Dimostrazione Esame

Fisica generale (Fisica generale I + Fisica generale II) (Gestionale) (12 crediti) (Università degli Studi di Bergamo)



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DIMOSTRAZIONI PER ESAME

$a = \frac{d^2 l}{dt^2} = \frac{F}{m}$

GITTATA

$\begin{cases} a_x = \frac{F_x}{m} = 0 \\ a_y = -\frac{mg}{m} = -g \end{cases}$ $\begin{cases} V_x = V_{0x} \\ V_y = V_{0y} - gt \end{cases}$ $\begin{cases} x = V_{0x} t + x_0 \\ y = V_{0y} t + y_0 - \frac{1}{2} g t^2 \end{cases}$

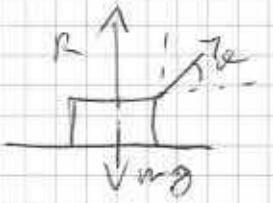
massima quota $V_{0y} = 0 \rightarrow V_y = V_{0y} - gt = 0 \rightarrow V_{0y} = gt$

$y = V_{0y} \cdot \frac{V_{0y}}{g} - \frac{1}{2} g \frac{V_{0y}^2}{g^2} = \frac{V_{0y}^2}{g} - \frac{1}{2} \frac{V_{0y}^2}{g} = \frac{V_{0y}^2}{2g}$ $t = \frac{V_{0y}}{g}$

Gittata $y = 0 \rightarrow x = S = \frac{2 V_{0x} V_{0y}}{g}$

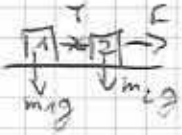
Moto lungo piano

$\begin{cases} a_x = \frac{F_x}{m} = \frac{F \cos\theta}{m} \\ a_y = \frac{-mg + R + F \sin\theta}{m} = 0 \end{cases}$ $\begin{cases} a_x \\ R = mg - F \sin\theta \end{cases}$



Moto lungo piano con fune

$\begin{cases} a_1 = \frac{T}{m_1} \\ a_2 = \frac{F - T}{m_2} \end{cases}$ $\begin{cases} T = m_1 a \\ a = \frac{F - T}{m_2} \end{cases}$ $\begin{cases} T = \frac{m_1}{m_2} F - \frac{m_1}{m_2} T \\ / \end{cases}$ $\begin{cases} T = \frac{F}{m_1 + m_2} \\ a = \frac{m_2}{m_1 + m_2} F \end{cases}$



PIANO INCLINATO

$\begin{cases} a_x = \frac{F_x}{m} = \frac{mg \sin\alpha}{m} \\ a_y = \frac{f_y}{m} = \frac{-mg \cos\alpha + R}{m} = 0 \end{cases}$



PENDOLO SEMPLICE

$\begin{cases} a_\parallel = -\frac{mg \sin\theta}{m} = -g \sin\theta \\ a_\perp = \frac{T - mg \cos\theta}{m} = \frac{v^2}{\ell} \end{cases} \rightarrow T = mg \cos\theta$

2CC centrifuga (solo teoricamente trascurabile)

$\sin\theta \sim \theta = \frac{s}{\ell}$

$a = -g\theta = -g \frac{s}{\ell}$ $a = -\omega^2 s \rightarrow \omega = \sqrt{\frac{g}{\ell}}$ $\Lambda = \ell\theta_0$

$T = 2\pi \sqrt{\frac{\ell}{g}}$

CORPO ATTACCATO A MOLLA

$F = -kx$ $a = \frac{F}{m} = -\frac{kx}{m} \rightarrow \omega = \sqrt{\frac{k}{m}}$ $T = 2\pi \sqrt{\frac{m}{k}}$

CARRUCOLA

$\begin{cases} a_1 = \frac{m_1 g - T}{m_1} \\ a_2 = + \frac{m_2 g - T}{m_2} \end{cases}$ $\begin{cases} m_1 a + m_2 a = m_1 g - m_2 g \\ T = + m_2 a_2 + m_2 g \end{cases}$ $\begin{cases} a = g \frac{(m_1 - m_2)}{m_1 + m_2} \\ T = + m_2 \left[g \frac{(m_1 - m_2)}{m_1 + m_2} \right] + m_2 g \end{cases}$

$= \frac{2 m_1 m_2}{m_1 + m_2} g$

FORZE ATTRITO FLUIDI

$f_a = -\beta v$ $\beta = \gamma \eta$

$a = \frac{mg - f_a}{m} = \frac{mg - \beta v}{m} = 0$ $v_{max} = \frac{mg}{\beta}$

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LAVORO

$L = F \cdot \Delta r = F \, \Delta r \cos\theta$

$L = \int_a^b F \, dr \rightarrow L = \int_a^b F \, ds$

per forza elastica $L = \int_{x_0}^{x} -k x \, dx = -\frac{1}{2} k (x_0^2 - x_0^2)$

POTENZA

$P = \frac{L}{\Delta t} \rightarrow P = \frac{dL}{dt} = F \cdot \frac{dr}{dt} = F \cdot v$

ENERGIA CINETICA

- F e a cost $L = F \cdot \Delta r = m a \cdot \Delta r$

  Siccome $a = \frac{(V - V_0)}{t}$ e $\Delta r = \frac{(V + V_0)}{2} t \rightarrow L = m \left[\frac{V - V_0}{t}\right] \cdot \left(\frac{V + V_0}{2}\right) t = \frac{1}{2} m V^2 - \frac{1}{2} m V_0$

