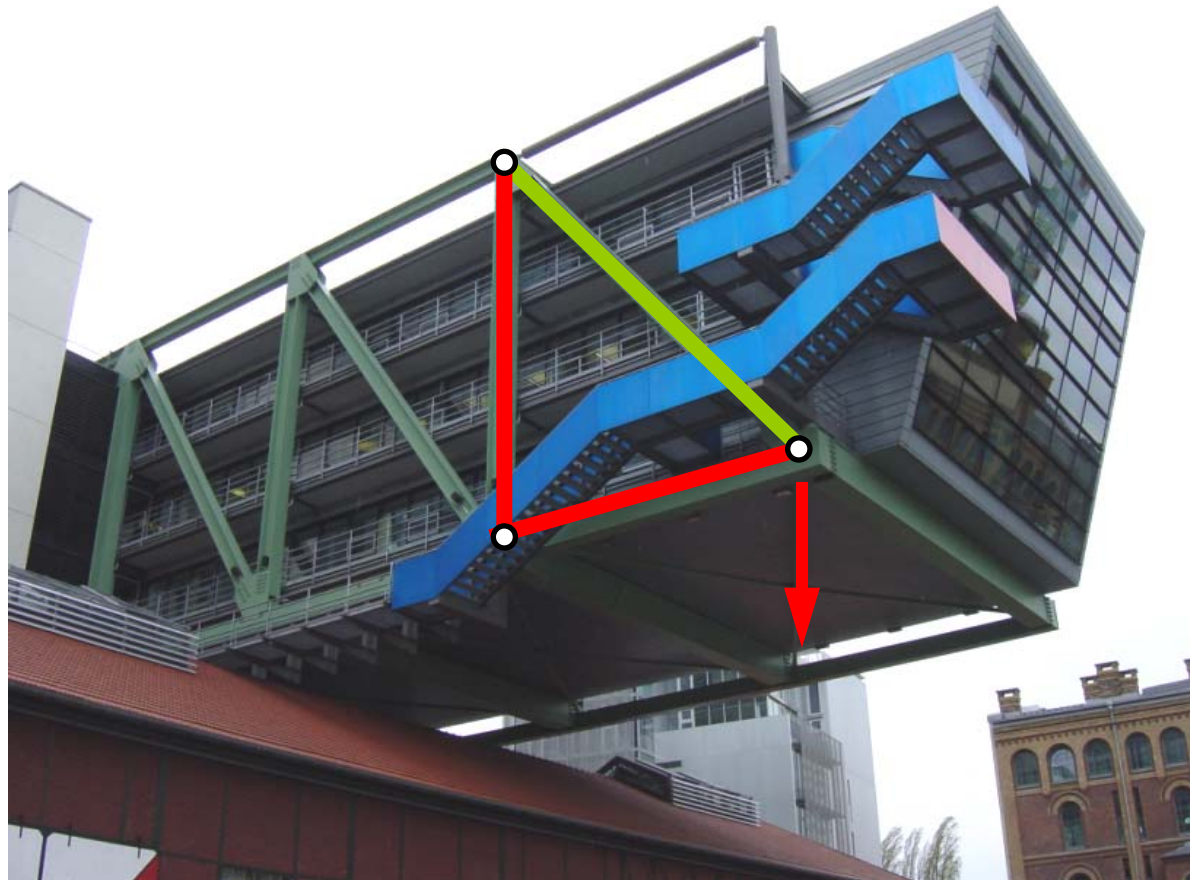


Chapter IV a

Simple Structures (Trusses)



Lecture Overview

- introduction, definition of trusses
- external forces vs internal forces
 - method of joints
 - method of sections



Definition of truss

Framework of two force members, pin-joined at their ends to form a rigid structure.

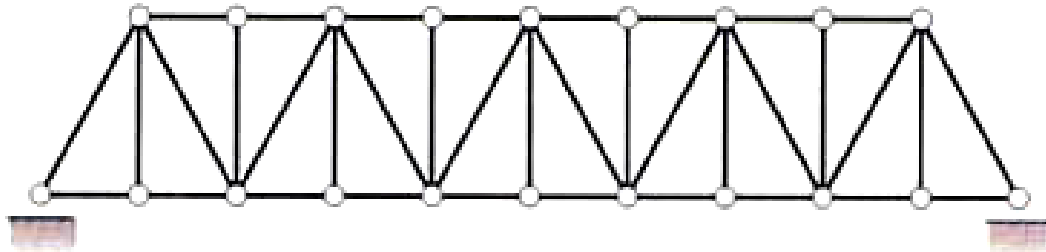


fig by J.L. Meriam, L.G. Kraige, Engineering Mechanics I



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Two force members

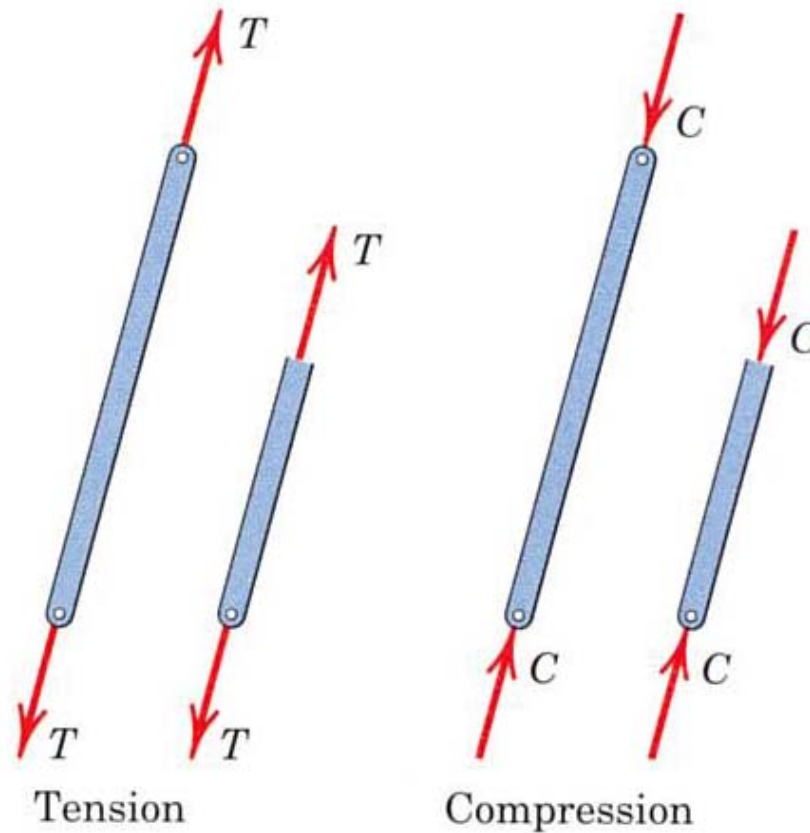


fig by J.L. Meriam, L.G. Kraige, Engineering Mechanics I



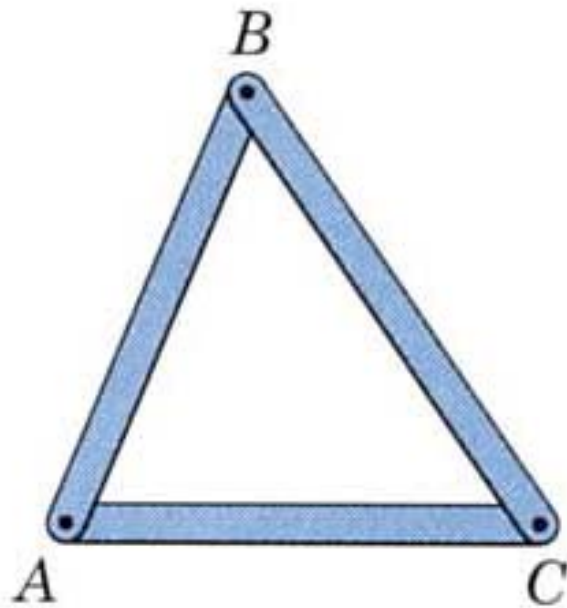
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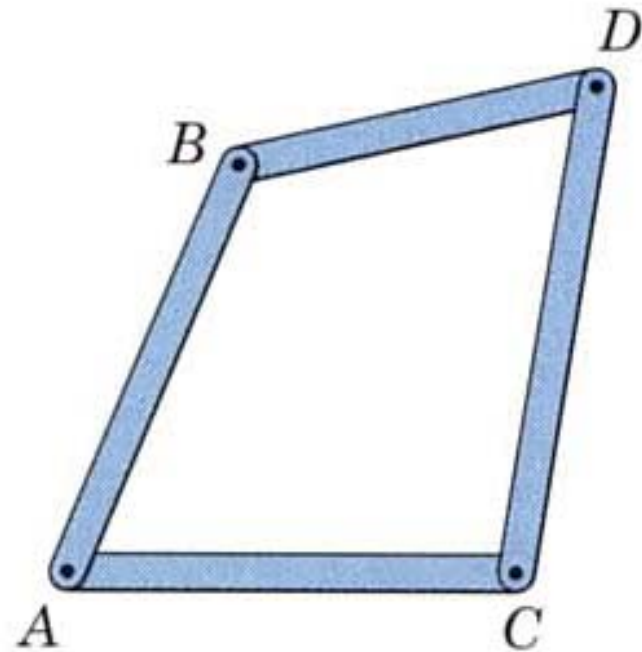
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Plane trusses, basic element

consisting of bars joined by pins



triangular element – rigid
(non-collapsible)



polygonal element – not rigid
(collapsible)

fig by J.L. Meriam, L.G. Kraige, Engineering Mechanics I



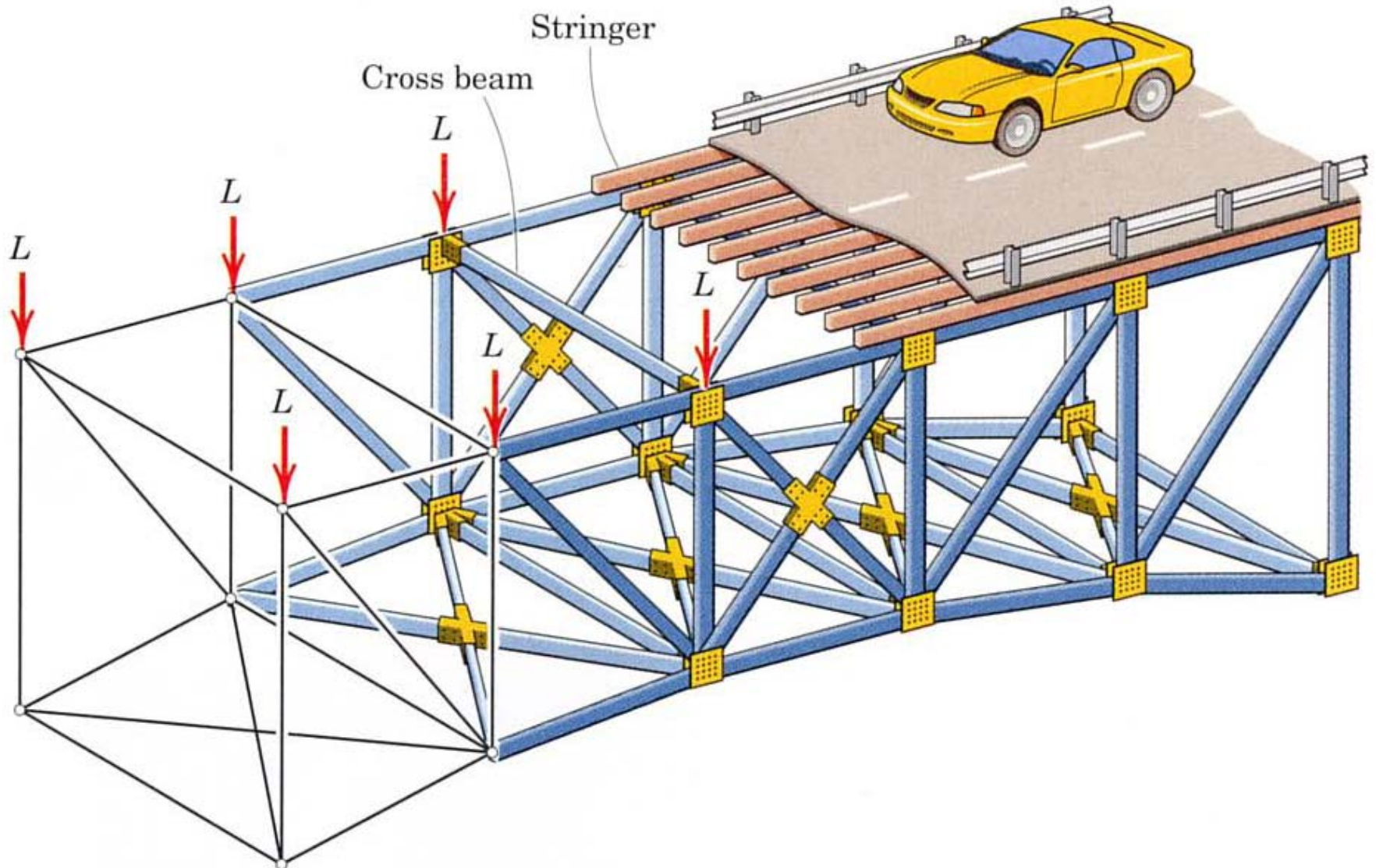


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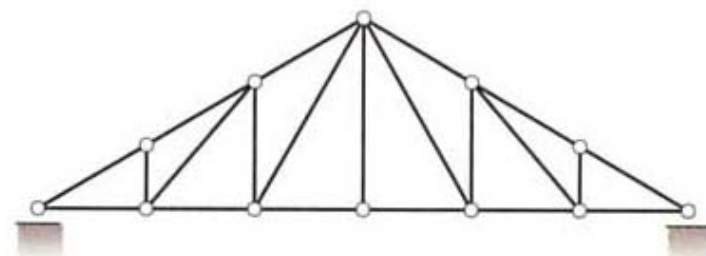
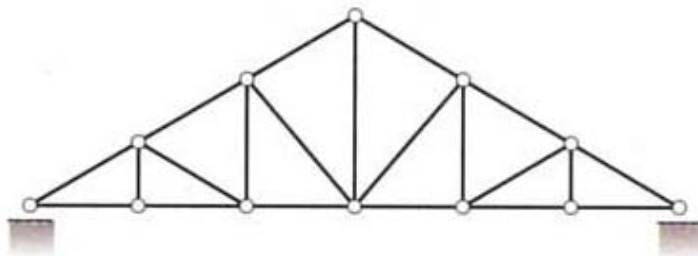
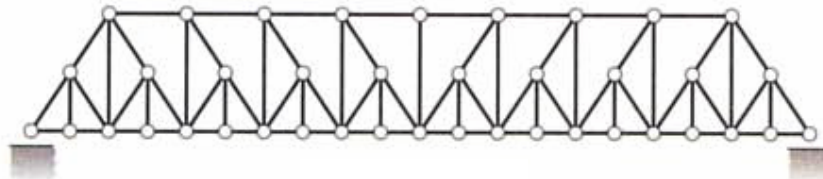
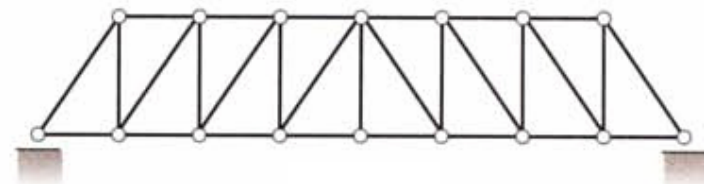
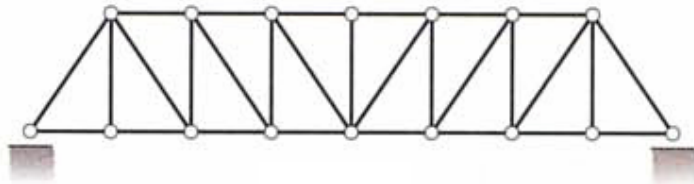


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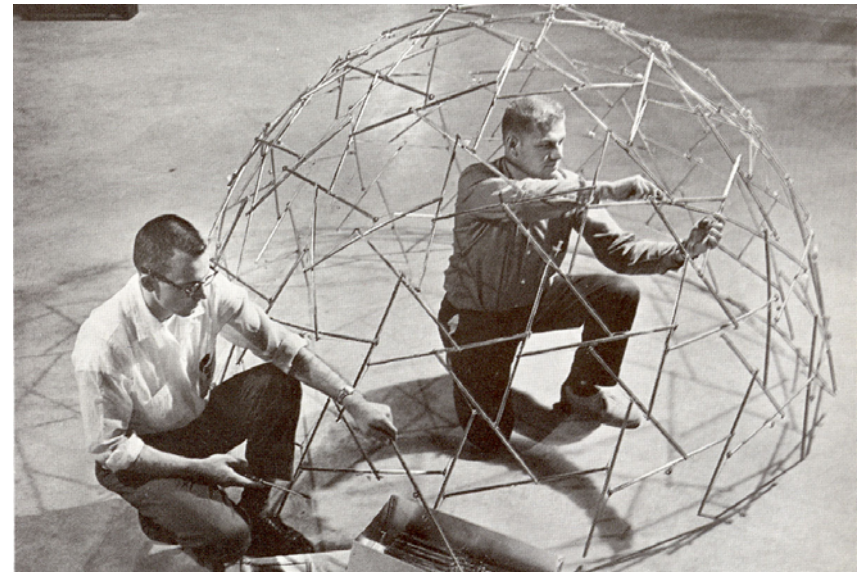
Project, University of Aachen



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Dome Structures,
Buckminster Fuller



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Exhibition Center Milano



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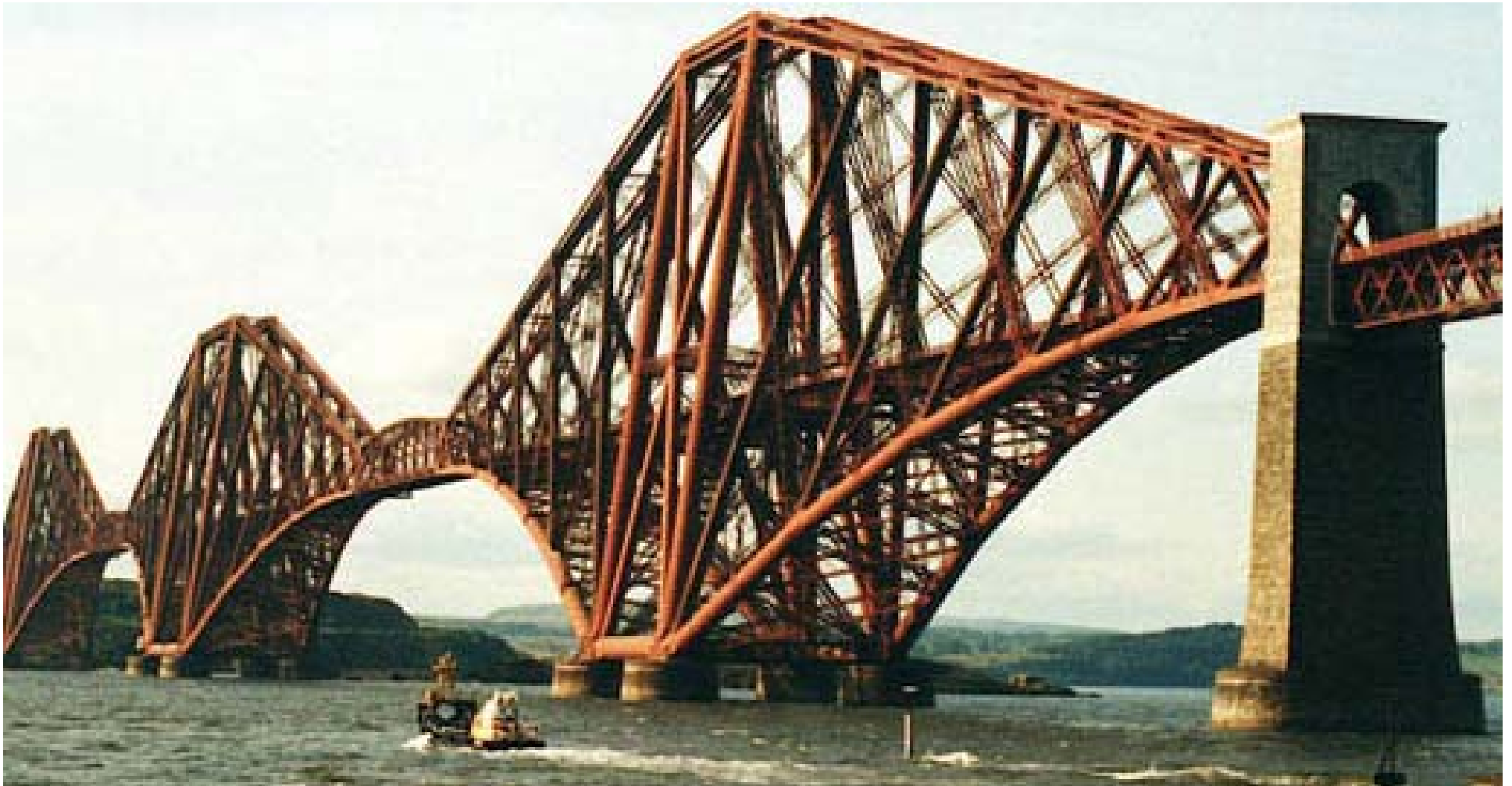
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Bridge over Firth of Tay (1879)





Firth of Forth Bridge 1880





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Cargolifter Shelter Brand

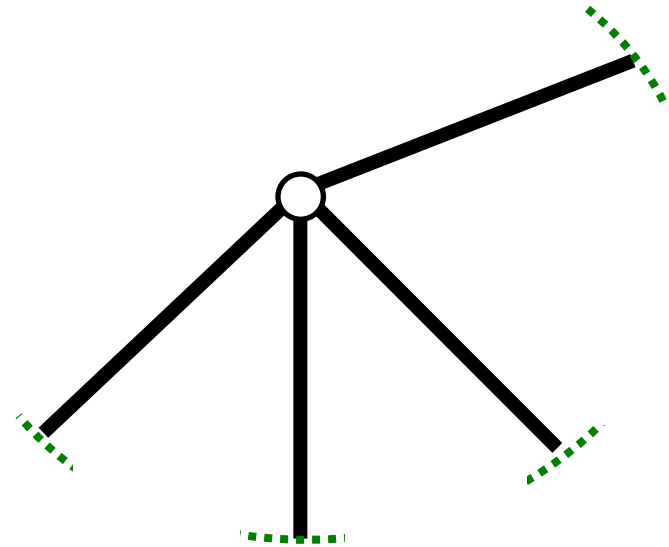
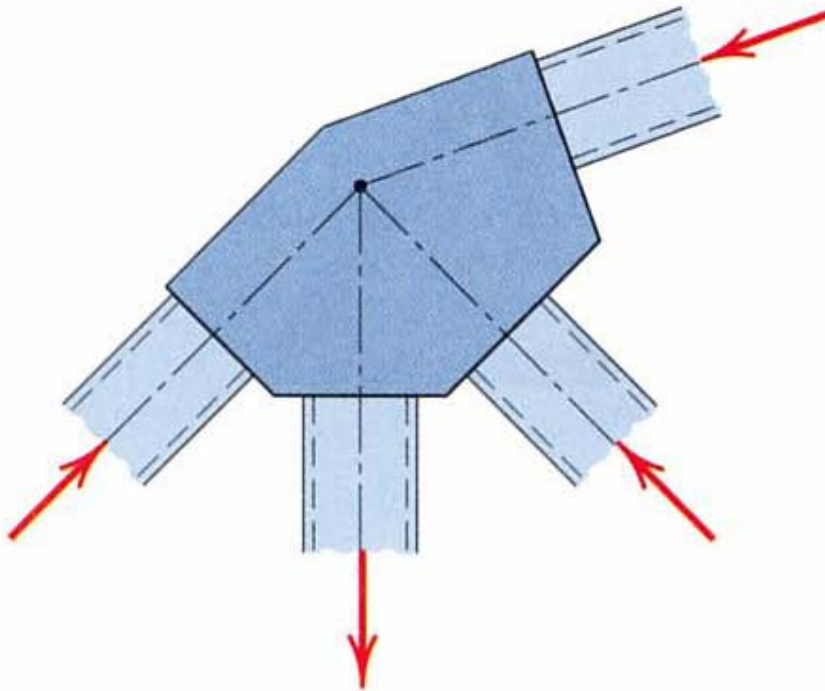


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Truss connections (pin joints)

