FM Unit 3 - pre mid and post

Business Analytics (Christ (Deemed To Be University))



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3

Chapter

TIME VALUE OF MONEY

Learning Objectives

After studying this chapter, you should be able to understand :

- *The concept of time value of money.*
- *Reasons for time preference of money.*
- *The techniques for adjusting the time value of money.*
- *Compounding technique and discounting or present value technique.*
- *Practical applications of time value techniques.*

■ INTRODUCTION

We have observed in the previous chapter that profit maximisation objective ignores the time value of money and does not consider the magnitude and timing of earnings. It treats all earnings as equal though they occur in different periods. Thus, maximisation of shareholder's wealth is considered to be an appropriate objective of a firm. Given the objective of wealth maximisation, the firm should, while making financial decisions, consider those things which are important to investors. Financial theory asserts that as the shareholders make a current sacrifice by investing their funds into the firm, they expect to get some future benefit, either as dividend or increased share price when the shares are sold or both. However, the expected benefit is a future one while the sacrifice is a current one. Hence, the investors expect a rate of return on their investment that compensates them the duration of the sacrifice deferral. This leads us to the study of the ratio of return of waiting or the time value of money.ZZ

Most of the financial decisions, such as acquisition of assets or procurement of funds affect firm's cash flows in different time periods. If a firm acquires an asset today, it will require an immediate cash outlay, but the benefit of this asset will be received in future. Thus, it will have an effect on future cash flows over the life of the asset will be received in future. Thus, it will have an affect on future cash flow over the life of the asset. Similarly, if funds are raised through borrowings for

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present needs, these will have to be returned in future as principal and interest. A firm may also raise additional funds through issue of share capital, but it will have to pay dividend on the share capital in future. While taking such financial decision, the firm will have to compare the total of cash inflows with the total cash outflows. The logical way is to recognise the time value of money and make appropriate adjustment for time otherwise it may take faulty decisions. The owner's utility will be maximised when net present worth is created from the financial decisions.

■ CONCEPT OF TIME VALUE OF MONEY

The simple concept of time value of money is that the value of the money received today is more than the value of same amount of money received after a certain period. In other words, money received in the future is not as valuable as money received today. the sooner one receives money, the better it is . Taking the case of a rational human being, given the option to receive a fixed amount of money at either of the two time periods, he will prefer to receive it at the earliest. If you are given choice of receiving ₹ 1000 today or after one year, you will definitely opt to receive today than after one year. This is because you value the current receipt of money higher than future receipt of money after one year. The phenomenon is referred to as *time preference for money*.

■ REASONS FOR TIME PREFERENCE OF MONEY

(i) The future is always uncertain and involves risk. An individual can never be certain of getting cash inflow in future and hence he will like to receive money today instead of waiting for the future.

(ii) People generally prefer to use their money for satisfying their present needs in buying more food, or clothes or another car, than deferring them for future. The present needs are considered urgent as compared to future needs, Moreover, there may also be a fear in one's mind that he may not be able to use the money in future for fear of illness or death.

(iii) Money has time value because of the opportunities available to invest money received at earlier dates at some interest or otherwise to enhance future earnings. For example, if you have ₹ 100 today, you can put it in your bank account and earn interest. After one year the interest would be ₹ 8 (taking rate of interest at 8% p.a.) and you would have ₹ 108 at the end of the year. So, if you have a choice between ₹ 100 today or after one year, it is the same as a choice between ₹ 108 next year or ₹ 100 next year. Any rational person would prefer the larger amount.

■ TECHNIQUES OF TIME VALUE OF MONEY

There are two techniques for adjusting the time value of money :

(i) Compounding Technique

(ii) Discounting or Present Value Technique.

■ (I) COMPOUNDING TECHNIQUE

The time preference for money encourages a person to receive the money at present instead of waiting for future. But he may like to wait if he is duly compensated for the waiting time by way of ensuring more money in future. For example, a person being offered ₹ 100 today may wait for a year

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if he is ensured of ₹ 110 at the end of one year (taking his preference for an interest of 10% p.a.) The cash flow of ₹ 100 at present or ₹ 110 after one year will be the same for this person.

The future value at the end of period 1 can be calculated by a simple formula given below :

$$V_1 = V_0 (1 + i)$$

where, V_1 = Future value at the period 1

V_0 = Value of money at time 0, *i.e.* original sum of money

i = Interest rate.

Taking the example given above,

$$V_1 = 100 (1+.10)$$

$$= ₹ 110.$$

In the example given above, we have only considered the future value after one period. But we may need to calculate future values over longer periods. For example, what will be the value of ₹ 100 after two years at 10% p.a. rate of interest if neither the principal sum of ₹, 100 nor interest is withdrawn a the end of one year? The answer to this question lies in understanding that the second year's interest will be paid on both original principal and the interest earned at the end of first year. This paying of interest is called *compounding*.

The value of money after 2 years can be calculated as :

$$V_2 = V_1 (1+ i)$$

$$= 110 (1+.10)$$

$$= ₹ 121.$$

Similarly, the value of ₹ 100 after 3 years shall be :

$$V_3 = V_2 (1 + i)$$

$$= 121 (1+.10)$$

$$= ₹ 133.10$$

And in the same manner we could calculate the time value of money after any number of years. We can generalise that the future value of a current sum of money at period n is:

$$V_n = V_0 (1+ i)^n$$

So, after 10 years the value of ₹ 100 at 10% rate of interest shall be :

$$V_{10} = 100 (1 +.10)^{10}$$

$$= 100 (1.10)^{10}$$

$$= ₹ 259.4$$

❒ Compound Factors Tables

We have noted above that as n becomes large, the calculation of $(1+i)^n$ becomes difficult. Such calculations can be made with the help of Compound Factor Tables given at the end of this book.