  Teor. forze vive $\rightarrow L = K - K_0 = \Delta K$

- F non costante $L = \int_a^b F \, dr = \int_{r_0}^{r} m a \cdot dr = \int_{t_0}^{t} m \frac{dv}{dt} v \, dt = \int_{V_0}^{V} m \, dv \cdot v = \frac{1}{2} m V^2 - \frac{1}{2} m V_0^2$

- Lavoro gravitazionale $L = \int_{r_0}^{r} F \cdot dr = \int_{r_0}^{r} mg \, dr = \int_{h}^{0} -mg \, dh = mgh$

  Siccome $L = \Delta K \rightarrow mgh = \frac{1}{2} m V_f^2 - \frac{1}{2} m V_0^2 = \frac{1}{2} m V_f^2$

  $\rightarrow V_f = \sqrt{2gh}$

ENERGIA POTENZIALE

$U = U(r)$ (1 dim $U = U(x)$)

$E = K + U = \frac{1}{2} m V^2 + U(x)$

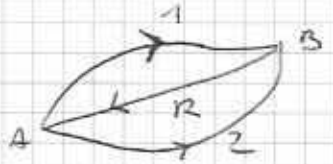
- E cost $K + U = K_0 + U_0 \rightarrow -U + V_0 = K - K_0$ Siccome $L = \Delta K \rightarrow L = U_0 - U$

  $U = U_0 - L$

$U(r) = -\int_{r_0}^{r} F \, dr + U_0$ $\left[1 \text{ dim} \rightarrow U(x) = -\int_{x_0}^{x} F \, dx + U_0\right]$

FORZE CONSERVATIVE

$\left.\begin{aligned} L_{AB}^{(1)} + L_{BA}^{(2)} = 0 \\ L_{AB}^{(2)} + L_{BA}^{(2)} = 0 \end{aligned}\right\} \rightarrow L_{AB}^{(1)} = L_{AB}^{(2)}$



NON CONSERVAZIONE ENERGIA MECCANICA

$\Delta E = \Delta K + \Delta U = L_{\text{non-cons}} < 0$

QUANTITÀ DI MOTO

$p = m V$

TEOREMA IMPULSO

$F = \frac{dp}{dt} \rightarrow F = ma = m \frac{dV}{dt} = \frac{d(mV)}{dt} = \frac{dp}{dt}$

CONSERVAZIONE QUANTITÀ DI MOTO

$F = 0 \rightarrow \frac{dp}{dt} = 0 \rightarrow p = \text{cost} \rightarrow p_0 = p_f$

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URTO elastico

$$\begin{cases} m_1 v_1 + m_2 v_2 = m_1 v_1' + m_2 v_2' \\ \frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2 = \frac{1}{2} m_1 v_1'^2 + \frac{1}{2} m_2 v_2'^2 \end{cases} \quad \begin{cases} m_1 (v_1 - v_1') = m_2 (v_2' - v_2) \\ m_1 (v_1 - v_1')(v_1 + v_1') = m_2 (v_2' - v_2)(v_2' + v_2) \end{cases}$$

$\hookrightarrow (v_1 + v_1') = (v_2' + v_2)$

$\Rightarrow v_1 - v_2 = v_2' - v_1'$

$$\Rightarrow \begin{cases} v_1' = \dfrac{(m_1 - m_2) v_1 + 2 m_2 v_2}{m_1 + m_2} \\ v_2' = \dfrac{(m_2 - m_1) v_2 + 2 m_1 v_1}{m_1 + m_2} \end{cases}$$

- Se $m_1 = m_2 \rightarrow v_1' = v_2$ e $v_2' = v_1$
- Se m_2 è un bersaglio $\Rightarrow v_2 = 0 \rightarrow v_1' = \dfrac{(m_1 - m_2) v_1}{m_1 + m_2}$ e $v_2' = \dfrac{2 m_1 v_1}{m_1 + m_2}$
  - Se $m_1 = m_2 \Rightarrow v_1' = 0$ e $v_2' = v_1$
  - Se $m_1 \gg m_2 \Rightarrow v_1' \approx v_1$ e $v_2' \approx 2 v_1$
  - Se $m_1 \ll m_2 \Rightarrow v_1' \approx -v_1$ e $v_2' \approx 0$

URTO ANAELASTICO

$$m_1 v_1 + m_2 v_2 = (m_1 + m_2) V \rightarrow V = \frac{m_1 v_1 + m_2 v_2}{m_1 + m_2}$$

$$Q = -\Delta K = -\left[\frac{1}{2} (m_1 + m_2) V^2 - \left(\frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2 \right) \right] = \frac{1}{2} \frac{m_1 m_2}{(m_1 + m_2)} (v_1 - v_2)^2$$

PENDOLO BALISTICO

$$m v_1 = (M + m) v_2 \Rightarrow v_2 = \frac{m v_1}{M + m}$$

$$K = \frac{1}{2} (M + m) v_2^2 = \frac{1}{2} \frac{m^2}{M + m} v_1^2 = U = (M + m) g h \rightarrow v_1 = \frac{M + m}{m} \sqrt{2gh}$$

