

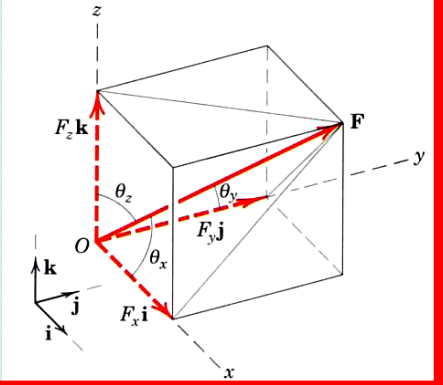
Chapter II b

Force Systems 3D

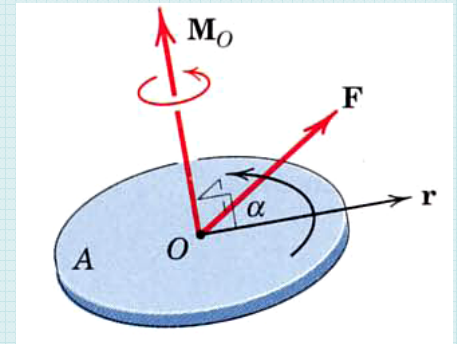


Lecture Overview

- 1.) - Rectangular Components
 - Specifications of a Vector in Space
 - Dot product, force components



- 2.) - Moment of a vector
 - Force couples
 - Resultant systems



Rectangular Components

general correlations

$$F_x = F \cos \theta_x$$

$$F_y = F \cos \theta_y$$

$$F_z = F \cos \theta_z$$

$$F = \sqrt{F_x^2 + F_y^2 + F_z^2}$$

$$\mathbf{F} = F_x \mathbf{i} + F_y \mathbf{j} + F_z \mathbf{k}$$

$$\mathbf{F} = F(\mathbf{i} \cos \theta_x + \mathbf{j} \cos \theta_y + \mathbf{k} \cos \theta_z)$$

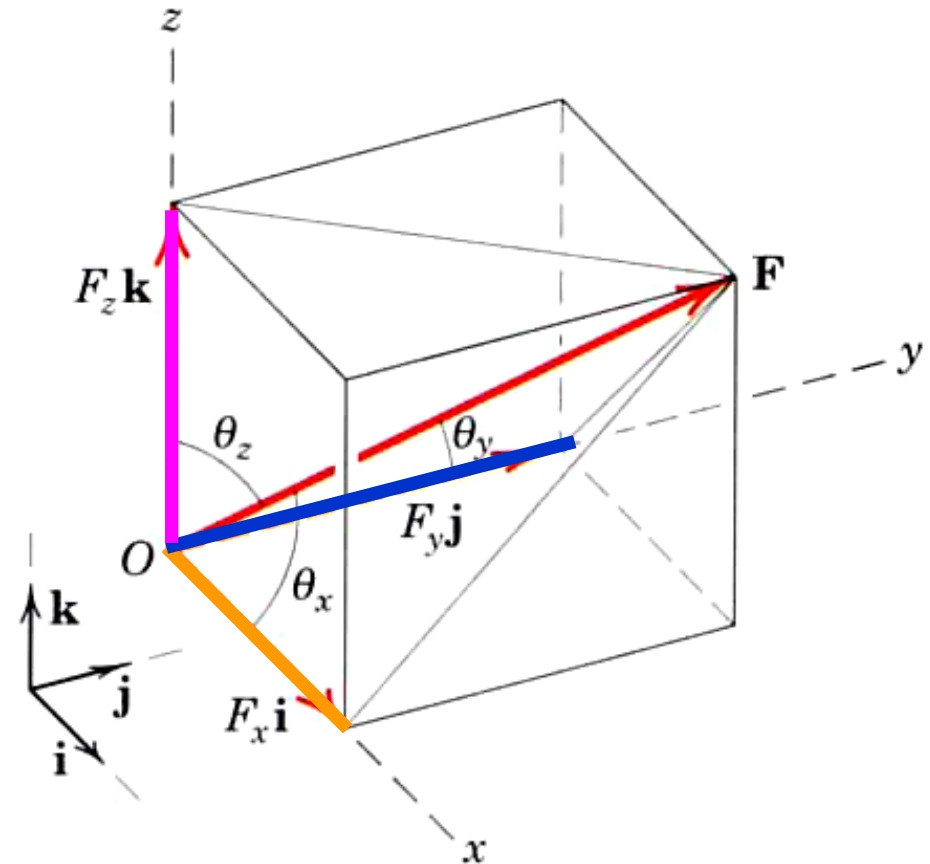


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Rectangular Components

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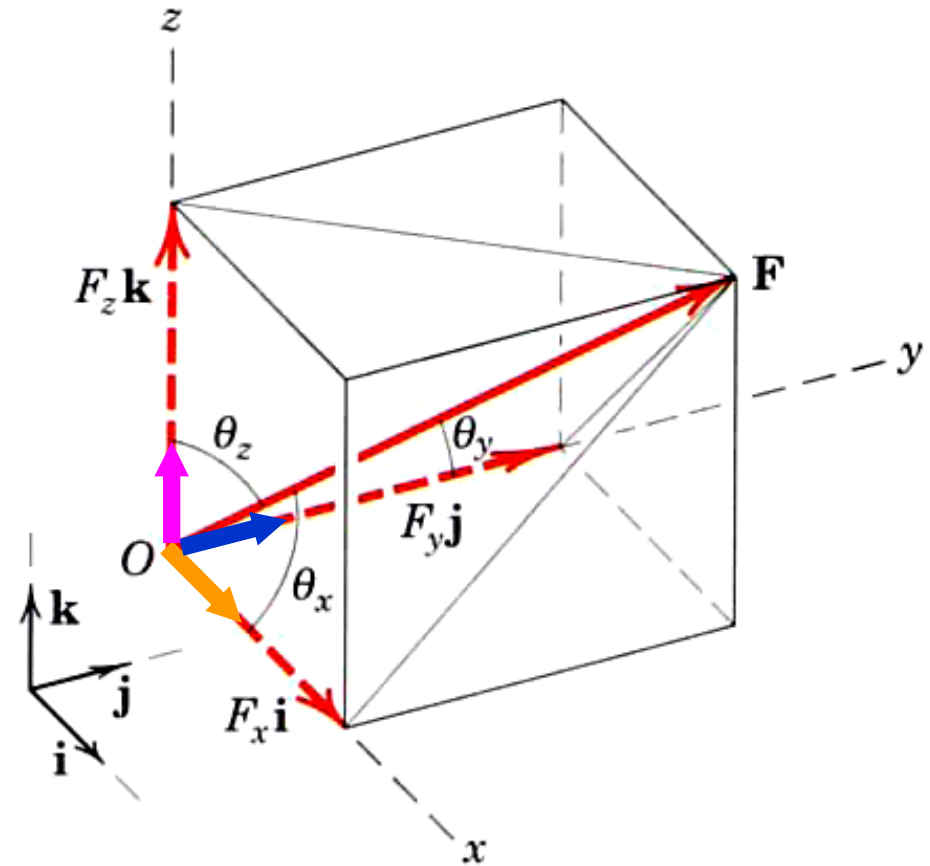


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$$\mathbf{F} = F(\mathbf{i} \cos \theta_x + \mathbf{j} \cos \theta_y + \mathbf{k} \cos \theta_z) = F \mathbf{n}_F$$

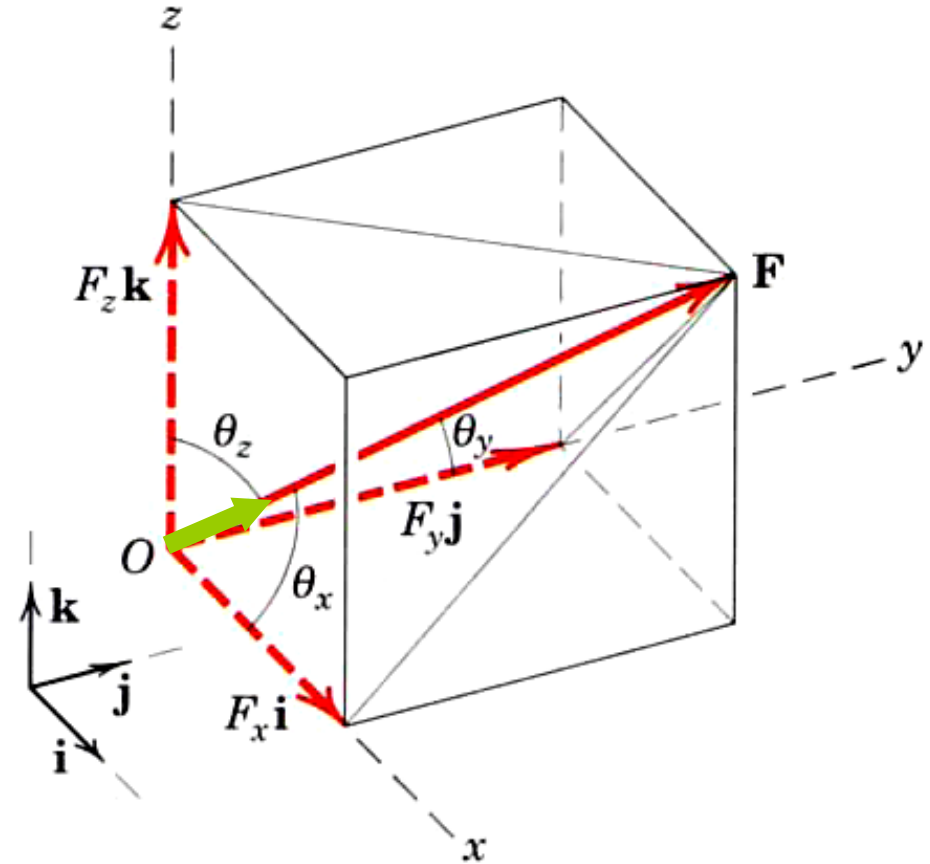


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Specification by 2 point on the line of action

given: point A and B, magnitude F

$$\mathbf{F} = F\mathbf{n}_F = F \frac{\overrightarrow{AB}}{AB} \quad \text{unit vector } \mathbf{n}_F$$
$$= F \frac{(x_2 - x_1)\mathbf{i} + (y_2 - y_1)\mathbf{j} + (z_2 - z_1)\mathbf{k}}{\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}}$$

