\dfrac{8}{9}i+\dfrac{4}{9}j-\dfrac{1}{9}k) = 32\mathbf{i}+16\mathbf{j}-4\mathbf{k}.$

ii) $\vec{F}_C = F_C.\vec{U}_{\overrightarrow{CB}} = F_C.\dfrac{\overrightarrow{CB}}{\overline{CB}} = 60\text{N}\,(\dfrac{8}{12}i-\dfrac{4}{12}j-\dfrac{8}{12}k) = 40\mathbf{i}-20\mathbf{j}-40\mathbf{k}.$

Thus, the angle between them is obtained using dot product.

$$\cos\theta = \frac{\vec{F}_A.\vec{F}_C}{F_A.F_C} = \frac{1120}{2160} = 0.519 \Rightarrow \theta = \cos^{-1}(0.519) = 58.7^0$$

b) The resultant is the sum of the two forces coordinate wise.

That is $\vec{R} = \vec{F}_A + \vec{F}_C = 72\mathbf{i}-4\mathbf{j}-44\mathbf{k}.$

Besides, its magnitude is $R = \sqrt{(72)^2+(-4)^2+(-44)^2} = \sqrt{7136} = 84.5\text{N}.$

Furthermore, for a force vector in space, its direction is specified by computing the cosine directions. That is

$$\begin{cases} \cos\theta_x = \dfrac{R_x}{R} = \dfrac{72}{84.5} = 0.852 \Rightarrow \theta_x = \cos^{-1}(0.852) = 31.6^0 \\ \cos\theta_y = \dfrac{R_x}{R} = -\dfrac{4}{84.5} = -0.047 \Rightarrow \theta_y = \cos^{-1}(-0.047) = 92.7^0 \\ \cos\theta_z = \dfrac{R_x}{R} = -\dfrac{44}{84.5} = -0.521 \Rightarrow \theta_z = \cos^{-1}(-0.521) = 121.4^0 \end{cases}$$

c) The moment at O is simply the sum of the moment due to the two forces. It is easily obtained using vector approach as follow:

$$\vec{M}_O = \vec{r}_{\overrightarrow{OB}} \times \vec{F}_A + \vec{r}_{\overrightarrow{OB}} \times \vec{F}_C = \begin{vmatrix} i & j & k \\ 8 & 3 & 0 \\ 32 & 16 & -4 \end{vmatrix} + \begin{vmatrix} i & j & k \\ 8 & 3 & 0 \\ 40 & -20 & -40 \end{vmatrix}$$

$$= -12i+32j+32k-120i+320j-280k = -132i+352j-248k$$

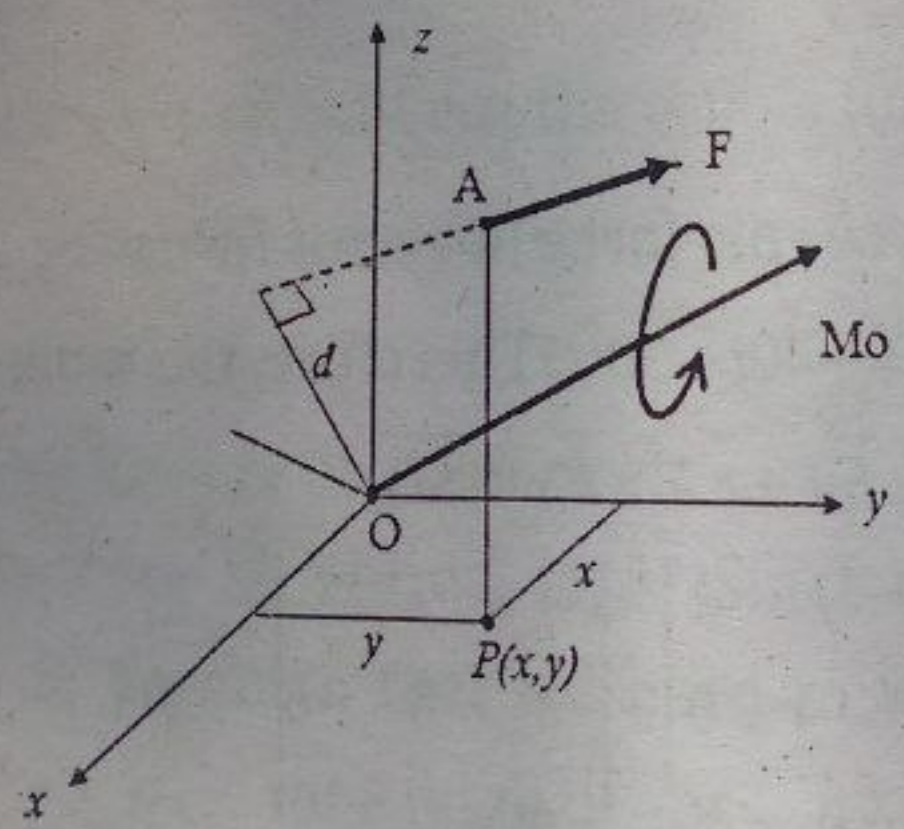
Therefore, the force-couple representation consists of a couple moment of $\vec{M}_O = -132i+352j-248k$ and a resultant force that we computed in part (b) with the directions computed.