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For clear understanding, a portion of the table is reproduced below :

Compound Factor Table*

Period (n)	Percent (i)				
	2	4	6	8	10
1.....	1.020	1.040	1.060	1.080	1.100
2.....	1.040	1.082	1.124	1.166	1.210
3.....	1.061	1.125	1.191	1.260	1.331
4.....	1.082	1.170	1.262	1.360	1.464
5.....	1.104	1.217	1.338	1.469	1.611
6.....	1.126	1.265	1.419	1.587	1.772
7.....	1.149	1.316	1.504	1.714	1.949
8.....	1.172	1.369	1.594	1.851	2.144
9.....	1.195	1.423	1.689	1.999	2.358
10.....	1.219	1.480	1.791	2.159	2.594

* For a more complete set of compound factors see Table A at the end of the book.

Using the Compound Factor Tables, the future value of money can be calculated as below :

$$V_n = V_0 (C F_{i, n})$$

(where $CF_{i, n}$ is compound factor at (i) percent and n periods.)

or $V_{10} = V_0 (2.594)$

$= ₹ 259.4$

❐ Doubling Period

Compound Factor Tables can be easily used to calculate the doubling period, i.e. the length of period which an amount is going to take to double at a certain given rate of interest. For example, we find from Table A that it takes about 7 year to double the amount at 10 per cent and about 6 years as 12 per cent rate of interest.

Doubling period can also be calculated by adopting the following Rules of Thumb:

1. Rule of 72

$$\text{Doubling Period} = \frac{72}{\text{Rate of Interest}}$$

2. Rule of 69

$$\text{Doubling Period} = 0.35 + \frac{69}{\text{Rate of Interest}}$$

ILLUSTRATION 1. If you deposit ₹ 5,000 today at 6 per cent rate of interest, in how many years will this amount double ? Work out this problem by using the Rule of 72 and Rule of 69.

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SOLUTION :

According to Rule of 72

$$\text{Doubling Period} = \frac{72}{\text{Rate of Interest}}$$

$$= \frac{72}{6} = 12 \text{ years}$$

According to Rule of 69

$$\text{Doubling Period} = 0.35 + \frac{69}{\text{Rate of Interest}}$$

$$= 0.35 + \frac{69}{6} = 0.35 + 11.50$$

$$= 11.85 \text{ years.}$$

❐ Multiple Compounding Periods

So far we have considered only the compounding of interest annually. But in many cases, interest may have to be compounded more than once a year. For example, banks may allow interest on quartely basis; or a company may allow compounding of interest twice a year on 30th June and 31st December every year.

The future value of money in such cases can be calculated as below :

$$V_n = V_0 \left(1 + \frac{i}{m}\right)^{m \times n}$$

where V_n = Future value of money after n years

V_0 = Value of money at time O, *i.e.* original sum of money .

i = Interest rate

m = Number of times (Frequency) of compounding per year.

ILLUSTRATION 2. Calculate the compound value of ₹ 10,000 at he end of 3 year at 12% rate of interest when interest is calculated on

(a) yearly basis, and (b) quarterly basis.

SOLUTION :

(*a*) When interest is compounded on yearly basis

$$V_n = V_0 (1+i)^n$$

$$= 10{,}000 (1+.12)^3$$

$$= 10{,}000 (1.405)$$

$$= ₹ 14{,}050$$

(*b*) When interest is compounded on quarterly basis

$$V_n = V_0 \left(1 + \frac{i}{m}\right)^{m \times n}$$

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$$= 10{,}000\ \left(1+\frac{.12}{4}\right)^{4\times3}$$

$$= 10{,}000\ (1.03)^{12}$$

$$= ₹\ 14{,}260.$$

Alternatively, we can use Compound Factor Table (A). The value of Re. 1 compounded at 3% (12÷4) rate of interest for 12 years (4×3) is ₹ 1.426. Thus, the value of ₹ 10,000 shall be : ₹ 10,000×1.426 = ₹ 14,260.

❐ Effective Rate of Interest In Case of Multi-Period Compounding

We have noticed above that amount grows faster in case of multi-period compounding, *i.e.* when frequency of interest compounding is more than once a year. It is so because the actual rate of interest realised, called effective rate in case of multi- period compounding is more than the apparent annual rate of interest called nominal rate. To illustrate, we can take a simple example of the future value of ₹ 100 at the end of one year at 10% rate of interest calculated (i) annually and (ii) on half yearly basis.

In case of yearly compounding, ₹ 100 shall grow to ₹ 110 at the end of the year.

But if compounding is made half-yearly, it will grow as below :

At the end of first six months to ₹ 105, *i.e.*

100 + 5% of ₹ 100.

At the end next six months to ₹ 110.25 *i.e.*

105 + 5% of ₹ 105.

Hence the total interest realised in case of half yearly compounding is ₹ 5 + 5.25 *i.e.* ₹ 10.25 or we can say that effective rate of interest is 10.25% while the nominal rate is 10%.

Effective rate of interest in case of multi-period compounding can also be calculated with the use of following formula :

$$EIR = \left(1+\frac{i}{m}\right)^{m} - 1$$

Where; EIR = Effective rate of interest

i = Nominal rate of interest

m = Frequency of compounding per year

ILLUSTRATION 3. A company offers 12 per cent rate of interest on deposits. What is the effective rate of interest if the compounding is done (i) half yearly, (ii) quarterly, and (iii) monthly ?

