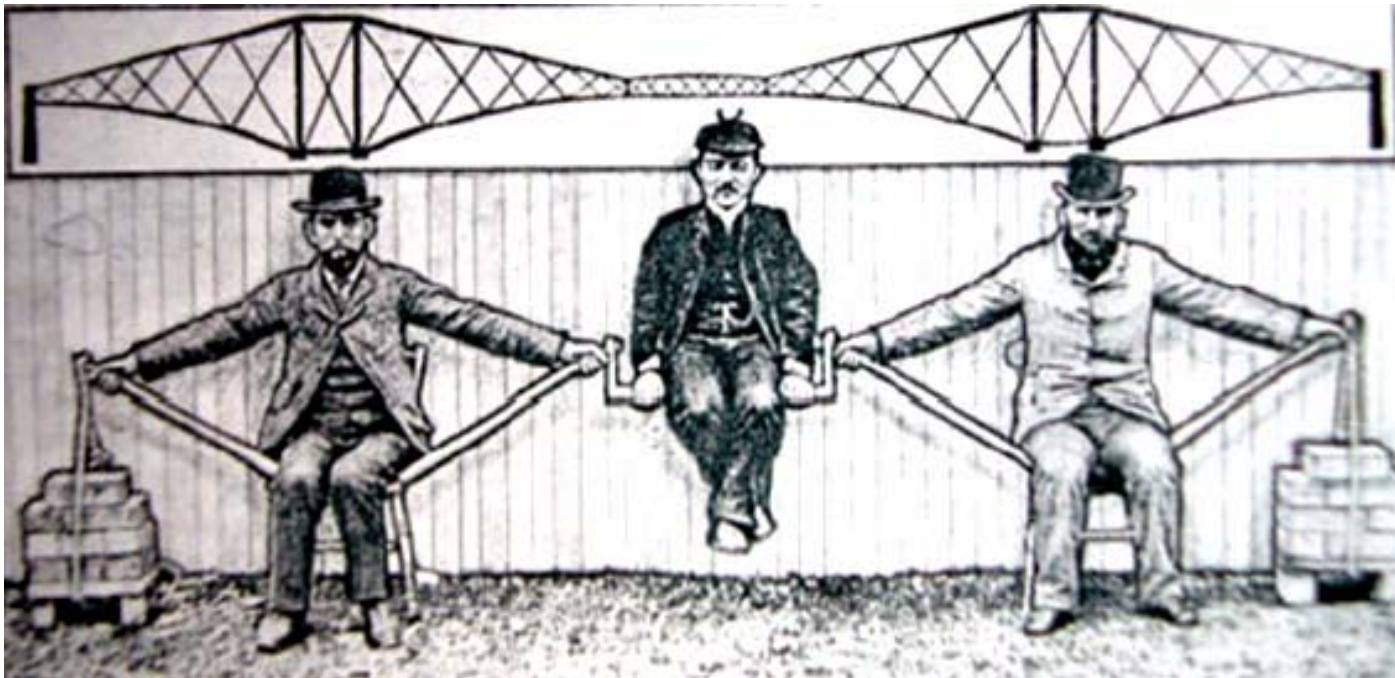


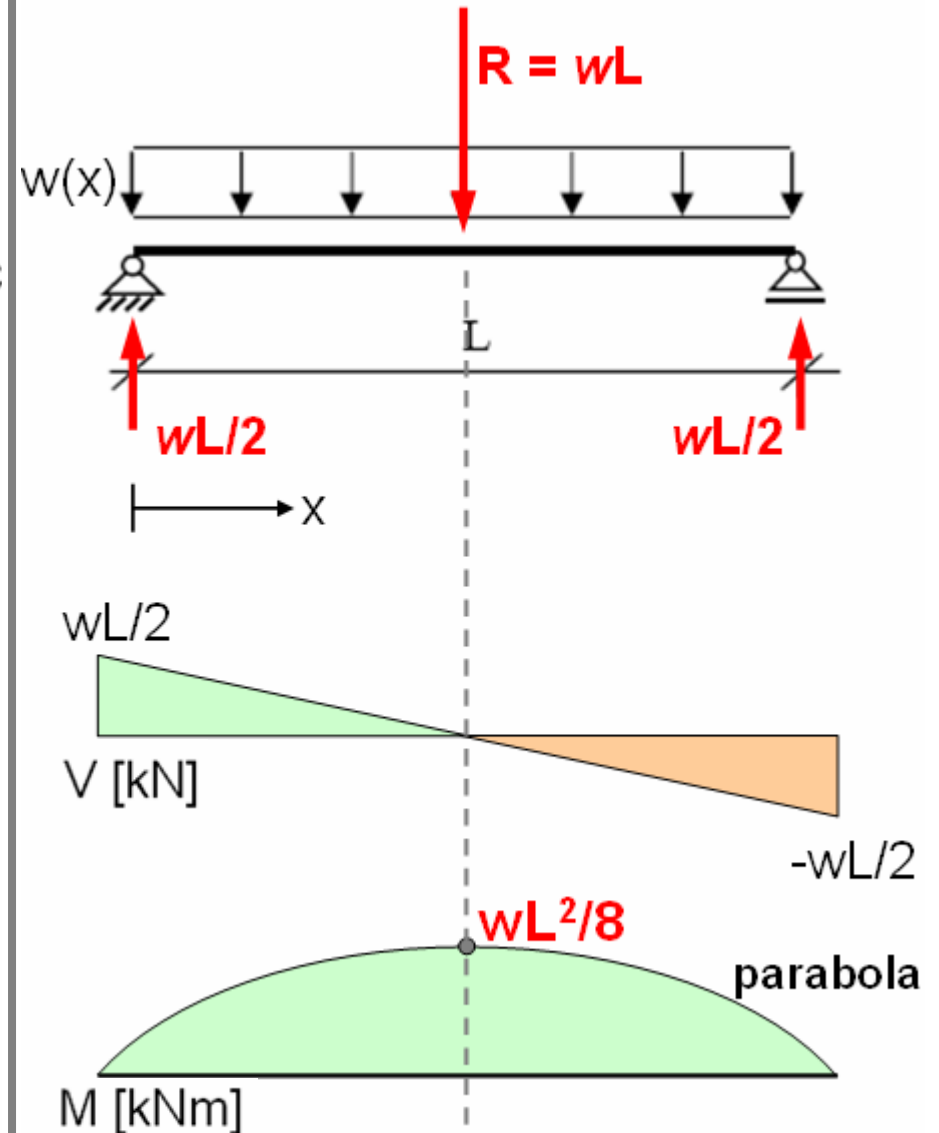
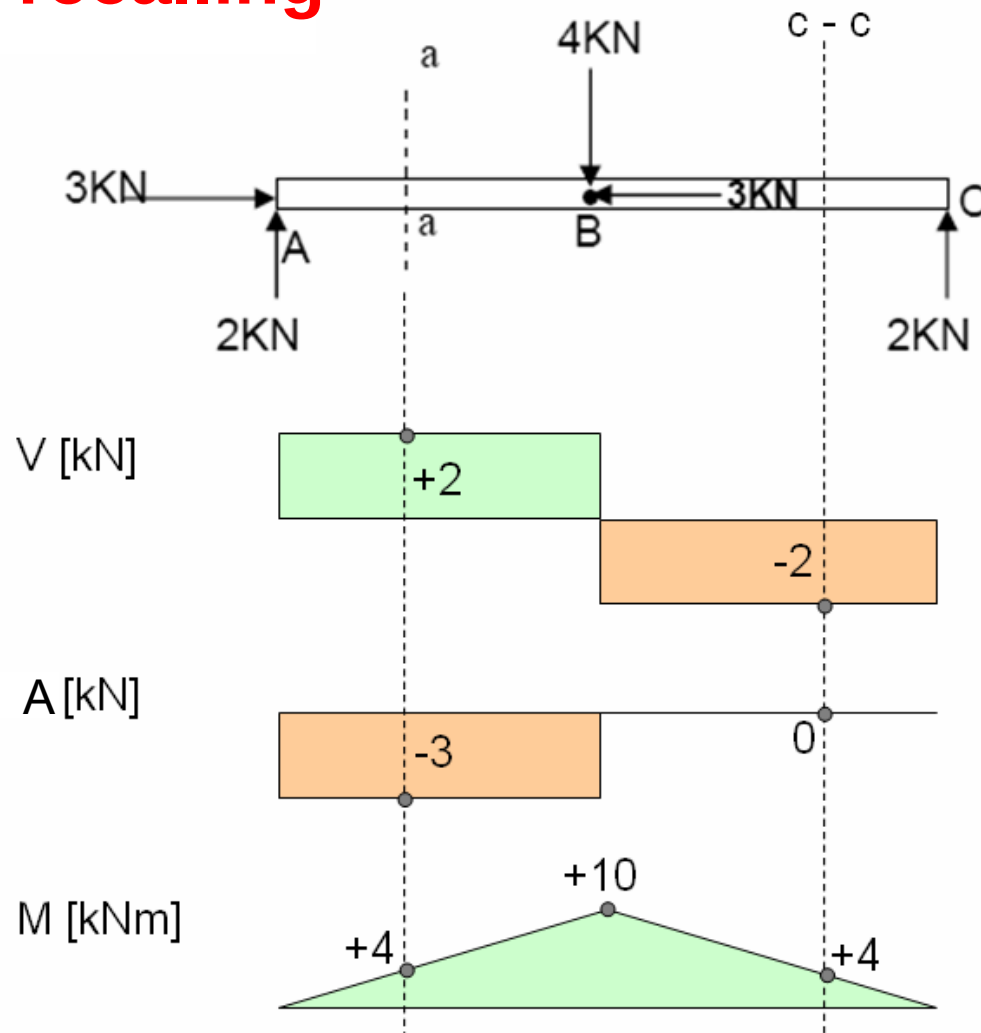
Chapter V b

Beams – internal actions

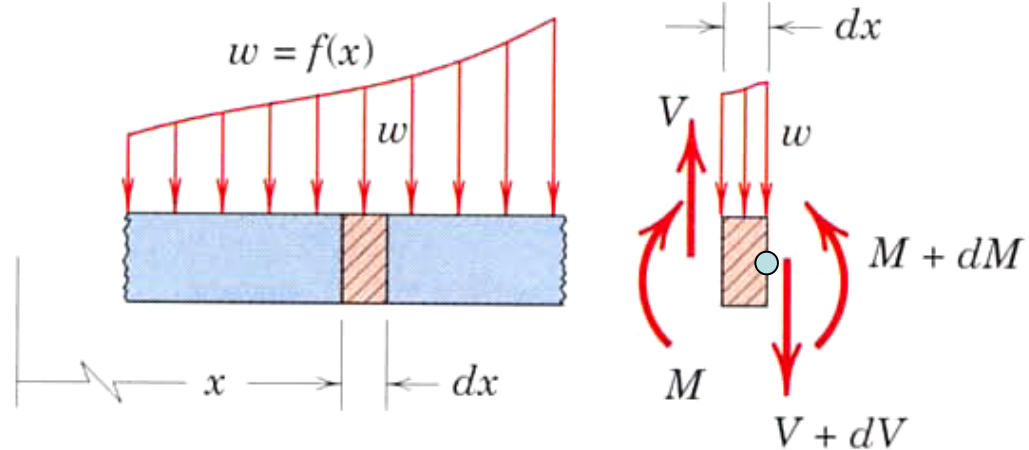


Comparison of internal force diagrams of concentrated and distributed load situations.

recalling



relation of shear force and bending moment functions



$$\sum F_y = 0 : dV = -w dx$$

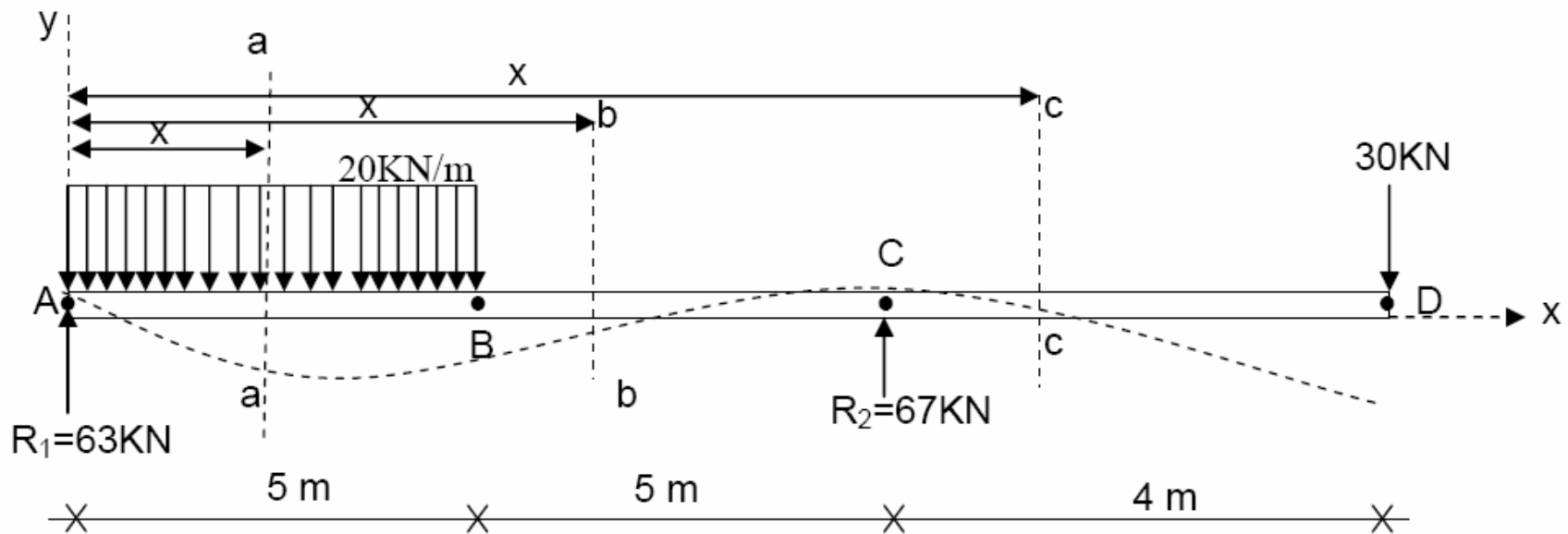
$$\sum M = 0 : -\cancel{M} - V dx - \overset{\text{neglectable higher order differential}}{w dx dx/2} + \cancel{M} + dM = 0$$