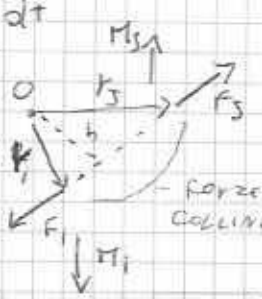
TEOREMA MOMENTO ANGOLARE

$$M = \frac{dL}{dt} \rightarrow \frac{dL}{dt} = \frac{d(r \times m v)}{dt} = \frac{dr}{dt} \times m v + r \times m \frac{dv}{dt} = v \times m v + r \times m a = r \times F = M$$

SISTEMI CON N PUNTI MATERIALI

$$F_{int}^{tot} = \sum_{i,j=1,1}^{N,N} F_{ij} = 0$$

$$M_{int}^{tot} = \sum_{i,j=1,1}^{N,N} r_i \times F_{ij} = 0$$



$F_j = F_i$ perché hanno lo stesso braccio

PRIMA EQUAZIONE CARDINALE DINAMICA DEI SISTEMI

$$F^{tot} = \frac{dP^{tot}}{dt} \rightarrow F^{tot} = \sum_{i=1}^{N} F_i = \sum_{i=1}^{N} m_i a_i = \sum_{i=1}^{N} m_i \frac{dv_i}{dt} = \frac{d}{dt} \left(\sum_{i=1}^{N} m_i v_i \right) = \frac{dP^{tot}}{dt}$$

SECONDA EQUAZIONE CARDINALE DINAMICA DEI SISTEMI

$$M^{tot} = \frac{dL^{tot}}{dt}$$

$$\Delta L^{tot} = L^{tot}_{(t_2)} - L^{tot}_{(t_1)} = \int_{t_1}^{t_2} M^{tot} \, dt$$

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CENTRO DI MASSA

$r_G = \frac{\sum_{i=1}^{N} m_i r_i}{m^{tot}}$ per sistemi continui $\Rightarrow r_G = \frac{1}{m^{tot}} \int_{m^{tot}} r \, dm$

se $\sigma = \frac{dm}{dV} \Rightarrow r_G = \frac{1}{m^{tot}} \int_{V^{tot}} \sigma r \, dV$

TEOREMA CENTRO DI MASSA

$V_G = \frac{p^{tot}}{m^{tot}} \Rightarrow \frac{dr_G}{dt} = V_G = \frac{1}{m^{tot}} \sum_{i=1}^{N} m_i \frac{dr_i}{dt} = \frac{1}{m^{tot}} \sum_{i=1}^{N} m_i V_i = \frac{p^{tot}}{m^{tot}}$

$a_G = \frac{F^{tot}}{m^{tot}} \Rightarrow \frac{dp^{tot}}{dt} = \frac{d(m^{tot} V_G)}{dt} = m^{tot} a_G$

per 1 eq. card. din $\Rightarrow \frac{dp^{tot}}{dt} = F^{tot} \Rightarrow F^{tot} = m^{tot} a_G$

CONSERVAZIONE MOMENTO ANGOLARE

$M = 0 \quad L = cost$

PRIMO TEOREMA KONIG

$K = K' + \frac{1}{2} m^{tot} V_G^2 \Rightarrow V_i = V_i' + V_G$

$K = \sum_i \frac{1}{2} m_i v_i^2 = \sum_i \frac{1}{2} m_i (V_i'^2 + V_G^2 + 2 V_i' V_G)$

$= \sum_i \frac{1}{2} m_i v_i'^2 + \sum_i \frac{1}{2} m_i V_G^2 + \sum_i V_i' V_G = K' + \frac{1}{2} V_G^2 \sum_i m_i + V_G \sum m_i V_i'$

$\Rightarrow K = K' + \frac{1}{2} m^{tot} V_G^2 + V_G p'^{tot}$

$= K' + \frac{1}{2} m^{tot} V_G^2 + V_G \cdot m^{tot} V_G'^{=0} \Rightarrow K = K' + \frac{1}{2} m^{tot} V_G^2$

SECONDO TEOREMA KONIG

$L_O = L_G + OG \times m^{tot} V_G$

MOMENTO DI INERZIA

discreta: $I = \sum_{i=1}^{N} m_i r_i^2$ continua: $I = \int_{m^{tot}} r^2 dm = \int_{V^{tot}} r^2 \sigma \, dV$

$K_{rot} = \frac{1}{2} I \omega^2$

TEOREMA HUYGENS - STEINER

$I = I_G + m^{tot} d^2$

TEOREMA MOMENTO ASSIALE

$L_z = I \omega \Rightarrow OP_i = OP_i' + P_i'P_i$

$OP_i' \times V_i = 0 \qquad P_i'P = r_i$

$L_z = \sum_i OP_i \times m V_i \Big|_z = \sum_i OP_i' \times m_i V_i \Big|_z + \sum_i P_i'P \times m_i V_i \Big|_z$

$= \sum_i r_i m_i V_i = \sum_i r_i m_i \omega r_i = \sum_i r_i^2 m_i \omega = I \omega$

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DINAMICA ROTAZIONALE

$$\frac{dL_z}{dt} = I\frac{d\omega}{dt} = I\alpha \quad \rightarrow \frac{dL_z}{dt} = \Pi_z \quad \rightarrow \Pi_z = I\alpha$$