SOLUTION :

(*i*) When compounding is done half-yearly, the effective rate of interest shall be:

$$EIR = \left(1+\frac{i}{m}\right)^{m} - 1$$

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$$= 1+\frac{.12}{2}^{2} - 1$$

$$= 0.1236 = 12.36\%$$

(*ii*) When compounding is done quarterly, the EIR shall be :

$$EIR = 1+\frac{.12}{12}^{4} - 1$$

$$= 0.1255 = 12.55\%$$

(*iii*) When compounding is done monthly, the EIR shall be :

$$EIR = 1+\frac{.12}{12}^{12} - 1$$

$$= 0.1268 = 12.68\%$$

❐ Future Value of a Series of Payments

So far we have considered only the future value of a single payment made at time zero. But in many instances, we may be interested to know the future value of a series of payments made at different time periods.

This can be calculated as below :

$$V_n = R_1 (1+ i)^{n-1} + R_2 (1+ i)^{n-2} +...(R_{n-1}) (1+i) + R_n$$

where, V_n = Future value at period n

R_1 = Payment after period 1

R_2 = Payment made after period 2

R_n = Payment made after period n

i = Rate of interest.

ILLUSTRATION 4. Calculate the future value at the end of five years of the following series of payments at 10% rate of interest :

R_1 = ₹ 1,000 at the end of the first year

R_2 = ₹ 2,000 at the end of the 2nd year

R_3 = ₹ 3,000 at the end of the 3rd year

R_4 = ₹ 2,000 at the end of the 4th year

R_5 = ₹ 1,500 at the end of the 5th year.

SOLUTION :

The future value of this series of payment will be :

$$V_5 = 1000 (1.10)^4 + 2000 (1.10)^3 + 3000(1.10)^2 + 2000 (1.10)+ 1500$$

$$= 1000 (1.464) + 2000 (1.331) + 3000 (1.210) + 2000 (1.100) + 1500$$

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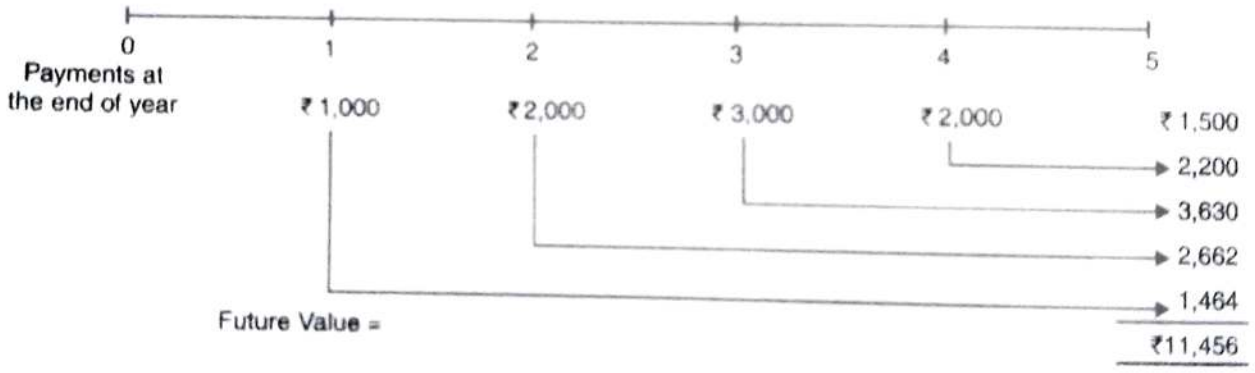
= ₹ 1464 + 2662 + 3630 + 2200 + 1500
= ₹ 11,456

It can also be calculated as given in the following table :

End of year	*Amount of Payment*	*Number of years Compounded*	*Compound Interest Factor*	*Future value*
1	2	3	4	5 = 2 x 4
	₹			₹
1	1,000	4	1.464	1464
2	2,000	3	1.331	2662
3	3,000	2	1.210	3630
4	2,000	1	1.100	2200
5	1,500	0	1.000	1500
	Future value at the end of 5 years			11,456

Time Scale for Future Value of Series of Payments

Future value of series of payment can also be shown on a time scale as below :



❐ Compounded Value of an Annuity

An annuity is a series of equal payments lasting for some specified duration. The premium payments of life insurance company, for example, are an annuity. When the cash flows occur at the end of each period the annuity is called a regular annuity or a **deferred annuity**. If the cash flows occur at the beginning of each period the annuity is called and **annuity due.**

Since the payments are equal in an annuity–

$R_1 = R_2 = R_3 \ldots . R_n = R$. The future value of an annuity can be calculated as below:

$$V_n = (R)\,(1+i)^{n-1} + (R)\,(1+i)^{n-2} + \ldots . (R)\,(1+i)^1 + R$$

$$\text{Or } V_n = (R)\,[(1+i)^{n-1} + (1+i)^{n-2} + \ldots . (1+i)^1 + 1]$$

❐ Annuity Compound Factor Tables

Compound value of an annuity can also be calculated with the help of Annuity Compound Factor Tables given at the end of book.