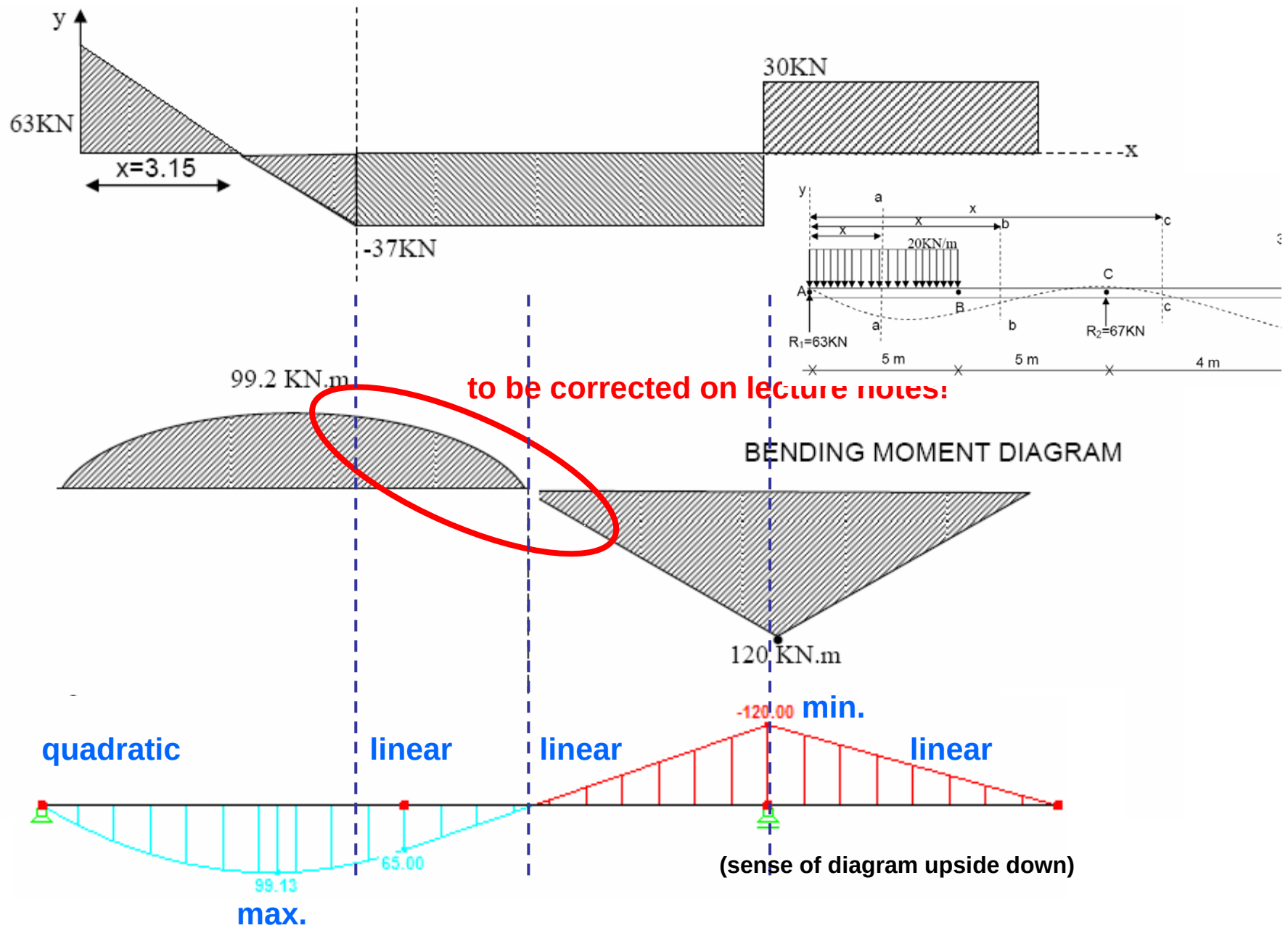
$$V' = \frac{dV}{dx} = -w$$

$$M' = \frac{dM}{dx} = V$$

fig by J.L. Meriam, L.G. Kraige, Engineering Mechanics I

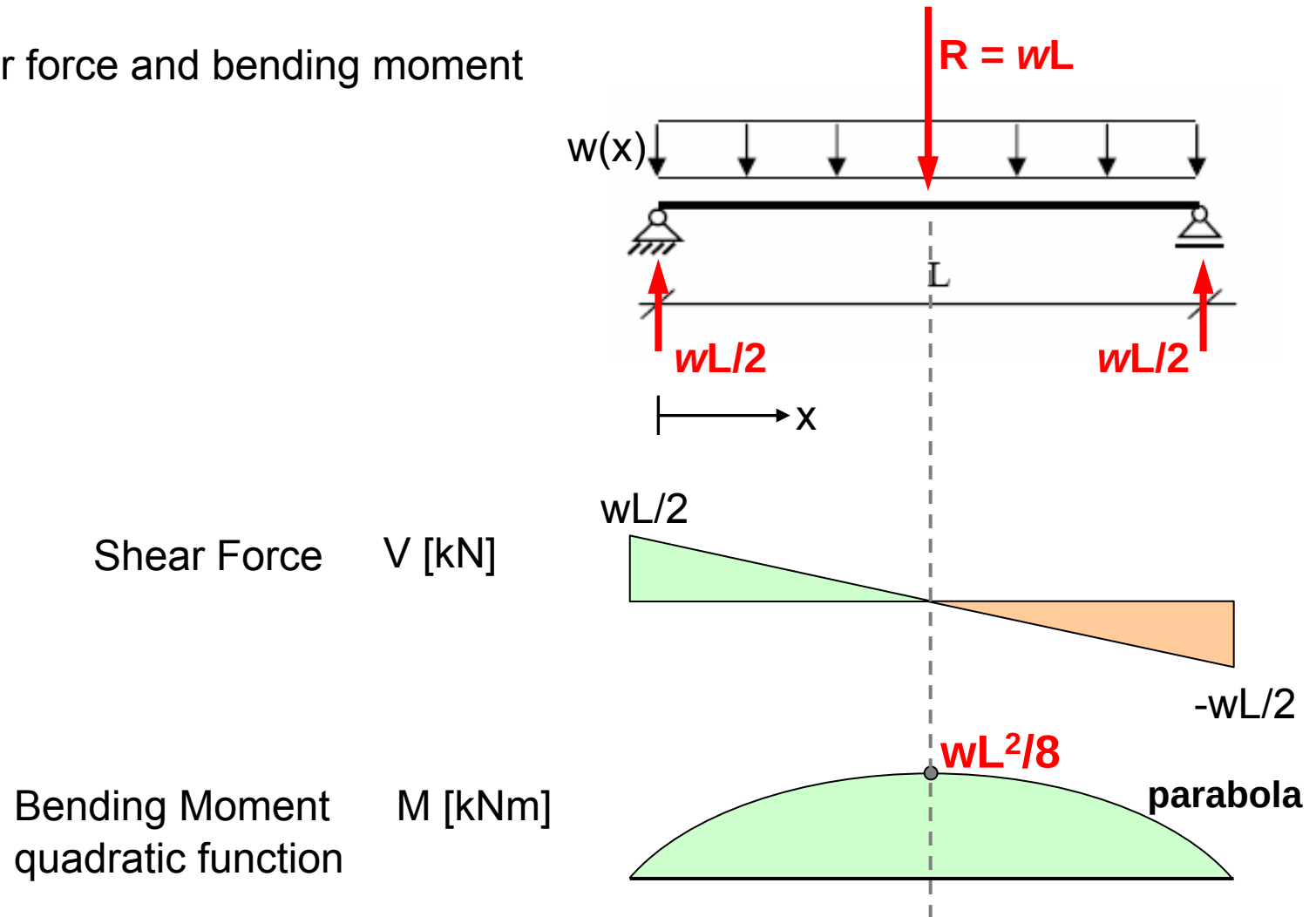






recalling example 5.2

Draw the shear force and bending moment diagrams.



example 5.3

Draw the shear force and bending moment diagram.

external equilibrium

$$\sum M_A = 0 : -w L (L/2) + M_A = 0$$

$$\rightarrow M_A = w L^2/2$$

$$\sum F_y = 0 : A_y - w L = 0$$

$$\rightarrow A_y = w L$$

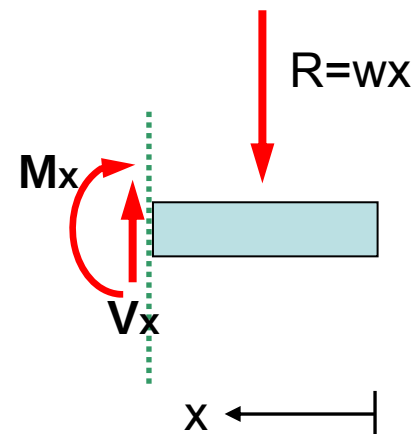
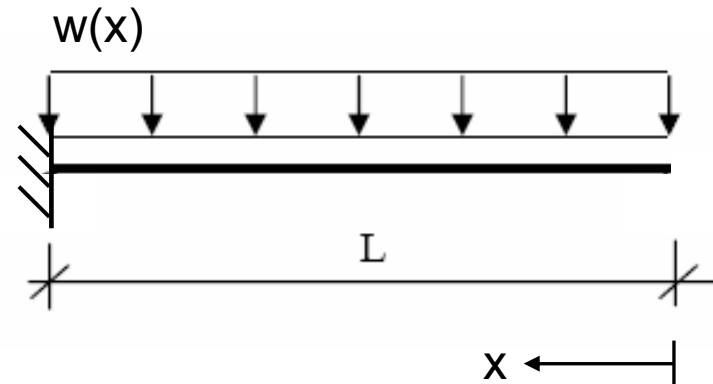
internal equilibrium