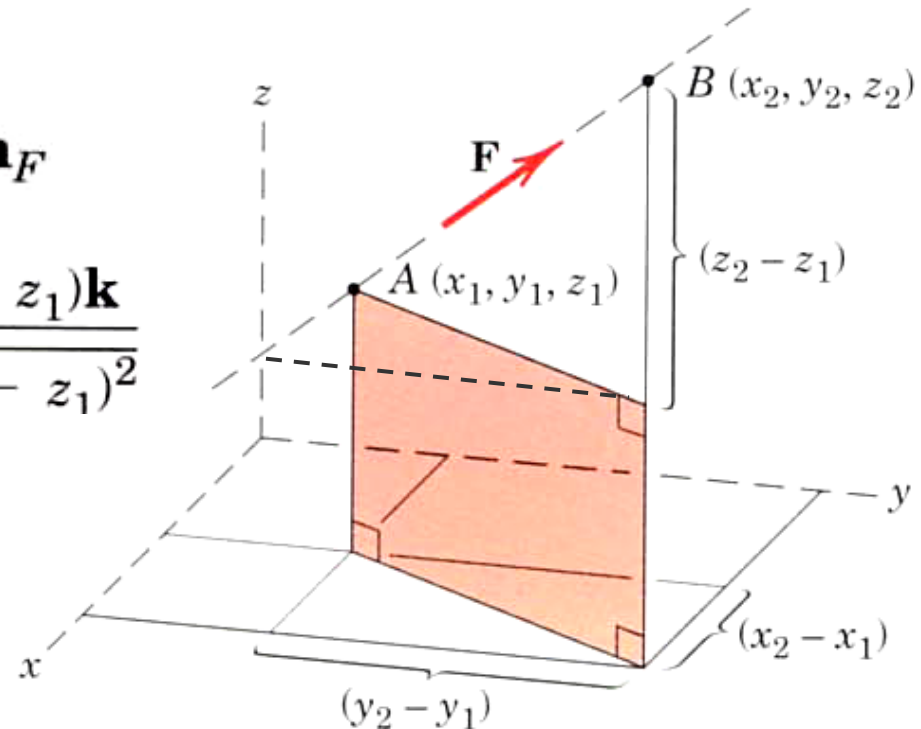


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$$= F \frac{(x_2 - x_1)\mathbf{i} + (y_2 - y_1)\mathbf{j} + (z_2 - z_1)\mathbf{k}}{\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}}$$

$$F_x = F \frac{(x_2 - x_1)}{\overline{AB}}$$

$$F_y = F \frac{(y_2 - y_1)}{\overline{AB}}$$

$$F_z = F \frac{(z_2 - z_1)}{\overline{AB}}$$

$$\mathbf{F} = F_x\mathbf{i} + F_y\mathbf{j} + F_z\mathbf{k}$$

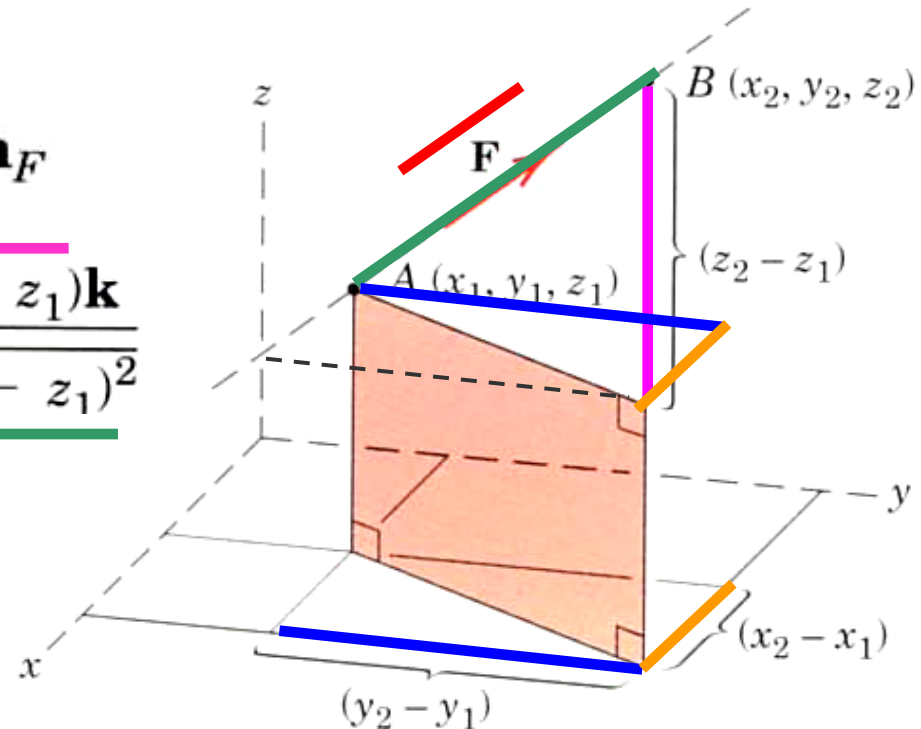


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Specification by 2 angles which orient the line of action

given: angle θ and ϕ , magnitude F

$$F_{xy} = F \cos \phi$$

$$F_z = F \sin \phi$$

$$F_x = F_{xy} \cos \theta = F \cos \phi \cos \theta$$

$$F_y = F_{xy} \sin \theta = F \cos \phi \sin \theta$$

$$\mathbf{F} = F_x \mathbf{i} + F_y \mathbf{j} + F_z \mathbf{k}$$

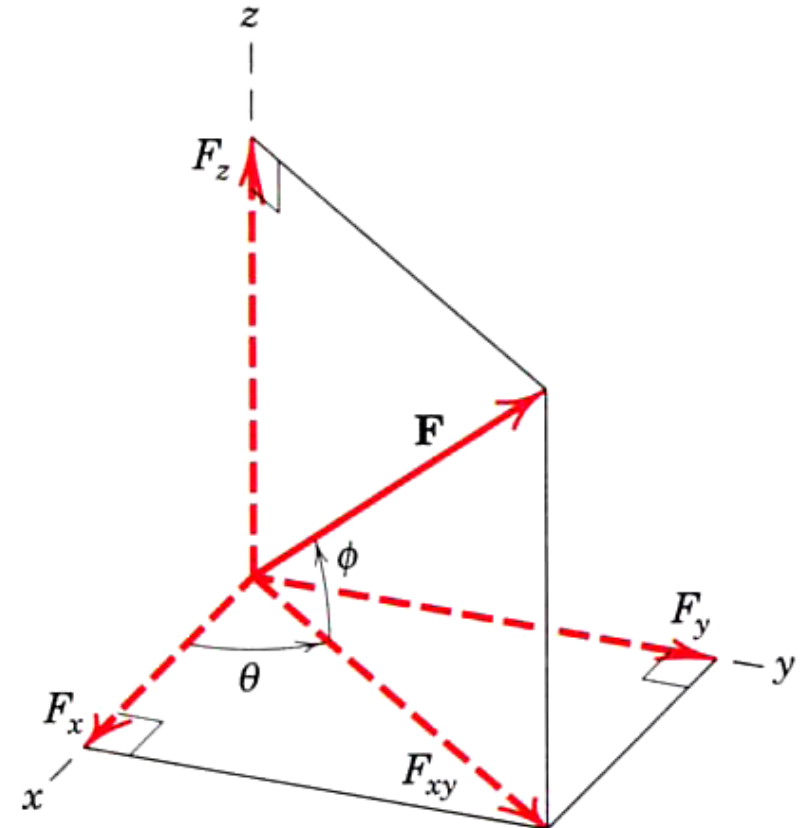


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Specification by 2 angles which orient the line of action

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$$\overline{F_{xy}} = F \cos \phi$$

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$$\overline{F_x} = F_{xy} \cos \theta = F \cos \phi \cos \theta$$

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$$\mathbf{F} = F_x \mathbf{i} + F_y \mathbf{j} + F_z \mathbf{k}$$

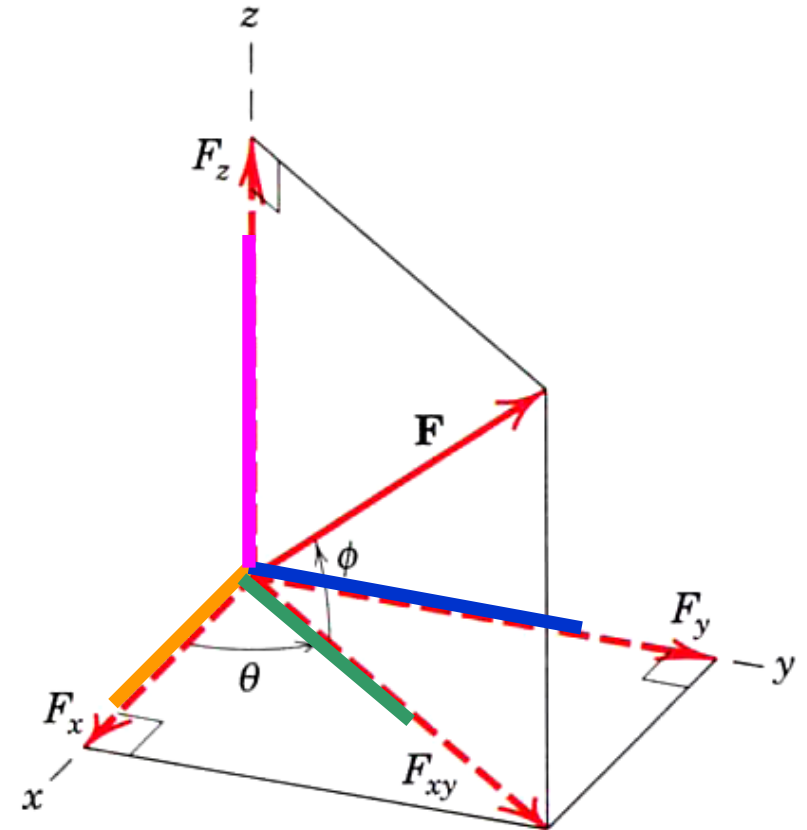
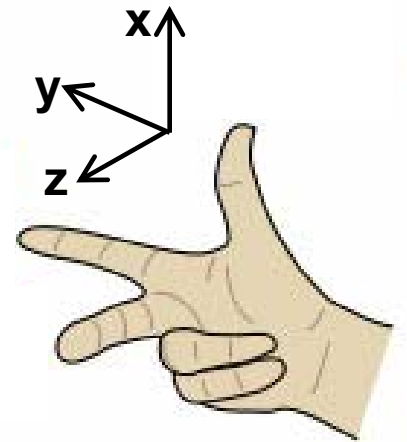


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solving a problem:

- **find appropriate coordinate system (convenience)**
- **choose the most convenient representation of the given forces concerning further operations**



Dot Product, force components

Projection of a force (component) in any arbitrary direction.

