B.C.A. (Part I) EXAMINATION, 2015
(Paper First)
DISCRETE MATHEMATICS

[Time : Three Hours] *[Maximum Marks : 50]*

Note : Attempt any two parts from each question. All questions carry equal marks.

UNIT - 1

1. (a) Prove that the following statement is a tautology :

$$P = [(p \vee q) \wedge (p \vee \sim q) \wedge (\sim p \vee q) \wedge (\sim p \vee \sim q)]$$

(b) Solve the following : $(p \vee q) \wedge \sim q$

(c) Draw a truth-table for the following statement :

$$(p \vee q) \wedge (\sim r) \Rightarrow q$$

UNIT - 2

2. (a) In Boolean algebra prove the following :
(i) Additive identity is unique
(ii) Multiplicative identity is unique

(b) Using Boolean algebra, prove the following :

$$(a \cdot b) + [(a + b') \cdot b]' = 1$$

(c) Draw the simple switching circuit for the following function:

$$f = (x, y) = x + x' y$$

UNIT - 3

3. (a) Find the disjunctive normal form for the following function:

$$f\ (x, y, z) = x \cdot y' \cdot z + x' \cdot z$$

(b) State and prove Boolean's Expansion theorem.

(c) Construct a Binomial Network for four variables x, y, z and w.

UNIT - 4

4. (a) If

$$A = \{1, 2, 3, 4\}$$
$$B = \{a, b, c - 5, 4, \sin 30°\}$$

then find $A \cup B$ and draw the Venn-diagram.

(b) If A,B,C are three non-empty sets, then prove the following : $A \times (B \cup C) = (A \times B) \cup (A \times C)$

(c) Let R be an equivalence relation in set A, then prove that R^{-1} also be an equivalence relation in set A.

UNIT - 5

5. (a) In a graph prove that the number of odd degree vertices is

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always even.

(b) Draw a binary tree for the 5 vertices having maximum possible height.

(c) Show that the maximum number of edges in a simple graph with n vertices is $\frac{n(n-1)}{2}$.

DISCRETE MATHEMATICS - 2014

Note : Attempt any two parts from each question. All questions carry equal marks.

UNIT - 1

1. (a) Prove that each of the following statements is a tautology :

$$(p \Leftrightarrow q) \wedge (q \Leftrightarrow r) \Rightarrow (p \Leftrightarrow r)$$

(b) Explain the following :

(i) Universal Quantifier (ii) Existential Quantifier

(c) Write the following predicate into symbolic language :

(i) For every real number there is a greater real number.

(ii) Every irrational number is a real number.

(iii) The number divisible by an even number is even.

(iv) Every teacher of a college in learned.

(v) All students are not wise

UNIT - 2

2. (a) Let B be the set of all positive divisors of 30 i.e. :

$$B = \{1, 2, 3, 5, 6, 10, 15, 30\}$$

and the operations $\vee$ and $\wedge$ on B are defined as follows :

$a \vee b$ = L C M of a and b

$a \wedge b$ = H C F of a and b

Prove that $(B, \vee, \wedge)$ is a Boolean Algebra.

(b) To prove that, for any two elements a, b of a Boolean algera:

a + b = least upper bound of a and b

i,e., a + b = lub {a, b}.

(c) Draw the logic circuit with inputs a,b,c and output y which corresponds to the Boolean expression :

$$Y = ab'c + abc' + ab'c'$$

UNIT - 3

3. (a) Show that the number of minimal Boolean function in n-

variables are 2^n.

(b) Express the following functions into disjunctive normal form :

$$f(x, y, z) = (x + y + z) \cdot (xy + x' \cdot z)'$$

(c) Design a tree-net in three variables for the flow function :

$$x \cdot y \cdot z + x' \cdot y \cdot z + x \cdot y' \cdot z + x' \cdot y' \cdot z$$

UNIT - 4

4. (a) If A,B,C are any three non-empty sets, then prove that :

$$(A - B) \times C = (A \times C) - (B \times C)$$

(b) Show that the set Q of rational numbers is countable.