b) Now, we need to represent the system by a force only. So, the problem is where this single force must be placed so that it gives the same moment at O as the general force system given.
Let the point where the resultant force crosses the xy-plane be $P=(x,y,0)$. Then, the moment by the resultant force equivalent to the given system is $\overrightarrow{M}_R=\vec{r}_{\overrightarrow{OP}}\times\overrightarrow{F}_R=\begin{vmatrix} i & j & k \\ x & y & 0 \\ 0 & 0 & 200 \end{vmatrix}=200yi-200xj$.

For the systems to be equivalent, we have
$\overrightarrow{M}_R=\overrightarrow{M}_O\Rightarrow 200yi-200xj=800i-240j\Rightarrow y=4m,\ x=1.2m$
Hence, the resultant force crosses the xy-plane at $P=(1.2m,4m,0)$.

27. The force $F=6\text{N}i+8\text{N}j+10\text{N}k$ acting as shown in the diagram below produces a moment of $M_o=-38i+6j+18k$ at O.
If the force F passes through point $P(x,1,z)$,

a) Determine the x and z coordinates of the point P
b) Give the perpendicular distance from O to the line of action of F



Solution:

$M_O=\vec{r}_{OP}\times F=-38i+6j+18k\Rightarrow z=6,x=3$

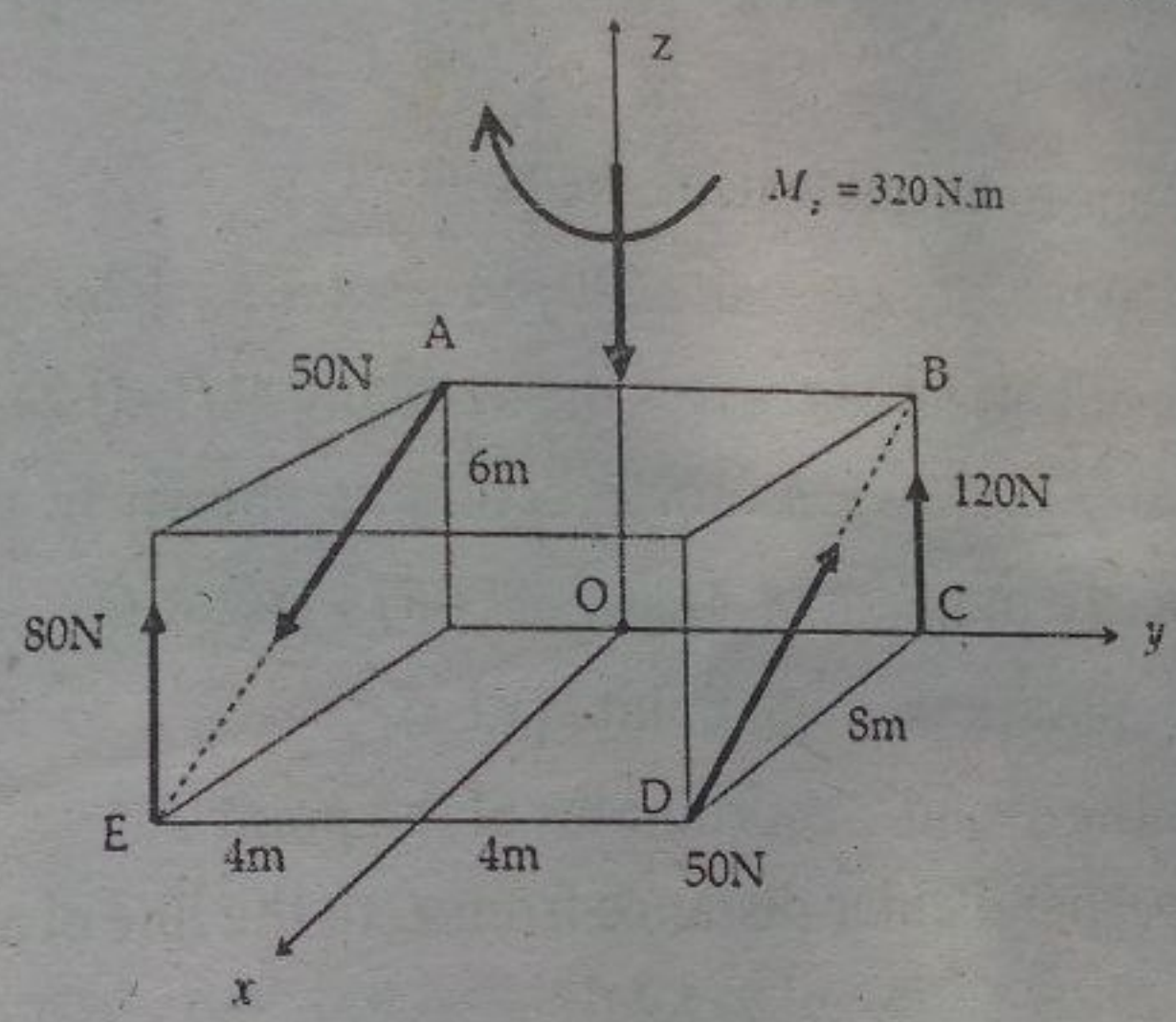
The perpendicular distance is $\|M_O\|=d.\|F\|\Rightarrow d=3m$.