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Engineering Mechanics
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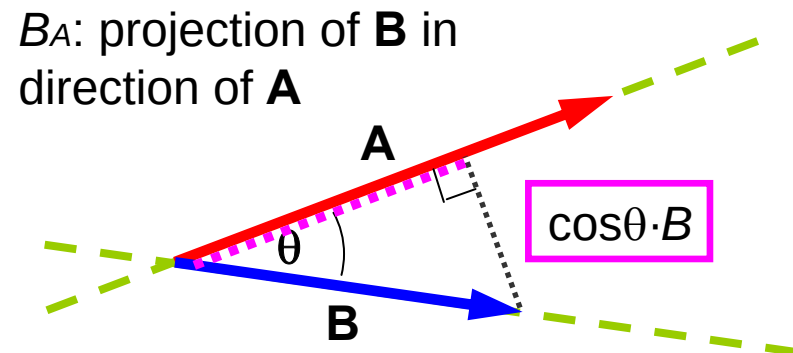
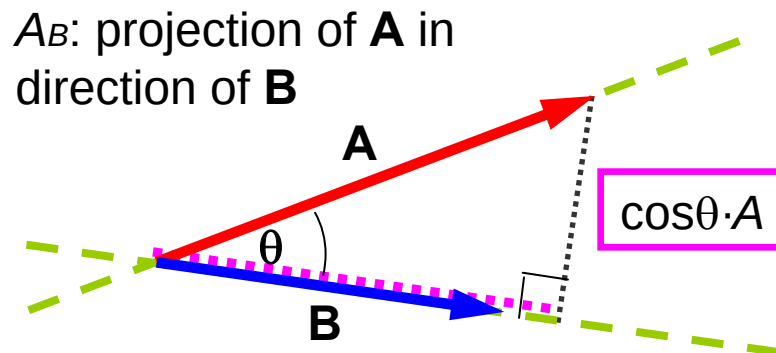
Department of
Civil Engineering

Recalling: Dot Product (scalar product)

Vectors **A** and **B** are θ inclined from each other

$$\vec{A} \cdot \vec{B} = \|\vec{A}\| \|\vec{B}\| \cos\theta = A B \cos\theta \quad \text{result = scalar}$$

$$\vec{A} \cdot \vec{B} = a_x b_x + a_y b_y + a_z b_z$$



Dot Product, force components

given: vector $\mathbf{F}$, $\mathbf{n}$ (any unit vector)

projection of $\mathbf{F}$ in direction of $\mathbf{n}$:

$$F_n = \mathbf{F} \cdot \mathbf{n}$$

magnitude of $\mathbf{F}_n$
= dot product

$$\mathbf{F}_n = (\mathbf{F} \cdot \mathbf{n})\mathbf{n}$$

vector $\mathbf{F}_n$

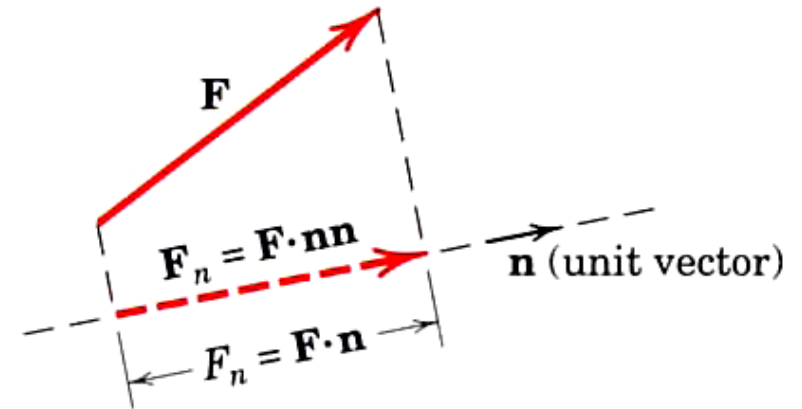
$$\mathbf{n} = \alpha\mathbf{i} + \beta\mathbf{j} + \gamma\mathbf{k}$$

α, β, γ = direction cosines of $\mathbf{n}$

$$\mathbf{F} = F(l\mathbf{i} + m\mathbf{j} + n\mathbf{k})$$

l, m, n = direction cosines of $\mathbf{F}$

$$F_n = \mathbf{F} \cdot \mathbf{n} = F(l\alpha + m\beta + n\gamma)$$



e.g.: component of $\mathbf{F}$ in x-direction ($\mathbf{i}$):

$$F_x = F \cos \theta_x$$

$$F_x = \mathbf{F} \cdot \mathbf{i} \quad \text{= dot product}$$

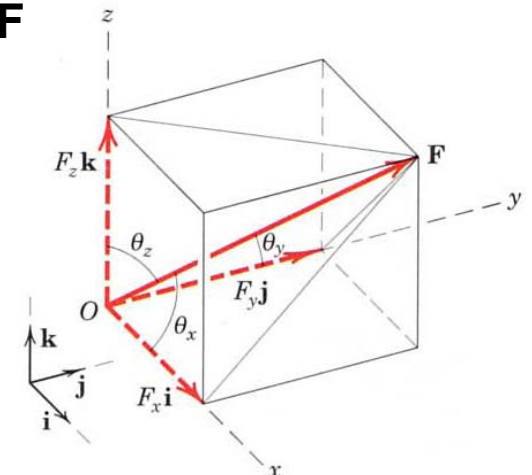


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Dot Product, angle between any 2 vectors

given: vector **P**, **Q**

$$\vec{P} \cdot \vec{Q} = P Q \cos \alpha$$

$$\alpha = \cos^{-1} \frac{\vec{P} \cdot \vec{Q}}{PQ}$$

$$\alpha = \cos^{-1} \frac{\vec{P} \cdot \vec{n}}{P}$$

concerning a unit vector **n**

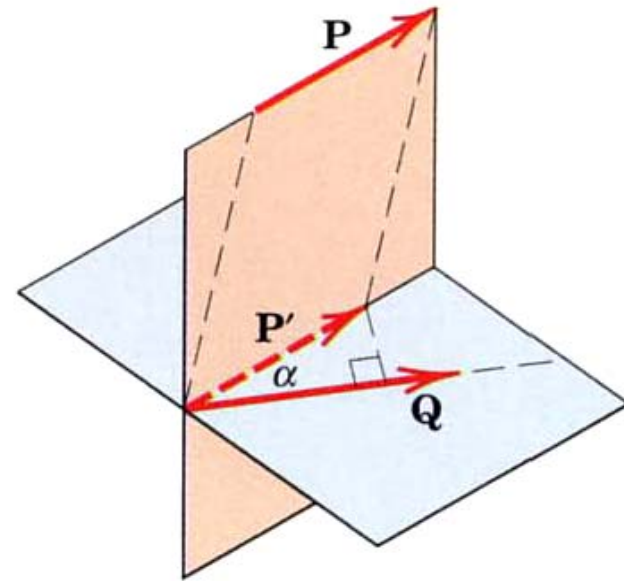


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example 2.5 (from meriam textbook, s.p. 2/9)

A force $\mathbf{F}$ with a magnitude of 100 N is applied at the origin O of the axes x - y - z as shown. The line of action of $\mathbf{F}$ passes through point A .

a) Determine the x , y and z components of $\mathbf{F}$

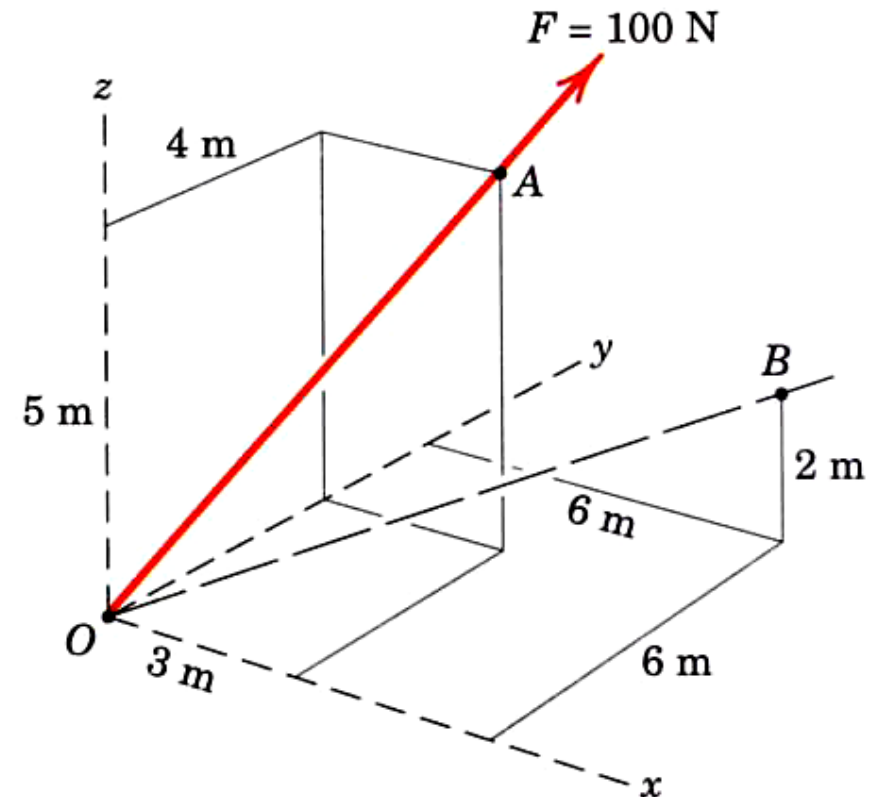


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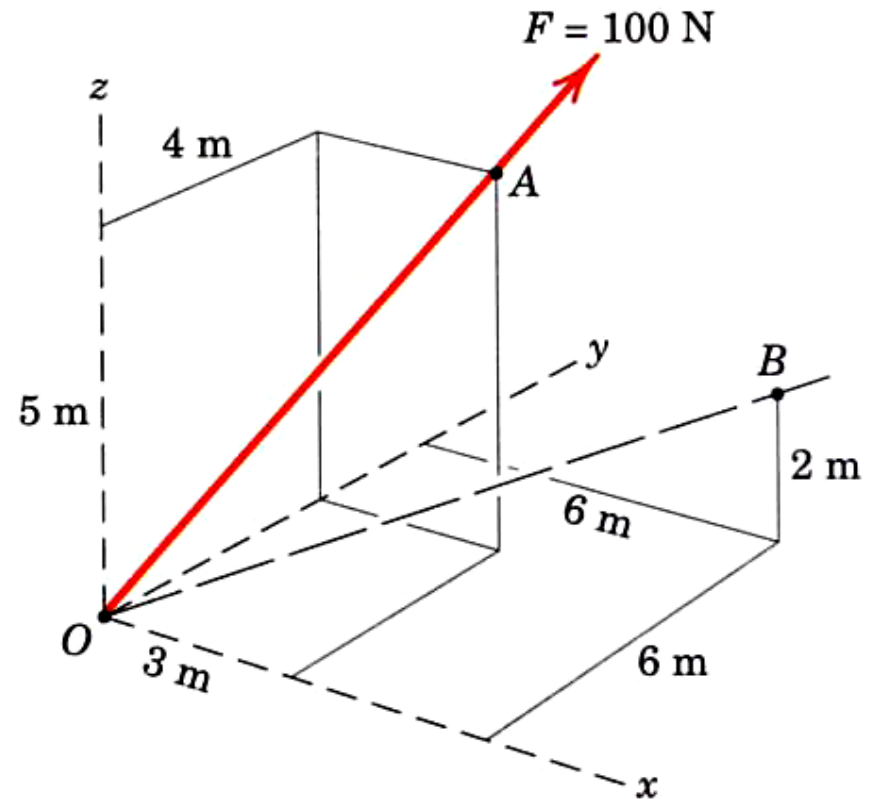
a) Determine the x , y and z components of $\mathbf{F}$

$$\vec{\mathbf{F}} = F \vec{\mathbf{n}}_{OA} = 100 \vec{\mathbf{n}}_{OA}$$

$$\vec{\mathbf{n}}_{OA} = \frac{\vec{OA}}{OA} = \frac{(3\vec{\mathbf{i}} + 4\vec{\mathbf{j}} + 5\vec{\mathbf{k}})}{\sqrt{3^2 + 4^2 + 5^2}}$$

$$\vec{\mathbf{F}} = \frac{100}{\sqrt{50}} (3\vec{\mathbf{i}} + 4\vec{\mathbf{j}} + 5\vec{\mathbf{k}})$$

$$\vec{\mathbf{F}} = 42.4\vec{\mathbf{i}} + 56.6\vec{\mathbf{j}} + 70.7\vec{\mathbf{k}} \text{ N}$$



$$F_x = 42.4 \text{ N} \quad F_y = 56.6 \text{ N} \quad F_z = 70.7 \text{ N}$$

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A force $\mathbf{F}$ with a magnitude of 100 N is applied at the origin O of the axes x - y - z as shown. The line of action of $\mathbf{F}$ passes through point A .