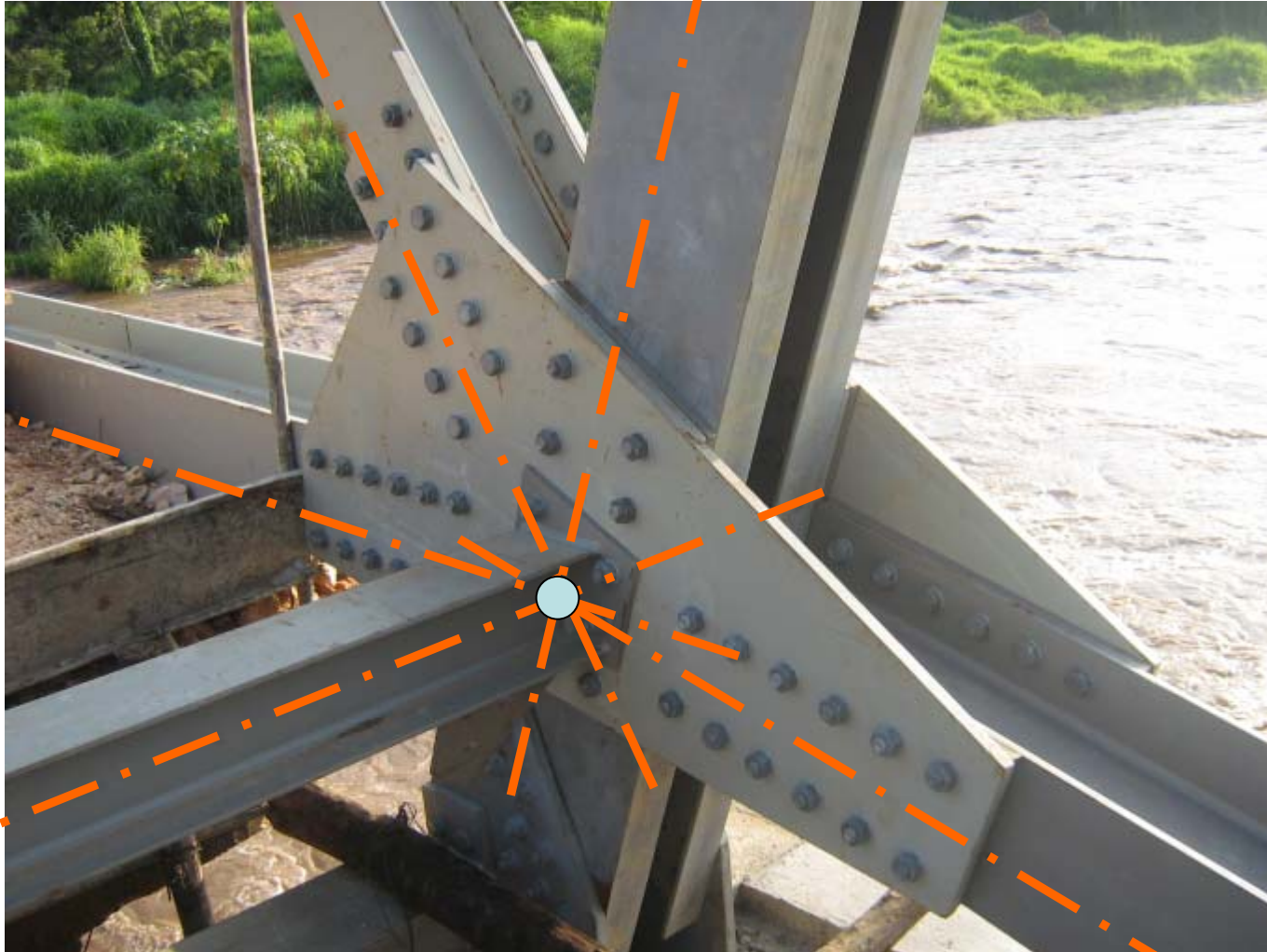




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Internal forces

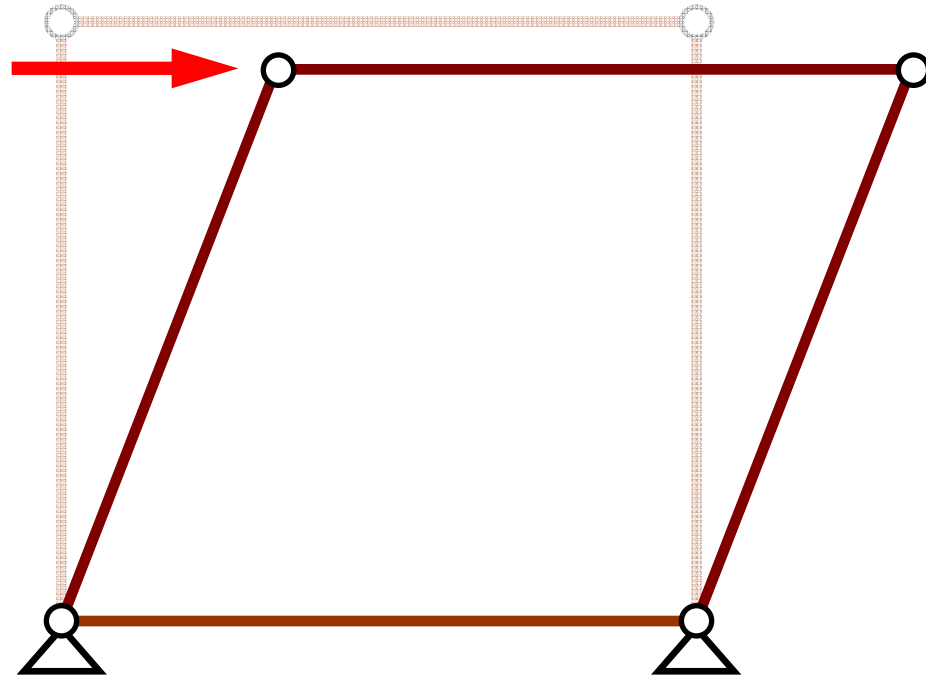


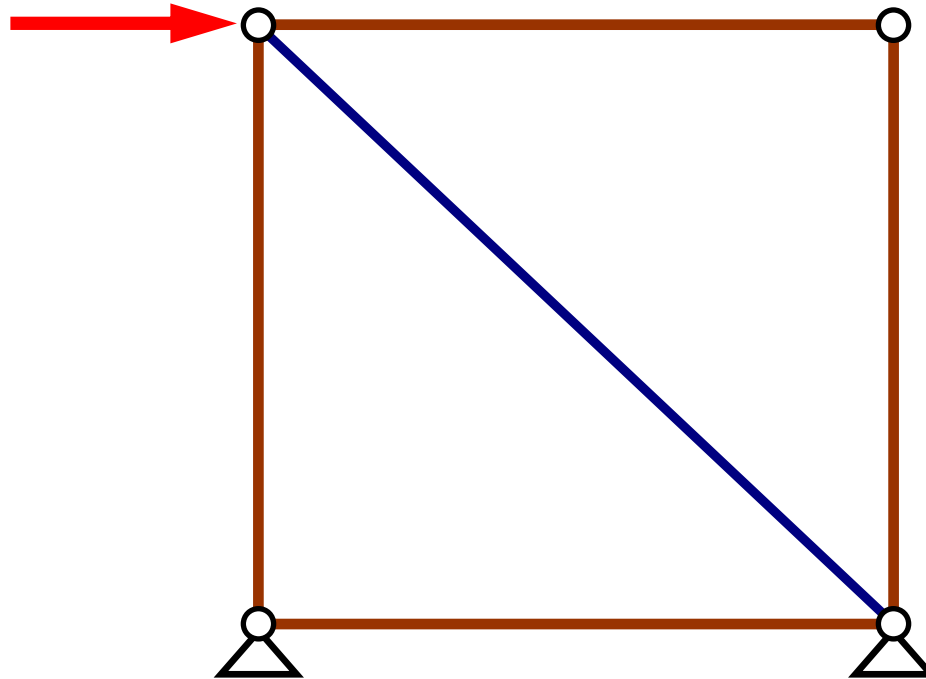
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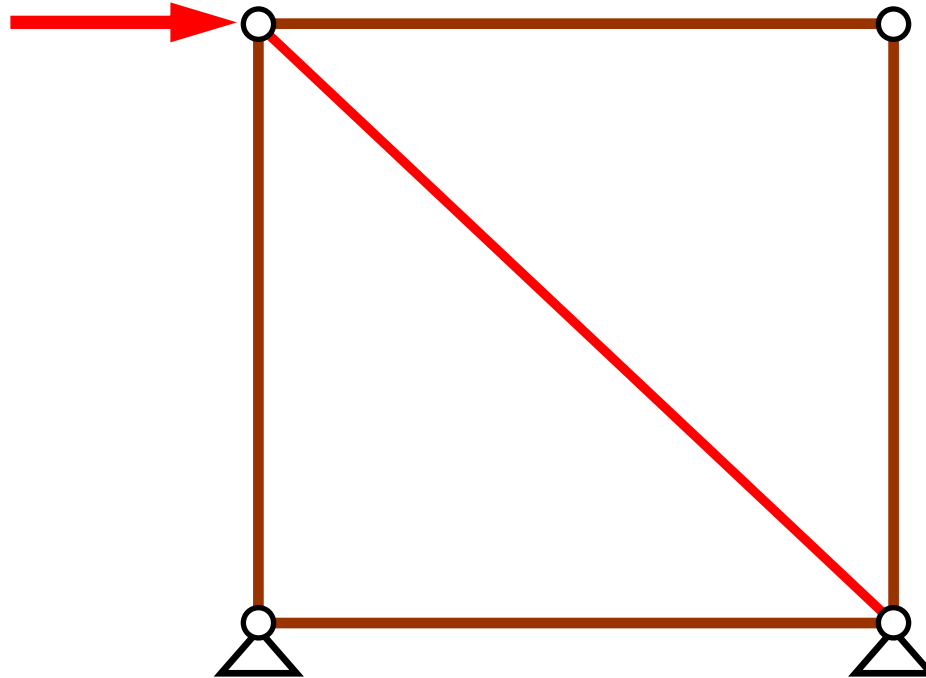
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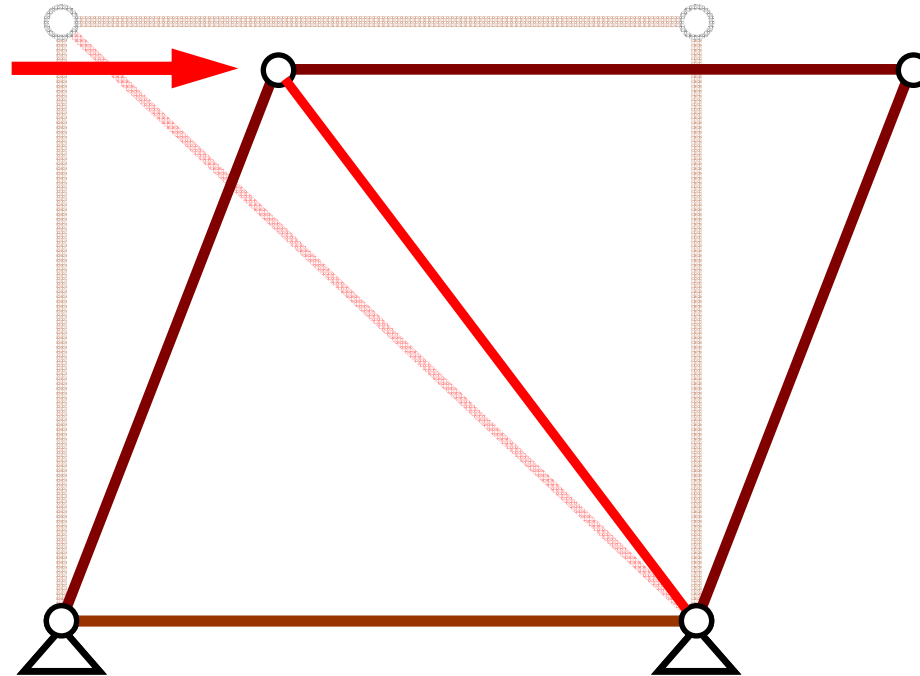


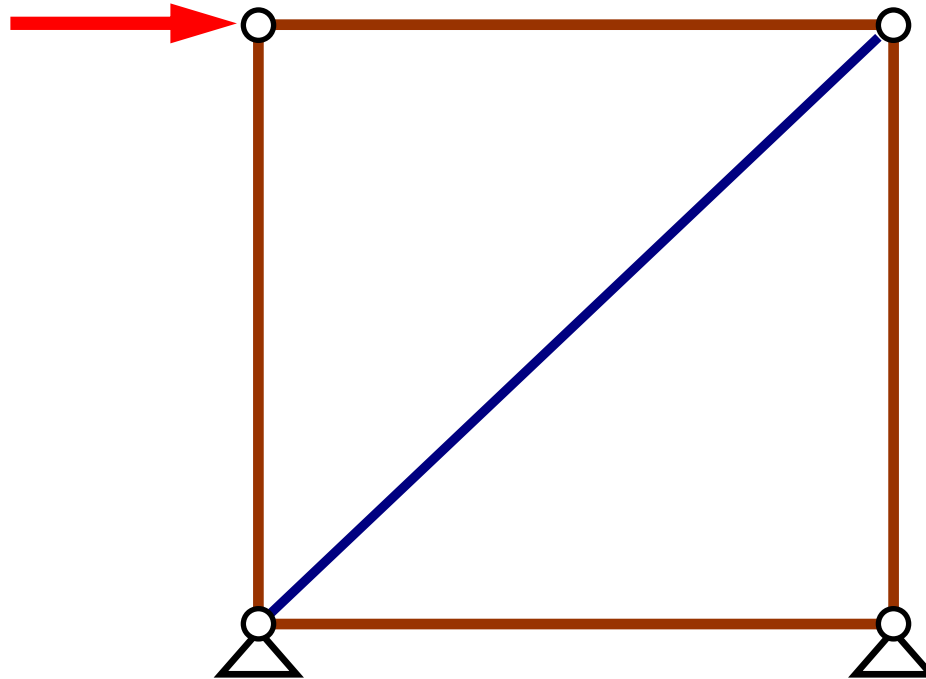


compression

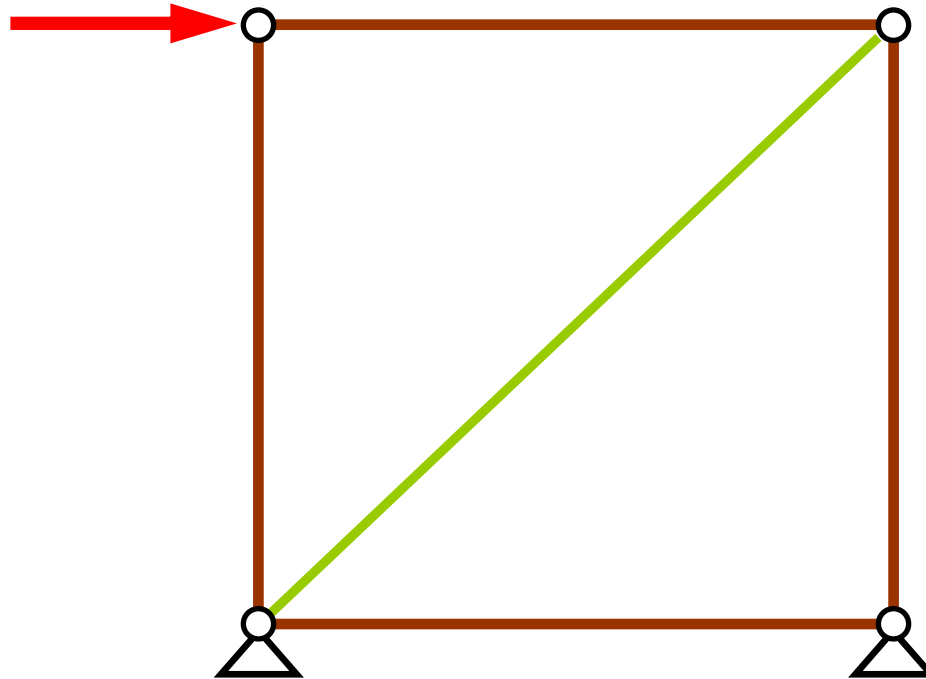


compression

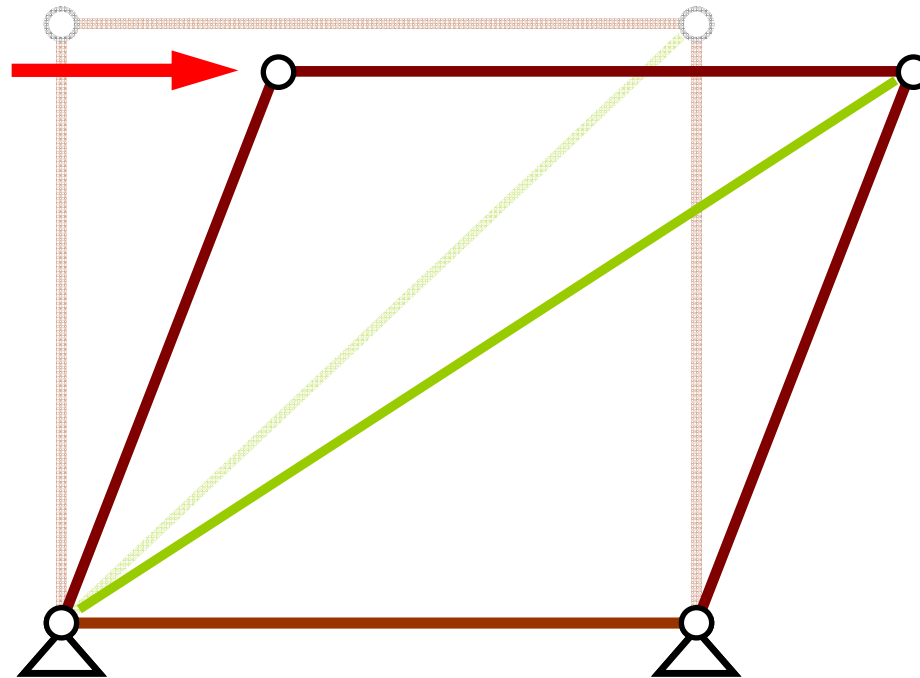




tension



tension





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Truss Crane, Harbour of Duisburg

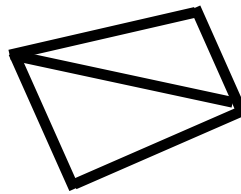


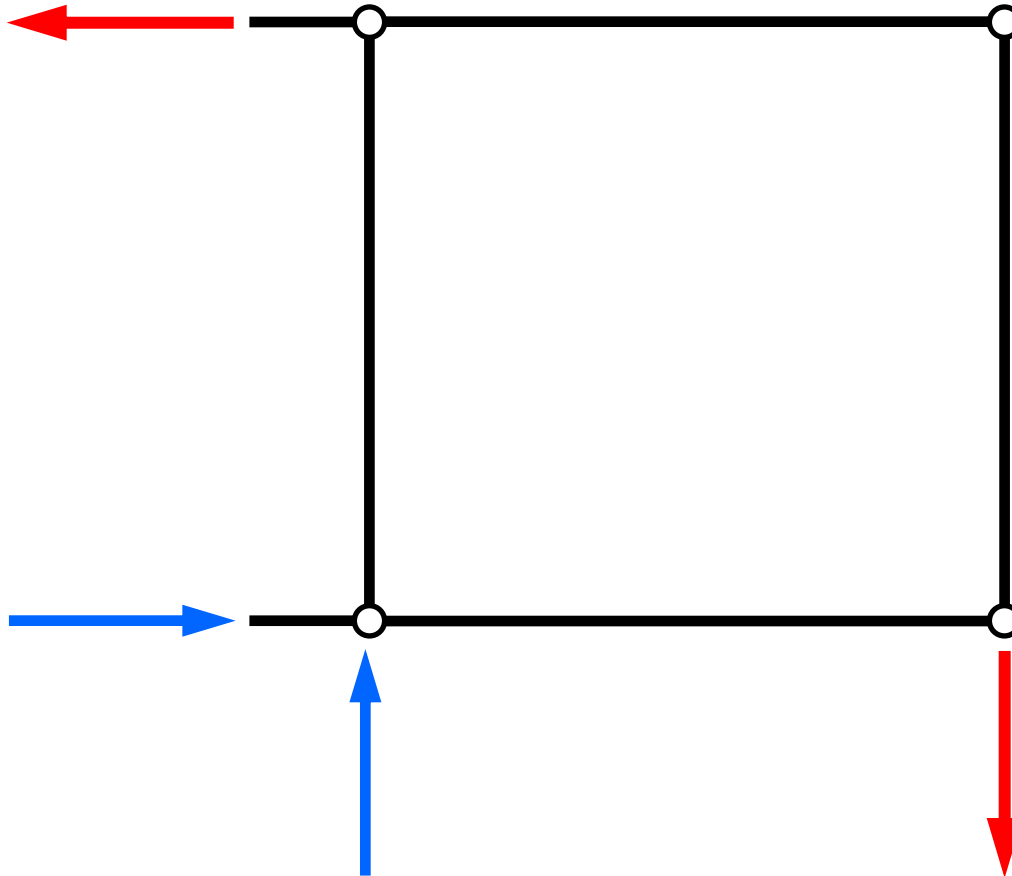


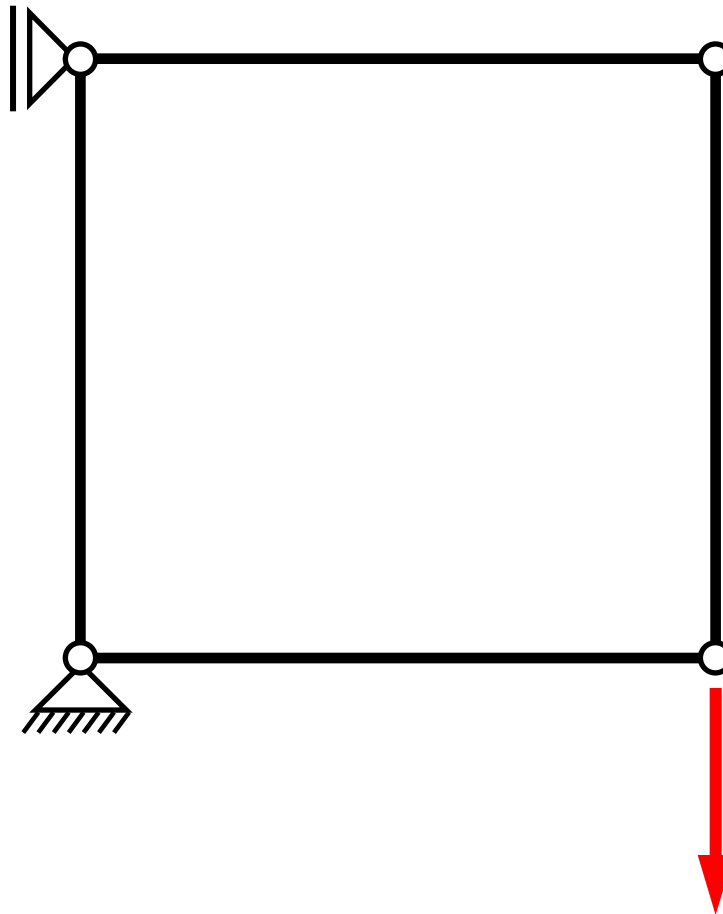
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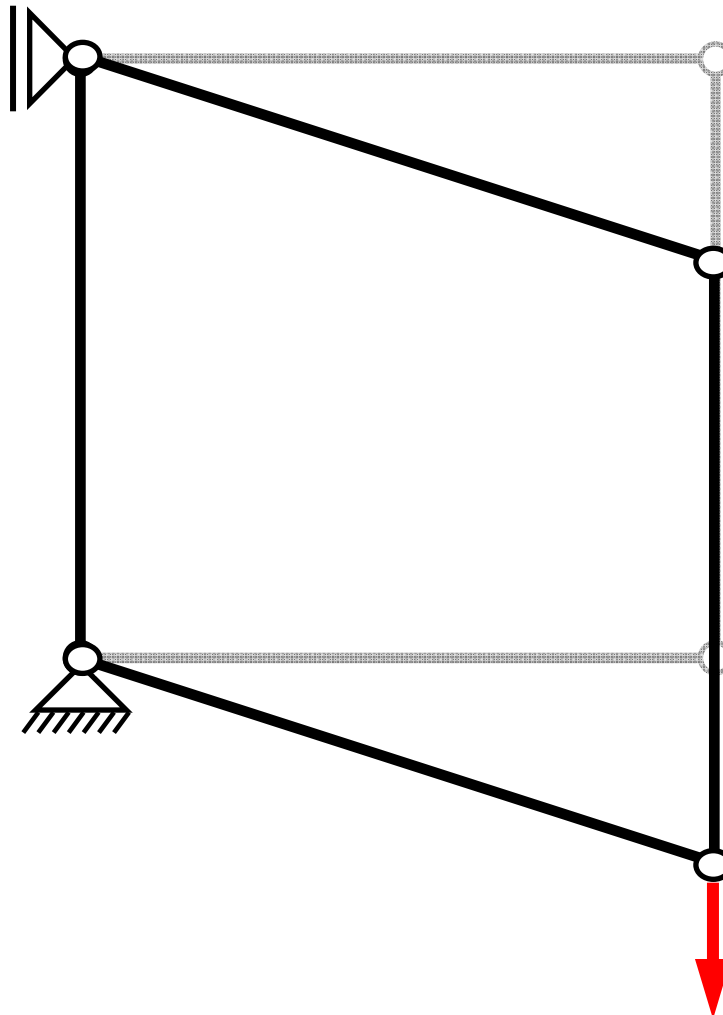
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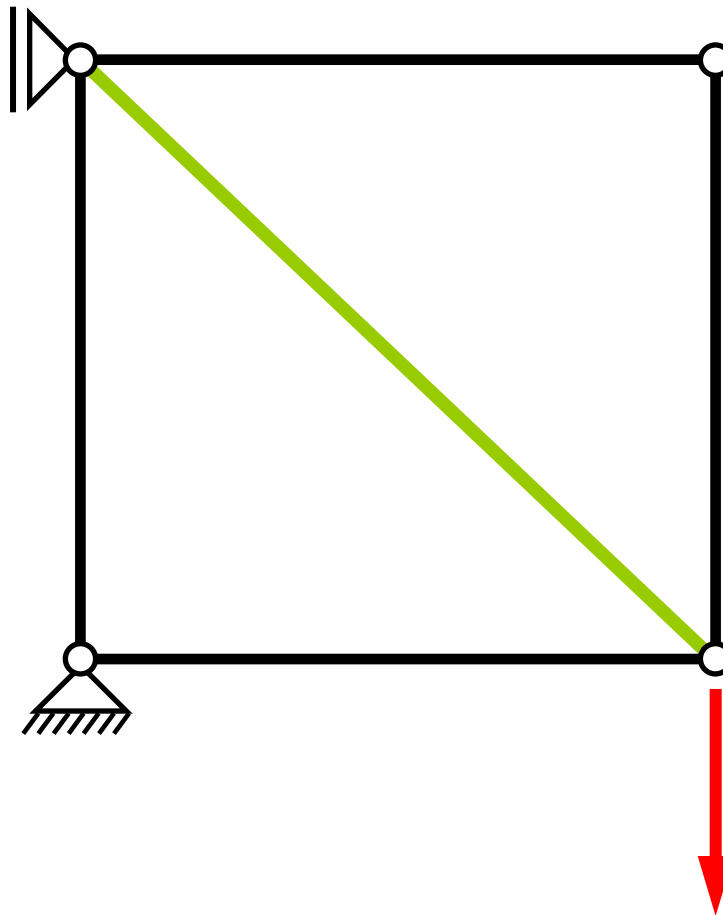