(c) Show that the mapping $f : R_+ \to R$ defined by $f(x) = \log x$, $x \in R_+$ is one-one onto where R_+ is the set of positive real numbers and R is the set of real numbers.

UNIT - 5

5. (a) Explain the basic concept of Graph theory.

(b) Write short notes on the following :

(i) Binary trees (ii) Spanning trees

(c) Write short notes on the following:

(i) Euler circuit (ii) Hamiltonian graph

DISCRETE MATHEMATICS - 2013

Note : Attempt any two parts from each question. All questions carry equal marks.

UNIT - 1

1. (a) Prove that each of the following statements is a contradiction :

$$[(p \wedge r) \vee (q \wedge \sim r)] \Leftrightarrow [(\sim p \wedge r) \vee (-q \wedge -r)]$$

(b) Explain the Algebra of Proposition.

(c) Write the following sentences into symbols :

(i) Two non-parallel coplaner straight lines have a common point.

(ii) If there is no prize, then a person does not purchase a ticket.

UNIT - 2

2. (a) Simplify the following by using Boolean algebra :

(i) $[a + (a' \cdot b)] \cdot [a' + (a \cdot b)]$

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(ii) $a \cdot b + [(a + b') \cdot b]'$

(b) Define partial order relation. Prove that the order relation $\leq$ is partial order relation in a Boolean algebra.

(c) Draw a simplified circuit of the expression :

$$F(x, y, z) = x \cdot y' \cdot z + (z + y) \cdot x'$$

UNIT - 3

3. (a) Explain the Boole's Expansion theorem.

(b) Express the following function in conjunctive normal form : $f(x, y, z) = (xy' + xz)' + x'$

(c) Design a 3-terminal circuit which gives the real forms to the following both functions :

$$f = xzw + y'zw$$
$$g = zxw + y'zw + x'y'z$$

UNIT - 4

4. (a) Let I be the set of non-zero integers and a relation R be defined by xRy if $x^y - y^x$, where $x, y \in I$. Is the relation R an equivalence relation ?

(b) If X, Y, Z, W are four sets and f, g, h be three mappings given by :

$$f : X \to Y,\ g : Y \to Z,\ h : Z \to X$$

then prove that $(h \circ g) \circ f = h \circ (g \circ f)$.

(c) If $f : A \to B$ is one-one and onto, then prove that $f^{-1} : B \to A$ is also one-one and onto.

UNIT - 5

5. (a) Show that the sum of the degrees of all vertices in a graph G is equal to twice the number of in a graph G is equal to twice the number of edges in G.

(b) If a graph G = (V, E) is defined by :

$$V = \{v_1, v_2, v_3\}$$
$$E = \{[v_1, v_2], [v_2, v_3], [v_1, v_3]\}$$
$$|V| = 3,\ |E| = 3$$

then find the adjacency matrix and the incidence matrix of the graph G.

(c) Explain the planar graphs.

DISCRETE MATHEMATICS - 2012 SUPPLE

Note : Attempt any two parts from each question. All questions carry equal marks.

1. (a) Define tautology and contradiction with an example. Prove that the following statement is tautology :

$$(p \Leftrightarrow \wedge r) \Leftrightarrow (\sim r \Rightarrow \sim p)$$

(b) Prove that following statements are logically equivalent :

$$p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$$

(c) Define the following :

(i) Universal quantifiers

(ii) Existential quantifiers

(iii) Negation of a quantifier with examples

2. (a) In a Boolean algebra prove that :

(i) $a + [(a' \cdot c) + b] = (a + b) + c$

(ii) $a \cdot b + [(a + b') \cdot b]' = 1$

(b) Draw the switching circuit for the switching function :

$$F(x, y, z) = x \cdot y \cdot z + (x + y) \cdot (x + z)$$

and replace it by a simpler one.