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26. Two forces $F_C = 120\text{N}$, $F_E = 80\text{N}$ and a couple $F = 50\text{N}$ are acting on the rectanguiar box as shown below. If a couple moment of $M_z = 320\text{N.m}$ is also induced about the z-axis,

a) Replace the system by a force-couple system at the origin O

b) Represent the system by a single force and give the point where the line of action of the stand alone force intersects the xy-plane.



Solution:

a) First express the couple force in vector form.

That is $\vec{F}_D = F.\vec{U}_{\overline{DB}} = -40i + 30k$. Therefore, the moment at O becomes

$$\vec{M}_O = \vec{r}_{\overline{OC}} \times \vec{F}_C + \vec{r}_{\overline{OE}} \times \vec{F}_E + \vec{r}_{\overline{AD}} \times \vec{F}_D + \vec{M}_z$$

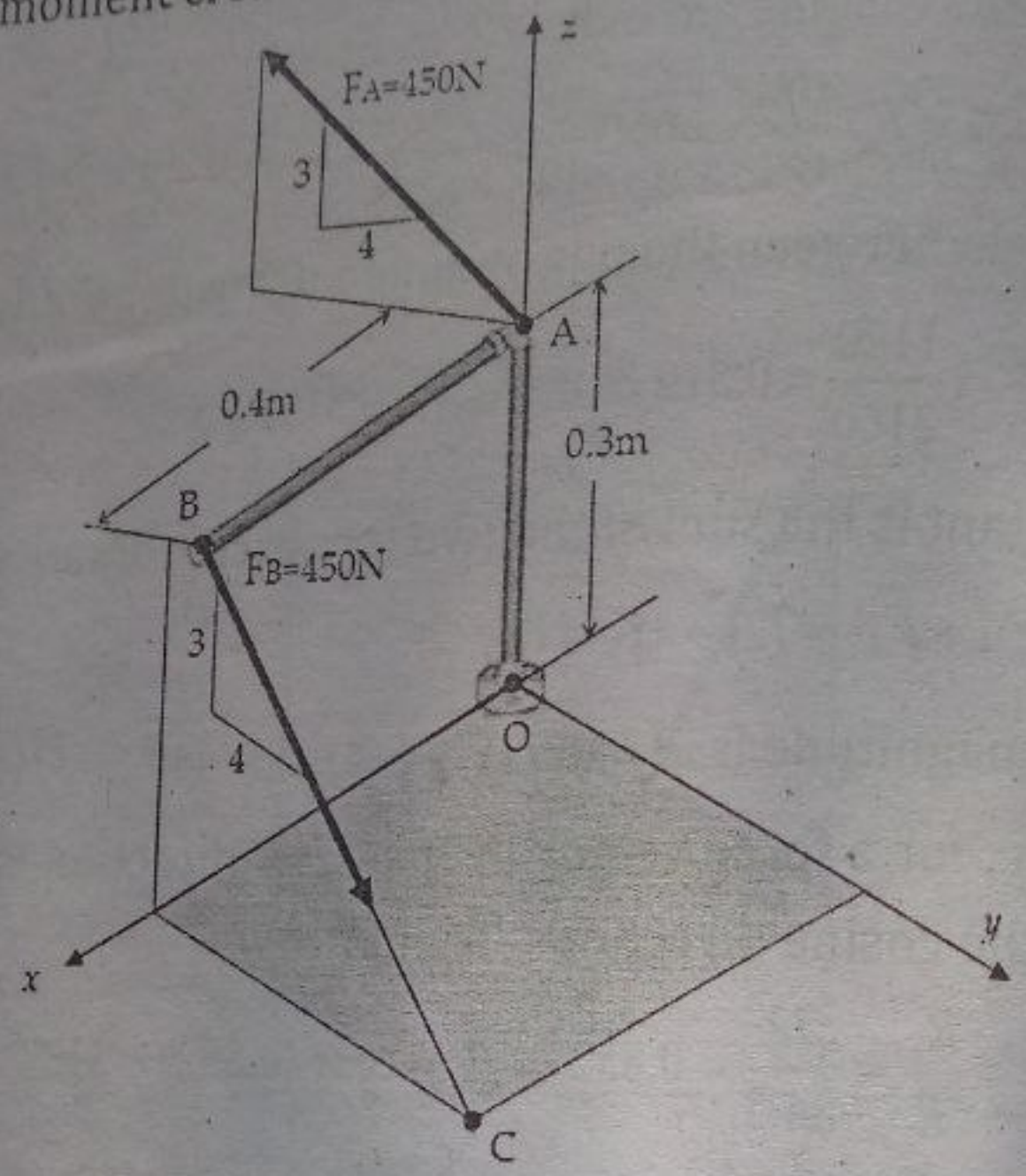
$$= \begin{vmatrix} i & j & k \\ 0 & 4 & 0 \\ 0 & 0 & 170 \end{vmatrix} + \begin{vmatrix} i & j & k \\ 8 & -4 & 0 \\ 0 & 0 & 30 \end{vmatrix} + \begin{vmatrix} i & j & k \\ 8 & 8 & -6 \\ -40 & 0 & 30 \end{vmatrix} - 320k$$

$$= 680i - 120i - 240j + 240i + 320k - 320k = 800i - 240j$$

Besides, the resultant force is $\vec{F}_R = 200\text{k}$. Hence, the system can be replaced by a force-couple system at O where the force is $\vec{F}_R = 200\text{k}$ and the couple moment is $\vec{M}_O = 800i - 240j$.

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25. The couple forces of $F = 450N$ are acting as shown below. Find the couple moment created by these forces.



Solution:

First: Find a position vector between the couple forces.

Let's find $\vec{r}_{\overline{AB}}$. From the given coordinates, $A=(0,0,0.3)$, $B=(0.4,0,0.3)$.

So, the position vector is $\vec{r}_{\overline{AB}} = B - A = 0.4i$.