b) Determine the projection F_{xy} of $\mathbf{F}$ on the x - y plane

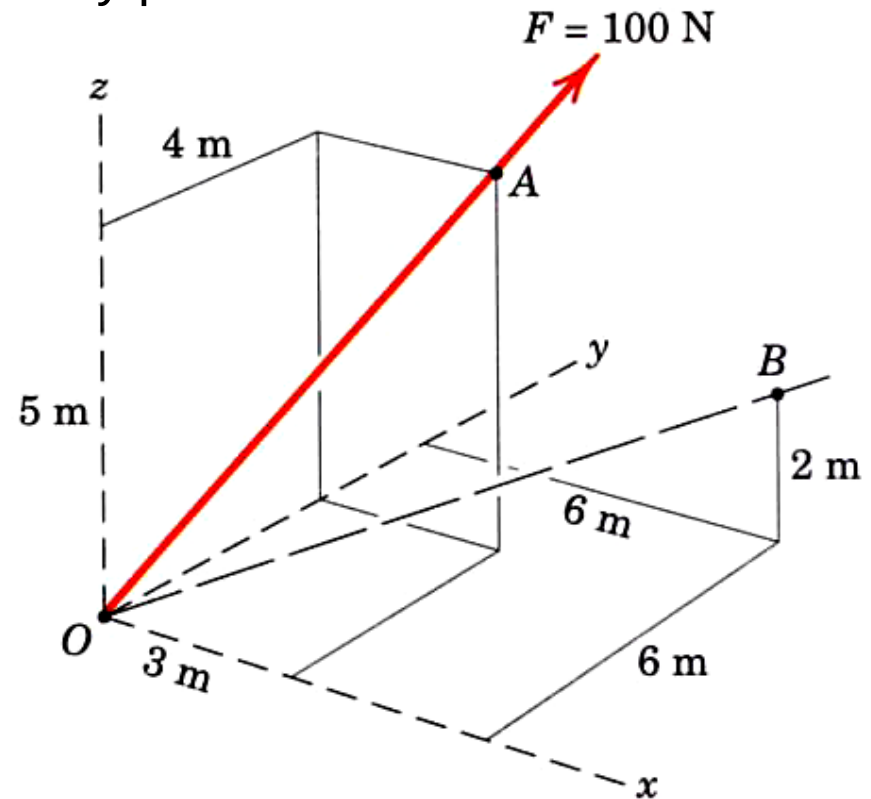


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$$F_{xy} = F \cos \theta_{xy}$$

$$\cos \theta_{xy} = \frac{\sqrt{3^2 + 4^2}}{\sqrt{3^2 + 4^2 + 5^2}} = 0.707$$

$$F_{xy} = 100 \cdot 0.707 \text{ N} = 70.7 \text{ N}$$

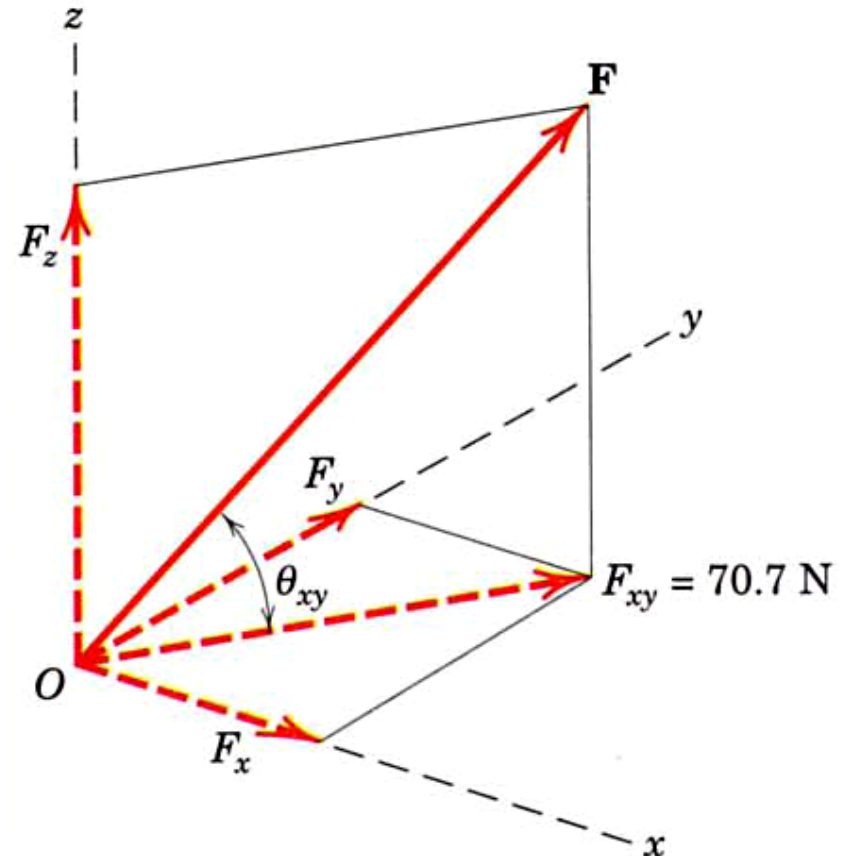


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A force $\mathbf{F}$ with a magnitude of 100 N is applied at the origin O of the axes x - y - z as shown. The line of action of $\mathbf{F}$ passes through point A .

c) Determine the projection F_{OB} of $\mathbf{F}$ along the line OB

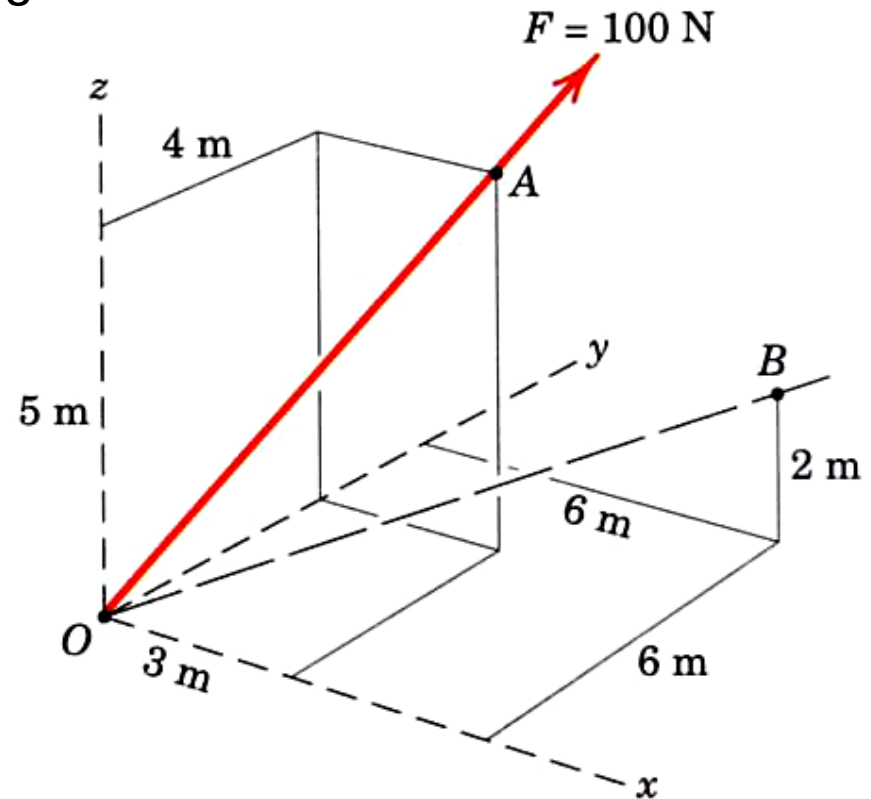


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c) Determine the projection F_{OB} of $\mathbf{F}$ along the line OB

$$F_{OB} = \vec{\mathbf{F}} \cdot \vec{\mathbf{n}}_{OB} \quad \boxed{\vec{\mathbf{F}} = 42.4\vec{\mathbf{i}} + 56.6\vec{\mathbf{j}} + 70.7\vec{\mathbf{k}} \text{ N}}$$

$$\vec{\mathbf{n}}_{OB} = \frac{\vec{OB}}{OB} = \frac{(6\vec{\mathbf{i}} + 6\vec{\mathbf{j}} + 2\vec{\mathbf{k}})}{\sqrt{6^2 + 6^2 + 2^2}}$$

$$\vec{\mathbf{n}}_{OB} = 0.668\vec{\mathbf{i}} + 0.668\vec{\mathbf{j}} + 0.229\vec{\mathbf{k}}$$

$$\begin{aligned} F_{OB} &= \vec{\mathbf{F}} \cdot \vec{\mathbf{n}}_{OB} \\ &= (42.4\vec{\mathbf{i}} + 56.6\vec{\mathbf{j}} + 70.7\vec{\mathbf{k}}) \cdot (0.688\vec{\mathbf{i}} + 0.688\vec{\mathbf{j}} + 0.229\vec{\mathbf{k}}) \\ &= (42.4)(0.688) + (56.6)(0.688) + (70.7)(0.229) \\ &= 84.4 \text{ N} \end{aligned}$$