$$\sum M_x = 0 : M_x + w x (x/2) = 0$$

$$\rightarrow M_x = -w x^2/2$$

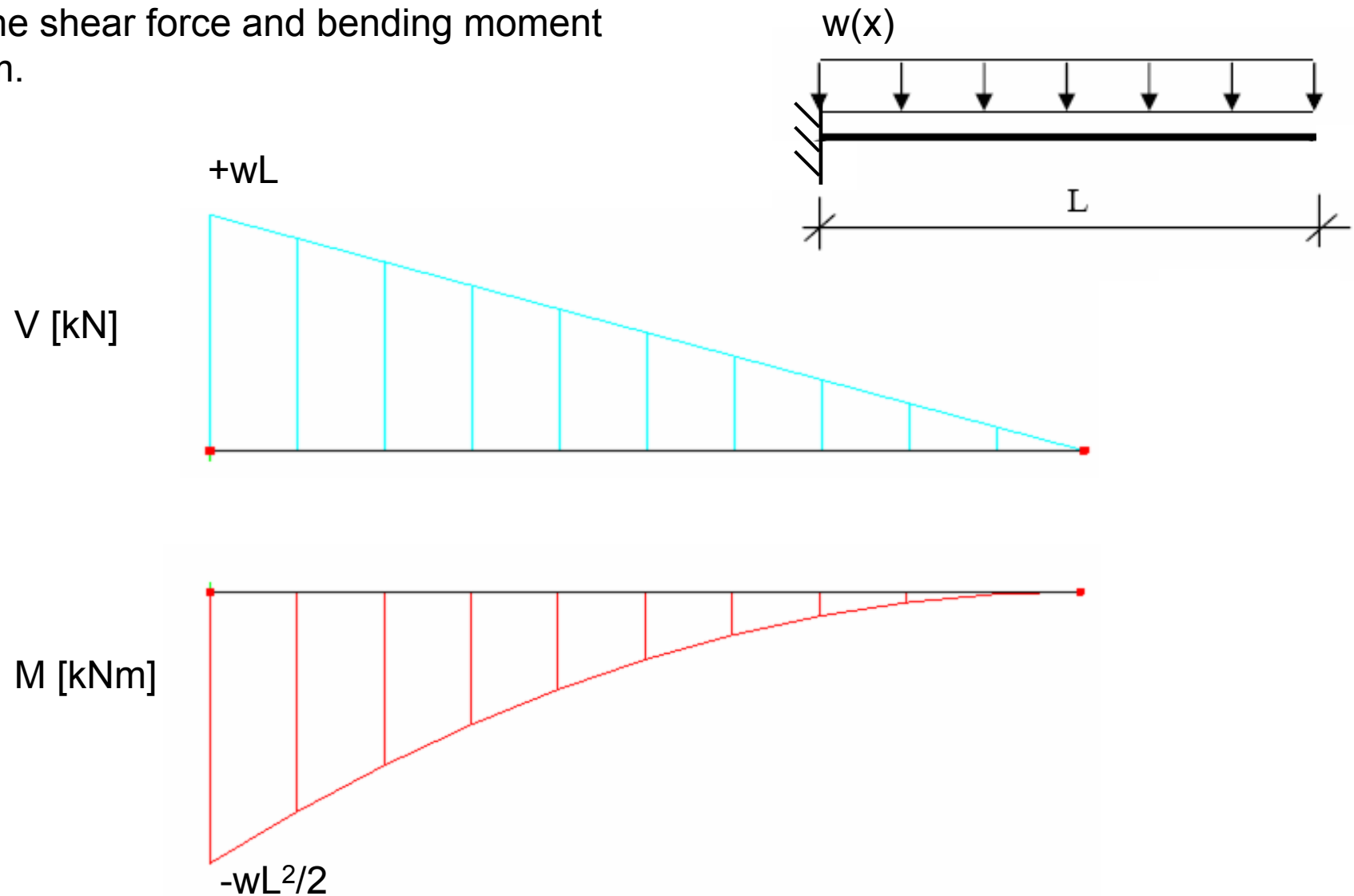
$$\sum F_y = 0 : V_x - w x = 0$$

$$\rightarrow V_x = w x$$



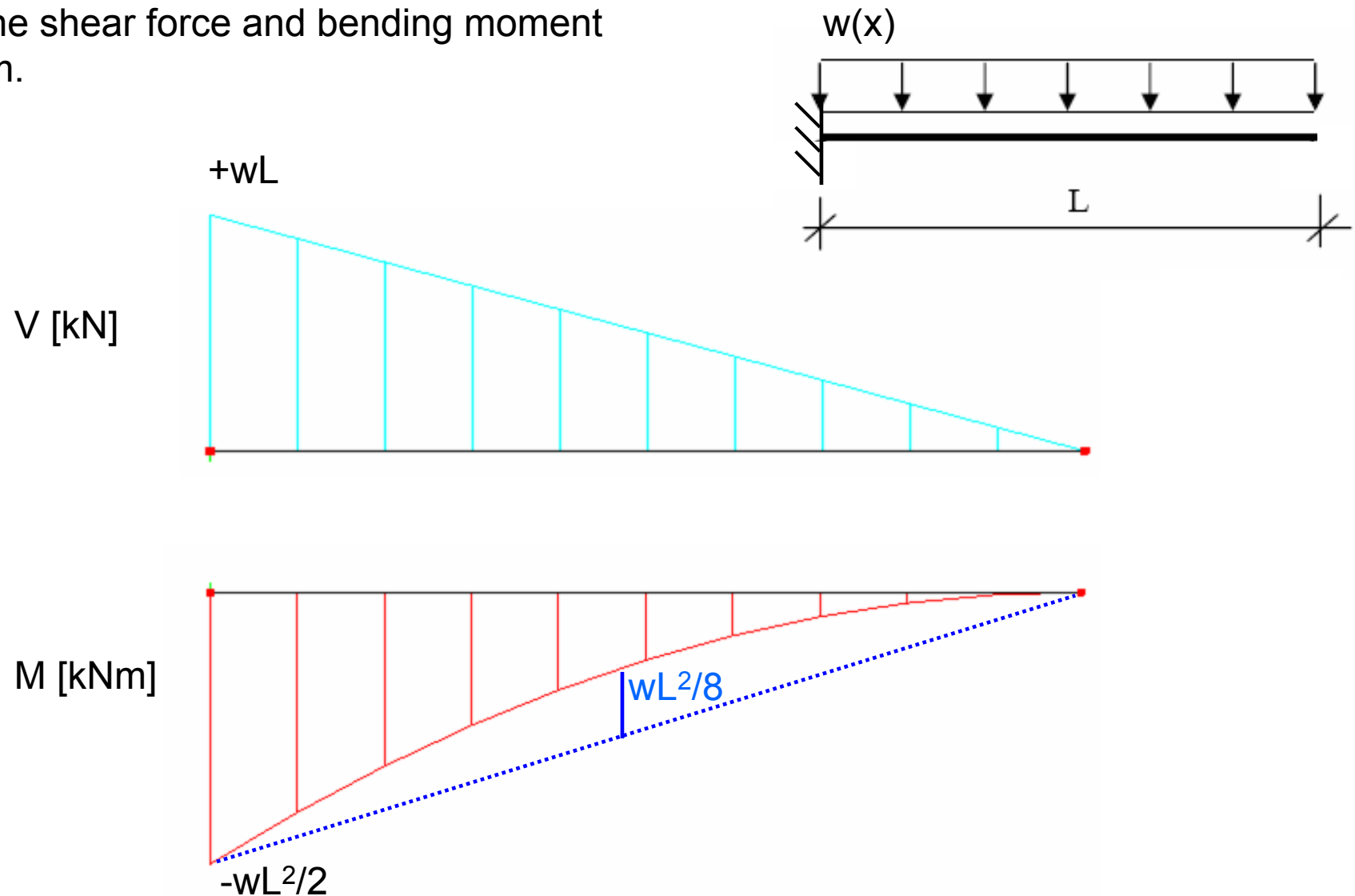
example 5.3

Draw the shear force and bending moment diagram.



example 5.3

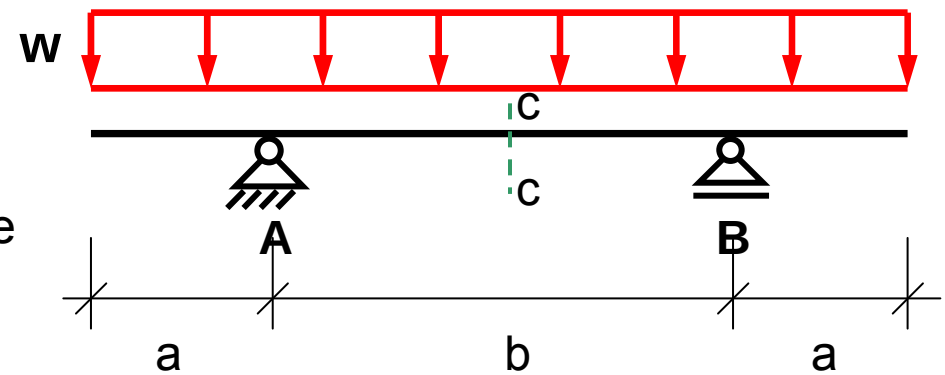
Draw the shear force and bending moment diagram.



example 5.4

Determine the max. and min. bending moment and draw the moment diagram.

Determine the appropriate ratio of $a : b$ concerning the most economic design of the beam having a constant cross section.



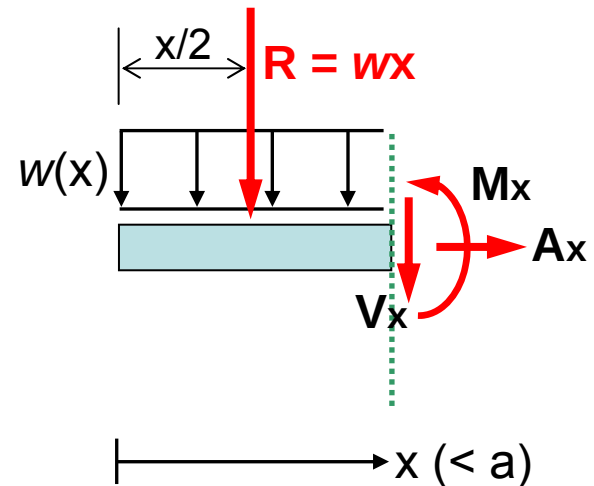
From external equilibrium:

$$A = B = w(2a + b) / 2$$

From internal equilibrium:

$$M_A = M_B = -w a^2 / 2$$

$$M_c = -w(a + b/2)^2 / 2 + A b/2 = w/2 (b^2/4 - a^2)$$



example 5.4

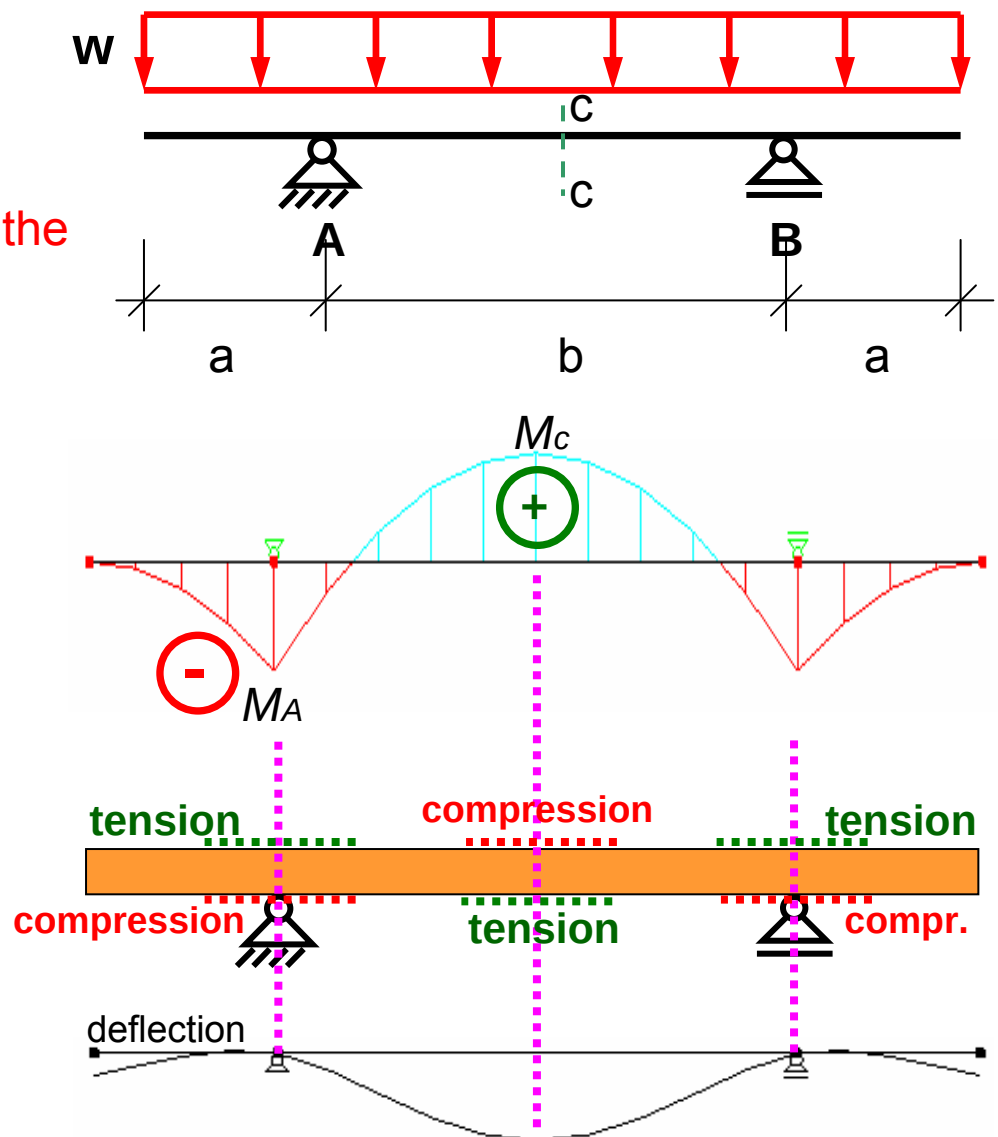
Determine the max. and min. bending moment and draw the moment diagram.

Determine the appropriate ratio of $a : b$ concerning the **most economic design of the beam having a constant cross section**.

From internal equilibrium:

$$M_A = M_B = -w a^2 / 2$$

$$M_c = w/2 (b^2/4 - a^2)$$



example 5.4

Determine the max. and min. bending moment and draw the moment diagram.

Determine the appropriate ratio of $a : b$ concerning the **most economic design of the beam having a constant cross section**.