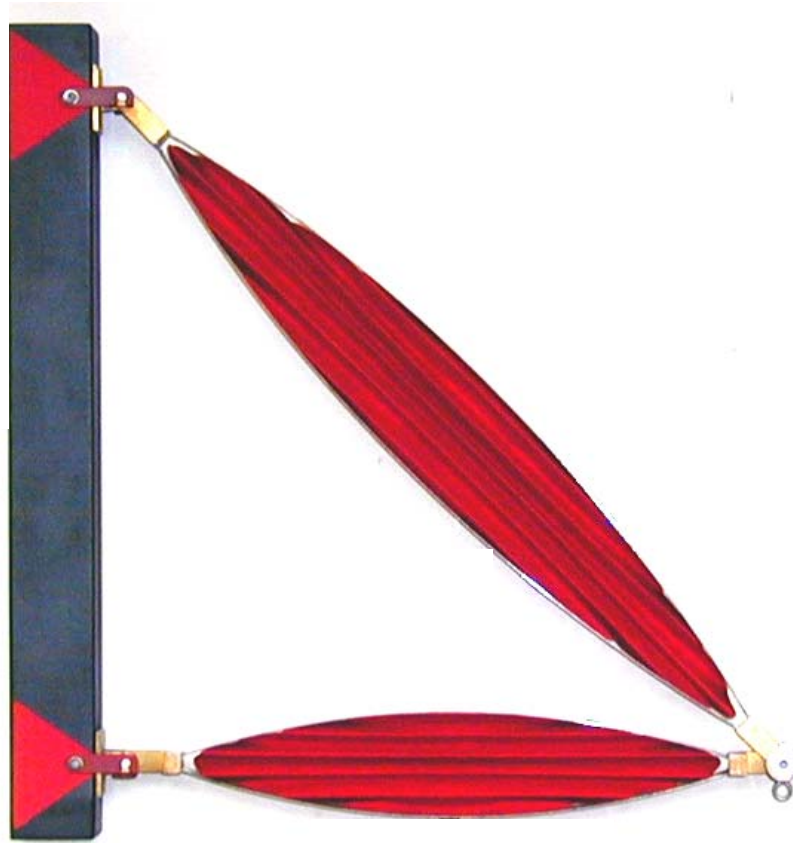


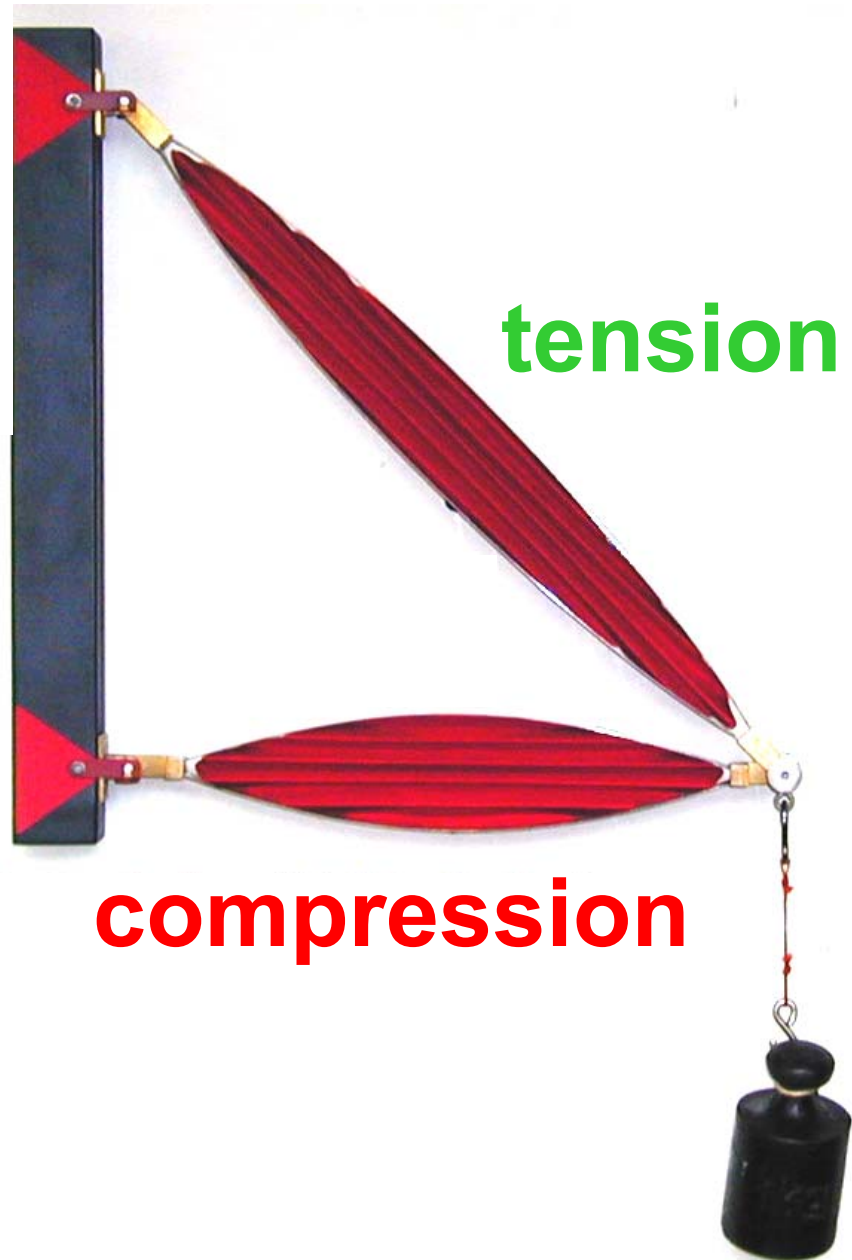




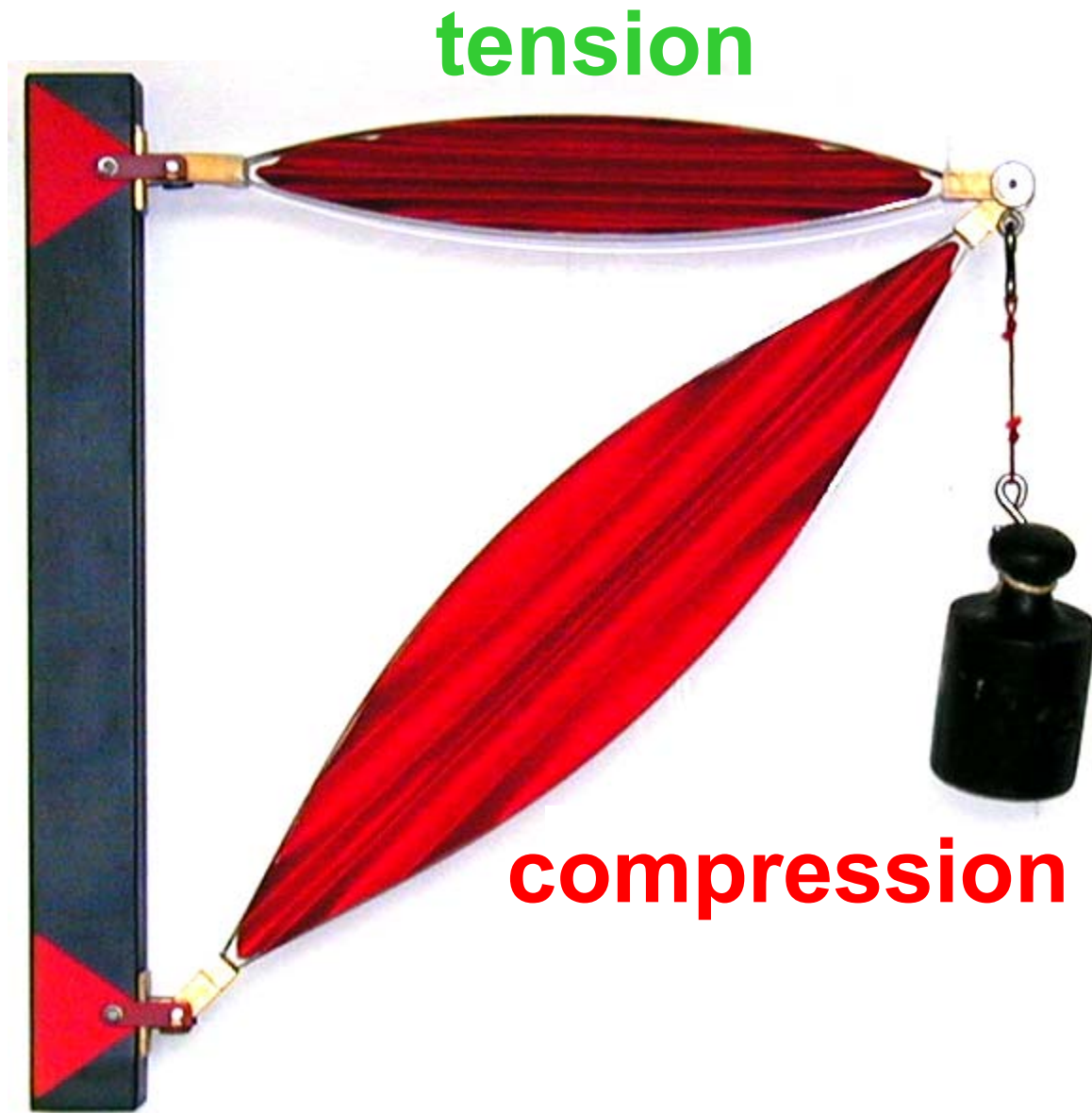
tension

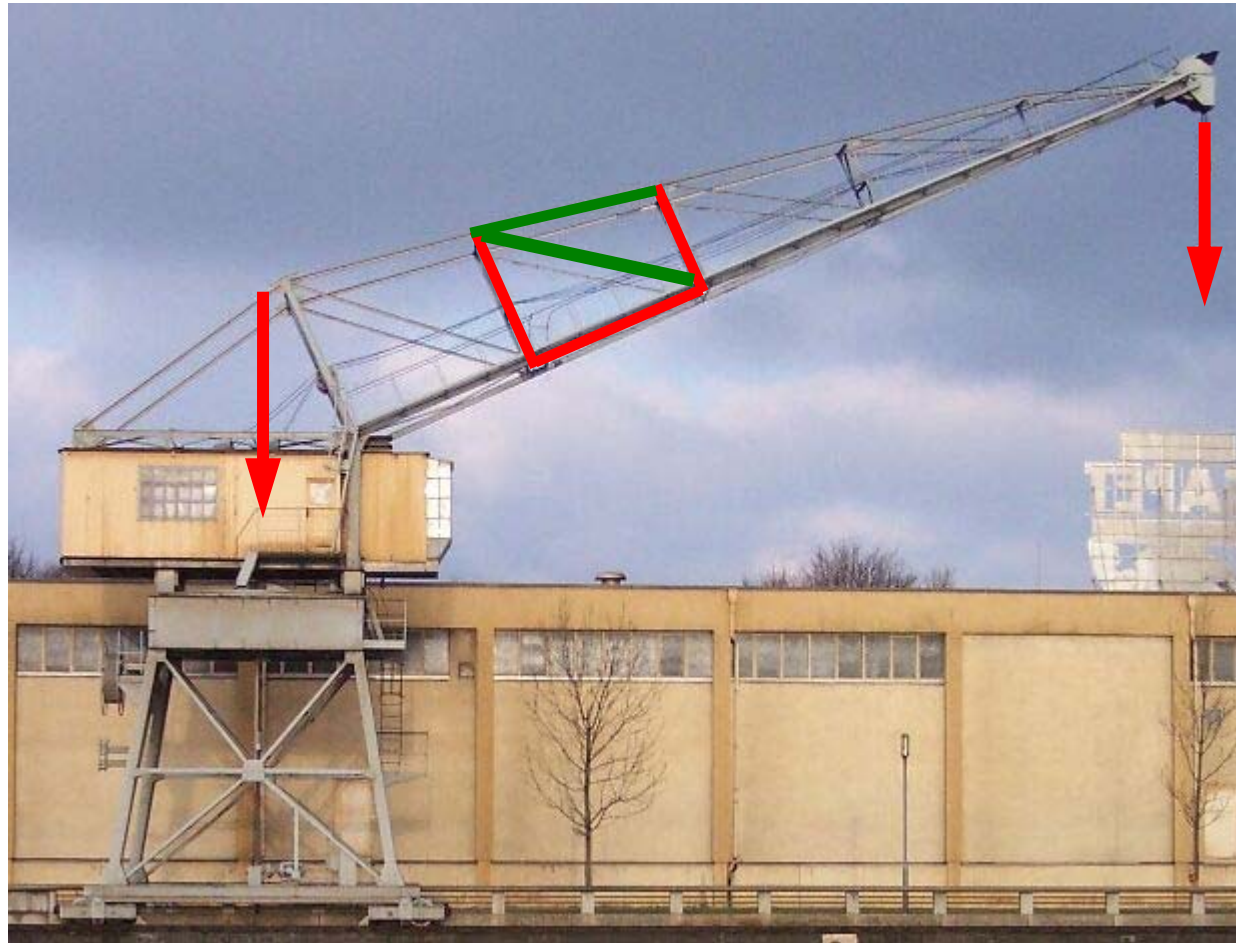










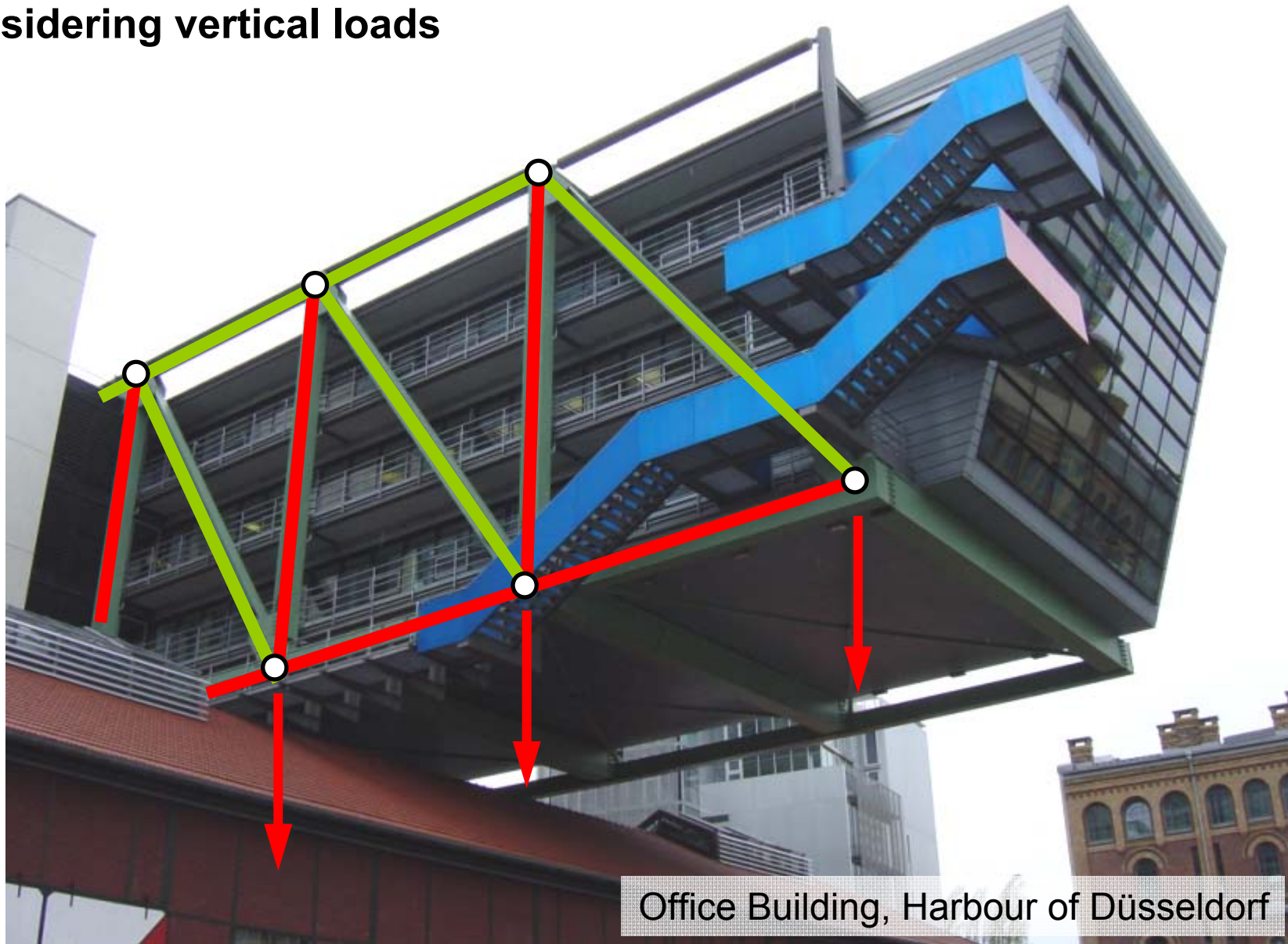


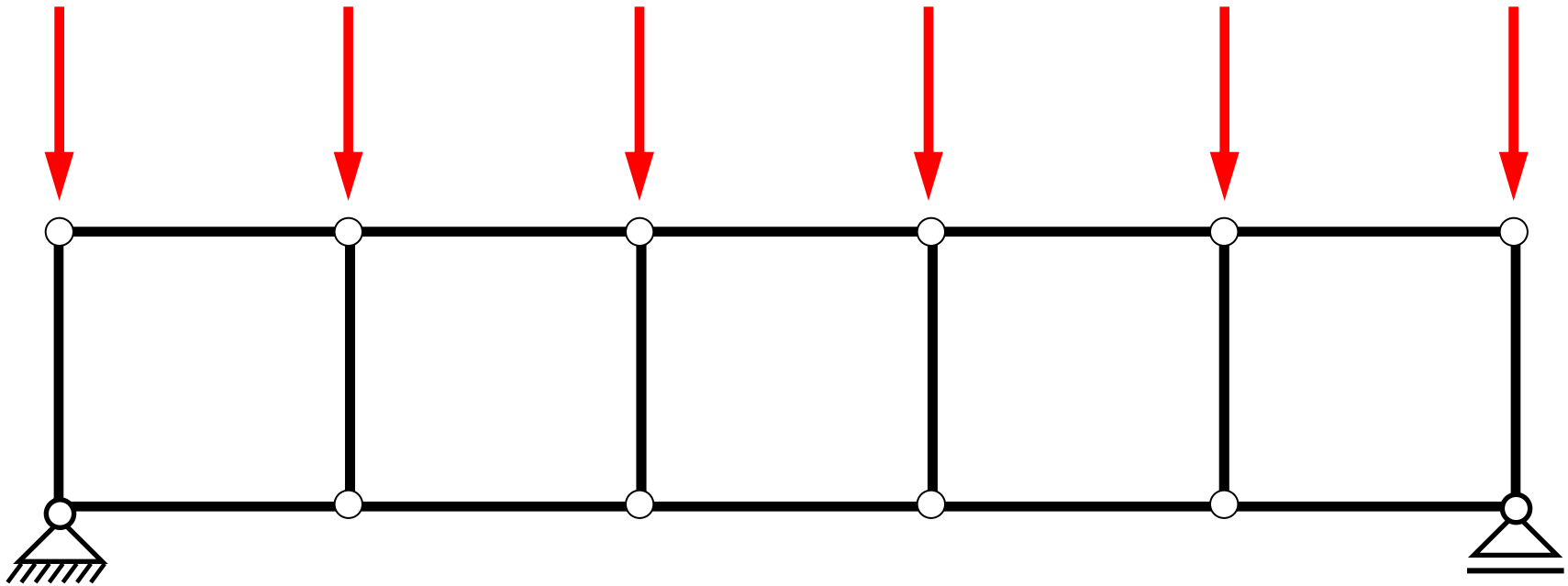
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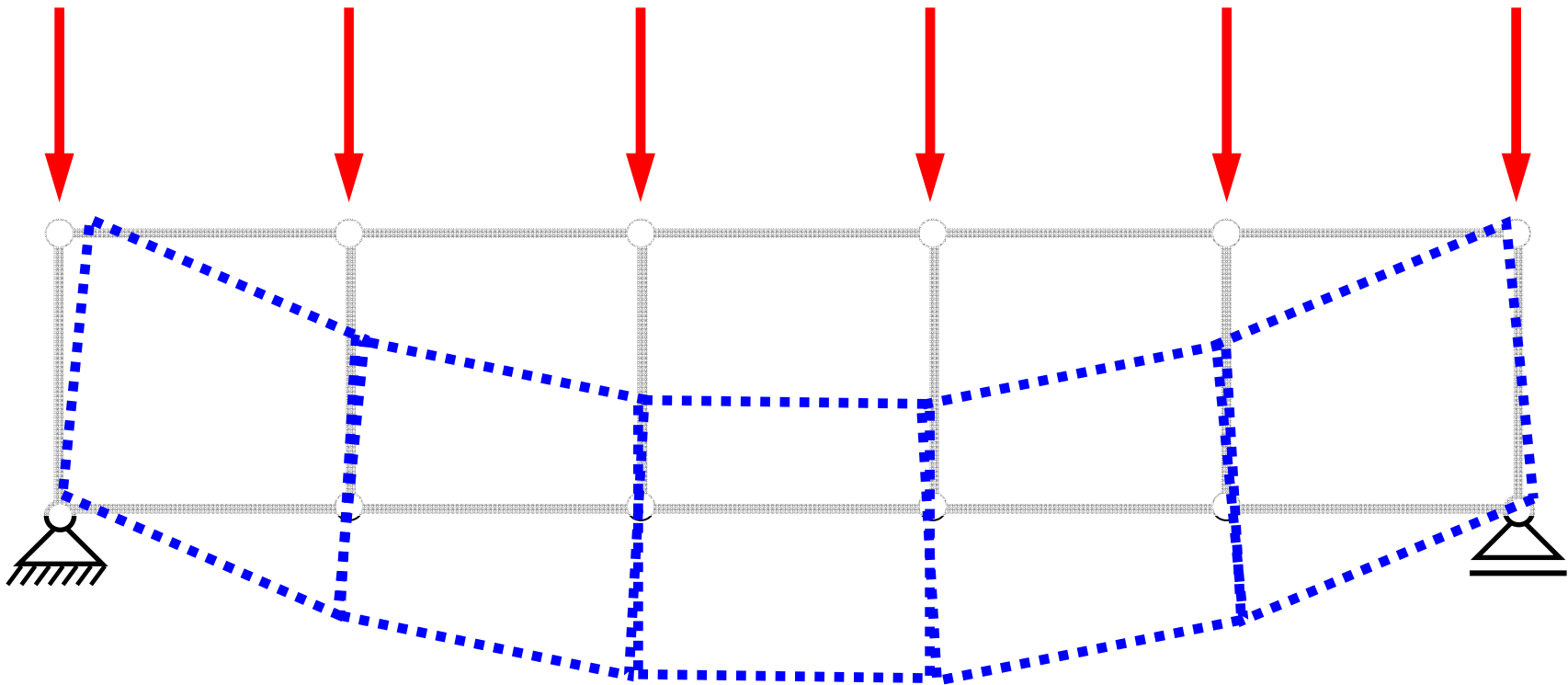
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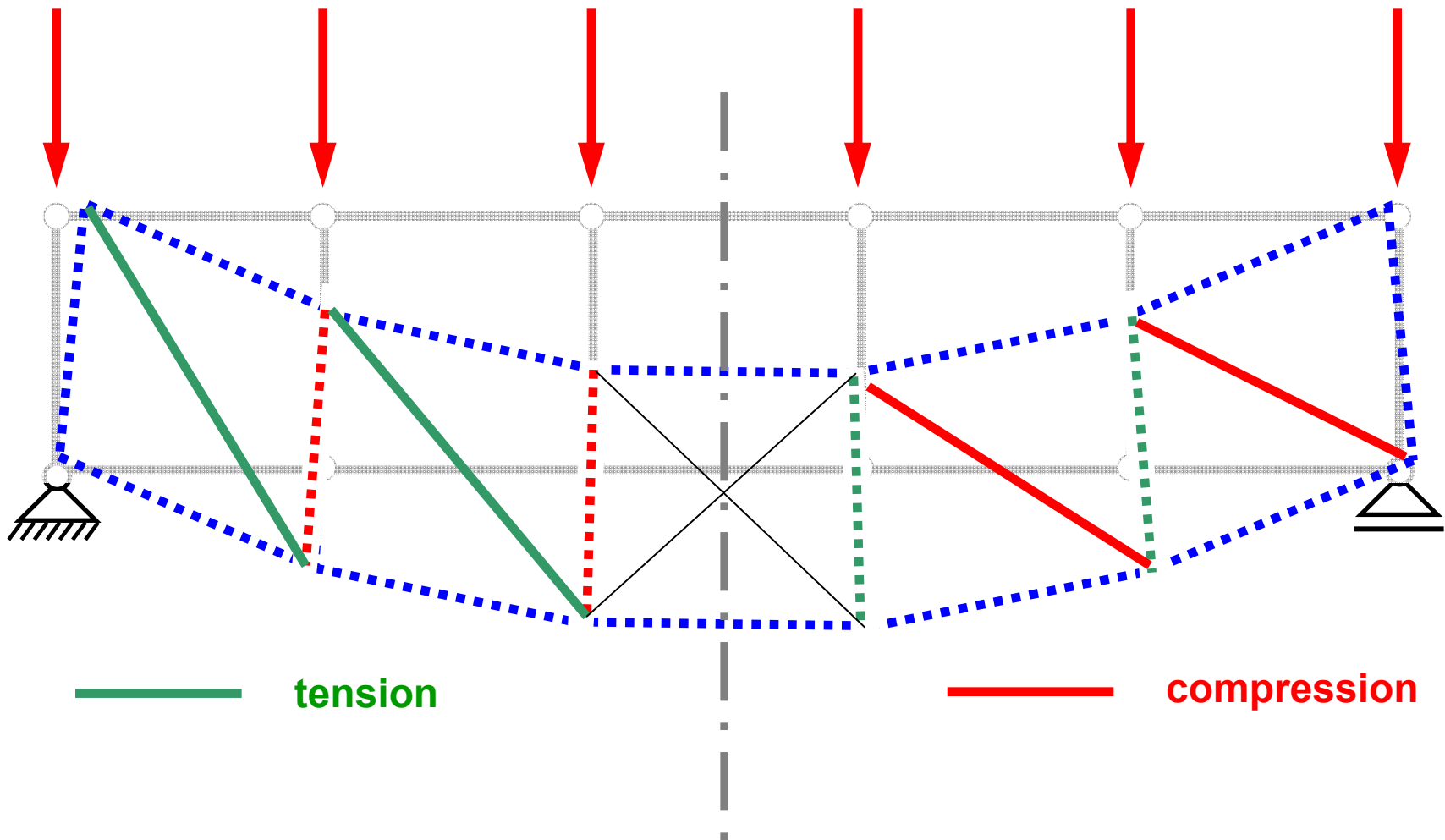
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Position of **tensile** and **compressive** members in a cantilever structure considering vertical loads