LAVORO ROTAZIONALE

$$\Pi_z = I\frac{d\omega}{dt} \quad \rightarrow \Pi_z\, d\theta = I\frac{d\theta}{dt}\cdot d\omega = I\omega\, d\omega$$

$$\int_{\theta_i}^{\theta_f} \Pi_z\, d\theta = \int_{\omega_i}^{\omega_f} I\omega\, d\omega = \frac{1}{2}I\omega_f^2 - \frac{1}{2}I\omega_i^2 = K_f - K_i$$

$$L = \Delta K = \int_{\theta_i}^{\theta_f} \Pi_z\, d\theta$$

TEOREMA POTENZA ROTAZIONALE

$$\frac{dL_{rot}}{dt} = \Pi_z\frac{d\theta}{dt} = \Pi_z\,\omega \quad \rightarrow P_{rot} = \Pi_z\,\omega$$

PRINCIPIO CONSERVAZIONE ENERGIA PER CORPO RIGIDO

$$E = \frac{1}{2}m^{tot}v_c^2 + \frac{1}{2}I\omega^2 + U(r_1, r_2, \dots r_N) = cost$$

PENDOLO COMPOSTO

$$\Pi_z = -mgd\sin\theta \sim -mgd\theta \qquad (\sin\theta \sim \theta)$$

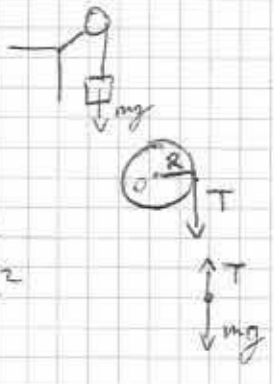
$$\alpha = \frac{d^2\theta}{dt^2} = \frac{\Pi_z}{I} = -\frac{mgd\theta}{I} = -\Omega^2\theta$$

$$T = \frac{2\pi}{\Omega} = 2\pi\sqrt{\frac{I}{mgd}} = 2\pi\sqrt{\frac{\ell^*}{g}} \qquad \ell^* \text{ lunghezza ridotta}$$

$$\ell^* = \frac{I}{mg} = \frac{I_G + md^2}{mg}$$

VOLANO

$$\begin{cases} a = \frac{F}{m} = \frac{mg - T}{m} \\ \alpha = \frac{\Pi_z}{I} = \frac{TR}{I} \end{cases} \quad \begin{cases} a = g = \frac{I}{m} \\ \alpha = \frac{TR^2}{I} \end{cases} \quad \begin{cases} T = \frac{mgI}{I + mR^2} \\ a = \frac{mgI}{I+mR^2}\frac{R^2}{I} = \frac{mgR^2}{I+mR^2} \end{cases}$$



- $L = \int_{\theta_0}^{\theta} \Pi_z\, d\theta = \Pi_z(\theta - \theta_0) = TR(\theta - \theta_0) = \frac{mgI}{I+mR^2}R(\theta - \theta_0)$

  con $\theta = \frac{1}{2}\alpha t^2 + \omega_0 t + \theta_0$ e $\omega_0 = \omega(0)$ e $\alpha = \frac{a}{R} = \frac{mgR}{I+mR^2} = cost$

- $P = \Pi_z\,\omega = TR\omega = TR(\alpha t + \omega_0) = TR\left(\frac{\Pi_z}{I}t + \omega_0\right) = TR\left(\frac{TR}{I}t + \omega_0\right) =$

$$= \frac{mgIR}{I+mR^2}\left(\frac{mgR}{I+mR^2}t + \omega_0\right)$$

STATICA

$$F^{tot} = 0 \qquad \Pi^{tot} = 0$$

MACCHINE STATICHE

$$b_{ant} : b_{res} = f_{res} : F_{ant}$$

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PRINCIPIO PASCAL

$p = \frac{F_1}{A_1} = \frac{F_2}{A_2} \rightarrow \frac{F_2}{F_1} = \frac{A_2}{A_1}$

LEGGE STEVINO

$\Delta p = \rho g \Delta h \rightarrow F = S \Delta p \rightarrow S \Delta p = mg = \rho V g = \rho S \Delta h g \rightarrow \Delta p = \rho g \Delta h$

PRINCIPIO VASI COMUNICANTI

$p_1 = p_2 \Rightarrow p_0 + \rho_1 g h_1 = p_0 + \rho_2 g h_2 \rightarrow \rho_1 h_1 = \rho_2 h_2 \rightarrow \frac{\rho_1}{\rho_2} = \frac{h_2}{h_1}$

LEGGE ARCHIMEDE

$F_a = p_{inf} S - p_{sup} S = \Delta p S = \rho_{FL} g \Delta h S = \rho_{FL} g V = m_{FL} g = F_{p\,FL}$

$F_a = -F_{pFL}$

$F_a = F_{pFL} = m_{FL} g = \rho_{FL} V_{imm} g = F_p = mg = \rho V g$

$\Rightarrow \rho_{FL} V_{imm} = \rho V \rightarrow \frac{V_{imm}}{V} = \frac{\rho}{\rho_{FL}}$

CONSERVAZIONE PORTATA

$dm = dV \rho = \rho S dl = \rho S v dt = cost$

$Q_m = \frac{dm}{dt} \rightarrow \rho_{IN} S_{IN} V_{IN} = \rho_F S_F V_F \rightarrow Q_m = \rho S V = cost$