From internal equilibrium:

$$M_A = M_B = -w a^2 / 2$$

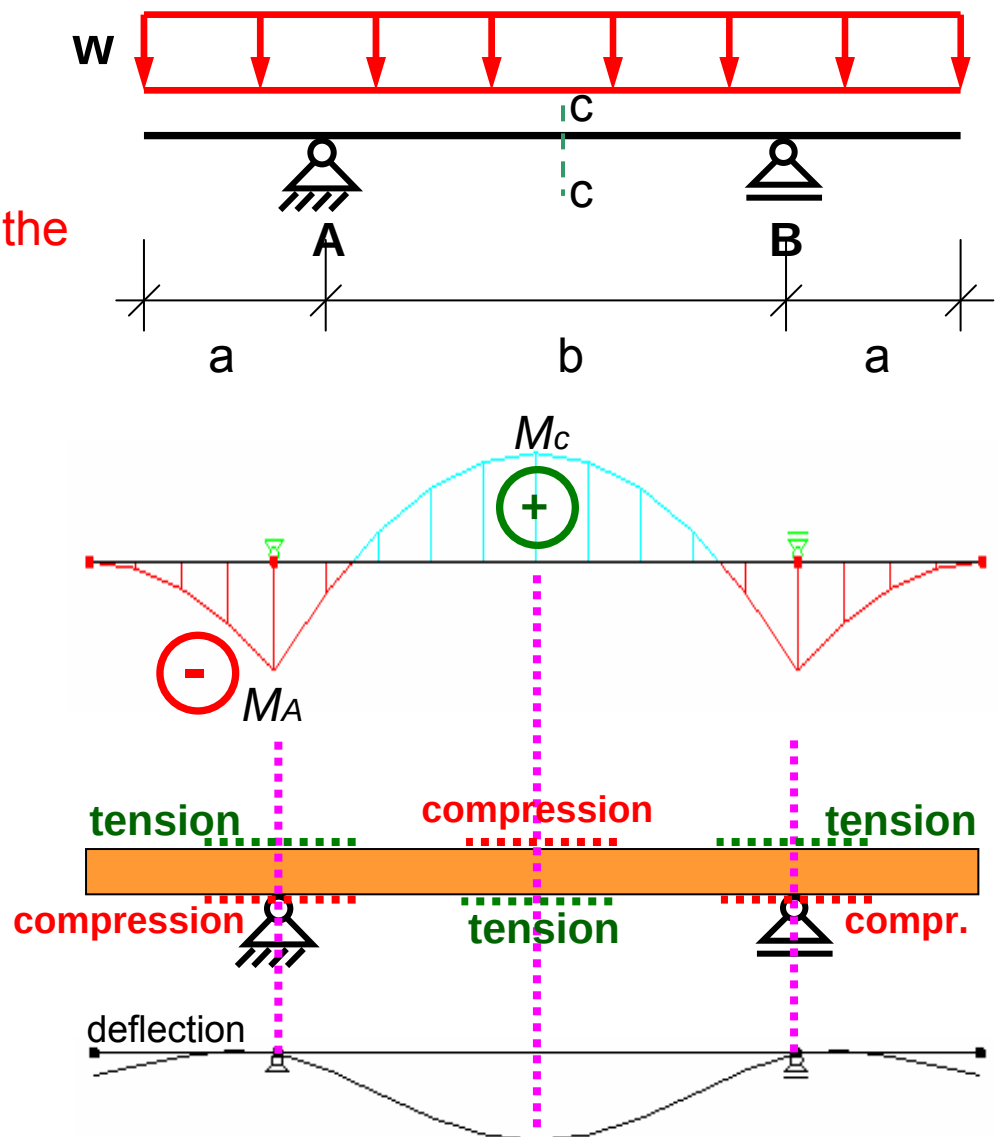
$$M_C = w/2 (b^2/4 - a^2)$$

Economic design demands:

$$|M_A| = |M_C| : w a^2 / 2 = w/2 (b^2/4 - a^2)$$

$$a^2 = b^2/8$$

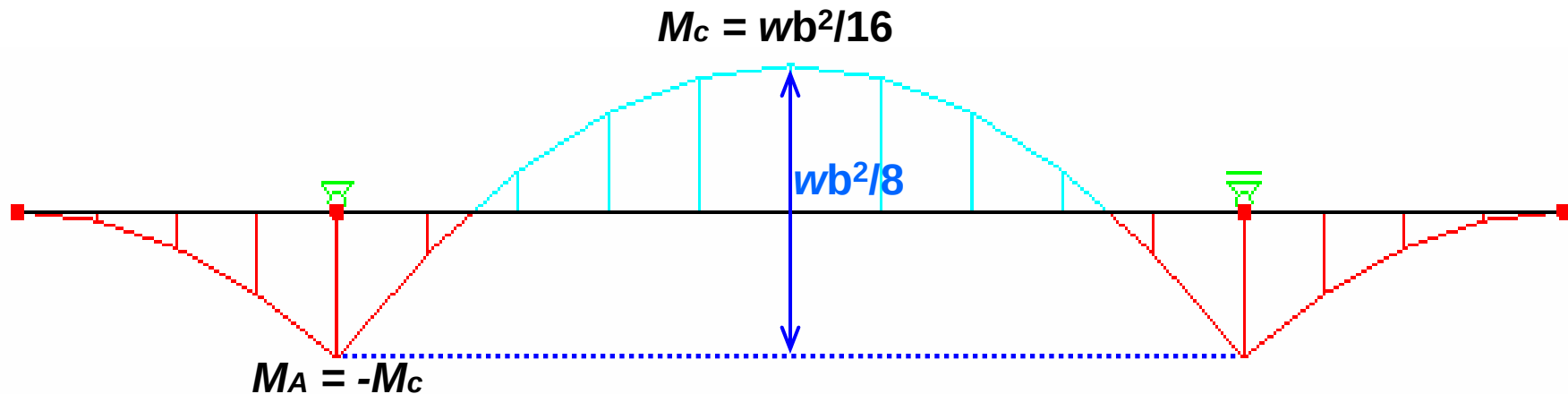
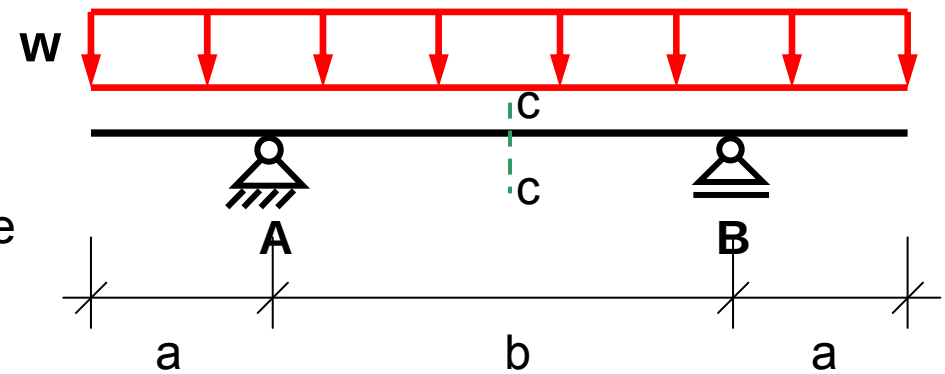
$$a = 0.354 b$$



example 5.4

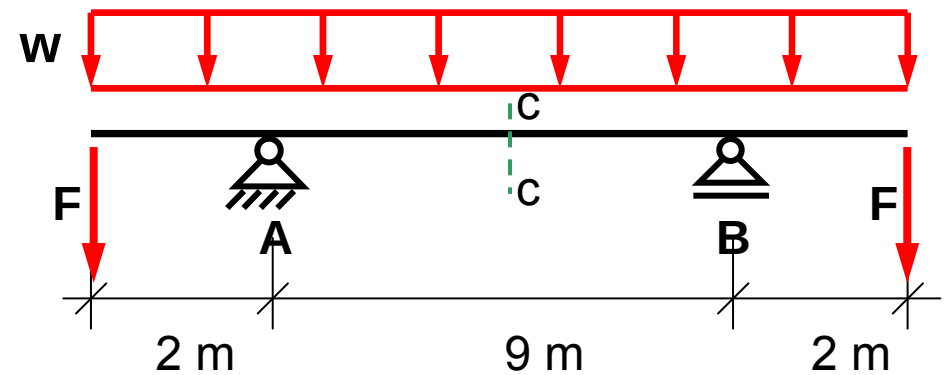
Determine the max. and min. bending moment and draw the moment diagram.

Determine the appropriate ratio of $a : b$ concerning the most economic design of the beam having a constant cross section.



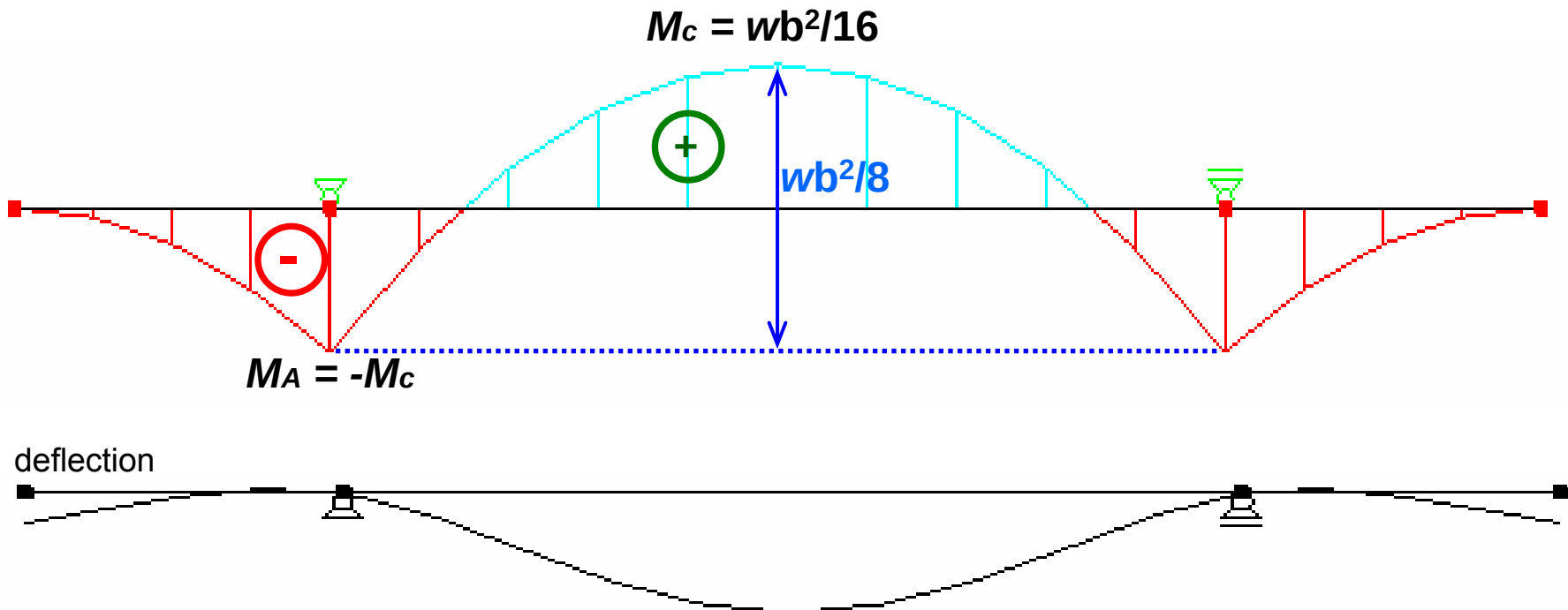
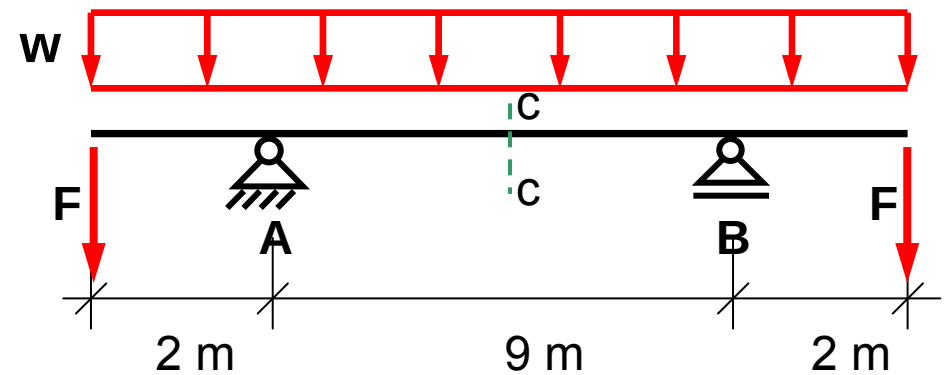
example 5.5

Determine the prestress force F in the façade structure for the most economic design of the slab support beam with a constant cross section if $w = 10$ kN/m.