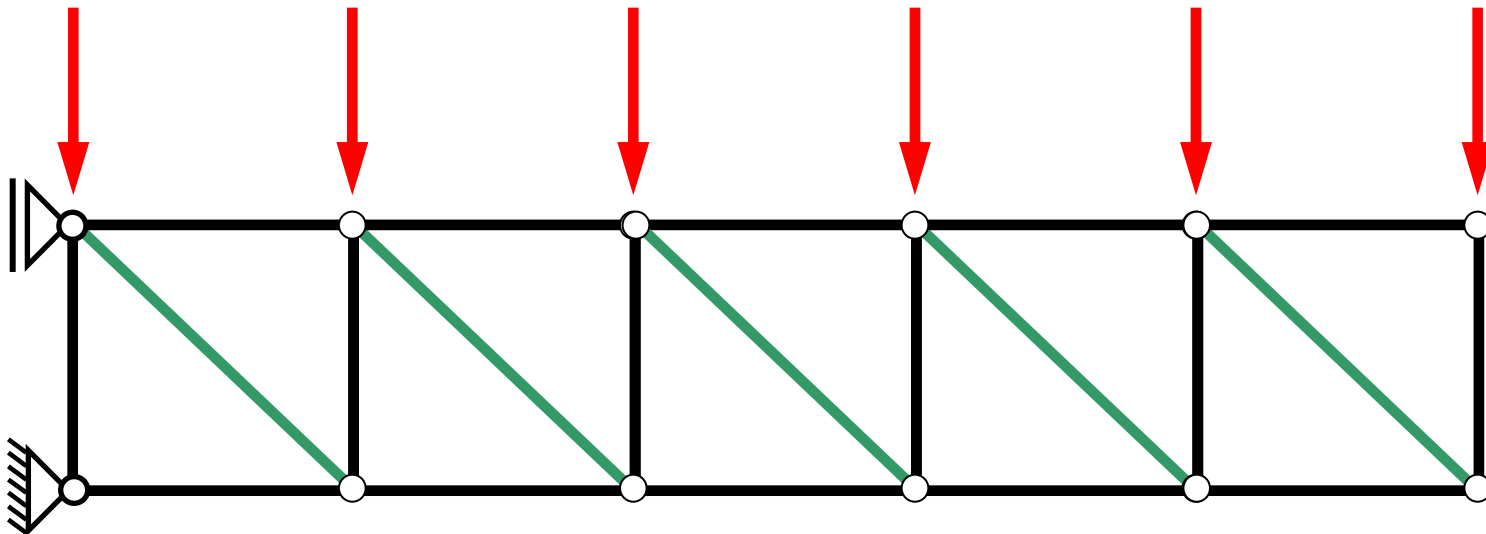
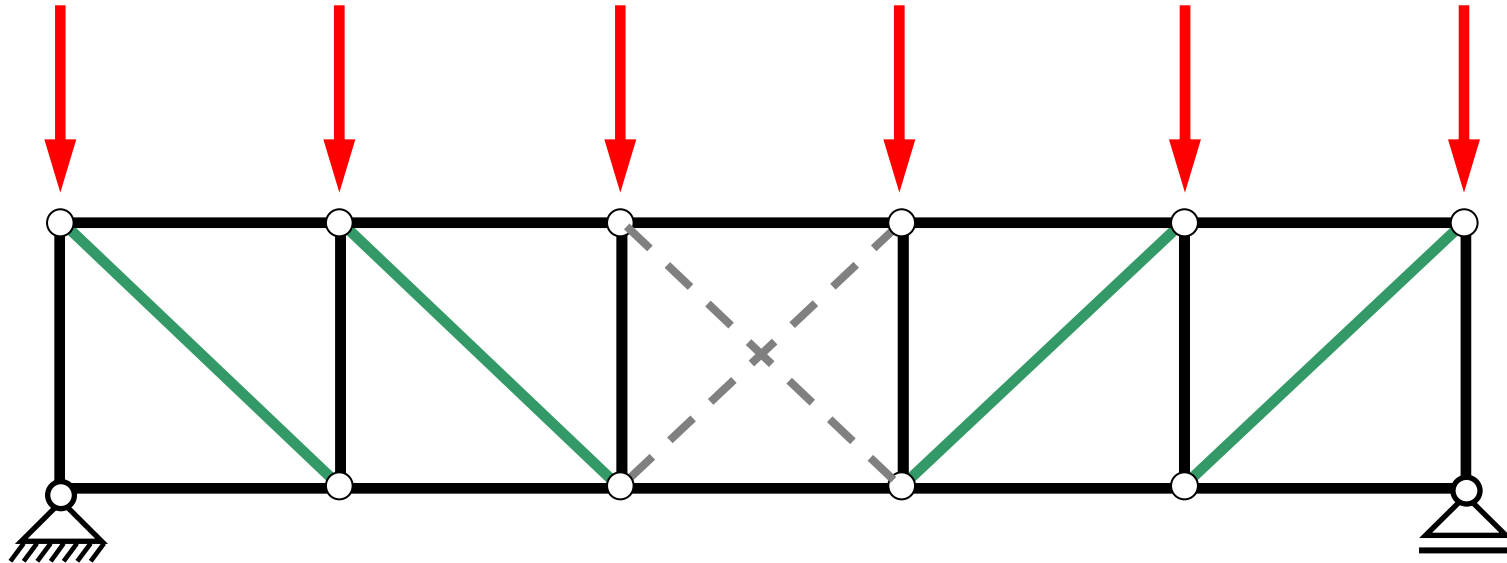








Position of tensile diagonal members in bridge and cantilever beams





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Harbour Bridge, Sidney

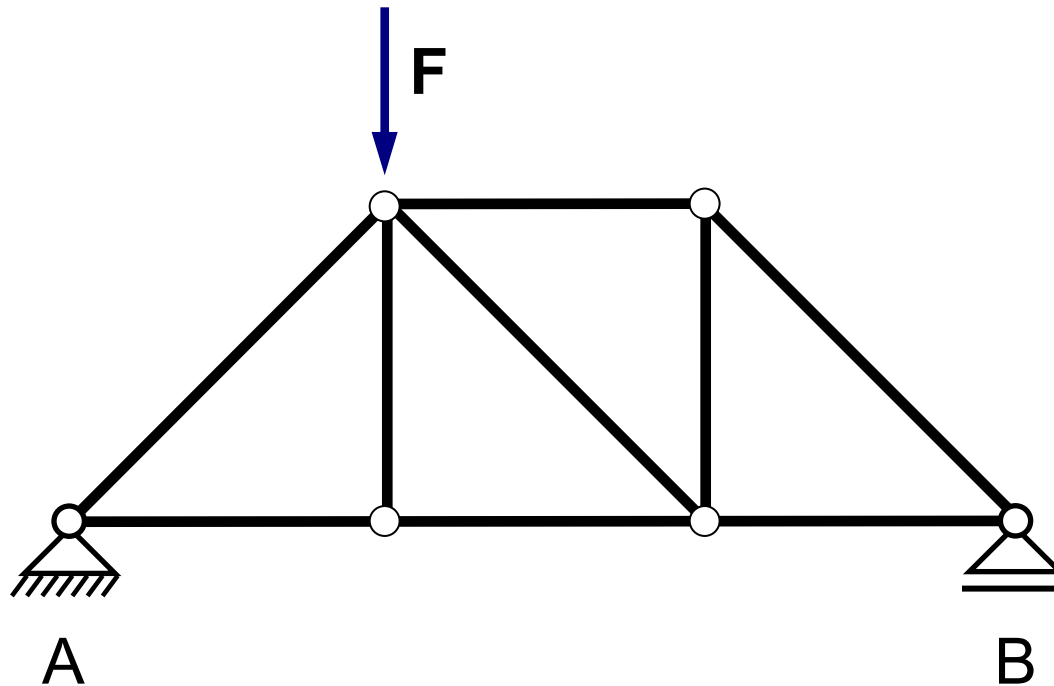


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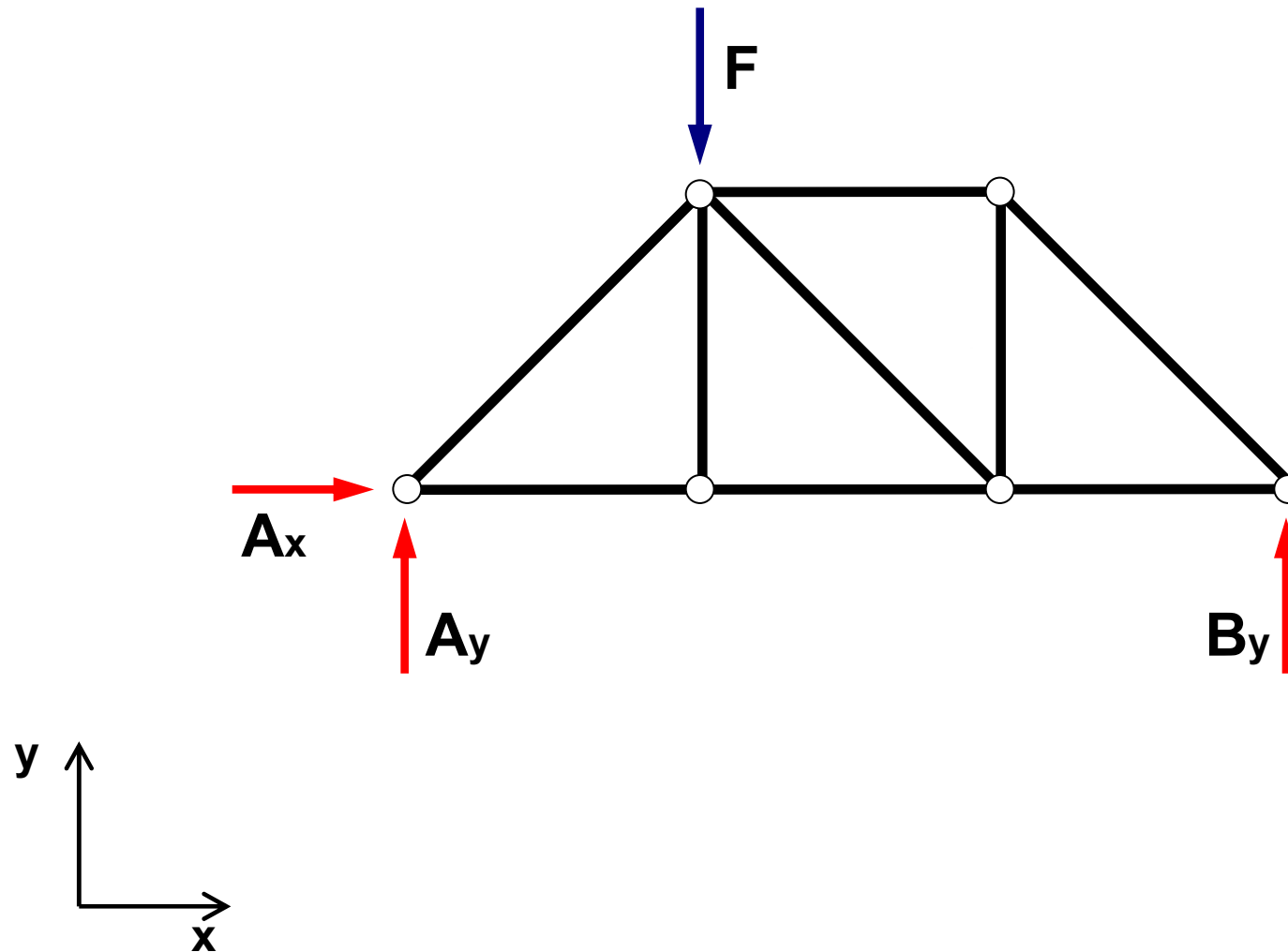
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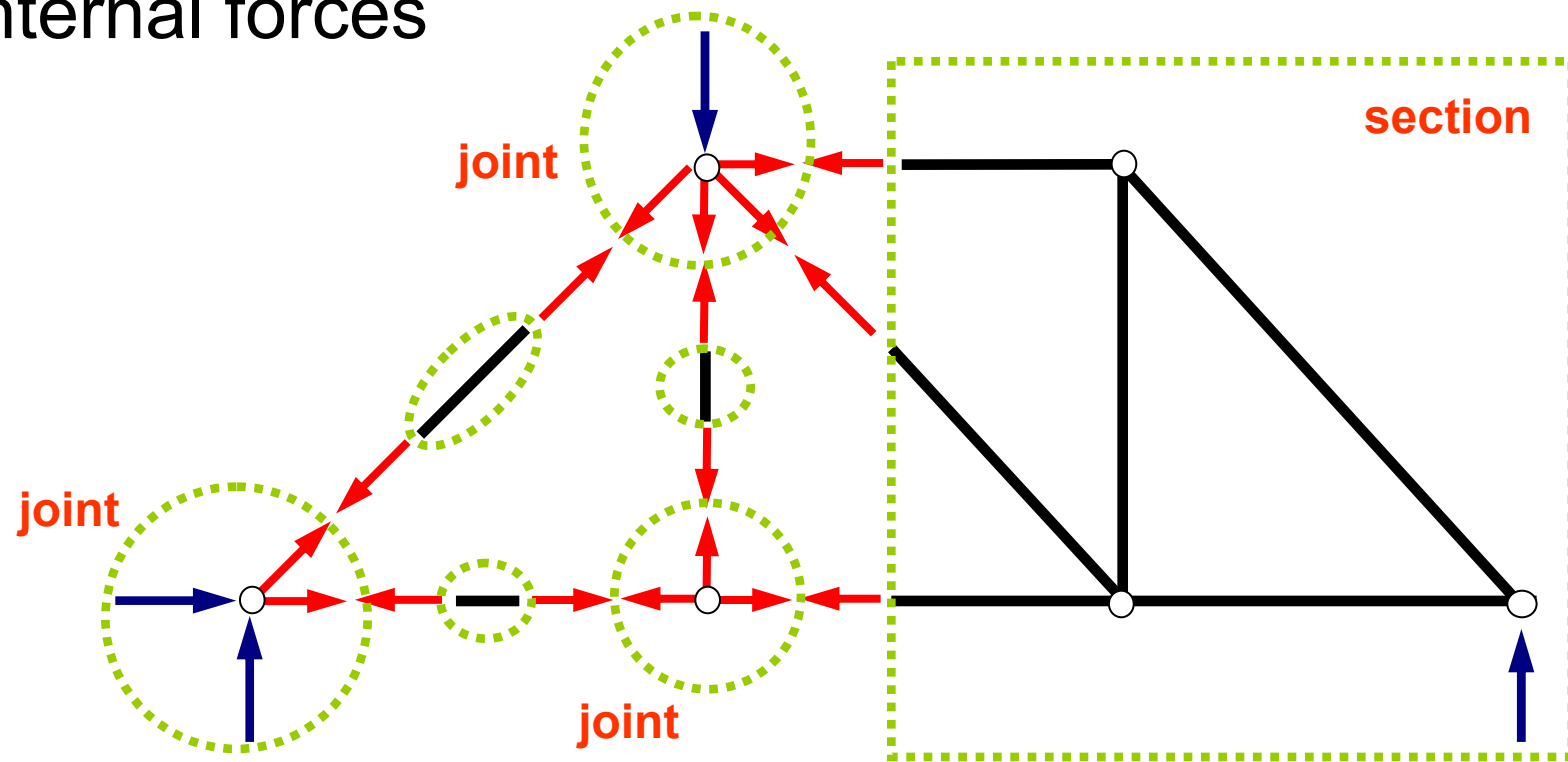
given situation



all external forces



internal forces

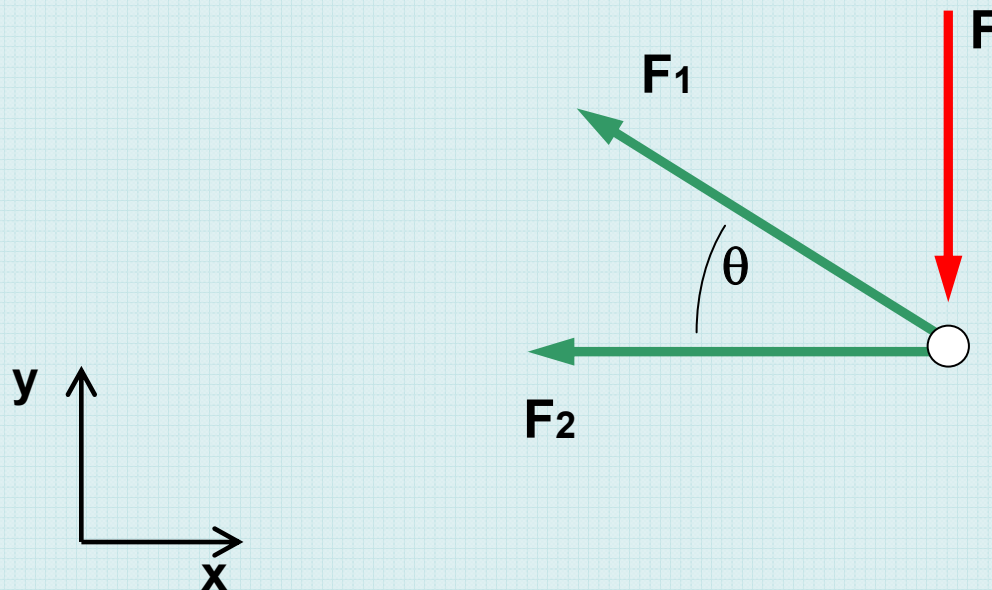


-  isolated particle
-  external forces (known)
-  internal forces (unknown, tension assigned as positive sense)



method of joints

determination of external and internal forces of a truss structure - equilibrium at joints



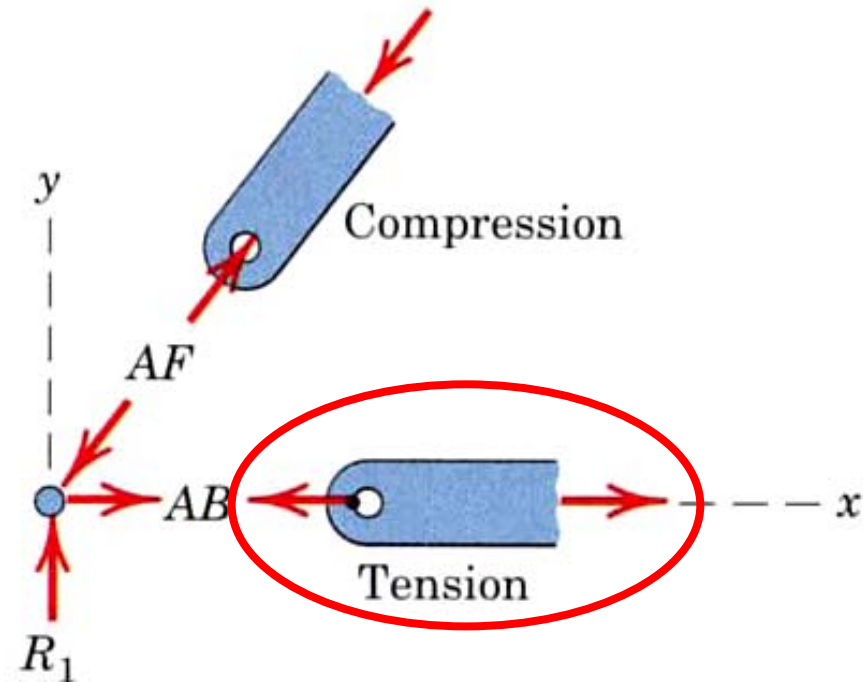
equilibrium at particle (pin joint)

$$\mathbf{R} = \sum \mathbf{F} = \mathbf{0}$$

$$\sum F_x = 0$$

$$\sum F_y = 0$$

concurrent force situation

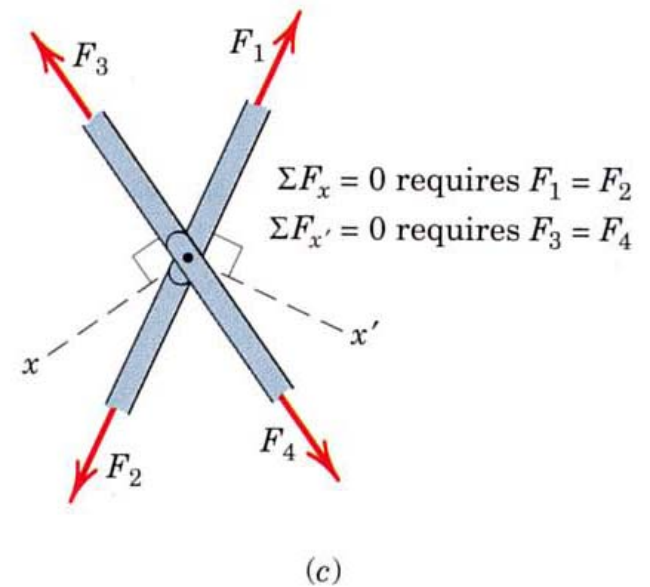
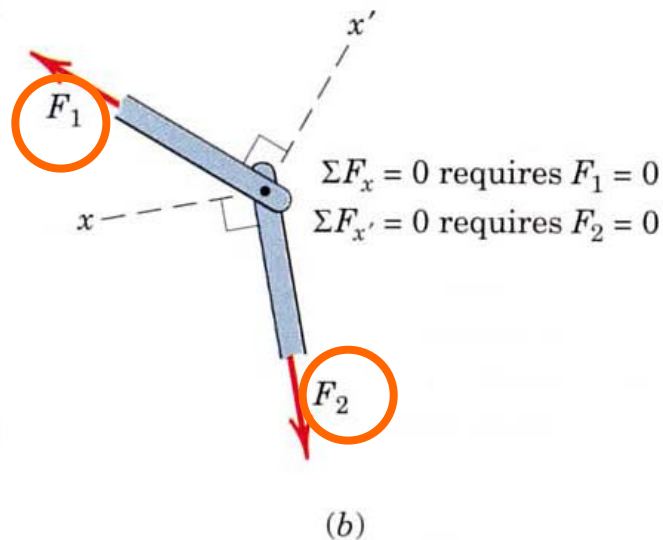
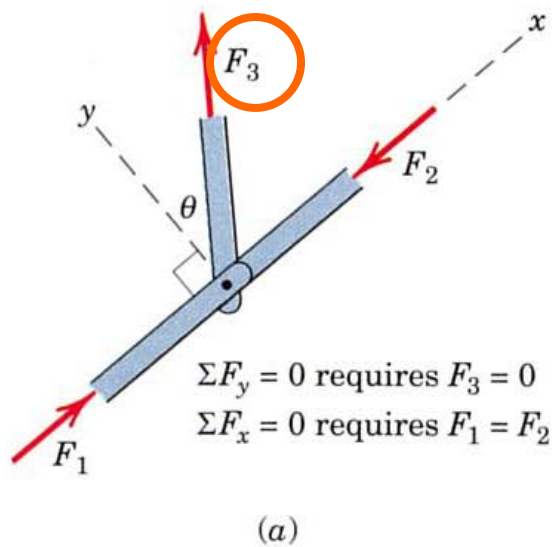


hint: assume unknown forces as tensile forces!

fig by J.L. Meriam, L.G. Kraige, Engineering Mechanics I



special conditions



 zero-force members

fig by J.L. Meriam, L.G. Kraige, Engineering Mechanics I



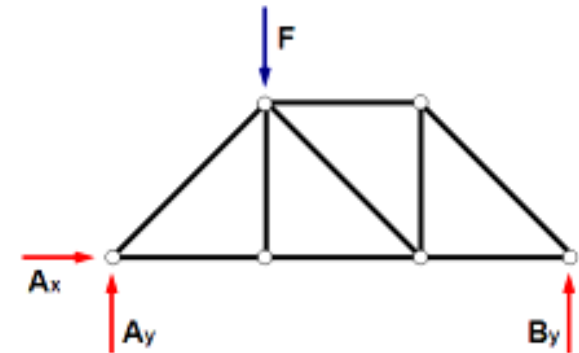
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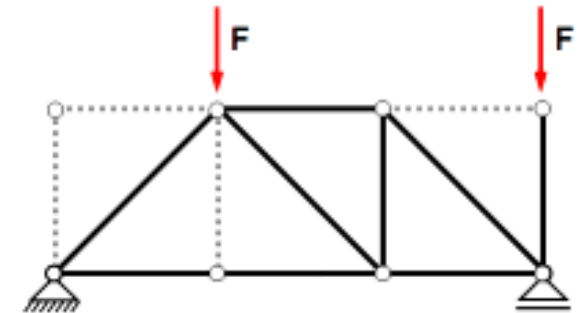
-method of joints

free body diagram of entire structure

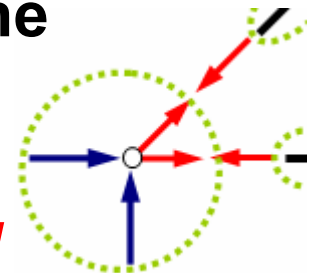


determination of all **external** forces (actions and reactions) by applying the 3 equilibrium conditions on the structure

elimination of all zero-force members in the given load situation



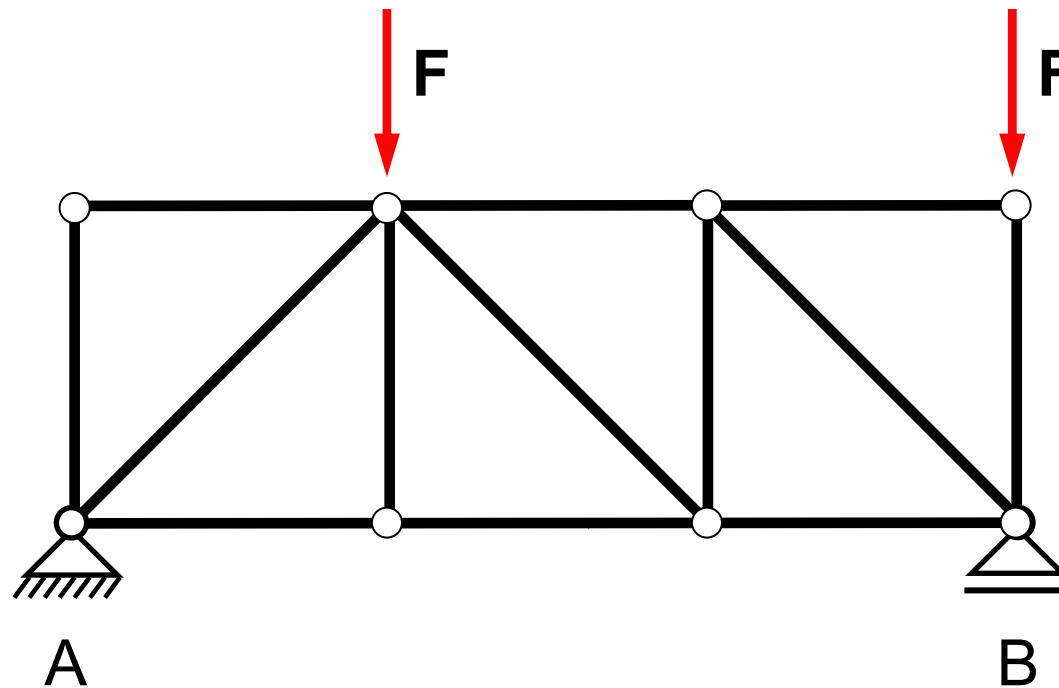
isolating each joint and applying **2 equilibrium** conditions (F_x , F_y) to determine the **internal** forces acting in the attached members



hint: begin with any joint with at least one known force!

