

To analyze About system (18)

(1)

Que

NET-15

The critical point of the system

$$\frac{dx}{dt} = -4x - y, \quad \frac{dy}{dt} = +x - 2y \quad \text{has}$$

✓ (1). Asymptotically stable node.

(2). Unstable node (X0)

✗ (3). Asymptotically stable spiral. (bcz not complex)

✗ (4). Unstable spiral (NOT complex)

Soln.

$$AX = \frac{d}{dt}(X), \quad A = \begin{bmatrix} -4 & -1 \\ 1 & -2 \end{bmatrix}, \quad X = \begin{bmatrix} x \\ y \end{bmatrix}$$

$$\Rightarrow \text{Ch}(A) = x^2 + 6x + 9 = (x+3)^2 \Rightarrow x = -3, -3$$

$$\Rightarrow d_1 = d_2 = -3$$

NB:(1) If both E-value are same sign & $\in \mathbb{R}$
— Node (eg. -2, -3 or 9, 5)(2) If both E-value are neg. & $\in \mathbb{R}$
— Asymptotically stable(3) If $d_1, d_2 \in \mathbb{R}$, $d_1 \neq d_2$, & opposite sign
— saddle point eg. (-3, 2, 9, -10)(4) If $d_1, d_2 \in \mathbb{C}$, $d_k = a_k + i b_k$, $a_k \neq 0$
— spiral point(5) (a) $d_k = a_k + i b_k$, if $a_k = 0$ i.e. $d_k \in i\mathbb{R}$
— Centre

(5)

stability

(2)

① Node ($d_1, d_2 \in \mathbb{R}$ & of same sign)

$d_1, d_2 > 0$, — unstable node

$d_1, d_2 < 0$, — Asymptotically stable

②

NET-JB

Consider the system of DEs —

$$\frac{dx}{dt} = 2x - 7y, \quad \frac{dy}{dt} = 3x - 8y$$

Then the critical pt. $(0,0)$ of the system is an —

- (1). Asymptotically stable node
- (2). Unstable node
- (3). Asymptotically ~~un~~ stable spiral
- (4). Unstable spiral

Solⁿ

∴ first construct matrix

$$A = \begin{bmatrix} 2 & -7 \\ 3 & -8 \end{bmatrix}$$

$$\text{Tr}(A) = -6$$

$$\det(A) = \frac{-16 + 21}{5}$$

$$\text{Ch. eqn: } x^2 + 6x + 5 = 0$$

$$\Rightarrow (x+1)(x+5) = 0$$

$$\Rightarrow x = -1, -5. \quad \text{i.e. } d_1 = -1, -5$$

Since all E. value are of same sign and both E. value are neg.

$\Rightarrow$ system is 'asymptotically stable node'