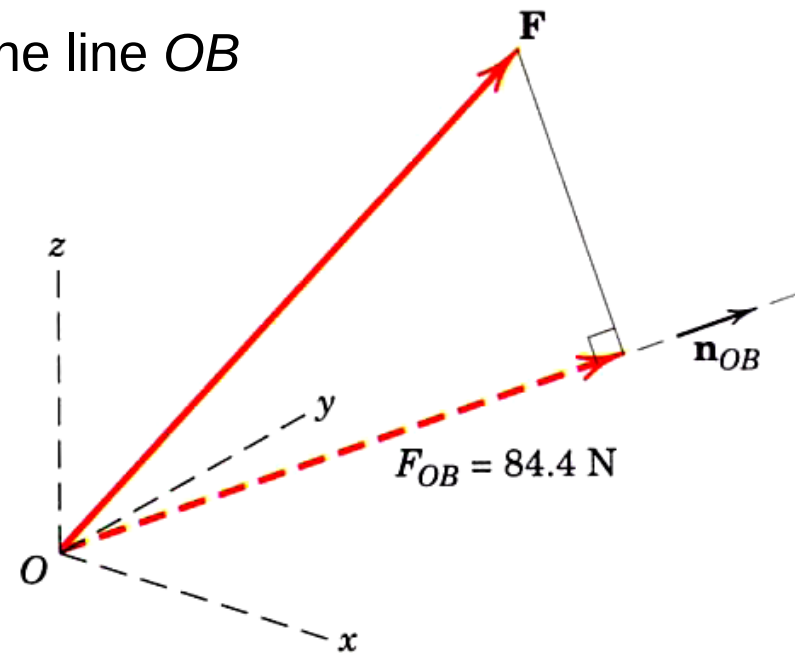


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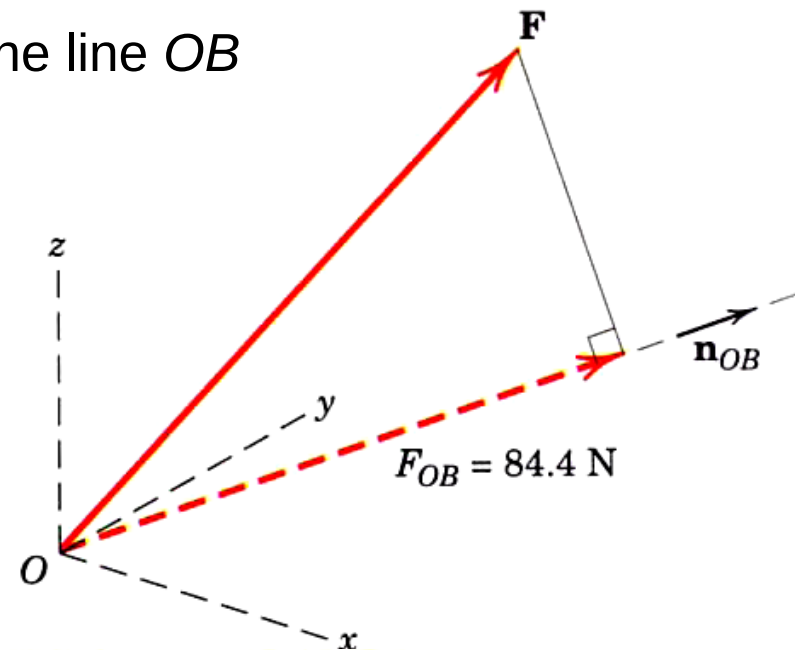
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$$\vec{\mathbf{n}}_{OB} = \frac{\vec{OB}}{OB} = \frac{(6\vec{\mathbf{i}} + 6\vec{\mathbf{j}} + 2\vec{\mathbf{k}})}{\sqrt{6^2 + 6^2 + 2^2}}$$

$$\vec{\mathbf{n}}_{OB} = 0.668\vec{\mathbf{i}} + 0.668\vec{\mathbf{j}} + 0.229\vec{\mathbf{k}}$$



If we wish to express the projection as a vector, we write

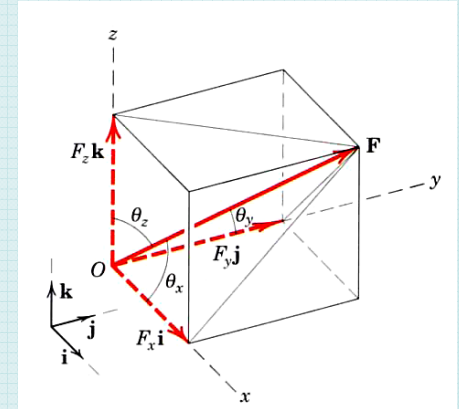
$$\begin{aligned} \mathbf{F}_{OB} &= \mathbf{F} \cdot \mathbf{n}_{OB} \mathbf{n}_{OB} \\ &= 84.4(0.688\mathbf{i} + 0.688\mathbf{j} + 0.229\mathbf{k}) \\ &= 58.1\mathbf{i} + 58.1\mathbf{j} + 19.35\mathbf{k} \text{ N} \end{aligned}$$

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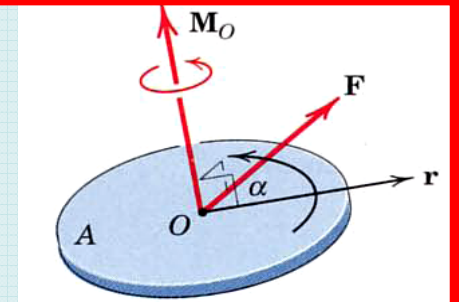


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- 1.) - Rectangular Components
 - Specifications of a Vector in Space
 - Dot product



- 2.) - Moment of a vector
 - Force couples
 - Resultant systems



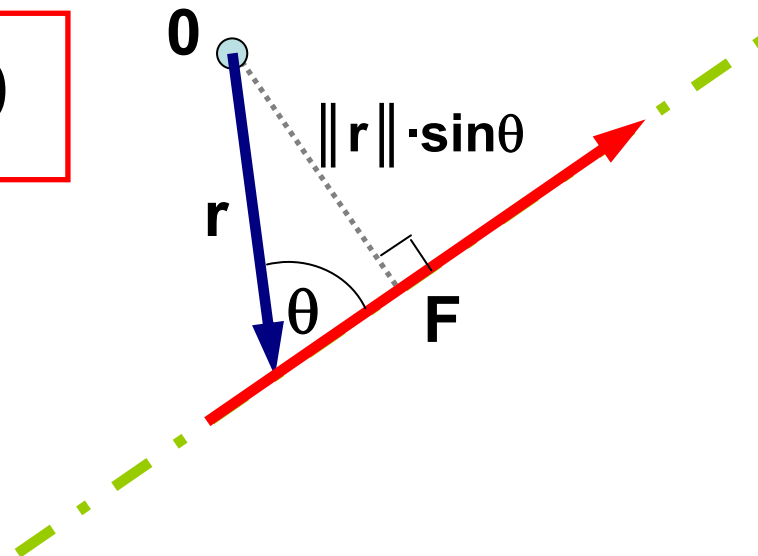
Recalling: Moment - Cross Product (vector product)

Moment of a vector $\mathbf{V}$ about any point 0

$\mathbf{r}$ is a position vector from reference point 0 to any point on the line of action of vector $\mathbf{F}$.

$$\vec{M}_0 = \vec{r} \times \vec{F} = \|\vec{F}\| \|\vec{r}\| \sin\theta$$

$$\vec{M}_0 = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ r_x & r_y & r_z \\ F_x & F_y & F_z \end{vmatrix}$$



Moment of a vector 3D

given: rigid body, force $\mathbf{F}$ and line of action, any point O , vector $\mathbf{r}$

$$M_O = Fd \quad \text{moment of } \mathbf{F}_n \text{ about point } O$$

$$= (r \sin \alpha)F$$

$$\mathbf{M}_O = \mathbf{r} \times \mathbf{F} \quad (\mathbf{F} \times \mathbf{r} = -\mathbf{M}_O)$$

$$\mathbf{M}_O = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ r_x & r_y & r_z \\ F_x & F_y & F_z \end{vmatrix} \quad \text{determinant form}$$

$$\mathbf{M}_O = (r_y F_z - r_z F_y)\mathbf{i} + (r_z F_x - r_x F_z)\mathbf{j} + (r_x F_y - r_y F_x)\mathbf{k}$$

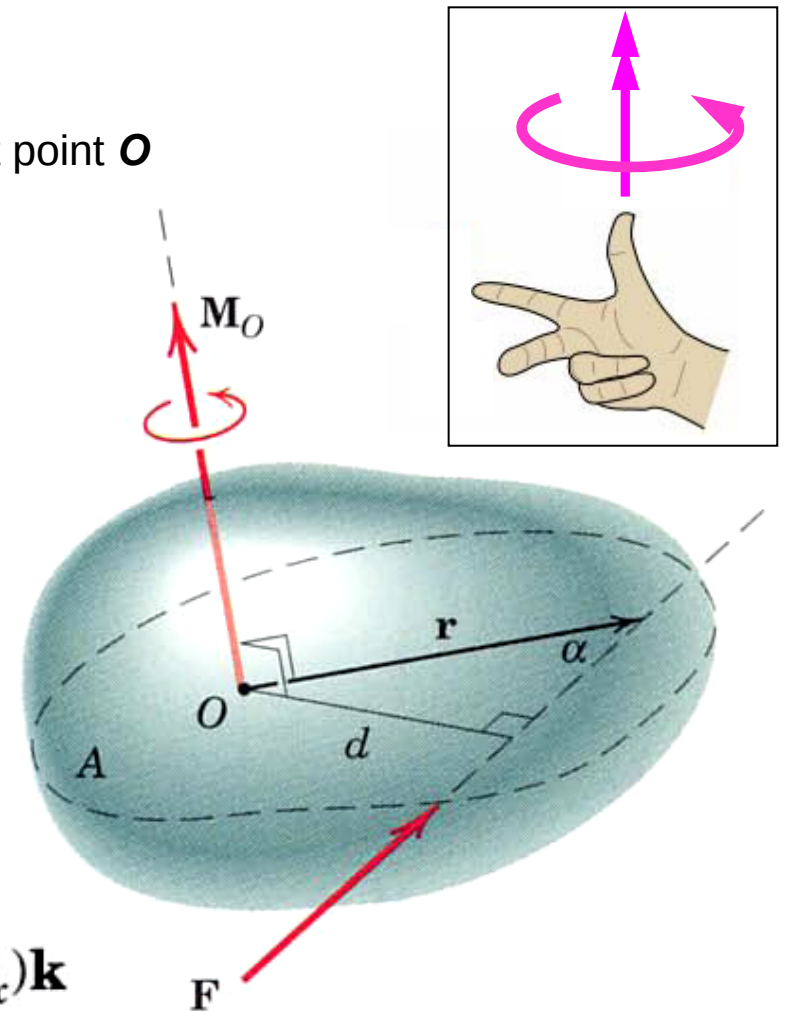


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Moment of a vector – rectangular components

$$\mathbf{M}_O = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ r_x & r_y & r_z \\ F_x & F_y & F_z \end{vmatrix}$$

$$\mathbf{M}_O = (r_y F_z - r_z F_y)\mathbf{i} + (r_z F_x - r_x F_z)\mathbf{j} + (r_x F_y - r_y F_x)\mathbf{k}$$

Recalling Varignon's Theorem:

$$M_x = r_y F_z - r_z F_y$$

$$M_y = r_z F_x - r_x F_z$$

$$M_z = r_x F_y - r_y F_x$$

$$\begin{aligned} \mathbf{M}_O &= \mathbf{r} \times (\mathbf{F}_1 + \mathbf{F}_2 + \mathbf{F}_3 + \cdots) \\ &= \mathbf{r} \times \Sigma \mathbf{F} \end{aligned}$$