TEOREMA BERNOULLI

$L_{pres} + L_{GRAV} = \Delta K$

$p_1 S_1 dx_1 - p_2 S_2 dx_2 + dm g (y_1 - y_2) = \frac{1}{2} dm V_2^2 - \frac{1}{2} dm V_1^2$

$p_1 dV_1 - p_2 dV_2 + dm g (y_1 - y_2) = p_1 \frac{dm}{\rho} - p_2 \frac{dm}{\rho} + dm g (y_1 - y_2) = \frac{1}{2} dm V_2^2 - \frac{1}{2} dm V_1^2$

$p_1 + \rho g y_1 + \frac{1}{2} \rho V_1^2 = p_2 + \rho g y_2 + \frac{1}{2} \rho V_2^2$

$p + \rho g h + \frac{1}{2} \rho V^2 = cost$

$\frac{p}{\rho g} + y + \frac{V^2}{2g} = cost$

LEGGE STEVINO

$p_1 + \rho g h_1 = p_2 + \rho g h_2 \rightarrow \Delta p = \rho g h$

TEOREMA TORRICELLI

$p_{atm} + \rho g h_1 = p_{atm} + \rho g h_2 + \frac{1}{2} \rho V^2 \rightarrow V = \sqrt{2gh}$

SCORRIMENTO ORIZZONTALE

$p_1 + \frac{1}{2} \rho V_1^2 = p_2 + \frac{1}{2} \rho V_2^2 \rightarrow p + \frac{1}{2} \rho \frac{Q^2}{S^2} = costante$

TUBO DI VENTURI

$V_1 = V_2 \frac{S_2}{S_1} \rightarrow p_1 - p_2 = \frac{1}{2} \rho (V_2^2 - V_1^2) = \frac{1}{2} \rho V_2^2 \left(1 - \frac{S_2^2}{S_1^2}\right) \rightarrow Q = V_2 S_2 = \sqrt{\frac{2(p_1 - p_2)}{\rho \left(\frac{1}{S_2^2} - \frac{1}{S_1^2}\right)}}$

MOTO DEI FLUIDI REALI

$\Delta p = \frac{8 \eta L Q}{\pi r^4} \rightarrow Q = \frac{\Delta p}{R}$

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SCALE TERMOMETRICHE

$T_{(°C)} = T_{(K)} - 273,15$ $\qquad T_{(F)} = \frac{9}{5} T_{(°C)} + 32$

LEGGE DI GAY-LUSSAC

$V = V_0 (1 + \alpha T_{(°C)}) = V_0 \alpha T_{(K)}$

$p = p_0 (1 + \alpha T_{(°C)}) = p_0 \alpha T_{(K)}$

CALORIMETRIA

$Q = c m \Delta T$ $\qquad c = c(T) = \lim_{\Delta T \to 0} \frac{Q}{m \Delta T} = \frac{1}{m} \frac{dQ}{dT} \Rightarrow Q = \int_{T_1}^{T_2} c m \, dT$

$c_p = \frac{1}{m} \frac{dQ}{dT}\Big|_{p=cost}$ $\qquad c_v = \frac{1}{m} \frac{dQ}{dT}\Big|_{V=cost}$

$C = cm \Rightarrow Q = C \Delta T$

$Q_{2\to1} = -Q_{1\to2} \Rightarrow c_1 m_1 (T_\infty - T_1) = c_2 m_2 (T_\infty - T_2)$

$c_1 m_1 T_\infty + c_2 m_2 T_\infty = c_1 m_1 T_1 + c_2 m_2 T_2$

$T_\infty = \frac{c_1 m_1 T_1 + c_2 m_2 T_2}{c_1 m_1 + c_2 m_2}$

CAMBIAMENTI DI STATO

$T_{liq} = T_{ev}$ $\qquad T_{FUS} = T_{SOL}$ $\qquad T_{SUBL} = T_{SUBL}$

$Q_{CS} = \lambda_{CS} m \Rightarrow \lambda_{CS} = \frac{T_{CS} \Delta V}{m} \frac{dp}{dT_{CS}}$

CONDUZIONE TERMICA

$\frac{dQ}{dT_{(K)}} = -\lambda S \frac{dT_{(K)}}{dl}$ $\qquad R = \frac{l}{\lambda} \Rightarrow Q = -\frac{S \Delta T}{R}$

CONVEZIONE

$Q = h S \Delta T$

IRRAGGIAMENTO

$\mathcal{E} = \sigma T^4$

REGOLA DELLE FASI

$n = c + 2 - f$

LAVORO FORZE DI PRESSIONE

$L = \int_{x_1}^{x_2} F_{est} \, dx \Rightarrow L = \int_{x_1}^{x_2} p_{est} S \, dx = \int_{V_1}^{V_2} p_{est} \, dV$

EQUAZIONE DI STATO GAS PERFETTI

Se $T = cost$, $pV = cost \Rightarrow \frac{V}{T} = cost \quad (V = V_0 \alpha T)$

$\frac{p}{T} = cost \quad (p = p_0 \alpha T)$

$\begin{cases} p_1 V_1 = p_m V_m = p_2 V_m \\ \frac{V_m}{T_m} = \frac{V_m}{T_1} = \frac{V_2}{T_2} \end{cases} \quad \begin{cases} p_1 V_1 = p_2 V_m = p_2 \frac{V_2 T_1}{T_2} \\ V_m = \frac{V_2 T_1}{T_2} \end{cases} \quad \begin{cases} \frac{p_1 V_1}{T_1} = \frac{p_2 V_2}{T_2} \end{cases} \Rightarrow pV = cT$