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A portion of the Annuity Compound Factor Tables is given below for ready use:

Period (n)	Annuity Compound Factor Table*				
	Percent (i)				
	2	4	6	8	10
1.....	1.000	1.000	1.000	1.000	1.000
2.....	2.020	2.040	2.060	2.080	2.100
3.....	3.060	3.122	3.184	3.246	3.310
4.....	4.122	4.246	4.375	4.506	4.641
5.....	5.204	5.416	5.637	5.867	6.105
6.....	6.308	6.633	6.975	7.336	7.716
7.....	7.434	7.898	8.394	8.923	9.487
8.....	8.583	9.214	9.897	10.637	11.436
9.....	9.755	10.583	11.491	12.488	13.579
10.....	10.950	12.006	13.181	14.487	15.937

* For a more complete set of annuity compound factors see Table B at the end of the book.

Making use of the annuity compound factor tables, we can calculate the future value of an annuity as :

$$V_n = (R)\,(ACF_{i,\,n})$$

ILLUSTRATION 5. Mr. A deposits ₹ 1,000 at the end of every year for 4 years and the deposit earns a compound interest @ 10% p.a. Determine how much money he will have at the end of 4 years ?

SOLUTION :

(a) Using Compound Factor Tables

$$V_n = (R)\,(1+i)^{n-1} + (1+i)^{n-2} + (1+i)^{n-3} + 1]$$

$$= 1{,}000\,[(1+.10)^{4-1} + (1+.10)^{4-2} = (1+.10)^{4-3} + 1]$$

$$= 1{,}000[(1.10)^3 + (1.10)^2 + (1.10)^1 + 1]$$

$$= 1{,}000\,[4.641]$$

$$= ₹\,4{,}641.$$

(b) Using Annuity Compound Factor Tables

$$V_n = (R)\,(ACF_{i,\,n})$$

$$= (1{,}000)(4.641)$$

$$= ₹\,4{,}641.$$

❒ Compound Value of an Annuity Due

When the cash flows occur at the beginning of each period the annuity is called an annuity due.

The future value of an annuity due can be calculated as below :

$$V_n = (R)\,(1+i)^n + (R)\,(1+i)^{n-1} + \ldots + (R)\,(1+i)^1$$

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$$= R \frac{(1+i)^n - 1}{i} (1 + i)$$

Making use of the annuity compound factor tables, we can calculate the future value of an annuity due as :

$$V_n = (R) (ACF_{i, n}) (1+i)$$

ILLUSTRATION 6. Mr. X deposits ₹ 5,000 at the beginning of each year for 5 years in a bank and the deposit earns a compound interest @8% p.a. Determine how much money he will have at the end of 5 years?

SOLUTION :

(a) Using Compound Factor Tables

$$V_n = (R)\ (1+i)^n + (R)\ (1+i)^{n-1} + \ldots (R)\ (1+i)^1$$

$$= 5000[(1+.08)^5 + (1 +.08)^4 + (1+.08)^3 + (1+.08)^2 + (1.08)^1]$$

$$= 5000[1.469 + 1.360 + 1.260 + 1.166 + 1.080]$$

$$= 5000[6.335)$$

$$= ₹\ 31,675$$

(b) Using Annuity Compound Factor Tables

$$V_n = (R)\ [(ACF_{i, n})\ (1+i)]$$

$$= (5000)\ [(5.867)\ (1.08)]$$

$$= 5000[6.33636]$$

$$= ₹\ 31,681$$

■ (ii) DISCOUNTING OR PRESENT VALUE TECHNIQUES

Present value is the exact opposite of compound or future value. While future value shows how much a sum of money becomes at some future period, present value shows what the value is today of some future sum of money. In compound or future value approach the money invested today appreciates because the compound interest is added to the principal. The present value of money to be received on future date will be less because we have lost the opportunity of investing it at some interest. Thus, the present value of money to be received in future will always be less. It is for this reason that the present value technique is called discounting.

Suppose, for example, you have an opportunity to buy a debenture today and you will get back ₹ 1,000 after one year. What will you be willing to pay for the debenture today if your time preference for money is 10 percent per annum ?

We can calculate the present value of ₹ 1,000 to be received after one year at 10% time preference rate as below :

$$V_n = V_0 (1 + i)$$

Where V_n = Future value n period

and V_0 = Present Value)

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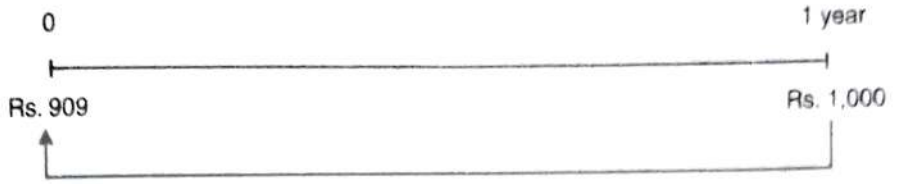
or, $$V_0 = \frac{V_n}{(1+i)}$$

$$= \frac{1000}{1.10}$$

$$= ₹\ 909$$

The concept of present value can be represented on time scale as shown below:

Present value Time Scale (i = 10 percent per annum)



❐ Present Value or Discount Factor Tables

We have observed in compounding technique that as n becomes large, the calculation of $(1+i)^n$ becomes difficult. To calculate the present values in such cases we can make use of Present Value or Discount Factor Tables given at the end of this book.