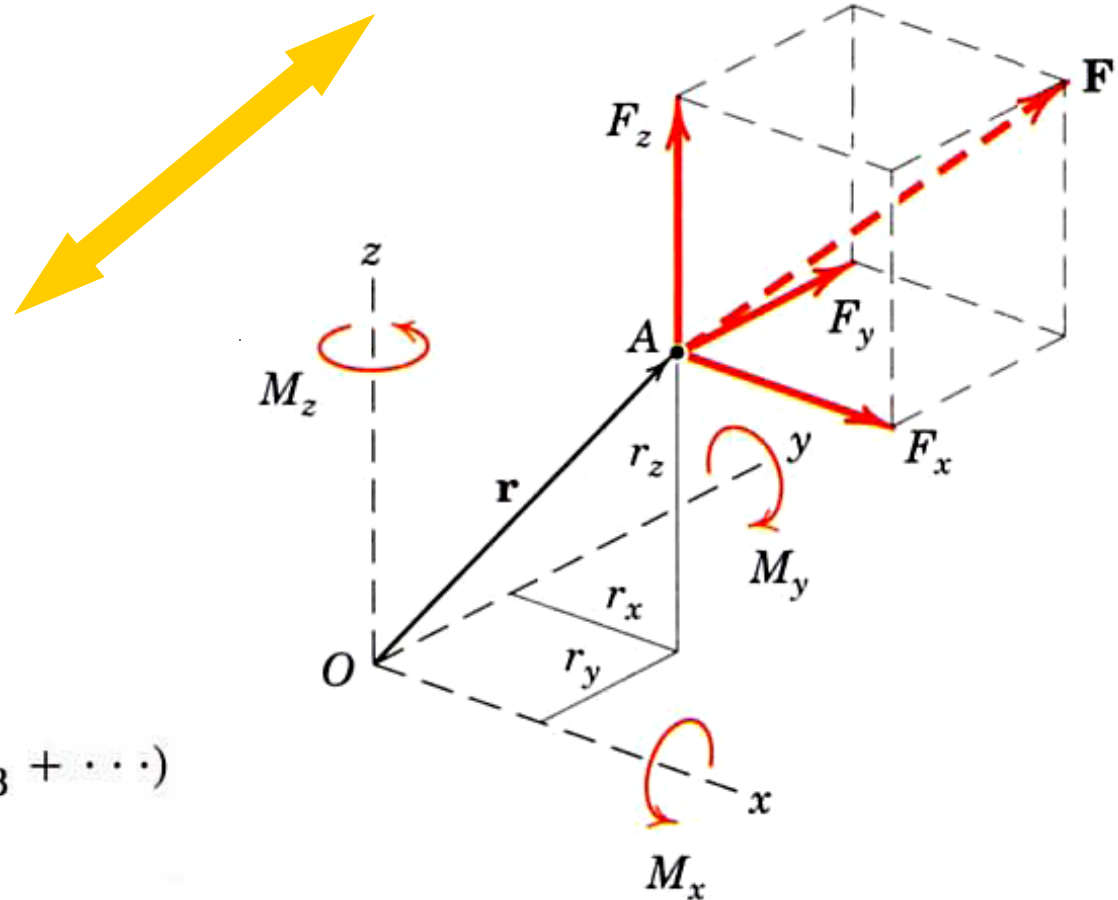


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Moment about an arbitrary axis

$$\mathbf{M}_\lambda = (\mathbf{r} \times \mathbf{F} \cdot \mathbf{n})\mathbf{n}$$

cross product = M_0 dot product

$$\mathbf{M}_\lambda = (\mathbf{r} \times \mathbf{F} \cdot \mathbf{n})\mathbf{n}$$

scalar result M_λ

$$|\mathbf{M}_\lambda| = M_\lambda = \begin{vmatrix} r_x & r_y & r_z \\ F_x & F_y & F_z \\ \alpha & \beta & \gamma \end{vmatrix}$$

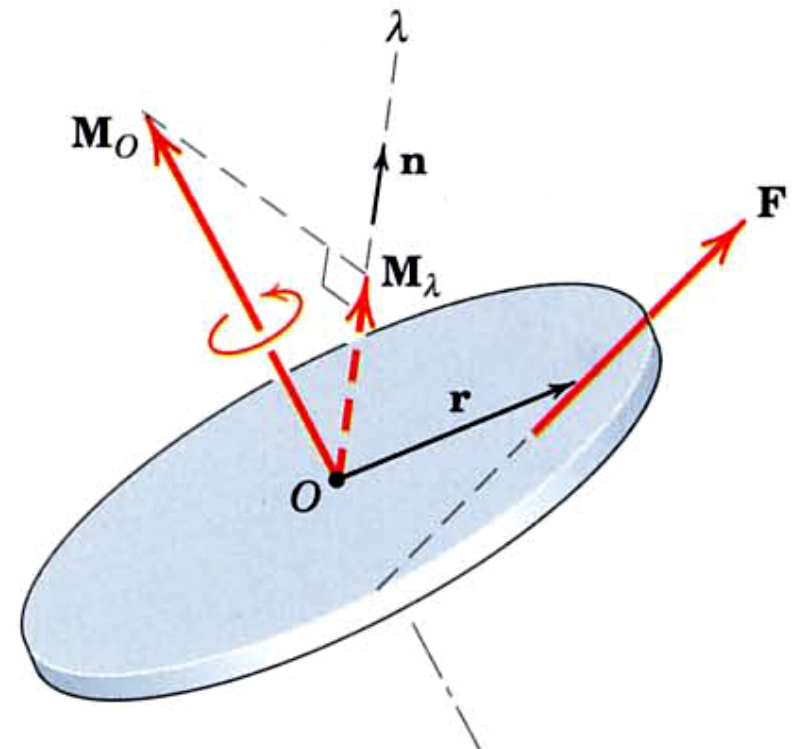


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where α , β , γ are the direction cosines of the unit vector $\mathbf{n}$



Moment – force couples 3D

given: force couple at distance d , point O

$$\begin{aligned}\mathbf{M} &= \mathbf{r}_A \times \mathbf{F} + \mathbf{r}_B \times (-\mathbf{F}) \\ &= (\mathbf{r}_A - \mathbf{r}_B) \times \mathbf{F}\end{aligned}$$

$$\mathbf{r}_A - \mathbf{r}_B = \mathbf{r}$$

$$\mathbf{M} = \mathbf{r} \times \mathbf{F}$$

$$M = Fd$$

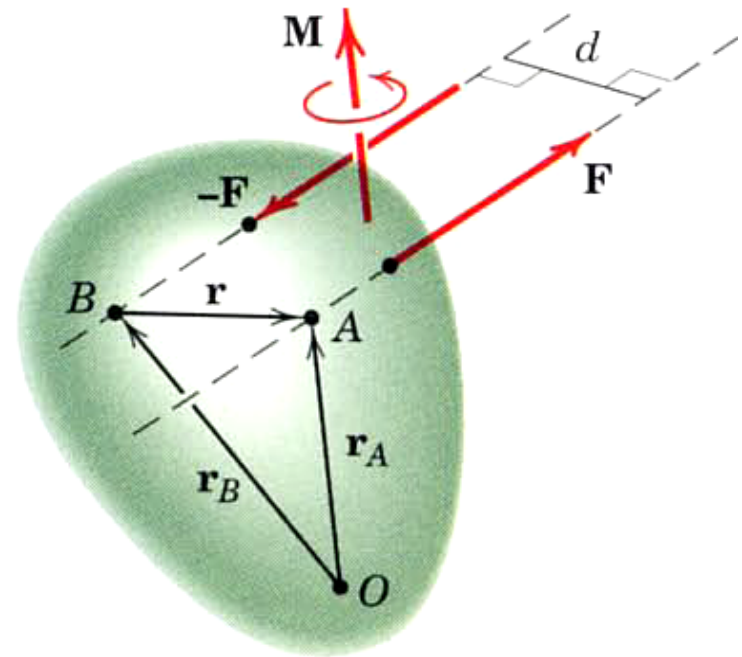


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Moment – force couples 3D

replacing a force by an equivalent force-couple system

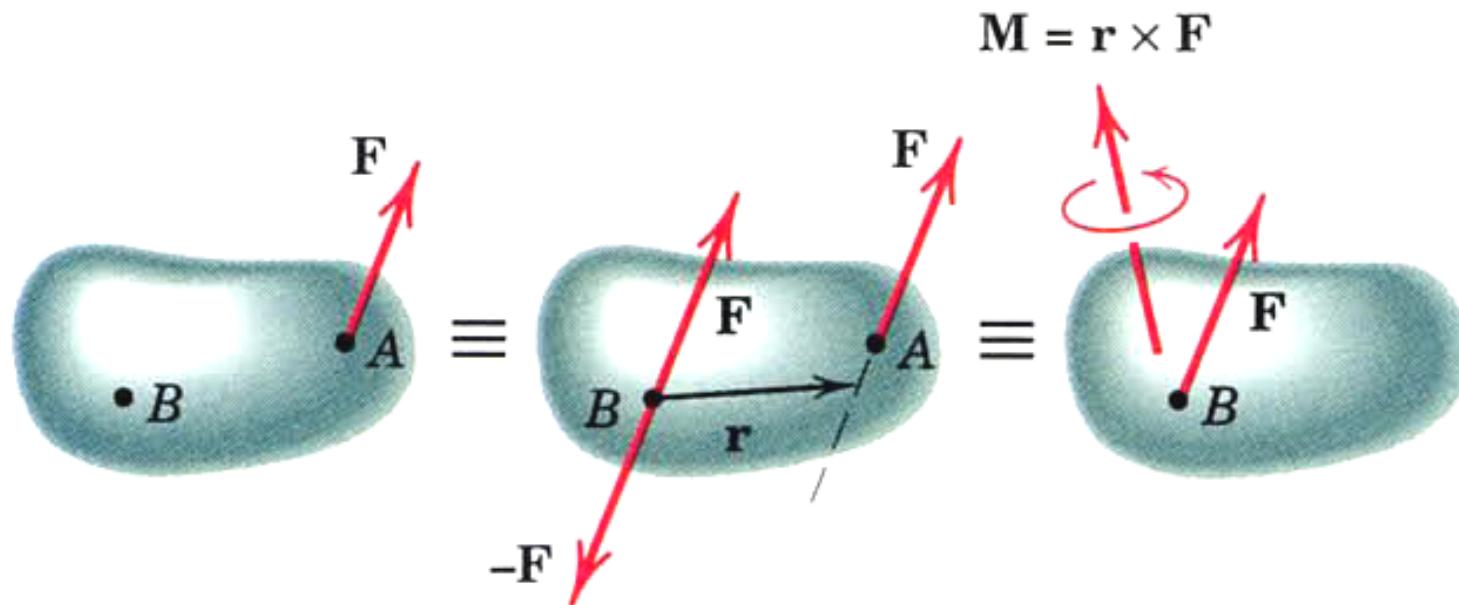


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example 2.6 (from meriam textbook, s.p. 2/10)

A tension $\mathbf{T}$ of magnitude of 10 kN is applied to the cable attached to the top A of the rigid mast and secured to the ground at B .

Determine the moment M_z of $\mathbf{T}$ about the z - axis.