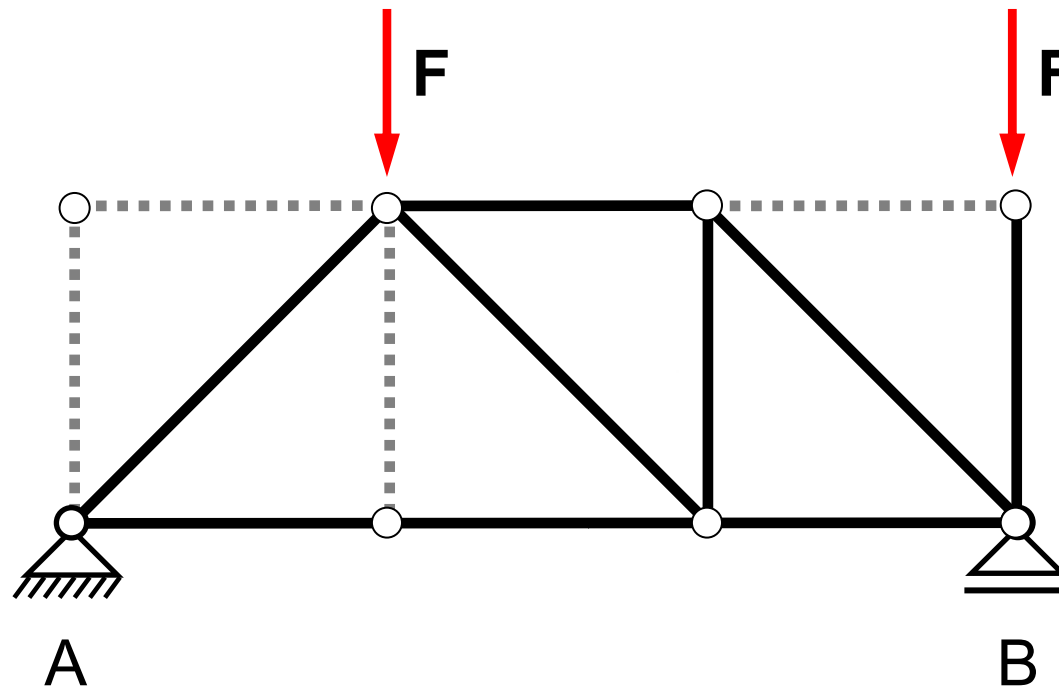


example



example

zero-force members

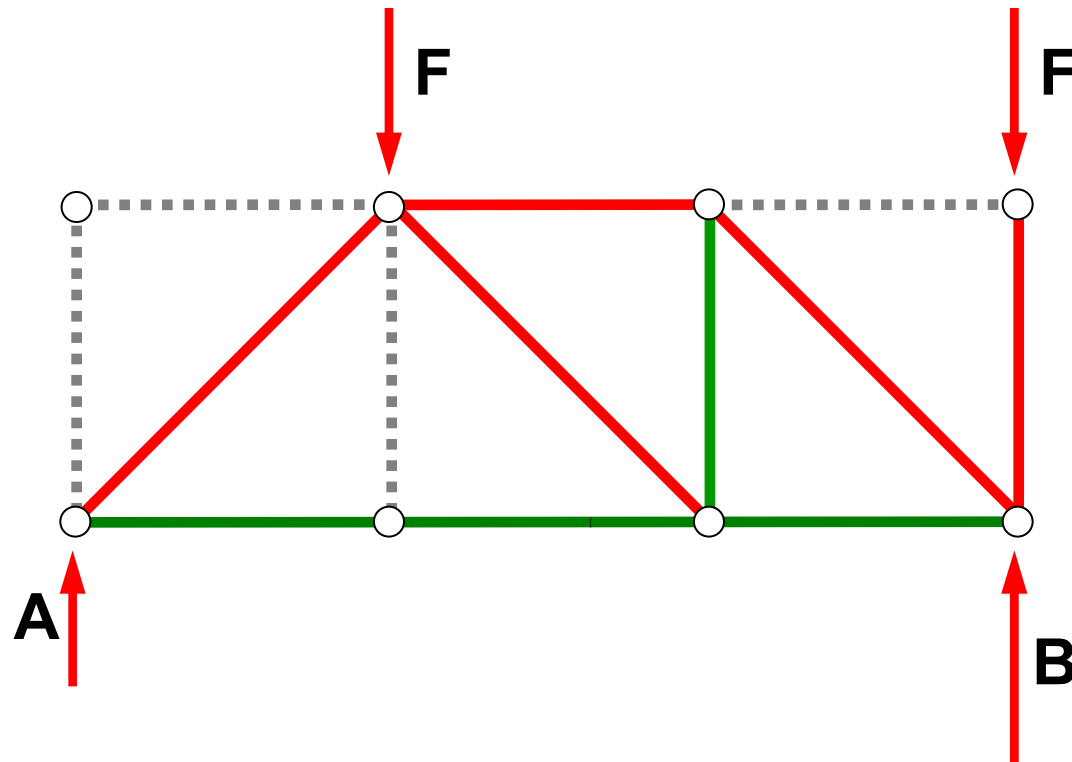


Representing the three possible situations for the pre-determination of zero-force members.

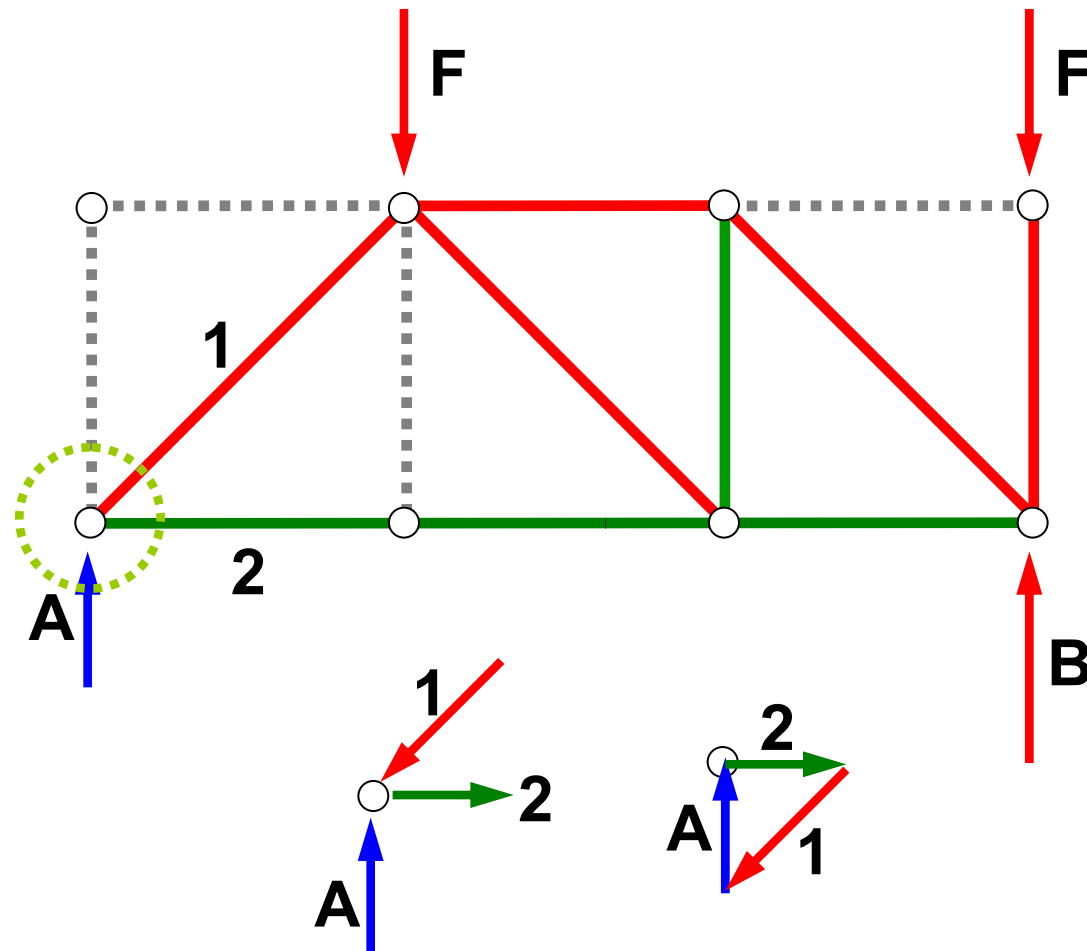


example

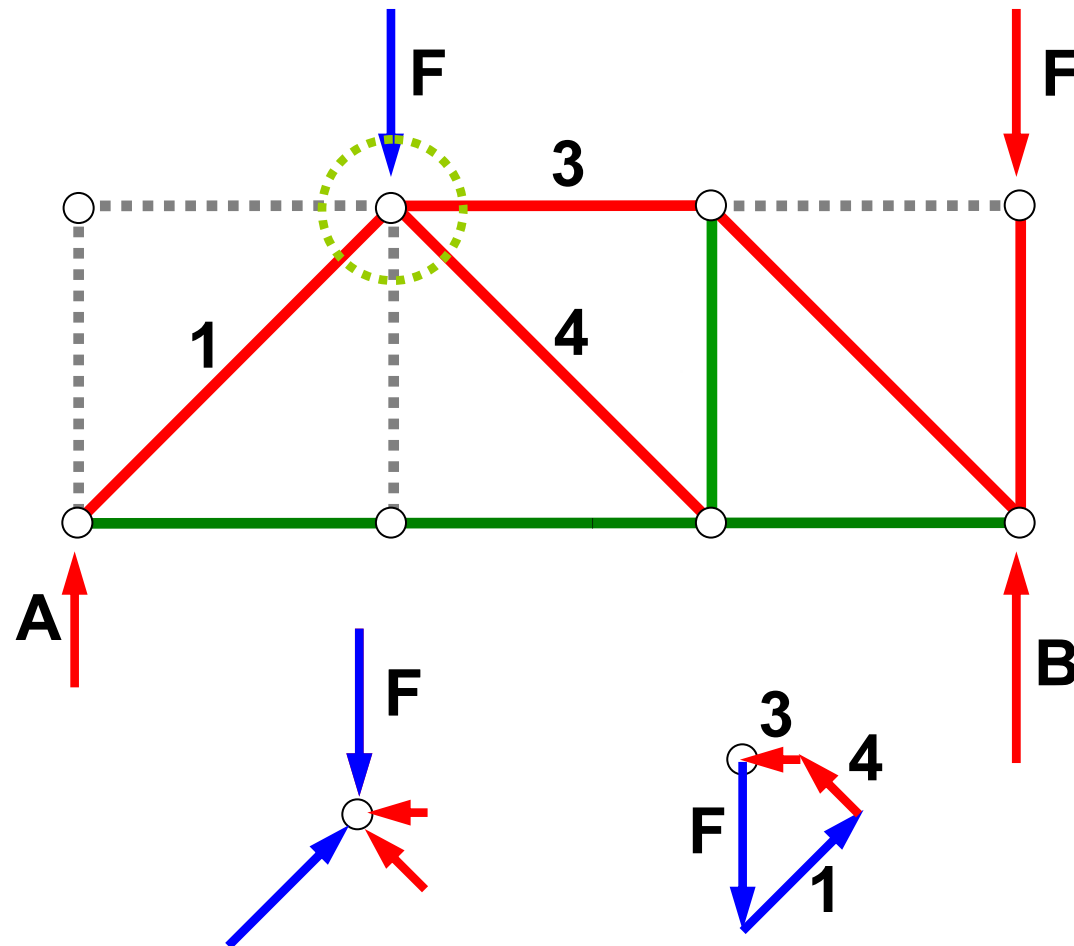
external forces, tensile and compressive members



example



example

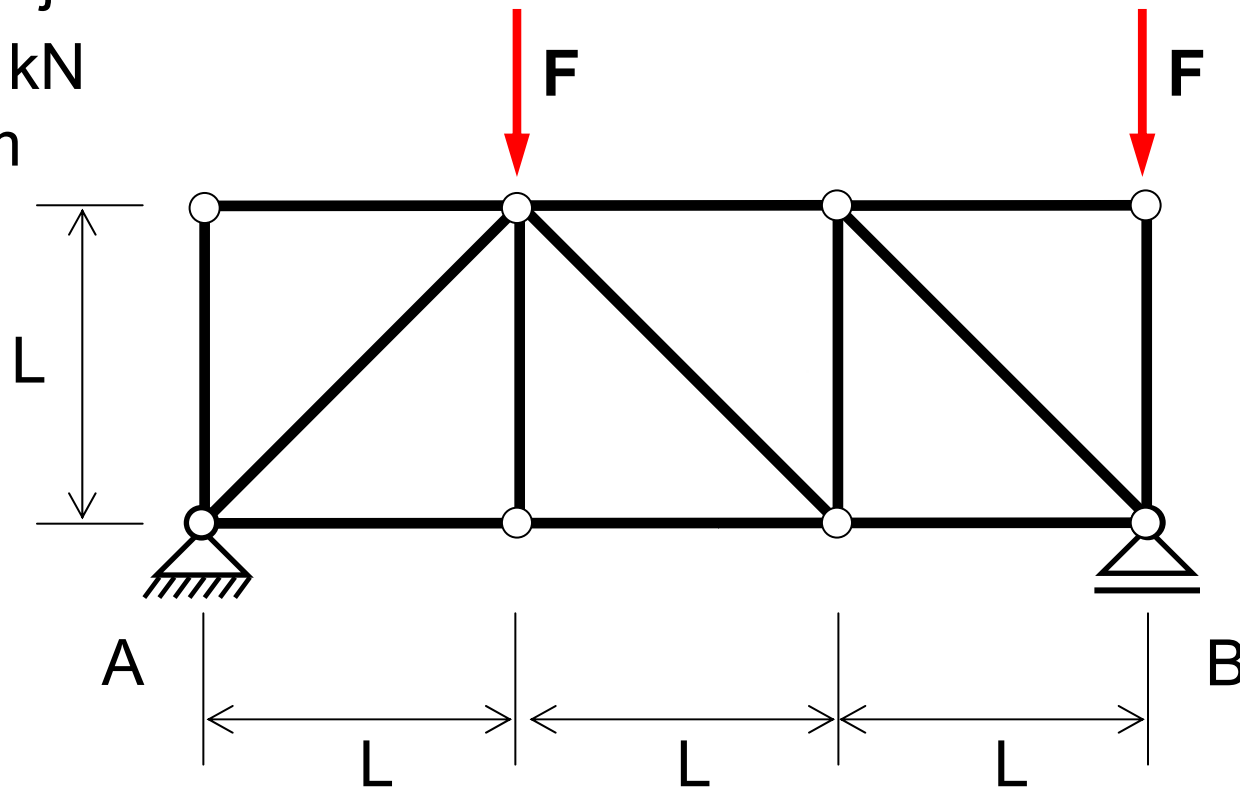


example 4.1

Determine all internal forces in the truss members applying the method of joints.

$$F = 10.0 \text{ kN}$$

$$L = 3.0 \text{ m}$$

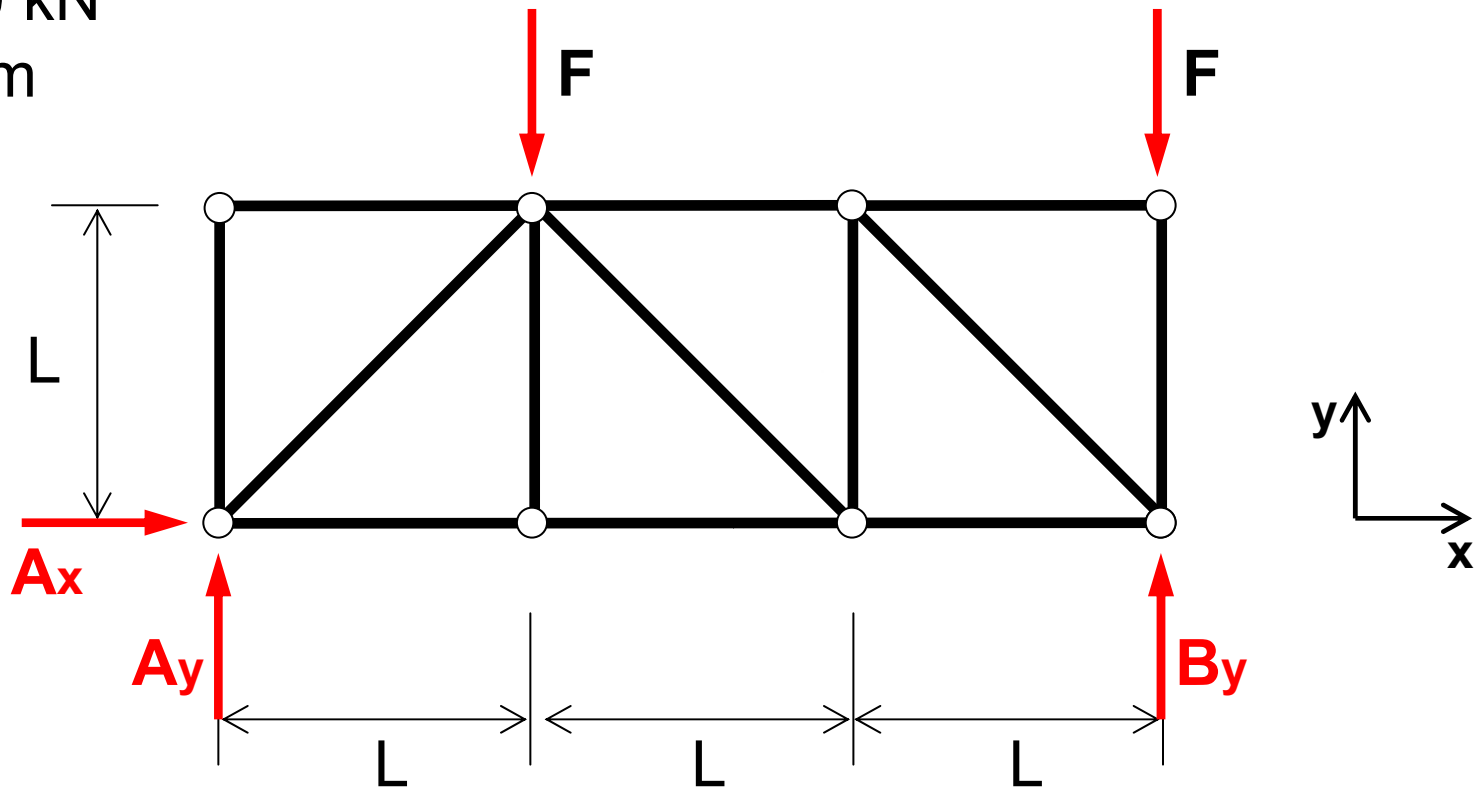


example

1. External Equilibrium – Free Body Diagram

$$F = 10.0 \text{ kN}$$

$$L = 3.0 \text{ m}$$

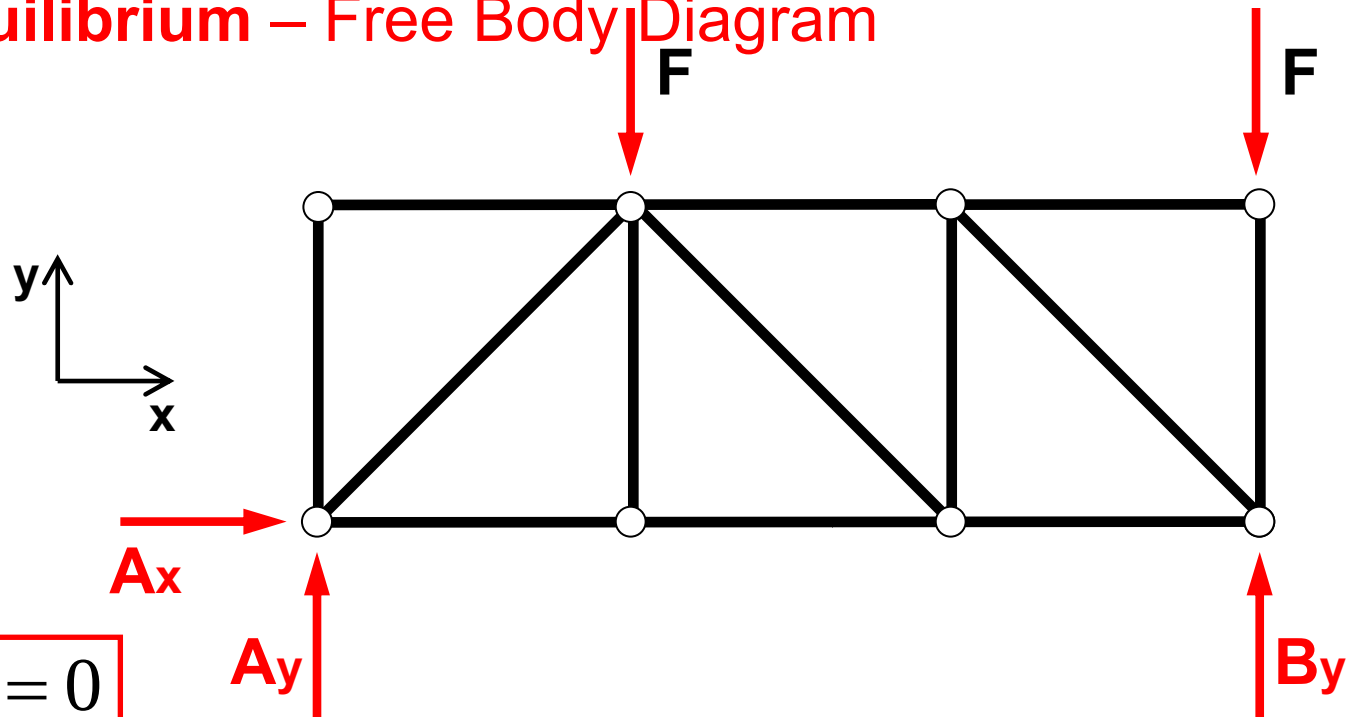


example

1. External Equilibrium – Free Body Diagram

$$F = 10.0 \text{ kN}$$

$$L = 3.0 \text{ m}$$



$$\sum F_x = 0: A_x = 0$$

$$\sum M_A = 0: B_y \cdot 9\text{m} - 10\text{kN} \cdot 9\text{m} - 10\text{kN} \cdot 3\text{m} = 0$$

$$\Rightarrow B_y = 13.33\text{kN}$$

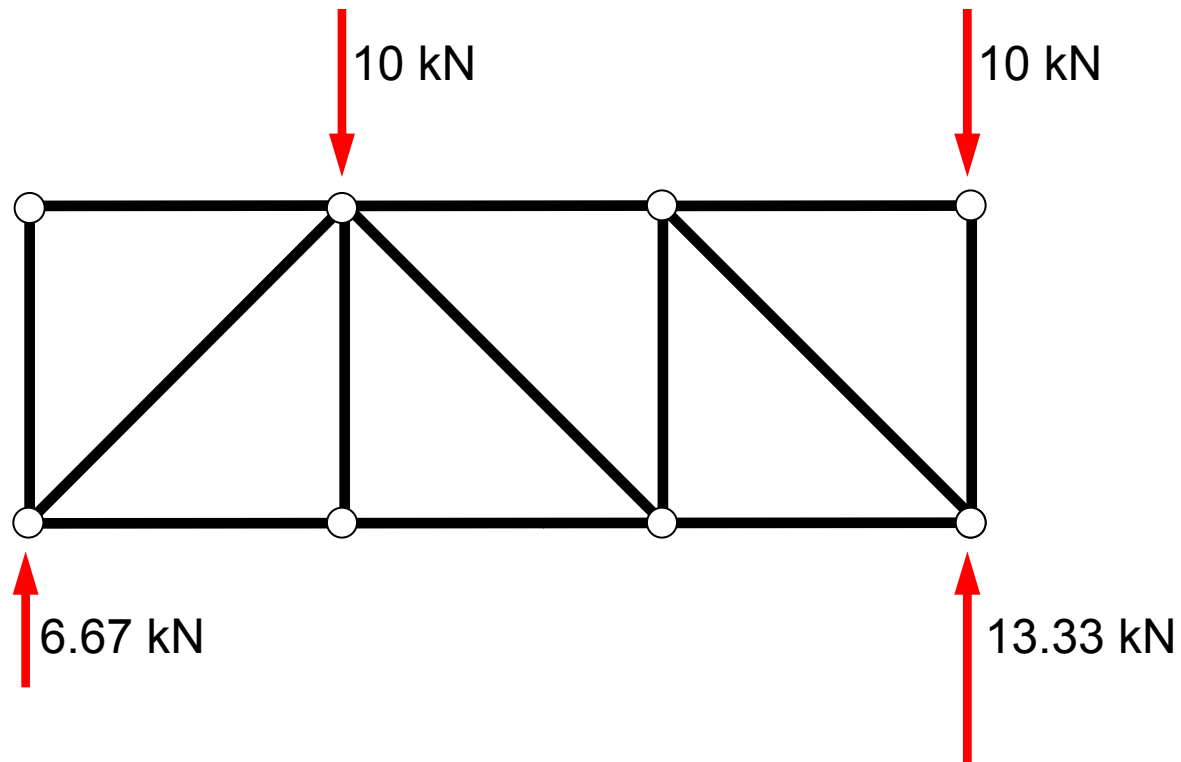
$$\sum F_y = 0: A_y + B_y - 2F = 0$$

$$\Rightarrow A_y = 6.67\text{kN}$$



example

External load situation:

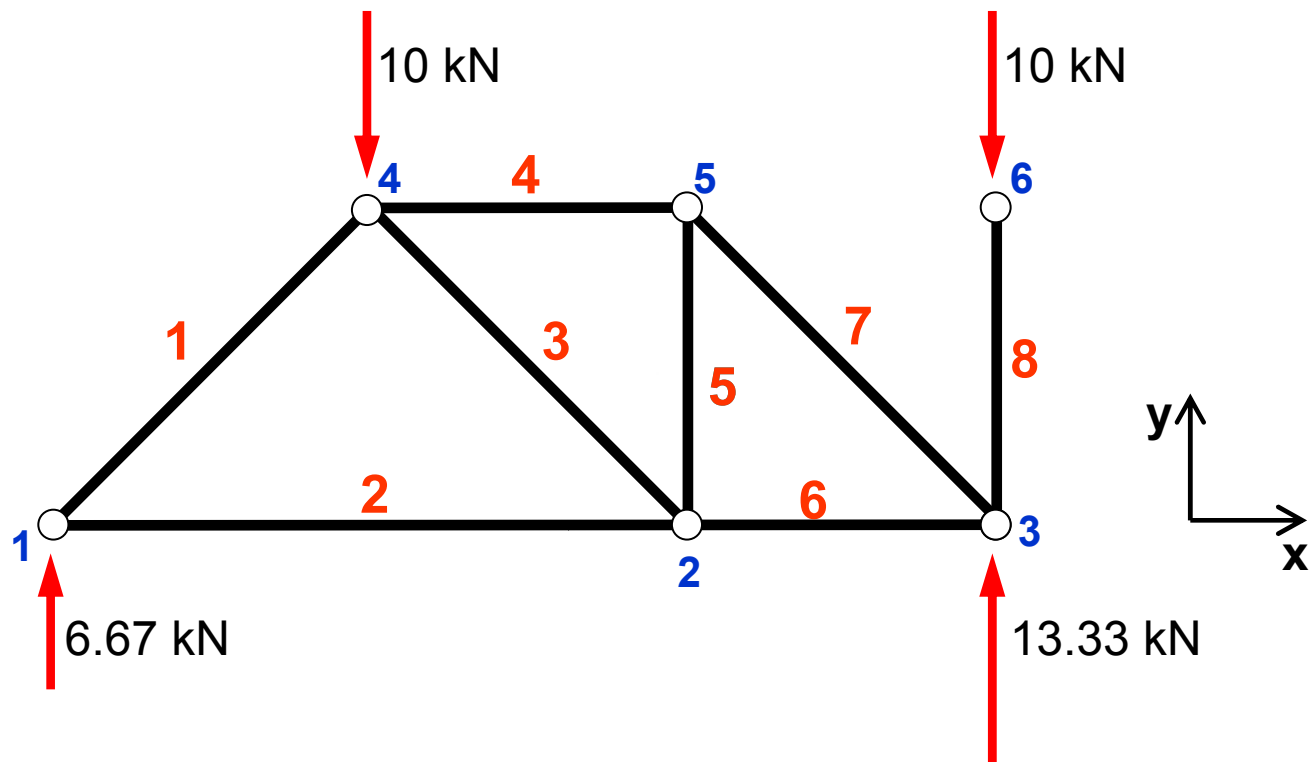


2. Elimination of zero-force members:



example

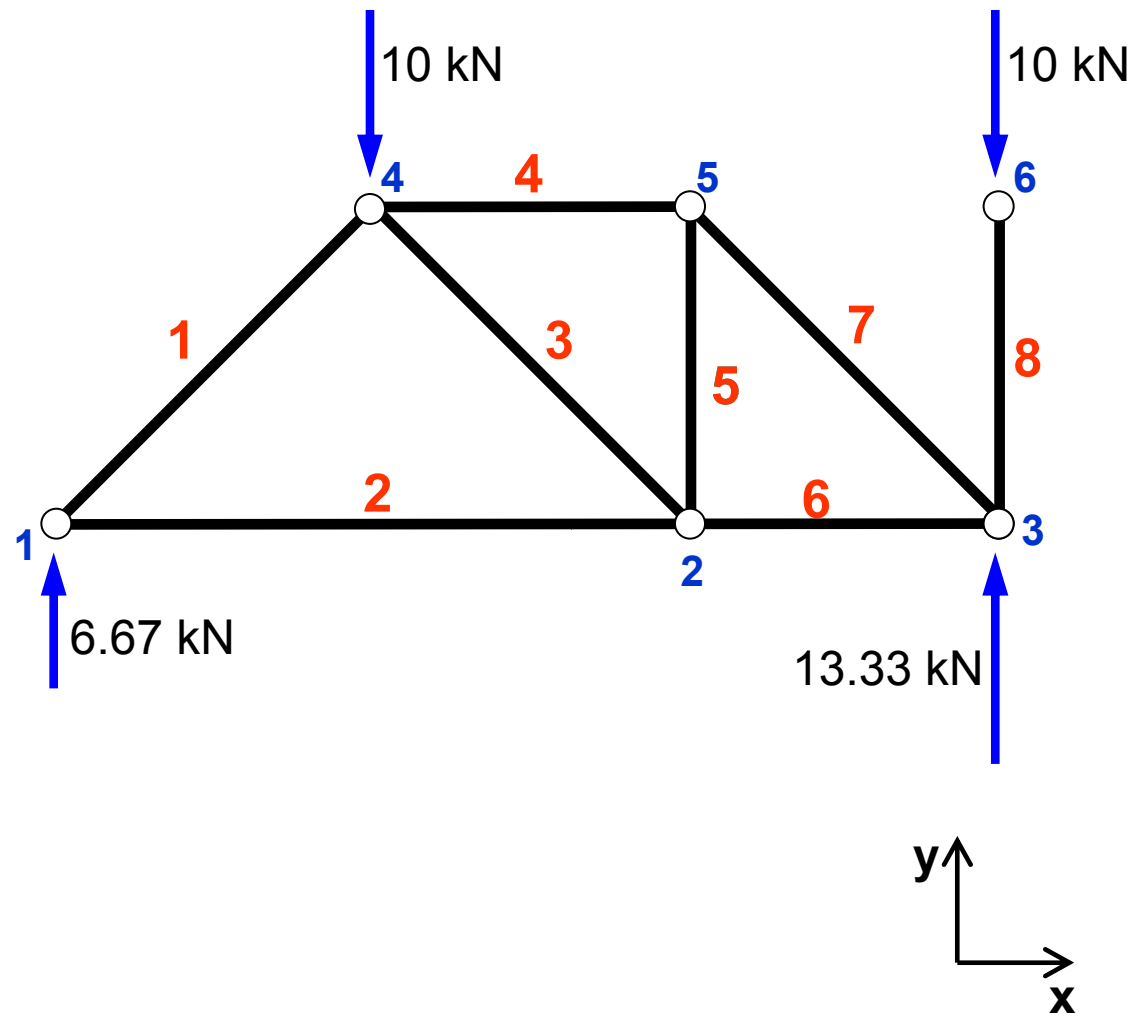
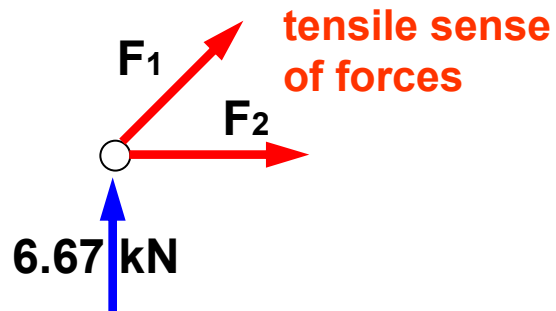
Applying numbers for joints (nodes) and members.



example

Internal Equilibrium

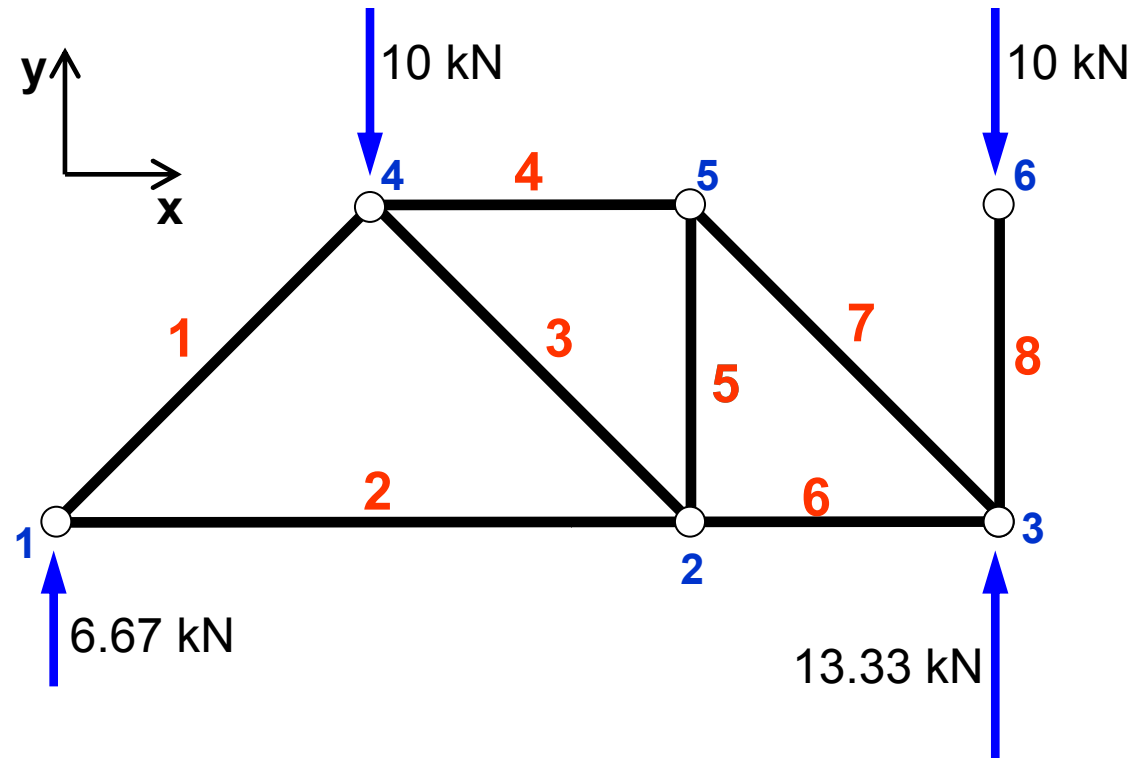
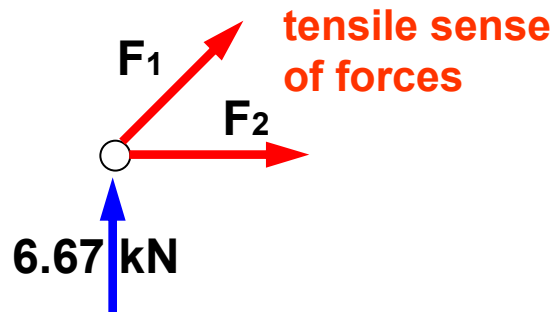
Equilibrium at joint 1:



example

Internal Equilibrium

Equilibrium at joint 1:



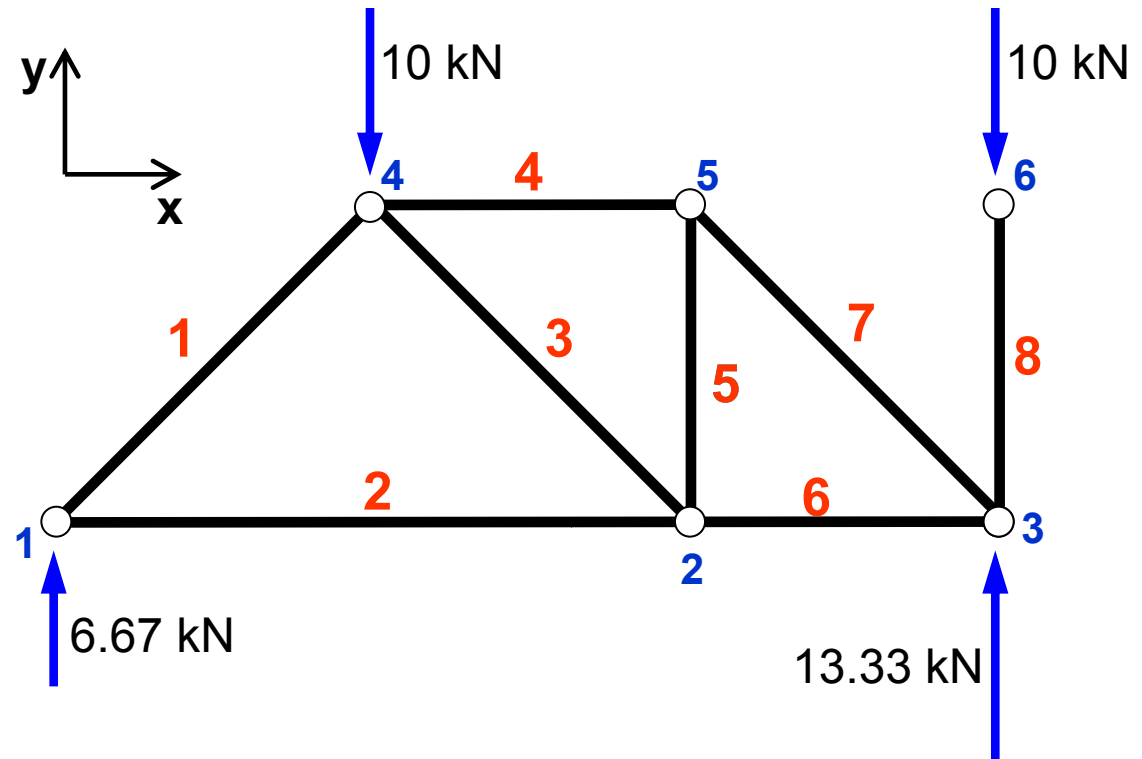
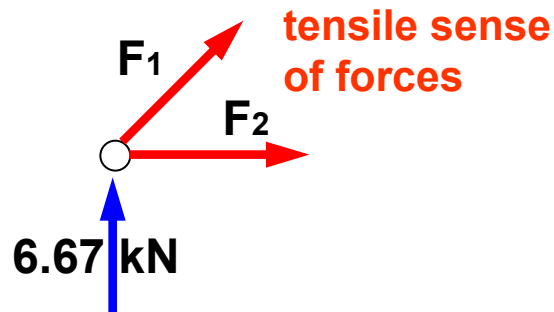
$$\sum F_y = 0: F_1 \sin 45^\circ + 6.67 \text{ kN} = 0 \quad \boxed{F_1 = -9.43 \text{ kN}} \quad \text{compression (-)}$$



example

Internal Equilibrium

Equilibrium at joint 1:



$$\sum F_y = 0: F_1 \sin 45^\circ + 6.67 \text{ kN} = 0 \quad \boxed{F_1 = -9.43 \text{ kN}} \quad \text{compression (-)}$$

$$\sum F_x = 0: F_1 \cdot \cos 45^\circ + F_2 = 0 \quad \boxed{F_2 = 6.67 \text{ kN}} \quad \text{tension (+)}$$

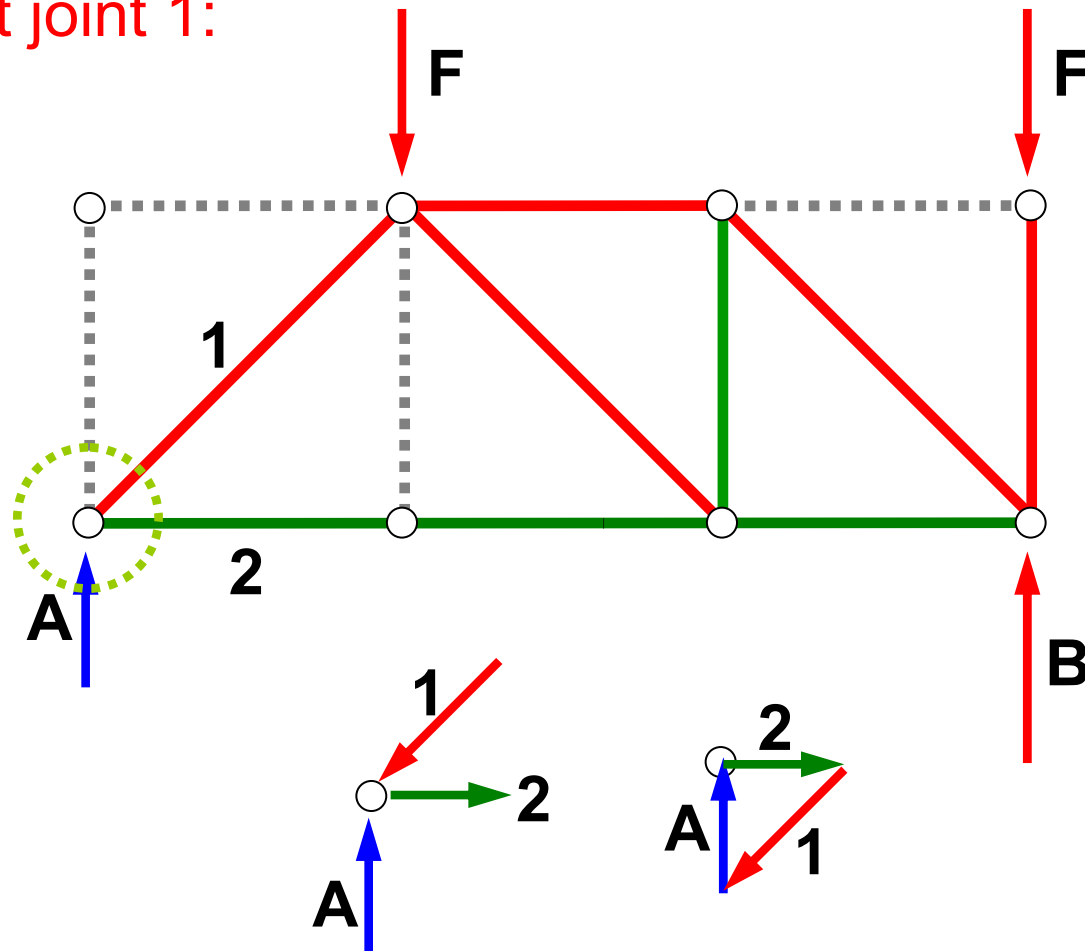


example

Internal Equilibrium

graphic solution

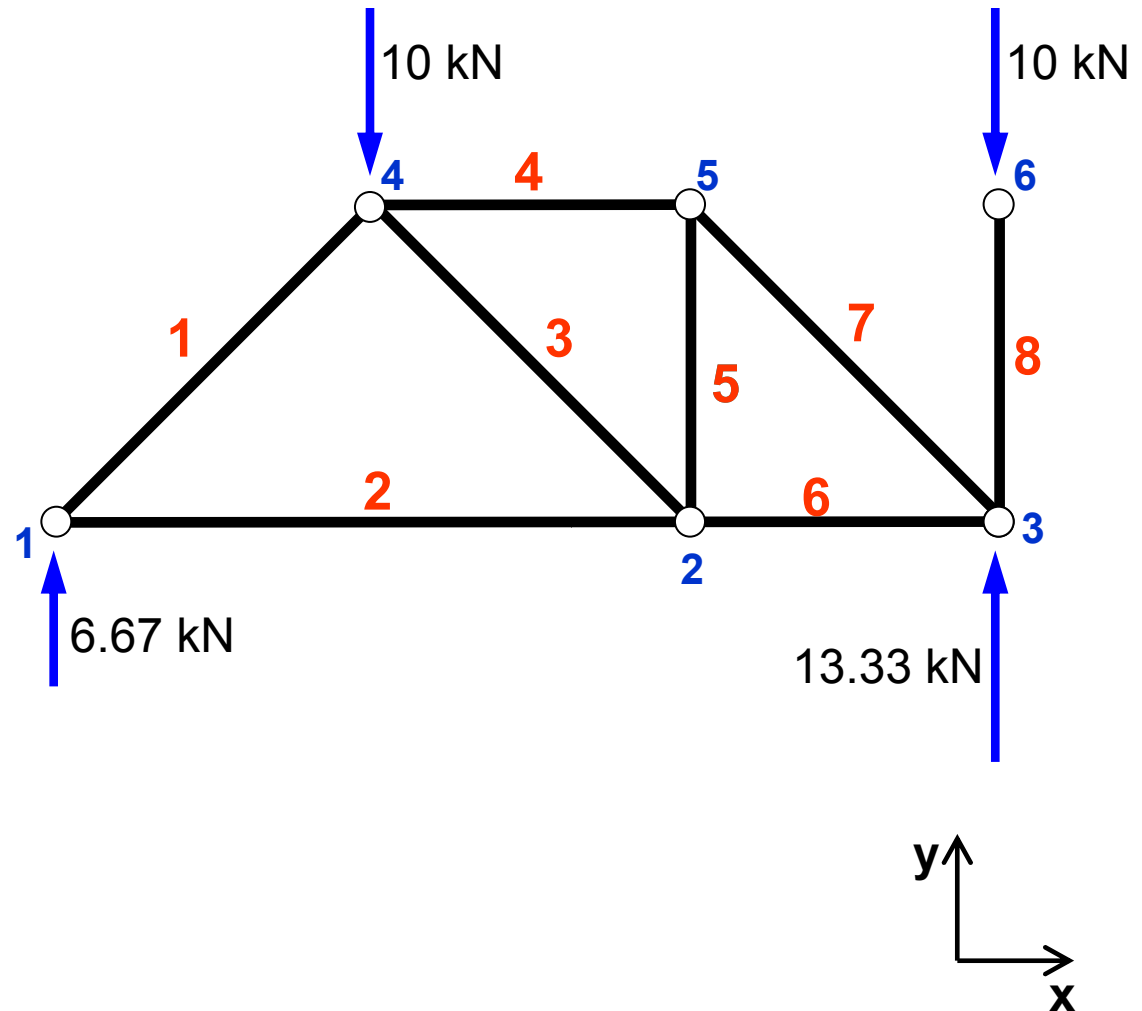
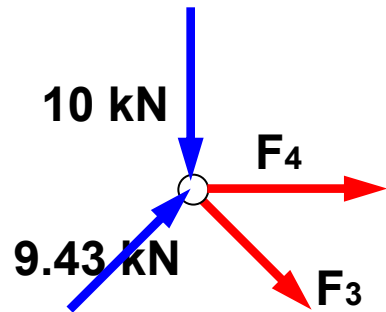
Equilibrium at joint 1:



example

Internal Equilibrium

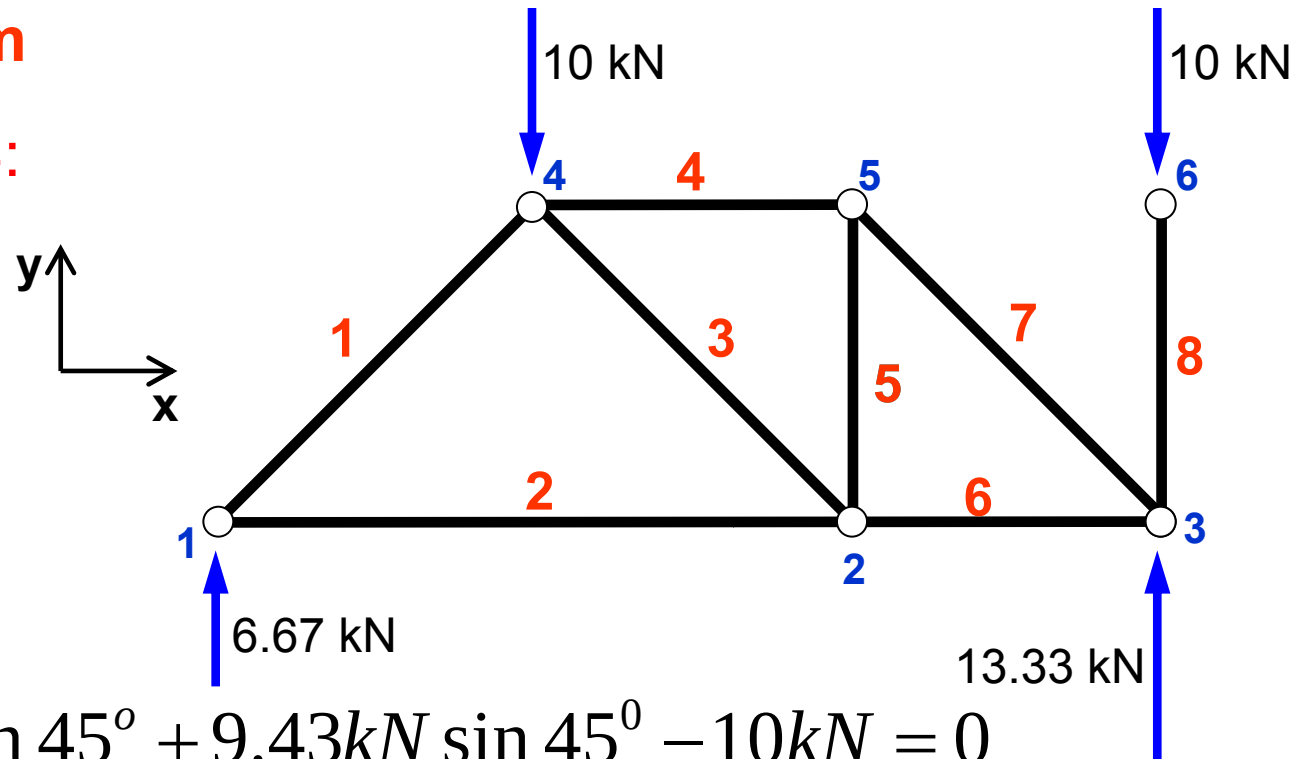
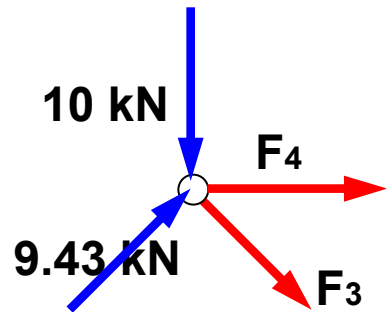
Equilibrium at joint 4:



example

Internal Equilibrium

Equilibrium at joint 4:



$$\sum F_y = 0: -F_3 \sin 45^\circ + 9.43 \text{ kN} \sin 45^\circ - 10 \text{ kN} = 0$$