$c = nR \Rightarrow pV = nRT$

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LAVORO COMPIUTO DA GAS PERFETTO IN TRASFORMAZIONE ISOBARA

$$L = \int_{V_i}^{V_f} p\,dV = p(V_f - V_i) = nR(T_f - T_i)$$

LAVORO COMPIUTO DA GAS PERFETTO IN TRASFORMAZIONE ISOTERMA

$$L = \int_{V_i}^{V_f} p\,dV = nRT \int_{V_i}^{V_f} \frac{1}{V}\,dV = nRT \ln\left(\frac{V_f}{V_i}\right) = nRT \ln\left(\frac{p_i}{p_f}\right)$$

LEGGE DI DALTON

$$p = p_A + p_B = \frac{(n_A + n_B)RT}{V}$$

CALORI SPECIFICI GAS PERFETTI

PRESSIONE COSTANTE $c_p = \frac{1}{n}\frac{dQ}{dT}\Big|_p \quad \Rightarrow Q = n c_p \Delta T$

VOLUME COSTANTE $c_V = \frac{1}{n}\frac{dQ}{dT}\Big|_V \quad \Rightarrow Q = n c_V \Delta T$

$$\gamma = \frac{c_p}{c_V}$$

EQUAZIONE STATO GAS REALI

$$\left(p + \frac{n^2 a}{V^2}\right)(V - nb) = nRT$$

CALCOLO CINETICO PRESSIONE

$$\Delta p_x = p_x^{fin} - p_x^{in} = -mv_x - mv_x = -2mv_x$$

$$\Delta t = \frac{2\ell}{v_x} \Rightarrow F = \frac{|\Delta p|}{\Delta t} = \frac{2mv_x}{2\ell} \cdot v_x = \frac{mv_x^2}{\ell}$$

$$F_{tot} = \sum_{i=1}^{i=N} \frac{mv_{ix}^2}{\ell} \Rightarrow p = \frac{F_{tot}}{S} = \frac{F_{tot}}{\ell^2} = \sum_{i=1}^{i=N} \frac{mv_{ix}^2}{\ell^3} = \frac{1}{V}\sum_{i=1}^{i=N} mv_{ix}^2$$

$$p = \frac{1}{V}\sum_{i=1}^{i=N} mv_{iy}^2 \qquad p = \frac{1}{V}\sum_{i=1}^{i=N} mv_{iz}^2$$

$$3p = \frac{1}{V}\sum_{i=1}^{i=N}\left(mv_{ix}^2 + mv_{iy}^2 + mv_{iz}^2\right) \Rightarrow p = \frac{1}{3V}\sum_{i=1}^{i=N} mv_i^2$$

$$\sqrt{\overline{v^2}} = \sqrt{\frac{1}{N}\sum_{i=1}^{i=N} v_i^2} \Rightarrow p = \frac{1}{3V} mN\overline{v^2} = \frac{2}{3V} N\left(\frac{1}{2}m\overline{v^2}\right) \Rightarrow pV = \frac{2}{3}N\overline{E_c}$$

$$\Rightarrow pV = \frac{2}{3}U$$

INTERPRETAZIONE CINETICA TEMPERATURA ASSOLUTA

$$nRT = \frac{2}{3}N\overline{E_c} \Rightarrow E_c = \frac{3}{2}\frac{n}{N}RT \Rightarrow E_c = \frac{3}{2}kT$$

VELOCITÀ QUADRATICHE MEDIE

$$v_{qm} = \sqrt{\overline{v^2}} = \sqrt{\frac{3kT}{m}} \Rightarrow \frac{v_{qm1}}{v_{qm2}} = \sqrt{\frac{m_2}{m_1}}$$

$$v_{qm} = \sqrt{\frac{3kT}{m}} = \sqrt{\frac{3N_A kT}{N_A m}} = \sqrt{\frac{3RT}{M}}$$

$$v_{qm} = \sqrt{\frac{3RT}{M}} = \sqrt{\frac{3pV}{nM}} = \sqrt{\frac{3pV}{m}} = \sqrt{\frac{3p}{\rho}}$$

$$\mathcal{R} = 4\pi n \overline{v} r^2$$

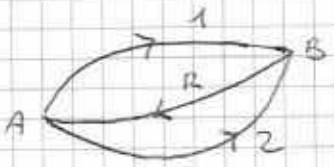
$$\overline{\ell} = \overline{v}\,\overline{\tau} = \frac{\overline{v}}{\mathcal{R}} = \frac{1}{4\pi n r^2}$$

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PRINCIPIO EQUIPARTIZIONE ENERGIA

$E_c = \frac{1}{2} f K T \Rightarrow U = \frac{1}{2} f N K T = \frac{1}{2} f n R T$

RELAZIONE 1° PRINCIPIO TERMODINAMICA e ESISTENZA FUNZIONE STATO ENERGIA INT.