Solution 1: finding z -component of $\mathbf{M}_O$
(by cross and dot product)

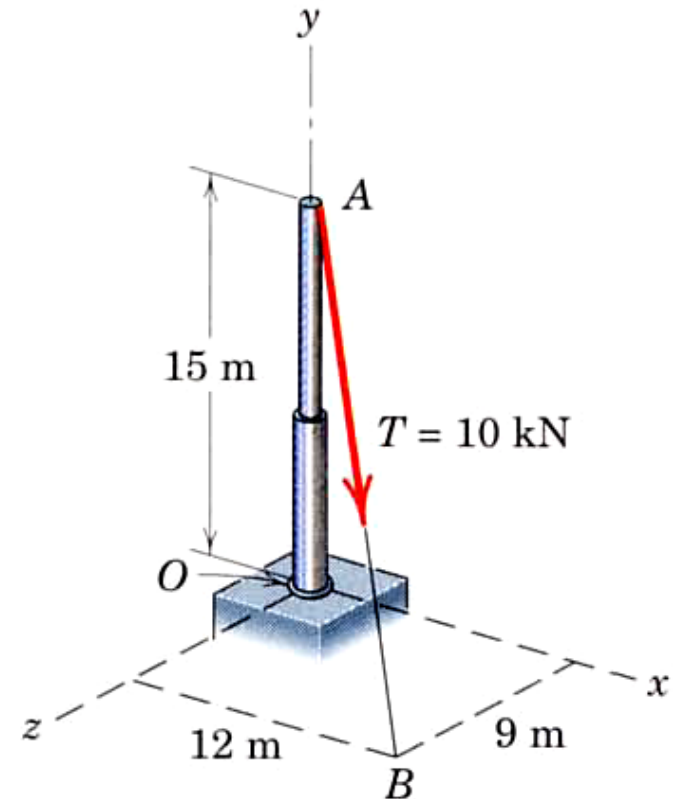
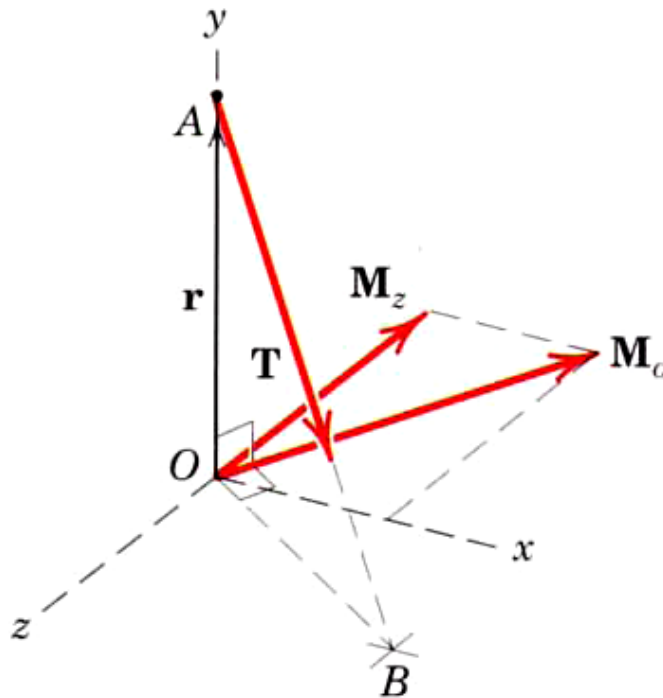


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Determine the moment M_z of $\mathbf{T}$ about the z - axis.

Solution 1: finding z -component of $\mathbf{M}_O$

$$\begin{aligned}\mathbf{T} &= T\mathbf{n}_{AB} = 10 \left[\frac{12\mathbf{i} - 15\mathbf{j} + 9\mathbf{k}}{\sqrt{(12)^2 + (-15)^2 + (9)^2}} \right] \\ &= 10(0.566\mathbf{i} - 0.707\mathbf{j} + 0.424\mathbf{k}) \text{ kN}\end{aligned}$$

$$[\mathbf{M}_O = \mathbf{r} \times \mathbf{T}] \quad \mathbf{r} = 15\mathbf{j} \quad \text{cross product}$$

$$\begin{aligned}\mathbf{M}_O &= 15\mathbf{j} \times 10(0.566\mathbf{i} - 0.707\mathbf{j} + 0.424\mathbf{k}) \\ &= 150(-0.566\mathbf{k} + 0.424\mathbf{i}) \text{ kN}\cdot\text{m}\end{aligned}$$

$$M_z = \mathbf{M}_O \cdot \mathbf{k} \quad \text{dot product}$$

$$M_z = 150(-0.566\mathbf{k} + 0.424\mathbf{i}) \cdot \mathbf{k} = -84.9 \text{ kN}\cdot\text{m}$$

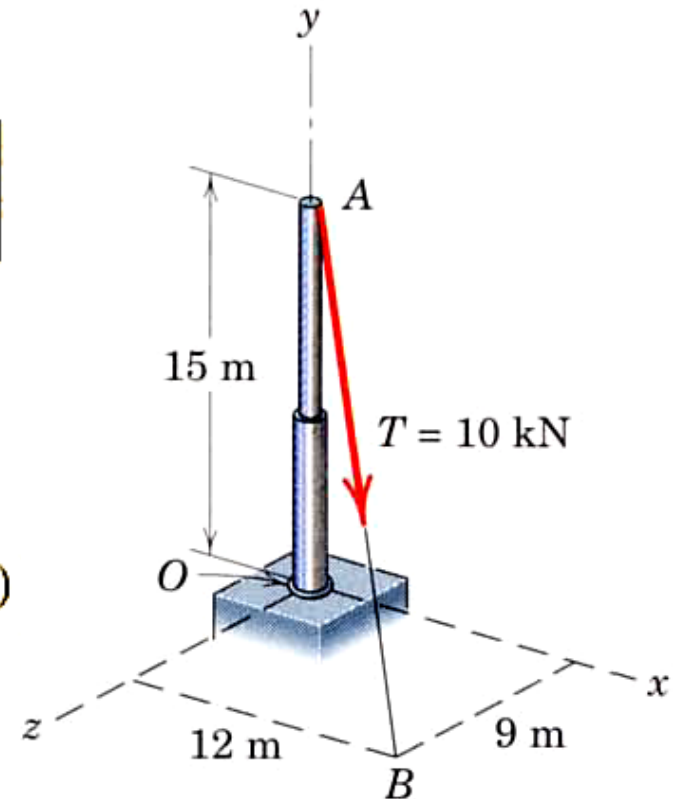


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example 2.6 (from meriam textbook, s.p. 2/10)

A tension $\mathbf{T}$ of magnitude of 10 kN is applied to the cable attached to the top A of the rigid mast and secured to the ground at B .

Determine the moment M_z of $\mathbf{T}$ about the z - axis.

Solution 2: finding M_z by T_x , T_y and T_z components

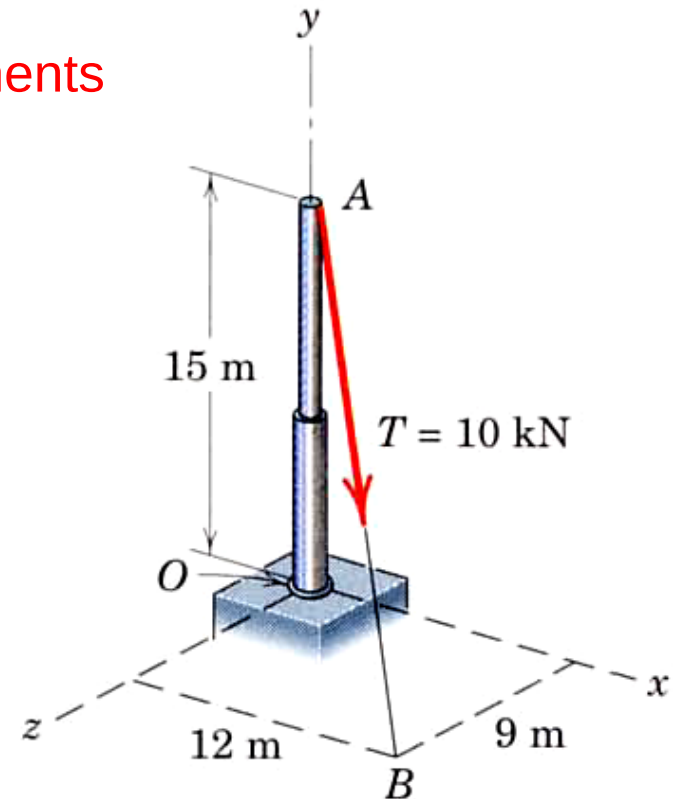
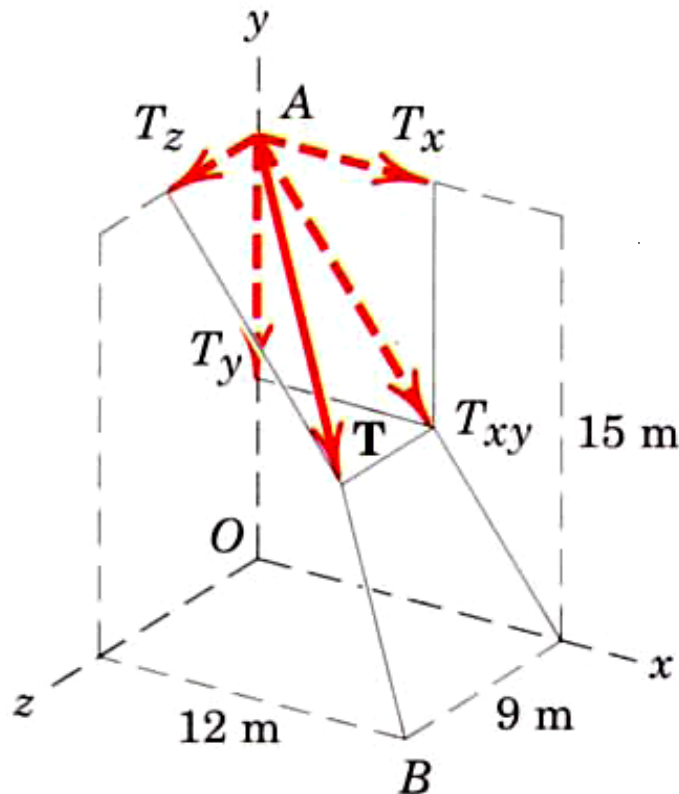


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T_y has no arm about the z -axis!