$$F_3 = -4.71 \text{ kN} \quad \text{compression (-)}$$

$$\sum F_x = 0: F_4 + F_3 \cos 45^\circ + 9.43 \text{ kN} \cos 45^\circ = 0$$

$$F_4 = -3.33 \text{ kN} \quad \text{compression (-)}$$

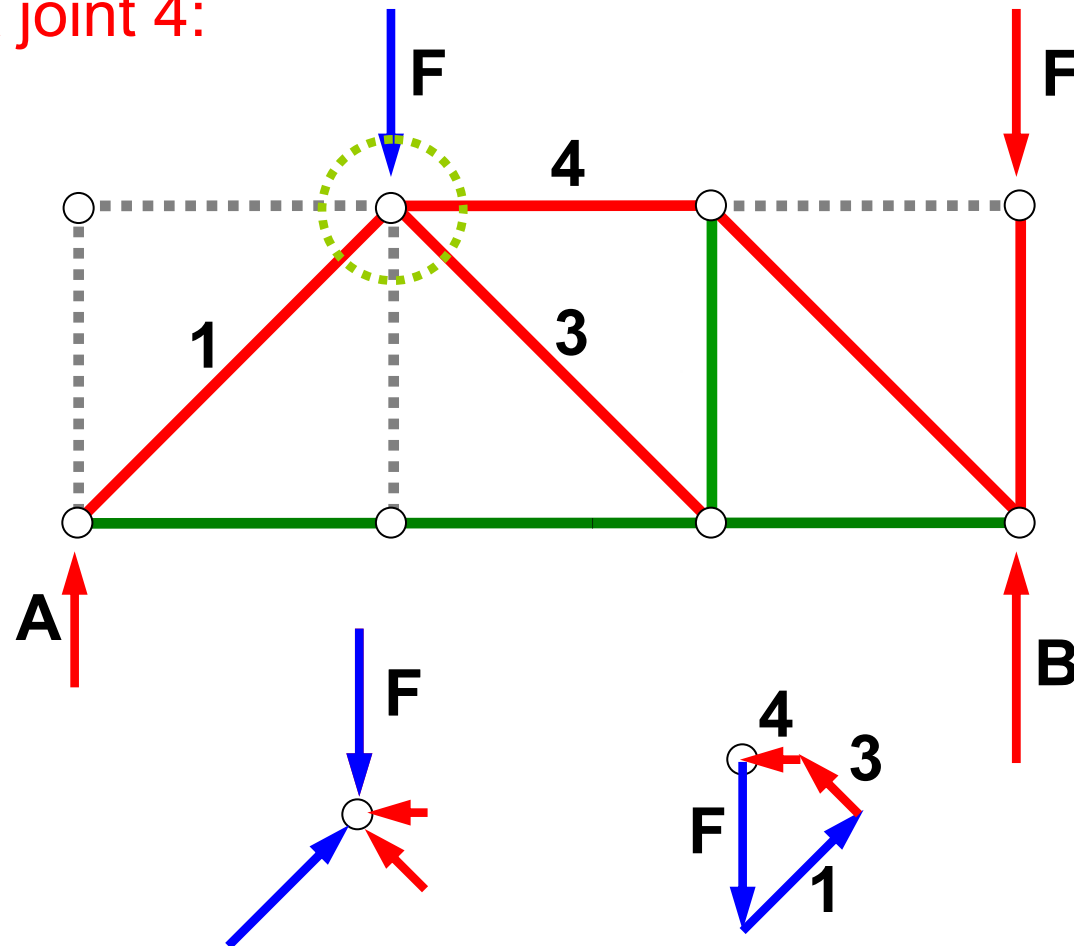


example

Internal Equilibrium

graphic solution

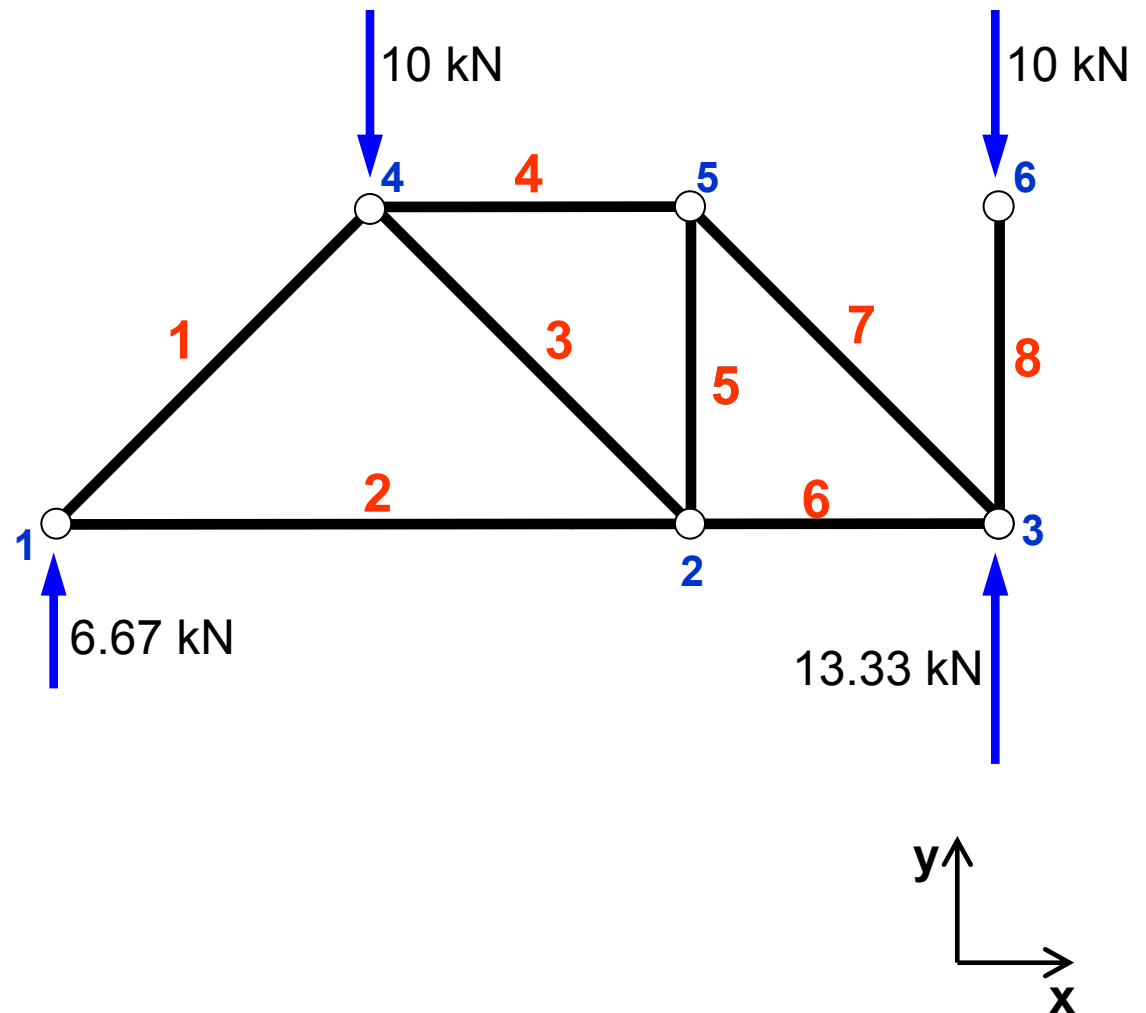
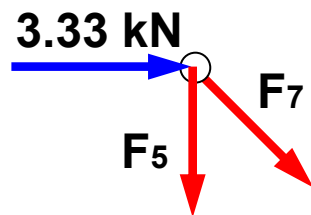
Equilibrium at joint 4:



example

Internal Equilibrium

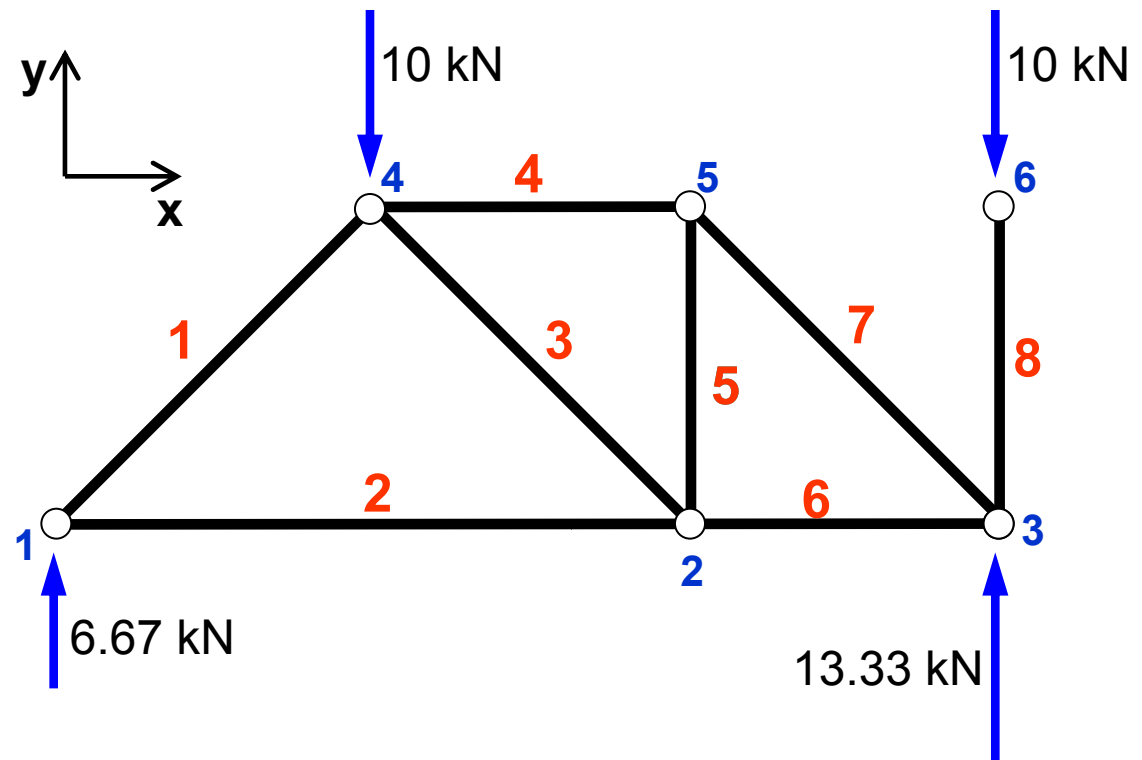
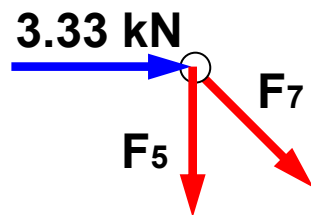
Equilibrium at joint 5:



example

Internal Equilibrium

Equilibrium at joint 5:



$$\sum F_x = 0: F_7 \cos 45^\circ + 3.33 \text{ kN} = 0 \quad \boxed{F_7 = -4.71 \text{ kN}} \text{ compression (-)}$$

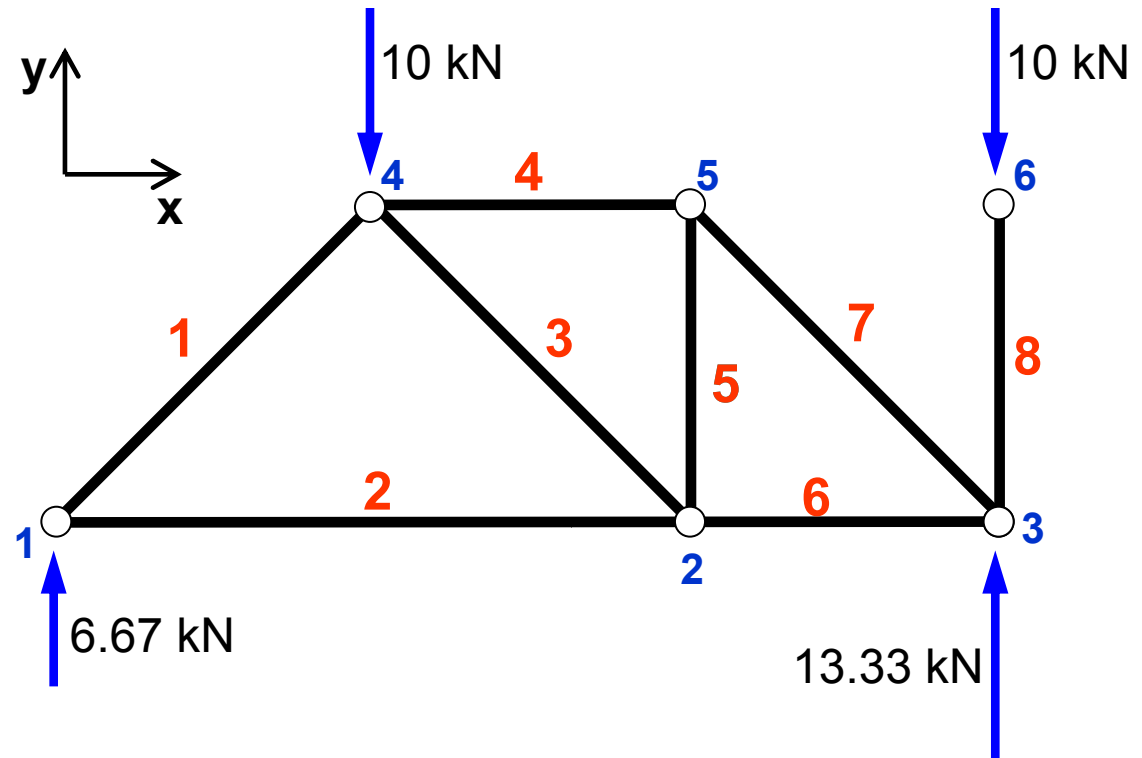
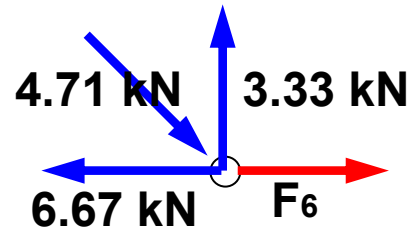
$$\sum F_y = 0: -F_7 \cdot \sin 45^\circ - F_5 = 0 \quad \boxed{F_5 = 3.33 \text{ kN}} \text{ tension (+)}$$



example

Internal Equilibrium

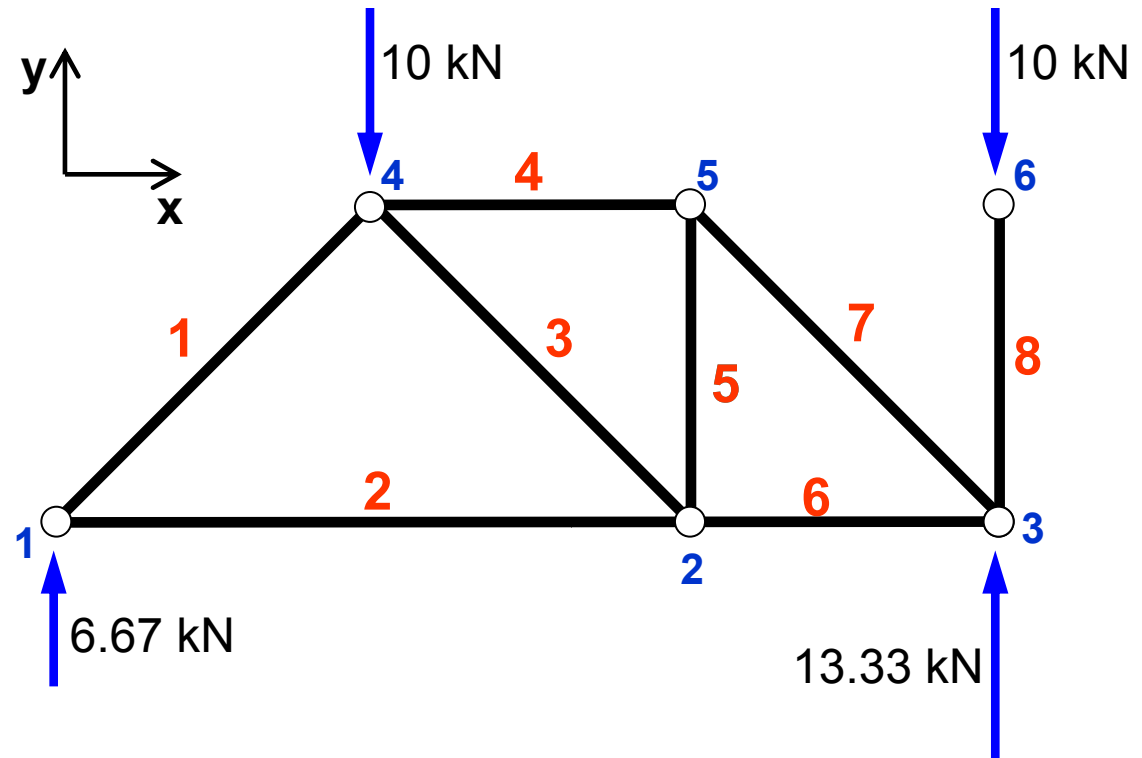
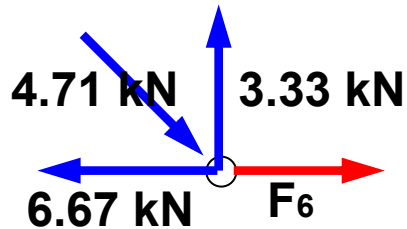
Equilibrium at joint 2:



example

Internal Equilibrium

Equilibrium at joint 2:



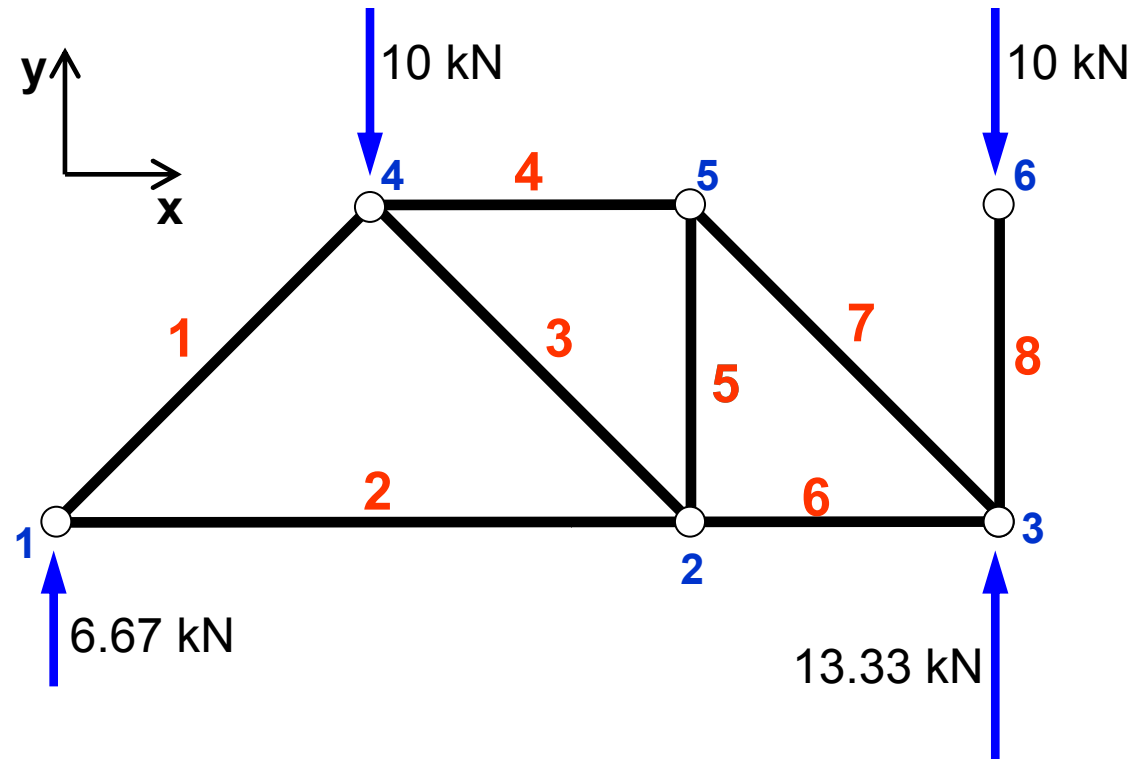
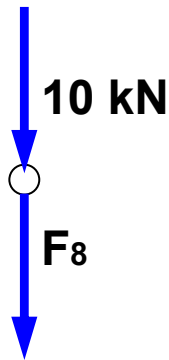
$$\sum F_x = 0: F_6 + 4.71 \cos 45^\circ - 6.67 \text{ kN} = 0$$

$$F_6 = 3.33 \text{ kN} \quad \text{tension (+)}$$



example

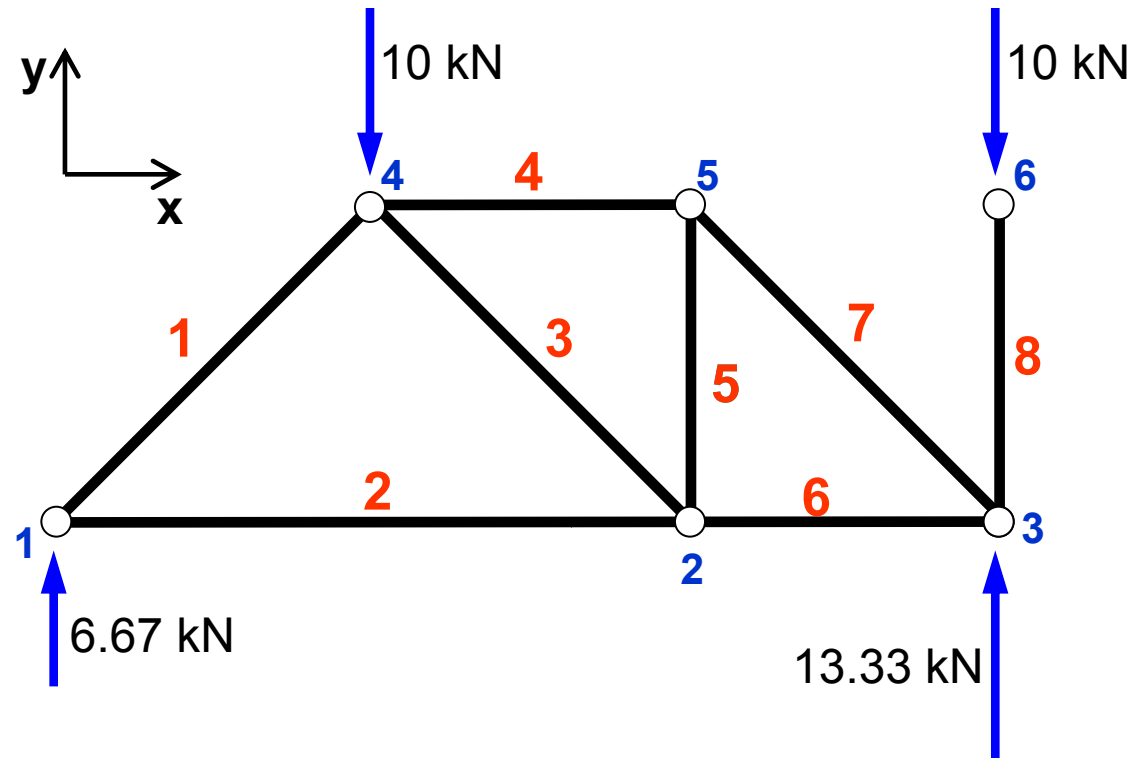
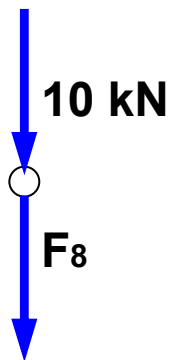
Internal Equilibrium Equilibrium at joint 6:



example

Internal Equilibrium

Equilibrium at joint 6:



$$\sum F_y = 0: -10\text{kN} - F_8 = 0$$

$$F_8 = -10.0\text{kN}$$

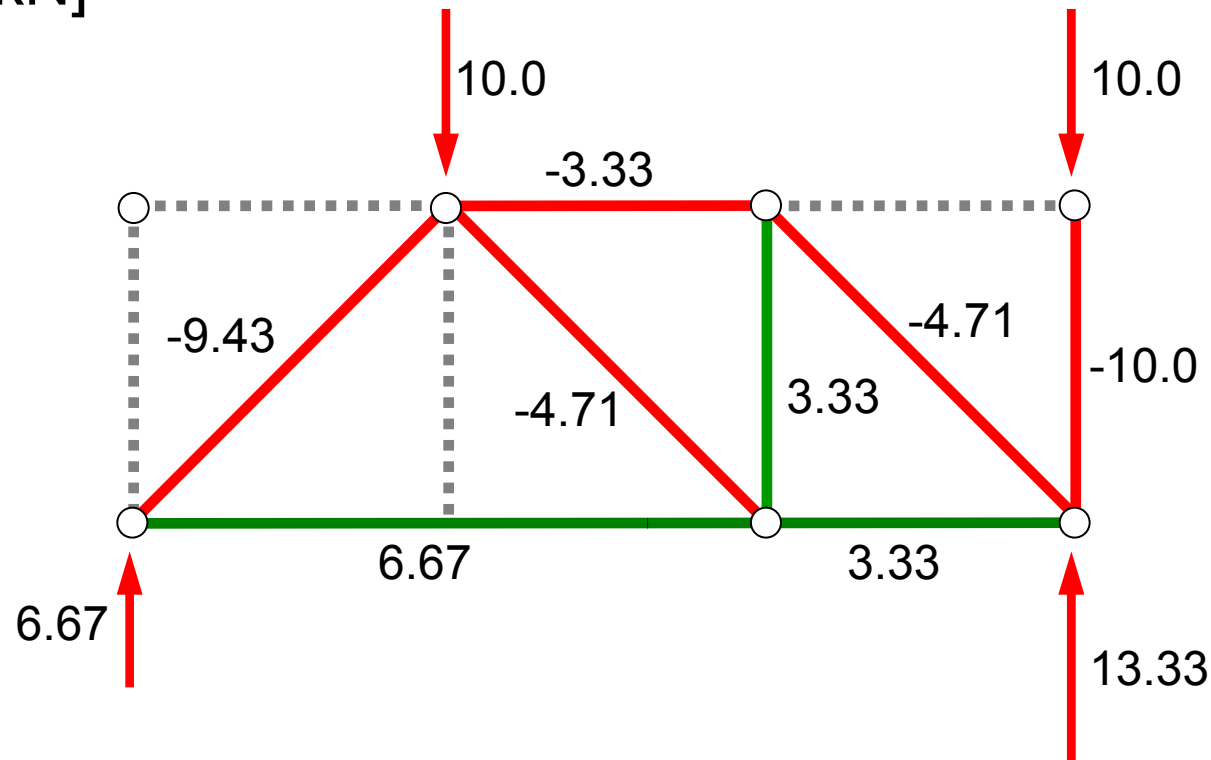
compression (-)

hint: use equilibrium on joint 3 to cross check results



example

Results [kN]



method of sections

determination of external and internal forces of a truss structure - equilibrium at portion of truss

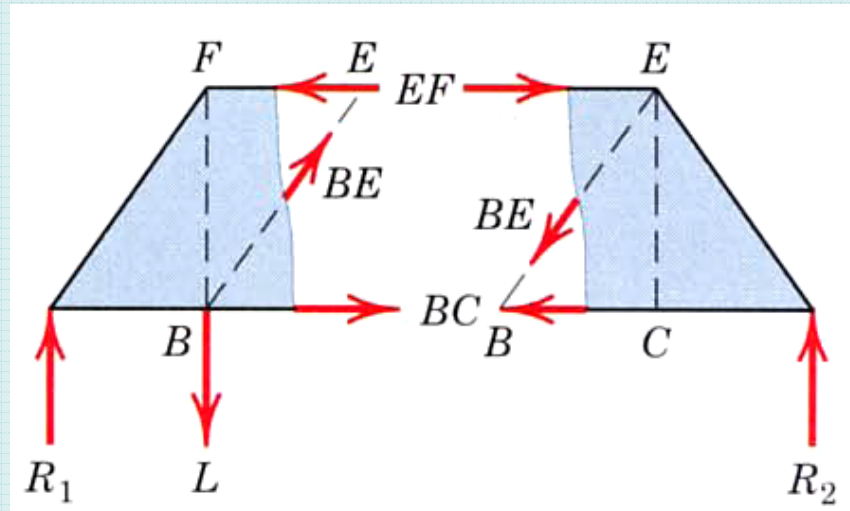
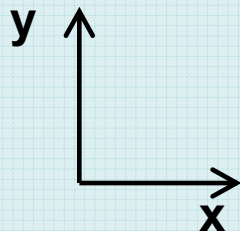


fig by J.L. Meriam, L.G. Kraige, Engineering Mechanics I



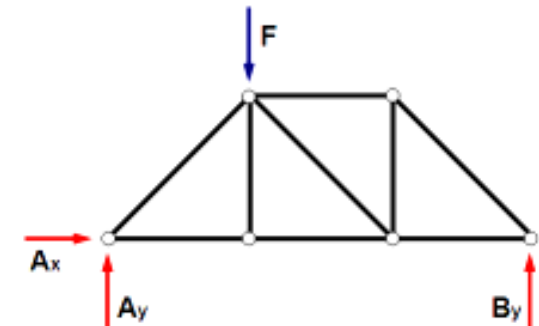
ADDIS ABABA UNIVERSITY
Faculty of Technology

Engineering Mechanics
material by Karsten Schlesier

Department of
Civil Engineering

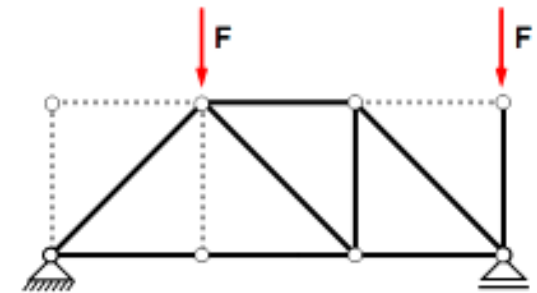
-method of sections

free body diagram of entire structure



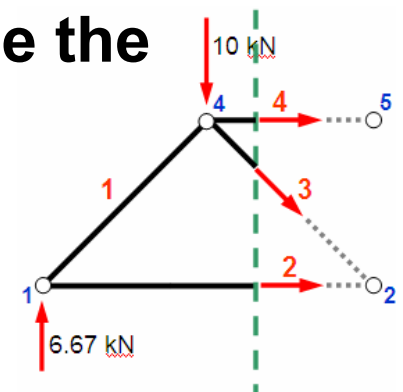
determination of all **external** forces (actions and reactions) by applying the 3 equilibrium conditions on the structure

elimination of all zero-force members in the given load situation



isolating any section of the truss system and applying the **3 equilibrium conditions** (M , F_x , F_y) to determine the **internal** forces acting in the cut members

*hint: choose moment centre (on or off the section)
eliminating max. amount of unknown forces!*

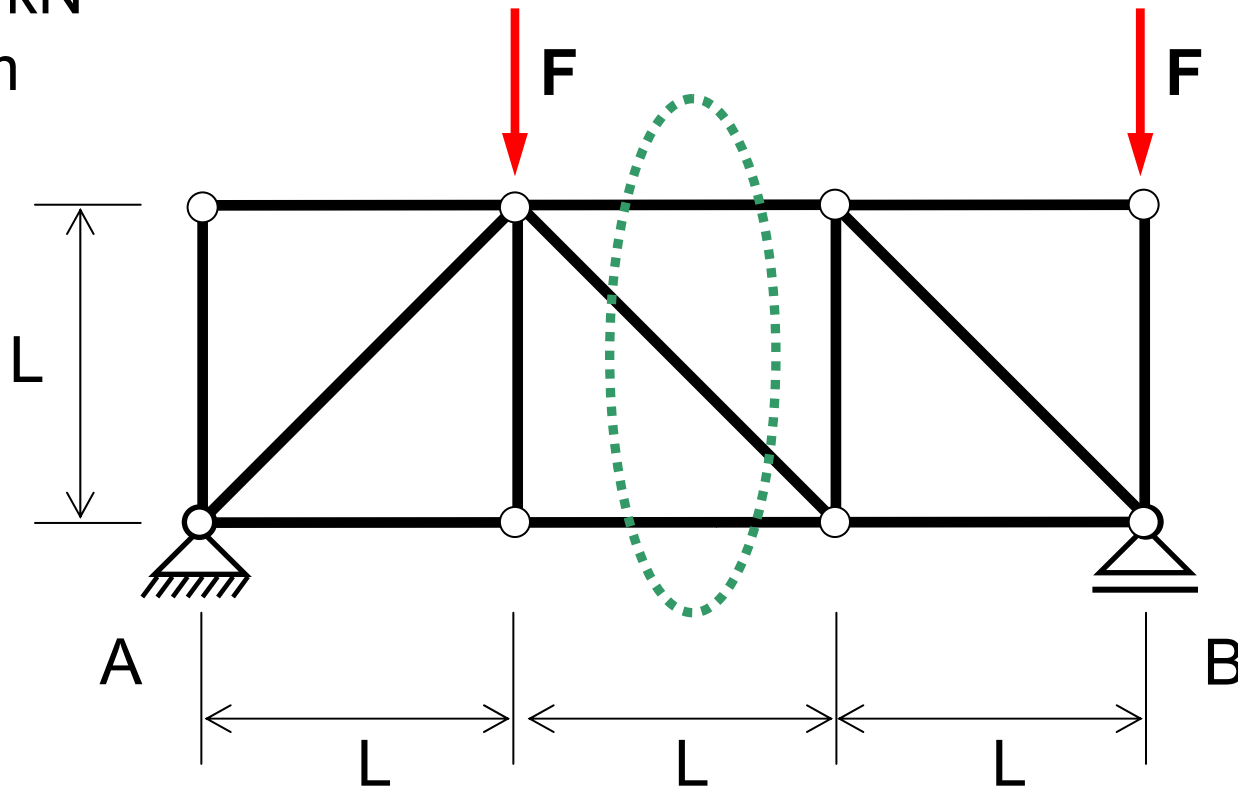


example 4.2

Determine the internal forces in the upper and lower truss chord such as the diagonal member at the middle section.