A potion of the table is reproduced below for ready use :

Discount Factor Table*

Period (n)	*Percent (i)*				
	2	*4*	*6*	*8*	*10*
1.....	0.980	0.962	0.943	0.926	0.909
2.....	0.961	0.925	0.890	0.857	0.826
3.....	0.942	0.889	0.840	0.794	0.751
4.....	0.924	0.855	0.792	0.735	0.683
5.....	0.906	0.822	0.747	0.681	0.621
6.....	0.888	0.790	0.705	0.630	0.564
7.....	0.871	0.760	0.665	0.583	0.513
8.....	0.853	0.731	0.627	0.540	0.467
9.....	0.837	0.703	0.592	0.500	0.424
10.....	0.820	0.676	0.558	0.463	0.386

* For a more complete set of discount factors see Table C at end of the book.

The discount factor for (i) percent interest and n periods is $DF_{1, n}$ which is equal to $\frac{1}{(1+i)^n}$

or $$DF_{i, n} = \frac{1}{(1+i)^n}$$

Present value (V_0) = Future Value (V_n) x $DF_{i, n}$

ILLUSTRATION 7. Mr. X is to receive ₹ 5,000 after 5 years from now. His time preference for money (rate of interest) is 10 percent per annum. Calculate its present value by using discount factor tables.

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SOLUTION

We look to the discount factor tables 10% column and the row for the 5th year. The discount factor is .621.

Precent Value (V_0) = Future Value $(V_n) \times DF_{i,n}$

$= 5{,}000 \times .621$

$= ₹\ 3{,}105$

❒ Present Value of a Series of Payments

So far, we have considered only present value of a single payment to be received or paid after a certain period. But in many instances we may have to calculate present value of several sums of money, each occurring at different point of time.

If series of payments is represented by R_1, R_2, R_3 etc. the present value of such a series of payments will be :

$$V_0 = \frac{R_1}{(1+i)} + \frac{R_2}{(1+i)^2} + \ldots\ldots \frac{R_{n-1}}{(1+i)^{n-1}} + \frac{R_n}{(1+i)^n}$$

or, $$V_0 = \sum_{t=1}^{n} \frac{R_t}{(1+i)^t}$$ where R_1 is the payment at period t.

We can also calculate the present value of a series of payments by finding out present values of such individual payments and then adding these present values. It has been explained in the following illustration.

ILLUSTRATION 8. Calculate present value of the following cash flows assuming a discount rate of 10%.

Year	*Cash Flows*
1	₹ 5,000
2	10,000
3	10,000
4	3,000
5	2,000

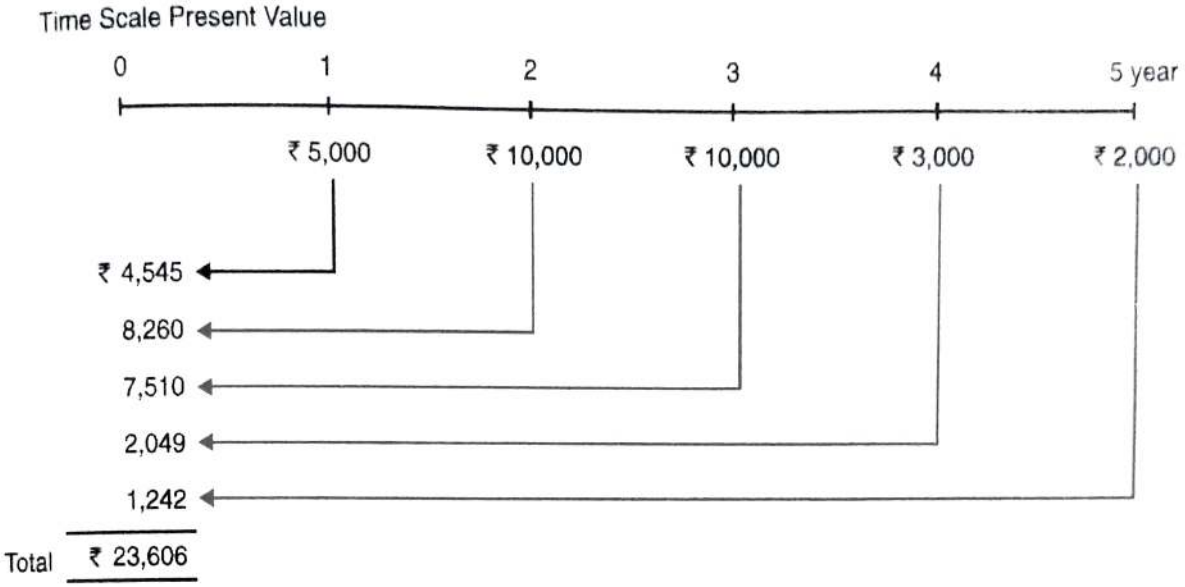
SOLUTION :

Calculation of Present Value

Year (1)	*Cash Flows* (2)	*Present Value Factor (at 10%)* (3)	*Present Value of Cash Flows* (4=2x3)
1	₹ 5,000	909	₹ 4,545
2	10,000	.826	8,260
3	10,000	.751	7,510
4	3,000	.683	2,049
5	2,000	.621	1,242
	Total present value of cash flows		₹ 23,606

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Time Scale Present Value



Annuity Discount Factor Table*

Period (n)	Percent (i)				
	2	4	6	8	10
1	0.980	0.962	0.943	0.926	0.909
2	1.942	1.886	1.833	1.783	1.736
3	2.884	2.775	2.673	2.577	2.487
4	3.808	3.630	3.465	3.312	3.170
5	4.713	4.452	4.212	3.993	3.791
6	5.601	5.242	4.917	4.623	4.355
7	6.472	6.002	5.582	5.206	4.868
8	7.325	6.733	6.210	5.747	5.335
9	8.162	7.435	6.802	6.247	5.759
10	8.983	8.111	7.360	6.710	6.145

For a more complete set of annuity discount factors see Table D at the end of the book.