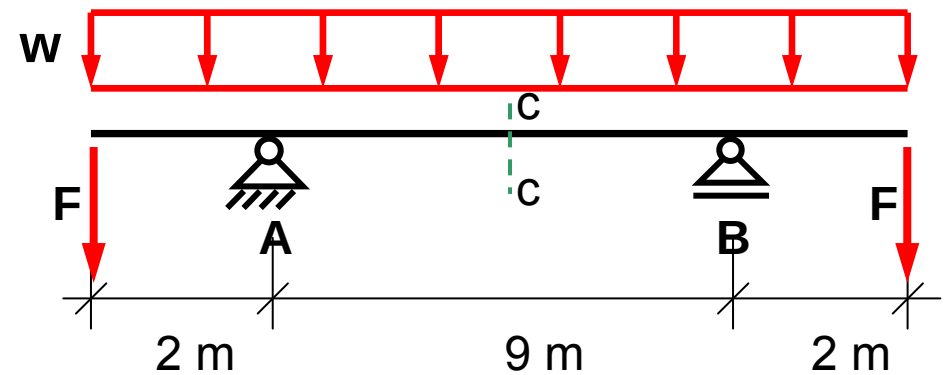
example 5.5

Determine the prestress force F in the façade structure for the most economic design of the slab support beam with a constant cross section if $w = 10 \text{ kN/m}$.



example 5.5

Determine the prestress force F in the façade structure for the most economic design of the slab support beam with a constant cross section if $w = 10 \text{ kN/m}$.



From internal equilibrium:

$$M_A = M_B = -F \cdot 2.0 \text{ m} - w (2.0 \text{ m})^2 / 2 = -F \cdot 2.0 \text{ m} - 20 \text{ kNm}$$

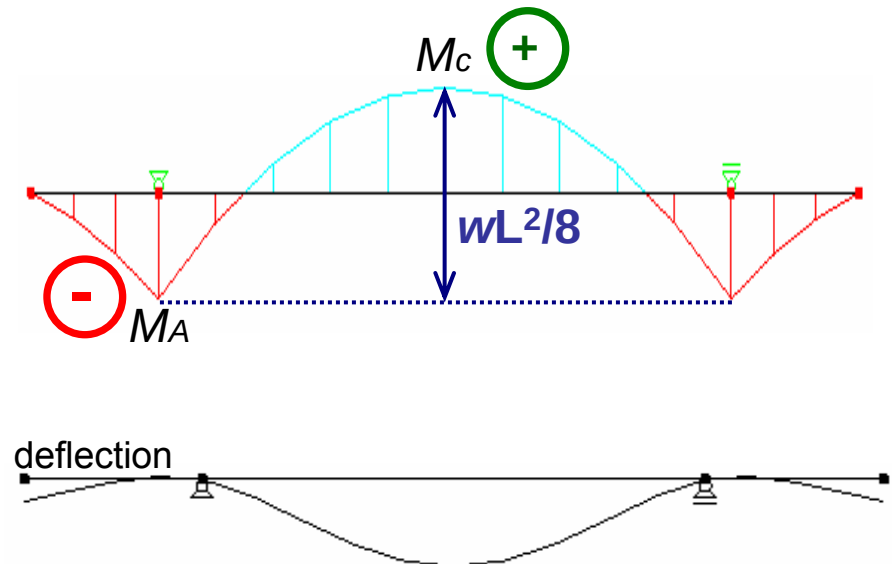
$$M_C = M_A + w (9.0 \text{ m})^2 / 8$$

Economic design demands:

$$M_C = -M_A$$

$$M_A = -w (9.0 \text{ m})^2 / 16 = -F \cdot 2.0 \text{ m} - 20 \text{ kNm}$$

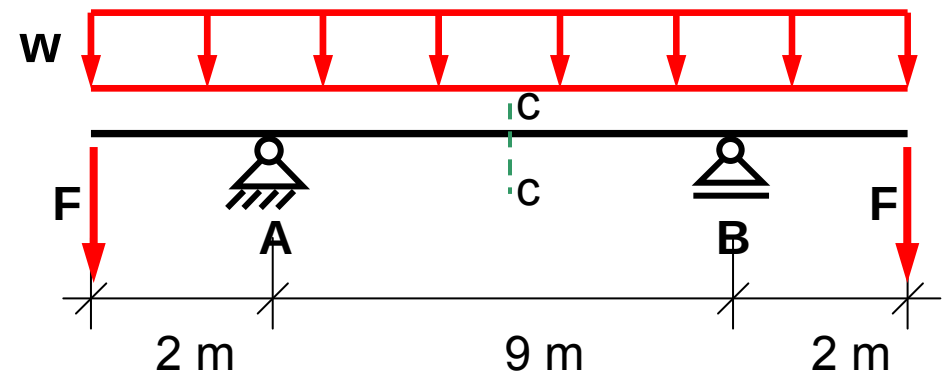
$$\rightarrow F = 15.3 \text{ kN}$$



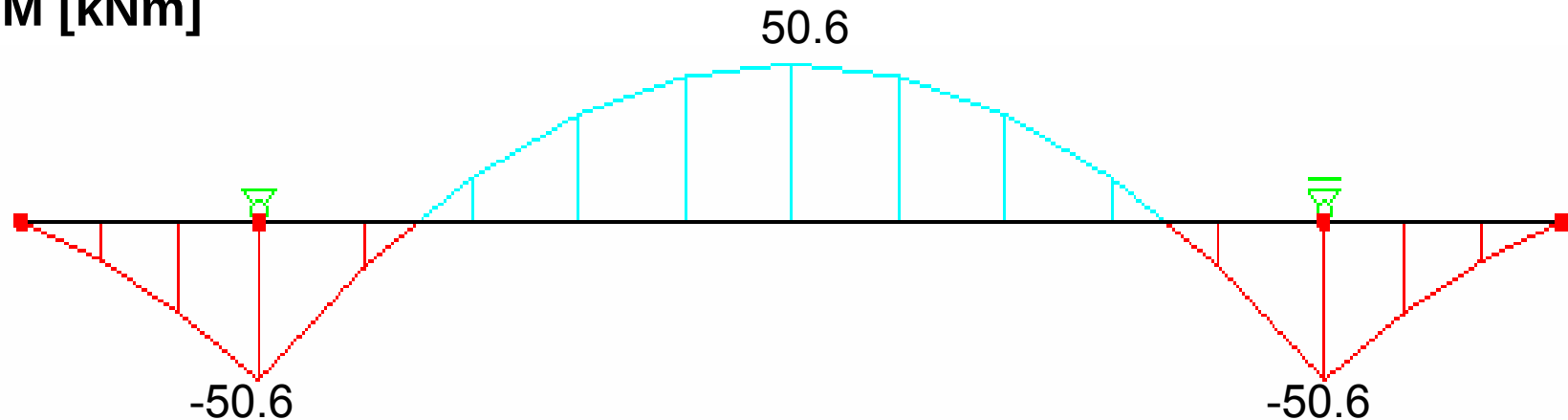
example 5.5

Determine the prestress force F in the façade structure for the most economic design of the slab support beam with a constant cross section if $w = 10 \text{ kN/m}$.

$$F = 15.3 \text{ kN}$$



$M \text{ [kNm]}$

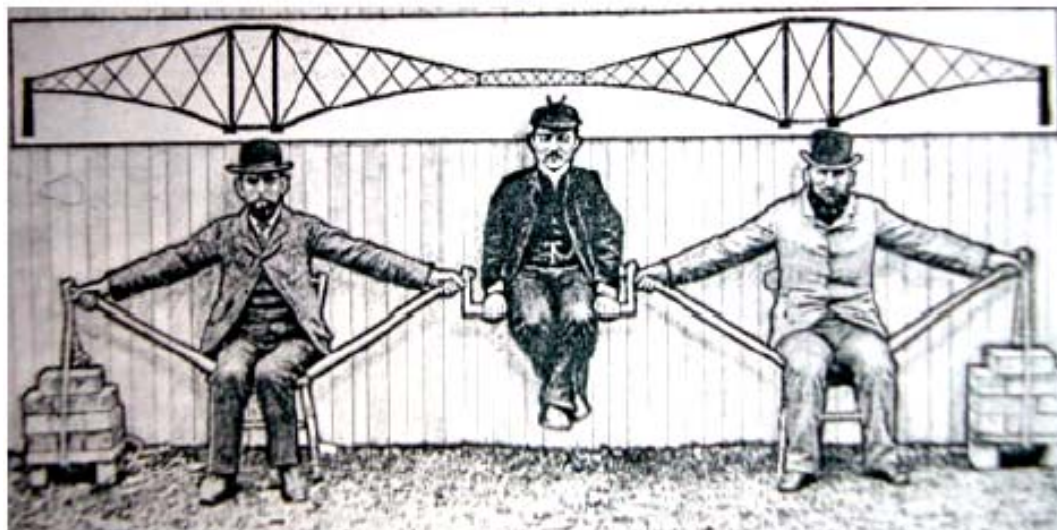
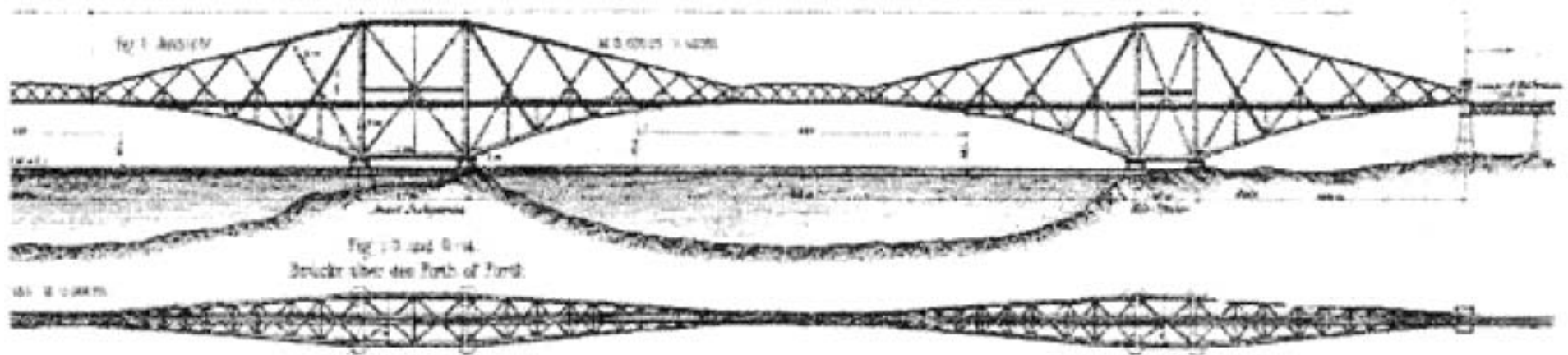




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Faculty of Technology

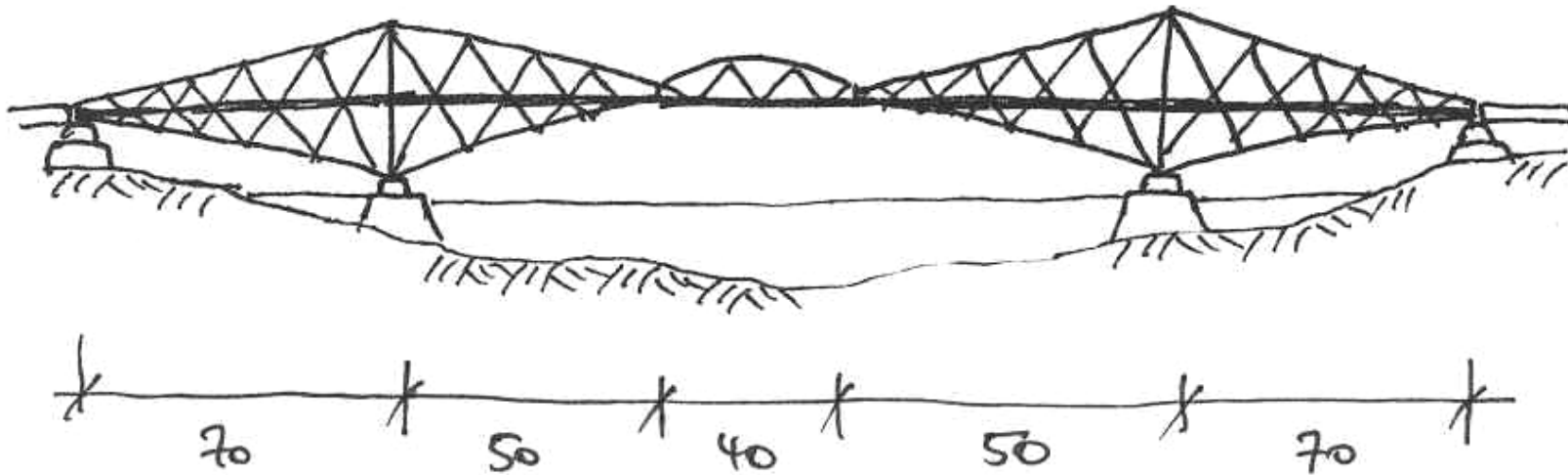
Engineering Mechanics
material by Karsten Schlesier