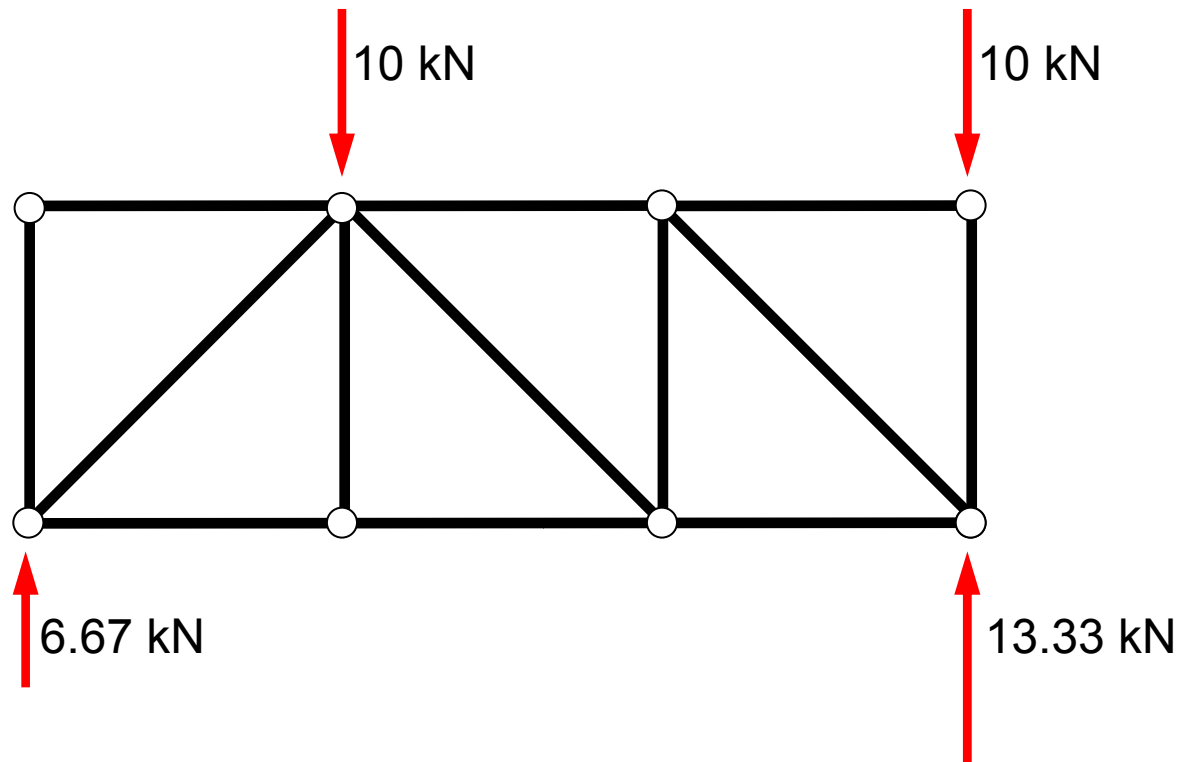
$$F = 10.0 \text{ kN}$$

$$L = 3.0 \text{ m}$$



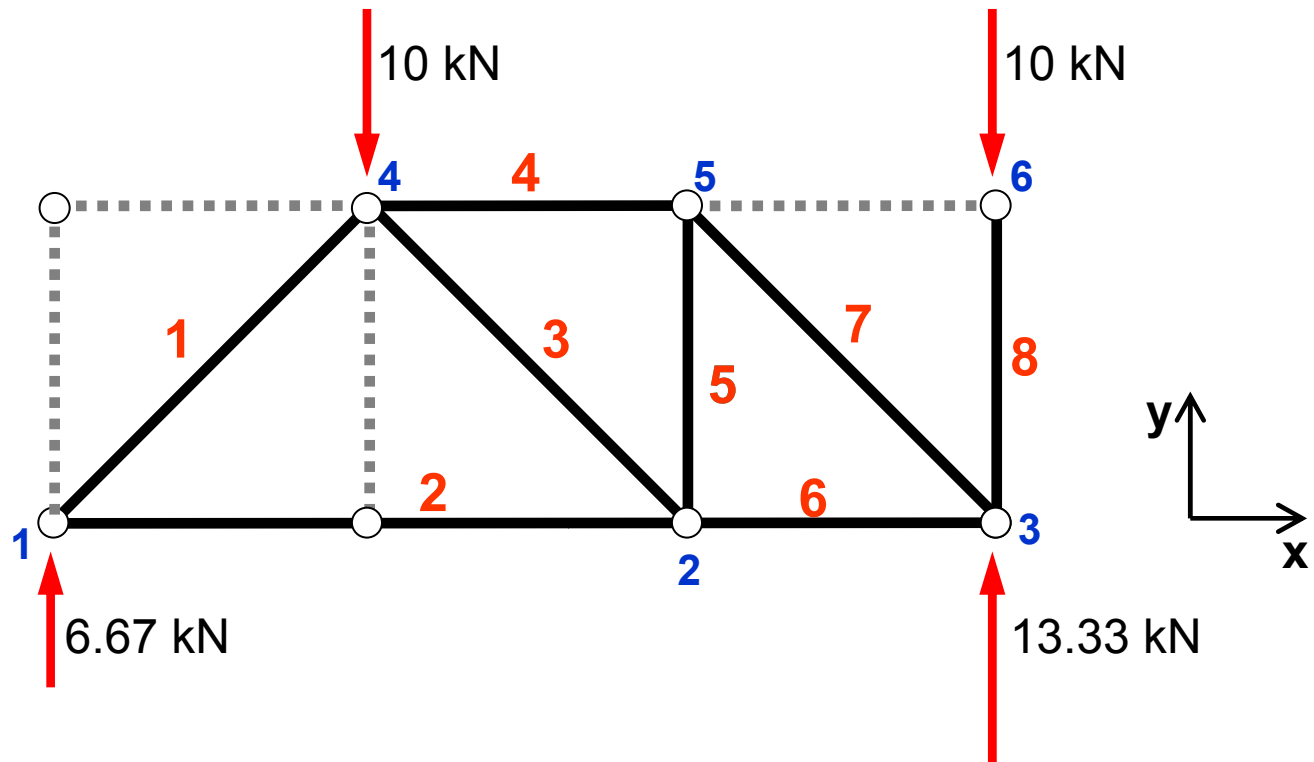
example

External load situation:



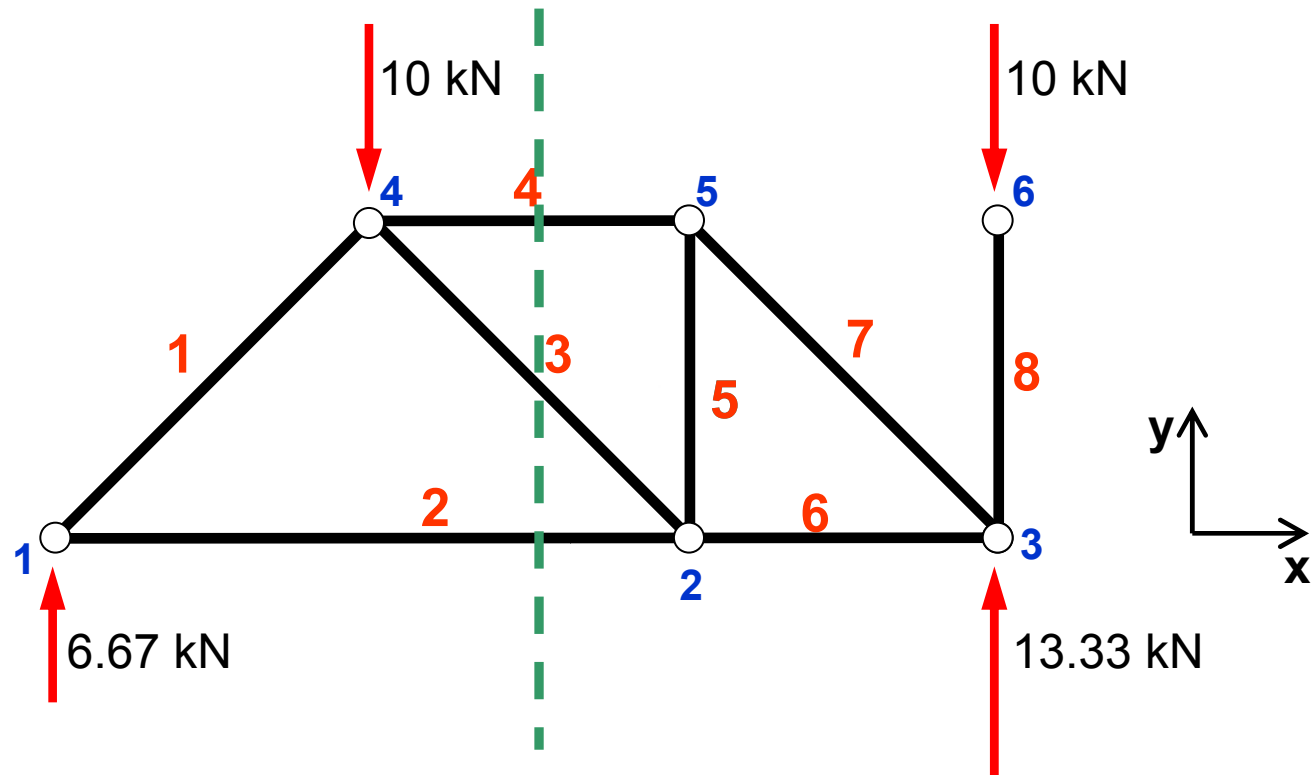
example

Zero-force members and numbers.



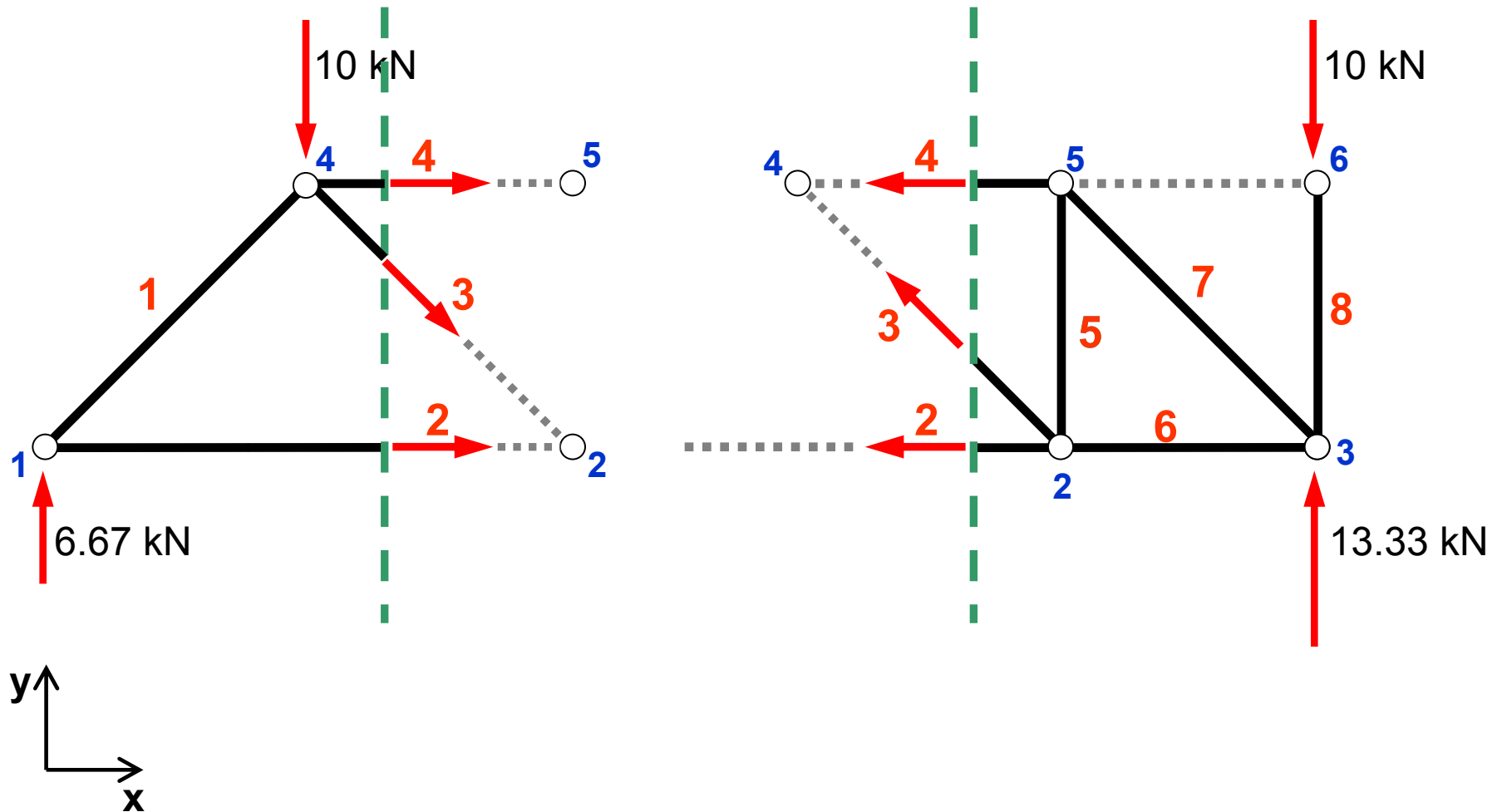
example

Defining the cutting plane, applying numbers for joints (nodes).



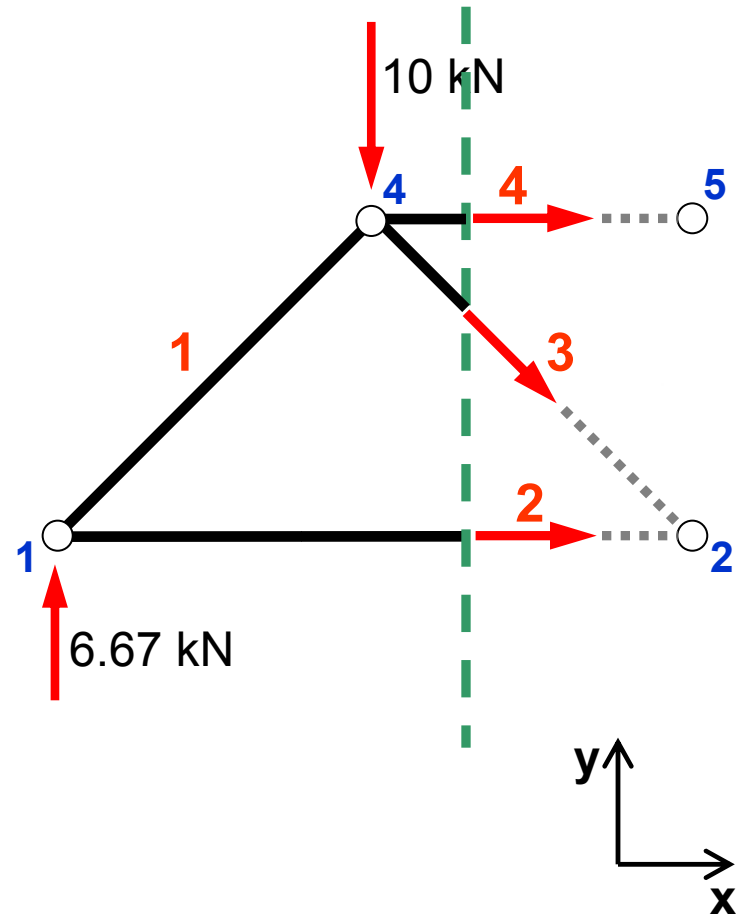
example

Sectioning of system by cutting 3 members, isolation of sections.



example

Determine the internal forces in the upper and lower truss chord such as the diagonal member at the middle section.

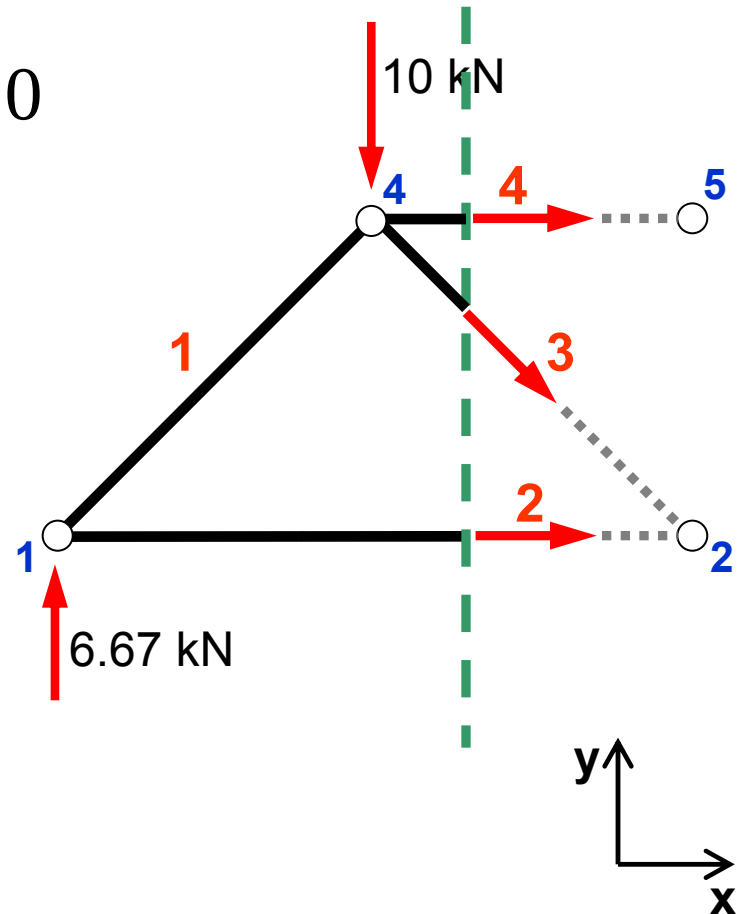


example

Determine the internal forces in the upper and lower truss chord such as the diagonal member at the middle section.

$$\sum M_4 = 0: F_2 \cdot 3m - 6.67kN \cdot 3m = 0$$

$$F_2 = 6.67kN$$



example

Determine the internal forces in the upper and lower truss chord such as the diagonal member at the middle section.

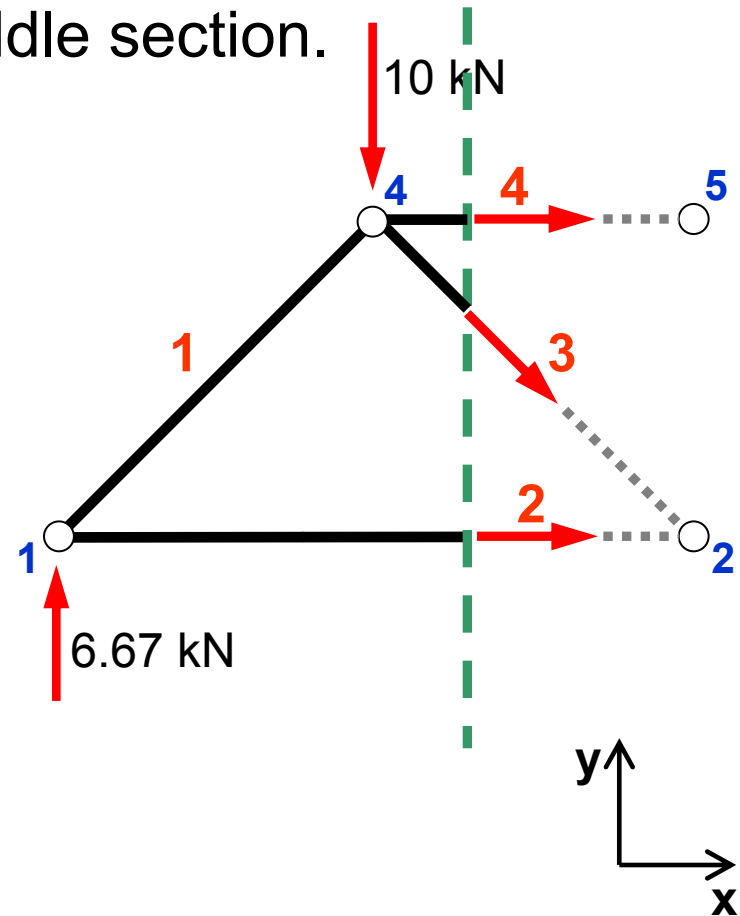
$$\sum M_4 = 0: F_2 \cdot 3m - 6.67kN \cdot 3m = 0$$

$$F_2 = 6.67kN$$

$$\sum M_2 = 0:$$

$$-F_4 \cdot 3m - 6.67kN \cdot 6m + 10kN \cdot 3m = 0$$

$$F_4 = -3.33kN$$



example

Determine the internal forces in the upper and lower truss chord such as the diagonal member at the middle section.

$$\sum M_4 = 0: F_2 \cdot 3m - 6.67kN \cdot 3m = 0$$

$$F_2 = 6.67kN$$

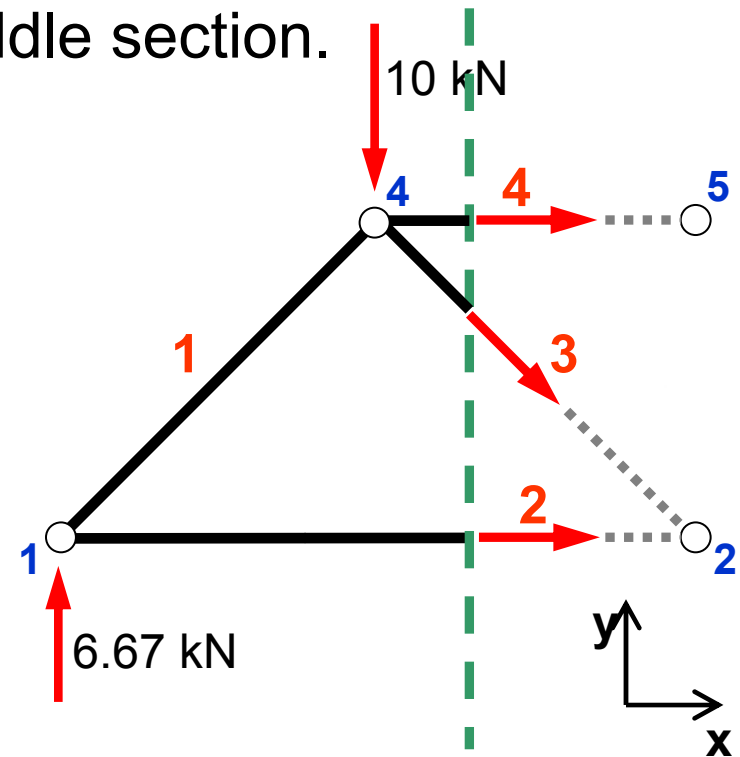
$$\sum M_2 = 0:$$

$$-F_4 \cdot 3m - 6.67kN \cdot 6m + 10kN \cdot 3m = 0$$

$$F_4 = -3.33kN$$

$$\sum F_y = 0: 6.67kN - 10kN - F_3 \sin 45^\circ = 0$$

$$F_3 = -4.72kN$$



three independent equations!





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