$Q_{A\to A} = L_{A\to A}$

$Q_{A\to B} + Q_{B\to A} = L_{A\to B} + L_{B\to A}$

$Q_{A\to B} - L_{A\to B} = L_{B\to A} + Q_{B\to A}$

$|Q-L|^{(1)}_{A\to B} = |L-Q|^{(2)}_{B\to A}$ $\quad |Q-L|^{(2)}_{A\to B} = |L-Q|^{(1)}_{B\to A}$

$|Q-L|^{(1)}_{A\to B} = |Q-L|^{(2)}_{A\to B} \Rightarrow \Delta U = Q - L$

. Per sol. e liqu. si $L \sim 0 \Rightarrow Q = \Delta U = mc\Delta T$

ENTALPIA

$H = U + pV \Rightarrow dH = dU + p dV + V dp$

Se $p = cost \Rightarrow dH = dU + p dV = dU + dL = dQ \Rightarrow \Delta H = Q_p = cost$

CALORE SPECIFICO A VOLUME COSTANTE DI UN GAS PERFETTO

$dL = p dV = 0 \Rightarrow$ 1° PRINCIPIO $dQ = dU \Rightarrow \frac{dQ}{dT}\Big|_V = \frac{dU}{dT}\Big|_V$

$U = \frac{1}{2} n f R T \Rightarrow C_V = \frac{1}{n}\frac{dQ}{dT}\Big|_V = \frac{1}{n}\frac{dU}{dT}\Big|_V = \frac{1}{2} f R$

$\Delta U = n C_V \Delta T$

RELAZIONE DI MAYER

$dQ = n C_p dT \Rightarrow dL = p dV \Rightarrow d(pV) = d(nRT)$

$p = cost \Rightarrow p dV + V dp = p dV = dL = nR dT$

$dU = n C_V dT \Rightarrow dL = dQ - dU$

$nR dT = n C_p dT - n C_V dT \Rightarrow R = C_p - C_V$

TRASFORMAZIONI ADIABATICHE GAS PERFETTO

$dQ = 0 \Rightarrow dL = -dU = -n C_V \Delta T$

$dL = p dV \Rightarrow -n C_V \Delta T = p dV$

$nR dT = p dV + V dp$

$-\frac{C_V}{R} = \frac{p dV}{p dV + V dp}$

$R p dV = -C_V (p dV + V dp)$

$(C_V + R) p dV = -C_V V dp \Rightarrow \frac{C_V + R}{C_V}\frac{dV}{V} = -\frac{dp}{p}$

$\Rightarrow \frac{C_p}{C_V}\frac{dV}{V} = -\frac{dp}{p} \Rightarrow \gamma \frac{dV}{V} = -\frac{dp}{p} \Rightarrow \gamma \int \frac{dV}{V} = -\int \frac{dp}{p}$

$\gamma \ln V = -\ln p + cost$

$\ln V^\gamma + \ln p = cost$

$\ln(pV^\gamma) = cost \Rightarrow pV^\gamma = e^{cost}$

$\Rightarrow pV^\gamma = cost \qquad TV^{\gamma-1} = cost \qquad pT^{\frac{\gamma}{1-\gamma}} = cost$

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TRASFORMAZIONE ISOCORA

$L = \int p dV = 0 \Rightarrow Q = L + \Delta U = \Delta U = n c_v \Delta T$

TRASFORMAZIONE ISOBARA

$L = \int p dV = p \Delta V = nR \Delta T \Rightarrow Q = L + \Delta U \Rightarrow nc_p \Delta T = nR\Delta T + nc_v \Delta T$

TRASFORMAZIONE ISOTERMA

$\Delta U = n c_v \Delta T = 0 \quad (\Delta T = 0)$

$L = nRT \ln\left(\frac{V_f}{V_i}\right) = nRT \ln\left(\frac{p_i}{p_f}\right) = Q$

TRASFORMAZIONE ADIABATICA

$Q = 0 \Rightarrow L = -\Delta U = -nc_v \Delta T$

ESPANSIONE LIBERA DI JOULE

$Q = 0, \; L = 0 \Rightarrow \Delta U = Q - L = 0 \Rightarrow \Delta U = nc_v \Delta T = 0 \Rightarrow \Delta T = 0$

MACCHINE TERMICHE

$Q_2 > 0$ (assorbito dalla sorgente calda)

$Q_1 < 0$ (ceduta alla sorgente fredda)

$\eta = \frac{L}{Q_2} \Rightarrow Q - \Delta U = Q_1 + Q_2 \Rightarrow \eta = \frac{Q_1 + Q_2}{Q_2} = 1 + \frac{Q_1}{Q_2} \quad (0 < \eta < 1)$

CICLO CARNOT GAS PERFETTO

$L = Q_1 + Q_2 = -|Q_1| + Q_2 \Rightarrow \eta = 1 - \frac{T_1}{T_2} \quad (<1)$

$TV^{\gamma-1} = \text{cost} \Rightarrow T_2 V_B^{\gamma-1} = T_1 V_C^{\gamma-1} \quad T_2 V_A^{\gamma-1} = T_1 V_D^{\gamma-1} \Rightarrow \frac{V_B}{V_A} = \frac{V_C}{V_D}$

$Q_1 = nRT_1 \ln\left(\frac{V_D}{V_C}\right) \qquad Q_2 = nRT_2 \ln\left(\frac{V_B}{V_A}\right)$

$\frac{Q_1}{Q_2} = \frac{nRT_1 \ln\left(\frac{V_D}{V_C}\right)}{nRT_2 \ln\left(\frac{V_B}{V_A}\right)} = \frac{nRT_1 \ln\left(\frac{V_A}{V_B}\right)}{nRT_2 \ln\left(\frac{V_A}{V_B}\right)} = -\frac{T_1}{T_2} \Rightarrow \eta = 1 + \frac{Q_1}{Q_2} = 1 - \frac{T_1}{T_2}$