Department of
Civil Engineering



example 5.6

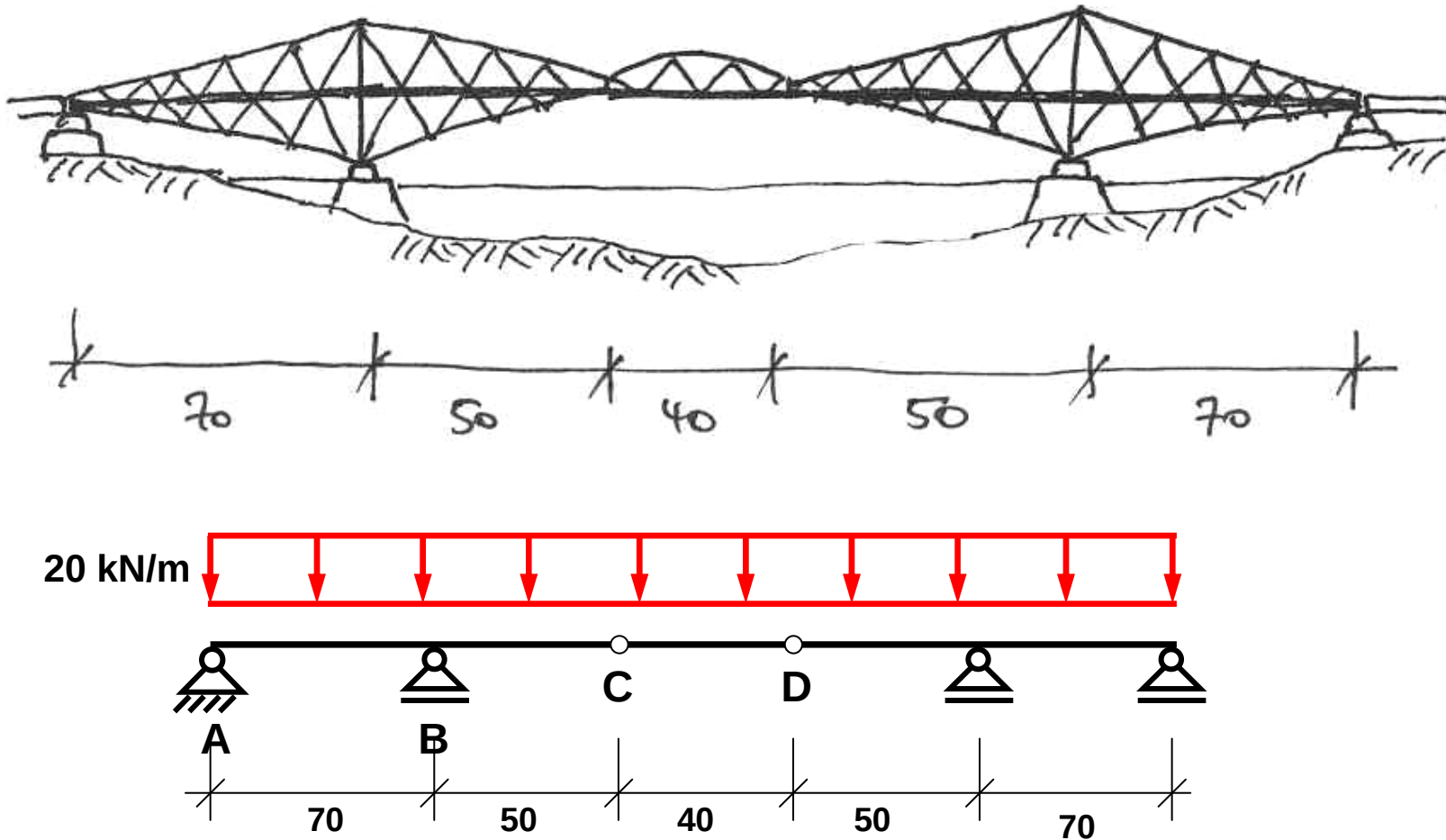
The given bridge structure is subjected to a distributed vertical load of $w = 20 \text{ kN/m}$. Determine the support reactions and the moment diagram.



example 5.6

The given bridge structure is subjected to a distributed load of $w = 20 \text{ kN/m}$.

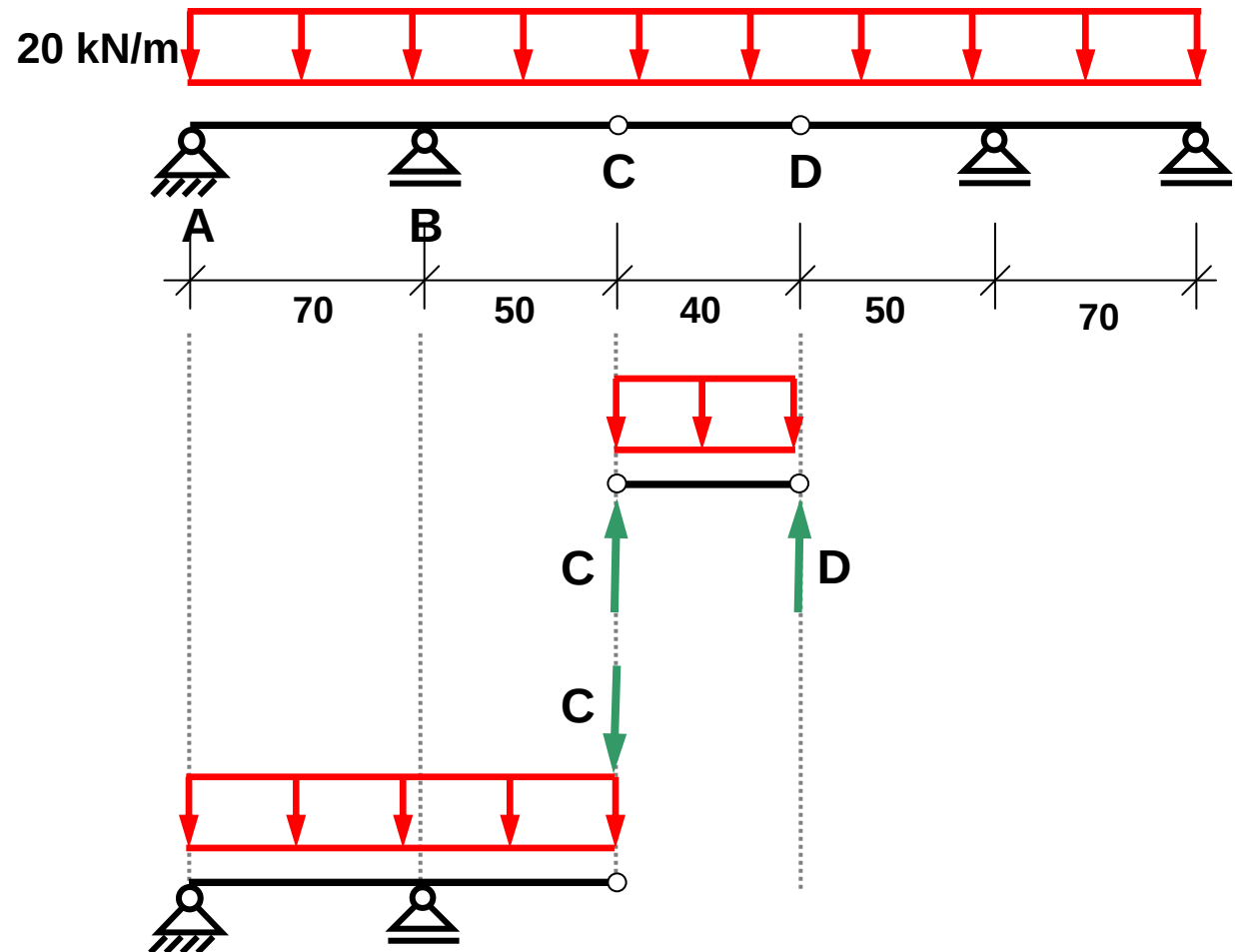
Determine the support reactions and the moment diagram.



example 5.6

The given bridge structure is subjected to a distributed load of $w = 20 \text{ kN/m}$.

Determine the support reactions and the moment diagram.



example 5.6

The given bridge structure is subjected to a distributed load of $w = 20 \text{ kN/m}$.

Determine the **support reactions** and the moment diagram.

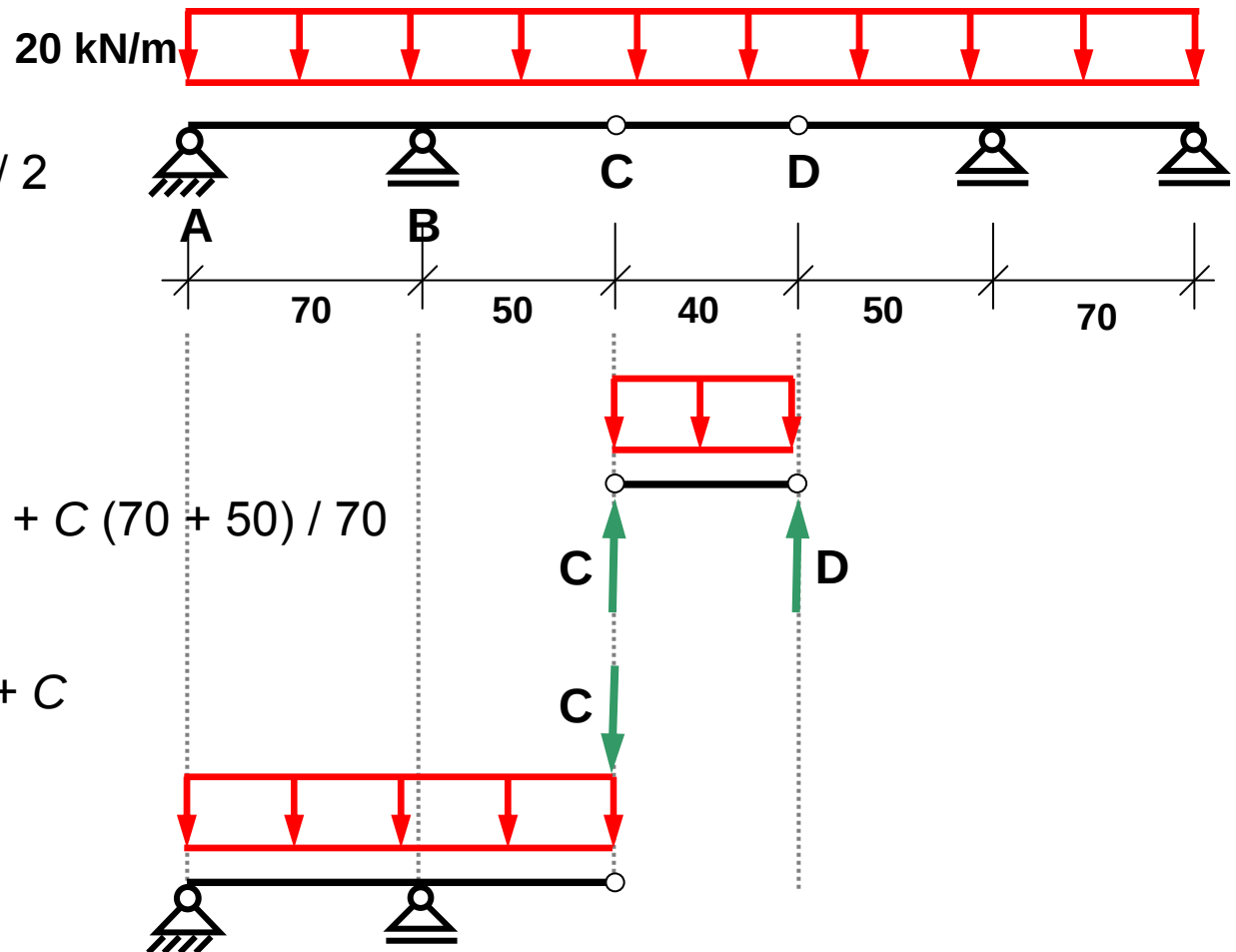
Part C – D:

$$\begin{aligned} C = D &= 20 \text{ kN/m} \cdot 40.0 \text{ m} / 2 \\ &= 400 \text{ kN} \end{aligned}$$

Part A – C:

$$\begin{aligned} B &= (20 (70 + 50)^2 / 2 + C (70 + 50) / 70 \\ &= 2742.9 \text{ kN} \end{aligned}$$

$$\begin{aligned} A &= 20 (70 + 50) - B + C \\ &= 57.1 \text{ kN} \end{aligned}$$



example 5.6

The given bridge structure is subjected to a distributed load of $w = 20 \text{ kN/m}$.

Determine the support reactions and the **moment diagram**.