Present Value of an Annuity

An annuity is a series of equal payments lasting for some specific period. If the amount of payment is R, the present value of an annuity can be calculated as :

$$V_0 = R \sum_{i=1}^{n} \frac{1}{(1+i)^t}$$

Using the *Annuity Discount Factor Tables,* The present value of annuity can be calculated by multiplying the annuity payment with the annuity discount factor.

$$V_0 = (R)(ADF_{i,n})$$

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ILLUSTRATION 9. Mr. X has to receive ₹ 2,000 per year for 5 years. Calculate the present value of the annuity assuming that he can earn interest on his investment at 10% p.a.

SOLUTION :

$$V_0 = (R)\,(ADF_{i,n})$$

In the given question, R = ₹ 2,000

From the Annuity Discount Tables we find that the annuity discount factor at (i), 10 percent and n, 5 years is 3.791

Present Value $V_0 = 2,000 \times 3.791$

$= ₹ 7,582$

❒ Present Value of an Annuity Due

The Present value of an annuity due, *i.e.* if the cash flows occur at the beginning of each year can be calculated by using the Present Value Tables as below :

$$V_0 = (R)\,(ADF_{i,n})\,(1 + i)$$

ILLUSTRATION 10. Mr. A has to receive ₹ 1000 at the beginning of each year for 5 years. Calculate the present value of the annuity due assuming 10% rate of interest.

SOLUTION :

$$V_0 = (R)\,(ADF_{i,n})\,(1 + i)$$

$$= 1000\,[3.791 \times 1.10]$$

$$= 1000\,[4.17]$$

$$= ₹ 4170$$

❒ Present Value of an Infinite Life Annuity

The present value of an infinite (a) life annuity can be calculated as :

$$V_0 = (R)\,(ADF_{i,\alpha})$$

or, $$V_0 = \frac{R}{i}$$

This is because as the length of time of the annuity increases, the discount factors increase, but when the length of time gets very long, this increase in annuity factor slows down. And as it becomes infinitely long the annuity discount factor reaches the upper limit, *i.e.*, $1/i$.

ILLUSTRATION 11. Calculate the present value of ₹ 1,000 received in perpetuity for an infinite period, taking discount rate of 10%.

SOLUTION :

$$V_0 = \frac{R}{i}$$

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$$= \frac{1,000}{.10} = ₹10,000$$

❒ Present Value of an Annuity Growing at a Constant Rate

When cash flows grow at a constant rate, we will have to calculate the series of cash flows first and then the present value of the series of payments as demonstrated in the following illustration.

ILLUSTRATION 12. Mr. X has rented out a portion of his house for 4 years at an annual rent of ₹ 6,000 with the stipulation that rent will increase by 10 per cent every year. If the required rate of return is 15 per cent, what is the present value of the expected series of rent ?

SOLUTION :

(a) Calculation of the Series of Payments of Rent

Year End	*Amount of Rent (₹)*
1	6000
2	6600 (6000 × 1.10)
3	7260 (6600 × 1.10)
4	7986 (7260 × 1.10)

(b) Calculation of Present Value

Year End	*Cash Flows*	*P.V. Factor*	*P.V. of Cash Flows*
	(₹)	*at 15%*	*(₹)*
1	6000	0.870	5,220
2	6600	0.756	4,990
3	7260	0.658	4,777
4	7986	0.572	4,568
			19,555

■ PRACTICAL APPLICATIONS OF TIME VALUE TECHNIQUES

The time value techniques of compounding and present value can be applied by the financial manager while taking a number of important financial decisions.

Some important applications are explained below :

❒ 1. Sinking Fund Problems

A financial manager may have to determine the amount of annual payments so as to accumulate a specified sum of money on a future date to redeem an existing liability or provide funds for replacement of an existing asset.

He can easily do so by making use of compound value technique :

$$V_n = (R)\ (ACF_{i,n})$$

or,

$$R = \frac{V_n}{ACF_{in}}$$

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ILLUSTRATION 13. A company has ₹ 2,00,000 6% Debentures outstanding today. The company has to redeem the debentures after 5 years and establishes a Sinking Fund to provide funds for redemption. Sinking Fund Investments earn interest @ 10% p.a. The investments are made at the end of each year. What annual payment must the firm make to ensure that the needed ₹ 2,00,000 is available on the designated date ?

SOLUTION :

$$R = \frac{V_n}{ACF_{in}}$$

Referring to the Annuity Compound Factor Tables, we find that at (*i*) 10 percent and $n = 5$ years, the compound value of an annuity of Re. 1, is 6.105.

$$R = \frac{2,00,000}{6.105}$$

$$= ₹ 32,760.03$$

Hence, the firm should make annual payment of ₹ 32,760.03 for 5 years.

❐ 2. Capital Recovery Problems

A financial manager may also be interested to know the amount of equal instalment to be paid every year to discharge a specified amount of loan raised from some financial institution at a given rate of interest and in a fixed period.

It can be easily calculated by applying the present value techniques as below :

$$V_n = (R)\ (ADF_{i,n})$$

or,

$$R = \frac{V_0}{ADF_{i,n}}$$

ILLUSTRATION 14. A company has raised a loan of ₹ 5,00,000 from a financial institution at 8% p.a. rate of interest. The amount has to be paid back in 5 equal annual instalments. What shall be the size of instalment?