(c) Prove that for any two elements a, b of a Boolean algebra :

(i) $a + b = \text{lub}\{a \cdot b\}$

(ii) $a \cdot b = g : b\{a \cdot b\}$

3. (a) Change the following Boolean function to disjunctive normal form :

$$f(x, y, z) = \{x + (x' + y)'\} \cdot [x + (y' \cdot z')']$$

(b) Desing a 3-terminal circuit which gives the real forms to the following both functions :

$$f = xzw + y'zw$$
$$g = xzw + y'zw + x'y'z$$

(c) Change the following function to conjuctive normal form in which minimum number of variables are used :

$$f = (x + y') \cdot (y + z') \cdot (z + x') \cdot (x' + y')$$

4. (a) Show that the relations 'is greater than' in the set N of positive integers is transitive but neither reflexive nor symmetric.

(b) Show that the mapping $f : R \to R$, $f(x) = \frac{1}{x}$, $x \neq 0$ and $x \in R$ is one-one onto, where R is the set of non-zero real numbers.

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(c) If $f : R \to R$, defined by $f(x) = x^2$, $x \in R$ and $g : R \to R$, defined by $g(x) = \sin x$ then show that $(g \circ f)x \neq (f \circ g)x$.

5. (a) Prove that the number of labelled trees with n-vertices $(n \geq 2)$ is n^{n-2}.

(b) Show that the maximum degree of any vertex in a simple graph with n-vertices is n -1

(c) Define the following terms with an example :

(i) Directed tree (ii) Planar graph

(iii) Hamiltonian graph (iv) Euler circuit

(v) Binary tree

DISCRETE MATHEMATICS - 2012

Note : Attempt any two parts from each question. All questions carry equal marks.

1. (a) Show that : $p \Rightarrow (q \Rightarrow r) \equiv (p \wedge q) \Rightarrow r$

(b) Construct converse, inverse and contrapositive of the direct statement, "If $4x - 2 = 10$, then $x = 3$".

(c) Write an essay on quantifiers.

2. (a) In a Boolean algebra, show that if $a + b = a + c$ and $ab = ac$, then $b = c$.

(b) In a Boolean algebra B, prove that $x \leq y$ if and only if $x + y = y$ where $x, y \in B$.

(c) Replace the Switching function :

$$F(x, y, z) = x \cdot y \cdot z + x \cdot y' \cdot z + x' \cdot y' \cdot z$$

by a simpler switching circuit.

3. (a) Show that the number of minimal Boolean function in n variable is 2^n.

(b) Express the function :

$$f(x, y, z) = [(xy')' + z'] \cdot [z + x']'$$

(c) Draw a binominal network for the following function :

$a \cdot b \cdot c + a' \cdot b \cdot c + a \cdot b' \cdot c + a \cdot b \cdot c' + a' \cdot b' \cdot c'$

4. (a) If A, B, C are any three non-empty sets, then prove that :

$$A \times (B \cap C) = (A \times B) \cap (A \times C)$$

(b) Show that the relation "x Ry $\Leftrightarrow$ x – y is divisible by 5", where x, y $\in$ I defined in the set of integers I is an equivalence relation.

(c) Show that the function $f : R \to R$ defined by $f(x) = 5x^3 - 1$ is one-one onto. Where R is the set of real numbers.

5. (a) Prove that, the maximum number of edges in a simple graph with n vertices in $\frac{n(n-1)}{2}$.

(b) If in a graph G there is one and only one path between every pair of vertices, then prove that G is a tree.

(c) Define Directed and Planar graph and prove that, a connected planar graph with n vertices and e edges has e – n + 2 regions.

DISCRETE MATHEMATICS - 2011

Note : Attempt any two parts from each question. All questions carry equal marks.

1. (a) Construct a truth table for the following statement :

$$(p \vee q \Rightarrow s) \Leftrightarrow (p \Rightarrow s) \vee (q \Rightarrow s)$$

(b) Find the truth value of each of the following compound statements :

(i) If $4 + 3 = 9$, then $5 + 5 = 10$.

(ii) If is not true that $3 + 3 = 7$ if and only if $6 + 6 = 14$

(iii) Paris is in England or London is in France.

(iv) It is not true that $2 + 2 = 5$ or $3 + 2 = 5$

(v) It is false that if Paris is in England then London is in France.

(c) Write the following sentences into symbols :

(i) The square of any rational number is not 2.