Part C – D:

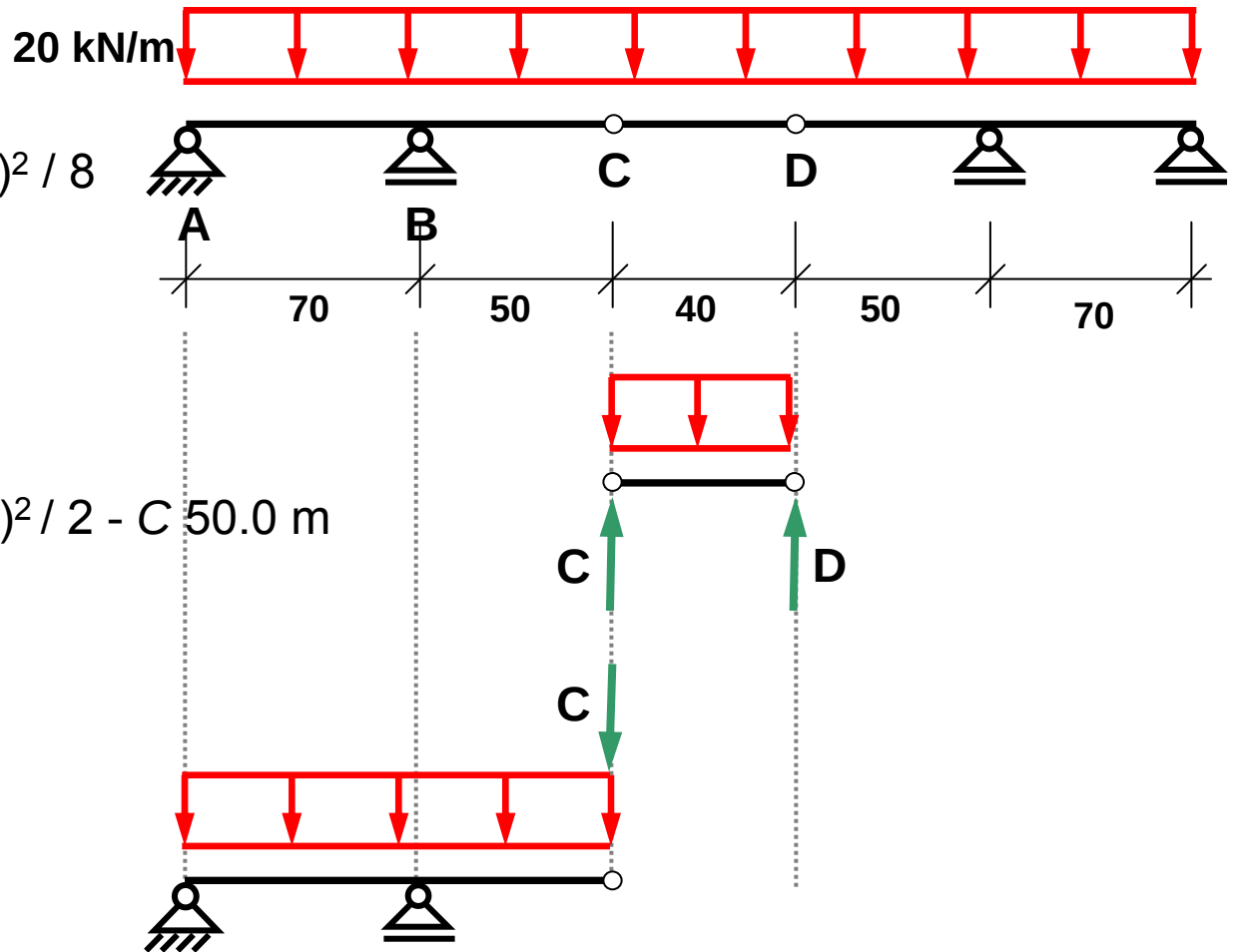
$$M_{CD,max} = 20 \text{ kN/m} (40.0 \text{ m})^2 / 8$$

$$= 4000 \text{ kNm}$$

Part A – C:

$$M_{B,min} = -20 \text{ kNm} (50.0 \text{ m})^2 / 2 - C \cdot 50.0 \text{ m}$$

$$= -45000 \text{ kNm}$$



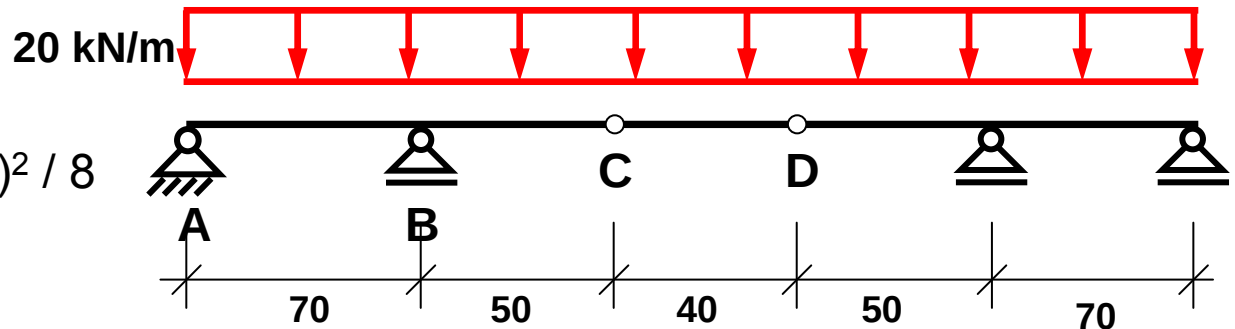
example 5.6

The given bridge structure is subjected to a distributed load of $w = 20 \text{ kN/m}$.

Determine the support reactions and the **moment diagram**.

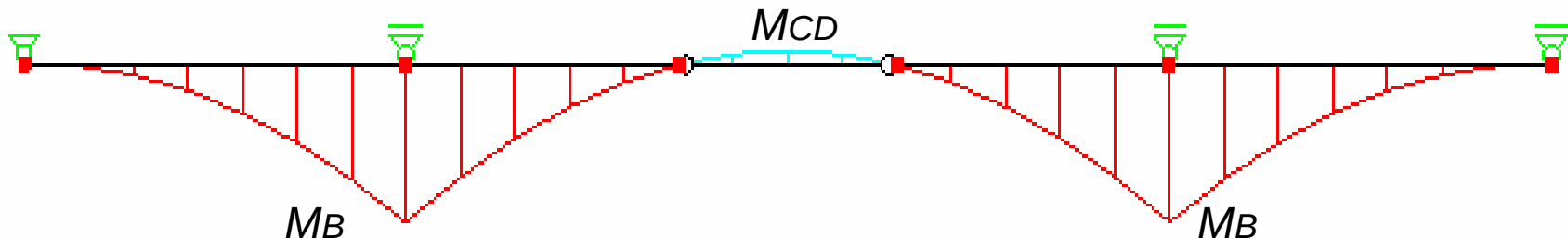
Part C – D:

$$\begin{aligned} M_{CD,max} &= 20 \text{ kN/m} (40.0 \text{ m})^2 / 8 \\ &= 4000 \text{ kNm} \end{aligned}$$



Part A – C:

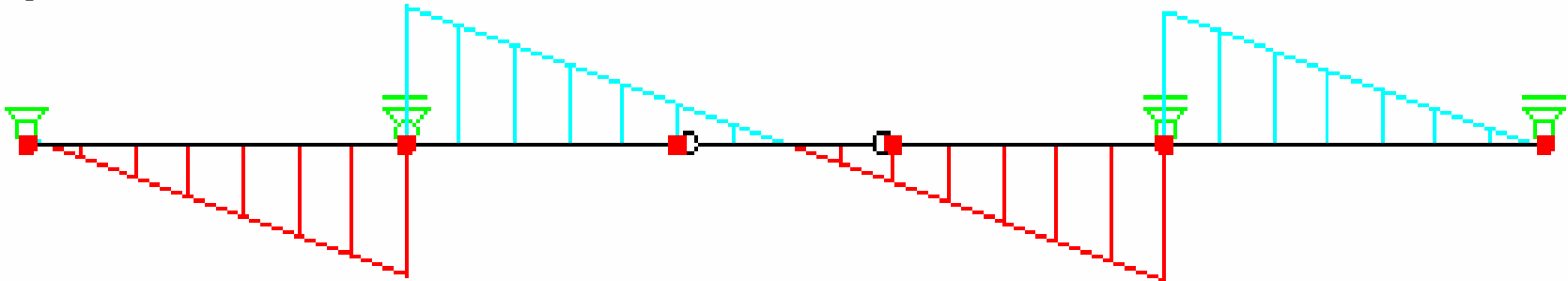
$$\begin{aligned} M_{B,min} &= - 20 \text{ kNm} (50.0 \text{ m})^2 / 2 - C 50.0 \text{ m} \\ &= - 45000 \text{ kNm} \end{aligned}$$



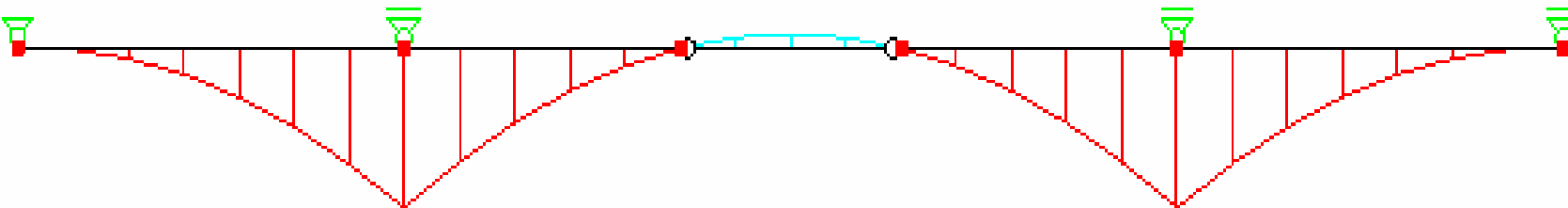
example 5.6

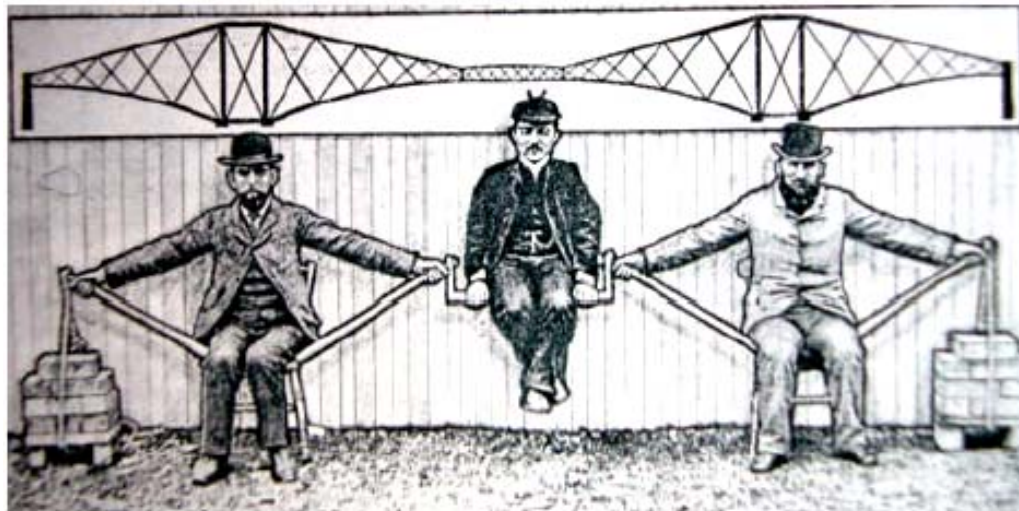
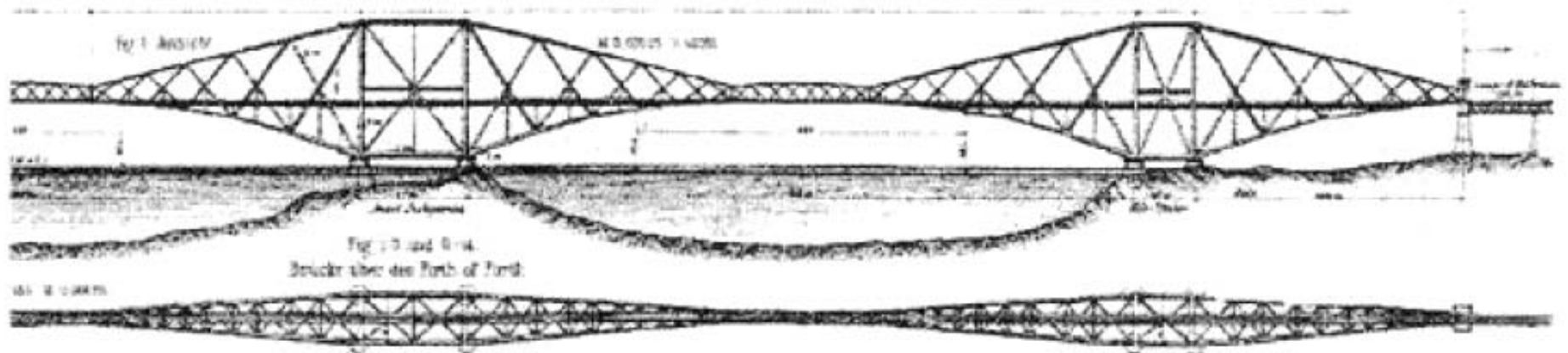
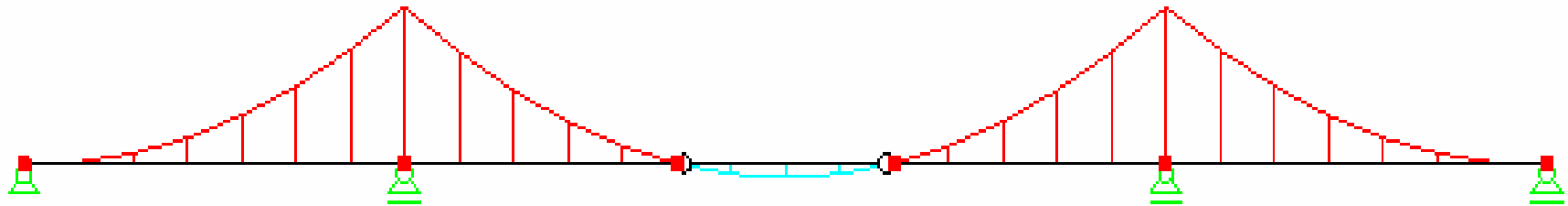
Shear force and moment diagram.