SOLUTION :

$$R = \frac{V_0}{ADF_{i,n}}$$

Referring to the Annuity Discount Factor Tables at (*i*) 8 percent and *n*, 5 years, we get 3.993

$$R = \frac{5,00,000}{3.993}$$

$$= ₹ 1,25,219.13$$

So, the firm should pay an annual instalment of ₹ 1,25,219.13 for 5 years.

❐ 3. Compound Growth Rate Problems

Sometimes a finance manager may have to calculate the compound rate of growth over a period

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of time, e.g.; for sales or profits. He can easily calculate such compound rate of growth by making use of Compound Factor Tables as demonstrated in the following illustration.

ILLUSTRATION 15. From the following data available for ABC Co. Ltd.; you are required to calculate compound rate of growth in profits.

Years	2010	2014	2015	2016	2017	2018
Profits (₹ in Lacs)	60	65	75	87	98	100

SOLUTION :

$$V_n = V_0 (ACF_{i,n})$$

where $CF_{i,n}$ is compound factor at (i) percent and (n) period.

or, $100 = 60 (CF_{i,n})$

or $$CF_{i,n} = \frac{100}{60} = 1.666$$

Using Compound Factor Tables for 6 years row we find that the nearest value to 1.666 is 1.67 at 9%. Hence the compound rate of growth is nearly 9%.

❐ 4. Interest Rate Problems

The time value techniques of compounding and present value can also be applied to calculate the implicit rate of interest in certain situations, as illustrated below.

ILLUSTRATION 16. An investment company offers to pay ₹ 20,304 at the end of 10 years to investors who deposit annually ₹ 1000. What interest rate is implicit in the offer ?

SOLUTION :

Future Value of an Annuity $V_n = V_0 (ACF_{i,n})$

where, R = Annual Payment

$ACF_{i,n}$ = Annuity Compound Factor at (i) rate of interest and (n) period.

Substituting the values, $20,304 = 1000 (ACF_{1,n = 10 \text{ years}})$

or, $$ACF_{in} = \frac{20,304}{1,000} = 20.304$$

Using Annuity Compound Factor Tables, we find that when n = 10 years and ACF = 20.304, the rate of interest is 15%

Hence the rate of interest implied in the offer is 15%.

ILLUSTRATION 17. A company offers to pay you ₹ 4007 annually for 5 years if you deposit ₹16,000 initially with the company. What interest rate do you earn on the deposit ?

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SOLUTION :

Using Present Value Annuity Tables

$$V_0 = (R)\,(ADF_{i,n})$$

or, $$16000 = 4007\,(ADF_{in})$$

or $$ADF_{i,n} = \frac{16000}{4007} = 3.993$$

From the Annuity Discount Tables, we find that for annuity discount factor 3.993 for a period of 5 years, the rate of interest is 8%

Hence, the rate of interest earned on deposit is 8% p.a.

❐ 5. Valuation Problems

The techniques of time value of money are also applied in dealing with the valuation problems of bonds, debentures, etc.

Summary of Formulae

S. No.	Purpose	Formula
1.	Future Value of a current sum of money at period n	$V_n = V_0\,(1 + i)^n$
2.	Future Value of current sum of money at period n, using	$V_n = V_n\,(CF_{i.\,n})$
3.	Doubling Period (Rule of 72)	$D.P. = \frac{72}{\text{Rate of Interest}}$
4.	Doubling Period (Rule of 69)	$D.P. = 0.35 + \frac{69}{\text{Rate of Int.}}$
5.	Future value of money in case of multiple compounding periods.	$V_n = V_0\ \left(1 + \frac{i}{m}\right)^{m \times n}$
6.	Effective rate of interest in case of multi-period compounding	$EIR = \left(1 + \frac{i}{m}\right) - 1$
7.	Future value of series of payments made at different time periods.	$V_n = R_1(1+i)^{n-1} + R_2(1+i)^{n-2}$
		$+ \ldots\ldots\ldots (R_n - 1)\,(1 + i) + R_n$
8.	Future value of an annuity	$V_n = (R)\,(ACF_{i,\,n})$
9.	Present value of future sums	$V_0 = V_n \cdot DF_{i,\,n}$
10.	Present value of an annuity	$V_0 = (R)\,(ADF_{i,\,n})$
11.	Present value of an infinite life annuity.	$V_0 = \frac{R}{i}$
12.	Annual payment required to accumulate a specific sum of money.	$R = \frac{V_n}{ACF_{i,\,n}}$

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13.	Amount of equal instalment to be paid to discharge a specified amount of loan	$R = \frac{V_0}{ADF_{i,n}}$
14.	Calculation of compound rate of growth	$V_n = V_0 (CF_{i,n})$
15.	Compound Value of an Annuity Due	$V_n = (R) (ACF_{i,n}) (1 + i)$
16.	Present Value of an Annuity Due	$V_0 = (R) (ADF_{i,n}) (1 + i)$

Review Questions

A. SHORT ANSWER TYPE QUESTIONS

1. Explain the term Time Value of Money within 2-3 sentences.
2. Define the following
   (*a*) Annuity
   (*b*) Discount Rate
   (*c*) Future Value
   (*d*) Present Value
   (*e*) Annuity Discount Factor.
3. What is time preference for money ?
4. Name the techniques of time value of money.
5. What is the concept of 'time value' ?