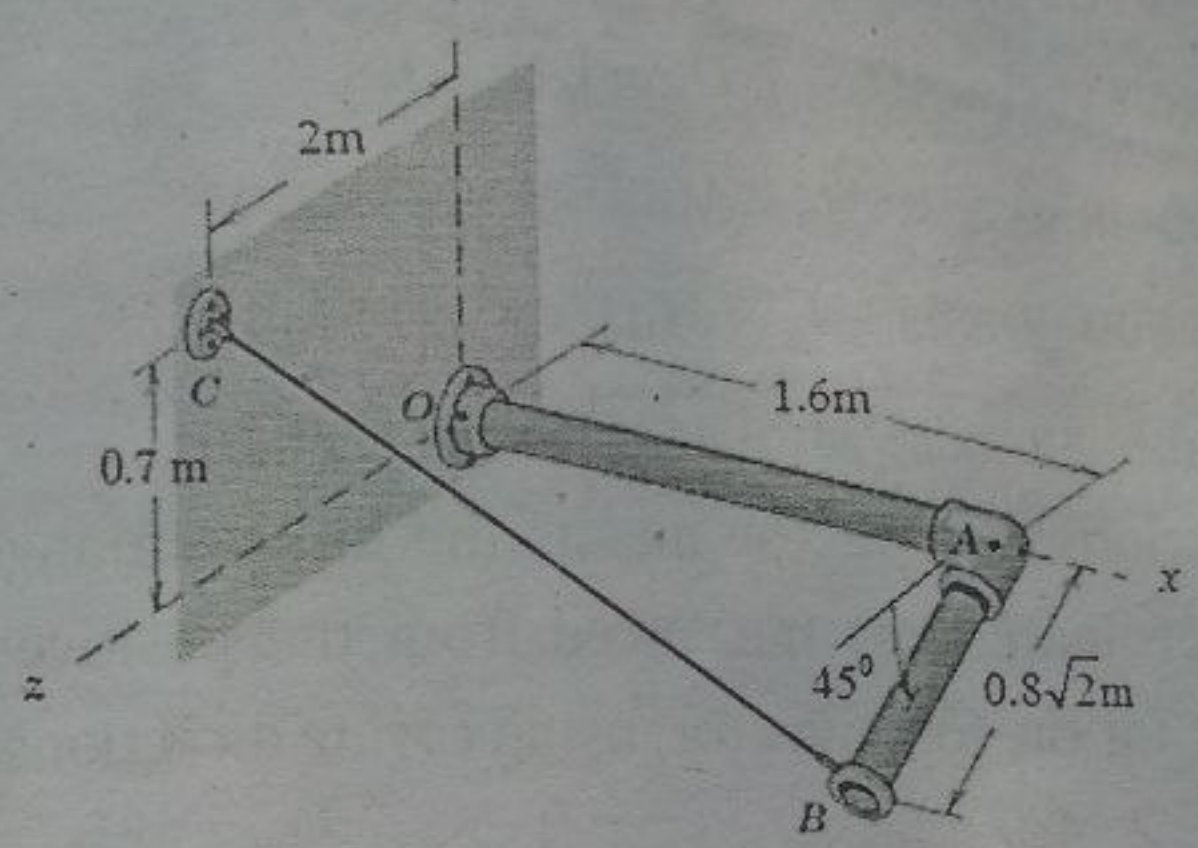
Second: Express the force in vector form.

Watch out! Which force should we take? Since our position vector points from A to B, the force that we need to express is the force acting at point B. But, if we take the position vector $\vec{r}_{\overline{BA}}$ (from B to A), then the force we need is the one acting at A. Hence, by using the given slopes, the force at B is expressed in vector form as $\overline{F} = F(\frac{4}{5}j - \frac{3}{5}k) = 360i - 270k$. Therefore, $M = \vec{r}_{\overline{AB}} \times \overline{F} = 108i + 144k$

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28. The right angle pipe OAB is subjected to a tension force of $T = 750\text{N}$ acting at point B.

a) Express this tension force in vector form using i, j, k unit vectors
b) Replace the tension force 750N by a force-couple system at O
c) Give the moments of this force about the three coordinate axes
d) Give the directions of the force and the resultant moment at O



Solution:

a) We need to express the tension force in vector form. To do so use the unit vector from B to C. Look carefully, how the angle is used to find the coordinates of B. Here, the coordinates are read to be

$B = (1.6, -0.8\sqrt{2}\sin 45^0 0.8\sqrt{2}\cos 45^0) = (1.6, -0.8, 0.8)$, $C = (0, 0.7, 2)$.

Thus, $\vec{U}_{\overline{BC}} = \dfrac{\overrightarrow{BC}}{\overline{BC}} = -\dfrac{1.6}{2.5}i + \dfrac{1.5}{2.5}j + \dfrac{1.2}{2.5}k = -\dfrac{16}{25}\mathbf{i} + \dfrac{3}{5}\mathbf{j} + \dfrac{12}{25}\mathbf{k}$.

Hence, the tension on cable BC is expressed in vector form as;

$$\vec{T} = \text{T}.\vec{U}_{\overline{BC}} = 750\text{N}\left(-\frac{16}{25}\mathbf{i} + \frac{3}{5}\mathbf{j} + \frac{12}{25}\mathbf{k}\right) = -480\text{N}\mathbf{i} + 450\text{N}\mathbf{j} + 360\text{N}\mathbf{k}.$$

b) We need the moment at O. That is

$$\vec{M}_O = \vec{r}_{OC} \times \vec{T} = \begin{vmatrix} i & j & k \\ 0 & 0.7 & 2 \\ -480 & 450 & 360 \end{vmatrix} = -648\mathbf{i} - 960\mathbf{j} + 336\mathbf{k}.$$

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