V



M





example 5.7

In the bridge structure (example 5.6) the pin joints are now moved symmetrically to the outer spans (see image).

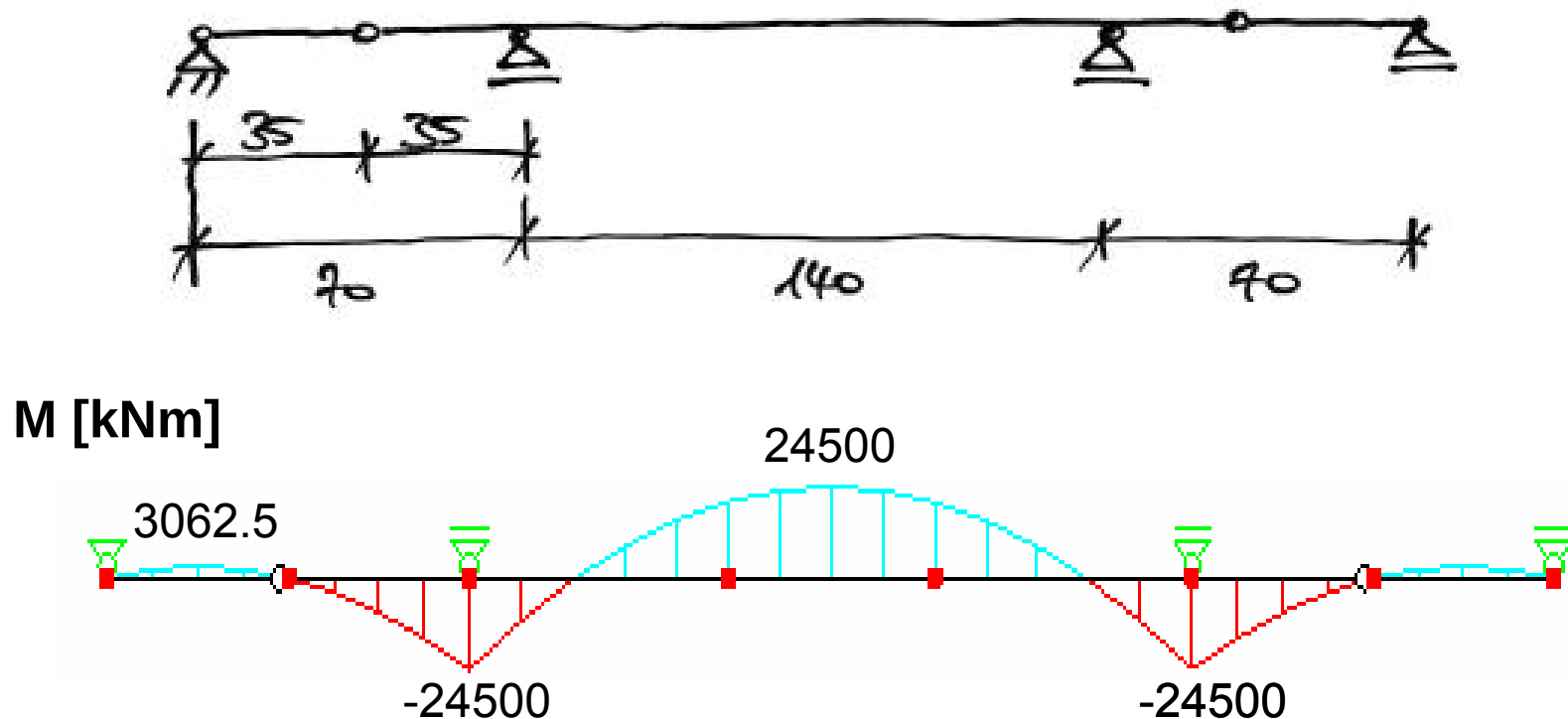
Determine the moment diagram and sketch an ideal shape for the bridge truss structure.



example 5.7

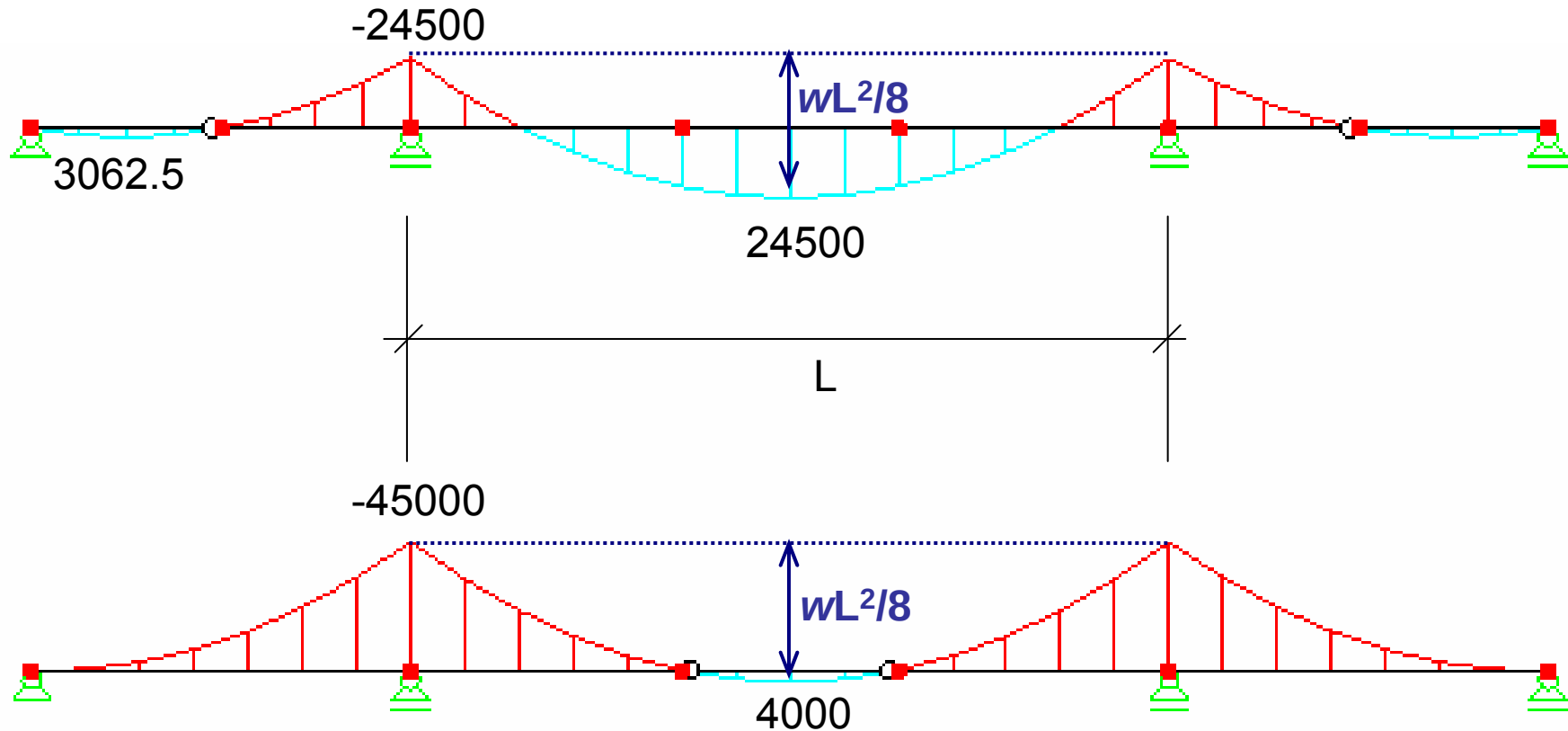
In the bridge structure (example 5.6) the pin joints are now moved symmetrically to the outer spans (see image).

Determine the moment diagram and sketch an ideal shape for the bridge truss structure.

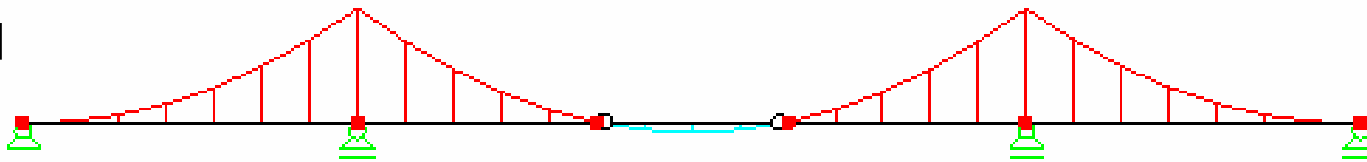


comparison of example 5.7 and 5.6:

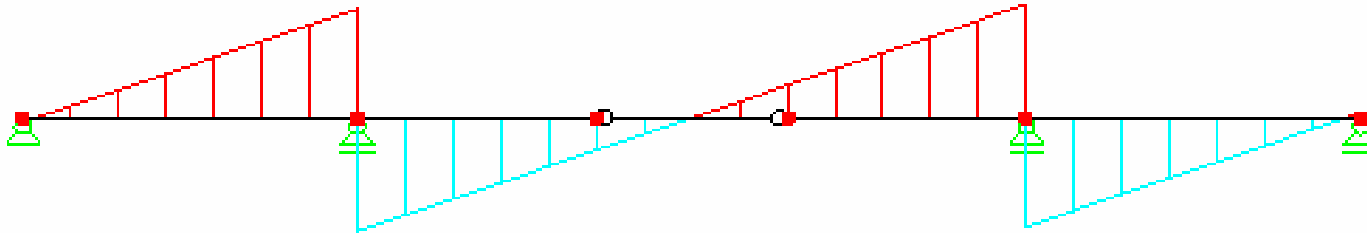
M [kNm]



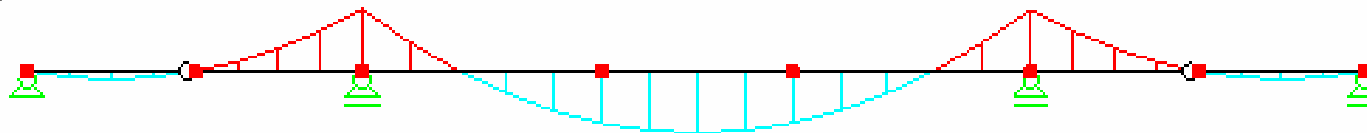
M



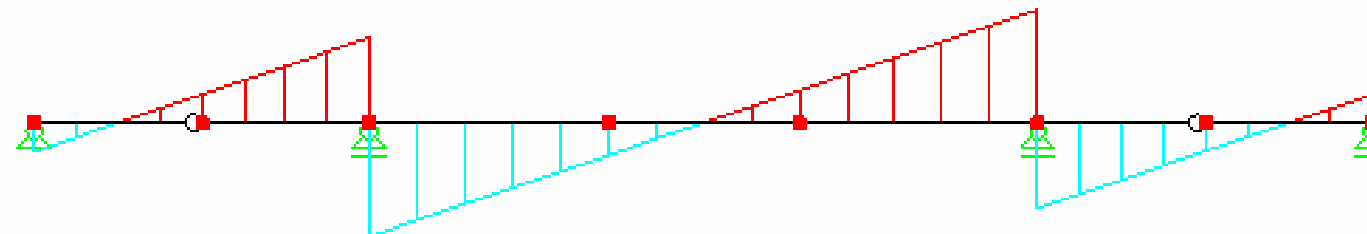
V

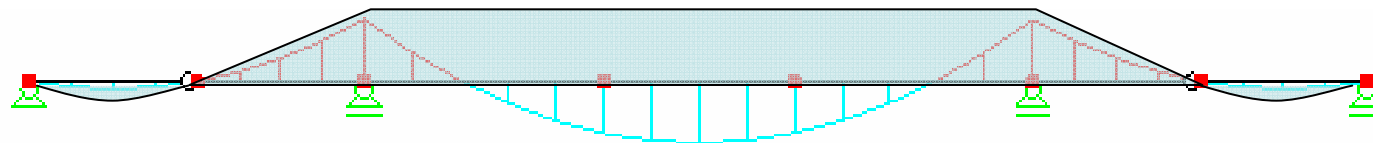
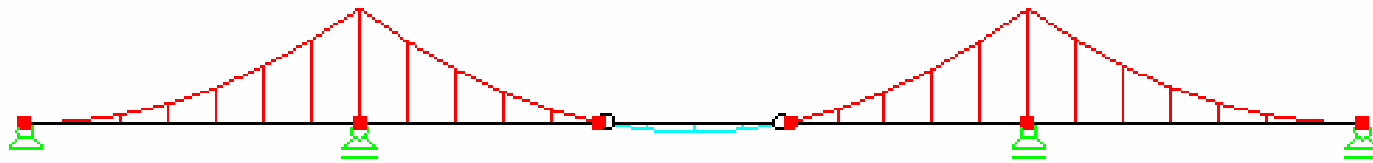


M



V





design proposal





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Faculty of Technology

Engineering Mechanics
material by Karsten Schlesier

Department of
Civil Engineering