(ii) Two non-parallel coplanar straight lines have a common point.

2. (a) In a Boolean algebra, show that if :

(i) $a + b = b$

(ii) $ab + bc + ca = (a + b)(b + c)(c + a)$

(b) Draw the switching circuit for the switching function :

$$F(x, y, z) = x \cdot y \cdot z + (x + y) \cdot (x + z)$$

and repalce it by a simpler one.

(c) In a Boolean algebra B, if $x \leq y$ and $y \leq x$ then prove that $x = y$.

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3. (a) Find complete disjunctive normal form in three variables and show that its value is 1.
   (b) Express the following function in conjunctive normal form : $f(x, y, z) = (xy' + xz') + x'$
   (c) Draw a binomial network for the following function :
   $a \cdot b \cdot c + a' \cdot b \cdot c + a \cdot b' \cdot c + a \cdot b \cdot c' + a' \cdot b' \cdot c'$
4. (a) If A, B, C are any three sets and $A \subseteq B$, then prove that : $A \times C \subseteq B \times C$
   (b) Prove that the relation "is equal to" in the set of all real numbers is an equivalence relation.
   (c) If $f : A \to B$ is one-one and onto, then prove that $f^{-1} : B \to A$ is also one-one and onto.
5. (a) The sum of the degrees of all vertices in a graph G is equal to twice the number of edges in G.
   (b) If a graph G = (V, E) is defined by :

$$V = \{v_1, v_2, v_3\} \cdot E = \{[v_1, v_2], [v_2, v_3], [v_1, v_3]\}$$

$$|V| = 3, |E| = 3$$

   then find the adjacency matrix and the incidence matrix of the group G.
   (c) Show that every tree with two or more vertices is 2-chromatic.

DISCRETE MATHEMATICS - 2010

Note : Attempt any two parts from each question. All questions carry equal marks.

1. (a) Prove that : $(p \Rightarrow q) \vee r \equiv (p \vee r) \Rightarrow (q \vee r)$
   (b) Construct converse, inverse and contrapositive of the direct statement "if $4x - 2 = 10$, then $x = 3$".
   (c) Define quantifiers. Also define :
   (i) Universal quatifier
   (ii) Existential amplifier with an example
2. (a) In a Boolean algebra, show that, if :
   (i) $a + b = a + c$ and $ab = ac$, then $b = c$
   (ii) $a + x = b + x$ and $a + x' = b + x$ then $a = b$.
   (b) Draw a circuit for the following Boolean function and replace it by a simple one :

$$F(x, y, z) = x \cdot z + [y \cdot (y' + z) \cdot (x' + x \cdot z)]$$

   (c) For any two elements a and b of Boolean algebra B, prove that :
   (i) $(a + b)' = a' + b$
   (ii) $(a + b)' = a + b' \ \forall \ a, b \in B$
3. (a) Prove that the number of minimal Boolean functions in n- variables are 2^n.
   (b) Change the following Boolean function to disjunctive normal form : $f = (x, y, z) = [x + (x + y)'] \cdot [x + (y' \cdot z')']$
   (c) Design a 4-terminal circuit which gives the real forms to the following three functions :

$$f = x\,y'\,z + (xy' + x'y)\,zw$$
$$g = xy'\,zw' + xyzw'$$
$$h = x'y + (xy' + x'y)\,(z' + w')$$

4. (a) Show that the relation $R = \{(a, b) : a, b \in I$ and $a - b$ is divisible by 5$\}$ i.e. $x\,R\,y \Leftrightarrow x - y$ is divisible by 5 is an equivalence relation.
   (b) Show that the function $f : R \Rightarrow R$ defined by $f(x) = 5x^3 - 1$ is one-one onto, where R is the set of real numbers.
   (c) Show that the set Q of rational numbers in countable.
5. (a) Prove that a tree with n- vertices has (n-1) edges.
   (b) Prove that a group G with n vertices, (n -1) edges, and no circuits is connected.
   (c) Define the following terms with examples :
   (i) Binary tree (ii) Spanning tree
   (iii) Planar graph (iv) Hamilton graph

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