B. ESSAY TYPE QUESTIONS

1. "A rational human being has a time preference for money." What are the reasons for such a preference ?
2. "Cash flows of different periods in absolute terms are incomparable." Discuss.
3. Examine the various techniques employed to adjust the time value of money.
4. "The finance manager should take into consideration the time value of money in order to take correct financial decisions." Elucidate.
5. Explain the mechanics of calculating present value of cash flows giving suitable examples.
6. Examine the various techniques employed to adjust the time value of money.

Exercises

Ex. 1. Calculate the future value of ₹ 20,000 invested now for a period of 5 years at a time preference rate of 8%. **[Ans. ₹ 29,380]**

Ex. 2. Determine the future value at the end of 5 years of the following series of payments at 5% rate of interest :

At the end of 1st year = ₹ 1,000

2nd year = ₹ 2,000

3rd year = ₹ 3,000

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4th year = ₹ 4,000

5th year = ₹ 5,000

[Ans. ₹ 16,041]

Ex. 3. Mr. X deposits ₹ 5,000 at the end of every year for 5 years and the deposit earns a compound interest @ 8% p.a. Determine how much money he will have at the end of 5 years ? [Ans. ₹ 29,335]

Ex. 4. Calculate the present value of ₹ 10,00,000 to be received after 5 years from now assuming 6% time preference for money. [Ans. ₹ 7,470]

Ex. 5. Calculate the present value of the following cash flows assuming a discount rate of 8%.

Year	*Cash Flow*
1	₹ 10,000
2	₹ 20,000
3	₹ 10,000
4	₹ 5,000

[Ans. ₹ 38,015]

Ex. 6. Mr. A has to receive ₹ 5,000 per year for 6 years. Calculate the present value of the annuity assuming that he can earn interest on his investment at 12% p.a. [Ans. ₹ 20,555]

Ex. 7. A company needs ₹ 10,00,000 after 5 years from now for replacement of its fixed assets. It has established a Sinking Fund for the purpose. The investments are to be made at the end of each year. What annual payment must be made to ensure the needed ₹ 10,00,000 after 5 years. Assume 10% interest per year on investments. [Ans. ₹ 1,63,800.16]

Ex. 8. Determine the amount of equal payment to be made for a loan of ₹ 2,00,000 taken for a period of 4 years at 10% rate of interest. [Ans. ₹ 63,091.48]

Ex. 9. If you deposit ₹ 10,000 today at 12 per cent rate of interest, in how many years will this amount grow to ₹ 80,000 ? Work out this problem by using the Rule of 72. Do not use the Compound Factor Tables. [Ans. 18 years]

Ex. 10. Mr. Ramesh purchased 100 shares of a company for ₹ 10 each on 1-1-1991. Te market value per share on 31.12.2005 is ₹ 47.85. What is the compound rate of growth in the value of the shares? [Ans. 11%]

Ex. 11. An investment company pays 12 per cent rate of interest and compound it quarterly. If ₹ 5,000 are deposited initially, how much shall it grow to in 5 years. [Ans. ₹ 9030]

Ex. 12. A company offers to pay you ₹ 5,002 annually for 6 years if you deposit ₹ 20,000 today with the company. What interest rate do you earn on the deposit. [Ans. 13%]

Ex. 13. An investment company offers to pay ₹ 60,965 at the end of 15 years to investors who deposit annually ₹ 1000. What interest rate is implicit in the offer. [Ans. 18%]

Ex. 14. If you deposit ₹ 20,000 today at 12 per cent rate of interest, in how many years will this amount grow to ₹ 80,000. Work out this problem by using Rule of 72. Do not use the compound Factor Tables. [Ans. 12 years]

Ex. 15. Mr. A offers to pay you ₹ 12,000 after 15 years in exchange of ₹ 1,000 today. What interest rate is implicit in the offer. [Ans. 18%]

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Ex. 16. Mr. Rakesh receives a sum of ₹ 2,00,000 as provident fund on his retirement. He deposits it in a bank which pays 12 per cent interest. If he withdraws annually ₹ 29,364, how long can he do so ? [**Ans.** 15 years]

Ex. 17. At the time of his retirement Mr. X is given a choice between two alternatives; (*i*) an annual pension of ₹ 10,000 as long as he lives, and (*ii*) a lump sum payment of ₹ 60,000. If Mr. X expects to live for 15 years and rate of interest is 15 per cent, which alternative should he select? [**Ans.** Lump sum, because the present value of ₹ 10,000 received for 15 years at 15 percent rate of interest is ₹ 58470 which is less than ₹ 60,000]

Ex. 18. Mr. Rajan is to receive ₹ 5,000 after five years from now. His time preference for money is 10% p.a. Calculate its present value, if the discount factor is 0.621. [**Ans.** ₹ 3,105]

Ex. 19. A 10, payment annuity of ₹ 5,000 will begin 7 years hence. What is the value of this annuity now if the discount rate is 12% ? [**Ans.** ₹ 14,315]

Ex. 20. A person invests ₹ 5000 at the end of each year at 10% rate of interest per year. State what amount he will receive at the end of 4 years. [**Ans.** ₹ 23,205]

Ex. 21. Mr. X is to receive from Indira Vikas Patra ₹ 20,000 after 5 years from now. His time preference for money is 10% p.a. Calculate its present value if the discount factor is 0.621. [**Ans.** ₹ 12,420]

Ex. 22. X deposits ₹ 1,00,000 on retirement in a bank which pays 10% annual interest. How much can he withdraw annually for a period of 10 years if PVgFA 10% is 6.145? [**Ans.** ₹ 16,273.39]

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