T_z can not cause a moment about the z -axis!

$$T_x = T \cos \theta_x$$

$$\cos \theta_x = 12 / \sqrt{9^2 + 12^2 + 15^2} = 0.566$$

$$T_x = 10(0.566) = 5.66 \text{ kN}$$

$$M_z = T_x d_x$$

$$M_z = 5.66(15) = 84.9 \text{ kN} \cdot \text{m}$$

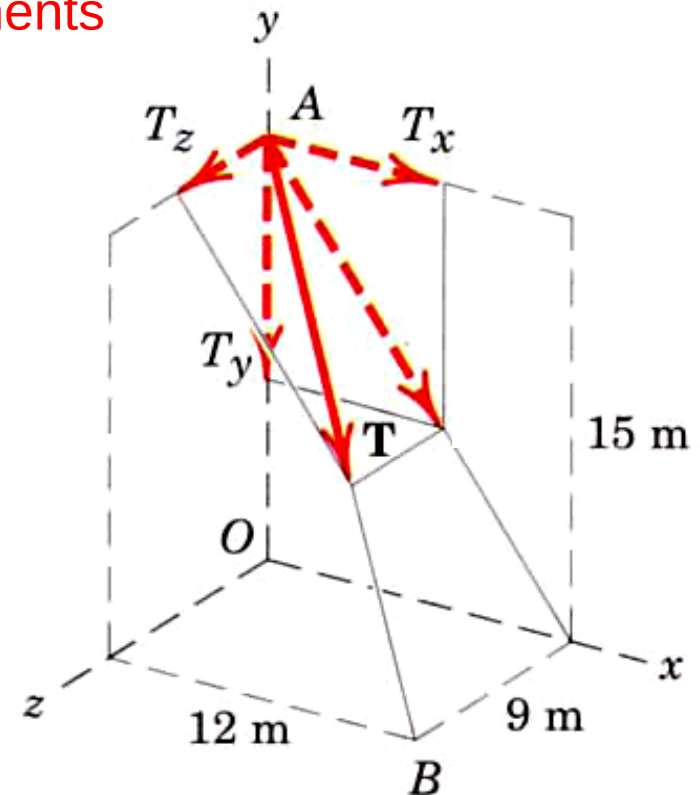
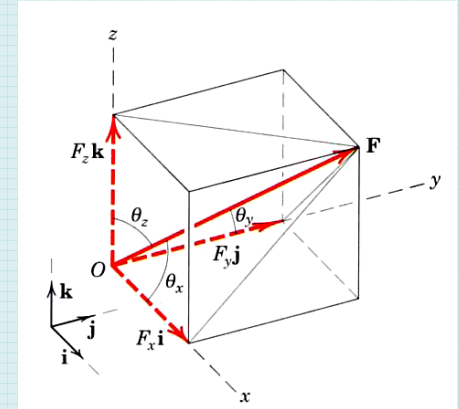


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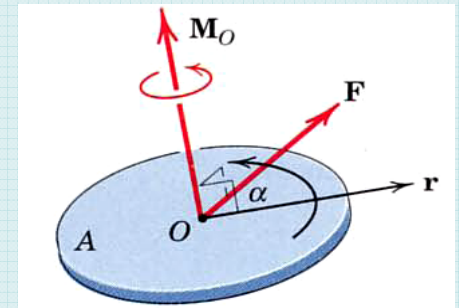


Lecture Overview

- 1.)
 - Rectangular Components
 - Specifications of a Vector in Space
 - Dot product
-



- 2.)
 - Moment of a vector
 - Force couples
 - Resultant systems



Resultants – 3D - moment as a vector

$$\mathbf{R} = \mathbf{F}_1 + \mathbf{F}_2 + \mathbf{F}_3 + \cdots = \Sigma \mathbf{F}$$

$$\mathbf{M} = \mathbf{M}_1 + \mathbf{M}_2 + \mathbf{M}_3 + \cdots = \Sigma(\mathbf{r} \times \mathbf{F})$$

$$R_x = \Sigma F_x \quad R_y = \Sigma F_y \quad R_z = \Sigma F_z$$

$$R = \sqrt{(\Sigma F_x)^2 + (\Sigma F_y)^2 + (\Sigma F_z)^2}$$

$$\mathbf{M}_x = \Sigma(\mathbf{r} \times \mathbf{F})_x \quad \mathbf{M}_y = \Sigma(\mathbf{r} \times \mathbf{F})_y \quad \mathbf{M}_z = \Sigma(\mathbf{r} \times \mathbf{F})_z$$

$$M = \sqrt{M_x^2 + M_y^2 + M_z^2}$$

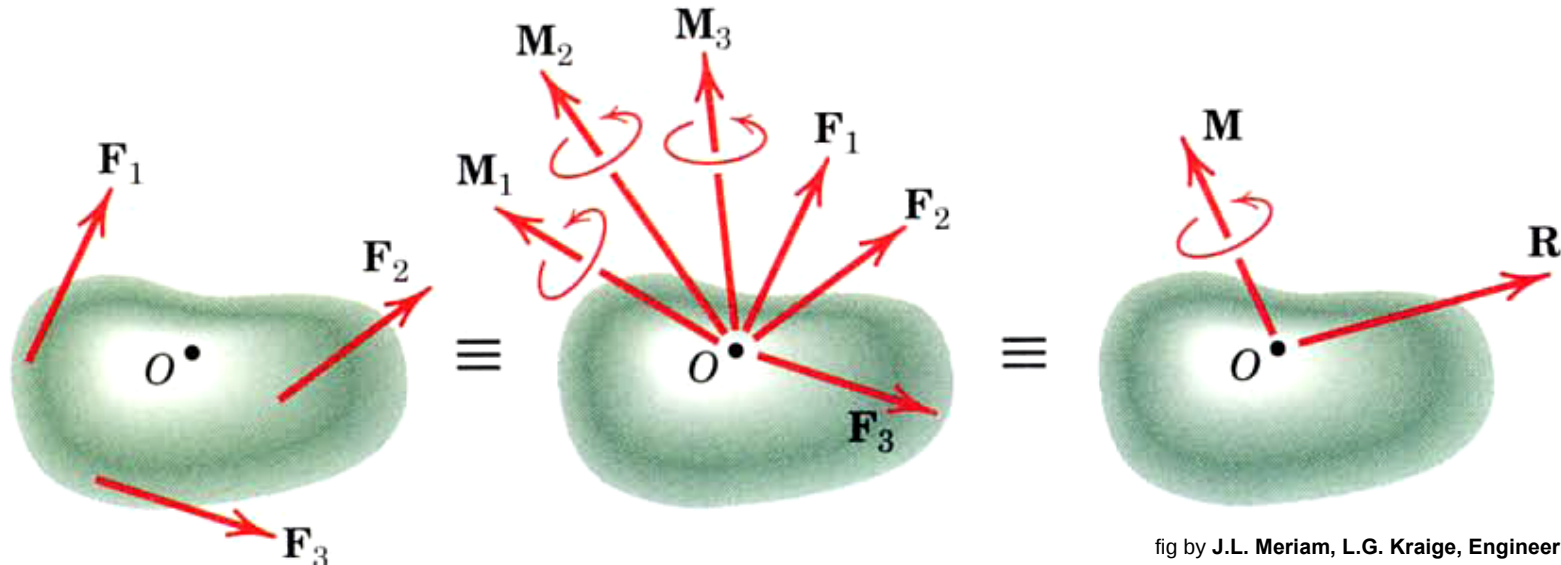
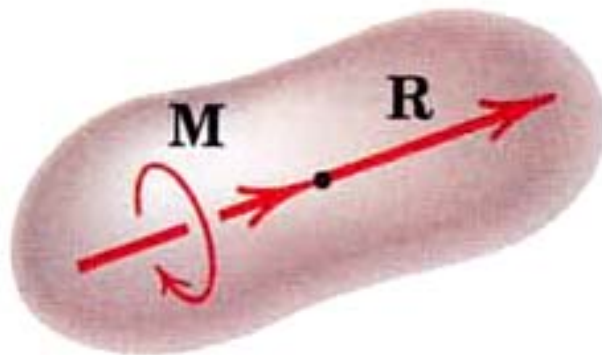


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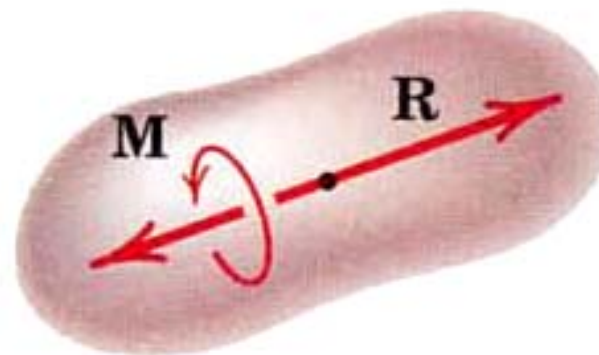


Wrench Resultant

When the resultant couple vector M is parallel to the resultant force R , the system is called a wrench.



Positive wrench



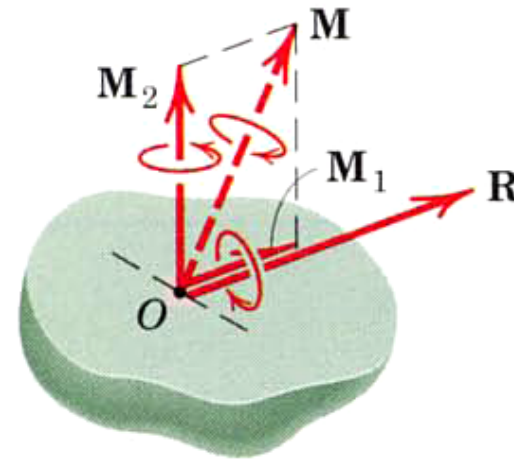
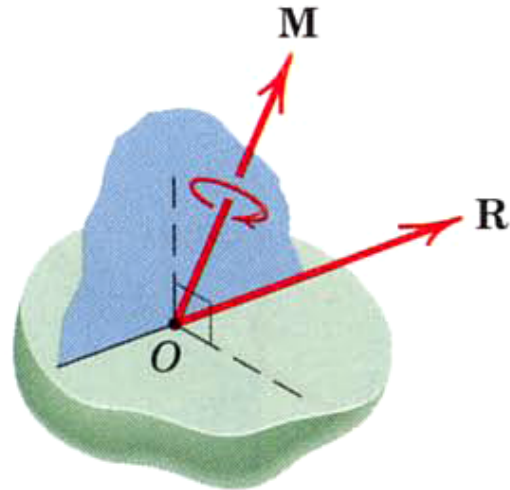
Negative wrench

example: screw driver

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Wrench Resultant



A given force system can be represented by a wrench applied along a unique line of action.

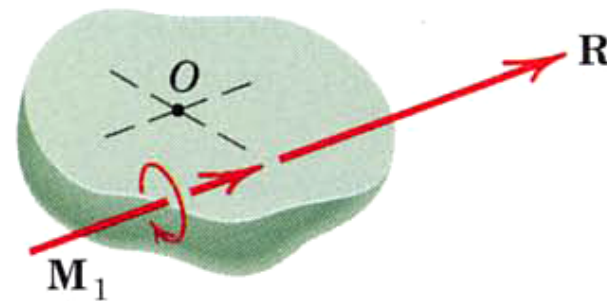
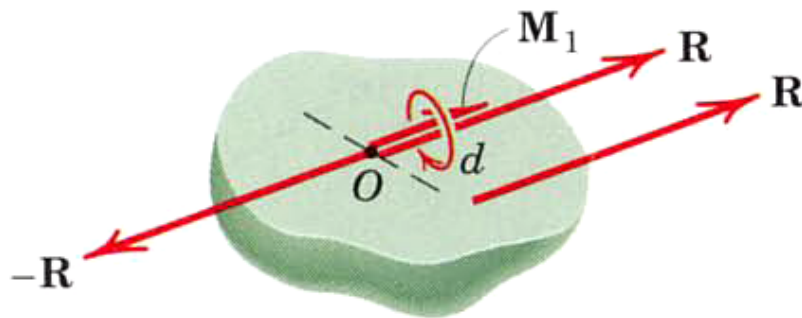


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