$\frac{Q_1}{T_1} + \frac{Q_2}{T_2} = 0$

TEOREMA CARNOT MACCHINE REVERSE

$\eta_{rev} = 1 - \frac{T_1}{T_2} \qquad \eta_{irr} < \eta_{rev} = 1 - \frac{T_1}{T_2} \Rightarrow \frac{Q_1}{T_1} + \frac{Q_2}{T_2} \leq 0$

$\sum_{i=1}^{n} \frac{Q_i}{T_i} \leq 0 \qquad \oint \frac{dQ}{T} \leq 0$

MACCHINE FRIGORIFERE

$\varepsilon = \frac{Q_1}{|L|} \qquad \varepsilon_{irr} < \varepsilon_{rev} \; (= \varepsilon_{max})$

$\varepsilon_{rev} = \frac{1}{\left|1 + \frac{Q_2}{Q_1}\right|} = \frac{1}{\left|1 - \frac{T_2}{T_1}\right|} = \frac{1}{\frac{T_2}{T_1} - 1} = \frac{T_1}{T_2 - T_1}$

ENTROPIA

$\int_{A}^{B}{}_{rev} \frac{dQ}{T} = \Delta S = S(B) - S(A)$

$S_{tot} = \sum_{i=1}^{N} S_i$

$dQ = T dS \Rightarrow Q = \int T dS$

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VARIAZIONE ENTROPIA TRASFORMAZIONE ISOTERMA REVERSIBILE

$$\Delta S = \int \frac{dQ}{T} = \frac{1}{T}\int dQ = \frac{Q}{T}$$

$$L = Q - \Delta U = T\Delta S - \Delta U = \Delta F \quad \Rightarrow F = U - TS$$

$$\Delta S = \frac{Q_1}{T_1} + \frac{Q_2}{T_2} = 0 \quad \Rightarrow \eta = 1 - \frac{T_2}{T_1}$$

Se $\Delta U = 0 \Rightarrow L = T\Delta S \Rightarrow \Delta S = \frac{L}{T} = \int_A^B \frac{p}{T} dV = \int_A^B \frac{nR}{V} dV = nR \ln\left(\frac{V_B}{V_A}\right) = nR \ln\left(\frac{p_A}{p_B}\right)$

$$\Delta S = \frac{Q_{cs}}{T_{cs}} = \frac{\lambda_{cs}\, m}{T_{cs}}$$

VARIAZIONE ENTROPIA TRASFORMAZIONE ADIABATICA REVERSIBILE

$$\Delta S = \int \frac{dQ}{dT} = 0$$

VARIAZIONE ENTROPIA TRASFORMAZIONE ISOCORA REVERSIBILE

$$\Delta S = \int_A^B \frac{dQ}{T} = \int_A^B \frac{nc_v\, dT}{T} = nc_v \ln\left(\frac{T_B}{T_A}\right) = nc_v \ln\left(\frac{p_B}{p_A}\right)$$

VARIAZIONE ENTROPIA TRASFORMAZIONE ISOBARA REVERSIBILE

$$\Delta S = \int_A^B \frac{dQ}{T} = \int_A^B \frac{nc_p\, dT}{T} = nc_p \ln\left(\frac{T_B}{T_A}\right) = nc_p \ln\left(\frac{V_B}{V_A}\right)$$

CASO GENERALE TRASFORMAZIONE GAS PERFETTO

$$dS = \frac{dQ}{T}\Big|_{rev} = \frac{dU}{T} + \frac{dL}{T} = \frac{nc_v\, dT}{T} + \frac{p\, dV}{T} \quad \Rightarrow \Delta S = nc_v \ln\left(\frac{T_B}{T_A}\right) + nR \ln\left(\frac{V_B}{V_A}\right)$$

SOLIDI E LIQUIDI

$$dQ = cm\, dT \quad \Rightarrow \Delta S = \int_B^B \frac{cm\, dT}{T} = cm \ln\left(\frac{T_B}{T_A}\right) \qquad (S = cm \ln T)$$

ESPANSIONE LIBERA JOULE

$$\Delta S = nR \ln\left(\frac{V_{FIN}}{V_{IN}}\right) > 0$$

LEGGE ACCRESCIMENTO ENTROPIA

$$\oint \frac{dQ}{T}\Big|_{irr} \leq \oint \frac{dQ}{T}\Big|_{rev} = 0 \quad \Rightarrow \int_A^B \frac{dQ}{T}\Big|_{irr} + \int_B^A \frac{dQ}{T}\Big|_{rev} \leq \int_A^B \frac{dQ}{T}\Big|_{rev} + \int_B^A \frac{dQ}{T}\Big|_{rev}$$

$$\int_A^B \frac{dQ}{T}\Big|_{irr} \leq \int_A^B \frac{dQ}{T}\Big|_{rev} = S(B) - S(A)$$

$$\int_A^B \frac{dQ}{T}\Big|_{irr} = 0 \leq S(B) - S(A) \Rightarrow S(B) \geq S(A)$$

DEGRADAZIONE ENERGIA

$$\eta_{max} = 1 - \frac{T_1}{T_2}$$

$$\Delta S = \frac{Q_2}{T_2} + \frac{Q_1}{T_1}$$

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